Income and Consumption Dynamics:
Panel Data Models, Nonlinear Persistence and Partial Insurance

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[Updated papers and references on my web page]

Lecture, November, 2017
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> Here I abstract (to begin with) from labor supply and non-separabilities (see recent papers Blundell, Pistaferri and Saporta, 2016, 2017). Instead focus on *nonlinear persistence and partial insurance*. 
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In particular, the aim here is:

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2. To explore the nonlinear nature of income shocks and the implications for consumption dynamics.
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In particular, the aim here is:

1. To consider alternative ways of modelling persistence, and
2. To explore the nonlinear nature of income shocks and the implications for consumption dynamics.

⇒ e.g. US Household Panel data and Norwegian Population Register data.
New data on consumption and family income sources

I. Administrative linked data: e.g. Norwegian population register.

- Linked registry databases with unique individual identifiers.
  - Containing records for **every Norwegian from 1967 to 2006**.
  - Detailed demographic and socioeconomic information (market income, cash transfers). Recent links to real estate and assets; and to hours of work. New consumption measurements.

- Family identifiers allow to match spouses and children.
  - see Blundell, Graber and Mogstad (2015).
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II. Newly designed panel surveys: e.g. PSID since 1999.
   - Collection of consumption and assets had a major revision in 1999
     - ~70% of consumption expenditures. Good match with NIPA
     - The sum of food at home, food away from home, gasoline, health, transportation, utilities, clothing etc.
   - Earnings and hours for all earners; Assets measured in each wave.
     - see Blundell, Pistaferri and Saporta-Eksten (2016).
A prototypical “canonical” panel data model of (log) family net (earned) income \( y_{it} \) is:

\[
y_{it} = \eta_{it} + \varepsilon_{it}, \quad i = 1, \ldots, N, \quad t = 1, \ldots, T.
\]

where \( y_{it} \) is net of a systematic component, \( \eta_{it} \) is a random walk with innovation \( \nu_{it} \),

\[
\eta_{it} = \eta_{it-1} + \nu_{it}, \quad i = 1, \ldots, N, \quad t = 1, \ldots, T.
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and \( \varepsilon_{it} \) is a transitory shock.
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where $y_{it}$ is net of a systematic component, $\eta_{it}$ is a random walk with innovation $v_{it}$,

$$\eta_{it} = \eta_{it-1} + v_{it}, \quad i = 1, ..., N, \quad t = 1, ..., T.$$ 

and $\epsilon_{it}$ is a transitory shock.

Consumption growth is then related to income shocks:

$$\Delta c_{it} = \phi_t v_{it} + \psi_t \epsilon_{it} + \nu_{it}, \quad i = 1, ..., N, \quad t = 1, ..., T.$$ 

where $c_{it}$ is log total consumption net of a systematic component,
> $\phi_t$ is the transmission of persistence shocks $v_{it}$, and
> $\psi_t$ the transmission of transitory shocks;
- the $\nu_{it}$ are taste shocks, assumed to be independent across periods.
More specifically, to account for the impact of income shocks on the evolution of consumption inequality we introduce *transmission* or *partial insurance* parameters, writing consumption growth as:

\[ \Delta \ln C_{it} \approx \gamma_{it} + \Delta Z_{it} \phi + \phi_t v_{it} + \psi_t \varepsilon_{it} + \zeta_{it} \]

\(\phi_t\) and \(\psi_t\) provide the link between the consumption and income distributions - \(v_{it}\) the permanent and \(\varepsilon_{it}\) the transitory shock to income.
Linking Income Dynamics to Consumption Inequality

More specifically, to account for the impact of income shocks on the evolution of consumption inequality we introduce transmission or partial insurance parameters, writing consumption growth as:

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\Delta \ln C_{it} \equiv \gamma_{it} + \Delta Z_{it}' \varphi + \phi_t \nu_{it} + \psi_t \varepsilon_{it} + \zeta_{it}
\]

\(\phi_t\) and \(\psi_t\) provide the link between the consumption and income distributions - \(\nu_{it}\) the permanent and \(\varepsilon_{it}\) the transitory shock to income.

- For a simple benchmark intertemporal consumption model for consumer of age \(t\), BLP (2013) show

\[
\phi_t = (1 - \pi_{it}) \text{ and } \psi_t = (1 - \pi_{it}) \gamma_{Lt}
\]

where

\[
\pi_{it} \approx \frac{\text{Assets}_{it}}{\text{Assets}_{it} + \text{Human Wealth}_{it}}
\]

and \(\gamma_{Lt}\) is the annuity value of a temporary shock to income for an individual aged \(t\) retiring at age \(L\).

[Easily extend to ARMA processes for income.]
This “standard” framework implies a set of covariance restrictions for panel data on consumption and income,

allowing the insurance parameters and variances to depend on age and education is key.

can show (over-)identification and efficient estimation via nonlinear GMM, see Blundell, Preston and Pistaferri (AER, 2008) and Blundell, Pistaferri and Saporta (AER, 2016) - who also examine wage shocks and introduce family labor supply, nonseparability and taxes.
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⇒ can show (over-)identification and efficient estimation via nonlinear GMM, see Blundell, Preston and Pistaferri (AER, 2008) and Blundell, Pistaferri and Saporta (AER, 2016) - who also examine wage shocks and introduce family labor supply, nonseparability and taxes.

• Linearity of the income (or wage) process simplifies identification and estimation.

▷ However, by construction, it rules out the nonlinear transmission of shocks.
Motivation

• The aim here is to step back and take a different tack - develop *an alternative approach to modeling persistence* in which the impact of past shocks on current incomes/earnings can be altered by the size and sign of new shocks.

• This new framework draws on a flurry of recent work on nonlinearity and heterogeneity in the dynamics of inequality and income risk (full references in Arellano, Blundell and Bonhomme, 2017).

• The idea is to have a framework allows:

⇒ “unusual” shocks to wipe out the memory of past shocks, and
⇒ future persistence of a current shock to depend on the future shocks.
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The idea is to have a framework allows:

⇒ “unusual” shocks to wipe out the memory of past shocks, and
⇒ future persistence of a current shock to depend on the future shocks.

