Reconciling Hayek’s and Keynes’ views of recessions

Paul Beaudry∗ Dana Galizia† Franck Portier‡
February 2015

Abstract

Recessions often happen after periods of rapid accumulation of houses, consumer durables and business capital. This observation has led some economists, most notably Friedrich Hayek, to conclude that recessions often reflect periods of needed liquidation resulting from past over-investment. According to the main proponents of this view, government spending should not be used to mitigate such a liquidation process, as doing so would simply result in a needed adjustment being postponed. In contrast, ever since the work of Keynes, many economists have viewed recessions as periods of deficient demand that should be countered by activist fiscal policy. In this paper we reexamine the liquidation perspective of recessions in a setup where prices are flexible but where not all trades are coordinated by centralized markets. We show why and how liquidations can produce periods where the economy functions particularly inefficiently, with many socially desirable trades between individuals remaining unexploited when the economy inherits too many capital goods. In this sense, our model illustrates how liquidations can cause recessions characterized by deficient aggregate demand and accordingly suggests that Keynes’ and Hayek’s views of recessions may be much more closely linked than previously recognized. In our framework, interventions aimed at stimulating aggregate demand face the trade-off emphasized by Hayek whereby current stimulus postpones the adjustment process and therefore prolongs the recessions. However, when examining this trade-off, we find that some stimulative policies may nevertheless remain desirable even if they postpone a recovery.

Key Words: Business Cycle, Unemployment, Liquidations ; JEL Class.: E32

The authors thank Daron Acemoglu, Francesco Caselli, Michael Devereux, Martin Ellison, Giovanni Gallipoli, Francisco Gonzalez, Allen Head, Amartya Lahiri, Shouyong Shi, Ivan Werning, Alwyn Young for discussions and the participants at seminars at the NBER summer institute, Toulouse School of Economics, the London School of Economics, University of Oxford, the Bank of Canada, the Bank of England, HEC Montreal and University of Waterloo for useful comments.

∗Vancouver School of Economics, University of British Columbia and NBER.
†Vancouver School of Economics.
‡Toulouse School of Economics and CEPR
1 Introduction

There remains considerable debate regarding the causes and consequences of recessions. Two views that are often presented as opposing, and which created controversy in the recent recession and its aftermath, are those associated with the ideas of Hayek and Keynes.\(^1\) The Hayekian perspective is generally associated with viewing recessions as a necessary evil. According to this view, recessions mainly reflect periods of liquidation resulting from past over-accumulation of capital goods. A situation where the economy needs to liquidate such an excess can quite naturally give rise to a recession, but government spending aimed at stimulating activity, it is argued, is not warranted since it would mainly delay the needed adjustment process and thereby postpone the recovery. In contrast, the Keynesian view suggests that recessions reflect periods of deficient aggregate demand where the economy is not effectively exploiting the gains from trade between individuals. According to this view, policy interventions aimed at increasing investment and consumption are generally desirable, as they favor the resumption of mutually beneficial trade between individuals.\(^2\)

In this paper we reexamine the liquidationist perspective of recessions in an environment with decentralized markets, flexible prices and search frictions. In particular, we examine how the economy adjusts when it inherits from the past an excessive amount of capital goods, which could be in the form of houses, durable goods or productive capital. Our goal is not to focus on why the economy may have over-accumulated in the past,\(^3\) but to ask how it reacts to such an over-accumulation once it is realized. As suggested by Hayek, such a situation can readily lead to a recession as less economic activity is generally warranted when agents want to deplete past over-accumulation. However, because of the endogenous emergence of unemployment risk in our set-up, the size and duration of the recession implied by the need for liquidation is not socially optimal. In effect, the reduced gains from trade induced by the need for liquidation creates a multiplier process that leads to an excessive reduction in activity. Although prices are free to adjust, the liquidation creates a period of deficient aggregate demand where economic activity is too low because people spend too cautiously due to increased unemployment risk. In this sense, we argue that liquidation and deficient aggregate demand should not be viewed as alternative theories of recessions but instead should be seen as complements, where past over-accumulation may be a key driver of periods of deficient aggregate demand. This perspective also makes salient the trade-offs faced by policy. In particular, a policy-maker in our environment faces an unpleasant trade-off between the prescriptions emphasized by Keynes and Hayek. On the one hand, a policy-maker would want to stimulate economic activity during a liquidation-induced recession because precautionary savings is excessively high. On the other hand, the policy-maker also

---

1 In response to the large recession in the US and abroad in 2008-2009, a high-profile debate around these two views was organized by Reuters. See http://www.reuters.com/subjects/keynes-hayek. See also Wapshott [2012] for a popular account of the Hayek-Keynes controversy.

2 See Caballero and Hammour [2004] for an alternative view on the inefficiency of liquidations, based on the reduction of cumulative reallocation and inefficient restructuring in recessions.

3 There are several reason why an economy may over-accumulate capital. For example, agents may have had overly optimistic expectations about future expected economic growth that did not materialize, as in Beaudry and Portier [2004], or it could have been the case that credit supply was unduly subsidized either through explicit policy, as argued in Mian and Sufi [2010] and Mian, Sufi, and Trebbi [2010], or as a by-product of monetary policy, as studied by Bordo and Landon-Lane [2013].
needs to recognize that intervention will likely postpone recovery, since it slows down the needed depletion of excess capital. The model offers a simple framework where both of these forces are present and can be compared.

One potential criticism of a pure liquidationist view of recessions is that, if markets functioned efficiently, such periods should not be socially very painful. In particular, if economic agents interact in perfect markets and realize they have over-accumulated in the past, this should lead them to enjoy a type of holiday paid for by their past excessive work. Looking backwards in such a situation, agents may resent the whole episode, but looking forward after a period of over-accumulation, they should nonetheless feel content to enjoy the proceeds of the past excessive work, even if it is associated with a recession. In contrast, in our environment we will show that liquidation periods are generally socially painful because of the multiplier process induced by precautionary savings and unemployment risk. In effect, we will show that everyone in our model economy can be worse off when they inherit too many capital goods from the past. This type of effect, whereby abundance creates scarcity, may appear quite counter-intuitive at first pass. To make as clear as possible the mechanism that can cause welfare to be reduced by such abundance, much of our analysis will focus on the case where the inherited capital takes the form of a good that directly contributes to utility, such as houses or durable goods. In this situation we will show why inheriting too many houses or durables can make everyone worse off.

A second potential criticism of a pure liquidationist view of recessions is that it often fails to explain why the economy does not simply reallocate factors to non-durable good producing sectors during the liquidationist period, and thereby maintain high employment. This criticism of the liquidationist view has been made forcefully, among others, by Krugman (1998). In particular, this line of criticism argues that since recessions are generally characterized by decreased production in almost all sectors, this constitutes clear evidence against the liquidationist view. In this paper we show why the coordination problem that arises initially in the durable goods sector due to past over-accumulation can create a contagion effect in the market for non-durables, leading both the consumption of durable and non-durable goods to decrease simultaneously. The force that links the markets, and makes them function as complements instead of substitutes, is precautionary behavior. Once there is less demand in the durable goods sector, agents fearing unemployment reduce demand in both sectors, thereby increasing unemployment risk overall.

On a more general note, a key contribution of this paper is to illustrate why the efficiency properties of a decentralized economy may depend on the extent of potential gains from trade within the economy. In particular, our framework will clarify why a decentralized economy can work quite efficiently when the gains from trade between agents are high but, in contrast, function very inefficiently when the gains from trade are low. The reason for this dichotomy is that, when the gains from trade are high, unemployment risk will tend to be small and this will render minor or non-operative the coordination problem associated with non-simultaneous trade, allowing the economy to function as if trade was simultaneous. In comparison, when gains from trade are low, non-simultaneous trade will ignite a coordination problem whereby households hold back on purchases because they fear not being able to find work, which in turn increases joblessness. In the particular liquidation setting we analyze, the level of gains from trade is determined by the level of past purchases, with past excess purchases resulting in lower current gains from trade. There are, however, other potential
reasons why gains from trade may be low, and many of our key results could be extended naturally to those settings. For this reason, we view our paper as proposing a more general theory of when deficient demand is likely to arise.\footnote{In addition to situations of past excessive accumulation, our framework would suggest that deficient demand is more likely to arise when technological progress is slow, or in a more distorted economy.}

The structure of our model builds on the literature related to search models of decentralized trading. In particular, we share with Lucas \cite{1990} and Shi \cite{1998} a model in which households are composed of agents that act in different markets without full coordination. Moreover, as in Lagos and Wright \cite{2005} and Rocheteau and Wright \cite{2005}, we exploit alternating decentralized and centralized markets to allow for a simple characterization of the equilibrium. However, unlike those papers, we do not have money in our setup. The paper also shares key features with the long tradition of macro models emphasizing strategic complementarities, aggregate demand externalities and multipliers, such as Diamond \cite{1982} and Cooper and John \cite{1988}, but we do not emphasize multiple equilibrium. Instead we focus on situations where the equilibrium remains unique, which allows standard comparative statics exercises to be conducted without needing to worry about equilibrium-selection issues. The multiplier process derived in the paper therefore shares similarities with that found in the recent literature with strategic complementarities such as Angeletos and La'O \cite{2013}, in the sense that it amplifies demand shocks. However, the underlying mechanism in this paper is very different, operating through unemployment risk rather than through direct demand complementarities as in Angeletos and La'O \cite{2013}.

Unemployment risk and its effects on consumption decisions is at the core of our model. The empirical relevance of precautionary saving related to unemployment risk has been documented by many, starting with Carroll \cite{1992}. For example, Carroll and Dunn \cite{1997} have shown that expectations of unemployment are robustly and negatively correlated with every measure of consumer expenditure (non-durable goods, durable goods and home sales). Carroll, Sommer, and Slacalek \cite{2012} confirm this finding and show why business cycle fluctuations may be driven to a large extent by changes in unemployment uncertainty. Alan, Crossley, and Low \cite{2012} use U.K. micro data to show that increases in saving rates in recessions appear largely driven by uncertainty related to unemployment.\footnote{Using these empirical insights, Challe and Ragot \cite{2013} have recently proposed a tractable quantitative model in which uninsurable unemployment risk is the source of wealth heterogeneity.} There are also recent theoretical papers that emphasized how unemployment risk and precautionary savings can amplify shocks and cause business cycle fluctuations. These papers are the closest to our work. In particular, our model structure is closely related to that presented in Guerrieri and Lorenzoni \cite{2009}. However, their model emphasizes why the economy may exhibit excessive responses to productivity shocks, while our framework offers a mechanism that amplifies demand-type shocks. Our paper also shares many features with Heathcote and Perri \cite{2012}, who develop a model in which unemployment risk and wealth impact consumption decisions and precautionary savings. Wealth matters in their setup because of financial frictions that make credit more expensive for wealth-poor agents. They obtain a strong form of demand externality that gives rise to multiple equilibria and, accordingly, they emphasize self-fulfilling cycles as the important source of fluctuations.\footnote{The existence of aggregate demand externalities and self-fulfilling expectations is also present in the work of Farmer \cite{2010} and in the work of Chamley \cite{2014}. In a model with search in both labor and goods}

\footnote{Finally, the work by Ravn and Sterk \cite{2012}...}
emphasizes as we do how unemployment risk and precautionary savings can amplify demand shocks, but their mechanism differs substantially from ours since it relies on sticky nominal prices.\footnote{See also Den Haan, Rendahl and Riegler (214).}

While the main mechanism in our model has many precursors in the literature, we believe that our setup illustrates most clearly (i) how unemployment risk gives rise to a multiplier process for demand shocks even in the absence of price stickiness or increasing returns, (ii) how this multiplier process can be ignited by periods of liquidation, and (iii) how fiscal policy can and cannot be used to counter the process.

The remaining sections of the paper are structured as follows. In Section 2, we present a static model where agents inherit from the past different levels of capital goods, and we describe how and why high values of inherited capital can lead to deficient demand and poor economic outcomes. The static setup allows for a clear exposition of the nature of the demand externality that arises in our setting with decentralized trade. We focus on the case where the inherited capital is in the form of a good which directly increases utility so as to make clear how more goods can reduce welfare. In an appendix we redo the analysis for the case where the inherited good is in the form of productive capital. We begin the analysis with a one good model. We then extend the model to the case of both a durable good and a non-durable good to show why inheriting many durable goods can also cause a reduction in non-durable purchases even when preferences are separable between the two types of goods. In Section 3 we discuss a set of robustness checks. In particular, we discuss how our analysis extends to different matching technologies and bargaining protocols, including directed search, and we compare outcomes with the constrained social optimum. In Section 4, we extend the model to an infinite-period dynamic setting and emphasize how the economy’s behavior changes when it is close versus far from its steady state. Finally, in Section 5, we discuss the trade-offs faced by a policy-maker in our setup, while Section 6 concludes.

\section{Static model}

In this section, we present a very stripped-down static model in order to illustrate as simply as possible why an economy may function particularly inefficiently when it inherits a large stock of capital from the past. In particular, we will want to make clear why agents in an economy can be worse off when the stock of inherited capital goods is too high. For the mechanism to be as transparent as possible, we begin by making several simplifying assumption that we eventually relax. For example, in our baseline model, we adopt a random matching setup with a particular matching function, while we later extend to more general matching functions and we show the robust of our main results to allowing for different bargaining protocols, including direct search. We also begin with the case where the inherited capital produces services which directly enter agents’ utility functions. Accordingly, this type of capital can be considered as representing houses or other durable consumer goods. In an appendix we discuss how the analysis carries over to the case of productive capital.
In our model, trades are decentralized, and there are two imperfections which cause unemployment risk to emerge. First, there is the matching friction in the spirit of Diamond-Mortensen-Pissarides, which will create the possibility that a household may not find employment when looking for a job. Second, there will be adverse selection in the insurance market that will limit the pooling of this risk. Since the adverse selection problem can be analyzed separately, we will begin the presentation by simply assuming that unemployment insurance is not available. Later we will introduce an adverse selection problem that rationalizes the missing market, and use the implied information problem to formulate the social planner’s problem. This will allow us to compare decentralized outcomes with the constrained efficient outcome. The key exogenous variable in the static model will be a stock of consumer durables that households inherit from the past. Our goal is to show why and when high values of this stock can cause the economy to function inefficiently and cause a decrease in welfare.

2.1 Setup

Consider an environment populated by a mass $L$ of households indexed by $j$. In this economy there are two sub-periods. In the first sub-period, households buy good 1, which we will call clothes, and try to find employment in the clothing sector. We refer to this good as clothes since in the dynamic version of the model it will represent a durable good. The good produced in the second sub-period, good 2, will be referred to as household services since it will have no durability. As there is no money in this economy, when the household buys clothes its bank account is debited, and when (and if) it receives employment income its bank account is credited. Then, in the second sub-period, households balance their books by repaying any outstanding debts or receiving a payment for any surplus. These payments are made in terms of good 2, which is also the numeraire in this economy.\(^8\)

Preferences for the first sub-period are represented by

$$U(c_j) - \nu(\ell_j)$$

where $c$ represents consumption of clothes and $\ell$ is the labor supplied by households in the production of clothes. The function $U(\cdot)$ is assumed to be increasing in $c$, strictly concave and $U''' > 0$.\(^9\) The dis-utility of work function $\nu(\cdot)$ is assumed to be increasing and convex in $\ell$, with $\nu(0) = 0$. The agents are initially endowed with $X_j$ units of clothes, which they can either consume or trade. We assume symmetric endowments, so that $X_j = X \forall j$.\(^10\) In the dynamic version of the model, $X$ will represent the stock of durable goods and will be endogenous.

The key assumption of the model is that exchange is not centralized and simultaneous. Instead trade in this economy will be subject to a potential coordination problem because exchanges made in the goods market and in the labor market are not simultaneous. To capture this idea, we assume that at the beginning of the first sub-period, the household needs

---

\(^8\) We remain agnostic about the precise details of how good 2 is produced for the time being. One possible interpretation is discussed in the following sub-section.

\(^9\) We will also assume when needed that $\lim_{c \to \infty} U'' \leq 0$.

\(^10\) In what follows, we will drop the $j$ index except where doing so may cause confusion.
to place its order of clothes before it knows its outcome in the labor market. After placing its order of goods, the household searches for employment opportunities in the labor market. The market for clothes functions in a Walrasian fashion, with both buyers and firms that sell clothes taking prices as given. The market for labor in this first sub-period is subject to a matching friction, with sellers of labor searching for employers and employers searching for labor. The important assumption is that households do not know, when choosing their consumption of clothes, whether they will secure a match in the labor market. This assumption implies that buyers will worry about unemployment risk when making purchases of clothes.

There is a large set of potential clothes firms in the economy who can decide to search for workers in view of supplying clothes to the market. Each firm can hire one worker and has access to a decreasing-returns-to-scale production function \( \theta F(\ell) \), where \( \ell \) is the number of hours worked for the firm and \( \theta > 0 \) is a technology shift factor. Production also requires a fixed cost \( \theta \Phi \) in terms of the output good, so that the net production of a firm hiring \( \ell \) hours of labor is \( \theta [F(\ell) - \Phi] \). \( \Phi \) can be thought as a vacancy posting cost, as it is incurred before a search can be conducted. For now, we will normalize \( \theta \) to 1, and will reintroduce \( \theta \) in its general form when we want to talk about the effects of technological change and balanced growth. We will also assume throughout that \( F(0) = 0 \) and that \( \Omega(\ell) \equiv F'(\ell)\ell \) is increasing in \( \ell \).\(^{11}\) Moreover, we will assume that \( \Phi \) is sufficiently small such that there exists an \( \ell^* > 0 \) satisfying \( F'(\ell^*) = \Phi \). These restrictions on the production technology are always satisfied if, for example, \( F(\ell) = A\ell^\alpha \), with \( 0 < \alpha < 1 \).

We begin by assuming that search is conducted in a random fashion, and later explore the case of directed search. Given the random search setup, when a firm and a worker match, they need to jointly decide on the number of hours worked and on the wage to be paid. There are many ways the surplus from the match can be divided, as long as it remains within the bargaining set. For the greatest clarity of results, we begin by following Lucas and Prescott (1974) and Lorenzoni and Guerrieri (2009) by assuming that the determination of the wage and hours-worked is done through a type of competitive pricing process. In effect, upon a match, one can view a Walrasian auctioneer as calling out a wage \( w \) that equilibrates the demand and supply of labor among the two parties in the match. Given the wage, the demand for labor from the firm is therefore given by the marginal productivity condition

\[ pF'(\ell) = w \]

where \( p \) is the relative price of clothes in terms of the non-durable good produced in the second sub-period.\(^{12}\) The supply of labor is chosen optimally by the household in a manner to be derived shortly.

This competitive bargaining process has the feature of limiting any within-pair distortions that could muddle the understanding of the main mechanisms of the model. In the following section, we show how our results extend to the case where wages and hours-worked are instead determined by a Nash bargaining process. As we shall see, Nash bargaining introduces additional elements into the analysis that are most easily understood after our baseline framework is presented.

---

\(^{11}\) Because we assume free-entry for clothes firms, the quantity \( \theta \Omega(\ell) \) will equal net output of clothes (after subtracting firms’ fixed costs) by a single employed worker.

\(^{12}\) As will become clear, \( p \) can be given an interpretation as an interest rate.
Letting \( N \) represent the number of firms who decide to search for workers, the number of matches is then given by the constant-returns-to-scale matching function \( M(N, L) \), with \( M(N, L) \leq \min\{N, L\} \). The equilibrium condition for the clothes market is given by

\[
L \cdot (c - X) = M(N, L)F(\ell) - N\Phi
\]

where the left-hand side is total purchases of new clothes and the right-hand side is the total available supply after subtracting search costs.

Firms will enter the market up to the point where expected profits are zero. The zero-profit condition can be written as

\[
\frac{M}{N} [pF(\ell) - w\ell] = \frac{M}{N} [pF(\ell) - pF'(\ell)\ell] = p\Phi
\]

At the end of the first sub-period, household \( j \)'s net asset position \( a_j \), expressed in units of good 2, is given by \( w\ell_j - p(c_j - X) \). We model the second sub-period so that it is costly to arrive in that sub-period with debt. For now, we can simply denote the value of entering the second sub-period with assets \( a_j \) by \( V(a_j) \), where we assume that \( V(\cdot) \) is increasing, with \( V'(a_1) > V'(a_2) \) whenever \( a_1 < 0 < a_2 \); that is, we are assuming that the marginal value of a unit of assets is greater if one is in debt than if one is in a creditor position. In the following sub-section we specify preferences and a market structure for the second sub-period that rationalizes this \( V(\cdot) \) function.

Taking the function \( V(a) \) as given, we can specify the household’s consumption decision as well as his labor-supply decision conditional on a match. The buyer’s problem in household \( j \) is given by

\[
\max_{c_j} U(c_j) + \mu V(w\ell_j - p(c_j - X)) + (1 - \mu) V(-p(c_j - X))
\]

where \( \mu \) is the probability that a worker finds a job and is given by \( \mu \equiv M(N, L)/L \). From this expression, we can see that the consumption decision is made in the presence of unemployment risk.

The worker’s problem in household \( j \) when matched, taking \( w \) as given, can be expressed as choosing a level of hours to supply in the first sub-period so as to solve

\[
\max_{\ell_j} -\nu(\ell_j) + V(w\ell_j - p(c_j - X))
\]

2.2 Deriving the value function \( V(a) \)

\( V(a) \) represents the value function associated with entering the second sub-period with a net asset position \( a \). In this subsection, we derive such a value function by specifying primitives in terms of preferences, technology and market organization. We choose to model this sub-period in such a way that if there were no friction in the first sub-period, there would be no trade between agents in the second sub-period. For this reason let us call “services” the good produced in the second period household, with preferences given by

\[
\tilde{U}(\tilde{c}) - \tilde{v}(\tilde{\ell})
\]

\footnote{We assume that searching firms pool their ex-post profits and losses so that they make exactly zero profits in equilibrium, regardless of whether they match.}
where \( \bar{c} \) is consumption of these services, \( \bar{U}(\cdot) \) is increasing and strictly concave in \( \bar{c}, \bar{\ell} \) is the labor used to produced household services, and \( \bar{\nu}(\cdot) \) is increasing and convex in \( \bar{\ell} \).

To ensure that a unit of net assets is more valuable when in debt than when in surplus, let us assume that households in the second sub-period can produce services for their own consumption, using one unit of labor to produce \( \bar{\theta} \) unit of services. However, if a household in the second sub-period has to produce market services – that is, services that can be sold to others in order to satisfy debt – then to produce \( \bar{\theta} \) units of market services requires them to supply \( 1 + \tau \) units of labor, \( \tau > 0 \). To simplify notation, we can set \( \bar{\theta} = 1 \) for now and return to the more general formulation when talking about effects of technological change.

The continuation value function \( V(a) \) can accordingly be defined as

\[
V(a) = \max_{\bar{c}, \bar{\ell}} \bar{U}(\bar{c}) - \bar{\nu}(\bar{\ell})
\]

subject to

\[
\bar{c} = \bar{\ell} + a \text{ if } a \geq 0
\]

and

\[
\bar{c} = \bar{\ell} + a(1 + \tau) \text{ if } a < 0
\]

It is easy to verify that \( V(a) \) is increasing in assets and concave. If \( \bar{\nu}(\bar{\ell}) \) is strictly convex, then \( V(a) \) will be strictly concave, regardless of the value of \( \tau \), with the key property that \( V'(a_1) > V'(a_2) \) if \( a_1 < 0 < a_2 \); that is, the marginal value of an increase in assets is greater if one is in debt than if one is in surplus.\(^{14}\) In the case where \( \bar{\nu}(\bar{\ell}) \) is linear, then \( V(a) \) will be piecewise linear and will not be differentiable at zero. Nonetheless, it will maintain the key property that \( V'(a_1) > V'(a_2) \) if \( a_1 < 0 < a_2 \). We will mainly work with this case, and in particular, will assume that \( \bar{\nu}(\bar{\ell}) = v \cdot \bar{\ell} \), which implies that \( V(a) \) is piecewise linear with a kink at zero.

### 2.3 Equilibrium in the first sub-period

Given the function \( V(a) \), a symmetric equilibrium for the first sub-period is represented by five objects: two relative prices (the price of clothing \( p \) and the wage rate \( w \)), two quantities (consumption of clothes by each household \( c \) and the amount worked in each match \( \ell \)), and a number \( N \) of active firms, such that

1. \( c \) solves the buyer’s problem taking \( \mu, p, w \) and \( \ell \) as given.
2. The labor supply \( \ell \) solves the worker’s problem conditional on a match, taking \( p, w \) and \( c \) as given.
3. The demand for labor \( \ell \) maximizes the firm’s profits given a match, taking \( p \) and \( w \) as given.
4. The goods market clears; that is, \( L \cdot (c - X) = M(N, L)F(\ell) - N\Phi \).

\(^{14}\) To avoid backward-bending supply curves, we will also assume that \( \bar{\nu}(\cdot) \) and \( \bar{U}(\cdot) \) are such that \( V'''(a) \geq 0 \). This assumption is sufficient but not necessary for later results. Note that a sufficient condition for \( V'''(a) \geq 0 \) is that both \( \bar{U}'''(\cdot) \geq 0 \) and \( \bar{\nu}'''(\cdot) < 0 \).
5. Firms’ entry decisions ensure zero profits.

