A Model of a Vehicle Currency with Fixed Costs of Trading

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March 7, 2005

The international financial system is very far from the ideal symmetric mechanism that is often described in theoretical models. Countries differ greatly in market size, financial openness, and asset positions. One of the most obvious asymmetries in the financial system concerns the role of currencies. It is widely acknowledged that the US dollar occupies a central role in the international economy. The dollar represents an international unit of account, in that it operates as an invoicing currency for commodity and asset trade, it represents a store of value, in that official reserves assets of central banks are predominantly held in US dollars, and it represents a international means of exchange, in that foreign exchange transactions are overwhelmingly conducted using the US dollar as one side of the transaction. In this latter function, the US dollar as a means of exchange, it has been noted (Krugman 1979, see also Rey 1998) that the dollar acts as a ‘vehicle currency’ for international trade.

In general bilateral markets for smaller country currencies are thin or non-existent. To engage in currency trade between the currencies of small countries, generally the US dollar is used as a vehicle. We would expect this to have some efficiency gains if there are fixed costs to setting up trading technologies. On the other hand, it would

\footnote{These represent an unfinished set of preliminary notes. Please do not circulate.}
seem to give the US dollar and US monetary policy a predominance over the rest of the world in a way that it would not necessarily always be beneficial, particularly in light of the fact that US monetary policy is focused on domestic rather than international goals.

This paper develops a simple dynamic general equilibrium model of a vehicle currency. We build a multi-country monetary exchange economy model. The money of a particular country is required to finance purchases in that country, through a cash-in-advance constraint. But the way in which agents acquire foreign currencies may differ. We model foreign exchange trade as a costly process that takes place through ‘trading post’ technologies. Trading posts have been modelled by Starr (2000) and Howitt (2005). They represent locations where agents can go in order to buy or sell one currency for another; that is, they facilitate bilateral trade in currencies. But trading posts are costly to set up. In a purely symmetric world, there would be one trading post for all possible bilateral pair of currencies. This would give agents and countries ‘equal treatment’ in the world economy. But this would also involve significant real resources used up in setting up trading posts.

An alternative equilibrium is where one country operates as a ‘vehicle currency’. That clearly Pareto dominates the symmetric equilibrium. But at the same time, it confers significant benefits on the vehicle currency issuer. The main object of the paper is to explore this trade off. We find that for a small number of countries, it is never desirable (for peripheral countries) to have a vehicle currency. The greater is center
country monetary growth, the less benefit does the periphery get from the vehicle currency. We find also that only one vehicle currency can exist at any one time.

The paper is not only motivated by the analysis of how a vehicle currency system works. We are also interested in determining how such situations come about. Historically, different currencies have acted as the standard in the international economy during different epochs. The pound sterling represented the central international currency in the pre WWI period, and to a lesser extent in the interwar period. More contemporaneously, now the euro offers a viable alternative international standard, what forces would lead it to supplant the US dollar as a vehicle currency? In a later version of the paper, we explore the robustness of a vehicle currency equilibrium.

1 The Model

1.1 Technology and Preferences

We assume that time is discrete, indexed by \( t = 0, 1, \ldots \). There are \( N \geq 3 \) identical countries, indexed by \( i = 1, 2, \ldots, N \), and every country has the same population which is normalized to 1. Each country has its own currency. Let \( M_t \) be the total stock of currency \( i \) in period \( t \) and the gross rate of growth of currency \( i \) be \( \mu_{it} = M_{it}/M_{it-1} \). Each country has a given endowments of its own good, so each household in a country \( i \) is endowed with \( y_{it} \) units of good \( i \) in period \( t \). All goods are perishable within a period.
Residents of a country receive lump-sum transfers from their monetary authority. Each household in country $i$ receives an amount, $(\mu_{it} - 1)M_{it-1}$, of currency $i$ at the beginning of period $t$. There are no fiscal transfers across countries.

To enforce a major role for international trade, we assume that preferences are identical across countries, and residents of each country wish to consume all country’s goods. A country $i$ household’s preferences are written:

$$\sum_{t=0}^{\infty} \beta^t \left[ \sum_{j=1}^{3} u(c_{ijt}) \right],$$

where $\beta \in (0, 1)$ is the discount factor;

1.2 Monetary Exchange

We impose a cash in advance constraint at the national level. That is, purchases of country $i$’s goods must use only currency $i$. Therefore, in order to consume country $j$’s good, a household within country $i$ must obtain currency $j$. How currency trade takes place is the main focus of interest in the paper.

We assume that currency trade is organized in bilateral trading posts. That is, at a trading post, one currency is exchanged for another. We order the two currencies at a post in ascending order and call a trading post with currencies $k$ and $j$ as post $kj$, where $k < j$;

Anyone can set up a trading post, but doing so involves fixed costs. In order to
set up trading post $kj$, the manager of a trading post must incur a fixed cost $\phi_k$ in good $k$ and $\phi_j$ in good $j$. There is also a cash-in-advance constraint on trading posts - the fixed cost in each country’s good needs to be paid in that country’s money.

The managers of each trading post announce two prices for a pairwise trade, one for sale of a currency (ask) for another currency, and one for purchase (bid) of a currency for another currency. We assume that potential entry into a trading post leads each manager to follow a Bertrand pricing rule. In equilibrium the bid and ask prices announced by the manager of the trading post is just sufficient to cover the fixed costs of setting up the trading post, given the buyers and sellers of the currency pair over which the trading post operates. These prices then represent the equilibrium nominal exchange rates for each currency pair.