In this lecture I will show that the presence of “unusual” shocks matches the data and has a key impact consumption and saving over the life cycle.
Background papers

- Blundell, Pistaferri and Preston [BPP] ‘Consumption inequality and partial insurance’ (AER, 2008)
- Blundell, Low and Preston [BLP] ‘Decomposing changes in income risk using consumption data’ (QE, 2013)
- Arellano, Blundell and Bonhomme [ABB] ‘Earnings and consumption dynamics: a nonlinear framework’ (Ecta, 2017)

maybe finding time to reference:
- Blundell, Pistaferri and Saporta-Eksten [BPS1/2] ‘Consumption inequality and family labor supply’ (AER, 2016; JPE, 2017)

on my website http://www.ucl.ac.uk/~uctp39a/pub.html
Consider a cohort of households, $i = 1, \ldots, N$, and denote age as $t$. Let $y_{it}$ denote log-labor income, net of age dummies

$$y_{it} = \eta_{it} + \varepsilon_{it}, \quad i = 1, \ldots, N, \quad t = 1, \ldots, T.$$ 

$\eta_{it}$ follows a general first-order Markov process (can be generalised).

Denoting the $\tau$th conditional quantile of $\eta_{it}$ given $\eta_{i,t-1}$ as $Q_t(\eta_{i,t-1}, \tau)$, we specify

$$\eta_{it} = Q_t(\eta_{i,t-1}, u_{it}), \quad \text{where} \ (u_{it} | \eta_{i,t-1}, \eta_{i,t-2}, \ldots) \sim \text{Uniform}(0, 1).$$

$\varepsilon_{it}$ has zero mean, independent over time.

The conditional quantile functions $Q_t(\eta_{i,t-1}, u_{it})$ and the marginal distributions $F_{\varepsilon_t}$ are all age ($t$) specific.
A measure of nonlinear persistence

• The model allows for nonlinear dynamics of income.

• To see this, consider the following measure of persistence

\[ \rho_t(\eta_{i,t-1}, \tau) = \frac{\partial Q_t(\eta_{i,t-1}, \tau)}{\partial \eta}. \]

⇒ \( \rho_t(\eta_{i,t-1}, \tau) \) measures the persistence of \( \eta_{i,t-1} \) when, at age \( t \), it is hit by a shock \( u_{it} \) that has rank \( \tau \). Measures the persistence of histories.

▷ Allows a general form of conditional heteroscedasticity, skewness and kurtosis.
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▷ Allows a general form of conditional heteroscedasticity, skewness and kurtosis.

- In the “canonical model” \( \eta_{it} = \eta_{i,t-1} + v_{it} \), with \( v_{it} \) independent over time and independent of past \( \eta \)'s,

\[ \eta_{it} = \eta_{i,t-1} + F_{v_{t}}^{-1}(u_{it}) \quad \Rightarrow \quad \rho_t(\eta_{i,t-1}, \tau) = 1 \text{ for all } (\eta_{i,t-1}, \tau). \]
A measure of nonlinear persistence

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- To see this, consider the following measure of persistence

  \[ \rho_t(\eta_i,t-1, \tau) = \frac{\partial Q_t(\eta_{i,t-1}, \tau)}{\partial \eta}. \]

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- Allows a general form of conditional heteroscedasticity, skewness and kurtosis.

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  \[ \eta_{it} = \eta_{i,t-1} + F_{v_t}^{-1}(u_{it}) \Rightarrow \rho_t(\eta_{i,t-1}, \tau) = 1 \text{ for all } (\eta_{i,t-1}, \tau). \]

- But what is the evidence for such nonlinearities in persistence?
Some motivating evidence: Quantile autoregressions of log-earnings

\[ \frac{\partial Q_{yt|y_{t-1}}(y_{i,t-1}, \tau)}{\partial y} \]

PSID data  Norwegian administrative data

Note: Pre-tax household labor earnings, Age 30-60 1999-2009 (US), Age 30-60 2005-2006 (Norway). Estimates of the average derivative of the conditional quantile

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Conditional skewness, Norwegian administrative data

Note: Skewness measured as a nonparametric estimate of

\[
\frac{Q_{y_t|y_{t-1}}(y_{i,t-1,.9}) + Q_{y_t|y_{t-1}}(y_{i,t-1,.1}) - 2Q_{y_t|y_{t-1}}(y_{i,t-1,.5})}{Q_{y_t|y_{t-1}}(y_{i,t-1,.9}) - Q_{y_t|y_{t-1}}(y_{i,t-1,.1})}.
\]

Age 30-60, years 2005-2006.
Life-cycle model simulations and model specification

Identification

Data and estimation strategy

Empirical results
Life-cycle model: illustrative simulation

- Calibration based on Kaplan and Violante [KV] (2010). Households enter the labor market at age 25, work until 60, and die with certainty at age 90.

- A single risk-free, one-period bond with return $1 + r$ ($r = .03$),

$$A_t = (1 + r)A_{t-1} + Y_{t-1} - C_{t-1}.$$ 

- Log-earnings are $\ln Y_t = \kappa_t + \eta_t + \epsilon_t$, where $\kappa_t$ is a deterministic age profile. In period $t$ agents know $\eta_t$, $\epsilon_t$ and their past values, but not $\eta_{t+1}$ or $\epsilon_{t+1}$ (no advance information).

- Period-$t$ optimization

$$V_t(A_t, \eta_t, \epsilon_t) = \max_{C_t} u(C_t) + \beta \mathbb{E}_t \left[ V_{t+1}(A_{t+1}, \eta_{t+1}, \epsilon_{t+1}) \right],$$

where $u(\cdot)$ is CRRA ($\gamma = 2$), and $\beta = 1/(1 + r) \approx .97$.

- We compare the results for the canonical earnings process used by KV, with our nonlinear process.
Simulation results

Consumption (age 37) by decile of $\eta_{t-1}$

Average consumption over the life-cycle

Note: Blue is nonlinear earnings process, Green is canonical earnings process.
• Let $c_{it}$ and $a_{it}$ denote log-consumption and assets (beginning of period) net of age dummies.

