OPTIMAL INFORMATION TRANSMISSION IN A HOLD-UP PROBLEM

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Abstract. This article examines the optimal contract in a bilateral trade model with unobservable relationship-specific investment and renegotiation. In such a setting, a contract plays an additional role that it does not have in the standard hold-up model, namely, that of transmitting information between the parties. The article shows that a partial-disclosure contract may be optimal and describes the optimal contract. If the investment is cooperative and the information between the trading parties is asymmetric, the optimal contract generally cannot result in the first-best, but dispensing with either of these assumptions makes the first-best achievable.

1. Introduction

Consider a trade relationship between two parties, one of which can make an investment that increases the joint surplus from the relationship. Such investments may include improving quality and supply reliability, tailoring a product to the other party's needs, or simply putting more effort into production. This investment is costly, and the cost is borne entirely by the investor. If the investment is noncontractible and relationship-specific and the parties cannot commit to abstain from renegotiation, this gives rise to the hold-up problem: the investor gets less than the social return on his investment, so he invests less than is socially optimal.

The hold-up problem has attracted much attention, starting from Klein, Crawford and Alchian (1978), Grout (1984) and Williamson (1985). Chung (1991), Aghion, Dewatripont and Rey (1994), Nöldeke and Schmidt (1995) and Edlin and Reichelstein (1996) proved that if investment is observable by both parties, although noncontractible, it is possible to write a simple contract that induces efficient investment, even if the parties cannot prevent renegotiation. Rogerson (1992) proved that efficiency can also be achieved in a model where investment is unobservable, but the parties can prohibit renegotiation. More recently, Schmitz (2002b) showed that this observation generalizes to a model with unobservable investment and ex post efficient renegotiation.

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All the articles mentioned above assume that investment is "selfish", i.e. it has a direct effect only on the payoff function of the investing party. This assumption is violated in many applications. For example, the level of employee's effort or the amount of time he invests in learning how to execute his tasks more efficiently can have a direct impact on both the employee's production cost and the employer's profit. The same is true of a seller's investment in supply reliability or in tailoring the product to the buyer's needs. The contribution of this article is to characterize the optimal contract in a model with asymmetric information, cooperative investment and renegotiation.

Models of cooperative investment and asymmetric information have been recently analyzed by Schmitz (2002a), Hori (2006) and Zhao (2008). Schmitz (2002a) observes that the first-best is generally unachievable; Hori (2006) and Zhao (2008) characterize the optimal contract for the case of indivisible and perfectly divisible good, respectively. The main difference between these articles and the present one is that they assume that the parties can commit not to renegotiate, whereas the present article does not.

Allowing for renegotiation adds a significant amount of complexity to the problem. When renegotiation is present and information is imperfect, a contract assumes a new role: it affects the beliefs of the parties at the renegotiation stage. These beliefs affect the renegotiation outcome and thus the share of the surplus that the investing party receives, which in turn determines the investment level. It has been noted before (Tirole, 1986; Gul, 2001; Gonzalez, 2004) that uncertainty about investment can protect the investing party from expropriation at the renegotiation stage and thus improve investment incentives.[1] In models without contracting, Hermalin and Katz (2009) show that observability of investment can either increase or reduce the equilibrium investment level and welfare, depending on the parameters; Lau (2008) shows that it may be optimal to disclose the information about investment only partially. The present article takes this idea one step further by solving for the optimal contract, which discloses just the right amount of information to maximize the joint welfare from the relationship.

This article considers a bilateral trade model where only one of the parties (the seller) can invest. Investment stochastically influences the state of nature, which can be 'high' or 'low'. The state determines both the buyer's valuation and the seller's cost. Both the investment level and the state are observable by the seller only. Before the investment is made, the parties can write a contract that aims at maximizing their joint welfare. However, they cannot commit not to renegotiate. At the renegotiation stage, which is the final stage of the game, the uninformed party (the buyer) can make a take-it-or-leave it offer of a price-quantity menu to the informed party (the seller).

[1]This observation has been made in the management literature as well: for example, Bakos and Brynjolfsson (1993) point out that suppliers can be unwilling to share their private information about the quality of the good in the fear that buyers will use it to exploit them.

We show that the first-best is not always achievable. As in Schmitz (2002a), Hori (2006) and Zhao (2008), the reason is that the parties face a trade-off between high investment and efficient trade. If the parties want to induce high investment, they can do it only by a contract that causes renegotiation to be inefficient; if they want to minimize welfare losses from ex-post inefficient trade, they have to content themselves with lower investment. This trade-off appears only if the assumptions of cooperativeness and imperfect information are combined. It is possible to write a contract that generates the first-best investment and trade if information is imperfect and investment is noncooperative or if information is perfect and investment has a cooperative element.

The article shows that a partial-disclosure contract (i.e. a contract that gives the buyer some, but not necessarily all, information about the state) is optimal if the parties favor high investment over efficient renegotiation. A contract influences the investment incentives for the seller through two channels: the terms of trade (price and quantity) that serve as the default for renegotiation, and the information that the buyer gets about the state. This is because the default terms of trade and the buyer's information affect the buyer's renegotiation offer, which in turn determines the seller's expected utility in both states. The seller's marginal benefit of investment equals his utility differential between the high and the low state. Thus the investment incentives are determined by the default terms of trade and the buyer's beliefs (in particular, the probability that he places on the high state). Optimizing over the default terms of trade for every possible level of buyer's beliefs allows us to represent the marginal benefit of investment as a function of the buyer's beliefs only.

Whether a partial-disclosure contract can induce higher investment than a no-disclosure contract depends on the shape of the marginal benefit of investment as a function of the buyer's beliefs. For example, if this function is convex, then the seller is effectively risk-loving with respect to the buyer's information: a contract that sends the buyer a noisy signal of the state may result in higher marginal benefit of investment than any contract without disclosure. Such a contract can be implemented with the help of a third party (a mediator), who can commit to ex ante public randomization schemes and enforce random default trades.

In approaching a contract as an information transmission device, this article is related to Calzolari and Pavan (2006a,b). The first of those articles solves for the optimal contract in a model of a monopoly with resale, where the seller aims at maximizing revenue in the primary market by optimally disclosing information to the secondary market. The primary seller may thus face a trade-off: on the one hand, he wants to sell to more types in the primary market; on the other hand, he wants to make the secondary market believe that the primary buyer's type is high, in order to increase the primary buyer's resale surplus and thus to be able to extract more rent from him upfront. It is shown that a partial-disclosure policy can help resolve this trade-off. Calzolari and Pavan (2006b) examine a model where the different principals contract with the same agent sequentially, and identify the conditions under which it is optimal for the upstream principal to release information about the terms of trade with the agent or the agent's type to the downstream principal. In both of these articles, disclosure of information is motivated by the desire to affect the rents from the downstream transaction in order to extract some of them upfront. In contrast, the present article assumes that the disclosure policy is designed to maximize the parties' joint surplus, and partial disclosure may be optimal because it provides the best investment incentives.[2]

The article is organized as follows. Section 2 presents the model. Section 3 analyzes the benchmark cases where the parties cannot contract, or the investment is noncooperative, or the state is perfectly observable. Section 4 presents the solution of the renegotiation stage for the general case. Section 5 looks at a particularly simple class of contracts called trade contracts. Section 6 allows for more general contracts and investigates the properties of the optimal contract. Section 7 presents an extension where the buyer can make repeated offers at the renegotiation stage. Section 8 discusses the results. Appendix contains the proofs unless stated otherwise.