In reality of course, currency traders do not just trade one currency for another. But there are clear limits on the number of exchange possibilities that exist. Few commercial currency exchanges are willing to buy or sell much more than about a half dozen currencies. Moreover, bid ask spreads are typically higher for smaller currencies. The use of trading posts allows us a simplified way to handle the frictions inherent in currency trading.

Note that with $N$ countries and trading posts for each pair of currencies, there are $N(N - 1)/2$ possible trading posts. But with each trading post incurring fixed costs, in principle this can be improved upon by using one currency as an an intermediate, and trading twice. When one currency plays the role of a ‘vehicle’, then only $N - 1$
trading posts need to exist in order to facilitate trade.

One immediate aspect of this environment should be clear. There will be many configurations of trading posts that constitute Nash equilibria. To see this note that if agents believe that a trading post is inactive, then no-one will have an incentive to bring a currency to buy or sell at this trading post. Hence it can remain inactive. This allows us to focus on a number of different equilibria of interest. The two equilibria that we investigate below are a) a Symmetric Trading equilibrium, in which there is a trading post for each pair of countries and there are $N(N - 1)/2$ trading posts, and b) a Vehicle Currency equilibrium, in which one currency is common to all trading posts, and there are only $N - 1$ trading posts.

Of course it is unsatisfactory merely to focus on alternative equilibria, since using this criterion, it is hard to rule out any trading configuration, however inefficient. In light of this, we investigate the robustness of the Vehicle Currency equilibrium in a later section of the paper.

The timing of events is as follows. At the beginning of a period, agents have any unspent cash balances in each currency. They receive their income from last period sales of their endowment, in domestic currency, plus a domestic currency transfer from the monetary authorities. In sum this gives $M_{ijt}$, which is country $i$ residents holdings of currency $j$ in time $t$. Agents then visit the trading posts of their choice in order to exchange currencies. By assumption, they can only visit a given trading post once in each period. After currency exchange at trading posts, they hold $M'_{ijt}$ of each currency.
After the currency trading is over, agents visit the goods market, with each household dividing into a shopper and a seller. At the end of the period, the households consumes all the goods purchased. The following period describes the timing:

\[
\begin{array}{cccc}
  t & m_{ijt} & \text{goods} & t + 1 \\
  \uparrow & \downarrow & \downarrow & \downarrow \\
  \text{measured} & \text{mkts} & \text{endowments,} & \text{currency} \\
  \text{m. transfers} & \text{trades} & \text{consume} \\
\end{array}
\]

Denote the nominal exchange rate for a buyer of currency \( j \) at a post \( kj \) in period \( t \) as \( S_{kjt}^a \). This the amount of currency \( k \) that can be bought with one unit of currency \( j \) at the post, or the ‘ask’ price of \( j \) in terms of \( k \). For a seller of currency \( j \), at trading post \( kj \), the exchange rate is \( S_{kjt}^b \). This is amount of \( k \) that can be obtained at the post, for one unit of \( j \), or the bid price of currency \( j \) in terms of \( k \). Clearly, \( S_{kjt}^a \geq S_{kjt}^b \) must hold, if trading posts are to be viable.

Let \( f_{ikt}^{kj} \) be the amount of currency \( k \) (normalized by the total stock of currency \( k, M_{kt} \)) brought to the post \( kj \) by a particular country \( i \) household, and let \( F_{ikt}^{kj} \) be the sum of \( f_{ikt}^{kj} \) over country \( i \) households. The trading restriction is that households cannot short on currencies at any post, that is, \( f_{ikt}^{kj} \geq 0 \) for all \( i, k, j \). The post \( kj \) is said to be active if at least one side of the post has a positive amount of currency, i.e., if \( \left( \sum_{i=1}^{N} F_{ikt}^{kj} \right) + \left( \sum_{i=1}^{N} F_{ijt}^{kj} \right) > 0. \)
1.3 Country i Household Decisions

Normalize all nominal variables associated with currency \( i \) by the total stock of currency \( i \), \( M_{it} \), which is measured immediately after the currency trades in period \( t \). Similarly, normalize the exchange rate by the relative stock of the two currencies involved, as follows:

\[
s_{kj} = \frac{S_{kj} M_{jt}}{M_{kt}}, \quad k < j, \quad z = a, b.
\]

We use the following general notation of monetary transfers \( \tau_{ij} \):

\[
\tau_{it} = \tau_{it} = (\gamma_{it} - 1)/\gamma_{it}; \quad \tau_{ijit} = 0 \text{ for } j \neq i.
\]

Similarly, \( y_{it} = y_{it} \) and \( y_{ijit} = 0 \) for \( j \neq i \).

In each period \( t \), the household’s choices \( h_{it} \), consist of the following variables (where the subscript \( t \) is suppressed):

- \( f_{kj} \): the amount of currency \( j \) brought to each post \( kj \), where \( k < j \);
- \( f_{jk} \), the amount of currency \( j \) brought to each post \( jk \), where \( j < k \);
- \( c_{ij} \): consumption of each good \( j \);
- \( m_{ijt+1} \): future holdings of money \( j \).

