Search Frictions and Wage Dispersion

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Abstract

We propose a way to measure the contribution of search frictions to the level of wage dispersion observed in the data. Using the data from the 1979 cohort of the National Longitudinal Survey of Youth we find that the variance of match qualities between workers and employers accounts for about 6% of the variance of log wages. Our method relies on a minimal set of assumptions, the main among them is that match quality is constant over the duration of a job. We show that this assumption can be verified empirically and is supported by the data.

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1 Introduction

A robust finding in countless empirical studies is that over a half of the observed wage variation cannot be accounted for by the observable worker characteristics. Yet, understanding the reasons for why observationally similar workers are paid differently is crucial for understanding the functioning of the labor market.

One possibility is that workers who look the same in the data available to an econometrician are actually different in their productivities and these productivity differences are reflected in wages. Moreover, workers’ productivity may evolve over the life-cycle and while the usual worker characteristics used in empirical work, such as age, gender, education, etc. may proxy relatively well for workers’ average productivities, they may not be flexible enough to describe the evolution of these productivities.

Alternatively, it is possible that observationally equivalent workers are the same in terms of their productivity and the differences in their wages reflect luck in locating productive job matches (Mortensen (2005) articulates this view). One line of research (e.g., Butters (1977), Burdett and Judd (1983), Mortensen (1991), Burdett and Mortensen (1998)) has determined conditions under which dispersion in wage policy is the only equilibrium outcome in the labor market with search frictions even if all employers and employees are identical in their productivity. In another class of models with labor market search (e.g., Dimond (1982), Mortensen (1982), Pissarides (1985, 2000), Mortensen and Pissarides (1994)) workers and firms bargain over match surplus implied by search frictions and this implies wage dispersion given productivity dispersion across worker-employer matches.

The objective of this paper is to assess empirically the contribution of search frictions to wage dispersion. Our approach is as follows. Suppose individual wages depend on general human capital accumulated with labor market experience and transferable across employers, employer-specific human capital accumulated with firm tenure, the quality of the match between the worker and the firm, and idiosyncratic worker productivity that contains a fixed and transitory components. Suppose we could obtain unbiased estimates of the returns to tenure, experience, and individual fixed effects (we discuss our approach below) and subtract their contribution from wages. The resulting residual wages contain only the match
quality term and the individual productivity realization (and, perhaps, the measurement error). We are interested in the variance of match qualities. It can be computed as the difference between the variance of residual wages and the variance of the individual productivity process. Assuming that match quality is constant on a job, the latter can be directly computed as the weighted average variance of wages within jobs (constant match quality does not affect the within-job wage variance). Applying this decomposition to the data from the U.S. National Longitudinal Survey of Youth we obtain that the variance of match qualities equals 0.016, or about 6% of the variance of log wages in the data.

This approach to assessing the role of search frictions is quite simple and yet quite general because it does not require any assumptions on, e.g., the distributions of wages or idiosyncratic productivity process, and it does not require us to take a stand on the value of parameters that are difficult to measure, such as the flow utility of unemployed workers. The only assumptions that this approach requires is that match quality is constant on a job and that wages reflect this match quality.

To assess whether the assumption that match quality is constant on the job is appropriate we consider the wage growth of workers who remain in the same match between two consecutive periods. If match quality is stochastic, workers who received negative innovations to their match quality are more likely to leave for other jobs. Thus, the sample of wage stayers is selected in favor of workers with positive innovations to their match quality. The strength of this selection effect depends on ease of locating alternative employers for workers with negative innovations. The probability of receiving an outside offer depends positively on the observed labor market tightness in most search models. Thus, if match quality is stochastic, wage growth of job stayers should depend positively on labor market tightness. We show that this is not the case in the data, implying that the model with constant match quality is a better description of the labor market (constant match quality does not affect the wage growth of job stayers and the selection effect is absent). This argument extends to models where the match quality is constant but is learned over time. Due to the same selection mechanism such models imply that wage growth of job stayers should be increasing in the probability of receiving an offer and we do not observe this in the data.
Second, the models featuring commitment power of workers or firms imply that various forms of contracts may decouple wages from the match quality. For example, the models of Postel-Vinay and Robin (2002) and Cahuc, Postel-Vinay, and Robin (2006) feature on-the-job search, just as our benchmark model, but also feature commitment of firms to matching outside offers received by the worker. In such models wages increase on the job when a worker receives an outside offer. Thus, the wage growth on the job is once again a function of the probability of receiving an outside offer. As we already discussed above, the data suggest that the wage growth is independent of the probability to receive an offer implying the lack of evidence for the importance of offer matching contracts in determining wages.

Finally, it is possible that the match quality is stochastic but firms smooth the fluctuations in the remuneration. We show that such insurance contracts leave our conclusions unaffected. In particular, in this paper we are only interested in decomposing the observed volatility of wages and not in measuring the amount of inefficiency that might be due to search frictions. Thus, the mechanism that translates differences in productivity into differences in wages (e.g., spot wages or insurance contracts) is not relevant for our analysis.

In an important related recent contribution, Hornstein, Krusell, and Violante (2009) study the ability of search models to generate frictional wage dispersion. They argue that the structure of the search model implies tight restrictions between the amount of wage dispersion the model can generate and the magnitude of labor market flows that can be directly measured. Their argument is strongest in models where workers cannot search on the job. In particular, they show that substantial wage dispersion due to search frictions is inconsistent with the observations that unemployment durations are very short in U.S. data. Given the plausible range of values for the disutility of being unemployed, short unemployment durations directly imply that the option value of waiting for a good offer is small.

Their approach is less powerful if workers are able to search on the job. And indeed, job-to-job flows are large in the data suggesting that this is an important restriction. If on-the-

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1Based on the monthly Current Population Survey (CPS) data, Fallick and Fleischman (2004), for example, estimate that in the U.S. about 2.7% of employed workers move job-to-job every month. Moscarini
job search is as efficient as search while unemployed, the duration of unemployment contains little information about the distribution of potential match qualities because unemployed workers take the first draw that dominates the value of non-market activity and continue searching while employed. More generally, the more efficient is on-the-job search, the less informative is the duration of unemployment for the distribution of matches. There is some information contained in the frequency of observed job-to-job moves but how this translates into the probability to receive an offer is strongly model-dependent. As a result, Hornstein, Krusell, and Violante (2009) show that the observed job-to-job moves imply fairly wide bounds on the extent of wage dispersion that may be attributed to search frictions. For example, wage dispersion is relatively small in the model of Burdett and Mortensen (1998) on the one hand and large in the models of Postel-Vinay and Robin (2002) and Cahuc, Postel-Vinay, and Robin (2006) on the other hand.

We view our approach as complementary to that of Hornstein, Krusell, and Violante (2009). While they study how much dispersion a search model is capable of generating, our objective is to assess how much dispersion is due to the variance of match qualities in the data. In this sense, our analysis provides a simple way to empirically discipline search models. If the model satisfies the assumptions underlying our analysis, we provide a target of how much dispersion in match qualities it should be generating. If the model does not satisfy our assumptions it nevertheless has to be consistent with the finding that the wage growth of job stayers is independent of the job finding rate.