• Our empirical specification is based on

$$c_{it} = g_t (a_{it}, \eta_{it}, \epsilon_{it}, \nu_{it}) \quad t = 1, \ldots, T,$$

where $\nu_{it}$ are independent across periods, and $g_t$ is a nonlinear, age-dependent function, monotone in $\nu_{it}$.

– $\nu_{it}$ may be interpreted a taste shifter that increases marginal utility. We normalize its distribution to be standard uniform in each period.
An Empirical Consumption Rule

• Let $c_{it}$ and $a_{it}$ denote log-consumption and assets (beginning of period) net of age dummies.

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> This consumption rule is consistent, in particular, with the standard life-cycle model on the earlier slide.

> Can allow for individual heterogeneity, advance information and habits.
Insurance coefficients

- With consumption specification given by

\[ c_{it} = g_t(a_{it}, \eta_{it}, \varepsilon_{it}, \nu_{it}), \quad t = 1, \ldots, T, \]

consumption responses to \( \eta \) and \( \varepsilon \) are

\[ \phi_t(a, \eta, \varepsilon) = \mathbb{E} \left[ \frac{\partial g_t(a, \eta, \varepsilon, \nu)}{\partial \eta} \right], \quad \psi_t(a, \eta, \varepsilon) = \mathbb{E} \left[ \frac{\partial g_t(a, \eta, \varepsilon, \nu)}{\partial \varepsilon} \right]. \]

\( \triangleright \) \( \phi_t(a, \eta, \varepsilon) \) and \( \psi_t(a, \eta, \varepsilon) \) reflect the transmission of the persistent and transitory earnings components, respectively.
Insurance coefficients

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\( \phi_t(a, \eta, \varepsilon) \) and \( \psi_t(a, \eta, \varepsilon) \) reflect the transmission of the persistent and transitory earnings components, respectively.

- The marginal effect of an earnings shock \( u \) on consumption is

\[ \mathbb{E} \left[ \frac{\partial}{\partial u} \bigg|_{u=\tau} g_t(a, Q_t(\eta, u), \varepsilon, \nu) \right] = \phi_t(a, Q_t(\eta, \tau), \varepsilon) \frac{\partial Q_t(\eta, \tau)}{\partial u}. \]
For $T = 3$, Wilhelm (2012) gives conditions under which the distribution of $\epsilon_{i2}$ is identified.

In particular, completeness of the pdfs of $(y_{i2}|y_{i1})$ and $(\eta_{i2}|y_{i1})$. This requires $\eta_{i1}$ and $\eta_{i2}$ to be dependent.

In this research we build on this result to establish identification of the earnings model.

Apply the result to each of the three-year sub-panels $t \in \{1, 2, 3\}$ to $t \in \{T - 2, T - 1, T\}$

The marginal distribution of $\epsilon_{it}$ are identified for $t \in \{2, 3, ..., T - 1\}$.

By independence the joint distribution of $(\epsilon_{i2}, \epsilon_{i3}, ..., \epsilon_{iT-1})$ is identified.

By deconvolution the distribution of $(\eta_{i2}, \eta_{i3}, ..., \eta_{iT-1})$ is identified.

The distribution of $\epsilon_{i1}$, $\eta_{i1}$, and $\epsilon_{iT}$, $\eta_{iT}$ are not identified in general.
Consumption: assumptions

- $u_{it}$ and $\varepsilon_{it}$ are independent of past earnings shocks and past asset holding, for $t \geq 1$, where $\eta_{it} = Q_t(\eta_{i,t-1}, u_{it})$.

- We let $\eta_{i1}$ and $a_{i1}$ be arbitrarily dependent.
  - This is important, because asset accumulation upon entry in the sample may be correlated with past persistent shocks.

- Denoting $\eta_i^t = (\eta_{it}, \eta_{i,t-1}, \ldots, \eta_{i1})$, we assume (in this talk) that: $a_{it}$ is independent of $(\eta_{i}^{t-1}, a_{i}^{t-2}, \varepsilon_{i}^{t-2})$ given $(a_{i,t-1}, c_{i,t-1}, y_{i,t-1})$.
  - Consistent with the accumulation rule in the standard life-cycle model with one single risk-less asset.
Consumption: initial assets

• Let $y = (y_1, ..., y_T)$. We have

$$f(a_1|y) = \int f(a_1|\eta_1, y)f(\eta_1|y)d\eta_1$$

$$= \int f(a_1|\eta_1)f(\eta_1|y)d\eta_1,$$

where we have used that $u_{it}$ and $\epsilon_{it}$ are independent of $a_{i1}$.

• Note that $f(\eta_1|y)$ is identified from the earnings process alone.

• If $f(\eta_1|y)$ is complete, then $f(a_1|\eta_1)$ is identified.

– Structure is as in the NPIV problem where $\eta_1$ is the endogenous regressor and $y$ is the instrument.
Consumption: first period

• We have

\[ f(c_1, a_1 | y) \equiv \int f(c_1, a_1 | \eta_1, y) f(\eta_1 | y) d\eta_1 \]

and given our assumptions

\[ f(c_1, a_1 | y) = \int f(c_1 | a_1, \eta_1, y_1) f(a_1 | \eta_1) f(\eta_1 | y) d\eta_1. \]

- \( f(a_1 | \eta_1) \) can be treated as known.

- Provided we have completeness in \((y_2, ..., y_T)\) of \( f(\eta_1 | y_1, y_2, ..., y_T) \), then \( f(c_1 | a_1, \eta_1, y_1) \), is identified.

• Intuition: \( y_{i2}, ..., y_{iT} \) are used as “instruments” for \( \eta_{i1} \).