2. The Model

A buyer is interested in trading with a seller. Before trade takes place, the seller can make an investment $e \in [0,1]$. Investment is costly for the seller, and the cost is equal to $\psi(e)$, such that $\psi(0) = 0$ and $\forall e > 0$, $\psi'(e) > 0$, $\psi''(e) > 0$, $\psi'''(e) \geq 0$. Investment stochastically affects the state of nature $i \in \{h, l\}$: in particular, $\Pr(i = h) = e$. Both the investment level and the state are observable only by the seller. The state affects both the buyer's valuation $V^i(q)$ and the seller's cost $c^i(q)$, where $q \geq 0$ is the quantity produced and traded. We assume that $\forall i \in \{h, l\}$, $V^i(0) = 0$, $V^i_q > 0$, $V^i_{qq} < 0$, $V^h_q \geq V^l_q$; $c^i(0) = 0$ and, whenever $q > 0$, $c^i_q > 0$, $c^i_{qq} \geq 0$, $c^h_q(q) < c^l_q(q)$. This means that the high state of nature is better for both parties than the low state, so that investment has both a selfish aspect (lower marginal cost for the seller) and a cooperative aspect (higher marginal valuation for the buyer).[3] Note that we allow for the possibility that investment is purely selfish, i.e. $V^h = V^l$. Examples of investments that have both selfish and cooperative aspects may be lowering transportation costs, improving communication between the buyer and the seller and modifying the product to better fit the buyer's needs.

Let

$$q_i^* \equiv \arg\max_{q \geq 0} \left(V^i(q) - c^i(q)\right), \ i = h, l$$

[2]A more distantly related strand of literature examines various mechanism design problems where the parties optimize ex ante over the amount of information that the informed party should receive before the mechanism is played (see, for example, Bergemann and Pesendorfer, 2007; Hagedorn, 2009; Ivanov, 2010).

[3]Che and Hausch (1999) call such investments 'hybrid'.

be the efficient level of trade in state i. We will assume that $0 \le q_l^* < q_h^*$. The first-best investment level satisfies

$$e^* \equiv \arg\max_{e\in[0,1]} \left(e\left(V^h(q_h^*) - c^h(q_h^*)\right) + (1-e)\left(V^l(q_l^*) - c^l(q_l^*)\right) - \psi(e)\right)$$

Let

$$\Delta c(q) \equiv c^l(q) - c^h(q).$$

We will make the following technical assumptions.

Assumption 1.

$$\frac{V^h_{qq}(q) - c^h_{qq}(q)}{V^h_q(q) - c^h_q(q)} > \frac{\Delta c_{qq}(q)}{\Delta c_q(q)},\ \forall q > q_h^*;$$
$$\frac{V^l_{qq}(q) - c^l_{qq}(q)}{V^l_q(q) - c^l_q(q)} < \frac{\Delta c_{qq}(q)}{\Delta c_q(q)},\ \forall q < q_l^*$$

Assumption 2.

$$\psi'(0) < V^h(q_h^*) - c^h(q_h^*) - \left(V^l(q_l^*) - c^l(q_l^*)\right);$$
$$\psi'(1) > \Delta c(q_h^*)$$

Assumption 3.

$$\lim_{q\to\infty} \Delta c(q) = \infty,\quad \lim_{q\to\infty}(V^h(q) - c^h(q)) = -\infty;$$
$$\lim_{q\to\infty}\left|\frac{\Delta c_q(q)}{V^h_q(q) - c^h_q(q)}\right| = M,\ \textit{for some } M \in (0,\infty).$$

Assumption 1 guarantees that the first-order conditions are sufficient to determine the optimal renegotiation offer. Assumption 2 is made to avoid corner solutions (note that the first part of the assumption implies that $e^* > 0$). Assumption 3 is needed to guarantee the existence of an optimal contract.

We will illustrate the results with the following two examples.

Example 1 *Suppose that* $V^h(q) = \theta q$, $V^l(q) = q$, $c^h(q) = \frac{1}{2}q^2$, $c^l(q) = \frac{1}{2}(1+\alpha)q^2$, *and* $\psi(e) = \beta\frac{e^2}{2}$, *where* $\theta > 1$, $\alpha > 0$ *and* $\beta > 0$. *The parameters* θ *and* α *can be thought of as measuring the cooperative and the selfish components of investment, respectively. Then* $q_h^* = \theta$, $q_l^* = \frac{1}{1+\alpha}$, $e^* = \min\left\{1, \frac{1}{2\beta}(\theta^2 - \frac{1}{1+\alpha})\right\}$. *To satisfy Assumption 2, we will assume that* $\alpha\theta^2 < 2\beta$.

Example 2 *Suppose that* V^h, V^l *and* ψ *are the same as in Example 1, and* $c^h(q) = \exp(\frac{1}{2}q^2) - 1$, $c^l(q) = (1+\alpha)\left(\exp(\frac{1}{2}q^2) - 1\right)$. *Then* q_h^* *solves the equation* $q\exp(\frac{1}{2}q^2) = \theta$, q_l^* *solves* $q\exp(\frac{1}{2}q^2) = \frac{1}{1+\alpha}$, *and* $e^* = \min\left\{1, \frac{1}{\beta}\left(\theta q_h^* - \exp\left(\frac{1}{2}(q_h^*)^2\right) - q_l^* + (1+\alpha)\exp\left(\frac{1}{2}(q_l^*)^2\right) - \alpha\right)\right\}$. *To satisfy Assumption 2, we will assume that* $\alpha\left(\exp\left(\frac{1}{2}(q_h^*)^2\right) - 1\right) < \beta$.

Before investment and trade takes place, the parties write a contract, to be described in more detail in the next two sections, that aims at maximizing the total ex-ante surplus. The timeline is as follows:

1. The parties sign a contract;
2. The seller makes unobservable investment e in quality and incurs investment cost $\psi(e)$;
3. The state of nature $i \in \{h, l\}$ is realized and observed by the seller only;
4. The level of trade and the price are determined according to the contract;
6. The buyer and the seller renegotiate the terms of trade;
7. The payoffs are realized.

At the renegotiation stage, the buyer offers the seller a price-quantity menu $((q_h, p_h), (q_l, p_l))$. The seller can choose a price-quantity pair from this menu or reject it altogether, in which case the terms of trade are those prescribed by the contract at stage 4. This renegotiation game is chosen for two reasons. First, the fact that the uninformed party is making the renegotiation offer makes the effects of information transmission at the contract execution stage more clear-cut. Second, this renegotiation game is easy to solve. A more general approach to renegotiation will be considered in Section 7.