For given endowments of money stocks \( (m_{ij(-1)})_{j=1}^{N} \), the household chooses a sequence \( \{h_{it}\}_{t=0}^{\infty} \) to solve the following problem:

\[
\max \sum_{t=0}^{\infty} \beta^{t} \left[ \sum_{j=1}^{N} u(c_{ijt}) \right]
\]
subject to the following constraints:

(i) Restriction at the currency posts (all currency j brought to the posts must come from left-over currency j in the last period, sales of goods in the last period, or monetary transfers in this period):

\[ \sum_{k<j} f_{ijkt}^{kj} + \sum_{k>j} f_{ijkt}^{jk} \leq m_{ijt}, \]

\[ m_{ijt} = \frac{1}{\gamma_{jt}} \left[ m_{ijt-1} - p_{jt-1} - c_{ijt-1} + p_{jt-1} y_{ijt-1} \right] + \tau_{ijt}. \]

Recall that m is measured immediately before currency trades and m’ is measured immediately after currency trades. The money growth \( \gamma_{jt} \) is applied to the amount of money carried over from the last period because \((f_{ijkt}^{kj}, f_{ijkt}^{jk}, \tau_{ij})\) are normalized by \( M_{jt} \) but \((m'_{ijt-1}, p_{jt-1})\) are normalized by \( M_{jt-1} \). \( p_{jt} \) is the normalized price nominal price of good j in currency j.

(ii) Households cannot short on currencies:

\[ f_{ijkt}^{kj}, f_{ijkt}^{jk} \geq 0, \quad \text{all } k \neq j; \]

(iii) Law of motion of money holdings:

\[ m'_{ijt} = m_{ijt} + \sum_{k<j} \left( \frac{1}{s_{kjt}^a} f_{ikjt}^{kj} - f_{ijkt}^{kj} \right) + \sum_{k>j} \left( s_{jkt}^b f_{ikjt}^{jk} - f_{ijkt}^{jk} \right), \]

where the first summation is the net amount of currency j that the household exchanges using currencies \( k < j \), and the second summation is the net amount of currency j that the household exchanges using currencies \( k > j \).
To clarify this notation, take the example of household $i$ who supplies currency $k$ in the amount $f^{kj}_{ik}$ to the $kj$ trading post, where $k < j$. This household is then denoted the seller of currency $k$ and pays the (normalized) ask price of $j$, $s^a_{kjt}$.

On the other hand, if the household supplies $f^{jk}_{ik}$ to the $jk$ trading post, where $j < k$, then this household is denoted the buyer of currency $j$ and receives the (normalized) bid price of $k$, $s^b_{jk}$. (iv) Cash in advance in the goods market:

$$p_{jt}c_{ijt} \leq m^l_{ijt}.$$

We now focus on two leading configurations of trading posts. In the first, all bilateral trading posts are active, while in the second, only $N - 1$ posts are active.

2 Symmetric Trading Equilibrium

Under this configuration, there is a trading post for each possible pair of currencies. We first examine the optimal choices of households, taking exchange rates as given, and then look at the equilibrium exchange rates which ensures that trading posts are viable. We denote this case the ‘Symmetric Trading’ equilibrium. The key assumption we make is that all cash-in-advance constraints are binding. This means that households have no foreign currency left over when they enter a period, and must visit all trading posts in order to ensure that they can consume all goods. Recall that there is only one session of currency trading per period.
2.1 Households

For households in country $i$, the constraints in this case can be reduced to the following:

$$m_{iit} = \frac{1}{\gamma_{it}} \left( m'_{iit-1} - p_{iit-1} c_{iit-1} + p_{iit-1} y_{iit-1} \right) + \tau_{it}$$

$$m'_{iit} = m_{iit} - \sum_{i<j} f_{s_{ij}}^{ij} - \sum_{i>j} f_{s_{ij}}^{ji}$$

$$m_{ijt} = \frac{1}{\gamma_{jt}} \left( m'_{ijt-1} - p_{ijt-1} c_{ijt-1} \right)$$

$$m'_{ijt} = m_{ijt} + \frac{1}{s_{ijt}} f_{s_{ijt}}^{ij}, \quad i < j$$

$$m'_{ijt} = m_{ijt} + s_{s_{ijt}}^{ij} f_{s_{ijt}}^{ij}, \quad i > j$$

$$m'_{ijt} = p_{ijt} c_{ijt}, \quad j = 1..N$$

Let $\lambda_{ijt}$ be the current-value multiplier on country $i$'s currency $j$ cash-flow, $\psi_{ijt}$ be the multiplier on its currency $j$ cash in advance constraint, and $\eta_{ijt}$ be the multiplier on its portfolio constraint for currency $j$. Then we may show that household $i$'s optimal choice can be described as:

$$\frac{u'(c_{ijt})}{P_{jt}} = \beta \lambda_{ijt+1} + \psi_{ijt}$$

$$\lambda_{ijt} = \eta_{ijt}$$

$$\eta_{ijt} = \beta \lambda_{ijt+1} + \psi_{ijt}$$

$$\eta_{iit} = \frac{\eta_{ijt}}{s_{ijt}^{a}}, \quad i < j$$
\[ \eta_{it} = \frac{\eta_{ijt}}{s_{jit}^b}, \quad i > j \]

As discussed above, we make the assumption that all cash in advance constraints bind, so that \( \psi_{ijt} > 0 \), for all \( j \) and \( t \). Then we may reduce these conditions to

\[
\frac{u'(c_{it})}{p_{it}} = \frac{u'(c_{ijt})}{s_{ijt}^a p_{jt}}, \quad i < j
\]

\[
\frac{u'(c_{it})}{s_{jit}^b p_{it}} = \frac{u'(c_{ijt})}{p_{jt}}, \quad i > j
\]

These conditions just say that, when all trading posts are active, and cash-in-advance constraints bind, agents in all countries choose an optimal consumption plan that according to a standard a-temporal trade-off at prevailing prices and exchange rates.