• Subsequent periods discussed in ABB (2017), briefly here...
Consumption: subsequent periods

• We have

\[
\begin{align*}
  f(a_2|c_1, a_1, y) &= \int f(a_2|c_1, a_1, \eta_1, y_1)f(\eta_1|c_1, a_1, y)d\eta_1 \\
  f(c_2|a_2, c_1, a_1, y) &= \int f(c_2|a_2, \eta_2, y_2)f(\eta_2|a_2, c_1, a_1, y)d\eta_2.
\end{align*}
\]

• By induction it can be shown that the joint density of \( \eta \)'s, consumption, assets, and earnings is identified provided, for all \( t \geq 1 \), the distributions of \((\eta_{it}|c_{it}, a_{it}, y_i)\) and \((\eta_{it}|c_{it-1}, a_{it}, y_i)\) are complete in \((c_{it-1}, a_{it-1}, y_{it-1}, y_{it+1}, \ldots, y_{iT})\).

• Intuition: lagged consumption and assets, as well as lags and leads of earnings, are used as instruments for \( \eta_{it} \).
Identification: extensions

- Similar techniques can be used in the presence of *advance information*, e.g.
  \[ c_{it} = g_{t} \left( a_{it}, \eta_{it}, \eta_{i,t+1}, \varepsilon_{it}, \nu_{it} \right) \]

  or *consumption habits*, e.g.
  \[ c_{it} = g_{t} \left( c_{i,t-1}, a_{it}, \eta_{it}, \varepsilon_{it}, \nu_{it} \right) \]

  ▶ also cases where the consumption rule depends on lagged \( \eta \), or when \( \eta \) follows a second-order Markov process. (See Section 3 in ABB, 2017).
• Similar techniques can be used in the presence of \textit{advance information}, e.g.

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▷ also cases where the consumption rule depends on lagged $\eta$, or when $\eta$ follows a second-order Markov process. (See Section 3 in ABB, 2017).

• Households differ in their initial productivity $\eta_1$ and initial assets, the panel data provide opportunities to allow for additional, \textit{unobserved heterogeneity} in earnings and consumption.

▷ For example: heterogeneity $\tilde{\zeta}_i$ in discounting or preferences, or heterogeneity $\tilde{\zeta}_i$ in the Markovian transitions of $\eta_{it}$.
• Consumption rule with *unobserved heterogeneity*:

\[ c_{it} = g_t (a_{it}, \eta_{it}, \varepsilon_{it}, \xi_i, \nu_{it}) . \]

• We assume that \( u_{it} \) and \( \varepsilon_{it} \), for \( t \geq 1 \), are independent of \( (a_{i1}, \xi_i) \).

• The distribution of \( (a_{i1}, \xi_i, \eta_{i1}) \) is unrestricted.

• A combination of the above identification arguments and the main result of Hu and Schennach (08) identifies:

  – the period-\( t \) consumption distribution \( f(c_t | a_t, \eta_t, y_t, \tilde{\xi}) \), and

  – the distribution of initial conditions \( f(\eta_1, \xi, a_1) \).
Data: PSID

- (New) PSID 1999 - 2009, we use 6 waves (every other year), as in BPS.

- $C_{it}$: Information on food expenditures, rents, health expenditures, utilities, car-related expenditures, education, and child care. Impute rent for home owners, see BPS. [Recreation and clothing are missing before 2004.]

- $A_{it}$: Assets holdings are the sum of financial assets, real estate value, pension funds, and car value, net of mortgages and other debt. (Net worth)

- $y_{it}$ are residuals of log total pre-tax household labor earnings on a set of demographics. $c_{it}$ and $a_{it}$ are residuals, using the same set of demographics as for earnings.

- $\Delta$ cohort and calendar time dummies, family size and composition, education, race, and state dummies.

- As in BPS, we select married male heads aged between 25 and 60.
Empirical specification: income

- The quantile function of $\eta_{it}$ given $\eta_{i,t-1}$ is specified as

$$Q_t(\eta_{t-1}, \tau) = Q(\eta_{t-1}, age_t, \tau) = \sum_{k=0}^{K} a_k^Q(\tau) \varphi_k(\eta_{t-1}, age_t),$$

where $\varphi_k, k = 0, 1, \ldots, K$, are polynomials (Hermite).

- In addition, the quantile functions of $\epsilon_{it}$ and $\eta_{i1}$ are

$$Q_{\epsilon}(age_t, \tau) = \sum_{k=0}^{K} a_k^{\epsilon}(\tau) \varphi_k(age_t),$$

$$Q_{\eta_1}(age_1, \tau) = \sum_{k=0}^{K} a_k^{\eta_1}(\tau) \varphi_k(age_1).$$
Empirical specification: consumption

• We specify the (log) consumption function as:

\[ g_t(a_t, \eta_t, \varepsilon_t, \tau) = g(a_t, \eta_t, \varepsilon_t, \text{age}_t, \tau) = \sum_{k=1}^{K} b^g_k \phi_k(a_t, \eta_t, \varepsilon_t, \text{age}_t) + b^g_0(\tau) \]

– additivity in the taste shifters, though not essential, is convenient given the sample size.

• In addition, the conditional quantiles of \( a_{i1} \) given \( \eta_{i1} \) and \( \text{age}_{i1} \) are

\[ Q^{(a)}(\eta_1, \text{age}_1, \tau) = \sum_{k=0}^{K} b^a_k(\tau) \phi_k(\eta_1, \text{age}_1). \]
Implementation choices

• Model $a_k^Q(\tau)$ as piecewise-linear interpolating splines (Wei and Carroll, 2009) on a grid $0 < \tau_1 < \tau_2 < \ldots < \tau_L < 1$,
  – convenient as the likelihood function is available in closed form.

• We extend the specification of the intercept coefficient $a_0^Q(\tau)$ on $(0, \tau_1]$ and $[\tau_L, 1)$ using a parametric model: exponential ($\lambda$).

• In practice, we take $L = 11$ and $\tau_\ell = \ell / L + 1$. $\varphi_k$ and $\tilde{\varphi}_k$ are low-dimensional tensor products of Hermite polynomials.

• We set $b_0(\tau) = \alpha + \sigma \Phi^{-1}(\tau)$, where $(\alpha, \sigma)$ are to be estimated.
Estimation algorithm

- The first estimation step recovers estimates of the income parameters $\theta$.

- The second step recovers estimates of the consumption parameters $\mu$, given a previous estimate of $\theta$.