3. The Benchmark Cases

First, let us consider what happens without a contract. In this case, the outside option at the renegotiation stage is no trade. Let e be the equilibrium investment level, taken as given at the renegotiation stage. Then the buyer's offer solves a standard monopolistic screening problem:

$$\max_{(q_h, p_h, q_l, p_l)} e\left(V^h(q_h) - p_h\right) + (1-e)\left(V^l(q_l) - p_l\right)$$

subject to

$$\begin{aligned}
&p_h - c^h(q_h) \geq p_l - c^h(q_l);\\
&p_l - c^l(q_l) \geq p_h - c^l(q_h);\\
&p_h - c^h(q_h) \geq 0;\\
&p_l - c^l(q_l) \geq 0,
\end{aligned}$$

The solution to the problem involves efficient trade in the high state and quantity under-provision in the low state:

$$((p_h, q_h), (p_l, q_l)) = ((c^h(q_h^*) + \Delta c(\hat{q}_l(e)), q_h^*), (c^l(\hat{q}_l(e)), \hat{q}_l(e))),$$

where $\hat{q}_l(e)$ is the value of q that solves

$$(1-e)\left(V_q^l(q) - c_q^l(q)\right) - e\Delta c_q(q) = 0 \tag{1}$$

If equation (1) does not have a nonnegative solution for a certain e, let $\hat{q}_l(e) = 0$. Note that the function $\hat{q}_l(e)$ is well-defined for $e \in [0,1]$, equals q_l^* when $e = 0$ and is decreasing whenever e is strictly positive.[4] The optimal renegotiation offer gives the seller zero utility in the low state and information rent $\Delta c(\hat{q}_l(e))$ in the high state. The equilibrium investment level $e_{\min}$ is determined by the first-order condition of the seller's problem:

$$\Delta c(\hat{q}_l(e_{\min})) = \psi'(e_{\min}) \tag{2}$$

Under no contract, there is always less investment than in the first-best: $e^* \geq e_{\min}$. This is because if $e^* < 1$, then the first-order condition for e^* is

$$V^h(q_h^*) - c^h(q_h^*) - (V^l(q_l^*) - c^l(q_l^*)) = \psi'(e^*)$$

and

$$\begin{aligned} V^h(q_h^*) - c^h(q_h^*) - (V^l(q_l^*) - c^l(q_l^*)) &> V^h(q_l^*) - c^h(q_l^*) - (V^l(q_l^*) - c^l(q_l^*)) \\ &\geq \Delta c(q_l^*) > \Delta c(\hat{q}_l(e)), \forall e > 0 \end{aligned}$$

Therefore, the null contract results in both ex ante inefficiency (underinvestment) and ex post inefficiency (quantity underprovision in the low state). However, if the investment is purely selfish, there exists a contract that restores efficiency by specifying appropriate outside options for the renegotiation stage.

Proposition 1. *If the investment is purely selfish (i.e. $V^h(q) \equiv V^l(q) \equiv V(q)$), then there exists a contract that achieves the efficient trade and investment.*

Proof. Consider the following "sell-out" contract: the seller can choose between producing q_h^* and receiving the payment $V(q_h^*)$ or producing q_l^* and receiving the payment $V(q_l^*)$; the seller's choice becomes common knowledge before the renegotiation stage. First, note that if the contract is incentive compatible (i.e. the seller chooses to produce q_i^* in state i), then it is renegotiation proof, because it results in efficient trade in

[4]Formally,

$$\frac{d\hat{q}_l(e)}{de} = -\frac{-\left(V_q^h(\hat{q}_l(e)) - c_q^l(\hat{q}_l(e))\right) - \Delta c_q(\hat{q}_l(e))}{(1-e)\left(\left(V_{qq}^l(\hat{q}_l(e)) - c_{qq}^l(\hat{q}_l(e))\right)\right) - e\Delta c_{qq}(\hat{q}_l(e))} < 0,$$

where the inequality follows from the fact that $\hat{q}_l(e) \leq q_l^*$ and Assumption 1.

both states. The contract is indeed incentive compatible, because

$$V(q_h^*) - c^h(q_h^*) \geq V(q_l^*) - c^h(q_l^*);$$

$$V(q_l^*) - c^l(q_l^*) \geq V(q_h^*) - c^l(q_h^*),$$

which follows from the definitions of q_h^* and q_l^*. Because the seller extracts the entire surplus in both states, he will choose the first-best investment level $e = e^*$. □

Results similar to Proposition 1 have been proven for environments with incomplete information by Schmitz (2002b) (Proposition 4), and for environments with complete information by Aghion, Dewatripont and Rey (1994) (Proposition 3.1). Note that the mechanism used in the proof need not work if the investment is cooperative: if $V^h(q)$ is higher than $V^l(q)$, the sell-out contract need not be incentive compatible in the low state.

As another benchmark, we can consider the case of complete information, where the state is observable by both parties, but unverifiable. The following proposition is a corollary of Proposition 2 of Che and Hausch (1999).

Proposition 2. *If the state is observable by both parties, then there exists a contract that achieves the efficient trade and investment.*

Example 1 continued *Let us construct the optimal contract for the case of complete information in the setting of Example 1. Let* $\bar{q} = \Delta c^{-1}(\psi'(e^*)) = \sqrt{\frac{2}{\alpha}\beta e^*}$. *Consider the contract whereby the seller has to produce* $\bar{q}$ *for the price of* $\bar{p} = c^l(\bar{q}) = \frac{1}{2}(1+\alpha)\bar{q}^2$. *If the state is observable by both parties, at the renegotiation stage the buyer will offer the terms of trade* $(p_h, q_h) = \left(\bar{p} - c^h(\bar{q}) + c^h(q_h^*), q_h^*\right) = \left(\frac{1}{2}\alpha\bar{q}^2 + \frac{1}{2}(q_h^*)^2, q_h^*\right)$ *in state* h *and* $(p_l, q_l) = \left(\bar{p} - c^l(\bar{q}) + c^l(q_l^*), q_l^*\right) = \left(\frac{1}{2}(1+\alpha)(q_l^*)^2, q_l^*\right)$ *in state* l*, extracting the entire renegotiation surplus. As a result, the seller will get the utility* $\bar{p} - c^h(\bar{q}) = \frac{1}{2}\alpha\bar{q}^2$ *in state* h *and* $\bar{p} - c^l(\bar{q}) = 0$ *in state* l*. The first-order condition for the seller at the investment stage will be* $\frac{1}{2}\alpha\bar{q}^2 = \beta e$*. By the definition of* $\bar{q}$*, the solution to this equation is* $e = e^*$*.*

Propositions 1 and 2 show that the first-best can be obtained by simple contracts if either the information between the parties is complete, or the investment is purely selfish. The next section will show that this is not generally the case when neither of these assumptions holds. If the investment is cooperative and the state is unobservable, the parties face a trade-off between high investment and efficient renegotiation, and this may prevent them from achieving the first-best.