Note that, since all cash-in-advance constraints bind, we may combine the portfolio allocation equations for agent \( i \) as follows;

\[
p_{it} c_{it} + \sum_{i<j} s_{ijt}^a p_{jt} c_{ijt} + \sum_{i>j} \frac{1}{s_{jit}^b} p_{jt} c_{ijt} = m_{iit}
\]

At the prices prevailing in all trading posts that are visited, the household divides up beginning of period cash-balances between expenditures on all goods.

### 2.2 Trading Posts

In this equilibrium, there is a firm at each trading post \( ij \). The firm sets prices \( s_{ij}^a \) and \( s_{ij}^b \) so as to just break even, after it incurs the fixed cost \( \phi_i \) in good \( i \) and \( \phi_j \) in good \( j \).

The firm must pay these fixed costs with currency. Hence, there is a cash in advance constraint applied to operators of trading posts as well as households. The firm must hold currency \( i \) (normalized) in the amount \( p_{it} \phi_i \) and currency \( j \) in the amount \( p_{jt} \phi_j \).
The implicit idea here is that if the firm were to make profits, then there is another firm in the background which would enter the $ij$ trading post. So the firm engages in Bertrand pricing (see Howitt 2005 for a formalization of this assumption) \(^3\)

Exchange rates in trading post $ij$ are set by the firm so as to satisfy two conditions. The first condition, determining the ask price of currency $j$, is written as:

$$s_{ijt}^a \left[ f_{jjt}^{ij} - p_{jt} \phi_j \right] = \frac{f_{it}^{ij}}{3}$$

This is explained as follows. In the Symmetric Trading equilibrium, trading post $ij$ receives total currency $j$ payments of $f_{jjt}^{ij}$ (since only country $j$ agents hold currency $j$ in this equilibrium). They must hold currency $p_{jt} \phi_j$ to pay the good $j$ fixed costs of setting up the trading post. It receives $f_{it}^{ij}$ deliveries of currency $i$ from country $i$ residents. It must set the asking price of currency $j$ that country $i$ residents will pay so that its holdings of currency $j$ are all paid out to country $i$ households. From this condition, $s_{ijt}^a$ exactly satisfies this property.

In a similar manner, to determine the bid price, $s_{ij}^b$, the trading post must satisfy the condition that deliveries of currency $i$ made by country $i$ households less required

\(^3\)There is some looseness in scaling here. We talk about there being one firm, which pays a fixed cost, but there being a large number of individuals, so that a single trading post manages transactions for all currency trade between two economies. We would anticipate the fixed cost of setting up a single firm to be miniscule relative to to the size of the economy. With some additional notation we could easily avoid this by assuming that each economy consisted of a large group of ‘Islands’ that were physically separate, and in each Island, there was a single trading post that acted in the same way as the trading posts modelled here.
currency holdings of \( p_i \phi_i \), must equal the deliveries of currency \( j \) by country \( j \) residents. This condition is:

\[
S_{ijt} f_{ij}^t = [f_{ii}^t - p_i \phi_i]
\]

Conditions (1) and (2) determine the structure of exchange rates in the Symmetric Trading equilibrium, conditional on currency deliveries made by households.

To determine \( f_{ij}^t \) and \( f_{jj}^t \), however, we need to solve the households problem more explicitly. First, note that total normalized money holdings for currency \( i \) are

\[
\sum_{j=1}^{N} m_{jit}' + 2P_i \phi_i = m_{it} = 1
\]

The last term on the left hand side indicates that there are two trading posts which use currency \( i \), and both posts need to hold \( p_i \phi_i \) in currency. But we know also, if all cash in advance constraints are binding, the left hand side of this equation may be written as

\[
\sum_{j=1}^{N} (P_{it} c_{jit} + 2 \phi_i) = P_{it} y_{it}
\]

Hence, we have \( P_{it} y_{it} = m_{it} = 1 \). Given country \( i \)’s budget constraint, with the cash-in-advance constraint binding, this implies that

\[
m_{it} = m_{it} = 1
\]

Combining this fact with conditions (1) above allow us to determine currency demands, implicitly.
Example:

To get further, we must make explicit assumptions about the form of preferences. Take the case of identical logarithmic preferences:

$$\sum_{j=1}^{N} U(c_{ij}) = \sum_{j=1}^{N} \ln(c_{ij})$$

Solving () for this utility function, it is easy to show that $f_{ij} = \frac{m_i}{N} = \frac{1}{N}$. Substituting into conditions () and () give the following solutions for exchange rates:

$$s_{ij}^b = (1 - N\hat{\phi}_{it})$$

where $\hat{\phi}_{it} = \frac{\phi_i}{N}$.

Likewise, we have:

$$s_{ij}^a = \frac{1}{(1 - N\hat{\phi}_{jt})}$$

So the bid-ask spread in the $ij$ market is

$$\frac{s_{ij}^a}{s_{ij}^b} = \frac{1}{(1 - N\hat{\phi}_{it})(1 - N\hat{\phi}_{jt})} \geq 1$$

The explanation for these expressions is straightforward. Say that trading posts could operate without costs, so that $\phi_i = 0$. Then, with log utility, the terms of trade between any two countries would be $\frac{m_i}{y_{jt}}$, and the normalized exchange rate would be unity among all pairs of currencies. But with fixed costs of setting up each trading post, and noting that in a symmetric economy agents in any country $j$ will spend an amount $\frac{1}{N}$ on each trading post that they visit, then the fixed cost relative to the cash deliveries is $\frac{\mu_i \phi_i}{N} = N\hat{\phi}_{it}$. To break even, the trading post must ensure that the bid
price of currency $j$ for each currency $i$ is reduced by this amount. A similar argument holds for the ask price. Put together, the bid-ask spread reflects the cost of the trading post in terms of both countries good.