- Our choice of a sequential estimation strategy, rather than joint estimation of $(\theta, \mu)$, is motivated by the fact that $\theta$ is identified from the income process alone.
Let $\theta$ be the income-related parameters with true values $\bar{\theta}$.

Let $\rho_\tau(u) = u(\tau - 1\{u \leq 0\})$ denote the “check” function of quantile regression, and let $a^Q_{k\ell}$ denote the value of $a^Q_k(\tau_{\ell})$ evaluated at the true $\bar{\theta}$. The model implies

$$
\left(a^Q_{0\ell}, \ldots, a^Q_{K\ell}\right) = \arg\min_{a^Q_{0\ell}, \ldots, a^Q_{K\ell}} \mathbb{E} \left[ \int \rho_{\tau_{\ell}} \left( \eta_{it} - \sum_{k=0}^{K} a^Q_{k\ell} \varphi_k(\eta_{i,t-1}, \text{age}_{it}) \right) f_i(\eta_i^T; \bar{\theta}) \, d\eta_T \right]
$$

with additional restrictions involving the other parameters in $\theta$.

In the above, $f_i$ denotes the posterior density of $(\eta_{i1}, \ldots, \eta_{iT})$ given the income data

$$
f_i(\eta_i^T; \bar{\theta}) = f(\eta_i^T | y_i^T, \text{age}_i^T; \bar{\theta}).
$$
Model’s restrictions: consumption

• Letting $\mu$ (true value $\bar{\mu}$) be the consumption-related parameters, the model implies

$$\left(\bar{\alpha}, \bar{b}_1^g, \ldots, \bar{b}_K^g\right) = \arg\min_{(\alpha, b_1^g, \ldots, b_K^g)} \mathbb{E} \left[ \int \left( c_{it} - \bar{\alpha} - \sum_{k=1}^{K} b_k^g \bar{\varphi}_k(a_{it}, \eta_{it}, y_{it} - \eta_{it}, age_{it}) \right)^2 g_i(\eta_{iT}; \bar{\theta}, \bar{\mu}) d\eta_{iT} \right],$$

and

$$\bar{\sigma}^2 = \mathbb{E} \left[ \int \left( c_{it} - \bar{\alpha} - \sum_{k=1}^{K} b_k^g \bar{\varphi}_k(a_{it}, \eta_{it}, y_{it} - \eta_{it}, age_{it}) \right)^2 \right],$$

with additional restrictions involving the other parameters in $\mu$.

• Here $g_i$ denotes the posterior density of $(\eta_{i1}, \ldots, \eta_{iT})$ given the earnings, consumption, and asset data

$$g_i(\eta_{iT}; \bar{\theta}, \bar{\mu}) = f(\eta_{iT} | c_{iT}, a_{iT}, y_{iT}, age_{iT} ; \bar{\theta}, \bar{\mu}).$$
Overview of estimation

- A compact notation for the restrictions implied by the income model is

\[
\bar{\theta} = \arg\min_{\theta} \mathbb{E} \left[ \int R(y_i, \eta; \theta) f_i(\eta; \bar{\theta}) d\eta \right].
\]

- We use a “stochastic EM” algorithm (in a non-likelihood setup).

Starting with \( \hat{\theta}^{(0)} \) we iterate on \( s=0,1,... \) the following two steps until convergence of the Markov Chain:

1. **Stochastic E-step:** draw \( \eta_{i}^{(m)} = (\eta_{i1}^{(m)}, \ldots, \eta_{iT}^{(m)}) \) for \( m = 1, \ldots, M \) from \( f_i(\cdot; \hat{\theta}^{(s)}) \). We use a random-walk Metropolis-Hastings sampler.

2. **M-step:** update

\[
\hat{\theta}^{(s+1)} = \arg\min_{\theta} \sum_{i=1}^{N} \sum_{m=1}^{M} R(y_i, \eta_{i}^{(m)}; \theta).
\]
• As the likelihood function is available in closed form, the E-step is straightforward.

• The M-step consists of a number of ordinary regressions and quantile regressions, such as

\[
\min (a_0^Q,\ldots,a_K^Q) \sum_{i=1}^N \sum_{t=2}^T \sum_{m=1}^M \rho_{\tau_\ell} \left( \eta_{it}^{(m)} - \sum_{k=0}^K a_{k\ell}^Q \varphi_k(\eta_{i,t-1}^{(m)}, \text{age}_{it}) \right), \quad \ell = 1, \ldots, L.
\]

• We compute \( \hat{\theta} \) as an average of \( \hat{\theta}^{(s)} \) across \( S \) iterations.

• We estimate \( \hat{\theta} \) and \( \hat{\mu} \) sequentially.
Nielsen (2000) studies the properties of this algorithm in a likelihood case. He provides conditions for the Markov Chain $\hat{\theta}^{(s)}$ to be ergodic (for a fixed sample size).

He also shows that $\sqrt{N} \left( \hat{\theta}^{(s)} - \bar{\theta} \right)$ converges to a Gaussian autoregressive process as $N$ tends to infinity.

Arellano and Bonhomme [AB] (2015) adapt Nielsen’s arguments to derive the form of the asymptotic variance in a non-likelihood case.

AB also study consistency as $K$ (number of polynomial terms) and $L$ (number of knots) tend to infinity with $N$. 
Empirical results
Nonlinear persistence of $\eta_{it}$ (PSID):

$$\rho_t(\eta_{i,t-1}, \tau) = \frac{\partial Q_{\eta_t|\eta_{t-1}}(\eta_{i,t-1}, \tau)}{\partial \eta}$$