The paper is organized as follows. In Section 2 we present a theoretical framework underlying our analysis. In Section 3 we develop our method to measure the dispersion of match qualities. We proceed in two steps. First, we construct residual wages by subtracting the contribution of tenure, experience, and individual fixed effects from wages. Various approaches can be employed to estimate these components of wages depending on the assumptions one is willing to make on the nature of the search process. The method we use relies on extending the approach in Abraham and Farber (1987), Altonji and Shakotko (1987), Topel (1991), and Altonji and Williams (2005). In the second step we decompose

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2In the earlier versions of the paper we followed the more complicated approach in Hagedorn and Thomsson (2007) show that a different treatment of missing observations in monthly CPS data raises this estimate to 3.2%.
the residual wage variance into the variance of match qualities and the variance of individual productivities. In Section 4 we discuss several possible generalizations of the benchmark model, including allowing for the stochastic match quality and various forms of contracts determining wages. In Section 5 we describe the data and present the results of the empirical evaluation of the contribution of match qualities, tenure and experience to wages. In Section 6 we calibrate our theoretical model and use it to evaluate the performance of our method in measuring job match qualities and the returns to tenure and seniority. Finally, we conduct counterfactual experiments to study the effects on wage distribution from hypothetically eliminating search frictions. Section 7 concludes.

2 Model with On-the-job search

A continuum of risk-neutral workers of measure one participates in the labor market. At a moment in time, each worker can be either employed or unemployed. An unemployed worker faces a probability $\lambda_\theta$ of getting a job offer. This probability depends exogenously on a business cycle indicator $\theta$ and is increasing in $\theta$. For example, a high level of $\theta$ (say, a high level of market tightness or low level of unemployment rate) means that it is easy to find a job, since $\lambda_\theta$ is high as well. Employed workers also face a probability $q_\theta$ of getting a job offer, which also depends monotonically on $\theta$. A worker who accepts the period $t$ offer, starts working immediately for the new employer in period $t + 1$. The unemployment rate in period $t$ is denoted $u_t$. The business cycle indicator $\theta_t$ is a stochastic process which is drawn from a stationary distribution.

A match between worker $i$ and a job/employer/firm $j$ at date $t$ is characterized by an idiosyncratic productivity level $\phi_{ijt}$. Each time a worker meets a new employer, a new value of $\phi$ is drawn, according to a distribution function $F$ with support $[\underline{\phi}, \bar{\phi}]$, density $f$ and expected value $\mu_\phi$. A worker switches if and only if the present value of wages (and thus lifetime utility) increases. The level of $\phi$ and thus productivity remain unchanged as long

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Manovskii (2010) and reached the same conclusions.

Workers thus maximize expected discounted income if the market interest rate $r$ and the discount factor $\beta$ satisfy $\frac{1}{1 + r} = \beta$. If instead the worker is risk averse and markets are complete, then maximizing expected utility and maximizing expected income are also equivalent (Rogerson, Shimer, and Wright (2005)).
as the worker does not switch, $\phi_{ijt-1} = \phi_{ijt}$.

The wage in the model is consistent with the standard specifications in empirical literature. The accumulation of firm-specific and general human capital increase wages. Wages increase by $e^{\beta_1 X_{ijt}}$ if total labor market experience equals $X_{ijt}$ reflecting the accumulation of general human capital. Wages also increase by $e^{\beta_2 T_{ijt}}$ if tenure equals $T_{ijt}$ reflecting the accumulation of firm-specific human capital. The wage also has a fixed individual specific component, $\mu_i$, a transitory individual specific component, $\epsilon_{it}$, and a fixed job match specific component, $\phi_{ij}$, and thus equals

$$e^{\beta_1 X_{ijt} + \beta_2 T_{ijt} + \mu_i \phi_{ijt} \epsilon_{it}},$$

so that the log wage equals

$$\log(W_{ijt}) = \beta_1 X_{ijt} + \beta_2 T_{ijt} + \log(\mu_i) + \log(\phi_{ijt}) + \log(\epsilon_{it}).$$

We abstract from time effects and from nonlinear terms in experience and tenure. Later we will allow for nonlinearity and show that is inconsequential for our results. To model time effects we could add the unemployment rate or labor market tightness in period $t$ to the model. Again this would not change our results since we allow for time dummies in the empirical implementation.

An employed worker faces an exogenous probability $s$ of getting separated and becoming unemployed. For every worker who left unemployment in period 0 and has worked continuously since then we first define an employment cycle. Assume that the worker switched employers in periods $1 + S_1, 1 + S_2, \ldots, 1 + S_k$, so that this worker stayed with his first employer between periods 0 and $S_1$, with the second employer between period $1 + S_1$ and $S_2$, and with employer $i$ between period $1 + S_{i-1}$ and $S_i$.

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4We show later that assumption describes the data best.
6To be precise, $X$ years of experience increase general human capital by $\beta_1 x$ and $T$ years of tenure increase firm specific human capital by $\beta_2 T$. Note that we assume that all benefits from human capital accrue to workers. For general human capital this seems reasonable as these skills are transferable across employers. For specific human capital we could instead assume that only a fraction $\chi$ accrues to workers and that the $\beta_2 = \chi \tilde{\beta}_2$, where $\tilde{\beta}_2$ is the true return to specific human capital. We could do the same for general human capital. All our results would remain unchanged.
3 Measuring Wage Dispersion

Let $T_{ijt}$ be completed tenure of individual $i$ in job $j$ at time $t$. We approximate $\phi_{ijt}^k$ as a function of $T_{ij}$ and error $E(\delta_{ij}) = 0$:

$$\log(\phi_{ijt}) = \alpha_0(T) + \alpha_1 T_{ij} + \alpha_2 T_{ij}^2 + \delta_{ijt},$$

where $T$ is tenure in the current job. We allow the mean of $\phi$ do change with tenure even after controlling for completed tenure, as we allow $\alpha_0$ do depend on $T$. The relationship between $\phi$ and completed tenure is just a statistical one. If $\phi$ was observable these coefficients could be obtained by an OLS regression in the data. For our theoretical arguments we can nevertheless use this regression. We proceed now in several steps. We first show how using completed tenure $T_{ijt}$ in an empirical wage regression leads to unbiased estimates of tenure and experience. We then show that completed tenure $T_{ijt}$ is positively correlated with match quality. Finally we use the residual wage variance for job stayers and switchers to compute the volatility of $\phi$.

3.1 Step 1: Unbiased Estimates of the Returns to Tenure and Experience

We first regress wages at the beginning of the employment cycle on experience at the beginning of the employment cycle (including individual fixed effects). This gives us the true coefficient on experience, $\beta_1$. The reason is that experience at the beginning of the employment cycle and match quality are uncorrelated since workers just leave unemployment.\(^7\) In a second step we first subtract the estimated contribution of experience from wages,

$$\hat{w} = w - \beta_1 X,$$

and regress this residual on tenure, completed tenure (and an individual fixed effect). As shown for example in Abraham and Farber (1987), this results in an unbiased estimate of $\beta_2$. This is easy to see. Conducting OLS is equivalent to first regress wages and tenure on completed tenure and then as a second step regress the residuals from this regression on

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\(^7\)We allow for such correlation and show how to control for the resulting bias in Appendix I.1.
each other, that is the wage residual on the tenure residual. The tenure residual is equal to
the deviation of tenure from its mean in the same job spell and is thus uncorrelated with
match quality. As a result the coefficient is unbiased. We thus have estimated unbiased
coefficients for $\beta_1$ and $\beta_2$.