Note also that the real cost of the trading post, and the bid-ask spread, is lower, the larger is output per capita, $y_{it}$. So agents buying currency $j$ will pay a lower ask price, the wealthier is country $j$.

Using the solutions for exchange rates and prices, we may determine consumption allocations under the Symmetric Trading equilibrium:

$$C_{iit} = \frac{y_{it}}{N}$$

$$C_{1jt} = \frac{Y_{jt}}{N}(1 - N\hat{\phi}_{jt})$$

The Symmetric Trading equilibrium is quite uncomplicated. Countries receive equal treatment. Each country consumes $\frac{1}{N}$ of its own good, and $\frac{1}{N}$ less the trading post costs of each other countries good. Thus, the trading posts induce an endogenous ‘home bias’ in consumption. But allocations and welfare are independent of money growth. Money is neutral, and there are obviously no international ‘spillovers’ of monetary policy.
3 Country 1 is a Vehicle Currency

We now focus on the polar opposite assumption about the configuration of trading posts. Now assume that all foreign exchange trading is conducted through currency 1 posts. Thus, currency 1 has active trading posts with all other currencies, but there are no bilateral posts other than those with country 1. Thus, currency 1 is a vehicle country. We will sometimes refer to Country 1 as the ‘center’ country, and all other countries as the ‘client’ countries.

In this case, residents of all countries \( j > 1 \) must engage in two foreign exchange transactions in order to consume goods other than their own or country 1’s good. Since there is only one round of trading in the currency markets in each time period, this means that, from the time of their decision to consume an additional unit of these goods, they must wait one period for consumption to take place.

The decision problem facing country 1 is identical to that described above, because country 1 has active trading posts with all other countries. But for country \( i > 1 \), the decision problems one of maximizing utility subject to the following altered set of constraints.

\[
m_{ijt} = \frac{1}{\gamma_{jt}} (m'_{ijt-1} - p_{jt-1} c_{ijt-1})
\]

\[
m'_{i1t} = m_{i1t} + s_{i1t} f_{i1t} - \sum_{j \neq i} f_{i1t}^{1j}
\]

\[
m_{i1t} \geq \sum_{j \neq i} f_{i1t}^{1j}
\]
\[m'_{ijt} = m_{ijt} + \frac{1}{s^a_{ijt}} f^1_{ijt}, \quad j \neq i, j > 1\]

\[m_{iit} = \frac{1}{\gamma_{it}} (m'_{iit-1} - p_{it-1} c_{iit-1} + p_{it-1} y_{it-1}) + \tau_{it}\]

\[m'_{iit} = m_{iit} - f^1_{iit}\]

\[m'_{ijt} \geq p_{jt} c_{ijt} \quad j = 1..N\]

We may explain these conditions as follows. Currency 1 is special, in that in order to consume any good \(j > 1, j \neq i\), country \(i\) residents must have either currency \(j\) itself, or currency 1, at the beginning of the period \(t\). Total purchases of currencies \(j, j \neq i\), is therefore constrained by the entering balances of currency 1. We may call this the 'vehicle currency constraint'. We will show below that for all currencies other than 1, cash in advance constraints will bind under the same set of conditions as described above. Therefore, in order to consume other country’s goods, agents need the vehicle currency.

First order conditions for country 2:

\[\frac{u'(c_{i1t})}{P_{1t}} = \frac{\beta}{\gamma_{1t+1}} \lambda_{i1t+1} + \psi_{i1t}\]

\[\lambda_{i1t} = \eta_{i1t} + \mu_{i1t}\]

\[\eta_{i1t} = \frac{\beta}{\gamma_{1t+1}} \lambda_{i1t+1} + \psi_{i1t}\]

\[\eta_{i1t} + \mu_{i1t} = \frac{\eta_{jt}}{s^a_{1jt}}\]

\[\frac{u'(c_{ijt})}{P_{jt}} = \frac{\beta}{\gamma_{jt+1}} \lambda_{ijt+1} + \psi_{ijt}, \quad j > 1\]
\[ \lambda_{ijt} = \eta_{ijt} \]

\[ \eta_{ijt} = \frac{\beta}{\gamma_{jt+1}} \lambda_{ijt+1} + \psi_{ijt} \]

\[ \eta_{i1t} = \frac{\eta_{i1t}}{s^b_{i1t}} \]

Here the multipliers are all as before, except that \( \mu_{i1t} \) is the multiplier on the vehicle currency constraint.