Note: Estimates of the average derivative of the conditional quantile function of $\eta_{it}$ on $\eta_{i,t-1}$ with respect to $\eta_{i,t-1}$, evaluated at percentile $\tau_{\text{shock}}$ and at a value of $\eta_{i,t-1}$ that corresponds to the $\tau_{\text{init}}$ percentile of the distribution of $\eta_{i,t-1}$. Evaluated at mean age in the sample.
Nonlinear persistence of $y_{it}$

$$\frac{\partial Q_{y_t|y_{t-1}}(y_{i,t-1}, \tau)}{\partial y}$$

PSID panel data

Nonlinear model

Note: Estimates of the average derivative of the conditional quantile function of $y_{it}$ given $y_{i,t-1}$ with respect to $y_{i,t-1}$, evaluated at percentile $\tau_{shock}$ and at a value of $y_{i,t-1}$ that corresponds to the $\tau_{init}$ percentile of the dist. of $y_{i,t-1}$. 
Nonlinear persistence of $y_{it}$

$$\frac{\partial Q_{yt|yt-1}(y_{i,t-1},\tau)}{\partial y}$$

Norwegian register data

Nonlinear model

Note: Estimates of the average derivative of the conditional quantile function of $y_{it}$ given $y_{i,t-1}$ with respect to $y_{i,t-1}$, evaluated at percentile $\tau_{\text{shock}}$ and at a value of $y_{i,t-1}$ that corresponds to the $\tau_{\text{init}}$ percentile of the dist. of $y_{i,t-1}$. 
Figure: Densities of persistent and transitory earnings components (PSID)

(a) Persistent component $\eta_{it}$  (b) Transitory component $\varepsilon_{it}$

Note: Nonparametric estimates of densities based on simulated data according to the nonlinear model, using a Gaussian kernel.
Nonlinear persistence of $y_{it}$ (cont.)

$$\frac{\partial Q_{y_{i,t-1}}(y_{i,t-1},\tau)}{\partial y}$$

PSID data  
Canonical model

Note: Estimates of the average derivative of the conditional quantile function of $y_{it}$ given $y_{i,t-1}$ with respect to $y_{i,t-1}$, evaluated at percentile $\tau_{shock}$ and at a value of $y_{i,t-1}$ that corresponds to the $\tau_{init}$ percentile of the dist. of $y_{i,t-1}$. 
Nonlinear persistence, 95% confidence bands

(a) Earnings, PSID data  (b) Earnings, nonlinear model

Note: Pointwise 95% confidence bands. Parametric bootstrap, 500 replications.
Figure: **Conditional skewness of log-earnings residuals and $\eta$ component**

(a) Log-earnings residuals $y_{it}$  
(b) Persistent component $\eta_{it}$

Note: Conditional skewness $sk(y, \tau)$ and $sk(\eta, \tau)$, for $\tau = 11/12$. Log-earnings residuals (left) and $\eta$ component (right). The x-axis shows the conditioning variable, the y-axis shows the corresponding value of the conditional skewness measure. Bootstrap confidence intervals in the Appendix.
Consumption response to $\eta_{it}$, by assets and age

$$\bar{\phi}_t(a) = \mathbb{E} \left[ \frac{\partial g_t(a, \eta_{it}, \varepsilon_{it}, \nu_{it})}{\partial \eta} \right]$$, nonlinear model

Note: Estimates of the average consumption response $\bar{\phi}_t(a)$ to variations in $\eta_{it}$, evaluated at $\tau_{\text{assets}}$ and $\tau_{\text{age}}$. 
Consumption responses to $y_{it}$, by assets and age

$$\mathbb{E} \left[ \frac{\partial}{\partial y} \mid y_{it} \mathbb{E} (c_{it} \mid a_{it} = a, y_{it} = y, age_{it} = age) \right]$$

PSID data  Nonlinear model

Note: Estimates of the average derivative of the conditional mean of $c_{it}$ given $y_{it}$, $a_{it}$ & $age_{it}$ with respect to $y_{it}$, evaluated at values of $a_{it}$ & $age_{it}$ corresponding to their $\tau_{assets}$ & $\tau_{age}$ percentiles, and averaged over the values of $y_{it}$. 
Figure: Household heterogeneity in earnings

(a) Nonlinear persistence of $\eta_{it}$

(b) Conditional skewness of $\eta_{it}$

Notes: (a) Estimates of the average derivative of the conditional quantile function of $\eta_{it}$ on $\eta_{i,t-1}$ with respect to $\eta_{i,t-1}$, based on estimates from the nonlinear earnings model with an additive household-specific effect.

(b) Conditional skewness $sk(\eta, \tau)$, for $\tau = 11/12$, based on the same model.
Consumption response to $\eta_{it}$, by assets and age, household heterogeneity

$$\bar{\phi}_t(a) = \mathbb{E} \left[ \frac{\partial g_t(a, \eta_{it}, \xi_{it}, \nu_{it})}{\partial \eta} \right],$$
nonlinear model

Note: Estimates of the average consumption response $\bar{\phi}_t(a)$ to variations in $\eta_{it}$, evaluated at $\tau_{\text{assets}}$ and $\tau_{\text{age}}$. 
Model’s simulation

• Simulate life-cycle earnings and consumption after a shock to the persistent earnings component (at age 37).

• We report the difference between:
  – Households that are hit by a “bad” shock ($\tau_{\text{shock}} = .10$) or by a “good” shock ($\tau_{\text{shock}} = .90$).
  – Households that are hit by a median shock $\tau = .5$.

• Age-specific averages across 100,000 simulations. At age 35 all households have the same persistent component (percentile $\tau_{\text{init}}$).
Impulse responses, earnings

Bad shock: $\tau_{shock} = .1$

$\tau_{init} = .1$

$\tau_{init} = .5$

$\tau_{init} = .9$

Good shock: $\tau_{shock} = .9$

$\tau_{init} = .1$

$\tau_{init} = .5$

$\tau_{init} = .9$
Impulse responses, consumption

**Bad shock:** $\tau_{\text{shock}} = .1$

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**Good shock:** $\tau_{\text{shock}} = .9$

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Impulse responses, consumption, household heterogeneity

Bad shock: $\tau_{\text{shock}} = .1$

$\tau_{\text{init}} = .1$

$\tau_{\text{init}} = .5$

$\tau_{\text{init}} = .9$

Good shock: $\tau_{\text{shock}} = .9$

$\tau_{\text{init}} = .1$

$\tau_{\text{init}} = .5$

$\tau_{\text{init}} = .9$
Impulse responses, consumption, linear assets rule

Nonlinear model $\tau_{\text{init}} = .1$

(a) $\tau_{\text{shock}} = .1$

(b) $\tau_{\text{shock}} = .9$

(e) $\tau_{\text{shock}} = .1$

(f) $\tau_{\text{shock}} = .9$

Note: Linear assets accumulation rule. Assets are constrained to be non-negative.
Impulse responses: canonical earnings and linear consumption model