4. Renegotiation Stage

At the renegotiation stage, the buyer offers the seller a menu $((p_h, q_h), (p_l, q_l))$.[5] By the revelation principle (Myerson, 1982), without loss of generality this menu is incentive compatible; it is also without loss of generality to assume that both types of the seller will participate. Suppose that the buyer believes that the state is high with probability $\bar{e}$, and the trade contract stipulates the terms of trade $(\bar{p}, \bar{q})$. Then the buyer's optimal offer solves the following problem:

$$\max_{(q_h, p_h, q_l, p_l)} \bar{e}\left(V^h(q_h) - p_h\right) + (1-\bar{e})\left(V^l(q_l) - p_l\right)$$

subject to

$$(\overline{IC}) \qquad p_h - c^h(q_h) \geq p_l - c^h(q_l);$$

$$(\underline{IC}) \qquad p_l - c^l(q_l) \geq p_h - c^l(q_h);$$

$$(\overline{IR}) \qquad p_h - c^h(q_h) \geq \bar{p} - c^h(\bar{q});$$

$$(\underline{IR}) \qquad p_l - c^l(q_l) \geq \bar{p} - c^l(\bar{q}).$$

This problem differs from the standard adverse selection problem in only one respect: the reservation payoff in the high state is higher than in the low state, unless $\bar{q} = 0$. Which constraints are binding in this problem depends on $\bar{q}$ and $\bar{e}$.

Let $\hat{q}_l(e)$ is the value of q that solves equation (1), and $\hat{q}_h(e)$ be the value of q that solves

$$e\left(V_q^h(q) - c_q^h(q)\right) + (1-e)\Delta c_q(q) = 0 \tag{3}$$

Equation (3) is the result of assuming that $\left(\overline{IR}\right)$ and $(\underline{IC})$ bind, substituting them into the objective function and taking the first-order condition with respect to q_h. A solution to this equation exists if e is high enough. The following lemma describes the solution to the problem as a function of $(\bar{e}, \bar{q})$. The proof is straightforward and therefore omitted.[6]

Lemma 1. *The $(\bar{e}, \bar{q})$-space can be separated into five regions, characterized as follows:*

1. ***Regions A and B:*** $0 \leq \bar{q} < q_l^*$. *In this case,* $(\underline{IR})$ *and* $\left(\overline{IC}\right)$ *bind, and* $(\underline{IC})$ *does not bind.*

[5]In principle, one could allow the buyer to use stochastic mechanisms (i.e. menus of lotteries over terms of trade); however, Assumption 1 guarantees that any non-deterministic mechanism would be strictly suboptimal for the buyer. Therefore, we will restrict attention to deterministic menus.

[6]This is an example of an adverse selection problem with type-dependent reservation values (see, for example, Lewis and Sappington (1989), Maggi and Rodriguez-Clare (1995) and Jullien (2000)). Laffont and Martimort (2002) provide the solution for the case of two types, which we use in what follows.

*In **Region A**, $0 \leq \bar{q} < \hat{q}_l(\bar{e})$. In this region, $\left(\overline{IR}\right)$ does not bind, and the optimal renegotiation offer is a "conditionally optimal" contract:*[7]

$$((p_h, q_h), (p_l, q_l)) = ((\bar{p} + c^h(q_h^*) - c^l(\bar{q}) + \Delta c(\hat{q}_l(\bar{e})), q_h^*), (\bar{p} - c^l(\bar{q}) + c^l(\hat{q}_l(\bar{e})), \hat{q}_l(\bar{e})))$$

*In **Region B**, $\hat{q}_l(\bar{e}) \leq \bar{q} \leq q_l^*$. In this region, $\left(\overline{IR}\right)$ binds, and the optimal offer is a "rent constrained" contract:*

$$((p_h, q_h), (p_l, q_l)) = ((\bar{p} - c^h(\bar{q}) + c^h(q_h^*), q_h^*), (\bar{p}, \bar{q}))$$

*2. **Region C**: $q_l^* \leq \bar{q} \leq q_h^*$. In this region, $\left(\overline{IR}\right)$ and $(\underline{IR})$ bind, and the optimal renegotiation offer is a "sell-out" contract:*

$$((p_h, q_h), (p_l, q_l)) = ((c^h\left(q_h^*\right) + \bar{p} - c^h\left(\bar{q}\right), q_h^*), (c^l\left(q_l^*\right) + \bar{p} - c^l\left(\bar{q}\right), q_l^*))$$

*3. **Regions D and E**: $\bar{q} > q_h^*$. In this case, $\left(\overline{IR}\right)$ and $(\underline{IC})$ bind, and $\left(\overline{IC}\right)$ does not bind.*

*In **Region D**, either $\bar{q} \leq \hat{q}_h(\bar{e})$ or $\hat{q}_h(\bar{e})$ does not exist. In this case, $(\underline{IR})$ binds, and the optimal offer is a "rent constrained" contract:*

$$((p_h, q_h), (p_l, q_l)) = ((\bar{p}, \bar{q}), (c^l(q_l^*) + \bar{p} - c^l(\bar{q}), q_l^*))$$

*In **Region E**, $\hat{q}_h(\bar{e}) < \bar{q}$. In this case, $(\underline{IR})$ does not bind, and the optimal offer is a "conditionally optimal" contract:*

$$((p_h, q_h), (p_l, q_l)) = ((\bar{p} + c^h(\hat{q}_h(\bar{e})) - c^h(\bar{q}), \hat{q}_h(\bar{e})), (c^l(q_l^*) + \bar{p} - c^h(\bar{q}) - \Delta c(\hat{q}_h(\bar{e})), q_l^*))$$

Figure 1 shows the general position of regions A - E in the $(\bar{e}, \bar{q})$-space. Region A corresponds to the textbook case, which is characterized by efficient production in the high state ($q_h = q_h^*$) and underprovision in the low state ($q_l = \hat{q}_l(\bar{e}) < q_l^*$). This case is valid as long as the seller's reservation payoff in the high state is not much higher than in the low state, i.e. $\bar{q}$ does not exceed $\hat{q}_l(\bar{e})$. When $\bar{q}$ exceeds $\hat{q}_l(\bar{e})$, the buyer has to increase q_l above $\hat{q}_l(\bar{e})$ in order to satisfy $(\underline{IR})$. As a result, he offers $q_l = \bar{q}$ to the low-state seller (region B), and the quantity underprovision in the low state is mitigated.

The production distortion disappears completely when $q_l^* < \bar{q} < q_h^*$ (region C). In this case, the menu that extracts the entire renegotiation surplus in both states is incentive compatible.

If $\bar{q} > q_h^*$ (regions D and E), the menu that extracts the entire renegotiation surplus from the seller is not incentive compatible in the low state. Intuitively, if $\bar{q}$ is high, the reservation payoff is much higher in the high state than in the low state, so, in order to induce participation, the buyer has to offer a much

[7]The terminology for the different types of optimal renegotiation offer is taken from Laffont and Tirole (1990).