3.2 Step 2: Completed Tenure and Match Quality

Recall that we approximate $\phi_{ijt}$ as a function of $T_{ij}$ and error $E(\delta_{ij}) = 0$:

$$\log(\phi_{ijt}) = \alpha_0(T) + \alpha_1 T_{ij} + \alpha_2 T_{ij}^2 + \delta_{ijt}.$$ 

We want to show that the covariance (and thus the correlation) of $\phi$ and $T$ is positive. For
now we do this for the case of no returns on tenure, $\beta_2 = 0$. It equals

$$E[(\phi - E(\phi))(T - E(T))] = E_\phi[(\phi - E(\phi))E(T - E(T) | \phi)].$$

Note that $E(T)$ is the unconditional expected duration of a job and that $E(T | \phi)$ is the
expected duration of a job conditional on type $\phi$.

Let the probability to receive an offer per period be $q$, then the probability not to leave
until period $T$ for type $\phi$ equals

$$\sum_{N=0}^{T} \left(\begin{array}{c} T \\ N \end{array}\right) q^N(1-q)^{T-N}F(\phi)^N = (1-q + qF(\phi))^T$$

and thus the probability to leave in period $T$ (and not before) equals

$$(1-q + qF(\phi))^{T-1}q(1 - F(\phi)).$$

The expected duration conditional on $\phi$ thus equals

$$\sum_{N=1}^{\infty} N(1-q + qF(\phi))^{N-1}q(1 - F(\phi)) = \frac{1}{q(1-F(\phi))},$$

which is increasing in $\phi$. The covariance thus equals

$$E[(\phi - E(\phi))\varphi(\phi)],$$

where $\varphi(\phi) = \frac{1}{q(1-F(\phi))} - E(T)$, which is increasing in $\phi$. Define $\Psi(\phi) = \int^{\phi} \varphi(\phi)d\phi$. It then
holds that the covariance equals (partial integration)

$$-\int^{\phi} \Psi(\phi)d\phi > 0.$$ 

since $\Psi(\phi) = \Psi(\bar{\phi}) = 0$ and thus $\Psi(\phi) \leq 0$. 

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3.3 Step 3: Volatility of $\phi$

In this section we measure the volatility of match quality $\phi$. The basic idea is to compare the volatility of wages of job stayers and switchers. As match quality is constant on the job and changes only if the worker changes jobs (because of search frictions), the difference in these volatilities is attributed to changes in $\phi$ and thus to search frictions.

We first define the wage residual, subtracting the returns to tenure and experience as well as the fixed effect from wages:

$$\log \tilde{w}_{it} = \log w_{it} - \beta_1 X_{it} - \beta_2 T_{it} - \log(\mu_i) = \log(\phi_{ijt}^k) + \log(\epsilon_{it}).$$

Behavior of Wages within Jobs

We start by looking at what happens within jobs. Let $v^{ij}$ be the variance of wages in job $j$, $n^{ij}$ - the number of observations in job $j$, and $N = \sum_{ij} (n^{ij} - 1)$ - the total number of observations minus the number of jobs. A consistent estimate of the within-job wage variance (Raab (1981)) is:

$$\text{Var}(\log \epsilon_{it}) = \sum_{ij} \frac{n^{ij}}{N} v^{ij}.$$

The variance of $\phi$

Once we know the variance of $\epsilon$, it is easy to compute the variance of $\phi$ by subtracting the variance of $\epsilon$ from the variance of the wage residual $\tilde{w}_{it}$:

$$\text{Var}(\log(\phi)) = \text{Var}(\log(\tilde{w})) - \text{Var}(\log(\log \epsilon)).$$

4 Extensions

In this section we discuss several possible generalizations of the benchmark model. In each case we demonstrate whether and how our analysis needs to be modified. We first allow for the possibility that match quality is not constant during a job as we assumed so far but that it instead follows a stochastic process even during a job. We show that the data
do not support this generalization and that the data are best described by a constant match quality. Second, we consider what type of contracts are (not) supported by the data and whether our analysis needs to be modified. Again we find that existing models are either not supported by the data or/and would not change our results. Finally we allow for occupational specific human capital (Kambourov and Manovskii (2009)).

4.1 Stochastic Match Quality

Suppose that (demeaned) match quality evolves as

$$\log(\phi_{ijt}) = \hat{\phi} + \rho(\log(\phi_{ijt-1}) - \hat{\phi}) + \epsilon_{ijt},$$  \hspace{1cm} (3)$$

where $\rho \in [0, 1]$ and $\epsilon_{ijt}$ is the period $t$ innovation. A constant $\phi$ is a special case for $\rho = 1$ and a zero volatility of $e$.

The match quality is approximated through an OLS regression as

$$\log(\phi_{ijt}) = \alpha_0(T) + \alpha_1 T_{ij} + \alpha_2 T^2_{ij} + \eta_{ijt},$$

Since we now allow the match quality to evolve over time, the $\alpha_0$ depends on tenure $T$. The mean of match quality with tenure $T$ equals $E_T(\alpha_1 T_{ij} + \alpha_2 T^2_{ij}) + \alpha_0(T)$, since $\eta_{ijt}$ has mean zero. We allow for a $T$-dependent variable $\alpha_0$ since the mean of match quality can evolve over the job.

Let us take a closer look at how match quality evolves over time during the job. The mean can change because the process is autoregressive if $\rho < 1$ and because workers with high realizations of $e$ tend to stay and workers with low realizations of $e$ tend to leave the current job. In particular the difference in match quality, which equals:

$$\log(\phi_{ijt}) - \log(\phi_{ijt-1}) = \alpha_0(T) + \alpha_1 T_{ijt} + \alpha_2 T^2_{ijt} + \eta_{ijt}$$

$$- \left(\alpha_0(T-1) + \alpha_1 T_{ijt-1} + \alpha_2 T^2_{ijt-1} + \eta_{ijt-1}\right)$$

$$= (\rho - 1)(\alpha_0(T - 1) + \alpha_1 T_{ijt-1} + \alpha_2 T^2_{ijt-1} - \hat{\phi} + \eta_{ijt-1}) + \epsilon_{ijt},$$

does not necessarily has mean zero. The first equality sign just uses the approximation of $\phi$ in (4). The second equality replaces $\log(\phi_{ijt})$ first using (3) and then applies the approximation in (4) to $\log(\phi_{ijt-1})$ only.
To understand this first difference in match quality, it is key to understand how \( e \) evolves, since \( e \) is the only unobservable variable in this equation which is exposed to selection. The expected value of \( e \) is subject to selection if the worker receives an offer. If the worker does not receive an offer \( e \) is a mean zero variable, say \( \tilde{e}_2 \). If however the worker receives an offer and this happens with probability \( q_t \), then the decision to stay or switch depends on \( \bar{T} \) and \( \alpha_0(T) \). Let \( p(e \mid \bar{T}, \alpha_0(T)) \) be the probability of type \( e \) of rejecting an offer conditional on \( \bar{T} \) and \( \alpha_0(T) \).