The equilibrium in the first sub-period can therefore be represented by the following system of five equations:

\[ U'(c) = p \left\{ \frac{M(N, L)}{L} V'(w\ell - p(c - X)) + \left[ 1 - \frac{M(N, L)}{L} \right] V'(-p(c - X)) \right\} \tag{1} \]

\[ \nu'(\ell) = V'(w\ell - p(c - X)) w \tag{2} \]

\[ pF'(\ell) = w \tag{3} \]

\[ M(N, L) F(\ell) = L(c - X) + N\Phi \tag{4} \]

\[ M(N, L) [pF(\ell) - w\ell] = Np\Phi \tag{5} \]

In the above system, equations (1) and (2) represent the first-order conditions for the household’s choice of consumption and supply of labor. Equations (3) and (5) represent a firm’s labor demand condition and its entry decision. Finally, (4) is the goods market clearing condition.

At this level of generality it is difficult to derive many results. Nonetheless, we can combine (1), (2) and (3) to obtain the following expression regarding one characteristic of the equilibrium,

\[ \frac{\nu'(\ell)}{U'(c)} \left\{ 1 + (1 - \mu) \left[ \frac{V'(p(c - X))}{V'(w\ell - p(c - X))} - 1 \right] \right\} = F'(\ell) \tag{6} \]

From equation (6), we see that as long as \( \mu < 1 \), the marginal rate of substitution between leisure and consumption will not be equal to the marginal productivity of work; that is, the labor market will exhibit a wedge given by

\[ (1 - \mu) \left[ \frac{V'(p(c - X))}{V'(w\ell - p(c - X))} - 1 \right] \]

In fact, in this environment, the possibility of being unemployed leads to precautionary savings, which in turn causes the marginal rate of substitution between leisure and consumption to be low relative to the marginal productivity of labor. As we will see, changes in \( X \) will cause this wedge to vary, which will cause a feedback effect on economic activity.

Our main goal now is to explore the effects of changes in \( X \) on equilibrium outcomes. In particular, we are interested in clarifying why and when an increase in \( X \) can actually lead to a reduction in consumption and/or welfare. The reason we are interested in this comparative static is that we are interested in knowing why periods of liquidations – that is, periods where agents inherit excessive levels of durable goods from the past – may be socially painful.

\[ \text{To ensure that an employed worker’s optimal choice of labor is strictly positive, we assume that } \lim_{\ell \to 0} U'(c) > \lim_{\ell \to 0} \frac{\nu'(\ell)}{F'(\ell)}. \]
To clarify the analysis, we make two simplifying assumptions. First, we assume that the matching function takes the form \( M(N, L) = \Lambda \min\{N, L\} \), with \( 0 < \Lambda \leq 1 \). The attractive feature of this matching function is that it has two regimes: one where congestion externalities are concentrated on workers, and one where the externalities are concentrated on firms. In the case where \( N > L \), which we refer to as a tight labor market, workers are the more scarce factor and only the firm matching rate is affected by changes in the ratio \( N/L \). In the case where \( L > N \), which we refer to as a slack labor market, jobs are scarce and only the worker matching rate is affected by changes in the ratio \( N/L \). While this particular matching function is not necessary for our main results, it allows us to cleanly compare the case where congestion externalities are stronger for firms to the case where they are stronger for workers. We will also assume that \( V(a) \) is piece-wise linear, with \( V(a) = va \) if \( a \geq 0 \) and \( V(a) = (1 + \tau)va \) if \( a < 0 \), where \( \tau > 0 \) and \( v > 0 \). This form of the \( V(\cdot) \) function corresponds to the case discussed in section 2.2 where the dis-utility of work in the second sub-period is linear. The important element here is \( \tau \). In effect, \( 1 + \tau \) represents the ratio of the marginal value of an extra unit of assets when one is in debt relative to its value when one is in surplus. A value of \( \tau > 0 \) can be justified in many ways, one of which is presented in section 2.2. Alternatively, \( \tau > 0 \) could reflect a financial friction that causes a wedge between borrowing and saving rates.

Under these two functional-form assumptions, the equilibrium conditions can be reduced to the following:

\[
U'(c) = \frac{\nu'(\ell)}{F'(\ell)} \left( 1 + \tau - \frac{\Lambda \min\{N, L\}}{L} \right) \tag{7}
\]

\[
\frac{\Lambda \min\{N, L\}}{L} = \frac{c - X}{F(\ell) - \Phi} \tag{8}
\]

\[
\frac{\Lambda \min\{N, L\}}{N}[F(\ell) - F'(\ell)\ell] = \Phi \tag{9}
\]

\[
w = \frac{\nu'(\ell)}{v} \tag{10}
\]

\[
p = \frac{\nu'(\ell)}{vF'(\ell)} \tag{11}
\]

This system of equations now has the feature of being block-recursive. Equations (7), (8) and (9) can be solved for \( c, \ell \) and \( N \), with equations (10) and (11) then providing the wage and the price. From equations (7) and (8), one can immediately notice the complementarity that can arise between consumption and employment in the case where \( N < L \) (i.e., where the labor market is slack). From (7) we see that, if \( N < L \), agents will tend to increase their consumption if they believe there are many firms looking for workers (\( N \) expected to be large). Then from equation (8) we see that more firms will be looking to hire workers if they believe that consumption will be high. Thus, greater consumption favors greater employment, which in turn reinforces consumption. This feedback effect arises as the result of consumption and employment playing the role of strategic complements. Workers demand higher consumption when they believe that many firms are searching to hire, as they view a high \( N \) as reducing their probability of entering the second sub-period in debt. It is important to notice that this multiplier argument is implicitly taking \( \ell \), the number of hours worked...
by agents, as given. But, in the case where the economy is characterized by unemployment, this is precisely the right equilibrium conjecture. In particular, from (9) we can see that if the economy is in a state of unemployment, then \( \ell \) is simply given by \( \ell^* \), the solution to the equation \( \Lambda[F(\ell^*) - F'(\ell^*)\ell^*] = \Phi \), and is therefore locally independent of \( X \) or \( c \). Hence, in the slack labor market regime, consumption and firm hiring will act as strategic complements. As is common in the case of strategic complements, multiple equilibria can arise. This possibility is stated in Proposition 1.

Proposition 1. There exists a \( \bar{\tau} > 0 \)\(^{16} \) such that (a) if \( \tau < \bar{\tau} \), then there exists a unique equilibrium for any value of \( X \); and (b) if \( \tau > \bar{\tau} \), then there exists a range of \( X \) for which there are multiple equilibria.

The proofs of all propositions are presented in Appendix A.

While situations with multiple equilibria may be interesting, in this paper we will focus on cases where the equilibrium is unique. Accordingly, Proposition 1 tells us that our setup will have a unique equilibrium if the marginal cost of debt is not too large. For the remainder of this section, we will assume that \( \tau < \bar{\tau} \). Proposition 2 focuses on this case and provides a first step in the characterization of the equilibrium.

Proposition 2. When \( \tau < \bar{\tau} \), there exists an \( X^* \) such that if \( X \leq X^* \) then the equilibrium is characterized by a tight labor market \((N \geq L)\), while if \( X > X^* \) it is characterized by a slack labor market \((N < L)\). Furthermore, there exists an \( X^{**} > X^* \) such that if \( X > X^{**} \), then employment is zero and agents simply consume their endowment \( (i.e., \ c = X) \)\(^{17} \).

The content of Proposition 2 is very intuitive as it simply states that if agents have a low endowment of the consumption good, then there are substantial gains from trade, and that will favor a tight labor market. In contrast, if the endowment is very high, this will reduce the demand for the good sufficiently as slack labor market. Finally, if \( X \) is extremely high, all trade among agents will stop as people are content to simply consume their endowment. Proposition 3 complements Proposition 2 by indicating how consumption is determined in each regime.

Proposition 3. When the labor market is tight \( (X^{**} > X > X^*) \), the level of consumption is given as the unique solution to

\[
c = U'^{-1} \left( \frac{\nu'(\ell^*)}{F'(\ell^*)} \left[ 1 + \tau - \frac{c - X}{F'(\ell^*)\ell^*} \right] \right)
\]

When the labor market is tight \( (X \leq X^*) \), consumption is the unique solution to

\[
c = U'^{-1} \left( \frac{\nu'(\Omega^{-1}(c - X))}{F'(\Omega^{-1}(c - X))} (1 + \tau (1 - \Lambda)) \right)
\]

Finally, when \( X \geq X^{**} \), consumption is given by \( c = X \).

---

\(^{16} \bar{\tau} = -U'' \left( U'^{-1} \left( \frac{\nu'(\ell^*)}{F'(\ell^*)} \right) \right) \frac{F'(\ell^*)[F(\ell^*)-\Phi]}{\nu'(\ell^*)}.

\(^{17} X^* = U'^{-1} \left( \frac{\nu'(\ell^*)}{F'(\ell^*)} \right) - F'(\ell^*)\ell^* \) and \( X^{**} = U'^{-1} \left( \frac{\nu'(\ell^*)}{F'(\ell^*)} (1 + \tau) \right)\).
Given the above propositions, we are now in a position to examine an issue of main interest, which is how an increase in $X$ affects consumption. In particular, we want to ask whether an increase in $X$, which acts as an increase in the supply of goods, can lead to a decrease in the actual consumption of goods. Proposition 4 addresses this issue.

**Proposition 4.** If $X^{**} > X > X^*$, then $c$ is decreasing in $X$. If $X \leq X^*$ or $X > X^{**}$, then $c$ is increasing in $X$.

The content of Proposition 4 is illustrated in Figure 1. Proposition 4 indicates that, starting at $X = 0$, consumption will continuously increase in $X$ as long as $X$ is compatible with a tight labor market. Then, when $X$ is greater than $X^*$, the economy enters the slack labor market regime and consumption starts to decrease as $X$ is increased. Finally, beyond $X^{**}$ trade collapses and consumption becomes equal to $X$ and hence it increases with $X$. The reason that consumption decreases with a higher supply of $X$ in the slack region is precisely because of the multiplier process described earlier. In this region, an increase in $X$ leads to a fall in expenditures on new consumption, where we define expenditures as $e \equiv c - X$. The decrease in expenditures reduces the demand for goods as perceived by firms. Less firms then search for workers, which increases the risk of unemployment. The increase in unemployment risk leads households to cut their expenditures further, which further amplifies the initial effect of an increase in $X$ on expenditures. It is because of this type of multiplier process that an increase in the supply of the good can lead to a decrease in its total consumption ($X + e$). Note that such a negative effect does not happen when the labor market is tight, as an increase in $X$ does not cause an increase in precautionary savings, which is the key mechanism at play causing consumption to fall.

**Figure 1: Consumption as function of $X$**

![Graph showing consumption as a function of $X$]

*Note: Example is constructed assuming the functional forms $U(c) = \log(c)$, $\nu(\ell) = \nu^{(1+\omega)}$, and $F(\ell) = A\ell^\alpha$, with parameters $\omega = 1$, $\nu = 0.5$, $\alpha = 0.67$, $A = 1$, $\Phi = 0.35$, $\Lambda = 1$ and $\tau = 0.4$.*
The link noted above between household $j$’s expenditure, which we can denote by $e_j \equiv c_j - X_j$, and its expectation about the expenditures by other agents in the economy, which can denote by $e$, can be captured by rewriting the relations determining $e_j$ implied by the elements of Proposition 3 as

$$e_j = Z(e) - X \quad (12)$$

with

$$Z(e) \equiv U'^{-1}(Q(e)) \quad (13)$$

and

$$Q(e) \equiv \begin{cases} \frac{\nu'(\ell^*)}{F'(\ell^*)} (1 + \tau - \frac{e}{\ell^*}) & \text{if } 0 < e < \Lambda e^* \\ \frac{\nu'(\Omega^{-1}(e))}{F'(\Omega^{-1}(e))} [1 + \tau(1 - \Lambda)] & \text{if } e \geq \Lambda e^* \end{cases} \quad (14)$$

Here, $e^* \equiv \Omega(\ell^*)$ is the level of output (net of firms’ search costs) that would be produced if all workers were employed, with hours per employed worker equal to $\ell^*$. In equilibrium we have the additional requirement that $e_j = e$ for all $j$.

The equilibrium determination of $e$ is illustrated in Figure 2, which somewhat resembles a Keynesian cross. In the figure, we plot the function $e_j = Z(e) - X$ for two values of $X$: a first value of $X$ which places the economy in an unemployment regime, and a second value of $X$ which places the economy in a tight labor market regime. An equilibrium in this figure corresponds to the point where the function $e_j = Z(e) - X$ crosses the 45° line. Note that changes in $X$ simply move the $e_j = Z(e) - X$ curve vertically.

There are several features to note about Figure 2. First, in the case where $X \in (X^*, X^{**})$, so that the equilibrium of the economy corresponds to a slack labor market with positive

---

**Figure 2: Equilibrium determination**

![Equilibrium determination graph](image)

**Note:** Example is constructed assuming the functional forms $U(c) = \log(c)$, $\nu(\ell) = \nu^{1+\omega} / (1+\omega)$, and $F(\ell) = A\ell^\alpha$, with parameters $\omega = 1$, $\nu = 0.5$, $\alpha = 0.67$, $A = 1$, $\Phi = 0.35$, $\Lambda = 1$ and $\tau = 0.4$. Values of $X$ used were $X = 0$ for the full-employment equilibrium and $X = 0.69$ for the unemployment equilibrium.
trade (i.e., \(0 < e < \Lambda e^*\)), the diagram is similar to a Keynesian cross. We can see graphically how an increase in \(X\) by one unit shifts down the \(Z(e) - X\) curve and, since the slope of \(Z(e) - X\) is positive and less than one, a multiplier process kicks in which causes \(e\) to fall by more than one. Because of this multiplier process, total consumption of clothes, which is equal to \(e + X\), decreases, which is the essence of the first part of Proposition 4.

Second, when \(X < X^*\), so that the labor market is tight (i.e., the equilibrium is such that \(e > \Lambda e^*\)), the diagram is different from the Keynesian cross. The most notable difference is the negative slope of the function \(Z(e) - X\) for values of \(e > e^*\). This reflects the fact that unemployment risk is not increasing in this regime. In fact, when \(X\) is sufficiently small so that the labor market is tight, an increase in \(X\) by one unit leads to a decrease in \(e\) that is less than one, compared to a decrease of greater than one as exhibited in the slack regime. Here, expenditure by others actually plays the role of a strategic substitute with one’s own expenditure – as opposed to playing the role of a strategic complement as is the case in the unemployment regime – through its effects on real wages and prices. Accordingly, in this region, an increase in \(X\) leads to an increase in total consumption of clothes. Another more subtle difference with the Keynesian cross is in how the intercept of \(Z(e) - X\) is determined. The intercept is given by \(U' - 1(F'(e^*) (1 + \tau)) - X\). The \(X\) term in the intercept can be interpreted as capturing a pure aggregate-demand effect, whereby higher values of \(X\) reduce aggregate demand. However, the remaining term, \(U' - 1(F'(e^*) (1 + \tau))\), reflects technology and preferences. In particular, we can generalize this term by re-introducing the technology parameter \(\theta\), in which case the intercept becomes \(U' - 1(F'(e^*) (1 + \tau))\). In this case, we see that an improvement in technology shifts up the intercept, and will lead to an increase in expenditures. This feature of the \(Z(e) - X\) curve illustrates its equilibrium nature, which incorporates both demand and supply effects, as opposed to a Keynesian cross that only reflects demand effects.

2.4 Is there deficient demand when the labor market is slack?

In the case where \(X\) is large enough for the economy to be in the slack labor market regime \((X^* < X < X^{**})\), we would like to know whether this regime should be characterized as suffering from deficient aggregate demand. For this, we need to first properly define the concept of deficient demand. In our definition we want to focus on a situation where economic activity is inefficiently low and where that low level of activity can be traced back at least in part to a lack of demand by others. In particular, we want our definition to exclude a situation where economic activity is inefficiently low simply because of price distortions that are unrelated to a lack of demand by others. For this reason, we define deficient demand as follows.

Definition. Deficient demand is a situation where increased demand by one agent would favor increased demand by other agents, and where a coordinated increased in demand by all agents would leave everyone better off.

\(^{18}\) Recall that an increase in \(\theta\) is associated with a proportional change in the search cost, so that \(\ell^*\) remains unchanged.
In this definition of deficient demand, we are considering how small deviations in consumption from households’ equilibrium strategies would affect outcomes (keeping the structure of all markets otherwise unchanged). The definition includes two elements. First, it requires that economic activity be inefficiently low, in the sense that if all households were to deviate from their equilibrium strategies by slightly increasing their demand in the first sub-period, this would create a Pareto improvement. However, we do not believe that this property is sufficiently restrictive on its own, since the reason for low economic activity may not be related in any way to demand effects. For this reason, we choose to be more restrictive in our definition by adding the requirement that agents perceive their low demand for goods as being at least in part the result of low demand by others. Using this definition, Proposition 5 indicates that the slack labor market regime is in fact characterized by deficient demand, while the full employment regime is not.

**Proposition 5.** When the labor market is slack \((X^* < X < X^{**})\) it exhibits deficient demand for all \(\tau > 0\), while if the labor market is tight \((X < X^*)\) the economy does not exhibit deficient demand.

### 2.5 Effects of changes in \(X\) on welfare

We have shown that when \(X\) is high enough, then the labor market will be slack, where a local increase in \(X\) will cause consumption to fall. We now want to ask how expected welfare is affected in these cases, where expected welfare is defined as

\[
U(c) + \mu \left[ -\nu(\ell) + V(w\ell - p(c - X)) \right] + (1 - \mu)V(-p(c - X))
\]

In particular, we want to ask whether welfare can decrease when the economy is endowed with more goods. Proposition 6 answers this question in the affirmative. Proposition 6 actually goes a step further and indicates two sufficient conditions for there to exist a range of \(X\) in the slack labor market regime where an increase in \(X\) leads to a fall in welfare.

**Proposition 6.** An increase in \(X\) can lead to a fall in expected welfare. In particular, if either (i) \(\tau\) is close enough to \(\overline{\tau}\) or (ii) the average cost of work \(\frac{\nu(\ell^*)}{\ell^*}\) is low enough relative to the marginal cost of work \(\nu'(\ell^*)\), then there is always a range of \(X \in [X^*, X^{**}]\) such that an increase in \(X\) leads to a decrease in expected welfare.

Proposition 6 provides an answer to whether more goods can make everyone worse off. In effect, the proposition indicates that the economy can function in a very perverse fashion when households have inherited many goods. We saw from Proposition 4 that an increase in \(X\) always leads to a decrease in consumption when we are in the slack regime. In comparison, Proposition 6 is weaker as it only indicates the possibility of a fall in welfare in the slack region when \(X\) rises. In response to a rise in \(X\) in the slack regime, there are three distinct channels through which expected welfare is affected. First, as discussed above, consumption falls, which tends to directly decrease welfare. Second, this fall in consumption is associated with a fall in the probability of being employed. It can be verified that the net benefit of being
employed is strictly positive, so that this second effect also tends to decrease welfare. Finally, a rise in $X$ means that a given quantity of consumption can be obtained with a lower level of expenditure, which increases assets for the employed and decreases debt for the unemployed, and therefore tends to increase welfare. Whether this final effect is outweighed by the first two depends on the factors discussed in Proposition 6.

As noted in Proposition 6, the effects of an increase in $X$ on welfare depends, among other things, on the difference between the marginal utility cost of work and the average utility cost of work. This distinction is relevant because an important component of the net benefit of being employed is the utility value of wages earned, net of the value of foregone leisure. In the current model, the average utility cost of work can be arbitrarily small relative to its marginal cost. When the average cost of work is low, the net benefit of being employed is large, and therefore a rise in the unemployment rate caused by a rise in $X$ will have a larger negative effect on welfare (i.e., the second channel discussed above becomes more important). Hence, in our model, when employment is not perceived as very painful, and we are in a slack labor market, then an increase in $X$ leads to decreased welfare. Figure 3 illustrates the change in welfare in our model for the parametric example considered in Figure 1. As can be seen, the decrease in welfare mimics closely the decrease in consumption plotted in Figure 1.

**Figure 3: Welfare as function of $X$**

\[\text{Note: Example is constructed assuming the functional forms } U(c) = \log(c), \quad \nu(\ell) = \frac{\nu^{1+\omega}}{1+\omega}, \text{ and } F(\ell) = A\ell^\alpha, \text{ with parameters } \omega = 1, \nu = 0.5, \alpha = 0.67, A = 1, \Phi = 0.35, \Lambda = 1 \text{ and } \tau = 0.4.\]

\[\text{19 The other component is the net welfare gain that stems from consumption expenditures being made in the positive-asset state rather than the more costly (in utility terms) negative-asset state.}\]
2.6 Adding a non-durable goods sector

In the previous section we showed that when trade in goods and labor is not simultaneous and there is risk of unemployment, the economy can function in a rather perverse fashion. In particular, we showed that inheriting a large amount of goods can result in deficient demand, with increases in inherited goods reducing both consumption and welfare. In this section we want to briefly explore the robustness of these results to allowing for a second sector (in the first sub-period) that can potentially expand when the market for durables contracts. This exploration is especially relevant given a common criticism of the liquidation views of recession, as expressed for example by Krugman (1998), that suggests that liquidationist models cannot explain why we see falls in the consumption of both durable and non-durable goods during most recessions. In order to explore this issue, let us extend the previous setup by making a few small changes. First let us allow utility in the first sub-period to take the form

\[ U^d(c^d) + U^n(c^n) - \nu(\ell), \]

where \( c^d \) is durable consumption (clothes) and is equal as before to \( X + e \), \( c^n \) is the added non-durable consumption good, and \( \ell \) is hours worked. We will assume that there is only one labor market, so that workers can switch frictionlessly across sectors of production. Both durable goods producers and non-durable goods producers will search for workers in this labor market. The markets for durable goods and non-durable good are assumed to be distinct, with each functioning in a Walrasian fashion. We treat producers of the different goods symmetrically, with production functions in the respective sectors denoted by \( F^d(\ell) \) and \( F^n(\ell) \) (maintaining the assumptions that \( F^j(\ell) \) is increasing in \( \ell \) in both sectors), and assuming that both types of firms face the fixed cost of entering the market given by \( \Phi \). Otherwise, we maintain the same structure as before, including the functional-form assumptions for \( M(N,L) \) and \( V(\cdot) \). In this extended model, the equilibrium conditions for quantities can be written

\[ U^d(c^d) = \frac{\nu'(\ell^d)}{F^d(\ell^d)} \left( 1 + \tau - \frac{\Lambda \min\{N^d + N^n, L\}}{L} \right) \]

\[ \frac{\Lambda \min\{N^d + N^n, L\}}{N^d + N^n} \left[ F^d(\ell^d) - F^d(\ell^d) \ell^d \right] = \Phi \]

\[ U^n(c^n) = \frac{\nu'(\ell^n)}{F^n(\ell^n)} \left( 1 + \tau - \frac{\Lambda \min\{N^d + N^n, L\}}{L} \right) \]

\[ \frac{\Lambda \min\{N^d + N^n, L\}}{N^d + N^n} \left[ F^n(\ell^n) - F^n(\ell^n) \ell^n \right] = \Phi \]

The issue we want to examine is the relationship between \( X \) and equilibrium outcomes for this economy. Proposition 7 establishes this.

20 The prices and wages are given by \( w^d = \frac{\nu'(\ell^d)}{v}, \quad p^d = \frac{\nu'(\ell^d)}{vF^d(\ell^d)}, \quad w^n = \frac{\nu'(\ell^n)}{v}, \quad \) and \( p^n = \frac{\nu'(\ell^n)}{vF^n(\ell^n)} \).
Proposition 7. In the economy with both a durable good and a non-durable good, if \( \tau \) is not too large, then for any value of \( X > 0 \) there exists a unique equilibrium. Moreover, if \( \Phi \) is not too small, there exists an \( X^* \) and an \( X^{**} > X^* \), such that:

1. For \( X < X^* \), the labor market is tight and the consumption of both durables \( (c^d = X + e) \) and non-durables \( (c^n) \) increases with \( X \).

2. For \( X \in [X^*, X^{**}] \), the labor market is slack with both \( c^d \) and \( c^n \) decreasing with \( X \).

3. For \( X > X^{**} \), \( c^d = X \) and \( c^n \) is invariant to \( X \).

The most interesting aspect of this proposition from our point of view is the existence of a slack labor market regime (when \( X \in [X^*, X^{**}] \)), where the consumption of durables, purchases of new durables, and purchases of non-durables all decrease in response to an increase in \( X \). Although workers can be hired by non-durable goods firms in response to an increase in \( X \), the proposition tells us that this substitution does not happen when the labor market is slack. To understand why this does not arise, it is helpful to examine the equilibrium conditions in the case where the labor market is slack, which can be combined to obtain

\[
U^{d\ell}(c^d) = \frac{\nu' (\ell_{ds})}{F^{d\ell} (\ell_{ds})} \left[ 1 + \tau - \left( \frac{c^d - X}{F^{d\ell} (\ell_{ds}) \ell_{ds}} + \frac{c^n}{F^{d\ell} (\ell_{ns}) \ell_{ns}} \right)^{\tau} \right]
\]

(21)

\[
U^{n\ell}(c^n) = \frac{\nu' (\ell_{ns})}{F^{n\ell} (\ell_{ns})} \left[ 1 + \tau - \left( \frac{c^d - X}{F^{d\ell} (\ell_{ds}) \ell_{ds}} + \frac{c^n}{F^{d\ell} (\ell_{ns}) \ell_{ns}} \right)^{\tau} \right]
\]

(22)

where \( \ell_{ds} \) and \( \ell_{ns} \) (hours worked in the unemployment regime) are defined implicitly by the conditions \( \Lambda[F^n (\ell_{ds}) - F^n (\ell_{ds}) \ell_{ds}] = \Phi \) and \( \Lambda[F^n (\ell_{ns}) - F^n (\ell_{ns}) \ell_{ns}] = \Phi \).