Earnings

\( \tau_{\text{shock}} = .1 \)

\[ \begin{array}{c}
\text{log-earnings} \\
\text{age}
\end{array} \]

\[ \begin{array}{c}
\text{log-earnings} \\
\text{age}
\end{array} \]

Consumption

\( \tau_{\text{shock}} = .1 \)

\[ \begin{array}{c}
\text{log-consumption} \\
\text{age}
\end{array} \]

\[ \begin{array}{c}
\text{log-consumption} \\
\text{age}
\end{array} \]
Impulse responses, by age and initial assets

Earnings

\( \tau_{\text{init}} = .9, \tau_{\text{shock}} = .1 \)
Young

Old

\( \tau_{\text{init}} = .1, \tau_{\text{shock}} = .9 \)
Young

Old

Consumption

\( \tau_{\text{init}} = .9, \tau_{\text{shock}} = .1 \)
Young

Old

\( \tau_{\text{init}} = .1, \tau_{\text{shock}} = .9 \)
Young

Old

Notes: Initial assets at age 35 (for “young” households) or 51 (for “old” households) are at percentile .10 (dashed curves) and .90 (solid curves). Linear assets accumulation rule. Assets are constrained to be non-negative.
Impulse responses by age and initial assets, linear assets rule

(a) Young

\[ \tau_{\text{init}} = 0.9, \quad \tau_{\text{shock}} = 0.1 \]

(b) Old

\[ \tau_{\text{init}} = 0.1, \quad \tau_{\text{shock}} = 0.9 \]

Notes: Initial assets at age 35 (for “young” households) or 51 (for “old” households) are at percentile .10 (dashed curves) and .90 (solid curves). Linear assets accumulation rule. Assets are constrained to be non-negative.
Conclusions and Next Steps

• New framework to shed new light on the nonlinear transmission of income shocks to consumption and the nature of insurance to income shocks.
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▷ A Markovian permanent-transitory model of household income, which reveals asymmetric persistence of unusual shocks in the PSID and in large administrative registers.
▷ An age-dependent nonlinear consumption rule that is a function of assets, permanent income and transitory income.
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• Provide conditions for nonparametric identification:

⇒ explain how a simulation-based sequential QR method is feasible.

• This framework leads to new empirical measures of the degree of partial insurance and the link between income and consumption inequality.
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• Provide conditions for nonparametric identification:
  ⇒ explain how a simulation-based sequential QR method is feasible.

• This framework leads to new empirical measures of the degree of partial insurance and the link between income and consumption inequality.

⇒ Next steps: Estimate on the full population register data. Generalize our nonlinear model to allow for other states or choices, such as evolution of household size and intensive/extensive margins of labor supply.
Identification when $T = 3$: Wilhelm (12)

- We work in $L^2$-spaces relative to suitable distributions.
- Let $g(y_2, y_3)$ such that there exists a $s(y_2)$ such that

\[
\mathbb{E}[g(Y_2, Y_3)|Y_1] = \mathbb{E}[s(Y_2)|Y_1].
\]

Under completeness of $Y_2|Y_1$, $s(\cdot)$ is unique.
- By conditional independence,

\[
\mathbb{E}[\mathbb{E}(g(Y_2, Y_3)|\eta_2)|Y_1] = \mathbb{E}[\mathbb{E}(s(Y_2)|\eta_2)|Y_1].
\]

- Under completeness of $\eta_2|Y_1$, it follows that

\[
\mathbb{E}[g(Y_2, Y_3)|\eta_2] = \mathbb{E}[s(Y_2)|\eta_2].
\]
The case $T = 3$ (cont.)

- Wilhelm (12) considers the functions $g_1(Y_3) = 1\{Y_3 \leq y_3\}$, and $g_2(Y_2, Y_3) = Y_2 1\{Y_3 \leq y_3\}$, for a given value $y_3$.
- This yields

\[
\begin{align*}
\mathbb{E} [1\{Y_3 \leq y_3\} | \eta_2] &= G(\eta_2) = \mathbb{E} [s_1(Y_2) | \eta_2] \\
\mathbb{E} [Y_2 1\{Y_3 \leq y_3\} | \eta_2] &= \eta_2 G(\eta_2) = \mathbb{E} [s_2(Y_2) | \eta_2].
\end{align*}
\]

- Hence, taking Fourier transforms (i.e., $\mathcal{F}(h)(u) = \int h(x) e^{iux} dx$),

\[
\mathcal{F}(G)(u) = \mathcal{F}(s_1)(u) \psi_{\epsilon_2}(-u)
\]

\[
i^{-1}d\mathcal{F}(G)(u)/du = \mathcal{F}(s_2)(u) \psi_{\epsilon_2}(-u),
\]

where $\psi_{\epsilon_2}(u) = \mathcal{F}(f_{\epsilon_2})(u)$ is the characteristic function of $\epsilon_2$, and $i = \sqrt{-1}$.
The case $T = 3$ (cont.)