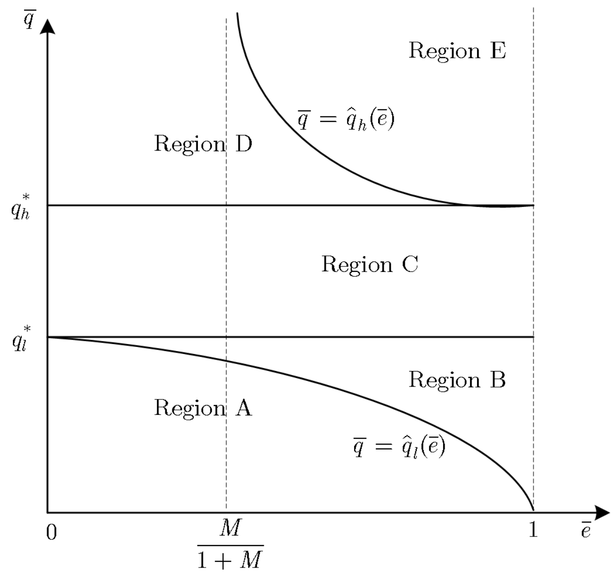

FIGURE 1. The Optimal Renegotiation Offer

higher transfer in the high state than in the low state. To achieve incentive compatibility in the low state, the buyer can either distort q_h upward from the first-best level, or increase p_l above $c^l(q_l^*) + \bar{p} - c^l(\bar{q})$. Thus the buyer faces a trade-off: trading more efficiently in the high state comes at a cost of having to increase p_l and losing profit in the low state. When $\bar{e}$ is high, the buyer cares more about trading efficiently in the high state, so he chooses q_h close to q_h^* and a high p_l. When $\bar{e}$ is low, the buyer wants to extract as much profit as possible in the low state. So he sets a low p_l and a high q_h.

The quantity $\hat{q}_h(\bar{e})$ represents the highest quantity the buyer is willing to offer the seller in the high state as a function of the buyer's beliefs $\bar{e}$. The function $\hat{q}_h(\bar{e})$ is decreasing: the higher $\bar{e}$, the smaller is the upward distortion in q_h that the seller is willing to tolerate, hence the lower $\hat{q}_h(\bar{e})$.[8] When $\bar{e} = 1$, $\hat{q}_h(\bar{e})$ equals q_h^*, and $\hat{q}_h(\bar{e})$ takes every value in $[q_h^*, \infty)$.[9]

Example 1 continued *Equations (1) and (3) imply that* $\hat{q}_l(\bar{e}) = \frac{1-\bar{e}}{1+\alpha-\bar{e}}$ *and* $\hat{q}_h(\bar{e}) = \frac{\theta\bar{e}}{\bar{e}(1+\alpha)-\alpha}$. *The latter is well-defined for* $\bar{e} > \frac{\alpha}{1+\alpha}$.

Example 2 continued *Equation (1) implies that* $\hat{q}_l(\bar{e})$ *solves*

$$q \exp\left(\frac{1}{2}q^2\right) = \frac{1-\bar{e}}{1+\alpha-\bar{e}}$$

[8]Formally,

$$\frac{d\hat{q}_h(\bar{e})}{d\bar{e}} = -\frac{V_q^l(\hat{q}_h(\bar{e})) - c_q^l(\hat{q}_h(\bar{e})) - \Delta c_q(\hat{q}_l(\bar{e}))}{\bar{e}\left(V_{qq}^h(\hat{q}_h(\bar{e})) - c_{qq}^h(\hat{q}_h(\bar{e}))\right) + (1-\bar{e})\,\Delta c_{qq}(\hat{q}_h(\bar{e}))} < 0,$$

where the inequality follows from the fact that $\hat{q}_h(\bar{e}) \geq q_h^*$ and from Assumption 1.

[9]Because $V_q^h(q) - c_q^h(q) < 0$ for any $q > q_h^*$ and $\Delta c_q(q) > 0$, the intermediate-value theorem implies that for every $q > q_h^*$, there exists a value of $\bar{e} \in (0,1)$ that solves (3).

Equation (3) implies that $\hat{q}_h(\bar{e})$ *solves*

$$q \exp\left(\frac{1}{2}q^2\right) = \frac{\theta\bar{e}}{\bar{e}(1+\alpha)-\alpha} \tag{4}$$

The function $\hat{q}_h(\bar{e})$ *is well-defined for* $\bar{e} > \frac{\alpha}{1+\alpha}$.

Let $\varphi^i(\bar{e},\bar{q})$ be the total surplus and $U^i(\bar{e},\bar{p},\bar{q})$ the seller's utility after renegotiation in state i. Lemma 1 implies that

$$\varphi^h(\bar{e},\bar{q}) = \begin{cases} V^h(q_h^*) - c^h(q_h^*), \text{ if } (\bar{e},\bar{q}) \in A \cup B \cup C \\ V^h(\bar{q}) - c^h(\bar{q}), \text{ if } (\bar{e},\bar{q}) \in D \\ V^h(\hat{q}_h(\bar{e})) - c^h(\hat{q}_h(\bar{e})), \text{ if } (\bar{e},\bar{q}) \in E \end{cases} \tag{5a}$$

$$\varphi^l(\bar{e},\bar{q}) = \begin{cases} V^l(\hat{q}_l(\bar{e})) - c^l(\hat{q}_l(\bar{e})), \text{ if } (\bar{e},\bar{q}) \in A \\ V^l(\bar{q}) - c^l(\bar{q}), \text{ if } (\bar{e},\bar{q}) \in B \\ V^l(q_l^*) - c^l(q_l^*), \text{ if } (\bar{e},\bar{q}) \in C \cup D \cup E \end{cases} \tag{5b}$$

and

$$U^h(\bar{e},\bar{p},\bar{q}) = \begin{cases} \bar{p} - c^l(\bar{q}) + \Delta c(\hat{q}_l(\bar{e})), \text{ if } (\bar{e},\ \bar{q}) \in A; \\ \bar{p} - c^h(\bar{q}), \text{ otherwise.} \end{cases} \tag{6a}$$

$$U^l(\bar{e},\bar{p},\bar{q}) = \begin{cases} \bar{p} - c^h(\bar{q}) - \Delta c(\hat{q}_h(\bar{e})), \text{ if } (\bar{e},\ \bar{q}) \in E; \\ \bar{p} - c^l(\bar{q}), \text{ otherwise.} \end{cases} \tag{6b}$$

Note that whenever $(\bar{e},\ \bar{q}) \in B \cup C \cup D$, the buyer extracts the entire renegotiation surplus in both states. Also, for any $(\bar{e},\bar{p},\bar{q})$, $U^h(\bar{e},\bar{p},\bar{q}) - U^l(\bar{e},\bar{p},\bar{q}) \in [\Delta c(\hat{q}_l(\bar{e})), \Delta c(\hat{q}_h(\bar{e}))]$. This observation is important because the difference in the seller's utilities between the high and the low states is what provides the incentives to invest for the seller.