Conditional on receiving an offer and staying (still in the same job in \( T \) as in \( T - 1 \), the innovation is named \( \tilde{e}_1 \), and has mean

\[
E_t(\tilde{e}_1) = \int_{\bar{e}}^{\bar{T}} e h(e) \frac{p(e \mid \bar{T}, \alpha_0(T))}{m(\bar{T}, \alpha_0(T))} \, de,
\]

where \( m(\bar{T}, \alpha_0(T)) \) is the probability to reject an offer and \( h(e) \) is the density of the innovation \( \tilde{e}_2 \). The expected value of \( e \) thus equals

\[
E_t(\tilde{e}_1) = q_t E_t(\tilde{e}_1) + (1 - q_t) E_t(\tilde{e}_2).
\]

Since \( E_t(\tilde{e}_1) > E_t(\tilde{e}_2) \), \( E_j(e_j) \) is increasing in \( q_t \).

If we implement an OLS regression of \( e \) on \( q \), completed tenure and a dummy, we get that

\[
e_{ijt} = \gamma_1 q_t + \gamma_2 (\alpha_1 \bar{T}_{ijt} + \alpha_2 \bar{T}_{ijt}^2) + \gamma_3 \alpha_0(T) + \nu_{ijt},
\]

where \( \gamma_1 > 0 \). The sign of the coefficients \( \gamma_2 \) and \( \gamma_3 \) is somewhat unclear but it is irrelevant anyway. Using this regression equation to replace \( e \) in the equation for the difference in \( \phi \),

\[
\log(\phi_{ijt}) - \log(\phi_{ijt-1})
= \gamma_1 q_t + (\gamma_2 + (\rho - 1)) (\alpha_1 \bar{T}_{ijt-1} + \alpha_2 \bar{T}_{ijt-1}^2) + (\rho - 1 + \gamma_3) \alpha_0(T - 1)
+ (\rho - 1)(\eta_{ijt-1} - \hat{\phi}) + \nu_{ijt},
\]

where \((\rho - 1)(\eta_{ijt-1} - \hat{\phi}) + \nu_{ijt}\) is just nuisance.

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\[8^8 \] \( E_t(\tilde{e}_1) - E_t(\tilde{e}_2) = \int_{\bar{e}}^{\bar{T}} e f(e)(\frac{p(e \mid \bar{T}, \alpha_0(T))}{m(\bar{T}, \alpha_0(T))}) \, de \). Let \( \Lambda(\tilde{e}, \bar{T}, \alpha_0(T - 1)) = \int_{\bar{e}}^{\bar{T}} f(e)(\frac{p_e(e \mid \bar{T}, \alpha_0(T - 1))}{m(\bar{T}, \alpha_0(T - 1))} - 1) \, de \),
then \( \Lambda(\tilde{e}, \bar{T}, \alpha_0(T - 1)) = \Lambda(\bar{T}, \bar{T}, \alpha_0(T - 1)) = 0 \). Since \( \frac{p_e(e \mid \bar{T}, \alpha_0(T - 1))}{m(\bar{T}, \alpha_0(T - 1))} - 1 \) is increasing in \( e \), it is negative if \( e < \bar{e} \) and positive if \( e \geq \bar{e} \), for some \( \bar{e} \). As a result \( \Lambda(\tilde{e}, \bar{T}, \alpha_0(T - 1)) < 0 \) so that (FOSD) \( E_j(\tilde{e}_1) > E_j(\tilde{e}_2) \).
The difference in wages equals

\[ \log(w_{ijt}) - \log(w_{ijt-1}) = \alpha^T_{ijt} + \alpha^E_{ijt} + \log(\phi_{ijt}) - \log(\phi_{ijt-1}), \]

where \( \alpha^T_{ijt} \) is the tenure dummy for the increase from \( t-1 \) to \( t \) and \( \alpha^E_{ijt} \) is the experience dummy for the increase from \( t-1 \) to \( t \). Plugging in the equation for \( \phi_{ijt} - \phi_{ijt-1} \) yields:

\[
\begin{align*}
\log(w_{ijt}) - \log(w_{ijt-1}) &= \alpha^T_{ijt} + \alpha^E_{ijt} \\
&+ \gamma_1 q_t + (\gamma_2 + (\rho - 1)) (\alpha^T_{1ijt} + \alpha^T_{2ijt}) \\
&+ (\rho - 1 + \gamma_3) \alpha_0(T) + (\rho - 1)(\eta_{ijt-1} - \hat{\phi}) + \nu_{ijt}.
\end{align*}
\]

We thus estimate the difference in wages on the dummies \( \alpha^T_{ijt} \) and \( \alpha^E_{ijt} \), a quadratic in \( T \) and a quadratic in \( q_t \). Note that \( \alpha_0(T) \) is not identified. Instead we estimate \( \alpha^T + (\rho - 1 + \gamma_3) \alpha_0(T) \), that is the returns to tenure would be biased if \( \rho \neq 1 \) or \( \gamma_3 \neq 0 \). Note that the coefficient in this regression on \( q_t \) is the same as in the regression for \( e \).

We test whether the estimated coefficients of the terms for \( T \) and \( q \) are zero. In this estimation we have to take into account that the wages depend on the business cycle already, even in the absence of stochastic match quality. As a result we have to take into account that the difference in wages depends on the change in business cycle conditions. We therefore add the first difference in \( q \) to the regression. Note that if match quality is stochastic, the level of \( q \) has to be (positively) significant. If the coefficients on \( q_t \) are zero, this implies that \( E_i(\hat{e}_1) = E_i(\hat{e}_2) \), that is selection is absent, what is possible only if \( e \) is identical to zero. As a result, \( \gamma_2 = 0 \) (and also \( \gamma_3 \)). Thus the finding that the coefficients on the terms of \( T \) are zero, \( \rho - 1 = 0 \), and thus \( \rho = 1 \).

In particular, the returns to tenure are unbiased since the potential bias, \( \rho - 1 + \gamma_3 = 0 \).
4.2 Contracts

In many models wages do not only reflect contemporaneous condition but instead contracts decouple current wages from current productivity, see for example Postel-Vinay and Robin (2002) and Cahuc, Postel-Vinay, and Robin (2006). A feature of wage formation in these models is that wages can be renegotiated upwards if a counteroffer is on the table. The probability that a worker and a firm engage in bargaining and increase the wage is increasing in the probability to receive a counteroffer. As a result the difference in wages, \( \log(w_{it}) - \log(w_{it-1}) \), is increasing in market tightness in these models. We thus implement the following regression:

\[
\log(w_{ijt}) - \log(w_{ijt-1}) = \alpha_T T + \alpha_E E_{ijt} + \chi_1 \log(q_{t-1}) + \chi_2 \log(q_{t-1})^2 + \nu_{ijt},
\]

(4)

where \( \alpha_T T \) is the tenure dummy for the increase from tenure level \( T - 1 \) to \( T \), \( \alpha_E E_{ijt} \) is the experience dummy for the increase from \( t - 1 \) to \( t \) and \( \nu_{ijt} \) is the residual of this regression. Again we have to take into account that the difference in wages depends on the change in business cycle conditions. We therefore add the first difference in \( q \) to the regression. Note that if such types of contracts are present, the level of \( q \) has to be (positively) significant.

