Appendix for "Private Pensions, Retirement Wealth and Lifetime Earnings"

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1 Introduction

This appendix discusses some additional details on the data, simulations and numerical methodology used in "Private Pensions, Retirement Wealth and Lifetime Earnings".

2 PSID Data

For comparison with Hendricks (2007), we also report the subsample of households from the PSID supplemental wealth files for 1984-2003. Overall, the sample characteristics are similar to those reported in Table 1 of Hendricks (2007).

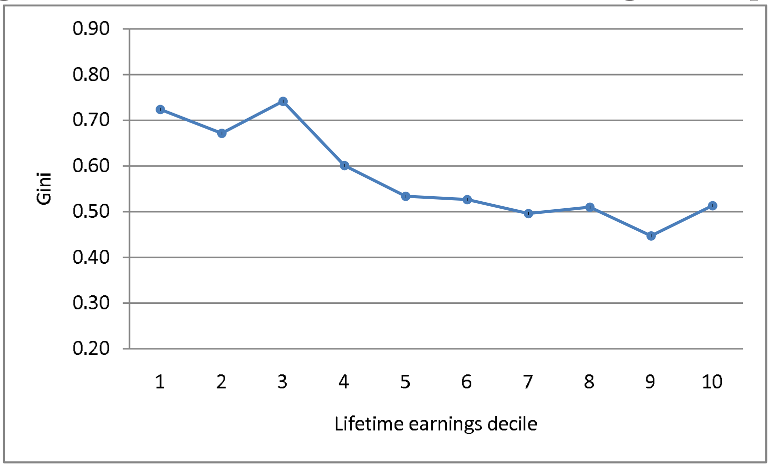
Table 1: Sample Statistics: 1984-2003 PSID

	Couples		Singles	
	Mean	Std.	Mean	Std
Number of observations	654	–	418	–
Birth year	1927.7	6.6	1928.5	6.5
Years of school	12.0	3.9	11.7	3.7
Earnings observations	25.8	5.5	26.6	5.3
Earnings at age 40-50	39.9	21.9	25.8	18.3
Lifetime earnings	3967.8	2088.0	2185.3	1332.5
Retirement wealth	385.8	806.0	156.4	532.2
Median retirement wealth	195.8	–	49.3	–

Note: Dollar figures are in thousands of detrended 1994 dollars.

Figure 1 shows the Gini coefficients of retirement wealth (net worth) across lifetime earnings deciles from the 1984-2005 PSID (7 waves). The larger sample generally resembles the sub-sample (1999-2005) (Figure 2 in the paper), although for some deciles in the 1999-2005 sample the Gini is roughly 0.05-0.1 higher.

Figure 1: Gini of Retirement Wealth: Larger Sample



3 Details on Experiments

In this section we report some additional details on experiments.

3.1 Benchmark Economy

The private pension economy misses some key features of the joint distribution of pension and retirement net worth. Figure 2 plots the distribution of retirement wealth including and excluding pension wealth in the model with private pensions and the PSID for the ninth lifetime earnings decile.[1] On the one hand, many low net worth households have

[1] We sort households by non-pension wealth. We do not report wealth/mean earnings for all wealth percentiles in the model. Instead, we only report the ratios in the model for those wealth percentiles found in the PSID data for a clear comparison.

pensions in the data – which matches the fact that in the model almost all low net worth households have pensions. However, there are two discrepancies between the data and the model predictions. First, a number of low net worth households in the data lack pensions. Second, some relatively high net worth households in the data have very large pensions – which results in the richest households holding more wealth than predicted by the model.