5. Trade Contracts

This section looks at a particularly simple class of contracts, the *trade contracts*. A trade contract works as follows. Before the investment takes place, the parties agree on a price-quantity pair $(\bar{p},\bar{q})$, where $\bar{p}$ is the price that the buyer pays the seller, and $\bar{q}$ is the quantity traded. Then, after the state is realized, the parties renegotiate, knowing that if renegotiation breaks down, the outside option is to trade at the terms $(\bar{p},\bar{q})$. An illustration is the contract in Example 1 after Proposition 2.

The contract determines the renegotiation outcome, which in turn affects the investment level. From the previous section, we know that choosing $\bar{q} \in [q_l^*, q_h^*]$ results in efficient renegotiation, whereas $\bar{q} > q_h^*$ ($\bar{q} < q_l^*$) results in inefficient renegotiation when the state is high (low).

Let the level of e that solves the equation

$$\Delta c\left(\hat{q}_h\left(e\right)\right) = \psi'\left(e\right) \tag{7}$$

be denoted by $e_{\max}$.[10] The following lemma summarizes the effect that the contract choice has on the investment level.

Lemma 2. *Investment level $e \in [0,1]$ is implementable by a trade contract if and only if $e \in [e_{\min}, e_{\max}]$, where $e_{\min}$ and $e_{\max}$ solve equations (2) and (7) respectively. In particular, a trade contract $(\bar{p}, \bar{q})$ induces $e = (\psi')^{-1}(\Delta c(\bar{q}))$ if $(\psi')^{-1}(\Delta c(\bar{q})) \in [e_{\min}, e_{\max}]$; $e = e_{\min}$, if $(\psi')^{-1}(\Delta c(\bar{q})) < e_{\min}$; and $e = e_{\max}$, if $(\psi')^{-1}(\Delta c(\bar{q})) > e_{\max}$.*

If $e \in [e_{\min}, e_{\max}]$, then $\psi'(e) \in [\Delta c(\hat{q}_l(e)), \Delta c(\hat{q}_h(e))]$. Therefore, if $\bar{q}$ solves $\Delta c(\bar{q}) = \psi'(e)$, then $\bar{q} \in [\hat{q}_l(e), \hat{q}_h(e)]$. Whenever a trade contract specifies such $\bar{q}$ and the buyer's probability of high state equals e, the buyer offers a menu that gives the seller his reservation payoffs in both states. At the investment stage, the seller equates the marginal cost of investment, which is equal to $\psi'(e)$, to the marginal benefit, which is equal to $U^h(\bar{e}, \bar{p}, \bar{q}) - U^l(\bar{e}, \bar{p}, \bar{q}) = \Delta c(\bar{q})$. Therefore the marginal benefit of investment, and thus the equilibrium investment level, increases in $\bar{q}$.

However, there is an upper bound on equilibrium investment levels achievable by increasing $\bar{q}$. Equations (6) imply that $U^h(\bar{e}, \bar{p}, \bar{q}) - U^l(\bar{e}, \bar{p}, \bar{q})$ (and thus the marginal benefit of investment) cannot exceed $\Delta c(\hat{q}_h(\bar{e}))$. In equilibrium, the buyer's belief $\bar{e}$ equals the investment level e. Therefore, the highest investment level a trade contract can induce solves equation (7). Symmetric reasoning implies that a trade contract cannot induce investment below $e_{\min}$, which is the investment level that occurs when no contract is in place.

The lemma implies a necessary and sufficient condition for the existence of a trade contract that achieves the first-best.

Corollary 1. *The first-best investment and trade in both states are achievable by a trade contract if and only if $\psi'(e^*) \leq \Delta c(q_h^*)$.*

Proof. By Lemma 2, a trade contract $(\bar{p}, \bar{q})$ achieves the first-best investment if $\psi'(e^*) = \Delta c(\bar{q})$. By Lemma 1, this contract achieves the first-best trade in both states if $(e^*, \bar{q}) \in C$, i.e. $\Delta c(\bar{q}) \in [\Delta c(q_l^*), \Delta c(q_h^*)]$. As shown in Section 3, the first-best level of investment e^* satisfies $\psi'(e^*) \geq \Delta c(q_l^*)$. Therefore, it is possible to achieve the first-best with a trade contract if and only if $\psi'(e^*) \leq \Delta c(q_h^*)$. □

[10]Note that $e_{\max}$ is unique and well-defined, because $\hat{q}_h(e)$ is continuous, decreasing, takes all values above q_h^* and Assumption 2 holds.

If $\psi'(e^*) > \Delta c(q_h^*)$, the parties face a trade-off: if they set $\bar{q} > q_h^*$, they generate high investment but inefficient renegotiation in the high state; if they set $\bar{q} \leq q_h^*$, they get lower investment and efficient renegotiation in the high state. The extent of this trade-off and the optimal way to resolve it depend on the parameters of the model, in particular, on the degree to which the buyer's payoff depends on the state.

Example 1 continued *Here* $e_{\min}$ *solves the equation* $\alpha\left(\frac{1-e}{1+\alpha-e}\right)^2 = \beta e$, *and* $e_{\max}$ *solves the equation* $\alpha\left(\frac{\theta e}{e(1+\alpha)-\alpha}\right)^2 = \beta e$. *It is easy to verify that if* $\alpha \geq \sqrt{1-\frac{1}{\theta^2}}$ *(i.e. the investment is sufficiently selfish), then* $\psi'(e^*) \leq \Delta c(q_h^*)$, *so that a trade contract can achieve the first-best investment and trade in both states. However, if* $\alpha < \sqrt{1-\frac{1}{\theta^2}}$, *then* $\psi'(e^*) > \Delta c(q_h^*)$, *so it is impossible to achieve the first-best trade and investment by a trade contract. Whether* e^* *is above or below* $e_{\max}$ *depends on* β*: in particular,* $e^* < e_{\max}$ *(and thus the first-best investment is achievable by a trade contract) if and only if* β *is high enough.*

Example 2 continued *Here* $e_{\min}$, *together with* $\hat{q}_l(e_{\min})$, *solves the system of equations*

$$\begin{cases} q\exp\left(\frac{1}{2}q^2\right) = \frac{1-e}{1+\alpha-e}; \\ \alpha\left(\exp\left(\frac{1}{2}q^2\right)-1\right) = \beta e \end{cases}$$

Similarly, $e_{\max}$, *together with* $\hat{q}_h(e_{\max})$, *solves the system of equations*

$$\begin{cases} q\exp\left(\frac{1}{2}q^2\right) = \frac{\theta e}{(1+\alpha)e-\alpha}; \\ \alpha\left(\exp\left(\frac{1}{2}q^2\right)-1\right) = \beta e \end{cases}$$

One can verify that, for any $\alpha > 0$, $\psi'(e^*) \leq \Delta c(q_h^*)$ *if* θ *is sufficiently close to 1. Therefore the first-best investment and trade are achievable with a trade contract if the cooperative effect of investment is weak enough.*

6. General Contracts

In this section, we consider more general contracts. This allows us to establish when it is optimal to transmit information about the seller's investment level to the buyer before the renegotiation stage.