From equations (21) and (22), we see that when the labor market is slack consumption in the two sectors act as strategic complements, and therefore they move in the same direction in response to a change in \( X \). As long as \( X \) is high enough to push the labor market into slack, any further increase in \( X \) decreases employment in the durable goods sector, which increases overall unemployment risk. Since this initial increase in unemployment risk causes households to act with precaution in all of their purchases, demand for both durable and non-durable goods falls, further increasing unemployment risk, and so on. Hence, this version of the model offers an explanation for why inheriting a high level of durables can lead to low activity in both the durable and non-durable goods sectors.\(^{21}\)

\(^{21}\) A key condition for the unemployment regime to emerge with high \( X \) is that \( \Phi \) not be too small. Specifically, we require that the value of \( c^n \) that solves

\[
U^{d\ell}(c^n) = \frac{\nu' (\ell_{ns})}{F^{n\ell} (\ell_{ns})} \left[ 1 + \tau - \frac{c^n}{F^{d\ell} (\ell_{ns}) \ell_{ns}} \right]
\]

be such that \( \frac{c^n}{F^{d\ell} (\ell_{ns}) \ell_{ns}} < \Lambda \). This property is guaranteed if \( \Phi \) is large enough.
2.7 Multiple equilibria

Let us briefly discuss how multiple equilibria can arise in this model when \( \tau > \bar{\tau} \). It can be verified that, when \( \tau > \bar{\tau} \), the equilibrium determination of expenditures can still be expressed as the solution to the pair of equations \( e_j = Z(e) - X \) and \( e_j = e \). The problem that arises is that this system may no longer have a unique solution. Instead, depending on the value of \( X \), it may have multiple solutions, an example of which is illustrated in Figure 4. In the figure, we see that, for this value of \( X \), there are three such solutions.

Figure 4: Equilibrium determination (multiple equilibria)

![Equilibrium determination diagram]

Note: Example is constructed assuming the functional forms \( U(c) = \log(c) \), \( \nu(\ell) = \frac{\nu^{1+\omega}}{1+\omega} \) and \( F(\ell) = A\ell^\alpha \), with parameters \( \omega = 1 \), \( \nu = 0.5 \), \( \alpha = 0.67 \), \( A = 1 \), \( \Phi = 0.35 \), \( \tau = 1.2 \), \( \Lambda = 1 \) and \( X = 0.35 \).

Figure 5 shows how the set of possible equilibrium values of consumption depends on \( X \) when \( \tau > \bar{\tau} \). As can be seen, when \( X \) is in the right range, there is more than one such equilibrium, with at least one in the slack regime and one in the tight regime. When this is the case, the selection of the equilibrium will depend on people’s sentiment. If people are pessimistic, they cut back on consumption, which leads firms to cut back on employment, which can rationalize the initial pessimism. In contrast, if households are optimistic, they tend to buy more, which justifies many firms wanting to hire, which reduces unemployment and supports the optimistic beliefs. This type of environment featuring multiple equilibria driven by demand externalities is at the core of many papers. On this front, this paper has little to add. The only novel aspect of the current paper in terms of multiple equilibria is to emphasize how the possibility of multiple equilibria may depend on the economy’s holding of capital goods.
3 Robustness of results to alternative assumptions

In our baseline model we have made several restrictive assumptions. For example, we worked with a matching function of the “min” form and we adopted a very particular process for the determination of wages and hours worked. These choices have allowed us to present the main mechanisms of interest in their simplest form. In this section we aim to highlight how our results carry over to more general frameworks. We start by discussing how these results can be extended to more general matching functions, then consider how the analysis changes as we adopt alternative processes for the determination of wages and hours worked. We complete this section by making explicit the informational constraint which limits unemployment insurance in our model, then use the explicit information constraint to formulate the social planner’s problem and compare it to the market solutions.\footnote{Throughout this section we will be assuming that we are in a region of the parameter space that guarantees uniqueness of equilibrium.}

3.1 Allowing for a more general matching technology

One of the important simplifying assumptions of our model is the use of a matching function of the “min” form. This specification has the nice feature of creating two distinct regimes: one where congestion externalities are on the worker’s side and one where they are on the firm’s side. However, this stark dichotomy, while useful, is not central to the main results of the model. In fact, as we now discuss, the important feature for our purposes is that there be one regime in which expenditures by individual agents play the role of strategic substitutes and another in which they play the role of strategic complements. To see this, it is helpful
to re-examine the equilibrium condition for the determination of expenditure for a general matching function. This is given by

\[ U'(X + e_j) = vp(e) \left[ 1 + \tau - \frac{M(N(e), L)}{L} \right] \]

(23)

where \( M(N, L) \) is a CRS matching function satisfying \( M(N, L) \leq \min\{N, L\} \). In (23), we have made explicit the dependence of \( N \) and \( p \) on \( e \), where this dependence comes from viewing the other equilibrium conditions as determining \( N, p \) \( w \) and \( \ell \) as functions of \( e \).\(^{23}\) Note that the other equilibrium conditions imply that \( p(e) \) and \( N(e) \) are always weakly increasing in \( e \). In (23) we have once again made clear that this condition relates the determination of expenditure for agent \( j \), \( e_j \), to the average expenditure of all agents, \( e \). From this equation, we can see that average expenditure can play the role of either strategic substitute or strategic complement to the expenditure decision of agent \( j \). In particular, through its effect on the price \( p \), \( e \) can play the role of a strategic substitute (if \( p'(e) > 0 \)), while through its effect on firm entry and, in turn, unemployment, it can play the role of strategic complement (if \( N'(e) > 0 \)). The sign of the net effect of \( e \) on \( e_j \) therefore depends on whether the price effect or the unemployment effect dominates. In the case where \( M(N, L) = \Lambda \min\{N, L\} \), the equilibrium features the stark dichotomy whereby \( p'(e) = 0 \) and \( \partial M(N(e), L)/\partial e > 0 \) for \( e < \Lambda e^* \), while \( p'(e) > 0 \) and \( \partial M(N(e), L)/\partial e = 0 \) for \( e > \Lambda e^* \). In other words, for low values of \( e \) the expenditures of others plays the role of strategic complement to \( j \)'s decision since the price effect is not operative, while for high values of \( e \) it plays the role of strategic substitute since the increased risk-of-unemployment channel is not operative. This reversal in the role of \( e \) from acting as a complement to acting as a substitute can be seen in Figure 2 or alternatively in Figure 6. In Figure 6, we first plot a cost-of-funds schedule for agents, defined by \( r = p(e) \left[ 1 + \tau - \frac{\min\{N(e), L\}}{L} \right] \), where \( r \) represents the total cost of funds to agent \( j \) when average expenditure is \( e \).\(^{24}\) Our notion of the total cost of funds reflects both the direct cost of borrowing, \( p(e) \), and the extra cost associated with the presence of unemployment risk. We superimpose on this figure the demand for \( e \) as a function of the total cost of funds, which is implicitly given by the function \( U'(X + e)/v = r \). This latter relationship, which can be interpreted as a type of aggregate demand curve, is always downward-sloping since \( U \) is concave. The important element to note in this figure is that the cost-of-funds schedule \( r = p(e) \left[ 1 + \tau - \frac{\min\{N(e), L\}}{L} \right] \) is first decreasing and then increasing in \( e \). Over the range \( e < e^* \), the cost of funds to an agent is declining in aggregate \( e \), since \( N \) is increasing while \( p \) is staying constant. Therefore, in the range \( e < e^* \), a rise in \( e \) reduces unemployment risk and makes borrowing less costly to

\[ \nu'(\ell) = vw \]

\[ pF'(\ell) = w \]

\[ M(N, L)F(\ell) = L(e - X) + N\Phi \]

\[ M(N, L)[pF(\ell) - w\ell] = Np\Phi \]

\(^{23}\) These remaining four equilibrium conditions can be written

\[^{24}\) Note that we are assuming that \( \Lambda = 1 \) in Figure 6.
agents. This is the complementarity zone. In contrast, over the range \( e \geq e^* \), the effect of \( e \) on the cost of funds is positive since the unemployment risk channel is no longer operative, while the price channel is. This is the strategic substitute zone. In the figure, a change in \( X \) moves the demand curve \( U'(X+e)/v \) without affecting the cost-of-funds curve. A change in \( X \) therefore has the equilibrium property \( \partial e/\partial X < -1 \) when \( e < e^* \) because the cost-of-funds curve is downward-sloping in this region, while \( \partial e/\partial X > -1 \) in the region \( e \geq e^* \) because the cost-of-funds curve is upward-sloping.

From the above discussion it become clear that our main results depend on the existence of two regions: one (associated with low levels of \( e \)) where the cost of funds is decreasing because the negative unemployment-risk channel dominates any potentially positive price channel, and a second (associated with high levels of \( e \)) where the price channel dominates the unemployment-risk channel. As we now show, this characterization of the economy holds for a larger class of matching functions, with one caveat: there may also exist an intermediate range of \( e \) with mixed properties. The class of matching functions we consider are those that satisfy the following assumption.

**Assumption 1.** The matching function \( M(N,L) \leq \min\{N,L\} \) is continuous, is weakly increasing and concave in both arguments, exhibits constant returns to scale, and satisfies

\[
\lim_{N \to 0} \frac{\partial M(1, \frac{L}{N})}{\partial N} = 0 \quad \text{and} \quad \lim_{N \to \infty} \frac{\partial M(\frac{N}{T}, 1)}{\partial L} = 0
\]  

While many of the properties of Assumption 1 are fairly standard, there are two worth emphasizing. First, a matching function clearly needs to satisfy \( M(N,L) \leq \min\{N,L\} \) in order to be admissible. Note that this rules out, for example, matching functions of the
Cobb-Douglas type. Second, since \( M(1, L/N) = M(N, L)/N \) is the firm matching rate, \( \partial M(1, L/N)/\partial N \leq 0 \) captures the congestion effect that additional firm entry has on that matching rate. The first part of (24) requires that this firm congestion effect disappears as \( N \) becomes small. Similarly, \( \partial M(N/L, 1)/\partial L \leq 0 \) denotes the worker congestion effect, so that the second part of (24) requires that worker congestion disappears as \( N \) becomes large. Simple examples of matching functions that satisfy Assumption 1 include the “min” function used above and the well known ball-urn matching function given by \( M(N, L) = N(1 - \exp\{-L/N\}) \).

In order to characterize equilibrium outcomes for the class of matching functions satisfying Assumption 1, it is useful to first define the cut-off level of \( X \), denoted \( X_{\text{max}} \), that would just cause trade in the economy to fall zero. This value is defined by \( X_{\text{max}} = U'(\nu'(\ell^*)/(1 + \tau)), \) where \( \ell^* \) is defined implicitly by \( [F(\ell^*) - F'(\ell^*)\ell^*]M_1(0, L) = \Phi \). We can now examine how the economy behaves with low versus high values of \( X \) for a more general specification of the matching function. This is given by Proposition 8.

**Proposition 8.** For any matching function satisfying Assumption 1, if \( F(\ell) = A\ell^\alpha \) then there exist \( \hat{X} \) and \( \hat{\hat{X}} \) satisfying \( \hat{X} \leq \hat{\hat{X}} < X_{\text{max}} \), such that equilibrium outcomes are characterized by

1. If \( X < \hat{X} \), then there is not deficient demand and \( \partial c/\partial X \geq 0 \).
2. If \( X \in (\hat{\hat{X}}, X_{\text{max}}) \), then there is deficient demand and \( \partial c/\partial X < 0 \).

When the production function is of the constant-elasticity form \( F(\ell) = A\ell^\alpha \), Proposition 8 generalizes results of Section 2 by indicating that, for our class of matching functions, the behavior of our model economy will again differ depending on whether the economy inherits a small or a large amount of goods from the past. In particular, the proposition states that, for large values of \( X \), the economy will again exhibit deficient demand – in the sense that a coordinated increase in \( c \) would increase welfare – and that in such a region the economy acts rather perversely with \( \partial c/\partial X < 0 \). In contrast, the economy would not exhibit deficient demand or act perversely if \( X \) were small.

The main differences between Proposition 8 and those of Section 2 is that, with the “min” matching function, two regions spanned all possible values of \( X < X_{\text{max}} \). However, this is not the case with Proposition 8. Implicit in Proposition 8 is the possible existence of a third region between \( \hat{X} \) and \( \hat{\hat{X}} \) where properties may be mixed. We have not been able to exclude the possibility of such a third region for this general class of matching functions. However, for most parametric examples we have been able to find simple sufficient conditions that guarantee the simple dichotomy, so that this third potential region is in fact empty (or, in other words, that \( \hat{X} = \hat{\hat{X}} \)). For example, if we assume that \( \nu \) is of the constant-elasticity form \( \nu(\ell) = \ell^{1+\omega} \) and that the matching function is of the ball-urn type, then the third region is empty. A second interesting example is the case where the matching function is CES with

\[ 26 \text{ We are again assuming that } \tau \text{ is sufficiently small to guarantee a unique equilibrium.} \]

\[ 25 \text{ More generally, this rules out any matching function of the CES form with elasticity of substitution greater than or equal to one.} \]
elasticity of substitution strictly less than one, i.e., where \( M(N, L) = (N^{-\gamma} + L^{-\gamma})^{-\frac{1}{\gamma}} \) with \( \gamma > 0 \). If \( \gamma > 1 \), it can be verified that this matching function satisfies Assumption 1, and further that our simple dichotomy (where \( X = \hat{X} \)) also holds.\(^{28}\) Although the ball-urn and CES matching functions are special parametric cases, these examples nicely illustrate that many of the results obtained using the simpler “min” matching function are not knife-edge, as they carry over to these alternate cases.\(^{29}\)

### 3.2 Changing the search and bargaining protocol

In this section, we return to the “min” specification of the matching function and examine how results are modified when we maintain random matching but change the bargaining protocol to Nash bargaining. We then discuss the implications of adopting directed search. In the remaining sections of the paper, we will simplify notation by setting \( \Lambda = 1 \) and thereby have \( M(N, L) = \min\{N, L\} \). This implies that a tight labor market will be characterized by full employment and a slack labor market with be characterized by unemployment. As should be clear from our previous analysis, the important distinction between the two regimes is not that the presence or absence of unemployment, but it is instead the fact that in a slack labor market \( (N < L) \) workers experience congestion effects while in a tight labor market \( (N > L) \) it is the firms that generate congestion effects.

#### 3.2.1 Nash bargaining

In our baseline model we assumed that, upon a successful search, wages and employment were determined by a Walrasian protocol in the spirit of that used by Lucas and Prescott (1974). This protocol gave rise to two results. First it implied that hours-worked satisfies a pair-wise efficient condition given by

\[
pF'(\ell) = \frac{\nu'(\ell)}{v}
\]

and second that the wage is equal to the marginal dis-utility of work,\(^{30}\) that is,\(w = \frac{\nu'(\ell)}{v}\)

As show in Appendix B, under Nash bargaining the within-pair efficiency condition \( pF'(\ell) = \nu'(\ell)/v \) remains. However, the determination of wages changes. In particular, under Nash bargaining the wage is given by

\[
w = \frac{\nu(\ell) - \tau p(c - X) + s [pF(\ell) - \nu(\ell) + \tau p(c - X)]}{l}, \quad 0 \leq s < 0
\]

\(^{27}\) This matching function was used in den Haan, Ramey, and Watson \([2000]\)

\(^{28}\) This result is based on maintaining the functional form assumption \( \nu(\ell) = \ell^{1+\omega} \).

\(^{29}\) The urn-ball function and CES function with \( \gamma > 1 \) belong to a more general class of matching functions for which we may obtain a simple sufficient condition to ensure the simple dichotomy when \( \nu(\ell) = \ell^{1+\omega} \). In particular, let \( \eta(N, L) \equiv N/M(N, L) \) be the inverse of the firm matching rate, and suppose \( \eta \) is convex in \( N \) (as is the case for the above functions). Then it may be verified that a sufficient (but not necessary) condition for Proposition 8 to hold with \( \hat{X} = \hat{X} \) is that \( \alpha \geq (1 + \omega)/2 \).

\(^{30}\) We could alternatively say here that wages are given by the marginal product condition \( w = pF'(\ell) \).
where \( s \) reflects the share of the match surplus that is attributed to the worker (an additional parameter). The total wage payment now reflects the reservation utility of the worker, which is given by \( \nu(\ell) - \tau p(c - X) \), plus a share of the match surplus, which is given by \( pF(\ell) - \nu(\ell) + \tau p(c - X) \). An important implication of this is that the wage is now decreasing in \( c \). If a worker enters a match with a greater \( c \), he is in a less desirable bargaining position since, if negotiations were to break down, the worker would be left with costly debt. This causes the worker to settle for a lower wage when he has committed to a high level of consumption. Our baseline formulation ruled out this mechanism in the determination of wages. As we shall see, this mechanism will tend to amplify a number of our previous results, since it will imply that \( p \) will be a decreasing function of \( c \), which will in turn cause the cost of funds schedule (plotted in Figure 6) to have an even more negative slope in the unemployment regime.

With Nash bargaining, the equilibrium determination of \( c, N \) and \( \ell \) reduces to the solution to equations (7), (8) and a new zero-profit condition given by (26), where we have used the new wage equation (25).\(^{31}\)

\[
(1 - s) \min\{N, L\} \left[ F(\ell) - \frac{\nu(\ell)F'(\ell)}{\nu'(\ell)} + \tau(c - X) \right] = \Phi \tag{26}
\]

Proposition 9 states that under Nash Bargaining we get a similar characterization of equilibrium outcomes as that obtained in our baseline model.\(^{32}\)

**Proposition 9.** When wages and hours worked are determined by Nash bargaining, there again exists an \( X^* \) and an \( X^{**} > X^* \) such that

1. If \( X < X^* \), then the labor market will be tight \((N > L)\), there is not deficient demand, and \( \frac{\partial c}{\partial X} \geq 0 \).
2. If \( X \in (X^*, X^{**}) \), then the labor market will be slack \((N \leq L)\), there is deficient demand, and \( \frac{\partial c}{\partial X} < 0 \).
3. If \( X \geq X^{**} \), then \( c = X \)

Although Proposition 9 indicates that many equilibrium properties remain unchanged as we switch from our baseline bargaining protocol to Nash bargaining, this does not imply that the equilibrium values of \( c \) and \( \ell \) do not change. For example, in the slack regime of our baseline model, the equilibrium had a recursive structure. The zero-profit condition determined \( \ell \), and hence the price \( p \), and then the optimal consumption decision was determined by the condition \( U''(c) = \frac{\nu'(\ell)F'(\ell)}{F(\ell)} \left[ 1 + \tau - \frac{c - X}{F(\ell) - \Phi} \right] \). Accordingly, in our baseline setup, the price of goods and hours worked did not vary as we changed \( X \) when the labor market was slack. In the

\(^{31}\) We will assume that the function \( sF'(\ell) + (1 - s)\frac{\nu(\ell)F'(\ell)}{\nu'(\ell)} \) is always increasing in \( \ell \), as can be easily verified to be the case under standard functional forms.

\(^{32}\) We have also derived sufficient conditions for an increase in \( X \) to lead to a decrease in welfare in the presence of Nash Bargaining. However, the expressions are rather complicated and not very informative, so we have omitted them here. Using numerical simulations, we have found it rather easy to find regions in the unemployment regime where an increase in \( X \) leads to a decrease in welfare.
case of Nash bargaining this recursive property is lost. The optimal consumption decision and the zero-profit condition must be solved jointly for $\ell$ and $c$, which in turn implies that the price also changes as $X$ changes. These effects are characterized in Proposition 10.

**Proposition 10.** If wages and hours-worked are determined by Nash bargaining, then

1. When the labor market is tight (i.e., for $X < X^*$), $\frac{\partial p}{\partial X} < 0$ and $\frac{\partial \ell}{\partial X} < 0$.

2. When the labor market is slack (i.e., for $X \in (X^*, X^{**})$), $\frac{\partial p}{\partial X} > 0$ and $\frac{\partial \ell}{\partial X} > 0$.

Propositions 9 and 10 together indicate that, when the labor market is slack, an increase in $X$ will lead to lower consumption and higher prices. The reason for this is that, as one cuts back on consumption due to higher $X$, the worker’s bargaining position improves, which puts downward pressure on firm profits. In order to maintain zero expected profits, matched firms must then hire more hours of labor, which in turn results in higher prices. This contrasts with our baseline model where an increase in $X$ in a slack market led to lower consumption at an unchanged price. The extra mechanism induced by Nash bargaining can therefore be seen as increasing the strength of the complementarity between the consumption decisions of the households in comparison to our baseline setup. Note that, in the slack regime, the consumption decision for household $j$ satisfies the relationship $U'(c_j) = p(1 + \tau - \tau \frac{c - X}{\Phi})$, where $c$ is the average consumption level of other agents. Recall that this condition holds regardless of whether we have Nash bargaining or the Walrasian protocol of our baseline model. In the slack regime of our baseline model, an exogenous increase in $c$ would not change $p$ or $\ell$, so from the household’s optimal consumption decision we can see easily that the consumption of other agents acts as a complement to one’s own consumption. In the case of Nash bargaining, this complementarity becomes even stronger, as in addition to the direct effects of others’ consumption on the probability of employment, when the consumption of other agents increases that tends to decrease the price and lower hours worked, and hence further increases one’s desire to consume. Since this additional mechanism is rather subtle, we opted to focus on the simpler and more direct mechanism in the baseline model, leaving us to clarify this additional channel here.

To conclude this section, we present in Figure 7 the behavior of consumption and welfare as a function of $X$ in the case of Nash bargaining. The parameters used in this example are similar to those used in Figures 1 and 3 for our baseline model. As can be seen, consumption and welfare are both increasing until $X$ reaches $X^*$, after which they begin to decline as the economy enters a region where the labor market is slack.

### 3.2.2 Directed search

Up to now, we have focused on environments where search is done in a random fashion. In this section, we explore how our results would change if we allowed for directed search. In particular, we examine whether the emergence of deficient demand when $X$ is high and the property that $\frac{\partial c}{\partial X} < 0$ in such cases are driven by the assumption of random search, or whether they are robust to allowing for directed search.

---

33 Directed search is also known as competitive search, see Moen [1997].
In the case of directed search, we can view the household’s problem as simultaneously choosing both a consumption level and a particular labor market in which to search for a job. There is a potential continuum of job markets, each specified as a triple composed of a wage, a number of hours worked, and a tightness level, where tightness level $\theta \equiv N/L$ translates into a job-finding rate for workers of $M(\theta, 1)$. The potential job markets available to households in equilibrium are all of the triples of characteristics $w$, $\ell$ and $\theta$ that leave firms with zero profits. The equilibrium outcome for this economy therefore maximizes household utility subject to the firm’s zero-profit condition (taking the price $p$ as given); that is, it solves

$$\max_{c, w, \ell, \theta} U(c) + M(\theta, 1) \left[ -\nu(\ell) + V(w\ell - p(c - X)) \right] + \left[ 1 - M(\theta, 1) \right] V(-p(c - X))$$

subject to

$$M \left( 1, \theta^{-1} \right) \left[ F(\ell) - \frac{w}{p} \ell \right] = \Phi$$

Maintaining the usual assumption on the form of $V(\cdot)$, solving this maximization problem yields the now-familiar conditions $U'(c) = pv(1 + \tau - \frac{M(N, L)}{L} \tau)$, $pF'(\ell) = \frac{\nu'(\ell)}{\nu}$, and $\frac{M(N, L)}{N} \left[ F(\ell) - \frac{w}{p} \ell \right] = \Phi$. The only new condition is the wage-determination equation, which is now given by

$$w = \frac{\nu(\ell) - \tau p(c - X) + \xi \left[ pF(\ell) - \nu(\ell) + \tau p(c - X) \right]}{\ell}$$

(27)

where $\xi$ is the elasticity of the matching function with respect to $L$, as given by $\xi = \frac{\Phi_2(N, L)}{\Phi(N, L)}$. 

Note: Solid line is consumption (left axis). Dash-dot line is welfare (right axis). Example is constructed assuming the functional forms $U(c) = \log(c)$, $\nu(\ell) = \nu\ell^{1+\omega}$ and $F(\ell) = Ae^{\alpha}$, with parameters $\omega = 1$, $\nu = 0.5$, $\alpha = 0.67$, $A = 1$, $\Phi = 0.35$, $\Lambda = 1$, $\tau = 0.1$ and $s = 0.5$. 

Figure 7: Consumption and welfare as functions of $X$ (Nash bargaining)
The set of equilibrium conditions is then completed with the usual market-clearing condition for the goods market, \( M(N, L)F(\ell) = L(c - X) + N\Phi \).