Figure 2: Retirement Wealth: Ninth Lifetime Earnings Decile

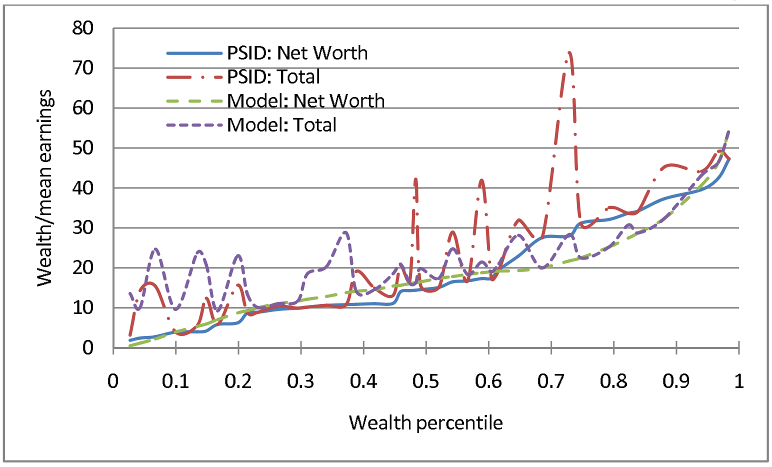


Figure 3 plots the Gini coefficient for each lifetime earning decile for total retirement wealth (net worth plus pension) in the benchmark economy with private pensions. The difference in the average Gini across deciles between the model with pensions and the data is only slightly smaller than the gap between the model with private pensions and the data.

Table 2 reports the distribution of social security wealth in the model. As can be seen from the Table, social security wealth in the model is much less unequal than private pensions wealth.

Figure 3: Gini Coefficient of Retirement Wealth (Total)

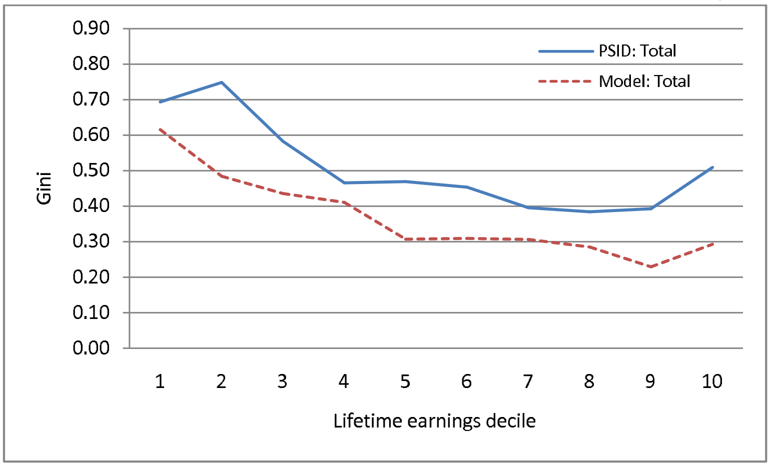


Table 2: Pension and Social Security Distribution

Wealth	Top 1%	1-5%	5-10%	10-20%	20-40%	40-60%	60-80%	80-100%	Gini
rw+SS	7.1	15.4	12.9	17.1	21.6	13.1	8.3	4.4	0.47
rw+SS+Pens	6.3	14.0	12.1	16.6	22.5	14.2	9.4	4.8	0.44
Pensions	7.8	22.8	19.1	22.6	22.2	5.4	0	0	0.72
S.S. benefits	1.5	5.9	7.2	14.0	24.6	20.3	15.9	10.6	0.19

3.2 Robustness: Bequest Motive and Borrowing

In this section, we briefly summarize the results of some robustness experiments, in which we introduce a bequest motive and allow households to borrow. We find that relaxing these assumptions does not significantly change our main results.

3.2.1 Robustness: Bequest Motive

One simplification in the benchmark model is the absence of an explicit bequest motive. Given that a bequest motive may affect households' consumption and saving behavior, one might worry that abstracting from this could significantly change our results.