In Section 5.6 we will document that the coefficients \( \chi_1 \) and \( \chi_2 \) are jointly insignificant. This implies that wages are not renegotiated. Contracts may still be present however by stipulating a constant wage. This is not relevant for our purposes though. We are interested in the volatility of wages due to search frictions.

The wage with contracts equals

\[
\log w^C_{it} = \beta^C_X X + \beta^C_T T + \log(\mu_i) + \log(\phi_{ij}) + \log(e^C_{it}),
\]

(5)

where \( \beta^C_T T \) is the remuneration to tenure and \( \beta^C_X X \) is the remuneration to experience (could be different from the spot wage model). There is an individual fixed effect \( \log(\mu_i) \) (also potentially be different) and \( \log(e^C_{it}) \) is the remuneration to the idiosyncratic effect. Finally \( \log(\phi_{ij}) \) is the remuneration to match quality, which due to the presence of contracts is constant on the job spell.

We again have to estimate the returns to tenure and experience first. Again we proceed in two steps. We first regress wages at the beginning of the employment cycle on experience
at the beginning of the employment cycle (including individual fixed effects). This gives us the true experience dummy for experience at the beginning of the job. The reason is again that experience at the beginning of the employment cycle and match quality are uncorrelated since workers just leave unemployment. In a second step we first subtract the estimated contribution of experience from wages,

\[ \hat{w} = w - \beta_1 X, \]

and regress this residual on tenure and completed tenure (and an individual fixed effect). As before in the model without contracts (but a Mincer wage regression), the returns to tenure are unbiased if we add completed tenure to the regression. The remaining procedure also remains the same. We compare the volatility of wages for switchers and stayers. Since match quality does not change during the job, the analysis for stayers is identical as only changes in \( \phi^C \) contribute to the volatility of wages during the job once the returns to tenure and experience have been accounted for. The volatility of wages for stayers is again different from the volatility of switchers because now match quality changes, even in the presence of contracts.

The procedure is the same in the presence and in the absence of contracts but theoretically the results can be different. Contracts translate an existing amount of dispersion in match quality into a different amount of wage dispersion than a model without contracts would do. From our measurement perspective however, this distinction does not matter. Whether an existing amount of volatility of wages is generated by a model without or with contracts does not affect our conclusion. We are just interested in the wage dispersion that is due to search frictions. If contracts eliminated any wage dispersion although productivity differences due to search frictions are large, we would conclude that there is no wage dispersion due to search frictions; although the amount of inefficiency due to search frictions is large.

4.3 Occupation-Specific Human Capital

Let \( \phi^P \) be the match quality, \( T^P \) be the tenure and \( T^P \) be completed tenure specific to an occupation. Overall match quality then equals \( \phi + \phi^P \). We use completed tenure in the
occupation to approximate occupation-specific match quality:

$$\phi^P = \alpha_0^P + \alpha_1^P T_{ij}^P + \alpha_2^P (T_{ij}^P)^2 + \delta_{ijt}^P$$

4.3.1 Estimating the returns to occupation-specific human capital

$$\log(W_{ijt}) = \beta_1 X_{ijt} + \beta_2 T_{ijt} + \beta_2^P T_{ijt}^P + \log(\mu_i) + \log(\phi_{ijt}).$$ (6)

The first step where we estimate the returns to initial experience is the same. The second step has to be adapted. As before we first subtract the estimated contribution of experience from wages,

$$\hat{w} = w - \beta_1 X.$$ 

We have now to take into account that one occupational spell can contain more than one job spell. Let these job spells start at 1, 1+T_1,...,1+T_m and thus end at times T_1,T_2,...,T_m. The job spells thus last from 1..T_1, from 1 + T_1..T_2 and from 1 + T_{m-1}..T_m. We then define the following adjusted completed duration variables for job spell k:

$$T_k = T_{k-1} + \frac{T_k - T_{k-1} + 1}{2}$$

It then holds that summarizing $T^P - T_k$ over job spell k yields:

$$\sum_{T^P=1+T_{k-1}}^{T_k} (T^P - T_k) = \sum_{T^P=1}^{T_k-T_{k-1}+1} (T^P - \frac{T_k - T_{k-1} + 1}{2}) = \frac{(T_k - T_{k-1})(T_k - T_{k-1} + 1)}{2} - \frac{(T_k - T_{k-1})(T_k - T_{k-1} + 1)}{2} = 0.$$ 

As a result using $T^P - T_k$ as an instrument yields unbiased estimates since this variable is uncorrelated with both $\phi$ and $\phi^P$ since both these variables are constant on each job spell.

4.3.2 Measure the volatility of $\phi^P$

The procedure is very similar to the benchmark case discussed above. All the differences arise for the switchers.
- Define the wage residual:

\[
\log \tilde{w}_{it}^P = \log w_{it} - \beta_1 X_{it} - \beta_2 T_{it} - \beta_2^P T_{it}^P - \log(\mu_i)
\]

\[
= \log(\phi_{ijt}) + \log(\phi_{ijt}^P) + \log(\epsilon_{it})
\]

\[
= \alpha_0 + \alpha_1 T_{ij} + \alpha_2 T_{ij}^2
\]

\[
+ \alpha_0^P + \alpha_1^P T_{ij}^P + \alpha_2^P (T_{ij}^P)^2
\]

\[
+ \delta_{ijt}^P + \log(\epsilon_{it})
\]

Note that we approximate the sum \(\log(\phi_{ijt}) + \log(\phi_{ijt}^P)\) with \(T_{ij}\) and \(T_{ij}^P\) (at the same time).