If we combine the wage determination equation (27) with the zero-profit condition, we get

\[
(1 - \xi) \frac{M(N, L)}{N} \left[ F(\ell) - \frac{\nu(\ell)F'(\ell)}{\nu'(\ell)} + \tau(c - X) \right] = \Phi
\]

(28)

This zero-profit condition is identical to the one obtained under Nash bargaining (equation (26)), except the fixed bargaining power \( s \) from the Nash bargaining setup has been replaced by the elasticity of the matching function \( \xi \), which is a standard result in the case of directed search. If we assume that the matching function is once again of the “min” form, we can derive similar results to those obtained for the random-search Nash bargaining case, as stated in Proposition 11.

**Proposition 11.** The properties stated in Proposition 9 also hold under directed search.

Proposition 11 indicates that, under directed search, the economy will again have a tendency to exhibit deficient demand and behave perversely when \( X \) is high, while this will not be the case if \( X \) is low.

### 3.3 Justifying the absence of unemployment insurance and formulating the social planner’s problem

In our analysis thus far, we have assumed that agents do not have access to unemployment insurance. It may be thought that allowing for the private provision of unemployment insurance would necessarily eliminate the mechanisms we have highlighted. For this reason, in this subsection we want to indicate how our analysis can be extended to allow for the private provision of unemployment insurance, but where the provision of this insurance will be constrained by an adverse selection problem. Once we have presented this adverse selection problem, we can then examine the more interesting question of how a social planner would allocate resources in an economy subject to two frictions: a search friction which creates unemployment, and an information friction that limits insurance. The solution to this social planner’s problem will then be used to clarify the fundamental nature of the inefficiency that arises in the de-centralized case.

#### 3.3.1 Adverse selection as a constraint on unemployment insurance

To explore the role of adverse selection, we return to our baseline model of Section 2, but now suppose that only a fraction \( \rho \) of households behave as the households we modeled in that Section. We will refer to these households as participant households. Suppose the remaining \((1 - \rho)\) fraction of households, which we call the non-participant households, do not value consumption of the first-period good and are unwilling to work at the market wage, but value consumption in the second sub-period in exactly the same way as the participant households. Now suppose that some private insurer wanted to offer unemployment insurance before the matching process, but could not differentiate between the two types of households.
In this case, the insurer will not be able to offer insurance contracts that are only attractive to participant households, because any contract with a positive net payment to unemployed individuals will be desirable to non-participants. Therefore, as indicated in Proposition 12, as long as $1 - \rho$ is sufficiently high this type of adverse selection problem implies that the only equilibrium outcome is one where no insurance is offered. Accordingly, in this setup, the mechanisms we have emphasized regarding the economy’s behavior when unemployment insurance is assumed not to exist will apply equally in an environment where the private provision of unemployment is allowed but is constrained by an adverse selection problem.

**Proposition 12.** *In the presence of both participant households and non-participant households, if $1 - \rho \geq \frac{1}{1 + \tau}$, i.e., if the fraction of non-participant households is sufficiently high, then no unemployment-insurance contracts are traded in equilibrium.*

### 3.3.2 The constrained social planner’s problem

In this section we want to show how a social planner would allocate resources in our environment if it simultaneously faces the search friction and the adverse selection problem. We will take the goal of the social planner to be the maximization of utility of participant households subject to the constraint that it offers contracts to participant households that are not attractive to non-participant households. We implicitly assume here that the social planner cannot distinguish between the two types of agents, and that non-participant households are sufficiently important in number that the social planner does not want to include them in any transfer scheme where they would be pure beneficiaries. We view the planner as offering a contract specifying four elements: an amount of the first-sub-period good, $e$; a number of hours worked conditional on finding employment, $l$; an amount of second-sub-period goods to receive if one is employed in the first sub-period, $d^e$; and an amount of second-sub-period goods that must be produced if one is unemployed in the first sub-period, $d^u$. In the absence of the adverse selection problem, the social planner problem would choose these elements, plus the number of firms to enter the market, by solving

$$
\max_{e, l, N, d^e, d^u} U(X + e) + \frac{M(N, L)}{L} [vd^e - \nu(\ell)] - \left[ 1 - \frac{M(N, L)}{L} \right] v(1 + \tau)d^u
$$

subject to the resource constraint in the first sub-period $M(N, L)F(\ell) = Le + NK$ and the resource constraint in the second sub-period $\frac{M(N, L)}{L}d^e = [1 - \frac{M(N, L)}{L}]d^u$. It is easy to verify that the solution to this problem involves $d^e = d^u$, that is, household would not bear any risk associated with not finding a job. The problem with this solution is that a non-participant household would in general want to participate in this scheme by accepting the offered first-sub-period goods, and then trading these goods to participant households in return for promises of second-sub-period goods. In fact, a non-participant households will want to take part in any scheme offered by the planner if it can manage to end up with a positive amount of second-sub-period goods (after deducting $d^u$, the number of goods transferred to the planner) by making such trades. Recognizing this, the social planner will be constrained
to choose values of \(e\) and \(d^u\) that will be unattractive to non-participant households.\(^{34}\) For this to be the case, it must be that a non-participant household cannot, by trading away all \(e\) of their first-sub-period good, obtain enough second-subperiod goods so that they are left with a positive amount after paying the required \(d^u\) to the planner. It can be verified that the relevant incentive compatibility constraint is given by\(^{35}\)

\[
\frac{U'(x + e)}{1 + \tau - \frac{M(N,L)}{L}} e \leq d^u \tag{29}
\]

On the left-hand side of this constraint is the number of second-sub-period goods that a non-participant could obtain by selling all of their first-sub-period goods. Here, \(U'(x + e)\) is the expected marginal value to participant households of first-sub-period goods, expressed in terms of second-sub-period goods. Accordingly, a non-participant household would accept any contract offered by the planner that satisfies \(d^u < \frac{eU'(x + e)}{1 + \tau - \frac{M(N,L)}{L}}\), since that household could guarantee itself a positive amount of second-sub-period goods after repaying the amount \(d^u\) agreed upon in the contract.

Once the constraint (29) is taken into account, the solution to the social planner’s problem is given by the two resource constraints, the incentive compatibility condition at equality (since it always binds), plus the two new conditions

\[
U'(X + e) = \frac{\nu'(\ell)}{F'(\ell)} \cdot \frac{1 + \tau - \frac{M(N,L)}{L}}{1 - \frac{M(N,L)}{L}} \cdot \frac{-U''(X + e)}{U'(X + e)} e \tau \tag{30}
\]

\[
(1 - \xi) \frac{M(N,L)}{N} \left\{ F(\ell) - \frac{\nu'(\ell) F'(\ell)}{\nu'(\ell)} + \left\{ 1 + \frac{\tau e}{1 - \frac{M(N,L)}{L}} \frac{-U''(X + e)}{U'(X + e)} e \tau \right\} \right\} = \\
\Phi + (1 - \xi) \frac{M(N,L)}{N} \frac{\tau^2 e}{1 + \tau} \left[ 1 - \frac{M(N,L)}{L} \right] \frac{1}{1 + \frac{\tau e}{1 - \frac{M(N,L)}{L}} \frac{-U''(X + e)}{U'(X + e)} e \tau} \tag{31}
\]

where, as before, \(\xi\) is the elasticity of the matching function with respect to \(L\). Equation (30) can be interpreted as the socially optimal condition for the determination of expenditures, while equation (31) can be interpreted as the socially optimal zero-profit condition. It is interesting to compare these two conditions to those we derived for a decentralized economy. One can see that, for all cases we considered, the decentralized conditions determining expenditures and entry decisions differ from the solution to the planner’s problem as long as \(\tau > 0\). The environment that more closely resembles the social planner’s solution is

\(^{34}\) Since they never find jobs, non-participant households do not care about the levels of \(\ell\) or \(d^c\) specified in the contract.

\(^{35}\) We assume that any trade between households occurs before the resolution of unemployment uncertainty in the first sub-period. Further, we assume that the realized employment state of one household cannot be verified by another, so that any promise of second-sub-period goods made by a household in the first sub-period cannot be contingent on the realized state of their employment.
the decentralized economy with directed search. Accordingly, we will focus our comparison here between directed search and the social optimum, noting that much of what we say also extends to our baseline analysis and to the case with Nash bargaining.

In comparison to the solution under directed search, the social planner would want to encourage more expenditure by households in the first sub-period while simultaneously limiting firm entry. This can be seen by the fact that the term in the denominator on the right-hand side of equation (30) is greater than one, and by the fact that the last term in equation (31) is positive. The social planner could easily implement his preferred outcome in a directed-search environment by the use of a subsidy on the purchase of first-sub-period goods, and by a tax on entry. If we go a step further and focus on the case where the matching function is of the “min” form, the we can see that the social planner’s solution and the decentralized solution with directed search actually become identical in the case of a tight labor market. However, they continue to depart when the labor market is slack, which will happen for high values of $X$. This comparison highlights that economic activity will be inefficiently low in the directed search environment when $X$ is high, but not when it is low. In other words, if the value of trading in the market is high, as is the case when $X$ is low, the decentralized economy can overcome the frictions and trade at the socially efficient level. However, when the gains from trade between individuals are rather low, as is the case when $X$ is high, the decentralized outcome under directed search will be inefficient relative to the constrained social optimum.

The reason that the social planner’s problem and the directed-search equilibrium do not coincide is due to a pecuniary externality. When agents are deciding how much to consume in the decentralized environment, they do not take into account the effect of their consumption on employment and prices. They do not recognize that, by consuming more, they would reduce wage demands, thereby favoring lower prices and more production. Since the extra production is socially desirable, as the economy tends to be in a situation of deficient demand with high $X$, it would be in the interest of agents to coordinate action by consuming more and favoring more entry.

## 4 Dynamics

In this section we want to explore a dynamic extension of our static durable-goods model where current consumption contributes to the accumulation of $X$. In particular, we want to consider the case where the accumulation of $X$ obeys the accumulation equation

\[ X_{t+1} = (1 - \delta)X_t + \gamma e_t \quad 0 < \delta \leq 1, \quad 0 < \gamma \leq 1 - \delta \]

(32)

where the parameter $\gamma$ represents the fraction of current consumption expenditures, $e_t = c_t - X_t$, which take the form of durable goods. Since we do not want to allow heterogeneity between individuals to expand over time, we will allow individuals to borrow and lend only within a period but not across periods; in other words, households are allowed to spend

---

36 In order to implement the social optimum, the subsidy would need to depend on the value of $X$.

37 The budget can be balanced by imposing a lump-sum tax on the employed as needed.

38 With the matching function of the “min” form, the free entry condition implies that $N = L$. 

31
more than their income in the first sub-period of a period, but must repay any resulting
debt in the second sub-period. The problem facing a household in the first sub-period
of a period is therefore to choose how much clothing to buy and, conditional on a match,
how much labor to supply. We model the second sub-period as in sub-section 2.2, where
households use labor to produce household services either for their own consumption or,
at a level of productivity that is lower by a factor $1 + \tau$, for the consumption of others. In
each second sub-period, then, the household chooses how much to consume of household
services and how much to produce of household services to both satisfy his needs and to pay
back any accumulated debt. In order to keep the model very tractable, we will continue to
assume that dis-utility of work in the second sub-period is linear (i.e., equal to $v \cdot \ell$). Under
this assumption, all households will choose the same level of consumption of household
services in each second sub-period, while the production of household services will vary
across households depending on whether they entered the sub-period in debt or in surplus.
Since there are no interesting equilibrium interactions in second sub-periods, we can maintain
most of our focus on equilibrium outcomes in the sequence of first sub-periods.

Relative to the static case, the only difference in equilibrium relationships (aside from
the addition of the accumulation equation (32)) is that the first-order condition associated
with the households’ choice of consumption of clothes is now given by the Euler equation

$$U'(X_t + e_t) - Q(e_t) = \beta [(1 - \delta - \gamma)U'(X_{t+1} + e_{t+1}) - (1 - \delta)Q(e_{t+1})]$$

(33)

where $Q$ is as defined in equation (14). In this dynamic setting, an equilibrium will be repre-
sented as a sequence of the previous equilibrium conditions (8) to (11) plus the accumulation
equation (32) and the Euler equation (33).

There are many complications that arise in the dynamic version of this model, which
makes characterizing equilibrium behavior difficult. In particular, there can be multiple
equilibrium paths and multiple steady-state solutions. Luckily, the problem can be simplified
if we focus on cases where $\delta$ is small; that is, on cases where the durability of goods is long.
In addition to simplifying the analysis, focusing on the low-$\delta$ case appears reasonable to us,
as many consumer durables are long-lived, especially if we include housing in that category.
In the case where $\delta$ is sufficiently small, as stated in Proposition 13, the economy will have
only one steady state and that steady state will have the property that the labor market will
be slack.

**Proposition 13.** If $\delta$ is sufficiently small, then the model has a unique steady state and this
steady state is characterized by a slack labor market ($N < L$).

Proposition 13 is very useful, as it will allow us to analyze the equilibrium behavior
around the steady state without worrying about equilibrium selection. Accordingly, for the
remainder of this section, we will assume that $\delta$ is sufficiently small so that Proposition 13
applies. However, before examining local properties in some generality, we believe that it is
helpful to first illustrate global equilibrium behavior for a simple case that builds directly
on our static analysis. The reason that we want to illustrate global behavior for at least

---

39 This lack of borrowing across periods can be rationalized if one assumes that the transaction cost of
intermediating loans across periods is greater than $1 + \tau$. 

32
one example is to emphasize that local behavior in our setup is likely to differ substantially and meaningfully from global behavior. Moreover, the example will allow us to gain some intuition on how the latter local results should best be interpreted.

Before discussing the transitional dynamics of the model, we first briefly discuss the conditions under which the model would exhibit a balanced growth path. In particular, suppose production in the first sub-periods is given by \( \theta_t F(\ell_t) \) where \( \theta_t \) is a technology index that is assumed to grow at a rate \( g_{\theta} \). Then it is easy to verify that our economy will admit an equilibrium growth path where both \( e \) and \( X \) grow at rate \( g_{\theta} \) if the following three conditions are satisfied: (i) the fixed cost of creating jobs grows at rate \( g_{\theta} \), (ii) the productivity of labor in the second sub-periods grows at rate \( g_{\theta} \), and (iii) the utility of consumption is represented by the log function. These conditions are not surprising, as they parallel those needed for a balanced growth path in many common macro models. The important aspect to note about this balanced-growth property is that the notion of high or low levels of capital should be interpreted as relative to the balanced growth path. In other words, the key endogenous state variable in the system should be viewed as the ratio of \( X_t \) to the growth component of \( \theta_t \).

4.1 Global dynamics for a simple case

The difficulty in analyzing the global dynamics for our model is related to the issue of multiple equilibria we discussed in the static setting. If the static setting exhibits multiple equilibria then the dynamic setting will likely exhibit multiple equilibrium paths. To see this, it is useful to recognize that our problem of describing equilibrium paths can be reduced to finding the household’s decision rule for consumption. Since the only state variable in the system is \( X_t \), the household’s decision rule for consumption will likely be representable by a relationship (which may be stochastic) of the form \( c(X_t) \). Given \( c(X_t) \), the equilibrium dynamics of the system are given by

\[
X_{t+1} = (1 - \delta - \gamma)X_t + \gamma c(X_t)
\]  

If the relationship \( c(X_t) \) is a function, then equilibrium dynamics are deterministic. However, if we consider the case with \( \beta = 0 \) – so that households are not forward-looking and thus the dynamic equilibrium is simply a sequence of static equilibria – we already know that the household’s decision rule \( c(X_t) \) may not be a function. For example, if \( \tau > \bar{\tau} \), then the household’s decision rule may be a correspondence of the form given in Figure 5. Therefore, even for the rather simple case where \( \beta = 0 \) and \( \tau > \bar{\tau} \) we know that the equilibrium dynamics need not be unique, in which case some equilibrium-selection device will be needed to solve the model. In contrast, for the case where \( \beta = 0 \) and \( \tau < \bar{\tau} \), then we know from Proposition 3 that \( c(X_t) \) is a function. Hence, in the case where \( \beta = 0 \) and \( \tau < \bar{\tau} \), we can describe the global dynamics of the system rather easily, and this is what we will do in this section. In particular, when \( \beta = 0 \) and \( \tau < \bar{\tau} \), the stock of durables evolves according to equation (34), with \( c(X_t) \) given by the value of \( c \) obtained using Proposition 3 with \( X_t \) in place of \( X \).

Figure 8 plots the equilibrium transition function for \( X \) for three cases; that is, it plots \((1 - \delta - \gamma)X_t + \gamma c(X_t)\) for different possible \( c(X_t) \) functions. The figure is drawn so that the
steady state is characterized by a slack labor market, which is consistent with a low value of $\delta$ as implied by Proposition 13. As can be seen from the figure, when $X_t$ is not too great

![Figure 8: $X_{t+1}$ as a function of $X_t$](image)

Note: Figure shows transition functions $X_{t+1}(X_t)$ for three different decision rules $c(X_t)$ that all yield the same steady state. Decision rules are identical when the economy features either full employment or zero employment, and differ when the economy features partial unemployment. Legend entries refer to value of $c'(X_t)$ in partial-unemployment regime.

(i.e., less than $X^*$) the economy is in the full-employment regime and $X_{t+1} > X_t$. So if the economy starts with a low value of $X_t$ it will generally go through a phase with a tight labor market. During this phase, we know from Proposition 3 that consumption is also increasing. Eventually, $X_t$ will exceed $X^*$ and the economy enters the unemployment regime, at which point the dynamics depend on the derivative of the equilibrium decision rule, i.e., $c'(X_t)$, where in this regime $c(X_t)$ solves

$$U''(c) = \frac{\nu' (\ell^*)}{F' (\ell^*)} \left( 1 + \tau - \tau \frac{c - X_t}{e^*} \right)$$

If $-c'(X_t) < \frac{1 - \delta - \gamma}{\tau}$ when $X^* < X_t < X^{**}$, then the transition function maintains a positive slope near the steady state and the economy will converge monotonically to its steady state. However, note that even if $X$ converges monotonically to its steady state in such a case, this will not be the case for consumption. Again, from Proposition 3 we know that consumption is decreasing in $X$ in the unemployment region. Hence, starting from $X = 0$, in this case
consumption would initially increase, reaching a maximum just as the economy enters the slack regime, then decline towards its eventual steady-state level which is lower than the peak obtained during the transition. If instead $-c'(X_t) > \frac{1-\delta-\gamma}{\gamma}$, then the transition function for $X$ will exhibit a negative slope in the slack regime. In this case, $X$ will no longer converge monotonically to the steady state. In fact, if the slope of this function (which depends on the elasticity of $c$ with respect to $X$ at the steady state) is negative but greater than -1, the system will converge with oscillations. However, if this slope is smaller than -1, which can arise for very large negative values of $c'(X_t)$, then the system will not converge and instead can exhibit rich dynamics, including cycles and chaos. In general, however, even in the case where $c'(X_t)$ is very negative, the system will not necessarily be explosive, since once it moves sufficiently far away from the steady state, forces kick in that work to push it back.

There are two main messages to take away from exploring the global dynamics in this special case with $\beta = 0$. First, the behavior of the state variable $X$ can be well-behaved, exhibiting monotonic convergence throughout. Second, the behavior of consumption (and therefore possibly welfare), can nonetheless exhibit interesting non-monotonic dynamics, with steady-state consumption actually being below the highest level it achieved during the transition. It is worth noting that if the steady state were to be in the tight labor market regime (due, for example, to a higher $\delta$), then from $X = 0$ both $X_t$ and $c_t$ would always converge monotonically to the steady state when $\beta = 0$ and $\tau < \bar{\tau}$.

The most interesting aspect about the global dynamics in this case is that it allows us to illustrate the following possibility: If the economy is near its steady state, then a small reduction in $X_t$ will increase consumption and can potentially increase welfare, while a large decrease in $X_t$ will certainly decrease welfare. In this sense, the model exhibits behavior around the steady state that can differ substantially from behavior far away from the steady state, with the behavior far away from the steady state being more akin to that generally associated with classical economics, while behavior in the unemployment regime being more similar to that suggested by a Keynesian perspective.\footnote{It is worth noting that this type of synthesis, which emphasizes differences between being near to the steady state versus far from the steady state, is substantially different from the new neo-classical synthesis, which emphasizes differences in the long run and the short run because of sticky prices.}

### 4.2 Local dynamics in the general case

In this subsection, we explore the local dynamics of the general model when $\beta > 0$, still assuming that $\delta$ is sufficiently small so that the steady state is unique and in the slack regime. From our analysis of the case with $\beta = 0$, we know that local dynamics can exhibit convergence or divergence depending on how responsive consumption is to $X$ around the steady state. The one question we could not address when $\beta = 0$ is whether dynamics could exhibit local indeterminacy. In other words, can forward-looking behavior give rise to an additional potential local source of multiple equilibria in our setup? Proposition 14 indicates that this is not possible; that is, the roots of the system around the unique steady state can not both be smaller than one.\footnote{In this section we only consider local dynamics around a unique unemployment-regime steady state. Nonetheless, it is straightforward to show that if the unique steady state is in the full-employment regime,}
Proposition 14. The local dynamics around the steady state can either exhibit monotonic convergence in \( c \) and \( X \), convergence with oscillations, or divergence. Locally indeterminacy is not possible.

Proposition 14 is useful as it tells us that the decision rule for consumption around the steady state is a function.\(^{42}\) Accordingly, we can now examine the sign of the derivative of this function. The question we want to examine is whether the decision rule for consumption around the steady state has the property that a larger \( X \) leads to a lower level of consumption, as was the case in our static model when in the unemployment regime. In other words, we want to know whether the results regarding the effect of \( X \) on consumption we derived for the static model extend to the steady state of the dynamic setting with \( \beta > 0 \). Proposition 15 indicates that if \( \tau \) is not too large, then local dynamics will exhibit this property. Note that the condition on \( \tau \) is a sufficient condition only.

Proposition 15. If \( \tau \) is sufficiently small, then in a neighborhood of the unique steady state, consumption is decreasing in \( X \), with the dynamics for \( X \) converging monotonically to the steady state.

From Proposition 15 we now know that, as long as \( \tau \) is not too big, our model has the property that when the economy has over-accumulated relative to the steady state (i.e., if \( X \) slightly exceeds its steady-state value), then consumption will be lower than in the steady state throughout the transition period toward the steady state, which we can refer to as a period of liquidation. In this sense, the economy is overreacting to its inherited excess of capital goods during this liquidation period, since it is reducing its expenditures to such an extent that people are consuming less even though there are more goods available to them in the economy. While such a response is not socially optimal, it remains unclear whether it is so excessive as to make people worse off in comparison to the steady state, since they are also working less during the liquidation phase. It turns out that, as in the static case, the welfare effect of such a liquidation period depends, among other things, on whether the average dis-utility of work is small enough relative to the marginal dis-utility. For example, if the average dis-utility of work is sufficiently low relative to its marginal value, then it can be verified that a liquidation period induced by inheriting an excess of \( X \) relative to the steady state will make average utility in all periods of the transition lower than the steady state level of utility. This result depends in addition on the unemployment rate not being too large in the steady state.

While we do not have a simple characterization of the global dynamics when \( \beta > 0 \), Propositions 14 and 15 suggest to us that the intuition we gained from the case where \( \beta = 0 \) likely extends to the more general problem as long as \( \tau \) is not too large and \( \delta \) is small. In particular, we take our analysis as suggesting that, starting from \( X = 0 \), the economy will generally go though a phase with a tight labor market, with both \( X \) and \( c \) increasing over time. The economy then enters into a range with a slack labor market once \( X \) is large enough. Then, as long as \( \tau \) is not too great, \( X \) will continue to monotonically increase, converging

---

42 This is a slight abuse of language since Proposition 14 does not rule out the existence of other equilibrium paths away from the steady state.
toward its steady state. In contrast to $X$, upon entering the slack regime, consumption starts to decrease as unemployment risk leads to precautionary savings which depresses activity. Eventually, the economy will reach a steady state where consumption, employment, and possibly period welfare are below the peak levels reached during the transition.

In the above discussion of liquidation, we have taken the level of inherited capital as given and have only examined how the economy responds over time to a situation where $X$ is initially above its steady state. In particular, we have shown that such a liquidation phase can be associated with excessively low consumption, low welfare and high unemployment, all relative to their steady state values. While the focus of the paper is precisely to understand behavior during such a liquidation phase, it nonetheless remains interesting to ask how welfare would behave if we were to view the whole cycle, both the over-accumulation phase and the liquidation phase together. To briefly examine this issue, we build on the news-noise literature and consider a case where agents in an economy start at a steady state and then receive information about productivity.\textsuperscript{43} Agents have to make their consumption decision based on the news, and we assume that they subsequently learn that the news is false. This leads to an initial high level of consumption during the period where agents are optimistic, followed by a period of low consumption during the liquidation phase after realizing that they had mistakenly over-accumulated. Details of this extension are presented in Appendix D.

In Figure 9 we report for illustration purposes two impulse responses associated with a simple calibration of such a noise-driven-boom-followed-by-liquidation model. We plot the dynamics for the stock of durables and the average period utility of households relative to the steady state. From the figure, we see that during the first period, when agents are acting on optimistic beliefs about productivity, their period welfare increases even if they are working hard to ramp up their stocks of durable goods. After one period, they realize their error since productivity has not actually improved, and consequently cut back on their expenditures to start a liquidation process. The welfare of households from the second period on is lower than in steady state because of the excessively cautious behavior of households, which stops the economy from taking advantage of the excessively high inherited capital stock.