To investigate this possibility, we follow De Nardi (2004) and introduce a "warm glow" type of voluntary bequest motive into the model. Households derive utility from leaving a bequest, and may receive at most one inheritance at age 40-60. The probability of receiving an inheritance and the level of the inheritance depend on the mortality risk of households aged 65-85 and the wealth level of those households.[2] We maintain the assumption that households have no information about future inheritance.[3] Formally, the Bellman equation of an age j household is given by:

$$V[k,e,\bar{y},pen,n_{db}] = \max_{c}\left\{\frac{c^{1-\sigma}}{1-\sigma} + \beta P_{j+1}E[V(k',e',\bar{y}',pen',n'_{db}] + (1-P_{j+1})\phi(b(k'))\right\} \tag{3.1}$$

where the utility derived from bequest b is defined as

$$\phi(b) = \phi_1\left(1+\frac{b}{\phi_2}\right)^{1-\sigma} \tag{3.2}$$

$$b(k') = k' - fc - \tau_b \cdot max(0, k' - fc - ex_b) \tag{3.3}$$

The term ϕ_1 reflects the household's concern about leaving bequests, ϕ_2 measures the extent to which bequests are a luxury good, fc denotes the funeral cost, and τ_b is the tax rate on estates that exceed the exemption level ex_b. To match the wealth level in the benchmark, the discount factor β is set at 0.948 in the presence of a bequest motive. We set fc at \$10,000 and other parameters are taken from De Nardi (2004).[4]

We run the program for several times to match the wealth distribution at age 65-85 to the inheritance distribution at age 40-60 by using the wealth distribution at age 65-85 as inheritance input. We obtain a Gini of lifetime inheritance, discounted to age 52, of 0.82 and roughly 51% of households receive inheritance at age 40-60. These numbers are close to those reported in Hendricks (2007).

The results of these experiments are reported in Table 3. Compared to the benchmark results, the inclusion of a bequest motive does not significantly increase wealth inequality

[2]This arrangement supplants the earlier assumption that households received an inheritance realization at age 52.

[3]If we assume that households can observe their parents' productivity and wealth level, it would make solving the model computationally more difficult since we need to add two more state variables.

[4]We add a funeral cost in order to limit the fraction of households receiving inheritance at age 40-60 (from households that died between ages 65-85).

in the model. For example, in the model without private pensions the Gini only increases from 0.56 to 0.57 when a bequest motive is added. Similar patterns also happen in the model with private pensions.[5] The small overall impact of the bequest motive is likely due to two reasons: (i) we assume that households have no information about future inheritance; and (ii) the introduction of a bequest motive has a small effect on the amount of retirement wealth at age 65 for most households, although it has a large impact on dissaving for the retirees as suggested by De Nardi (2004).

Table 3: Wealth at 65: Bequest Motive

	Wealth	Top 1%	1-5%	5-10%	10-20%	20-40%	40-60%	60-80%	80-100%	Gini
Model: No-pens.	R.W.	8.5	17.4	14.5	18.5	22.6	11.7	5.6	1.2	0.57
Model: Pens.	R.W.	9.4	18.9	15.2	19.1	22.0	10.6	4.4	0.4	0.61
Model: Pens.	R.W.+ Pens.	8.0	16.6	13.9	18.4	22.7	12.4	6.2	1.8	0.55

Note: The table reports the Lorenz curve of retirement wealth.

3.2.2 Robustness: Borrowing

Another simplification in the model is the absence of borrowing. Based on the augmented model with pensions, we run an experiment in which we allow households to borrow up to one year of mean earnings but they must repay the debt by age 51 since there is mortality risk at age 52. The borrowing rate is set at the rate of return plus 4%. Table 4 shows the results. Compared to the augmented model with pensions but no borrowing, we find that the effect of borrowing is very small.

3.3 Return and Profile Heterogeneity

Table 5 summaries the correlations coefficients for the life-cycle model with private pensions, return and earnings profile heterogeneity.

[5] When there is a bequest motive, the mean Gini of non-pension retirement wealth across lifetime earnings deciles is 0.43 in the model without private pensions, and 0.50 with private pensions.