- Wage growth for occupational switchers (from job \(k\) to job \(k'\) and from occupation \(m\) to \(m'\)):\(^{10}\)

\[
\kappa^{(k,k'),(m,m')} = \Delta \log(\epsilon_k^i) + \Delta \log(\phi_{k,k'}) + \Delta \log(\phi_{m,m'}^P)
\]

- Covariance of wages for job-to-job switchers (from job \(k\) to job \(k'\) and from occupation \(m\) to \(m'\)).

\[
\gamma^{(k,k'),(m,m')} = Cov(\log(\tilde{w}_{k',m'}), \log(\tilde{w}_{k,m}))
\]

\[
= Cov(\log(\epsilon_{k',m'}), \log(\epsilon_{k,m})) + Cov(\log(\phi_{k'}), \log(\phi_{m'}^P), \log(\phi_{k}) + \log(\phi_{m}^P))
\]

It then holds that

\[
Var(\kappa^{(k,k'),(m,m')})
\]

\[
= Var(\log(\epsilon_{k,m}) + \log(\phi_k^i) + \log(\phi_m^i)) + Var(\log(\epsilon_{k',m'}) + \log(\phi_{k'}^i) + \log(\phi_{m'}^i))
\]

\[
- 2\gamma^{(k,k'),(m,m')}.
\]

Thus since \(\epsilon\) and \(\phi\) are uncorrelated:

\[
Var(\log(\epsilon_{k',m'})) + Var(\log(\phi_{k'}^i)) + Var(\log(\phi_{m'}^i)) + Var(\log(\epsilon_{k,m})) + Var(\log(\phi_{k}^i)) + Var(\log(\phi_{m}^i))
\]

\[
= Var(\kappa^{(k,k'),(m,m')}) + 2\gamma^{(k,k'),(m,m')}.
\]

\(^{10}\)If \(m = m'\) there is no occupational switch, if \(m' = m + 1\), then the worker switched the occupation.
The RHS can be obtained from the data, so that the LHS is known. The volatility of $\epsilon$ is treated the same way as above as it refers to within job movements only. We thus have that

$$
\text{Var}(\log(\phi_k) + \log(\phi_m^P)) + \text{Var}(\log(\phi_{k'}) + \log(\phi_{m'}^P)) \\
= \text{Var}(\kappa^{(k,k'),(m,m')}) + 2\gamma^{(k,k'),(m,m')} - \text{Var}(\log(\epsilon_{k',m'}^i)) - \text{Var}(\log(\epsilon_{k,m}^i)).
$$

Since $T$ and $T^P$ on the one hand and $\delta^P$ on the other hand are uncorrelated,

$$
\text{Var}(\log(\phi_k) + \log(\phi_m^P)) + \text{Var}(\log(\phi_{k'}) + \log(\phi_{m'}^P)) \\
= \text{Var}(\alpha_1 T_k + \alpha_2 T_k^2 + \alpha_1^P T_{m'}^P + \alpha_2^P (T_{m'}^P)^2) \\
+ \text{Var}(\alpha_1 T_{k'} + \alpha_2 T_{k'}^2 + \alpha_1^P T_{m'}^P + \alpha_2^P (T_{m'}^P)^2) \\
+ \text{Var}(\log(\delta_{k,m})) + \text{Var}(\log(\delta_{k',m'})).
$$

That is we know about the variance of $\delta$ that

$$
\text{Var}(\log(\delta_{k,m})) + \text{Var}(\log(\delta_{k',m'})) \\
= \text{Var}(\kappa^{(k,k'),(m,m')}) - \text{Var}(g^i_l) + 2(\gamma^{(k,k'),(m,m')} - c) \\
- \text{Var}(\alpha_1 T_k + \alpha_2 T_k^2 + \alpha_1^P T_{m'}^P + \alpha_2^P (T_{m'}^P)^2) \\
- \text{Var}(\alpha_1 T_{k'} + \alpha_2 T_{k'}^2 + \alpha_1^P T_{m'}^P + \alpha_2^P (T_{m'}^P)^2).
$$
5 Empirical Results

Our empirical analysis is based on the National Longitudinal Survey of Youth described in detail below. NLSY is convenient because it contains detailed work-history data on its respondents in which we can track employment cycles.

5.1 National Longitudinal Survey of Youth Data

The NLSY79 is a nationally representative sample of young men and women who were 14 to 22 years of age when first surveyed in 1979. We use the data up to 2004. NLSY is convenient because it allows to measure all the variables we are interested in. In particular, it contains detailed work-history data on its respondents in which we can track employment cycles. Each year through 1994 and every second year afterward, respondents were asked questions about all the jobs they held since their previous interview, including starting and stopping dates, the wage paid, and the reason for leaving each job.

The NLSY consists of three subsamples: A cross-sectional sample of 6,111 youths designed to be representative of noninstitutionalized civilian youths living in the United States in 1979 and born between January 1, 1957, and December 31, 1964; a supplemental sample designed to oversample civilian Hispanic, black, and economically disadvantaged non-black/non-Hispanic youths; and a military sample designed to represent the youths enlisted in the active military forces as of September 30, 1978. Since many members of supplemental and military samples were dropped from the NLSY over time due to funding constraints, we restrict our sample to members of the representative cross-sectional sample throughout.

We construct a complete work history for each individual by utilizing information on starting and stopping dates of all jobs the individual reports working at and linking jobs across interviews. In each week the individual is in the sample we identify the main job as the job with the highest hours and concentrate our analysis on it. Hours information is missing in some interviews in which case we impute it if hours are reported for the same job at other interviews. We ignore jobs that in which individual works for less than 15 hours per week or that last for less than 4 weeks.\footnote{We have also experimented with the following more complicated algorithm with no impact on our}
We partition all jobs into employment cycles following the procedure in Barlevy (2008). We identify the end of an employment cycle with an involuntary termination of a job. In particular, we consider whether the worker reported being laid off from his job (as opposed to quitting). We use the workers stated reason for leaving his job as long as he starts his next job within 8 weeks of when his previous job ended, but treat him as an involuntary job changer regardless of his stated reason if he does not start his next job until more than 8 weeks later.\footnote{As Barlevy (2008) notes, most workers who report a layoff do spend at least one week without a job, and most workers who move directly into their next job report quitting their job rather than being laid off. However, nearly half of all workers who report quitting do not start their next job for weeks or even months. Some of these delays may be planned. Yet in many of these instances the worker probably resumed searching from scratch after quitting, e.g. because he quit to avoid being laid off or he was not willing to admit he was laid off.} If the worker offers no reason for leaving his job, we classify his job change as voluntary if he starts his next job within 8 weeks and involuntary if he starts it after 8 weeks. We ignore employment cycles that began before the NLSY respondents were first interviewed in 1979.

At each interview the information is recorded for each job held since the last interview on average hours, wages, industry, occupation, etc. Thus, we do not have information on, e.g., wage changes in a given job during the time between the two interviews. This leads us to define the unit of analysis, or an observation, as an intersection of jobs and interviews. A new observation starts when a worker either starts a new job or is interviewed by the NLSY and ends when the job ends or at the next interview, whichever event happens first. Thus, if an entire job falls in between of two consecutive interviews, it constitutes an observation. If an interview falls during a job, we will have two observations for that job: the one between the previous interview and the current one, and the one between the current interview and the next one (during which the information on the second observation would be collected).
Consecutive observations on the same job broken up by the interviews will identify the wage changes for job-stayers. Following Barlevy (2008), we removed observations with an reported hourly wage less than or equal to $0.10 or greater than or equal to $1,000. Many of these outliers appear to be coding errors, since they are out of line with what the same workers report at other dates, including on the same job. We also eliminated employment cycles where wages change by more than a factor of two between consecutive observations.

To each observation we assign a unique value of worker’s job tenure, labor market experience, race, marital status, education, smsa status, and region of residence, and whether the job is unionized. Since the underlying data is weekly, the unique value for each of these variables in each observation is the mode of the underlying variable (the mean for tenure and experience) across all weeks corresponding to that observation. The educational attainment variable is forced to be non-decreasing over time.