It is interesting to contrast this path with that which would happen if unemployment risk were perfectly insured or if matching frictions were absent. In such a case, the news would still lead to a boom, and the realization of the error would lead to a recession. However, the dynamics of period welfare would be very different. Instead of the boom being associated with high period welfare and the recession being associated with low period welfare, as in our model with unemployment risk, the opposite would happen. The boom would be associated with low period welfare, as agents would be working harder than normal, while in the recession welfare would be above the steady-state value since agents would take a vacation and benefit from past excess work. While evaluating welfare is certainly difficult, the path for period welfare in our model with unemployment risk appears to us as more in line with common perceptions about boom-bust cycles than that implied by a situation with no market frictions.

\textsuperscript{43} See Beaudry and Portier [2013] for a survey of this literature.
Figure 9: Response of economy to a noise shock

\[ \hat{X}_t \]

\[ \hat{u}_t \]

Note: Impulse is associated with a 10% overly-optimistic belief by shoppers in the first sub-period of \( t = 0 \). \( \hat{X}_t \) is the stock of durables and \( \hat{u}_t \) is average period utility across all households, both expressed in deviations from steady state. Example is constructed assuming the functional forms \( U(c) = \log(c) \), \( \nu(\ell) = \nu_1^{ \ell+\omega } \) and \( F(\ell) = A\ell^\alpha \), with parameters \( \beta = 0.9 \), \( \delta = 0.1 \), \( \gamma = 0.1 \), \( \omega = 1.2 \), \( \nu_1 = \nu_2 = 0.35 \), \( \alpha = 0.67 \), \( A = 1.2 \), \( \Phi = 0.5 \) and \( \tau = 0.3 \).
5 Policy trade-offs

In this last section, we turn to one of our motivating questions and ask whether or not stimulative policies should be used when an economy is going through a liquidation phase characterized by high unemployment. In particular, we consider the case where the economy has inherited from the past a level of $X$ above its steady-state value and, in the absence of intervention, would experience a period of liquidation, with consumption below its steady-state level throughout the transition. Obviously, the first-best policies in this environment would be to remove the sources of frictions or to perfectly insure agents against unemployment risk. However, for reasons such as adverse selection, such first-best policies may not be possible. We therefore want to consider the value of a more limited type of policy: one that seeks only to temporarily boost expenditures. In particular, we are interested in asking whether welfare would be increased by stimulating expenditures for one period, knowing that this would imply a higher $X$ tomorrow and therefore lower consumption in all subsequent periods until the liquidation is complete. This policy question is aimed at capturing the tension between the Keynesian and Hayekian prescriptions in recession. In answering this question, we will be examining the effects of such a policy without being very explicit about the precise policy tools used to engineer the stimulus, as we think it could come from several sources. However, it can be verified that the stimulus we consider can be engineered by a one period subsidy to consumption financed by a tax on the employed.

Examining how a temporary stimulus to expenditures affects welfare during a liquidation turns out to be quite involved. For this reason, we break down the question into two parts. First, we ask whether a temporary stimulus would increase welfare if the economy were initially in a steady state characterized by a slack labor market. Second, we ask whether the effect on welfare of such a stimulus would be greater if the economy were initially in a state of liquidation (i.e., with $X_0$ above its steady state) than in the case where it is initially at a steady state.

When looking at how a temporary boost in expenditures would affect welfare, one may expect it to depend on many factors, including the extent of risk-aversion and the dis-utility of work. However, since the level of expenditures represents a private optimum, the present discounted welfare effect of a temporary boost in expenditures turns out to depend on a quite limited set of factors. In particular, if the economy is initially at a steady state in the slack regime, then to a first-order approximation the direction of the cumulative welfare effect depends simply on whether the stimulus induces an increase or decrease in the presented discounted value of the output stream. This is stated in Proposition 16.

Proposition 16. Suppose the economy is in steady state with $N < L$. Then, to a first-order approximation, a (feasible) change in the path of expenditures from this steady state equilibrium will increase the present discounted value of expected welfare if and only if it increases the presented discounted sum of the resulting expenditure path, $\sum_{i=0}^{\infty} \beta^i e_{t+i}$.

The logic behind Proposition 16 derives mainly from the envelope theorem. Since the consumption stream is optimally chosen from the individual’s perspective, most of the effects of a change in the consumption path are only of second order and can therefore be neglected when the change is small. Moreover, in the slack regime, prices, wages and hours worked are
invariant to changes in expenditures. Hence the only effects needed to be taken into account for welfare purposes are the induced changes in the match probabilities times the marginal value of changing these match probabilities. When the economy is initially in a steady state, the marginal value of changing the match probabilities is the same at each point in time. Further, since the match probabilities are proportional to expenditures, this explains why welfare increases if and only if the perturbed path of expenditures has a positive presented discounted value.

With this result in hand, it becomes rather simple to calculate whether, starting from steady state, a one-period increase in expenditures followed by a return to equilibrium decision rules results in an increase in welfare. In particular, recall that the law of motion for $X$ is given by

$$X_{t+1} = (1-\delta)X_t + \gamma e(X_t) \quad 0 < \gamma < 1 - \delta$$

where the function $e(X_t)$ is the equilibrium policy function for $e_t$. Now, beginning from steady state, suppose at $t=0$ we stimulate expenditures by $\epsilon$ for one period such that the stock at $t=1$ is now given by

$$\tilde{X}_1 = (1-\delta)X_0 + \gamma (e + \epsilon)$$

As as result of this one-period perturbation, the path for expenditures for all subsequent periods will be changed even if there is no further policy intervention. The new sequence for $X$, which we denote $\tilde{X}$, will be given by $\tilde{X}_{t+1} = (1-\delta)\tilde{X}_t + \gamma e(\tilde{X}_t)$ for all $t \geq 1$. From Proposition 16, this perturbation increases present discounted welfare if and only if

$$\epsilon > - \sum_{t=1}^{\infty} \beta^t \left[ e(\tilde{X}_t) - e \right]$$

(35)

For $\epsilon$ small, we can use the linear approximation of the function $e(\cdot)$ around the steady state to make this calculation. Note that $e'(X) = -(1-\delta - \lambda_1)/\gamma$, where $\lambda_1$ is the smallest eigenvalue of the dynamic system in modulus.\textsuperscript{44} Thus, in this case, one may show that condition (35) becomes

$$\frac{1 - \beta(1-\delta)}{1 - \beta \lambda_1} > 0$$

If the system is locally stable, then $\lambda_1 < 1$, and therefore this condition will always hold. Hence, if we are considering a situation where the labor market in steady state is slack, and this steady state is locally stable, then a one-period policy of stimulating household expenditures will increase welfare. This arises even though most of the effect of the policy is to front-load utility by creating an initial boom followed by a liquidation bust.\textsuperscript{45} While we knew that the initial steady state was sub-optimal, and that a policy that increases expenditures in all periods would likely be desirable, it is interesting to learn that a policy that favors expenditure today over expenditure tomorrow – when in the economy is in the unemployment regime – tends to increase welfare.

---

\textsuperscript{44} See the proof of Proposition 15.

\textsuperscript{45} Note that this result does not depend on the welfare factors considered earlier in the static model, such as the magnitudes of $\tau$ and of the difference between the marginal and average disutility of work.
The question we now want to examine is whether the gains in welfare of a temporary stimulus are greater when the economy is initially in a liquidation phase than in steady state. We believe this is a relevant question since a case for stimulus during a liquidation can best be made if the gains are greater than when the economy is in steady state. Otherwise, there is no particular reason to favor stimuli more when unemployment is above normal than when it is at a normal level. Somewhat surprisingly to us, as long as $U'''$ is not too big, the answer to this question is negative, as stated in Proposition 17.

**Proposition 17.** Assuming the economy’s steady state has $N < L$ and $U'''$ is not too big, then, to a second-order approximation around the steady state, a temporary stimulus increases the presented discounted value of welfare less when implemented during a liquidation phase then when implemented at the steady state.

Although a period of liquidation is associated with a higher-than-normal level of unemployment, and the degree of distortion as captured by the labor wedge is higher in such periods when compared to the steady state, Proposition 17 indicates that the gains to a temporary stimulus are not greater during a liquidation period than in normal (steady-state) times. At first pass, one may be puzzled by this result, as one might have expected the gains to be highest when the marginal utility of consumption is highest. However, when the economy is in a liquidation phase, while the benefits from current stimulus are high, so are the costs associated with delaying the recovery. In fact, because consumption levels are at a private optimum, these two forces essentially cancel each other out. Moreover, when in the unemployment regime, the direct gain from employing one more individual – that is, the value of the additional production, net of the associated dis-utility of work – is the same regardless of whether unemployment is high or low. Hence, the only remaining difference between the value of stimulus in high- versus low-unemployment states relates to the net utility gain from employed workers entering the second sub-period in surplus rather than debt. In a lower-unemployment regime, households take less precaution, so that unemployed workers end up with more debt, which is costly. It is this force which makes postponing an adjustment particularly costly when in a liquidation phase.

With respect to the policy debate between the followers of Hayek and Keynes, we take our results are clarifying the scope of the arguments. On the one hand, we have found that a policy that stimulates current consumption at the cost of lower consumption in the future can often be welfare-improving when the economy features unemployment. However, at the same time, we have found that the rationale for such a policy does not increase simply because the level of unemployment is higher. Hence, if one believes that stimulus is not warranted in normal times (because of some currently un-modeled costs) and that normal times are characterized by excessive unemployment, then stimulus should not be recommended during liquidation periods. While this insight will likely not extinguish the debate on the issue, we believe it can help focus the dialogue.

---

46 Note that this condition on $U'''$ is sufficient but not necessary for this result.

47 There is an additional force at play here, which relates to the fact that the magnitude of the amplification mechanism will in general be different when the economy is away from the steady state. However, as long as $U'''$ is not too big, this effect can safely be ignored.
6 Conclusion

There are three elements that motivated us to write this paper. First, there is the observation that most deep recessions arise after periods of fast accumulation of capital goods, either in the form of houses, consumer durables, or productive capital. This, in our view, gives plausibility to the hypothesis that recessions may often reflect periods of liquidation where the economy is trying to deplete excesses from past over-accumulation.\footnote{Note that this is a fundamentalist view of recessions, in that the main cause of a recession is viewed as an objective fundamental (in this case, the level of capital relative to technology) rather than a sunspot-driven change in beliefs.}\footnote{An alternative interpretation of this observation is that financial imbalances associated with the increase in capital goods are the main source of the subsequent recessions.} Second, during these apparent liquidation-driven recessions, the process of adjustment seems to be socially painful and excessive, in the sense that the level of unemployment does not seem to be consistent with the idea that the economy is simply “taking a vacation” after excessive past work. Instead, the economy seems to be exhibiting some coordination failure that makes the exploitation of gains from trade between individuals more difficult than in normal times. These two observations capture the tension we believe is often associated with the Hayekian and Keynesian views of recessions. Finally, even when monetary authorities try to counter such recessions by easing policy, this does not seem to be sufficient to eliminate the problem. This leads us to believe that there are likely mechanisms at play beyond those related to nominal rigidities.\footnote{We chose to analyze in this paper in an environment without any nominal rigidities so as to clarify the potential role of real rigidities in understanding behavior in recessions. However, in doing so, we are not claiming that the economy does not also exhibit nominal rigidities or that monetary policy is ineffective. We are simply suggesting that explanations based mainly on nominal rigidities may be missing important forces at play that cannot be easily overcome by monetary policy.}

Hence, our objective in writing this paper was to offer a framework that is consistent with these three observations, and accordingly to provide an environment where the policy trade-offs inherent to the Hayekian and Keynesian views could be discussed.

A central contribution of the paper is to provide a simple macro model that explains, using real as opposed to nominal frictions, why an economy may become particularly inefficient when it inherits an excessive amount of capital goods from the past. The narrative behind the mechanism is quite straightforward. When the economy inherits a high level of capital, this decreases the desire for trade between agents in the economy, leading to less demand. When there are fixed costs associated with employment, this will generally lead to an increase in unemployment. If the risk of unemployment cannot be entirely insured away, households will react to the increased unemployment by increasing saving and thereby further depressing demand. This multiplier process will cause an excess reaction to the inherited goods and can be large enough to make society worse off even if – in a sense – it is richer since it has inherited a large stock of goods. Within this framework, we have shown that policies aimed at stimulating activity will face an unpleasant trade-off, as the main effect of stimulus will simply be to postpone the adjustment process. Nonetheless, we find that such stimulative policies may remain desirable even if they postpone recovery, but these gains do not increase simply because the rate of unemployment is higher. As noted, the mechanisms presented in the paper have many antecedents in the literature, but we believe that our framework offers a particularly tractable and clear way of capturing these ideas and of reconciling diverse
views about the functioning of the macro-economy.
References


CHALLE, E., AND X. RAGOT (2013): “Precautionary Saving over the Business Cycle,” PSE Working Papers hal-00843150, HAL.


Appendix

A Proofs of Propositions

For simplicity, when the matching function is of the form $M(N, L) = \Lambda \min\{N, L\}$ we prove results only for the case where $\Lambda = 1$. It is nonetheless straightforward to extend all proofs to the more general case.

Proof of Proposition 1

We first establish that there always exists an equilibrium of this model. Substituting equation (8) into equation (7) and letting $e \equiv c - X$ yields

$$U'(X + e) = \frac{\nu'(\ell)}{F'(\ell)} \left( 1 + \tau - \frac{e}{\Omega(\ell)} \right)$$

(A.1)

where $\Omega(\ell) \equiv F'(\ell)\ell$ is output net of search costs per employed worker, which is assumed to be strictly increasing. When $N < L$ (i.e., the full-employment constraint is not binding), equation (9) implies that $\ell = \ell^*$, and equation (8) implies that $e < e^*$, where $e^* \equiv \Omega(\ell^*)$. On the other hand, when $N > L$ (i.e., the full-employment constraint binds), equation (8) implies that $\ell = \Omega^{-1}(e)$. Further, since $\min\{N, L\} < N$ and $F(\ell) - F'(\ell)\ell$ is assumed to be strictly increasing in $\ell$, equation (9) implies that $\ell > \ell^*$, and thus, by strict increasingness of $\Omega$, we also have $e > e^*$. Substituting these results into equation (A.1) yields that $e > 0$ is an equilibrium of this model if it satisfies

$$U'(X + e) = Q(e)$$

(A.2)

where the function $Q(e)$, defined in equation (14), is the expected marginal utility cost of consumption when aggregate expenditures are $e = c - X$. Note that $Q$ is continuous, strictly decreasing on $[0, e^*]$, and strictly increasing on $[e^*, \infty)$.

Lemma A.1. If $U'(X) \leq Q(0)$, then there is an equilibrium with $e = 0$.

Proof. To see this, suppose aggregate conditions are that $e = 0$. Then the marginal utility of consumption when the household simply consumes its endowment is no greater than its expected marginal cost, and thus households respond to aggregate conditions by making no purchases, which in turn validates $e = 0$.

Lemma A.2. If $U'(X) > Q(0)$, then there is an equilibrium with $e > 0$.

Proof. We have that $\min Q(e) = \nu'(\ell^*)/F'(\ell^*) > 0$. Since we have assumed $\lim_{c \to \infty} U'(c) \leq 0$, it necessarily follows that for any $X$, there exists an $e$ sufficiently large that $U'(X + e) < \min Q(e)$, and therefore, by the intermediate value theorem, there must exist a solution $e > 0$ to equation (A.2).
Lemmas A.1 and A.2 together imply that an equilibrium necessarily exists. We turn now to showing under what conditions this equilibrium is unique for all values of $X$. As in equation (12), we may represent household $j$’s optimal expenditure when aggregate expenditure is $e$ as $e_j(e) = U'^{-1}(Q(e)) - X$, so that equilibrium is a fixed point $e_j(e) = e$. The function $e_j(e)$ is continuous everywhere, and differentiable everywhere except at $e = e^*$, with

$$e'_j(e) = \frac{Q'(e)}{U''(U'^{-1}(Q(e)))}$$

Note that $e'_j(e)$ is independent of $X$, strictly increasing on $[0, e^*]$ and strictly decreasing on $[e^*, \infty)$.

**Lemma A.3.** If

$$\lim_{e \uparrow e^*} e'_j(e) < 1$$

(A.3)

then $e'_j(e) < 1$ for all $e$.

**Proof.** Note first that $e'_j(e) < 0$ for $e > e^*$, so that this condition is obviously satisfied in that case. For $e < e^*$, note that

$$e''_j(e) = \frac{Q''(e) - U'''(X + e_j(e)) [e'_j(e)]^2}{U''(X + e_j(e))}$$

Since $Q''(e) = 0$ on this range and $U''' > 0$, we have $e''_j(e) > 0$, and thus $e'_j(e) < \lim_{e \uparrow e^*} e'_j(e)$, which completes the proof.

**Lemma A.4.** Inequality (A.3) holds if and only if

$$\tau < \bar{\tau} \equiv -U'' \left( U'^{-1} \left( \frac{\nu'(\ell^*)}{f'(\ell^*)} \right) \right) \frac{f'(\ell^*) [f(\ell^*) - \Phi]}{\nu'(\ell^*)}$$

**Proof.** We have that

$$\lim_{e \uparrow e^*} e'_j(e) = \frac{\nu'(\ell^*) \tau}{-U'' \left( U'^{-1} \left( \frac{\nu'(\ell^*)}{f'(\ell^*)} \right) \right) f'(\ell^*) [f(\ell^*) - \Phi]}$$

which is clearly less than one if and only if $\tau < \bar{\tau}$.

**Lemma A.5.** If $\tau < \bar{\tau}$, then there always exists a unique equilibrium regardless of the value of $X$. If $\tau > \bar{\tau}$, then there exists values of $X \in \mathbb{R}$ such that there are multiple equilibria.

**Proof.** We have already established that there always exists an equilibrium. Note that equilibrium occurs at the point where the $e_j = e_j(e)$ locus intersects with the locus characterizing the equilibrium condition, i.e., $e_j = e$. To see the first part of the lemma, suppose $\tau < \bar{\tau}$ so that inequality (A.3) holds. Then since the slope of the equilibrium locus is one, and the slope of the $e_j = e_j(e)$ locus is strictly less than one by Lemma A.3, there can be at most one intersection, and therefore the equilibrium is unique.
To see the second part of the lemma, suppose that \( \tau > \bar{\tau} \) and thus (A.3) does not hold. Then by strict convexity of \( e_j(e) \) on \((0, e^*)\), there exists a value \( \bar{e} < e^* \) such that \( e_j'(\bar{e}) > 1 \) on \((\bar{e}, e^*)\). Define \( X(e) \equiv U''(Q(e)) - e \), and note that \( e \) is an equilibrium when \( X = \tilde{X}(e) \).

We show that there are at least two equilibria when \( X = \tilde{X}(e) \) with \( e \in (\bar{e}, e^*) \). To see this, choose \( e_0 \in (\bar{e}, e^*) \), and note that, for \( X = \tilde{X}(e_0) \), \( e_j(e_0) = e_0 \) and \( e_j'(e_0) > 1 \) on \((e_0, e^*)\). Thus, it must also be the case that \( e_j(e^*) < e^* \). But since \( e_j(e) \) is continuous everywhere and strictly decreasing on \( e > e^* \), this implies that there exists some value \( e > e^* \) such that \( e_j(e) = e \), which would represent an equilibrium. Since \( e_0 < e^* \) is also an equilibrium, there are at least two equilibria.

This completes the proof of Proposition 1.

Proof of Proposition 2

Lemma A.6. If \( \tau < \bar{\tau} \) and \( X \) is such that \( e > 0 \), then \( \frac{de}{dX} < 0 \).

Proof. Differentiating equilibrium condition (A.2) with respect to \( X \) yields

\[
\frac{de}{dX} = \frac{U''(X + e)}{Q'(e) - U''(X + e)} \tag{A.4}
\]

From Lemma A.4, we see that \( Q'(e) > U''(U''^{-1}(Q(e))) \). In equilibrium, \( U''^{-1}(Q(e)) = X + e \), so that this inequality becomes \( Q'(e) > U''(X + e) \), and thus the desired conclusion follows by inspection.

Given Lemma A.6 and the fact that the economy exhibits unemployment when \( e < e^* \) and full employment when \( e \geq e^* \), it is clear that the economy will exhibit unemployment if and only if \( X \) is smaller than the level such that \( e = e^* \) is the equilibrium; that is, if \( X < X^* \), where

\[
X^* \equiv U''^{-1}\left( \frac{\nu'(\ell^*)}{F'(\ell^*)} \right) - F'(\ell^*)\ell^*
\]

This completes the proof of the first part of the proposition.

Next, from Lemma A.1, we see that there is a zero-employment equilibrium if and only if \( U'(X) \leq \frac{\nu'(\ell^*)}{F'(\ell^*)}(1 + \tau) \), which holds when \( X \geq X^{**} \), where

\[
X^{**} \equiv U''^{-1}\left( \frac{\nu'(\ell^*)}{F'(\ell^*)}(1 + \tau) \right)
\]

This completes the proof of Proposition 2.

Proof of Proposition 3

If \( X < X^{**} \), we know from Proposition 2 that \( e > 0 \), and therefore \( e \) solves equation (A.2). Substituting \( e = c - X \) for \( e \) yields the desired result in this case. From Proposition 2, we also know that if \( X \geq X^{**} \) then \( e = 0 \), in which case \( c = X \), which completes the proof.
Proof of Proposition 4

If $X > X^{**}$, so that the economy features zero employment and therefore $c = X$, then clearly $c$ is increasing in $X$. Thus, suppose $X < X^{**}$, so that $e > 0$. Differentiating the expression $c = X + e$ with respect to $X$ and using equation (A.4), we obtain

$$\frac{dc}{dX} = \frac{Q'(e)}{Q'(e) - U''(X + e)}$$ (A.5)

Since the denominator of this expression is positive (see the proof of Lemma A.6), the sign of $dc/dX$ is given by the sign of $Q'(e)$, which is negative if $e < e^*$ (i.e., if $X^* < X < X^{**}$) and positive if $e > e^*$ (i.e., if $X < X^*$). This completes the proof.

Proof of Proposition 5

Letting $U(e)$ denote welfare conditional on the coordinated level of $e$, we may obtain that

$$U(e) = U(X + e) + \mu(e) \left[ \ell^* - \frac{\nu'(\ell^*)}{F'(\ell^*)} e \right] - [1 - \mu(e)](1 + \tau) \frac{\nu'(\ell^*)}{F'(\ell^*)} e$$

where $\mu(e) = e/[F'(\ell^*)\ell^*]$ denotes employment conditional on $e$ and $\mathcal{L}^* \equiv \nu'(\ell^*)\ell^* - \nu(\ell^*) \geq 0$. Using the envelope theorem, it is straightforward to see that the only welfare effects of a marginal change in $e$ from its decentralized equilibrium value are those that occur through the resulting change in employment. Thus,

$$U'(e) = \left[ \mathcal{L}^* + \frac{\nu'(\ell^*)}{F'(\ell^*)} \tau e \right] \mu'(e) > 0$$

and therefore a coordinated rise in $e$ would increase expected utility of all households.

Proof of Proposition 6

Denote welfare as a function of $X$ by

$$U(X) \equiv U(X + e) + \mu \left[ -\nu(\ell) + V(w\ell - pe) \right] + (1 - \mu)V(-pe)$$

If $X < X^*$, so that the economy is in the full-employment regime, or if $X > X^{**}$, so that the economy is in the zero-employment regime, we may show that $U'(X) > 0$ always holds. Thus, we focus on the case where $X \in (X^*, X^{**})$. When this is true, some algebra yields

$$U(X) = U(X + e) + \nu'(\ell^*) \ell^* - \frac{\nu(\ell^*)}{\ell^*} + \frac{\nu'(\ell^*)}{F'(\ell^*)} e \right] \mu - \frac{\nu'(\ell^*)}{F'(\ell^*)}(1 + \tau) e$$

Using the envelope theorem, we may differentiate this expression with respect to $X$ to obtain

$$U'(X) = U'(X + e) + \left[ \mathcal{L}^* + \frac{\nu'(\ell^*)}{F'(\ell^*)} e \right] \frac{d\mu}{dX}$$ (A.6)

where $\mathcal{L}^* \equiv \nu'(\ell^*)\ell^* - \nu(\ell^*) \geq 0$. 

49
Lemma A.7. $U''(X) > 0$ on $(X^*, X^{**})$.

Proof. Substituting the equilibrium condition (A.2) into (A.6) and using the fact that

$$\frac{d\mu}{dX} = \frac{1}{F'(\ell^*)\ell^*} \frac{de}{dX}$$

after some algebra, we obtain

$$U'(X) = \frac{\nu'(\ell^*)}{F'(\ell^*)} \left[ 1 + \tau + \tau \mu \left( \frac{de}{dX} - 1 \right) \right] + \frac{L^*}{F'(\ell^*)\ell^*} \frac{de}{dX} \tag{A.7}$$

From (A.4), we may also obtain that

$$\frac{de}{dX} = \left( \frac{\nu'(\ell^*)\tau}{-U''(X + e)[F'(\ell^*)]^2 \ell^*} - 1 \right)^{-1}$$

and therefore

$$U''(X) = \frac{\nu'(\ell^*)}{F'(\ell^*)} \frac{d\mu}{dX} \left( \frac{de}{dX} - 1 \right) + \left[ \frac{\nu'(\ell^*)}{F'(\ell^*)} \tau \mu + \frac{L^*}{F'(\ell^*)\ell^*} \right] \frac{d^2e}{dX^2}$$

Since $de/dX < 0$, $d\mu/dx < 0$, and thus the first term is positive, as is the second term, and the proof is complete.