Table 4: Wealth at 65: Borrowing

	Wealth	Top 1%	1-5%	5-10%	10-20%	20-40%	40-60%	60-80%	80-100%	Gini
Augm. Model: Borr.	R.W.	12.4	22.5	16.8	19.3	18.7	7.7	2.6	0.2	0.69
Augm. Model: Borr.	R.W.+ Pens.	10.3	19.4	15.1	18.7	19.8	10.2	4.9	1.4	0.61

Note: The table reports the Lorenz curve of retirement wealth.

Table 5: Correlation Coefficients in Augmented Model with Private Pensions

		Earnings	Net Worth	Net Worth + Pension	Pensions
All Households	Earnings	1.00	0.72	0.77	0.54
	Net Worth	0.72	1.00	0.97	0.25
Top 50 % Earners	Earnings	1.00	0.72	0.75	0.35
	Net Worth	0.72	1.00	0.97	0.13
Bot. 50 % Earners	Earnings	1.00	0.18	0.24	0.45
	Net Worth	0.18	1.00	0.97	-0.02

3.3.1 Decomposing Effect of Return and Profile Heterogeneity

To better understand the contribution of earnings profile and rate of return heterogeneities in the augmented model, we shut down each channel individually. The results of these experiments are reported in Table 6.

The first three rows report the retirement wealth distribution for an economy with earnings profile heterogeneity but no return heterogeneity. In the model without private pensions, the Gini coefficient of non-pension wealth increases from 0.56 (in the benchmark) to 0.60 when households face different earnings profiles. In the model with private pensions, the Gini coefficients also increase significantly (compared to those in the benchmark) for retirement wealth with and without private pensions. For example, the Gini of non-pension retirement wealth increases from 0.62 to 0.66.[6] These experiments suggest

[6]The mean Gini of non-pension retirement wealth across lifetime earnings deciles is 0.46 in the model without private pensions and 0.55 in the model with private pensions. The mean Gini of total retirement wealth (including pensions) is 0.43. All are significantly higher than those in the benchmark.

that profile heterogeneity has a large impact on retirement wealth distribution. We also find that the inclusion of private pensions still improves the model's ability to account for retirement wealth inequality when we consider profile heterogeneity.

Table 6: Wealth at 65: Earnings Profile and Return

		Profile Hetero.									
		Wealth	Top 1%	1-5%	5-10%	10-20%	20-40%	40-60%	60-80%	80-100%	Gini
1	Model: No-pens.	R.W.	9.6	19.5	15.2	19.0	20.1	10.4	5.0	1.3	0.60
2	Model: Pens.	R.W.	11.1	21.7	16.4	18.7	19.5	8.7	3.5	0.4	0.66
3	Model: Pens.	R.W.+ Pens.	9.1	18.7	14.7	18.6	20.4	11.1	5.6	1.8	0.58
		Return Hetero.									
		Wealth	Top 1%	1-5%	5-10%	10-20%	20-40%	40-60%	60-80%	80-100%	Gini
4	Model: No-pens.	R.W.	8.9	18.6	15.0	19.2	21.4	10.7	5.0	1.1	0.60
5	Model: Pens.	R.W.	10.2	20.5	16.0	20.0	20.5	9.2	3.3	0.3	0.65
6	Model: Pens.	R.W.+ Pens.	8.6	18.0	14.5	18.7	21.4	11.3	5.8	1.7	0.57