• This yields the following first-order differential equation

$$\mathcal{F}(s_1)(-u) \frac{d\psi_{\varepsilon_2}(u)}{du} = \left[ \frac{d\mathcal{F}(s_1)(-u)}{du} - i\mathcal{F}(s_2)(-u) \right] \psi_{\varepsilon_2}(u).$$

• In addition, $\psi_{\varepsilon_2}(0) = 1$.
• This ODE can be solved in closed form for $\psi_{\varepsilon_2}(\cdot)$, provided that $\mathcal{F}(s_1)(u) \neq 0$ for all $u$ (which is another injectivity condition).
• As a result, the distribution of $\varepsilon_2$, and the distribution of $Y_3$ given $\eta_2$, are both nonparametrically identified.
### Descriptive statistics (means)

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<td>376,485</td>
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**Notes:** Balanced subsample from PSID, $N = 749$, $T = 6$.
- Compared to BPS (12), households in our balanced sample have higher assets, and to a less extent higher earnings and consumption.
Consumption response, two-period model

- CRRA utility. The Euler equation is (assuming $\beta(1 + r) = 1$)

$$C_1^{-\gamma} = \mathbb{E}_1 \left[ (1 + r) A_2 + Y_2 \right]^{-\gamma},$$

where $\gamma > 0$ is risk aversion and we have used the budget constraint

$$A_3 = (1 + r) A_2 + Y_2 - C_2 = 0.$$

- Let $X_1 = (1 + r) A_1 + Y_1$, $R = (1 + r) X_1 + \mathbb{E}_1 (Y_2)$, and $Y_2 = \mathbb{E}_1 (Y_2) + \sigma W$. Expanding as $\sigma \to 0$ we obtain

$$C_1 \approx \frac{(1 + r) X_1 + \mathbb{E}_1 (Y_2)}{2 + r} - \frac{\gamma + 1}{2R} \mathbb{E}_1 (W^2) + \frac{(2 + r)(\gamma + 1)(\gamma + 2)}{6R^2} \mathbb{E}_1 (W^3).$$

- certainty equivalent
- precautionary-variance
- precautionary-skewness
Nonlinear persistence, 95% confidence bands

(a) Earnings, PSID data  (b) Earnings, nonlinear model

(c) Persistent component $\eta_{it}$, nonlinear model

Note: Pointwise 95% confidence bands. Parametric bootstrap, 500 replications.
Conditional skewness of log-earnings residuals and $\eta$ component, 95% confidence bands

(a) Log-earnings residuals $y_{it}$

(b) Persistent component $\eta_{it}$

Note: Pointwise 95% confidence bands. Parametric bootstrap, 500 replications.
Consumption responses to earnings shocks, by assets and age, 95% confidence bands

(a) Consumption response to $\eta_{it}$
Nonlinear model

(b) Consumption response to $\varepsilon_{it}$
Nonlinear model

Note: Pointwise 95% confidence bands. Parametric bootstrap, 48 replications (preliminary).
Consumption response to $\varepsilon_{it}$, by assets and age

$$\overline{\psi}_{it}(a) = \mathbb{E} \left[ \frac{\partial g_t(a, \eta_{it}, \varepsilon_{it}, \nu_{it})}{\partial \varepsilon} \right], \text{ nonlinear model}$$

Note: Estimates of the average consumption response $\overline{\psi}_{it}(a)$ to variations in $\varepsilon_{it}$, evaluated at $\tau_{assets}$ and $\tau_{age}$.
Nonlinear persistence of $\eta_{it}$ (Norwegian register data):

$$\rho_t(\eta_{i,t-1}, \tau) = \frac{\partial Q_{\eta_t|\eta_{t-1}}(\eta_{i,t-1}, \tau)}{\partial \eta}$$

Note: Estimates of the average derivative of the conditional quantile function of $\eta_{it}$ on $\eta_{i,t-1}$ with respect to $\eta_{i,t-1}$, evaluated at percentile $\tau_{\text{shock}}$ and at a value of $\eta_{i,t-1}$ that corresponds to the $\tau_{\text{init}}$ percentile of the distribution of $\eta_{i,t-1}$. Evaluated at mean age in the sample.
A simple nonlinear parametric model

- A parametric model for $\eta_{it}$ is

$$
\eta_{it} = \rho_t(\eta_{i,t-1}, v_{it}) \eta_{i,t-1} + v_{it},
$$

where $\rho_t(\eta, v) = 1 - \delta$ if $(\eta > c_{t-1}, v < -b_t)$ or $(\eta < -c_{t-1}, v > b_t)$, and $\rho_t(\eta, v) = 1$ otherwise; and $v_{it} \sim \mathcal{N}(0, \sigma^2_t)$.

- Persistence is lower ($1 - \delta < 1$) when a bad shock hits a high earnings household (“individual disasters”), or a good shock hits a low earnings household.

- $\eta_{it}$ features conditional skewness: positive for low $\eta_{i,t-1}$, negative for high $\eta_{i,t-1}$.

- In the simulation results we set $\delta = .2$ and the probability of a high or low “unusual shock” set to 15%.
Simulated Variance of Consumption and Assets

Consumption variance over the life-cycle

Assets variance over the life-cycle

Note: Blue is nonlinear earnings process, Green is canonical earnings process.
Impulse responses, consumption

Bad shock: \( \tau_{\text{shock}} = .1 \)
- \( \tau_{\text{init}} = .1 \)
- \( \tau_{\text{init}} = .5 \)
- \( \tau_{\text{init}} = .9 \)

Good shock: \( \tau_{\text{shock}} = .9 \)
- \( \tau_{\text{init}} = .1 \)
- \( \tau_{\text{init}} = .5 \)
- \( \tau_{\text{init}} = .9 \)
A simple model of income dynamics

In this ‘permanent - transitory’ decomposition, log income for household $i$ in time period $t$, is written

$$y_{it} = Z_{it}' \varphi + \eta_{it} + \varepsilon_{it}$$
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- A key consideration is to allow the distributions of the latent persistent and transitory factors ($\eta_t$ and $\varepsilon_t$) to vary with age/time for each birth cohort.

- Simple but can be very revealing - detailed work on Norwegian population register panel data and PSID.
Partial Insurance

The transmission parameters $\phi_t$ and $\psi_t$ link the evolution of consumption inequality to income inequality.

- they indicate the degree of partial insurance, and will differ by age, assets and human capital.
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Using the PSID, BPP estimates of partial insurance, $\hat{\phi}_t$:

0.6423 (.09) overall,
0.9439 (.13) for the sample without college education, and
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- The estimate falls by more than 30% if we exclude taxes, EITC and food stamps (SNAP) for the no college group.
- For a low wealth sample $\hat{\phi}_t$ is .8489 and there are significant impacts of transitory fluctuations in income too.
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- For example, an unusually bad shock, to those on higher permanent income, can wipe out their permanent income history.
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