Lemma A.8. If

$$\tau > \bar{\tau} \equiv \frac{\nu(\ell^*)}{\nu'(\ell^*)\ell^*} \left( \frac{\bar{\tau}}{1 + \bar{\tau}} \right)$$

then there exists a range of $X$ such that $U'(X) < 0$.

Proof. Since $U$ is convex by Lemma A.7, $U'(X) < 0$ for some values of $X$ if and only if $\lim_{X \downarrow X^*} U'(X) < 0$. Taking limits of equation (A.7), and using the facts that

$$\lim_{X \downarrow X^*} \frac{de}{dX} = -\frac{\bar{\tau}}{\bar{\tau} - \tau}$$

and $\lim_{X \downarrow X^*} \mu = 1$, we obtain that

$$\lim_{X \downarrow X^*} U'(X) = \frac{\nu'(\ell^*)}{F'(\ell^*)} \left( 1 - \frac{\tau \bar{\tau}}{\bar{\tau} - \tau} \right) - \frac{L^*}{F'(\ell^*)\ell^*} \left( \frac{\bar{\tau}}{\bar{\tau} - \tau} \right)$$

Substituting in from the definition of $L^*$, straightforward algebra yields that this expression is less than one if and only if $\tau > \bar{\tau}$.

Note that, by convexity of $\nu(\ell)$ and the fact that $\nu(0) = 0$, we have $\nu(\ell^*) \leq \nu'(\ell^*)\ell^*$, and thus $\tau < \bar{\tau}$, so that there always exists values of $\tau$ such that $\bar{\tau} < \tau < \bar{\tau}$. From the definition of $\bar{\tau}$, we also see that, holding $\tau$ and $\nu'(\ell^*)$ constant, if $\nu(\ell^*)/\ell^*$ is small, this inequality is more likely to be satisfied.
Proof of Proposition 7

We first consider equilibria within three regimes separately, then establish the relationship between these regimes and different ranges of $X$. Note that we assume that $\lim_{c_n \to 0} U_n' (c_n) = \infty$, so that we will always have $N^n > 0$.

**Equilibrium Regime 1: $N \equiv N^d + N^n \geq L$**

Suppose that $N \equiv N^d + N^n \geq L$, so that $\mu = 1$. Assuming $\Phi$ is sufficiently small to ensure the existence of a regime with $\mu < 1$ (see footnote 21), it follows that we must have $N^d > 0$ when $\mu = 1$. Using $\psi \equiv N^d/N$ and $N$ instead of $N^d$ and $N^n$, and letting $p_j^l (\ell) \equiv \nu_j' (\ell) / F_j^l (\ell)$, the equilibrium conditions can be written

$$U^d (X + e) = p^d (\ell^d) \quad (A.8)$$
$$U^n (c^n) = p^n (\ell^n) \quad (A.9)$$
$$F^d (\ell^d) - F^d' (\ell^d) \ell^d = \frac{N}{L} \Phi \quad (A.10)$$
$$F^n (\ell^n) - F^n' (\ell^n) \ell^n = \frac{N}{L} \Phi \quad (A.11)$$
$$e = \psi \Omega^d (\ell^d) \quad (A.12)$$
$$c^n = (1 - \psi) \Omega^n (\ell^n) \quad (A.13)$$

**Lemma A.9.** There exists at most one equilibrium in Regime 1.

*Proof.* The zero-profit conditions (A.10) and (A.11) can be solved to obtain $\ell^d = \ell^d (N)$, with $\ell^d (N) > 0$. Substituting the resource constraint equations (A.12) and (A.13) into the demand equations (A.8) and (A.9) for $e$ and $c^n$, we may then reduce the system to two equations,

$$U^d (X + \psi \Omega^d (\ell^d (N))) = p^d (\ell^d (N)) \quad (A.14)$$
$$U^n ((1 - \psi) \Omega^n (\ell^n (N))) = p^n (\ell^n (N)) \quad (A.15)$$

in two unknowns, $N$ and $\psi$. Each of these equations maps out a locus of points in $(N, \psi)$-space, and anywhere they intersect represents an equilibrium. Taking total derivatives with respect to $N$ (holding $X$ constant) of each of these equations yields, respectively,

$$\frac{d\psi}{dN} = \frac{\ell^d (N)}{\Omega^d (\ell^d (N))} \left[ - \frac{p^d (\ell^d (N))}{U^d (X + \psi \Omega^d (\ell^d (N)))} - \psi \Omega^d (\ell^d (N)) \right] < 0$$
$$\frac{d\psi}{dN} = \frac{\ell^n (N)}{\Omega^n (\ell^n (N))} \left[ - \frac{p^n (\ell^n (N))}{U^n ((1 - \psi) \Omega^n (\ell^n (N)))} + (1 - \psi) \Omega^n (\ell^n (N)) \right] > 0$$

so that the locus associated with equation (A.14) is downward-sloping, and the locus associated with equation (A.15) is upward-sloping. Thus, if an equilibrium exists in this region, it is clearly unique.

\[\Box\]
Lemma A.10. In an equilibrium in Regime 1, as long as $N \equiv N^d + N^n > L$ we have $de/dX < 0$, $dc^d/dX > 0$ and $dc^n/dX > 0$.

Proof. A rise in $X$ results in a shift down of the locus associated with equation (A.14), so that $d\psi/dX < 0$ and $dN/dX < 0$. Since a fall in $N$ results in a fall in $\ell^i$ and thus also $p^i$, it follows from equations (A.8) and (A.9) that $dc^d/dX > 0$ and $dc^n/dX > 0$. Further, since $\psi$ and $\ell$ both fall, it follows from equation (A.12) that $de/dX < 0$. \hfill $\Box$

Equilibrium Regime 2: $N \equiv N^d + N^n \leq L$ and $N^d > 0$

Suppose now that $N \equiv N^d + N^n \leq L$ but with a positive output of durables ($N^d > 0$).

Lemma A.11. For $\tau$ sufficiently small, there exists at most one equilibrium in Regime 2.

Proof. Let

$$Z^n(e, c^n) \equiv U^{n-1}(\nu'(\ell_{ns}) F^{n'}(\ell_{ns}) \left[ 1 + \tau - \tau \left( \frac{e}{\Omega^d(\ell_{ds})} + \frac{c^n}{\Omega^n(\ell_{ns})} \right) \right])$$

$$= U^{n-1}(\nu'(\ell_{ns}) F^{n'}(\ell_{ns}) (1 + \tau - \tau \mu))$$

where, as before, $\Omega^j(\ell) \equiv F^{j'}(\ell) \ell$. For a given $e$, $c^n$ solves $c^n = Z^n(e, c^n)$. A sufficient condition for this solution to always be unique is that $Z^n_2(e, c^n) < 1$ for all combinations of $e$ and $c^n$ such that $N \leq L$. Since

$$Z^n_2(e, c^n) = \frac{1}{-U^n(Z^n(e, c^n))} \nu'(\ell_{ns}) \frac{\tau}{F^n(\ell_{ns}) \Omega^n(\ell_{ns})}$$

$Z^n_2(e, c^n)$ is maximized when $Z^n(e, c^n)$ is maximized, which in turn occurs when $\mu = 1$. Thus, a sufficient condition to ensure that $c^n$ is always uniquely determined given $e$ is that

$$\tau < \tilde{\tau}^n \equiv -U^n \left( U^{n-1}(\nu'(\ell_{ns}) F^{n'}(\ell_{ns})) \Omega^n(\ell_{ns}) \frac{\tau}{\nu'(\ell_{ns}) F^{n'}(\ell_{ns})} \right)$$

Assume henceforth that this is true, and let $c^n(e)$ denote the unique value of $c^n$ that solves $c^n = Z^n(e, c^n)$. Note that

$$c''(e) = \frac{\nu'(\ell_{ns}) \frac{\tau}{F^n(\ell_{ns}) \Omega^d(\ell_{ds})}}{-U^n(c^n(e)) - \nu'(\ell_{ns}) \frac{\tau}{F^n(\ell_{ns}) \Omega^n(\ell_{ns})}}$$

Since $\tau < \tilde{\tau}^n$, it may be verified that the denominator of this expression is strictly positive, so that $c''(e) > 0$. Further,

$$c'''(e) = \frac{\nu'(\ell_{ns}) \frac{\tau}{F^n(\ell_{ns}) \Omega^d(\ell_{ds})} U^n(c^n(e)) c''(e)}{\left[ -U^n(c^n(e)) - \nu'(\ell_{ns}) \frac{\tau}{F^n(\ell_{ns}) \Omega^n(\ell_{ns})} \right]^2} > 0$$

52
Next, let

$$Z^d(e) \equiv U^{dr-1} \left( \frac{\nu'(\ell^d)}{F^{dr}(\ell^d)} \left[ 1 + \tau - \tau \left( \frac{e}{\Omega^d(\ell^d)} + \frac{c^n(e)}{\Omega^n(\ell^n)} \right) \right] \right)$$

$$= U^{dr-1} \left( \frac{\nu'(\ell^d)}{F^{dr}(\ell^d)} (1 + \tau - \tau \mu) \right)$$

so that the equilibrium solves $e = Z^d(e) - X$. This equilibrium is unique for all $X$ if $Z^{dr}(e) < 1$ for all $e$. We have

$$Z^{dr}(e) = \frac{1}{-U^{dr}(Z^d(e))} \left[ \frac{\nu'(\ell^d)}{\Omega^d(\ell^d)} + \frac{c^n(e)}{\Omega^n(\ell^n)} \right] > 0$$

Since

$$Z^{dr}(e) = \frac{U^{ddn}(Z^d(e)) [Z^d(e)]^2}{-U^{dr}(Z^d(e))} + \frac{1}{-U^{dr}(Z^d(e))} \frac{\nu'(\ell^d)}{\Omega^d(\ell^d)} \frac{c^n(e)}{\Omega^n(\ell^n)} > 0$$

it follows that $Z^{dr}(e)$ is maximized at the maximum value of $e$ such that the economy is still in the unemployment regime, i.e., at the value of $e$ such that

$$\frac{e}{\Omega^d(\ell^d)} + \frac{c^n(e)}{\Omega^n(\ell^n)} = 1$$

Let $e^*$ denote the value of $e$ for which this is true, and note that

$$Z^d(e^*) \equiv U^{dr-1} \left( \frac{\nu'(\ell^d)}{F^{dr}(\ell^d)} \right)$$

Then a sufficient condition for there to always exist a unique equilibrium is that

$$\tau < \bar{\tau}^d \equiv \min \left\{ \bar{\tau}^n, -U^{ddn} \left( Z^d(e^*) \right) \left[ \frac{1}{\Omega^d(\ell^d)} + \frac{c^n(e^*)}{\Omega^n(\ell^n)} \right]^{-1} \frac{\nu'(\ell^d)}{\nu'(\ell^d)} \right\}$$

Lemma A.12. If $\tau < \bar{\tau}^d$, then in an equilibrium in Regime 1, as long as $\mu < 1$ we have $de/dX < 0$, $dc^d/dX < 0$ and $dc^n/dX < 0$.

Proof. Since $c^n(e) > 0$, it follows that $dc^n/dX < 0$ as long as $dc^d/dX < 0$. Differentiating the equilibrium condition $e = Z^d(e) - X$ with respect to $X$ and solving yields

$$\frac{dc}{dX} = -\frac{1}{1 - Z^{dr}(e)}$$

Since $\tau < \bar{\tau}^d$, we have $0 < Z^{dr}(e) < 1$, so that $de/dX < -1$ and thus $dc^d/dX < 0$. 

53
Equilibrium Regime 3: \( N \equiv N^d + N^n < L \) and \( N^d = 0 \)

Suppose finally that there is unemployment (\( \mu < 1 \)), but that there is no output of durables (\( N^d = 0 \)). Clearly then, \( e = 0 \), and thus \( c^d = X \).

**Lemma A.13.** If \( \tau < \bar{\tau} \), then there exists at most one equilibrium in Regime 3, and this equilibrium is independent of \( X \).

**Proof.** An equilibrium in this case is given by a solution to \( c^n = Z^n (0, c^n) \). As argued in the proof of Lemma A.11, this solution is unique if \( \tau < \bar{\tau} \) and given in that case by \( c^n (0) \). Further, since \( Z^n (0, c^n) \) does not depend on \( X \) in any way, this equilibrium is independent of \( X \).

Equilibrium as a function of \( X \)

Define

\[
X^{**} \equiv Z^d (0) \\
X^* \equiv Z^d (e^*) - e^*
\]

It is straightforward to verify that if \( X < X^* \) then the equilibrium is in Regime 1, if \( X^* \leq X < X^{**} \) then the equilibrium is in Regime 2, and if \( X \geq X^{**} \) then the equilibrium is in Regime 3. The properties of Proposition 7 then follow immediately.

**Proof of Proposition 8**

Let \( \theta \equiv N/L \) be labor-market tightness and \( \mu (\theta) \equiv M (\theta, 1) \) be the resulting employment rate. Then we can obtain

\[
\ell = \ell (\theta) \equiv \left[ \frac{\Phi \theta}{(1 - \alpha) A\mu (\theta)} \right]^{\frac{1}{\alpha}} \\
e = \frac{\alpha \Phi}{(1 - \alpha) \theta} \\
p = p (\theta) \equiv \frac{\nu' (\ell (\theta))}{\alpha A [\ell (\theta)]^{\alpha - 1}} \\
w = w (\theta) \equiv \nu' (\ell (\theta))
\]

Equilibrium is then given by a solution to the equation

\[
U' \left( X + \frac{\alpha \Phi}{(1 - \alpha) \theta} \right) = Q (\theta) \equiv p (\theta) [1 + \tau - \tau \mu (\theta)]
\]

for \( \theta \).

**Lemma A.14.** For \( X \) sufficiently small, \( dc/dX \geq 0 \). For \( X < X^{\text{max}} \) sufficiently large, \( dc/dX < 0 \).
Proof. Assuming $\tau$ is small enough so that the equilibrium is unique, it can be easily verified that $dc/dX < 0$ if $Q'(\theta) < 0$ and $dc/dX > 0$ if $Q'(\theta) > 0$. Further, since $d\theta/dX < 0$, showing that Lemma A.14 holds is equivalent to showing that $Q'(\theta) < 0$ for $\theta$ sufficiently small (i.e., $X$ sufficiently large) and $Q'(\theta) > 0$ for $\theta$ sufficiently large (i.e., $X$ sufficiently small).

We have

$$Q'(\theta) = p(\theta) \left\{ \frac{p'(\theta)}{p(\theta)} [1 + \tau - \tau \mu(\theta)] - \tau \mu'(\theta) \right\}$$

We may obtain that

$$\frac{p'(\theta)}{p(\theta)} = \left[ \omega(\theta) + 1 - \alpha \right] \frac{\ell'(\theta)}{\ell(\theta)}$$

where

$$\omega(\theta) \equiv \frac{\nu''(\ell(\theta)) \ell(\theta)}{\nu'(\ell(\theta))} \geq 0$$

and

$$\frac{\ell'(\theta)}{\ell(\theta)} = \frac{1 - E_{\mu \theta}(\theta)}{\alpha \theta}$$

where the notation $E_{fx}(x)$ denotes the elasticity $f'(x) x / f(x)$. Thus

$$Q'(\theta) = p(\theta) \frac{\theta}{\theta} \Gamma(\theta)$$

(A.16)

where

$$\Gamma(\theta) \equiv \frac{\omega(\theta) + 1 - \alpha}{\alpha} \left[ 1 - E_{\mu \theta}(\theta) \right] [1 + \tau - \tau \mu(\theta)] - \tau \mu'(\theta) \theta$$

Consider first the case where $\theta \to \infty$. From (A.16), we see that the sign of $Q'(\theta)$ is equal to the sign of $\Gamma(\theta)$. Since, by Assumption 1,

$$\lim_{N \to \infty} \frac{\partial M\left(\frac{N}{L}, 1 \right)}{\partial L} = 0$$

we may obtain that

$$\lim_{\theta \to \infty} \mu'(\theta) \theta = 0$$

which in turn implies that $\lim_{\theta \to \infty} E_{\mu \theta}(\theta) = 0$. Thus, letting $\bar{\mu} \equiv \lim_{\theta \to \infty} \mu(\theta)$, we have

$$\lim_{\theta \to \infty} \Gamma(\theta) = (1 + \tau - \tau \bar{\mu}) \lim_{\theta \to \infty} \frac{\omega(\theta) + 1 - \alpha}{\alpha} > 0$$

so that for $\theta$ sufficiently large ($X$ sufficiently small) we have $Q'(\theta) > 0$, and thus $dc/dX > 0$.

Next, consider the case where $\theta \to 0$. From (A.16), we see that the sign of $Q'(\theta)$ is equal to the sign of $\Gamma(\theta)/\theta$, so that $\text{sgn}(Q'(0)) = \text{sgn}(\lim_{\theta \to 0} \Gamma(\theta)/\theta)$. We have

$$\lim_{\theta \to 0} \frac{\Gamma(\theta)}{\theta} = \frac{\omega(0) + 1 - \alpha}{\alpha} (1 + \tau) \lim_{\theta \to 0} \left[ \frac{1 - E_{\mu \theta}(\theta)}{\theta} \right] - \tau \mu'(0)$$
Note that the basic restrictions on the matching function imply that $\mu(0) = 0$ and $0 < \mu'(0) \leq 1$.\textsuperscript{51} Thus,

$$\lim_{\theta \to 0} E_{\mu_\theta}(\theta) = \lim_{\theta \to 0} \frac{\mu'(\theta)}{\mu(\theta) / \theta}$$

Since the limit of the numerator is non-zero and bounded, and the limit of the denominator is equal to $\mu'(0)$ by definition (since $\mu(0) = 0$), which is also non-zero and bounded, we have $\lim_{\theta \to 0} E_{\mu_\theta}(\theta) = 1$. By Assumption 1,

$$\lim_{N \to 0} \frac{\partial M(1, L)}{\partial N} = 0$$

from which we may obtain

$$0 = -\frac{1}{L} \lim_{\theta \to 0} \frac{\mu(\theta) - E_{\mu_\theta}(\theta)}{\theta} = -\frac{\mu'(0)}{L} \lim_{\theta \to 0} \frac{1 - E_{\mu_\theta}(\theta)}{\theta}$$

Since $\mu'(0) > 0$, this can only be true if $\lim_{\theta \to 0} [1 - E_{\mu_\theta}(\theta)] / \theta = 0$. Thus,

$$\lim_{\theta \to 0} \frac{\Gamma(\theta)}{\theta} = -\tau \mu'(0) < 0$$

so that for $\theta$ sufficiently small ($X$ sufficiently large) $Q'(\theta) < 0$, and thus $dc/dX < 0$, which completes the proof.

\textbf{Lemma A.15.} For $X$ sufficiently small, there is not deficient demand. For $X < X^{\max}$ sufficiently large, there is deficient demand.

\textit{Proof.} Conditional on $\theta$, equilibrium welfare is given by

$$U(X, \theta) \equiv U(X + \frac{\alpha \Phi}{(1 - \alpha) \theta}) + \mu(\theta) [w(\theta) \ell(\theta) - \nu(\ell(\theta))] - Q(\theta) \frac{\alpha \Phi}{(1 - \alpha) \theta}$$

Using the envelope theorem and other results from above, at the equilibrium level of $\theta$ we may obtain that

$$U_2(X, \theta) = \mu'(\theta) [\nu'(\ell) \ell - \nu(\ell)] + \mu(\theta) \nu''(\ell) \frac{\ell^2}{\alpha \theta} [1 - E_{\mu_\theta}(\theta)] - Q'(\theta) \frac{\alpha \Phi}{(1 - \alpha) \theta}$$

If $Q'(\theta) < 0$, which occurs when $X$ is sufficiently large (see Lemma A.14), then clearly $U_2(X, \theta) > 0$, so that we have deficient demand. Suppose instead that $Q'(\theta) > 0$, which occurs when $X$ is sufficiently small. Substituting in for $Q'(\theta)$ we may obtain

$$U_2(X, \theta) = \frac{\mu(\theta)}{\theta} \nu'(\ell) \ell \left\{ E_{\mu_\theta}(\theta) \left[ 1 + \tau \mu(\theta) - \frac{\nu(\ell)}{\nu'(\ell) \ell} \right] - [1 - E_{\mu_\theta}(\theta)] \left[ \frac{\omega(\theta) + 1 - \alpha}{\alpha} \tau (1 - \mu(\theta)) + \frac{1 - \alpha}{\alpha} \right] \right\}$$

\textsuperscript{51} Technically, Assumption 1 does not rule out the possibility that $\mu'(0) = 0$. However, since $\mu$ is concave and non-decreasing, if $\mu'(0) = 0$ this would imply that $\mu(\theta) = 0$ for all $\theta$, i.e., employment is always zero. We ignore this uninteresting case.
Since \( \nu(\ell)/\nu'(\ell) \ell \leq 1 \) and \( \mathcal{E}_{\mu\theta}(\theta) \to 0 \) as \( \theta \to \infty \), the first term in braces approaches zero as \( \theta \to \infty \), while the second term approaches

\[
-\frac{1}{\alpha} \left\{ \tau \lim_{\theta \to \infty} \omega(\theta) [1 - \mu(\theta)] + 1 - \alpha \right\} < 0
\]

so that for \( \theta \) sufficiently large (\( X \) sufficiently small) \( U_2(X, \theta) \leq 0 \), so that there is not deficient demand.

\[ \square \]

**Proof of Proposition 9**

The arguments establishing that \( dc/dX \leq 0 \) (with strict equality as long as \( N > 0 \)) are nearly identical to in the Walrasian bargaining case, and are therefore omitted. Thus, there exist \( X^* \) and \( X^{**} \) such that for \( X < X^* \) the equilibrium satisfies \( N \geq L \) (labor market is tight), for \( X \in (X^*, X^{**}) \) the equilibrium satisfies \( 0 < N < L \) (labor market is slack), and for \( X \geq X^{**} \) we have \( N = 0 \).

**Lemma A.16.** Suppose \( X \in (X^*, X^{**}) \), so that the labor market is slack but there is strictly positive employment, i.e., \( 0 < N < L \). Then \( dc/dX < 0 \) and there is deficient demand.

**Proof.** As before, let \( \theta \equiv N/L \) be labor-market tightness and \( \mu(\theta) \equiv \min\{\theta, 1\} \) be the resulting employment rate, and note that since \( N < L \) we have \( \mu(\theta) = \theta \). Conditional on \( e \), we may obtain from equation (26) that

\[
\ell = \ell(e) \equiv \Omega^{-1} \left( \frac{\Phi}{1 - s} - \tau e \right)
\]

where

\[
\Omega(\ell) \equiv F(\ell) - \frac{\nu(\ell) F'(\ell)}{\nu'(\ell)}
\]

It may be easily verified that \( \Omega'(\ell) > 0 \), so that \( \Omega^{-1} \) is well-defined.

Letting \( W \equiv w\ell \) be the total wage bill, given \( e \) we may obtain

\[
p = p(e) \equiv \frac{\nu'(\ell(e))}{F'(\ell(e))} \\
\theta = \theta(e) \equiv \frac{e}{F(\ell(e)) - \Phi} \\
W = W(e) \equiv \nu(\ell) + p \frac{s\Phi}{1 - s} - \tau pe
\]

Equilibrium is then a solution to

\[
U'(X + e) = Q(e) \equiv p(e) [1 + \tau - \tau \theta(e)]
\]

for \( e \) (provided this solution satisfies \( \theta(e) \leq 1 \)). As usual, we will have \( dc/dX < 0 \) if and only if \( Q'(e) < 0 \). Since \( \ell'(e) < 0 \), it is easily verified that \( p'(e) < 0 \) and \( \theta'(e) > 0 \), so that \( Q'(e) < 0 \) necessarily holds.

\[ ^{52} \text{Note that this necessarily follows only under the maintained assumption that } \tau \text{ is sufficiently small such that a unique equilibrium exists.} \]
Next, welfare conditional on \( e \) is given by

\[
U(X, e) = U(X + e) + p(e) \left[ \frac{s\Phi}{1 - s} \theta(e) - (1 + \tau)e \right]
\]

Taking the derivative with respect to \( e \) and evaluating at the equilibrium, we may obtain

\[
U_2(X, e) = Q(e) + p'(e) \left[ \frac{s\Phi}{1 - s} \theta - (1 + \tau)e \right] + p(e) \left[ \frac{s\Phi}{1 - s} \theta'(e) - (1 + \tau) \right]
\]

It may be verified that

\[
\ell'(e) = -\frac{\tau p\ell}{\nu(\ell) E_{pl}(\ell)}
\]

\[
\theta'(e) = \frac{p}{\mathcal{W}} \left[ 1 + \tau \theta \frac{E_{pl}(\ell)}{E_{pl}(\ell)} \right]
\]

\[
p'(e) = -\frac{\tau p^2}{\nu(\ell)}
\]

where \( E_{pl}(\ell) \equiv \frac{\nu'((\ell)\ell)}{\nu((\ell))} - \frac{F'((\ell)\ell)}{F((\ell))} > 0 \) and \( E_{pl}(\ell) = \frac{\nu'((\ell)\ell)}{\nu((\ell))} > 0 \). Using these relationships, plus the fact that \( \theta = pe/\mathcal{W} \), yields (after some algebra)

\[
U_2(X, e) = \frac{\tau^2 p^2 e}{\nu(\ell)} (1 - \theta) + \frac{s\Phi}{1 - s} \frac{p^2}{\mathcal{W}} \left[ 1 + \tau \theta \frac{E_{pl}(\ell)}{E_{pl}(\ell)} \right] > 0
\]

so that there is deficient demand. \( \Box \)

**Lemma A.17.** Suppose \( X < X^* \), so that the labor market is tight, i.e., \( N > L \). Then \( dc/dX > 0 \) and there is not deficient demand.