Assume that \( \psi_{i1t} = 0 \), or the cash in advance constraints for currency 1, is not binding. This will never be binding in this instance, or country \( i \) could not consume any of the goods other countries except good 1. We can then rearrange these conditions to get:

\[ \frac{u'(c_{i1t})}{p_{1t}} = \frac{u'(c_{i1t})}{s^b_{i1t} p_{i1t}} \]

\[ \frac{u'(c_{i1t})}{s^b_{i1t} p_{i1t}} = \frac{\beta}{\gamma_{i1t+1} s^b_{i1t+1} p_{i1t+1}} = \frac{\beta}{\gamma_{i1t+1} s^b_{i1t+1} p_{i1t+1}}, \quad j \neq i \]

The trade-off between good 1 consumption and that of the domestic good is the same as before, for each country. But the trade-off involved in consumption of the domestic good and that of a third country is quite different. Giving up one dollar of domestic currency incurs a sacrifice of \( \frac{u'(c_{i1t})}{p_{i1t}} \) in terms of consumption of the domestic good. Converted into currency 1, this gives \( s^b_{i1t} \). But since the vehicle currency constraint is binding, this can only be converted into a third country’s currency in next period foreign exchange trading session. Hence the marginal benefit of this investment is \( s^b_{i1t} \frac{\beta}{\gamma_{i1t+1} s^b_{i1t+1} p_{i1t+1}} \). Hence, there are three basic features of the vehicle currency environment that impact on the decisions of peripheral countries. First, in
their consumption of third country goods, they must undertake two foreign exchange
transactions, accepting the bid price of their own currency, and paying the ask price of
currency 1 for the third country currency. Second, this involves a delay, which is costly
because agents discount future utility. Finally, it also involves a cost due to country
1 money growth, as country 1 inflation will reduce the real value of their currency 1
money holdings over time.

Again, maintaining the assumption that the vehicle currency constraint is bind-
ing, we may re-arrange the budget constraints to establish that, for all countries \( i > 1 \),
we have

\[
p_i t c_{i t} + \frac{1}{s_{1i t}^b} p_i t c_{i1 t} + \frac{1}{s_{1i t}^b} \sum_{j \neq i} s_{1j t+1}^o p_{jt+1} c_{j t+1} = m_i t
\]

Thus, a peripheral country arranges its spending in any time period so that the sum
of current valued spending on its own good and the good of the vehicle country, plus
the discounted value of future spending on all third country goods, just exhausts its
beginning of period cash balances (in its own currency).

For country 1 however, there is no necessity to engage in multiple foreign exchange
dealings. Its decisions are as before and it faces the same spending constraint. Now
however, it is no longer true that \( M_{11 t} = M_{1 t} \). Country 1 does not begin the period
holding all of its own currency, since all other countries must also be holding currency
1.

Note that total holdings of currency 1 add up to the total stock of currency 1:

\[
M'_{11 t} + \sum_{i=2}^{N} M'_{i1 t} + NP_{1t} \phi_1 = M_{1 t}
\]
But the left hand side of this equation is

\[ P_{1t}C_{11t} + \sum_{i=2}^{N} P_{it}C_{i1t} + \sum_{i=2}^{N} \sum_{j \neq i} p_{jt+1}C_{ijt+1} + NP_{1t}\phi_1 \]

By goods market clearing for country 1, this is equal to:

\[ P_{1t}Y_{1t} + \sum_{i=2}^{N} \sum_{j \neq i} p_{jt+1}C_{ijt+1} \]

So therefore, we have:

\[ M_{1t} = P_{1t}Y_{1t} + \sum_{i=2}^{N} \sum_{j \neq i} p_{jt+1}C_{ijt+1} \]

or, from country 1’s cash-flow for currency 1:

\[ M_{11t} = M_{1t} - M_{1t-1} + P_{1t-1}Y_{1t-1} = M_{1t} - \sum_{i=2}^{N} \sum_{j \neq i} p_{jt}C_{ijt} \]

\[ = P_{1t}Y_{1t} + \sum_{i=2}^{N} \sum_{j \neq i} p_{jt+1}C_{ijt+1} - \sum_{i=2}^{N} \sum_{j \neq i} p_{jt}C_{ijt} \]

The first equality holds because of the binding country 1 cash in advance constraint. The second and third come from applying the previous equation. An interpretation of this equation is that, so long as \( \gamma_{1t+1} > 0 \), nominal spending of peripheral countries using country 1 money must be rising over time. As a result, country 1, by providing the vehicle currency, may run a persistent current account deficit (its nominal spending will continuously exceed nominal income).
3.1 Trading Posts with a Vehicle Currency

In the case of a vehicle currency, all trading posts deal with currency 1. Hence, the deliveries of all other currencies to currency 1 trading posts must be larger than in the symmetric equilibrium. But it is also true that the deliveries of currency 1 to each trading post is higher, because both country 1 and all other country residents will wish to exchange currency 1 to purchase peripheral country goods.

We may describe the equations determining exchange rates in trading post $1i$, $i > 1$, as follows. Country $i$ residents bring $f_{iit}^{1i}$ to the $1i$ post at time $t$. Country 1 residents bring $f_{11i}^{1i}$ to the $1i$ market at time $t$.

Country $j$ residents, where $j > 1$, $j \neq i$, holding money from the last period, bring currency 1 to the $1i$ trading post, so as to purchase good $i$. Their supply of currency $i$ to the $1i$ trading post is $f_{j1i}^{1i}$. In total then, the ask price of currency $i$ is determined by:

$$s_{a1it} = \left[ f_{iit}^{1i} - P_{it}\hat{\phi}_i \right] = f_{11i}^{1i} + \sum_j f_{j1i}^{1i}, \quad j \neq i,$$

while the bid price is determined by:

$$s_{b1it} = f_{iit}^{1i} = f_{11i}^{1i} + \sum_j f_{j1i}^{1i} - P_{it}\hat{\phi}_1 \quad j \neq i$$