The problem of finding the optimal contract can be divided into three parts. First, we assume that it is possible to induce a given investment level with some contract, and look for a contract that induces it in the most efficient way. Proposition 3 shows that if an investment level is achievable by a trade contract, then the trade contract is the optimal way to achieve it. Proposition 4 describes the contract that is optimal to achieve investment above $e_{\max}$.

The second step is to find out what investment levels can be induced by a contract. We already know that investment e can be induced by a trade contract if and only if $e \in [e_{\min}, e_{\max}]$ (Lemma 2). If the parties hope to induce investment higher than $e_{\max}$, they should write a contract that transmits some information to the buyer. Lemma 3 gives a necessary and a sufficient condition for it to be possible to induce investment above $e_{\max}$.

The final step is to optimize over investment levels. The main result of the article, Proposition 5, shows that the optimal investment level is never below $e_{\min}$ and gives a necessary and sufficient condition for it to be above $e_{\max}$. If this condition is satisfied, the optimal contract transmits some information to the buyer before the renegotiation stage.

Definitions. In order to study the effects of information transmission, we will follow the mechanism design literature in assuming that the parties have access to a communication device that acts as a neutral trustworthy mediator. A contract works as follows. After the investment has been made and the state has realized, the seller privately communicates the state to the mediator, who cannot observe the state directly. Having received the seller's report, the mediator chooses a public signal, z, out of a set of allowable signals Z. Each signal z is accompanied by terms of trade (p_z, q_z). After the signal has been sent, the parties renegotiate the terms of trade using (p_z, q_z) as the outside option.

A contract includes a signal set Z; the default terms of trade (p_z, q_z) for every $z \in Z$; and a *disclosure policy*, which determines the probability of each signal conditional on each realization of the state as reported by the seller. Formally, a disclosure policy includes two probability distributions on Z, ζ^h and ζ^l. After the seller has reported that the state is i, the mediator randomizes over signals in Z according to ζ^i. We will denote the probability of message $z \in Z$ after the seller has reported state i by ζ^i_z (if ζ^i is a continuous probability distribution, then ζ^i_z is the probability density function evaluated at z). A contract of this type is called a direct revelation mechanism.

The immediate advantage of assuming that the parties have access to a mediator is that it makes the optimal contracting problem tractable. In particular, the revelation principle (Myerson, 1982) implies that without loss of generality we can restrict attention to equilibria where the seller reports the state truthfully. This makes the the equilibrium conditions for the seller easy to state. Another advantage of considering mediated communication is that, by the revelation principle, any communication protocol can be replicated by a direct revelation mechanism. For example, to replicate a trade contract with outside option $(\bar{p}, \bar{q})$, the mediator sends the same signal, which is accompanied by outside option $(\bar{p}, \bar{q})$, regardless of the seller's report. Another example is the sell-out contract from the proof of Proposition 1: to replicate it, the mediator sends a signal with outside option $(\bar{p}_i, \bar{q}_i) = (V(q^*_i), q^*_i)$ whenever the seller reports that the state is i.

Communication arrangements that use a third party are widely used in practice. For example, mediators and arbitrators are commonly employed in bargaining and dispute resolution, and it has been recognized in the literature (Brown and Ayres, 1994; Mitusch and Strausz, 2005) that one of the reasons of using them is to filter the information between the bargaining parties. Another setting where a third party can appear naturally in bargaining is transfer pricing in a firm with decentralized decision-making (Baldenius, 2006): the firm owner can facilitate bargaining between the division managers. Note also that a mediator is not always necessary to implement a direct revelation mechanism: for example, trade contracts are implementable without a mediator.[11]

In what follows, we will distinguish between two classes of contracts: contracts with no disclosure and with partial disclosure. In a contract with **no disclosure**, the probability distribution that the mediator uses to randomize over the signals is independent of the seller's report (formally, $\forall z \in Z, \zeta_z^h = \zeta_z^l$). As a result, the buyer does not learn any information about the state from the public signal. An example of a no-disclosure contract is a trade contract. Contracts that do not satisfy this definition will be called contracts with **partial disclosure**.

Note that partial-disclosure contracts, as well as no-disclosure contracts, can be either stochastic or deterministic. For example, the sell-out contract from the proof of Proposition 1 is a partial-disclosure contract that is deterministic. An example of a stochastic no-disclosure contract a contract whereby the mediator sends a normally distributed signal with mean 0 and variance 1 regardless of the seller's report. A trade contract is a deterministic no-disclosure contract.

Renegotiation Stage. If the equilibrium investment is equal to e_0, then, after the public signal $z \in Z$ has been sent, the buyer's posterior probability of the high state conditional on z is equal to

$$e_z \equiv \Pr(h|z) = \frac{\zeta_z^h e_0}{\zeta_z^h e_0 + \zeta_z^l (1-e_0)}$$

and the outside option is (p_z, q_z). The buyer's optimal offer as a function of e_z, p_z and q_z has been described in Section 4. In what follows, we will denote $U^i(e_z, p_z, q_z)$ by U_z^i and $\varphi^i(e_z, q_z)$ by φ_z^i.

The Contracting Problem. We can now formulate the optimal contracting problem. As it is commonly assumed in the literature, the optimal mechanism maximizes the expected total surplus net of investment costs. Formally, it solves the following problem:

$$\max_{\begin{pmatrix} Z,\ (p_z,\ q_z)_{z\in Z}, \\ \zeta^h,\ \zeta^l,\ e_0 \end{pmatrix}} e_0 \int_{z\in Z} \varphi^h(e_z, q_z)\, d\zeta^h + (1-e_0) \int_{z\in Z} \varphi^l(e_z, q_z)\, d\zeta^l - \psi(e_0) \qquad \textbf{(Problem 1)}$$

[11]More generally, it is possible to prove that any direct revelation mechanism can be replicated by a mechanism that does not require a mediator, if the set of public messages is finite and the probability of each message is a rational number (Krishna, 2007).

subject to

$$(IC_i) \qquad \int\limits_{z\in Z} U_z^i d\zeta^i \geq \int\limits_{z\in Z} U_z^i d\zeta^j, \quad i,j = h,l$$

$$(INV) \qquad \int\limits_{z\in Z} U_z^h d\zeta^h - \int\limits_{z\in Z} U_z^l d\zeta^l = \psi'(e_0);$$

$$(PR_i) \qquad \int\limits_{z\in Z} d\zeta^i = 1, \quad i = h,l;$$

$$(FEAS) \qquad q_z \geq 0;\ \zeta^i \geq 0;\ e_0 \in [0,1].$$

Conditions (IC_i) are incentive compatibility conditions: they say that the seller should not gain by misrepresenting the state to the mediator when the true state is i. Condition (INV) is the first-order condition for the seller choosing investment e_0. The second-order condition is satisfied, because the seller's benefit from investing is linear in his investment level (the buyer cannot detect a deviation from the investment level, so he computes posteriors using the equilibrium level of investment), and the cost of investment $\psi(e)$ is convex. Conditions (PR_i) say that ζ^i are probability measures.