**Proof.** Using the notation defined in the proof of Lemma A.16, we may obtain

\[
\ell = \ell(e) \equiv \Omega^{-1}([1 + \tau (1 - s)]e)
\]

\[
p(e) \equiv \frac{\nu'((\ell)\ell)}{\nu((\ell))}
\]

\[
\theta = \theta(e) \equiv \frac{F((\ell)\ell) - e}{\Phi
\]

\[
\mathcal{W} = \mathcal{W}(e) \equiv p(e) e
\]

where

\[
\bar{\Omega}(\ell) \equiv sF(\ell) + (1 - s) \frac{\nu((\ell)) F'((\ell))}{\nu'((\ell))}
\]

and, by assumption, \( \bar{\Omega}'(\ell) > 0 \) (see footnote 31) so that \( \bar{\Omega}^{-1} \) is well-defined. Equilibrium is then given by a solution to

\[
U'(X + e) = p(e)
\]

for \( e \) (provided this solution satisfies \( \theta(e) > 1 \)). We will clearly have \( dc/dX > 0 \) if and only if \( p'(e) > 0 \). Since \( \bar{\Omega}'(\ell) > 0 \), it is easily verified that \( p'(e) > 0 \), so that indeed \( dc/dX > 0 \).
Next, equilibrium welfare conditional on $e$ is given by

$$U(X, e) = U(X + e) - \nu(\ell(e))$$

Taking derivatives with respect to $e$ and evaluating at the equilibrium, we may obtain

$$U_2(X, e) = \frac{(1 - s)\tau + \frac{\xi_p(\ell)}{\tilde{\xi}_a(\ell)}}{1 - (1 - s)\frac{\xi_p(\ell)}{\tilde{\xi}_a(\ell)}}$$

It may be verified that the assumption $\bar{\Omega}'(\ell) > 0$ implies that the denominator of this expression is strictly positive, and thus $U_2(X, e) < 0$ so that there is not deficient demand.

**Proof of Proposition 10**

The proofs of Lemmas A.16 and A.16 establish that when $X \in (X^*, X^{**})$ we have $dp/de < 0$ and $d\ell/de < 0$, while when $X < X^*$ we have $dp/de > 0$ and $d\ell/de > 0$. Proposition 10 then follows immediately.

**Proof of Proposition 11**

The arguments establishing that $de/dX \leq 0$ (with strict equality as long as $N > 0$) are nearly identical to in the Walrasian bargaining case, and are therefore omitted. Thus, there exist $X^*$ and $X^{**}$ such that for $X < X^*$ the equilibrium satisfies $N \geq L$ (labor market is tight), for $X \in (X^*, X^{**})$ the equilibrium satisfies $0 < N < L$ (labor market is slack), and for $X \geq X^{**}$ we have $N = 0$.

It is easily verified that, when $X > X^*$, $\xi = 0$ and thus the system is identical to the Nash bargaining case with $s = 0$, and thus the desired properties in this case follow directly from Proposition 10. We thus focus only on the case where $X < X^*$.

**Lemma A.18.** There does not exist an equilibrium with $N > L$.

*Proof.* When $N > L$, we have $\xi = 1$. Since $\Phi > 0$, from the zero-profit condition (28) we see that this value of $\xi$ cannot be consistent with an equilibrium.

Thus, for $X < X^*$ we must have $N = L$. This implies

$$\ell = \ell(e) \equiv F^{-1}(e + \Phi) \quad (A.17)$$

$$p(e) = \frac{\nu'(\ell(e))}{F'(\ell(e))} \quad (A.18)$$

Since the matching function is not differentiable at the point $N = L$, any value $\xi < 1$ of the worker’s share of the match surplus can be consistent with zero firm profits as long as $e$ is
appropriately chosen. Equivalently, given $e$, we may obtain a worker’s share consistent with zero profit via

$$\xi = \xi (e) \equiv \frac{(1 + \tau) e - \frac{\nu(\ell(e))}{p(e)}}{(1 + \tau) e - \frac{\nu(\ell(e))}{p(e)} + \Phi} \in [0, 1)$$ (A.19)

Thus, an equilibrium is a solution to

$$U'(X + e) = p(e)$$

for $e$ (provided this solution satisfies $e \geq e^*$, where $e^*$ is the maximum value of $e$ below which the equilibrium features $N < L$), in which case $w$ is given by (27) with $\xi = \xi (e)$, i.e., $w = pe/\ell$. Since $\ell'(e) > 0$, we have $p'(e) > 0$, and thus $dc/dX > 0$.

Next, equilibrium welfare conditional on $e$ is given by

$$U(X, e) = U(X + e) - \nu(\ell(e))$$

Taking derivatives with respect to $e$ and evaluating at the equilibrium, it is straightforward to show that $U_2(X, e) = 0$, so that there is not deficient demand, which completes the proof.

**Proof of Proposition 12**

We suppose there is a competitive insurance industry offering a menu of unemployment insurance contracts. A typical contract is denoted $(h, q)$, where $h$ is the premium, paid in all states, and $q$ is the coverage, which the purchaser of the contract receives if and only if he is unemployed. Both $h$ and $q$ are expressed in units of good 1. Since insurance is only potentially useful when $0 < \mu < 1$, we henceforth assume that this is true. Note also that zero profit of insurers requires that $h = (1 - \mu)q$, where $\rho$ is the fraction of purchasers of the contract that are participant households. This implies that non-participant households will not purchase any such zero-profit contract featuring $q < 0$.

**Lemma A.19.** In any separating equilibrium, no contracts are purchased by participant households.\(^{54}\)

*Proof.\* Suppose there is a separating equilibrium, and let $(h_p, q_p)$ denote the contract purchased by participant households, and $(h_n, q_n)$ that purchased by non-participant households. From the insurer’s zero-profit condition, we must have $h_p = (1 - \mu)q_p$ and $h_n = q_n$. Since non-participant households will always deviate to any contract with $h_p < q_p$, this implies that we must have $q_p < 0$ in such an equilibrium.

Next, for any zero-profit separating contract, the assets of employed participant households are given by $A_e = w\ell - p[(1 - \mu)q_p + e]$ and of unemployed participant households by $A_u = p(\mu q_p - e)$. Note that, since $q_p < 0$ and from the resource constraint $wl > pc$, we must

\(^{53}\) This value is given implicitly by $\Omega (F^{-1} (e^* + \Phi)) + \tau e^* = \Phi$, where $\Omega (\ell) \equiv F(\ell) - \frac{\nu(\ell)F'(\ell)}{\nu'(\ell)}$.

\(^{54}\) Technically, agents are always indifferent between not purchasing a contract and purchasing the trivial contract $(0, 0)$. For ease of terminology, we will assume that this trivial contract does not exist.
have $A_u < 0 < A_e$. Also, the derivative of the household’s objective function with respect to $q_p$ along the locus of zero-profit contracts is given by

$$\frac{\partial \mathcal{U}}{\partial q_p} = p\mu(1 - \mu) [V'(A_u) - V'(A_e)] > 0$$

wherever such a derivative exists. Since $A_u < 0 < A_e$, this derivative must exist at the candidate equilibrium, and therefore in a neighborhood of that equilibrium the objective function is strictly increasing on $q_p < 0$. Thus, given any candidate zero-profit equilibrium contract with $q_p < 0$, there exists an alternative contract $(h_p', q_p')$ with $q_p' > q_p$ which satisfies that $h_p' - (1 - \mu)q_p'$ is strictly greater than but sufficiently close to zero so that participant households would choose it over $(h_p, q_p)$, while non-participant households would not choose it, and therefore insurers could make a positive profit selling it. Thus, $(h_p, q_p)$ cannot be an equilibrium contract. Since this holds for all $q_p < 0$, it follows that no separating equilibrium exists in which contracts are purchased by participant households.

Next, consider a pooling equilibrium, so that $\hat{\rho} = \rho$. As argued above, we must have $q \geq 0$ in any such equilibrium. Assets of an employed worker when choosing a zero-profit pooling contract $(h, q) = ((1 - \mu\rho)q, q)$ are given by $A_e = w\ell - p[(1 - \mu\rho)q + e]$, while $A_u = p(\mu\rho q - e)$ are those of an unemployed worker. Let $\mathcal{U}(q)$ denote the value of the household’s objective function when choosing such a zero-profit pooling contract.

**Lemma A.20.** If $\mathcal{U}(q)$ is strictly decreasing in $q$ whenever $A_e > A_u$, then a pooling equilibrium does not exist.

**Proof.** Note first that if $A_e \leq A_u$, then being unemployed is always strictly preferred to being employed by participant households, so that this cannot represent an equilibrium. Furthermore, as argued above, we must have $q \geq 0$ in any pooling equilibrium. Thus, suppose $A_e > A_u$ and $q > 0$. We show that such a $q$ cannot represent an equilibrium. To see this, let $(h', q')$ denote an alternative contract with $0 < q' < q$ and $h' = (1 - \mu\rho)q'$. Since $\mathcal{U}$ is strictly decreasing in $q$, this contract is strictly preferred by participant households. Furthermore, since non-participant households would get net payment $\mu\rho(q' - q) < 0$ from deviating to this new contract, only participant households would deviate to it, and therefore the expected profit to an insurer offering it would be $(1 - \rho)\mu q' > 0$. Thus, this deviation is mutually beneficial for participants and insurers, and so $q$ cannot be an equilibrium.

**Lemma A.21.** If $\rho < 1/(1 + \tau)$, then there is no equilibrium in which an insurance contract is purchased by participant households.

**Proof.** Note that $\mathcal{U}(q)$ is continuous, with

$$\mathcal{U}'(q) = p\mu[(1 - \mu)\rho V'(A_u) - (1 - \mu\rho) V'(A_e)]$$

wherever this derivative exists (i.e., whenever $A_e A_u \neq 0$). If $A_e A_u > 0$, then $V'(A_e) = V'(A_u)$, and therefore $\mathcal{U}'(q) = -p\mu(1 - \rho)V'(A_e) < 0$. Suppose on the other hand that $A_e A_u < 0$. If in addition $A_e > A_u$, we must have $A_u < 0 < A_e$, and therefore $\mathcal{U}'(q) = -p\mu\{1 - \rho[1 + \tau(1 - \mu)]\}$. Since $\rho < 1/(1 + \tau)$, it follows that $\mathcal{U}'(q) < 0$. Thus, $\mathcal{U}(q)$ is strictly decreasing whenever $A_e > A_u$, and therefore by Lemma A.20, no pooling equilibrium exists. Since, by Lemma A.19, there does not exist a separating equilibrium either, no equilibrium exists.
Proof of Proposition 13

It can be verified that the steady-state level of purchases $e$ solves

$$U'\left(\frac{\delta + \gamma}{\delta}e\right) = \zeta Q(e) \quad \text{(A.20)}$$

where

$$\zeta \equiv \frac{1 - \beta(1 - \delta)}{1 - \beta(1 - \delta) + \beta \gamma} \in (0, 1)$$

Lemma A.22. For $\delta$ sufficiently small, a steady state exists and is unique.

Proof. Similar to in the static case, we may express individual $j$’s optimal choice of steady-state expenditure $e_j$ given aggregate steady-state expenditure $e$ as

$$e_j(e) = \frac{\delta}{\delta + \gamma} U''^{-1}(\zeta Q(e))$$

As before, we can verify that $e_j'(e) < 0$ for $e > e^*$, while $e_j'(e) > 0$ and $e_j''(e) > 0$ for $e < e^*$. Thus, an equilibrium necessarily exists and is unique if $e_j'(e) < 1$ for $e < e^*$, which is equivalent to the condition that $\lim_{\tau \to e^*} e_j'(e) < 1$. This is in turn equivalent to the condition $\tau < \bar{\tau}$, where

$$\bar{\tau} \equiv -\frac{\delta + \gamma}{\delta \zeta} U'' \left( U''^{-1} \left( \zeta \nu'(\ell^*) \right) \right) \frac{F'(\ell^*) [F(\ell^*) - \Phi]}{\nu'(\ell^*)} \quad \text{(A.21)}$$

As $\delta \to 0$, $\bar{\tau}$ approaches infinity, and thus it will hold for any $\tau$, which completes the proof.

Note for future reference that if $e_j'(e) < 1$ then

$$(\delta + \gamma) U''(X + e) < \delta \zeta Q'(e) \quad \text{(A.22)}$$

Lemma A.23. For $\delta$ sufficiently small, there exists a steady state in the unemployment regime.

Proof. Since $U'(0) > Q(0)$ by assumption, we also have $U''(0) > \zeta Q(0)$. Thus, if

$$U'\left(\frac{\delta + \gamma}{\delta}e^*\right) < \zeta Q(e^*)$$

then by the intermediate value theorem, equation A.20 holds for at least one value of $e < e^*$. Note that

$$\lim_{\delta \to 0} \frac{\delta + \gamma}{\delta}e^* = \infty$$

and $\lim_{\delta \to 0} \zeta t = (1 - \beta)/(1 - \beta + \beta \gamma) > 0$. Thus, since $\lim_{\delta \to \infty} U'(e) \leq 0$ by assumption, it follows that

$$\lim_{\delta \to 0} U'\left(\frac{\delta + \gamma}{\delta}e^*\right) \leq 0 < \lim_{\delta \to 0} \zeta Z(e^*)$$

and thus the desired property holds for $\delta$ close enough to zero. \qed

Lemmas A.22 and A.23 together prove the proposition.
Proof of Proposition 14

Linearizing the system in $e_t$ and $X_t$ around the steady state and letting variables with hats denote deviations from steady state and variables without subscripts denote steady-state quantities, we have

$$\hat{X}_{t+1} = (1 - \delta)\hat{X}_t + \gamma \hat{e}_t$$

$$\hat{e}_{t+1} = -\frac{[1 - \beta(1 - \delta)(1 - \delta - \gamma)]U''X + e}{\beta [(1 - \delta)Q'(e) - (1 - \delta - \gamma)U''(X + e)]} \hat{X}_t$$

$$+ \frac{Q'(e) - [1 - \beta\gamma(1 - \delta - \gamma)]U''(X + e)}{\beta [(1 - \delta)Q'(e) - (1 - \delta - \gamma)U''(X + e)]} \hat{e}_t$$

or

$$\hat{x}_{t+1} \equiv \begin{pmatrix} \hat{X}_{t+1} \\ \hat{e}_{t+1} \end{pmatrix} = \begin{pmatrix} 1 - \delta & \gamma \\ a_{eX} & a_{ee} \end{pmatrix} \begin{pmatrix} \hat{X}_t \\ \hat{e}_t \end{pmatrix} \equiv A\hat{x}_t$$

where $a_{eX}$ and $a_{ee}$ are the coefficients on $\hat{X}_t$ and $\hat{e}_t$ in the expression for $\hat{e}_{t+1}$. The eigenvalues of $A$ are then given by

$$\lambda_1 \equiv \frac{1 - \delta + a_{ee} - \sqrt{(1 - \delta + a_{ee})^2 - 4\beta^{-1}}}{2}$$

$$\lambda_2 \equiv \frac{1 - \delta + a_{ee} + \sqrt{(1 - \delta + a_{ee})^2 - 4\beta^{-1}}}{2}$$

We may obtain that

$$\lambda_1\lambda_2 = \beta^{-1} > 1 \quad (A.23)$$

so that $|\lambda_i| > 1$ for at least one $i \in \{1, 2\}$. Thus, this system cannot exhibit local indeterminacy (see, e.g., Blanchard and Kahn (1980)), which completes the proof.

Proof of Proposition 15

Note for future reference that (A.23) implies that if the eigenvalues are real then they are of the same sign, with $\lambda_2 > \lambda_1$.

Lemma A.24. The system is saddle-path stable if and only if

$$|1 - \delta + a_{ee}| > \frac{1 + \beta}{\beta} \quad (A.24)$$

in which case the eigenvalues are real and of the same sign as $1 - \delta + a_{ee}$.

Proof. To see the “if” part, suppose (A.24) holds, and note that this implies

$$(1 - \delta + a_{ee})^2 > \left(\frac{1 + \beta}{\beta}\right)^2 > 4\beta^{-1}$$
and therefore the eigenvalues are real. If \( 1 - \delta + a_{ee} > (1 + \beta)/\beta \), then this implies that \( \lambda_2 > \lambda_1 > 0 \), and therefore the system is stable as long as \( \lambda_1 < 1 \), which is equivalent to the condition

\[
(1 - \delta + a_{ee}) - 2 < \sqrt{(1 - \delta + a_{ee})^2 - 4\beta^{-1}} \tag{A.25}
\]

Since \( 1 - \delta + a_{ee} > (1 + \beta)/\beta > 2 \), both sides of this inequality are positive, and therefore, squaring both sides and rearranging, it is equivalent to

\[
1 - \delta + a_{ee} > \frac{1 + \beta}{\beta} \tag{A.26}
\]

which holds by hypothesis. A similar argument can be used to establish the claim for the case that \(-(1 - \delta + a_{ee}) > (1 + \beta)/\beta\).

To see the “only if” part, suppose the system is stable. If the eigenvalues had non-zero complex part, then \( |\lambda_1| = |\lambda_2| > 1 \), in which case the system would be unstable. Thus, the eigenvalues must be real, i.e., \((1 - \delta + a_{ee})^2 > 4\beta^{-1}\), which in turn implies that

\[
|1 - \delta + a_{ee}| > 2\sqrt{\beta^{-1}}
\]

If \( 1 - \delta + a_{ee} > 2\sqrt{\beta^{-1}} \), then, reasoning as before, \( \lambda_2 > \lambda_1 > 0 \), and therefore if the system is stable then (A.25) must hold. Since \((1 - \delta + a_{ee}) > 2\sqrt{\beta^{-1}} > 2\), then again both sides of (A.25) are positive, and thus that inequality is equivalent to (A.26), which in turn implies (A.24). Similar arguments establish (A.24) for the case where \(-(1 - \delta + a_{ee}) > 2\sqrt{\beta^{-1}}\). \(\square\)

**Lemma A.25.** The system is saddle-path stable with positive eigenvalues if and only if

\[
(1 - \delta - \gamma)U''(X + e) < (1 - \delta)Q'(e) \tag{A.27}
\]

**Proof.** Note that the system is stable with positive eigenvalues if and only if (A.26) holds. We have that

\[
1 - \delta + a_{ee} - \frac{1 + \beta}{\beta} = \frac{[1 - \beta(1 - \delta - \gamma)][\delta\zeta Q'(e) - (\delta + \gamma)U''(X + e)]}{\beta(1 - \delta - \gamma)(\delta\zeta Q'(e) - (\delta + \gamma)U''(X + e))}
\]

Since the numerator is positive by (A.22), inequality (A.26) holds if and only if (A.27) holds. \(\square\)

**Lemma A.26.** If

\[
\tau < \tilde{\tau}^* \equiv -\frac{1 - \delta - \gamma}{1 - \delta} U'' \left( U''^{-1} \left( \zeta \frac{\nu'(\ell^*)}{F'(\ell^*)} \right) \right) \frac{F'(\ell^*)[F(\ell^*) - \Phi]}{\nu'(\ell^*)}
\]

then the system is saddle-path stable with positive eigenvalues.

**Proof.** Note that condition (A.27) always holds around a full-employment steady state. If the steady state is in the unemployment regime, then it can be verified that condition (A.27) holds if and only if

\[
e_j'(e) < \frac{\delta}{\delta + \gamma} \frac{1 - \delta - \gamma}{1 - \delta} \in (0, 1)
\]

where \( e_j'(e) \) is as defined in Lemma A.22. As before, this condition holds for all \( e \) if it holds for \( \lim_{e\uparrow \ell^*} e_j'(e) \), which it can be verified is equivalent to the condition \( \tau < \tilde{\tau}^* \). Note also that \( \tilde{\tau}^* < \tilde{\tau} \), where \( \tilde{\tau} \) was defined in equation (A.21), so that this condition is strictly stronger than the one required to ensure the existence of a unique steady state. \(\square\)
Lemmas A.25 and A.26 together establish that, for \( \tau \) sufficiently small (e.g., \( \tau < \bar{\tau}^* \)), the system converges monotonically to the steady state. It remains to show that consumption is decreasing in the stock of durables. Assuming \( \tau \) is sufficiently small so that the system is saddle-path stable with positive eigenvalues, it is straightforward to obtain the solution

\[
\begin{align*}
\dot{X}_t &= \lambda_1 \dot{X}_0 \\
\dot{\epsilon}_t &= \psi \dot{X}_t \\
\dot{\epsilon}_t &= (1 + \psi \dot{X}_t)
\end{align*}
\]

where \( \psi \equiv -(1 - \delta - \lambda_1)/\gamma \). Thus, consumption is decreasing in the stock of durables if and only if \( \psi < -1 \).

**Lemma A.27.** If (A.27) holds and the steady state is in the unemployment regime, then \( \psi < -1 \).

**Proof.** We may write

\[
1 - \delta - \gamma - \lambda_1 = \frac{\sqrt{[a_{ee} + 2\gamma - (1 - \delta)]^2 + 4\beta^{-1}[(1 - \delta - \gamma)(a_{ee} + \gamma) - 1] - [a_{ee} + 2\gamma - (1 - \delta)]}}{2}
\]

Now, \( a_{ee} + 2\gamma - (1 - \delta) > a_{ee} - (1 - \delta) > 0 \), so that \( 1 - \delta - \gamma - \lambda_1 \) is positive if and only if \( \beta(1 - \delta - \gamma)(a_{ee} + \gamma) > 1 \). We have

\[
\beta(1 - \delta - \gamma)(a_{ee} + \gamma) = \frac{[1 + \beta \gamma (1 - \delta)]Q'(e) - U''(X + e)}{(1 - \delta - \gamma)} Q'(e) - U''(X + e)
\]

Note by earlier assumptions that this expression is strictly positive, and that

\[
\frac{1 - \delta}{1 - \delta - \gamma} - [1 + \beta \gamma (1 - \delta)] = \gamma \frac{1 - \beta(1 - \delta)(1 - \delta - \gamma)}{1 - \delta - \gamma} > 0
\]

Thus, if \( Q'(e) < 0 \) (i.e., the steady state is in the unemployment regime) then \( \beta(1 - \delta - \gamma)(a_{ee} + \gamma) > 1 \), in which case \( 1 - \delta - \gamma - \lambda_1 > 0 \) and therefore \( \psi < -1 \). \( \square \)

**Proof of Proposition 16**

Without loss of generality, assume the alternative path begins at \( t = 0 \), and let \( \bar{\epsilon}_t(\Delta) \equiv e + \Delta \cdot \epsilon_t \) denote the alternative feasible path of expenditures, where \( \epsilon_t \) is the change in the path of expenditures, and \( \Delta \) is a perturbation parameter, which is equal to zero in the steady-state equilibrium and equal to one for the alternative path. Let \( \tilde{X}_t(\Delta) \) denote the associated path for the stock of durables, and note that \( \tilde{X}_0(\Delta) = X \), i.e., this alternative path does not affect the initial stock of durables. Welfare can then be written as a function of \( \Delta \) as

\[
U(\Delta) = \beta \left\{ U(\tilde{X}_t(\Delta) + \bar{\epsilon}_t(\Delta)) + \frac{\tilde{\epsilon}_t(\Delta)}{F'(\ell^*)} \left[ -\nu(\ell^*) + V(w^* \ell^* - p^* \bar{\epsilon}_t(\Delta)) \right] + \left(1 - \frac{\tilde{\epsilon}_t(\Delta)}{F'(\ell^*)} \right) V(-p^* \bar{\epsilon}_t(\Delta)) \right\}
\]
From the envelope theorem, beginning from the steady state path (i.e., $\Delta = 0$), for a marginal change in $\Delta$ the net effect on welfare through the resulting changes in $U$ and $V$ in each period is zero. Thus, we need only consider effects that occur through changes in the employment rate term, $\tilde{e}_t(\Delta)/[F'(\ell^*)\ell^*]$. A first-order approximation to $U(1)$ around $U(0)$ is therefore given by

$$U(1) \approx U(0) + \frac{1}{F'(\ell^*)\ell^*} \left[ \mathcal{L}^* + \frac{\nu'(\ell^*)}{F'(\ell^*)} \epsilon \right] \sum_{t=0}^{\infty} \beta^t \tilde{e}_t(0)$$

Substituting in $\tilde{e}_t(0) = \epsilon_t$, the desired result obtains.