Again, in order to derive more explicit expressions for exchange rates we make the assumption of log utility as in (). In this case, equations () can be expressed as:

$$s_{a1it}^a \left[ \frac{1 + \beta(N - 2)}{2 + \beta(N - 2)} - \hat{\phi}_{it} \right] =$$
\[
\frac{1}{N} \left[ 1 - \frac{\beta}{(1 + \gamma_{1t})(2 + \beta(N-2))} \sum_{j=2}^{N} s_{1jt-1}^b \right] + \frac{\beta}{(1 + \gamma_{1t})(2 + \beta(N-2))} \sum_{j>1, j \neq i}^{N} s_{1jt-1}^b
\]

\[
s_{1it}^b \left[ 1 + \beta(N-2) \right] = \frac{1}{N} \left[ 1 - N\hat{\phi}_{1t} - \frac{\beta}{(1 + \gamma_{1t})(2 + \beta(N-2))} \sum_{j=2}^{N} s_{1jt-1}^b \right]
+ \frac{\beta}{(1 + \gamma_{1t})(2 + \beta(N-2))} \sum_{j>1, j \neq i}^{N} s_{1jt-1}^b + \hat{\phi}_{1t} \left( 2 + \beta(N-2) \right) \sum_{j=2}^{N} s_{1jt}
\]

This represents a system of \(2(N-1)\) dynamic equations in the \(2(N-1)\) normalized exchange rates. To study the system further, we focus on a steady state, with constant \(\gamma_{1t}\) and \(\phi_{it}\), and where \(s_{1it}^k = s_{1it-1}^k\). Further, we assume that the fixed costs of a trading post are identical across all the peripheral countries, so that \(\hat{\phi}_{it} = \hat{\phi}^*\), for all \(i > 1\). This reduces () down to two equations in \(s^a\) and \(s^b\). The solutions are given by:

\[
s^b = \Gamma \frac{(1 - N\hat{\phi}_1)}{1 + \beta(N - 2) + \beta \left( \frac{N-1}{N} - (N - 2) \right) \frac{1}{1+\gamma} - \beta\hat{\phi}_1(N - 1)}
\]

and

\[
s^a = \Gamma \frac{1}{1 - \left( \frac{2 + \beta(N-2)}{1 + \beta(N-2)} \right) \left( \frac{(N-1)\gamma + N(N-2)}{1+\gamma} \right)} \frac{(1 - \frac{\beta\hat{\phi}_1}{1+\beta(N-2)}((N-1)\gamma + N(N-2)) \frac{1}{1+\gamma}))}{1 + \beta(N - 2) + \beta \left( \frac{N-1}{N} - (N - 2) \right) \frac{1}{1+\gamma} - \beta\hat{\phi}_1(N - 1)}
\]

where \(\Gamma \equiv \frac{(2 + \beta(N-2))}{N}\).

So the bid ask spread in the case of a vehicle currency may be written as

\[
\frac{s^a}{s^b} = \left( \frac{1 - \frac{\beta}{(1+\beta(N-2))} \hat{\phi}_1((N-1)\gamma + N(N-2)) \frac{1}{1+\gamma}}{1 - N\hat{\phi}_1((1 - \frac{2 + \beta(N-2)}{(1+\beta(N-2))} \phi^*))} \right)
\]
This is less than \( \frac{1}{(1-N\hat{\phi}_1)(1-N\hat{\phi}^*)} \), which is the bid ask spread in the economy with no vehicle currency. There are two factors that push down the bid ask spread in the vehicle currency economy. The most important factor is that there is substantially greater volume at each trading post, reducing the average cost of running the post. This is captured by the term \( \frac{(2+\beta(N-2))\hat{\phi}^*}{(1+\beta(N-2))\hat{\phi}^*} \) in the denominator of (), which contrasts with \( N\hat{\phi}^* \) in the symmetric economy. The second factor is more indirect, and is captured by the numerator of (). Because some currency 1 is held by peripheral countries, the price level in country 1 is lower than it would be in the symmetric equilibrium. As a result, the fixed cost of setting up a post in terms of good 1, \( p\hat{\phi}_1 \), is less. This pushed up the bid price of currency 1, reducing the bid ask spread.

Note that while the bid-ask spread is lower in the economy with a vehicle currency, it is raised by country 1 money growth.

It is clear that the Vehicle Currency equilibrium Pareto dominates the Symmetric Trading equilibrium. Less resources are used up in trading posts. But the presence of the vehicle country introduces a fundamental asymmetry into the allocation of world resources. Country 1 occupies a special role. As we see below, its role as a provider of the vehicle currency gives it three distinct advantages. First, it achieves a higher terms of trade in international trade. We can write the terms of trade for country 1 as:

\[
\frac{s^a}{p_1} = \frac{Y_1}{Y_2} \frac{1}{\Gamma} \left[ 1 - \frac{\beta\hat{\phi}_1}{1+\beta(N-2)} \left( \frac{(N-1)\gamma + N(N-2)}{1+\gamma} \right) \right] \frac{1 - \frac{(N-2)\gamma}{1+\gamma} \left( N - \frac{1}{N} \right) \gamma}{1+\gamma}
\]

In the no-vehicle currency case, the terms of trade is equal to \( \frac{Y_1}{Y_2} \frac{1}{1-N\hat{\phi}^*} \). It is easy
to see that this is higher than that in the vehicle currency equilibrium.

A second advantage conferred upon the center country is that it does not have to ‘wait’ to consume. That is, it can convert current income into consumption of all goods, without a one period delay, as is the case for the peripheral countries. When $\beta < 1$, this has a welfare cost for the peripheral countries.

Finally, the center country gains from positive money growth. Since center country money growth reduces the real value of peripheral country savings, it will reduce the peripheral countries consumption of other peripheral countries goods. This loss is a direct gain to the center country.