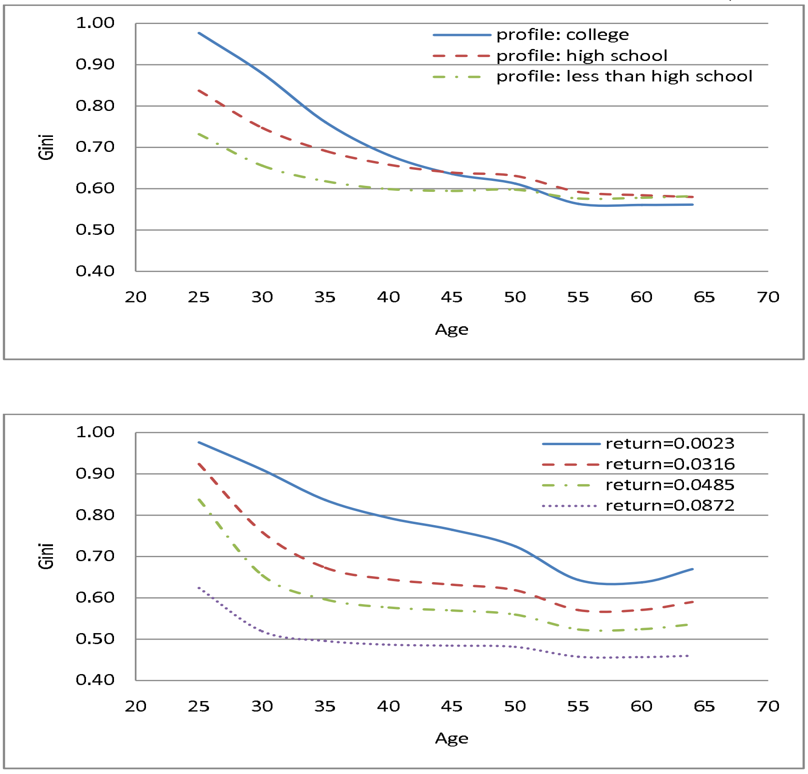
Note: The table reports the Lorenz curve of retirement wealth.

Rows 4-6 in Table 6 report the wealth distribution for the model with rate of return heterogeneity (and no earnings profile heterogeneity). Compared to the benchmark, we find that the inclusion of rate of return heterogeneity tends to reduce the discrepancies between model and data. For the mean Gini of non-pension retirement wealth across lifetime earnings deciles, it is 0.46 in the model without private pensions and 0.54 in the model with private pensions. The mean Gini of total retirement wealth (including pensions) across lifetime earnings deciles is 0.43, which is lower than 0.51 observed in the data. However, the results suggest that private pensions still make a big difference in accounting for non-pension retirement wealth distribution when we consider rate of return heterogeneity.

3.3.2 Life-cycle Wealth Inequality

Figure 4 plots the life-cycle Gini for retirement wealth (net worth) inequality discussed in the paper. It shows the effects of earnings profile heterogeneity and rate of return heterogeneity respectively.

Figure 4: Age-Gini Coefficient of Retirement Wealth (Net Worth)



4 Numerical Solution

We use numerical dynamic programming techniques to approximate the decision rules as well as the value function. In the model with private pensions, the dynamic program has five state variables in addition to period j: financial wealth k, earnings state e, average earnings over past periods $\bar{y}$, private pension status in current period pen, and years of pension coverage until current period n_{db}.

We discretize the state-space along the two continuous state variables, k and $\bar{y}$. The model is solved using backward induction. In the last period ($j = J$) the policy functions are trivial. In periods prior to J, we calculate optimal decision rules for each possible combination of nodes, using stored information about the subsequent period's decision rules and value function. We follow Tauchen (1986) to approximate the distributions of the innovations to the labor income process. For points which do not lie on the state-space grids, we evaluate the value function using a bi-cubic spline interpolation along the two dimensions. After computing the values of all the alternatives, we pick the maximum, thus obtaining the decision rules for the current period.

Once we determine the optimal decision rules for all possible nodes in each period, we simulate the income history of 20,000 households. All programs are parallelized and run on SHARCNET.[7]

References

De Nardi, M. C. (2004): "Wealth Inequality and Intergenerational Links," *Review of Economic Studies*, 71, 743–768.

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[7]SHARCNET is a multi-institutional High Performance Computing network that spans 17 academic institutions in Ontario, Canada.