We merge the individual data from the NLSY with the aggregate data on unemployment and vacancies. Aggregate unemployment rate, $u$, is constructed by the Bureau of Labor Statistics (BLS) from the Current Population Survey (CPS). The help-wanted advertising index, $v$, is constructed by the Conference Board. Both $u$ and $v$ are quarterly averages of monthly series. The ratio of $v$ to $u$ is the measure of the labor market tightness.

We use the underlying weekly data for each observation (job-interview intersection) to construct aggregate statistics corresponding to that observation. The current unemployment rate for a given observation is the average unemployment rate over all the weeks corresponding to that observation. Similarly, the current labor market tightness for a given observation is the average labor market tightness over all the weeks corresponding to that observation.

All empirical experiments that we conduct are based on the individual data weighted using custom weights provided by the NLSY which adjust both for the complex survey design and for using data from multiple surveys over our sample period. In practice, we found that using weighted or unweighted data has no impact on our substantive findings.

5.2 Results

The variance of log wages in our sample is 0.28.
Table 1: Estimating Returns to Experience. NLSY Data.

<table>
<thead>
<tr>
<th>Coefficient Estimates</th>
<th>Experience</th>
<th>Experience^2</th>
<th>Experience^3</th>
<th>Experience^4</th>
</tr>
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<tr>
<td></td>
<td>.0014259</td>
<td>-1.35e-06</td>
<td>9.43e-10</td>
<td>-2.07e-13</td>
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<tr>
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<td>(.0003532)</td>
<td>(1.20e-06)</td>
<td>(1.66e-09)</td>
<td>(7.15e-13)</td>
</tr>
</tbody>
</table>

Returns to Experience

<table>
<thead>
<tr>
<th>2 Years</th>
<th>5 Years</th>
<th>10 Years</th>
<th>15 Years</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.144207</td>
<td>1.343121</td>
<td>1.638015</td>
<td>1.936596</td>
</tr>
</tbody>
</table>

Note - Standard errors are in parentheses.

5.3 Wage Regression

We obtain residual wages following the two-step procedure described above. We first estimate returns to experience by regressing wages at the beginning of the employment cycle on a quartic in experience at the beginning of the employment cycle. The regression includes individual fixed effects. We also control for education, industry, region, union, and marital status dummies. The estimated coefficients on experience terms and the implied returns to experiences are reported in Table 1.

In a second step we first subtract the estimated contribution of experience from wages, and regress this residual on quartics in tenure and completed tenure. The regression includes individual fixed effects and dummies for education, industry, region, union, and marital status. The estimated coefficients on tenure and completed tenure and the implied returns to tenure are reported in Table 2. The residual wage variance not accounted for by the observables in this regression is 0.17.
Table 2: Estimating Returns to Tenure. NLSY Data.

<table>
<thead>
<tr>
<th></th>
<th>Coefficient Estimates</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Tenure</td>
<td>Tenure^2</td>
<td>Tenure^3</td>
<td>Tenure^4</td>
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<tr>
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<td>(7.83e-10)</td>
<td>(4.08e-13)</td>
</tr>
<tr>
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<td>Compl. Tenure</td>
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<td>Compl. Tenure^3</td>
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<tr>
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<td>(1.33e-09)</td>
<td>(6.52e-13)</td>
</tr>
<tr>
<td>Returns to Tenure</td>
<td>2 Years</td>
<td>5 Years</td>
<td>10 Years</td>
<td>15 Years</td>
</tr>
<tr>
<td></td>
<td>1.040294</td>
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<td>1.082636</td>
<td>1.091651</td>
</tr>
</tbody>
</table>

Note - Standard errors are in parentheses.

5.4 Volatility of $\phi$

To compute the volatility of match qualities we first subtract estimated returns to experience and tenure, and the estimated individual fixed effects from raw wages:

$$\log \tilde{w}_{it} = \log w_{it} - \beta_1 X_{it} - \beta_2 T_{it} - \log(\mu_i).$$

Using that $Var(\log(\phi)) = Var(\log(\tilde{w})) - Var(\log(\epsilon))$ we obtain that $Var(\log(\phi)) = 0.016$.

Comparing this to the total variance of log wages in our sample, we obtain that the fraction of wage dispersion accounted for by search frictions is

$$\frac{0.016}{28} = 0.057.$$

5.5 Stochastic Match Quality

As implied by the theoretical analysis above one can discriminate between models with stochastic and constant match quality by estimating a regression of the difference in wages
Table 3: Stochastic Match Quality? NLSY Data.

<table>
<thead>
<tr>
<th>Coefficient Estimates</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( \bar{T} )</td>
<td>( \bar{T}^2 )</td>
<td>( \bar{T}^3 )</td>
<td>( \bar{T}^4 )</td>
<td></td>
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<tr>
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<tr>
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<td>(9.61e-07)</td>
<td>(1.32e-09)</td>
<td>(5.83e-13)</td>
<td></td>
</tr>
<tr>
<td>( q_t )</td>
<td>( q_t^2 )</td>
<td>( q_t^3 )</td>
<td>( q_t^4 )</td>
<td></td>
</tr>
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<td>-.0091866</td>
<td>.0006233</td>
<td>-.0000139</td>
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<tr>
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<td>(.0133011)</td>
<td>(.0007109)</td>
<td>(.0000137)</td>
<td></td>
</tr>
</tbody>
</table>

Note - Standard errors are in parentheses.

on tenure and experience dummies \( \alpha_{ijt}^T \) and \( \alpha_{ijt}^E \), a quartic in completed tenure \( \bar{T} \) and in the probability to receive an offer \( q_t \) which we measure through the observable labor market tightness. The estimated coefficients on \( \bar{T} \) and \( q_t \) are reported in Table 3 and are all individually and jointly insignificant.

The fact that the coefficients on \( q_t \) are statistically equal to zero, implies that \( E_t(\tilde{\epsilon}_1) = E_t(\tilde{\epsilon}_2) \), that is selection is absent, what is possible only if the variance of innovations in the match quality is equal to zero. As a result, \( \gamma_2 = 0 \) and \( \gamma_3 = 0 \). Thus the finding that the coefficients on the terms of \( \bar{T} \) are zero implies that the persistence of the match quality is equal to one. Thus, the model with a constant match quality is more consistent with the data.

Note as well, that since we found \( \rho = 1 \) and \( \gamma_3 = 0 \) the estimated returns to tenure are unbiased.

### 5.6 Contracts

To asses whether the data is better described by a model in which employers committ to matching outside offers, as in, e.g., Postel-Vinay and Robin (2002) and Cahuc, Postel-Vinay, and Robin (2006), we check whether whether wage growth is increasing in the labor market tightness (the probability to receive an offer and a counteroffer). To do so we regress wage
growth on on tenure and experience dummies $\alpha_{ijt}^T$ and $\alpha_{ijt}^E$, and a quartic in the probability to receive an offer $q_t$ which we measure through the observable labor market tightness. The estimated coefficients on $q_t$ are reported in Table 4 and are all individually and jointly insignificant.

### 6 Model Simulations

To be written

### 7 Conclusion

- We proposed a method to measure the dispersion of match qualities in the data.