We may write out the equilibrium consumption rates of the country 1 and all other countries. For country 1, consumption of all other countries is equal. Thus, we have:

$$c_{11} = \frac{1}{N} \frac{m_{11}}{p_1} = \frac{1}{N} \frac{1}{\left[1 - \frac{\beta(N-1)}{2+\beta(N-2)} \frac{1}{1+\gamma} b\right]}$$

$$c_{1i} = \frac{m_{11}}{s^a p_2} = \frac{1 + \beta(N-2)}{2 + \beta(N-2)} y_2 \frac{1}{N} \frac{1}{\left[1 - \frac{\beta(N-1)}{2+\beta(N-2)} \frac{1}{1+\gamma} b\right]} \left(1 - \frac{2 + \beta(N-2)}{1 + \beta(N-2)} \hat{\phi}_1\right), \quad i > 1$$

For all countries $i > 1$, there are three separate consumption rates; consumption of the domestic good, consumption of good 1, and consumption of all other countries. These are given by:

$$c_{ii} = \frac{1}{N} y_i$$

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\[ c_{11} = y_1 \frac{1 - N \hat{\phi}_1}{N + \beta(N(N - 2) - (N - 1))^{1+\gamma}} \]

\[ c_{ij} = y_j \frac{\beta}{2 + \beta(N - 2)} \frac{s^b}{s^a} \frac{1}{1 + \gamma} \quad j > 1 \]

These solutions may be illustrated in Figures 1 and 2. Figure 1 gives the consumption of the domestic good and all other goods for country 1 in the vehicle currency equilibrium, and contrasts this with the consumption rates under the symmetric equilibrium. In this case, we let \( N = 5 \), so that there is one center and four peripheral countries. We let \( \beta = .997 \). Along the horizontal axis, we measure the rate of money growth for country 1. Clearly, country 1 is better off absolutely in the vehicle currency equilibrium. With zero money growth, consumption of its own good is the same as before. Consumption of peripheral goods is higher however. This arises directly from the increased supply of the peripheral good, as trading cost savings increase output of the peripheral good.

Increased money growth exacerbates the gain for country 1, increasing its consumption of both the domestic and the peripheral good.

Figure 2 illustrates the analogous outcomes for the peripheral country. The Figure shows consumption of good 1, and consumption of other peripheral country goods, for a range of center country money growth rates. Again, we assume 5 countries and \( \beta = .997 \). Note that consumption of the domestic good, for each peripheral country, is unchanged across regimes, so we omit this from the Figure. For zero money growth, the peripheral country’s consumption of good 1 is approximately unchanged, relative
to the symmetric equilibrium. Consumption of other peripheral goods is higher. But note that this is quickly eroded by money growth in the center country.

From this analysis, it is unclear whether the peripheral countries will gain or lose from the vehicle currency equilibrium. The fundamental trade-off is between the gains from greater efficiency in foreign exchange trading, relative to the losses from reduced terms of trade, delayed consumption, and center country money growth. Note that a key aspect of the vehicle currency equilibrium is that money is not neutral. Center country money growth determines the distribution of gains of moving to the vehicle currency equilibrium.

The net welfare effects of the vehicle currency for the peripheral countries depends critically on the number of countries-currencies in the world economy. For a small number of countries, the gains from reduced trading posts are quite slight, and the peripheral countries tend to lose, even for zero center country money growth. For instance, we may show that when $N = 3$, and trading costs are equal for all countries, the peripheral countries always lose. More than 100 percent of the efficiency gains from a vehicle currency go to the center country. But this case is very special, because it involves efficiency gains from shutting down only 1 trading post (i.e. the 23 trading post). As the number of countries increase, the gains from the vehicle currency rise. Figure 3 shows the gains to peripheral countries (in terms of percentage of permanent consumption) as a function of the number of countries. The gains are increasing in $N$, because the efficiency gains from fewer trading posts increase in $N$. Note however, that
higher center country money growth still shrink these gains. Figure 4 shows that, for a center country money growth rate equal to 5 percent, peripheral countries will lose from a vehicle currency equilibrium unless N is greater than 10.

An interesting feature that arises in Figure 4 is that the relationship between the the number of countries and the gains from a vehicle currency is non-monotonic. As we increase the number of countries in the presence of vehicle currency money growth, initially each country tends the loss relative to the symmetric equilibrium tends initially to rise. After some point though, this turns around, and more and more countries lead to relative gains, for an individual country. The intuition behind this non-monotonic process is as follows. As we increase the number of countries, using our preference specification, each country becomes more open, since preferences are assumed equally weighted towards all country’s goods. Hence, for each peripheral country, there is greater exposure to country 1 money growth, as the number of countries increases. Thus, the losses from adopting a vehicle currency tends to rise, as N increases. Offsetting this however, is the fundamental efficiency mechanism, leading to a greater gain from the adopting a vehicle currency, the greater the number of bilateral trading posts that are closed down by its adoption, i.e. the more countries there are. So these two forces conflict with one another. For a relatively small number of countries, the first force tends to dominate, and increasing numbers tends to reduce the gains to a vehicle currency. For a much larger number of countries, the second force is predominant, and the gains to a vehicle currency begin to rise and become positive.
4 Conclusions

To follow

5 References

To follow
Figure 2 Peripheral Country Consumption

Country 1 Money Growth

$C_{j1 \text{ vc}}$ $C_{ji \text{ vc}}$ $C_{j1 \text{ se}}$