**Proof of Proposition 17**

Let $\tilde{e}_t(\epsilon)$ and $\tilde{X}_t(\epsilon)$ denote alternative paths for expenditure and the stock of durables, with $\tilde{e}_t(\epsilon) \equiv e(\tilde{X}_t(\epsilon)) + \epsilon_t$ and $\tilde{X}_{t+1}(\epsilon) = (1-\delta)\tilde{X}_t(\epsilon) + \gamma \tilde{e}_t(\epsilon)$. Here, $e(\cdot)$ is the equilibrium policy function for expenditures, while $\epsilon_0 = \epsilon$ and $\epsilon_t = 0$ for $t \geq 1$. Letting $U(X_0, \epsilon)$ denote the corresponding welfare as a function of $X_0$ and $\epsilon$, we may write a second-order approximation to this function around $(X_0, \epsilon) = (X, 0)$ as

$$U(X_0, \epsilon) \approx U(X, 0) + U_X \tilde{X}_0 + U_\epsilon \epsilon + \frac{1}{2} \left[ U_{XX} \tilde{X}_0^2 + U_{\epsilon\epsilon} \epsilon^2 \right] + U_{X\epsilon} \tilde{X}_0 \epsilon$$

where variables with hats indicate deviations from steady state and partial derivatives of $U$ are evaluated at the point $(X_0, \epsilon) = (X, 0)$. Clearly, to a second-order approximation, the welfare effect of a temporary stimulus is smaller when the economy is in a liquidation phase if and only if $U_{X\epsilon} < 0$.

Next, using the envelope condition as in the proof of Proposition 16, it is straightforward to obtain that

$$U_t(X_0, 0) = \frac{1}{F'(\ell^*)\ell^*} \sum_{t=0}^{\infty} \beta^t \left[ \mathcal{L}^* + \frac{\nu'(\ell^*)}{F'(\ell^*)} e(X_t) \right] \tilde{e}_t(0)$$

where $X_t = \tilde{X}_t(0)$ is the stock of durables that would occur in the absence of stimulus. One may also obtain that

$$\tilde{e}_t(0) = \begin{cases} 1 & : t = 0 \\ \gamma e'(X_t) \{ \prod_{i=1}^{t-1} [1 - \delta + \gamma e'(X_{t-i})] \} & : t \geq 1 \end{cases}$$

so that

$$U_t(X_0, 0) = \frac{1}{F'(\ell^*)\ell^*} \left\{ \left[ \mathcal{L}^* + \frac{\nu'(\ell^*)}{F'(\ell^*)} e(X_0) \right] + \gamma \sum_{t=1}^{\infty} \beta^t \left[ \mathcal{L}^* + \frac{\nu'(\ell^*)}{F'(\ell^*)} e(X_t(X_0)) \right] \right\}$$

where $X_t(X_0)$ indicates the equilibrium value of $X_t$ given $X_0$. Taking the derivative of this expression with respect to $X_0$ and evaluating at $X_0 = X$ yields

$$U_{Xt}(X, 0) = \frac{1}{F'(\ell^*)\ell^*} \frac{\nu'(\ell^*)}{F'(\ell^*)} \cdot \frac{1 - \beta \lambda_1 (1-\delta)}{1 - \beta \lambda_1^2} \psi + \Xi e''(X)$$
where \( \psi \equiv e'(X) < 0 \), which was computed above, and \( \Xi \) is some strictly positive number. Since \( \lambda_1 < 1 \), the first term on the right-hand side of this expression is clearly negative. Thus, there is a strictly positive number \( \xi \) such that if \( e''(X) < \xi \) we will have \( U_{Xt}(X, 0) < 0 \), which is the desired result.

Letting \( \chi(X_t) \) denote the equilibrium value of \( X_{t+1} \) given \( X_t \), we may re-express the equilibrium equations governing the dynamics of the system (i.e., equations (32) and (33)) as

\[
\chi(X_t) = (1 - \delta)X_t + \gamma e(X_t)
\]

and

\[
U'(X_t + e(X_t)) - Q(e(X_t)) = \beta [(1 - \delta - \gamma)U'(\chi(X_t) + e(\chi(X_t))) - (1 - \delta)Q(e(\chi(X_t)))]
\]

Taking derivatives of both sides of these equations twice with respect to \( X_t \), evaluating at \( X_t = X \) and solving for \( e''(X) \), we may obtain that \( e''(X) = bU'''(X + e) \), where \( b \) is some number that does not depend on \( U'''(X + e) \). Thus, if \( U''' \) is sufficiently close to zero, \( e''(X) < \xi \) and the desired result holds.

**Proof of Proposition C.1**

The following result will be useful.

**Lemma A.28.** Let \( \varepsilon_{X,M} \) denote the elasticity of substitution between \( X \) and \( M \) embodied in \( g \). Then

\[
\varepsilon_{X,M} = \frac{g_X(X, M)g_M(X, M)}{g_{XM}(X, M)g(X, M)}
\]  

(A.28)

**Proof.** Letting \( H^k \) denote homogeneity of degree \( k \), note first that, since \( g \) is \( H^1 \), for \( a, b \in \{X, M\} \), \( g_a \) is \( H^0 \) and \( g_{ab} \) is \( H^{-1} \).

Next, by definition, we have

\[
\varepsilon_{X,M}^g \equiv \left[ \frac{d \log (g_X(X, M)/g_M(X, M))}{d \log (M/X)} \right]^{-1}
\]

Letting \( \tilde{M} \equiv M/X \) and using \( H^0 \) of \( g_X \) and \( g_M \), we may obtain

\[
\varepsilon_{X,M}^g = \frac{g_X(1, \tilde{M})}{g_1(1, \tilde{M})} \left[ \frac{d}{d \tilde{M}} \left( \frac{g_X(1, \tilde{M})}{g_M(1, \tilde{M})} \right) \right]^{-1}
\]

\[
= \frac{g_X(1, \tilde{M})g_M(1, \tilde{M})}{\tilde{M} [g_{XM}(1, \tilde{M})g_1(1, \tilde{M}) - g_X(1, \tilde{M})g_{MM}(1, \tilde{M})]}
\]

\[
= \frac{g_X(X, M)g_M(X, M)}{\tilde{M} [g_{XM}(X, M)g_M(X, M) - g_X(X, M)g_{MM}(X, M)]}
\]

where the last line follows from \( H^0 \) of \( g_a \) and \( H^{-1} \) of \( g_{ab} \). Adding and subtracting \( g_{XM}(X, M)g_X(X, M)X \) in the denominator and grouping terms yields

\[
\varepsilon_{X,M}^g = \frac{g_X(X, M)g_M(X, M)}{g_{XM}(X, M) [g_X(X, M)X + g_M(X, M)M] - g_X(X, M) [g_{XM}(X, M)X + g_{MM}(X, M)M]}
\]
The first bracketed term in the denominator equals $g(X, \mathcal{M})$ by $H^1$ of $g$, while the second bracketed term equals 0 by $H^0$ of $g$, and thus equation (A.28) follows.

Next, let $W(X, \mathcal{M}) \equiv U(g(X, \mathcal{M}))$. Then the equilibrium condition (C.1) can be written

$$W_M(X, \mathcal{M}) = Q(\mathcal{M})$$

(A.29)

Note that

$$W_{MM}(X, \mathcal{M}) = [g_M(X, \mathcal{M})]^2 U''(g(X, \mathcal{M})) + g_{MM}U'(g(X, \mathcal{M})) < 0$$

so that the left-hand side of equation (A.29) is strictly decreasing in $\mathcal{M}$. To ensure the existence of an equilibrium with $\mathcal{M} > 0$, we assume that $W_M(X, 0) > Q(0)$. We further assume that $g_{MMM}(X, \mathcal{M}) \geq 0$, which ensures that $Q_{MMM} > 0$, and therefore, similar to in the durable-goods model, there are at most three equilibria: at most two in the unemployment regime, and at most one in the full-employment regime. Additional conditions under which we can ensure that there exists a unique equilibrium are similar in flavor to in the durable-goods case, though less easily characterized explicitly. We henceforth simply assume conditions are such that the equilibrium is unique, and note that this implies that

$$W_{MM}(X, \mathcal{M}) < Q'(\mathcal{M})$$

(A.30)

at the equilibrium value of $\mathcal{M}$. Define also

$$\mathcal{E}_M^Q \equiv \frac{Q'(\mathcal{M})\mathcal{M}}{Q(\mathcal{M})}$$

as the elasticity of $Q$ with respect to $\mathcal{M}$.

**Lemma A.29.** $dc/dX < 0$ if and only if

$$-\mathcal{E}_M^Q \mathcal{E}_X^g > 1$$

(A.31)

**Proof.** Differentiating the equilibrium condition (A.29) with respect to $X$ yields that

$$\frac{d\mathcal{M}}{dX} = \frac{W_{XM}(X, \mathcal{M})}{Q'(\mathcal{M}) - W_{MM}(X, \mathcal{M})}$$

(A.32)

Doing the same with the equilibrium condition $c = g(X, \mathcal{M})$ yields

$$\frac{dc}{dX} = g_X(X, \mathcal{M}) + g_M(X, \mathcal{M}) \frac{d\mathcal{M}}{dX}$$

$$= \frac{g_X(X, \mathcal{M}) [Q'(\mathcal{M}) - W_{MM}(X, \mathcal{M})] + g_M(X, \mathcal{M})W_{XM}(X, \mathcal{M})}{Q'(\mathcal{M}) - W_{MM}(X, \mathcal{M})}$$

where the second line has used (A.32). By (A.30), the denominator is positive, so that this expression is of the same sign as the numerator. Substituting in for $W_{MM}$ and $W_{XM}$ and using the equilibrium condition (A.29), we may obtain that $dc/dX < 0$ if and only if

$$\left[ \frac{g_{XM}(X, \mathcal{M})}{g_X(X, \mathcal{M})} - \frac{g_{MM}(X, \mathcal{M})}{g_M(X, \mathcal{M})} \right] \mathcal{M} < -\mathcal{E}_M^Q$$

(A.33)
The term in square brackets, meanwhile, can be written as
\[
g_{XM}(X,M)[g_X(X,M)X + g_M(X,M)M] - g_X(X,M)[g_{XM}(X,M)X + g_{MM}(X,M)M]
g_X(X,M)g_M(X,M)M
\]
By \(H^0\) of \(g_{MM}\), the second term in the numerator equals zero, and thus by \(H^1\) of \(g\), we have
\[
g_{XM}(X,M) - g_{MM}(X,M) = \frac{g_{XM}(X,M)g(X,M)}{g_X(X,M)g_M(X,M)M}
\]
Substituting this into (A.33) and using (A.28) yields (A.31).

If the economy is in the full-employment regime, \(\mathcal{E}_M^Q > 0\) and therefore, since \(\mathcal{E}_{XM}^Q > 0\), condition (A.31) cannot hold. Thus, from Lemma A.29, if the economy is in the full-employment regime, \(dc/dX > 0\). If instead the economy is in the unemployment regime, then \(\mathcal{E}_M^Q < 0\), and therefore condition (A.31) can hold as long as \(\mathcal{E}_{XM}^Q\) is sufficiently large, which completes the proof of the proposition.

**Proof of Proposition C.2**

Let \(y = g(X_1, \mathcal{M})\) denote output of the final good in the first period. Furthermore, let \(B(X_2) \equiv U''^{-1}(R(X_2)) + X_2\) denote the total resources (output plus undepreciated first-period capital) that would be required for the choice \(X_2\) to satisfy the constraints (C.2) and (C.3) as well as the intertemporal optimality condition (C.5), and note that
\[
B'(X_2) = \frac{R'(X_2)}{U''(c)} + 1 > 1 \tag{A.34}
\]
where the inequality follows from the assumption made that \(R'(X_2) < 0\). Since total resources actually available are \((1-\delta)X_1 + g(X_1, \mathcal{M})\), we have \(X_2 = B^{-1}((1-\delta)X_1 + g(X_1, \mathcal{M}))\), and therefore from condition (C.4) equilibrium can be characterized by a solution to
\[
G(X_1, \mathcal{M}) = Q(\mathcal{M}) \tag{A.35}
\]
for \(\mathcal{M}\), where \(G(X, \mathcal{M}) \equiv g_M(X, \mathcal{M})R(B^{-1}((1-\delta)X + g(X, \mathcal{M})))\). Note that
\[
G_M(X_1, \mathcal{M}) = g_{MM}(X_1, \mathcal{M})R(X_2) + \frac{R'(X_2) [g_M(X_1, \mathcal{M})]^2}{B'(X_2)} < 0
\]
Similar to in the static case, we assume that \(G(X, 0) > Q(0)\) so that there is an equilibrium with \(\mathcal{M} > 0\), and further, conditions are such that this equilibrium is unique, which implies that
\[
G_M(X_1, \mathcal{M}) < Q'(\mathcal{M}) \tag{A.36}
\]
at the equilibrium value of \(\mathcal{M}\).

**Lemma A.30.** If \(dX_2/dX_1 < 0\) then \(dc/dX_1 < 0\) and \(di/dX_1 < 0\).
Proof. Since in equilibrium $c + X_2 = B(X_2)$, we have that
\[
\frac{dc}{dX_1} = [B'(X_2) - 1] \frac{dX_2}{dX_1}
\]
Since $B'(X_2) > 1$, if $dX_2/dX_1 < 0$ then $dc/dX_1 < 0$. Further, if $X_2$ falls when $X_1$ rises, from the capital accumulation equation (C.2) we see that $i$ must also fall.

**Lemma A.31.** $dX_2/dX_1 < 0$ if and only if
\[
\left\{-\mathcal{E}_M^Q + \frac{(1 - \delta)g_M(X, M)}{g_X(X, M)[g_X(X, M) + 1 - \delta]}\right\} \mathcal{E}_M^g > 1 \tag{A.37}
\]

**Proof.** Differentiating the equilibrium condition (A.35) with respect to $X_1$ yields that
\[
\frac{dM}{dX_1} = \frac{G_X(X_1, M)}{Q'(M) - G_M(X_1, M)} \tag{A.38}
\]
Doing the same with $y = g(X, M)$ yields
\[
\frac{dy}{dX_1} = g_X(X_1, M) + g_M(X_1, M) \frac{dM}{dX_1} \tag{A.39}
\]
while differentiating $X_2 = B^{-1}((1 - \delta)X_1 + g(X_1, M))$ yields
\[
\frac{dX_2}{dX_1} = \frac{1}{B'(X_2)} \left(1 - \delta + \frac{dy}{dX_1}\right)
\]
\[
= \frac{[1 - \delta + g_X(X_1, M)] [Q'(M) - G_M(X_1, M)] + g_M(X_1, M)G_X(X_1, M)}{B'(X_2) [Q'(M) - G_M(X_1, M)]}
\]
where the second line has used equations (A.38) and (A.39). Since the denominator of this expression is positive by (A.34) and (A.36), the sign of $dX_2/dX_1$ is given by the sign of the numerator. Substituting in for $G_M$ and $G_X$ and using (A.35), some algebra yields that this expression is negative if and only if condition (A.37) holds.

Lemmas A.30 and A.31 together indicate that $dc/dX_1 < 0$ and $di/dX_1 < 0$ both hold if and only if condition (A.37) holds. Further, for a given equilibrium level of $M$, it is clear that the minimum level of $\mathcal{E}_M^g$ needed to satisfy (A.37) is (weakly) greater than that needed to satisfy (A.31) in the static case.
B Nash bargaining

Here we consider the static model of section 2 and replace the “competitive” determination of \( w \) and \( \ell \) within a match by Nash bargaining.

The gain from a match for a firm is \( pF(\ell) - w\ell \) while outside option is zero. The gain for the household is \( -\nu(\ell) + V(\ell) - p(c - X) \) while the outside option is \( V(-p(c - X)) \). Using the piecewise linear specification for \( V \), the Nash-Bargaining criterion \( W \) is:

\[
W = \left( pF(\ell) - w\ell \right)^{1-s} \left( -\nu(\ell) + vw\ell + v\tau p(c - X) \right)^s
\]

Maximizing \( W \) w.r.t. \( \ell \) and \( w \) gives the following F.O.C.:

\[
\frac{(1-s)W}{pF(\ell) - w\ell} \left( \frac{pF'(\ell) - w}{pF(\ell) - w\ell} \right) = \frac{sW}{-\nu(\ell) + vw\ell + v\tau p(c - X)} \left( vw - \nu'(\ell) \right)
\]

\[
\frac{W}{pF(\ell) - w\ell} = \frac{sW}{-\nu(\ell) + vw\ell + v\tau p(c - X)} \nu'(\ell)
\]

Rearranging gives the two equations

\[
vpF'(\ell) = \nu'(\ell)
\]

\[
vw\ell = spF(\ell) + (1-s)\nu(\ell) - (1-s)v\tau p(c - X)
\]

Thus, an equilibrium is given by a solution to the five equations:

\[
u'(c) = \frac{\nu'(\ell)}{F'(\ell)} \left( 1 + \tau - \frac{M(N,L)}{L} \right)
\]

\[
w\ell = spF(\ell) + \frac{1-s}{v}\nu(\ell) - (1-s)v\tau p(c - X)
\]

\[
vpF'(\ell) = \nu'(\ell)
\]

\[
M(N,L)F(\ell) = L(c - X) + N\Phi
\]

\[
M(N,L)(pF(\ell) - w\ell) = pN\Phi
\]
C A version with productive capital

We have shown how a rise in the supply of the capital good $X$, by decreasing demand for employment and causing households to increase precautionary savings, can perversely lead to a decrease in consumption. While thus far we have considered the case where $X$ enters directly into the utility function, in this section we show that Proposition 4 can be extended to the case where $X$ is introduced as a productive capital good. To explore this in the simplest possible setting, suppose there are now two types of firms and that the capital stock $X$ no longer enters directly into the agents’ utility function. The first type of firm remains identical to those in the first version of the model, except that instead of producing a consumption good they produce an intermediate good, the amount of which is given by $\mathcal{M}$. There is also now a continuum of competitive firms who rent the productive capital good $X$ from the households and combine it with goods purchased from the intermediate goods firms in order to produce the consumption good according to the production function $g(X, \mathcal{M})$. We assume that $g$ is strictly increasing in both arguments and concave, and exhibits constant returns to scale. Given $X$, it can be verified that the equilibrium determination of $\mathcal{M}$ will then be given as the solution to

$$g_{\mathcal{M}}(X, \mathcal{M})U'(g(X, \mathcal{M})) = Q(\mathcal{M}) \quad (C.1)$$

where $Q(\cdot)$ is defined in equation (14).

Note the similarity between condition (C.1) and the corresponding equilibrium condition for the durable-goods version of the model, which can be written $U'(X + e) = Q(e)$. In fact, if $g(X, \mathcal{M}) = X + \mathcal{M}$, so that the elasticity of substitution between capital and the intermediate good $\mathcal{M}$ is infinite, then the two conditions become identical, and therefore $X$ affects economic activity in the productive-capital version of the model in exactly the same way as it does in the durable-goods model. Thus, a rise in $X$ leads to a fall in consumption when the economy is in the unemployment regime. In fact, as stated in Proposition C.1, this latter result will hold for a more general $g$ as long as $g$ does not feature too little substitutability between $X$ and $\mathcal{M}$.55

Proposition C.1. If the equilibrium is in the full-employment regime, then an increase in productive capital leads to an increase in consumption. If the equilibrium is in the unemployment regime, then an increase in productive capital leads to a decrease in consumption if and only if the elasticity of substitution between $X$ and $\mathcal{M}$ is not too small.

The reason for the requirement in Proposition C.1 that the elasticity of substitution be sufficiently large relates to the degree to which an increase in $X$ causes an initial impetus that favors less employment. If the substitutability between $X$ and $\mathcal{M}$ is small, so that complementarity is large, then even though the same level of consumption could be achieved at a lower level of employment, a social planner would nonetheless want to increase employment. Since the multiplier process in our model simply amplifies – and can never reverse – this

55 We assume throughout this section that an equilibrium exists and is unique. Conditions under which this is true are similar to the ones obtained for the durable-goods model, though the presence of non-linearities in $g$ makes explicitly characterizing them less straightforward in this case.
initial impetus, strong complementarity would lead to a rise in employment and therefore a rise in consumption, rather than a fall. In contrast, if this complementarity is not too large, then an increase in $X$ generates an initial impetus that favors less employment, which is in turn amplified by the multiplier process, so that a decrease in consumption becomes more likely.\footnote{Note that a rise in $X$ also increases output for any given level of employment. To ensure that consumption falls in equilibrium, we require that the substitutability between $X$ and $\mathcal{M}$ be large enough so that the drop in employment more than offsets this effect.}

Let us emphasize that the manner in which we have just introduced productive capital into our setup is incomplete – and possibly unsatisfying – since we are maintaining a static environment with no investment decision. In particular, it is reasonable to think that the more interesting aspect of introducing productive capital into our setup would be its effect on investment demand. To this end, we now consider extending the model to a simple two-period version that features investment. The main result from this endeavor is to emphasize that the conditions under which a rise in $X$ leads to a fall in consumption are weaker than those required for the same result in the absence of investment. In other words, our results from the previous section extend more easily to a situation where $X$ is interpreted as physical capital if we simultaneously introduce an investment decision. The reason for this is that, in the presence of an investment decision, a rise in $X$ is more likely to cause an initial impetus in favor of less activity.

To keep this extension as simple as possible, let us consider a two-period version of our model with productive capital (where there remains two sub-periods in each period). In this case, it can be verified that the continuation value for household $j$ for the second period is of the form $R(X_2) \cdot X_{2,j}$, where $X_{2,j}$ is capital brought by household $j$ into the second period and $X_2$ is capital brought into that period by all other households. In order to rule out the possibility of multiple equilibria that could arise in the presence of strategic complementarity in investment, we assume we are in the case where $R'(X_2) < 0$. The description of the model is then completed by specifying the capital accumulation equation,

$$X_2 = (1 - \delta)X_1 + i \quad (C.2)$$

where $i$ denotes investment in the first period and $X_1$ is the initial capital stock, as well as the new first-period resource constraint,

$$c + i = g(X_1, \mathcal{M}) \quad (C.3)$$

Given this setup, we need to replace the equilibrium condition from the static model (equation (C.1)) with the constraints (C.2) and (C.3) plus the following two first-order conditions,

$$g_\mathcal{M}(X_1, \mathcal{M})U'(c) = Q(\mathcal{M}) \quad (C.4)$$

$$U'(c) = R(X_2) \quad (C.5)$$

Equation (C.4) is the household’s optimality condition for its choice of consumption, and is similar to its static counterpart (C.1), while equation (C.5) is the intertemporal optimality condition equating the marginal value of consumption with the marginal value of investment.
Of immediate interest is whether, in an unemployment-regime equilibrium, a rise in $X_1$ will produce an equilibrium fall in consumption and/or employment in the first period. As Proposition C.2 indicates, the conditions under which our previous results extend are weaker than those required in Proposition C.1 for the static case, in the sense that lower substitution between $X$ and $M$ is possible.

**Proposition C.2.** *In the two-period model with productive capital,*\(^57\) an increase in capital leads to a decrease in both consumption and investment if and only if the elasticity of substitution between $X$ and $M$ is not too small. Furthermore, for a given level of equilibrium employment, this minimum elasticity of substitution is lower than that required in Proposition C.1 in the absence of investment decisions.

The intuition for why consumption and investment fall when the elasticity of substitution is high is similar to in the static case. The addition of the investment decision has the effect of making it more likely that an increase in $X$ leads to a fall in consumption because the increase in $X$ decreases investment demand, which in turn increases unemployment and precautionary savings.

---

\(^{57}\) We are again assuming that the equilibrium exists, is unique, and is in the unemployment regime.


D Noise shock extension

For the extension discussed at the end of Section 4.2, we re-introduce the first-sub-period \((\theta)\) and second-sub-period \(\tilde{\theta}\) productivity factors to the model, and assume that \(\tilde{\theta}_t = \theta_t\). We assume that the economy is always in the unemployment regime, and that all agents come into the first sub-period of period \(t\) with the same belief about the value of \(\theta_t\), but that after the household splits to go to market, the true value is revealed to the workers and firms, while the shoppers retain their initial belief.

To abstract from issues relating to uncertainty about the true value of \(\theta_t\), we assume that all agents are subjectively certain – though possibly incorrect – about the entire stream of productivity values \(\theta_t\), only updating such a belief if they receive some information that contradicts it. One may verify that, in the unemployment regime, shoppers’ prior beliefs are never contradicted until re-uniting with the workers after making their purchases. We denote agents’ belief about \(\theta_t\) at the beginning of date \(s\) by \(\tilde{\theta}_{t|s}\).

In the example constructed, we assume that productivity is constant at \(\theta_t = 1\) for all \(t \in \mathbb{Z}\), but that at the beginning of \(t = 0\), agents receive information such that \(\tilde{\theta}_{t|0} = \theta > 1\) for all \(t \geq 0\), i.e., that productivity has risen permanently. After the households split, workers and firms learn that in fact productivity has not changed, nor will it in the future. Shoppers do not receive this information until after making their purchases, so that for one shopping period they are overly optimistic. In all subsequent periods \(s \geq 1\), however, we have \(\tilde{\theta}_{t|s} = \theta_t = 1\).