- A model with constant match quality on the job appears to be a good description of the data.

- A large class of models seems inconsistent with properties of the wage growth of job stayers.

- Search frictions account for about 6% of wage dispersion.
References


27

APPENDICES

I Proofs and Derivations

I.1 Endogenous selection w.r.t. X0

Consider the following wage equation in the first job:

\[ \log(w) = \beta_1 X_0 + \beta_2 T + \phi, \]

where we assume that tenure \( T \) is already instrumented and thus uncorrelated with \( \phi \). Thus the expected wage conditional on \( X_0 \) and \( T \) equals\(^{13}\)

\[ E(\log(w) \mid X_0, T) = \beta_1 X_0 + \beta_2 T + E(\phi \mid X_0, T) = \beta_1 X_0 + \beta_2 T + E(\phi \mid X_0) \]

The expected value of \( \phi \) equals

\[ E(\phi \mid X_0) = \int_{\delta(X_0)}^{\tau} \epsilon dF(\epsilon), \]

where \( \delta(X_0) \) is the threshold function determining participation. Allowing for business cycles with unemployment \( u \) as an indicator we get that

\[ E(\phi \mid X_0, u_0) = \int_{\delta(X_0, u_0)}^{\tau} \epsilon dF(\epsilon), \]

where \( u_0 \) is the initial unemployment rate where \( \delta(X_0, u_0) \) is the threshold function determining participation (as a function of \( X_0 \) and \( u_0 \)). The probability to participate conditional on \( X_0 \) and \( u_0 \) equals

\[ p = \int_{\delta(X_0, u_0)}^{\tau} 1 dF(\epsilon) = 1 - F(\delta(X_0, u_0)), \]

Now we regress the probability to participate on \( X_0 \) and \( u_0 \):

\[ p = \chi_X X_0 + \chi_u u_0, \]

\(^{13}\)We can also add a business cycle indicator \( u_t \) to capture changes in wages due to aggregate fluctuations. We can also add controls such as region, marital status etc.
where
\[ \chi_X = -f(\delta(X_0, u_0))\delta_X(X_0, u_0) \]
\[ \chi_u = -f(\delta(X_0, u_0))\delta_u(X_0, u_0). \]

Thus we have that
\[ \frac{\chi_X}{\chi_u} = \frac{\delta_X(X_0, u_0)}{\delta_u(X_0, u_0)}. \]

Now regress wages on \( X_0, T \) and \( u_0 \):
\[ \log(w) = \rho_X X_0 + \rho_T T + \rho_u u_0, \]
where
\[ \rho_X = \beta_1 - f(\delta(X_0, u_0))\delta_X(X_0, u_0)\delta(X_0, u_0) \]
\[ \rho_u = -f(\delta(X_0, u_0))\delta_u(X_0, u_0)\delta(X_0, u_0). \]

Thus we can finally recover \( \beta_1 \) as
\[ \beta_1 = \rho_X - \rho_u \frac{\chi_X}{\chi_u}. \]

Note that if either \( \rho_u = 0 \) or \( \chi_u = 0 \), then the estimate of wages on \( X_0 \) is unbiased. Otherwise we have derived the appropriate correction using \( u_0 \). Note furthermore that this identification is different from the one in Dustmann and Meghir (2005), or in the Heckman selection model, the control function approach etc. These approaches all feature some form of exclusion restriction, i.e. an variable that affects the decision to participate but not wages. This is not true for the variable here, \( u_0 \). This variable affects both the decision to participate and wages. However, theory implies some cross-equation restrictions. The probability to participate depends on both \( X_0 \) and \( u_0 \) but not in an arbitrary way. By the same way, wages depend on both \( X_0 \) and \( u_0 \) but not in an arbitrary way. Both \( X_0 \) and \( u_0 \) affect the match quality \( \phi \) only through the threshold function \( \delta \). Because of this, the participation decision and wages are also related. This what is used above. Cross equation restrictions implied by theory instead of (made-up) exclusion restrictions.
We can extend this to the quadratic case. Let the probability to participate on $X_0$ and $u_0$:

$$p = \chi X_0 + \frac{\chi^2}{\lambda_u} X_0^2 + \chi_u u_0 + \frac{\chi^2}{\lambda_u} u_0^2,$$

and wages be

$$\log(w) = \rho X_0 + \frac{\rho^2}{\lambda_u} X_0^2 + \rho_T T + \frac{\rho^2 T}{\lambda_u} T^2 + \rho_u u_0 + \frac{\rho^2 u_0}{\lambda_u} u_0^2.$$

It holds that

$$\rho_X^2 = \beta_2 - f'(\delta(X_0, u_0)) \delta_{X_0}(X_0, u_0) \delta(X_0, u_0)$$

$$- f(\delta(X_0, u_0)) \delta_{X_0}(X_0, u_0) \delta(X_0, u_0) - f(\delta(X_0, u_0)) \delta_{X_0}^2(X_0, u_0) \delta(X_0, u_0).$$

$$\rho_u^2 = - f'(\delta(X_0, u_0)) \delta_{u_0}(X_0, u_0) \delta(X_0, u_0)$$

$$- f(\delta(X_0, u_0)) \delta_{u_0}(X_0, u_0) \delta(X_0, u_0) - f(\delta(X_0, u_0)) \delta_{u_0}^2(X_0, u_0),$$

so that

$$\rho_X^2 - \rho_u^2 \frac{\lambda_X^2}{\lambda_u^2}$$

$$= \beta_2 - f(\delta(X_0, u_0)) \delta_{X_0}(X_0, u_0) \delta(X_0, u_0) - f(\delta(X_0, u_0)) \delta_{u_0}(X_0, u_0) \frac{\lambda_X^2}{\lambda_u^2} \delta(X_0, u_0).$$

For the coefficients on the quadratic terms in the participation equation, it holds:

$$\chi_X^2 = - f'(\delta(X_0, u_0)) \delta_{X_0}^2(X_0, u_0) - f(\delta(X_0, u_0)) \delta_{X_0}^2(X_0, u_0).$$

$$\chi_u^2 = - f'(\delta(X_0, u_0)) \delta_{u_0}^2(X_0, u_0) - f(\delta(X_0, u_0)) \delta_{u_0}^2(X_0, u_0).$$

Furthermore,

$$\delta(X_0, u_0) = \frac{\rho_u}{\chi_u},$$

so that

$$\rho_X^2 = \beta_2 - \rho_u \frac{\lambda_X^2}{\chi_u^2} + (\chi_u^2 \frac{\lambda_X^2}{\chi_u^2} - \chi_X^2) \delta(X_0, u_0)$$

$$= \beta_2 - \rho_u \frac{\lambda_X^2}{\chi_u^2} + (\chi_u^2 \frac{\lambda_X^2}{\chi_u^2} - \chi_X^2) \frac{\rho_u}{\chi_u}.$$

This means we know the bias in $\beta_2$, $\rho_u \frac{\lambda_X^2}{\chi_u^2} + (\chi_u^2 \frac{\lambda_X^2}{\chi_u^2} - \chi_X^2) \frac{\rho_u}{\chi_u}$, since it is a function of known parameters.