Implementing a Given Investment Level. The first question we can answer is what contract is optimal in the class of contracts inducing a given investment level e_0.

Definition 1. *A contract $G = \left(Z,\ p_z,\ q_z,\ \zeta_z^h,\ \zeta_z^l\right)$ is* ***optimal given*** $\mathbf{e}_0$ *if it is feasible, achieves investment e_0 in equilibrium and no other contract achieving e_0 yields strictly higher expected surplus.*

First, let us look at $e_0 \in [e_{\min}, e_{\max}]$. We know that these investment levels can be achieved by trade contracts (Lemma 2). The following proposition states that trade contracts are the optimal way to achieve such investment levels.

Proposition 3. *If $e_0 \in [e_{\min}, e_{\max}]$ (i.e. e_0 can be achieved by a trade contract), then a trade contract is optimal given e_0.*

The intuition for the proof is that disclosure causes uncertainty about the quantity offered to the high-type seller, and the joint surplus is concave. By Jensen's inequality, if a trade contract can be used to achieve e_0, it yields higher surplus than any other contract achieving e_0.

Now suppose that $e_0 > e_{\max}$. Then e_0 is unachievable by trade contracts (Lemma 2). Suppose, however, that e_0 can be achieved by a partial-disclosure contract. The following proposition describes the qualitative features of the contract that is optimal to achieve e_0.

Proposition 4. *Suppose that* $e_0 > e_{\max}$ *and* e_0 *can be achieved by a partial-disclosure contract. Then there exists a contract* $G = \left(Z,\ p_z,\ q_z,\ \zeta_z^h,\ \zeta_z^l\right)$ *that is optimal given* e_0 *and has the following properties:*

(I) *Z includes either 3 or 4 messages;*

(II) $\forall z \in Z, p_z = c^l(q_z)$*;*

(III) *Z can be partitioned into two subsets,* Z_l *and* Z_h*, where:*

(i) $|Z_l| \geq 1$, $|Z_h| \geq 2$*;*

(ii) *if* $z \in Z_l$*, then either* $(e_z, q_z) = (0, q_l^*)$ *or* $(e_z, q_z) \in B$*;*

(iii) *if* $z \in Z_h$*, then either* $(e_z, q_z) = (1, q_h^*)$ *or* $(e_z, q_z) \in D$ *and* $q_z = \hat{q}_h(e_z)$*;*

(iv) *if* $z \in Z_l$ *and* $z' \in Z_h$*, then* $e_z < e_{z'}$*.*

Condition (I) of Proposition 4 seemingly contradicts Proposition 1 of Bester and Strausz (2001), which shows that in adverse selection problems with limited commitment one can limit the cardinality of the message space to that of the state space (2 in our case). The difference in results stems from different informational assumptions. In particular, Bester and Strausz' model does not include a mediator: the agent makes a direct report to the principal, who is then making the renegotiation offer. As a result, it need not be possible to induce the agent to report the state truthfully; the agent may randomize between his messages in equilibrium in order to prevent the principal from extracting the entire renegotiation surplus. In our model, the principal (the buyer) cannot observe the report that the agent (the seller) makes to the mediator, and the noise in the buyer's information comes from the mediator's disclosure policy, rather than the seller's mixed strategy. The mediator can replicate any equilibrium of Bester and Strausz' direct communication game; however, Proposition 1 of Bester and Strausz implies that some mediated mechanisms (for instance, the ones that use more than two public messages) cannot be replicated by one-shot communication without a mediator.

Figure 2 shows an example of a contract that satisfies conditions $(I) - (III)$ of Proposition 4. The horizontal axis measures e_z, and the vertical axis measures q_z. The contract in Figure 2 includes three signals. If the seller reports that the state is low, the mediator either sends the signal z_1, which corresponds to default quantity q_1, or z_2, which corresponds to default quantity $q_2 > q_1$, or z_0, which corresponds to the default quantity q_l^*. If the seller reports that the state is high, the mediator sends either z_1 or z_2. The disclosure policy is such that upon receiving z_2 the buyer's posterior probability of the high state, e_2, is lower than upon receiving z_1.

To understand how the optimal contract works, first note that without loss of generality we can restrict attention to contracts such that after every signal, the optimal renegotiation offer extracts the entire renegotiation surplus from the seller: that is, $\forall z \in Z$, $(e_z, q_z) \in B \cup C \cup D$ (Lemma 6 in the Appendix). The

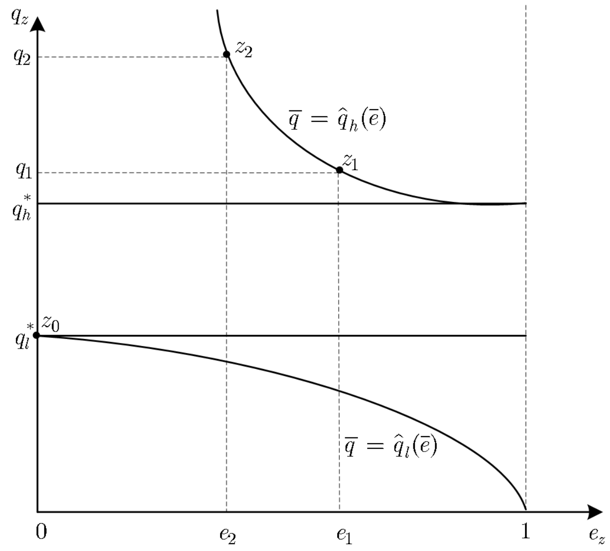

FIGURE 2. An Optimal Partial-Disclosure Contract

reason is that if the default terms of trade (p_z, q_z) leave positive rent to the seller in one of the states, it is always possible to adjust them so that the buyer extracts the entire renegotiation surplus, but the seller's utility in both states, as well as the welfare, remains unchanged. Therefore, we can assume that after any signal z, the seller's utility in state i equals his default utility, $p_z - c^i(q_z)$.

The next thing to note is that, without loss of generality, the default prices can be chosen to drive the seller's utility in the low state to zero: that is, $\forall z \in Z$, $p_z = c^l(z)$ (Lemma 7 in the Appendix). It follows that the seller's utility in the high state conditional on signal z equals $\Delta c(q_z)$, and the marginal benefit of investment for the seller equals his expected utility in the high state, $\sum_{z \in Z} \Delta c(q_z)\zeta_z^h$. Also, this means that the seller's utility in the high state conditional on signal z cannot exceed $\Delta c(\hat{q}_h(e_z))$, which decreases in e_z. This is because the less likely is the high state conditional on z, the higher is the upward distortion in quantity that the buyer is willing to tolerate in the high state, and thus the higher is the upper bound on the seller's utility in the high state.

These observations imply that it is impossible to implement any investment above $e_{\max}$ by a full-disclosure contract that completely reveals the state to the buyer. Indeed, if the buyer is informed of the state perfectly, renegotiation is efficient, and the utility that the seller can get in the high state cannot exceed $\Delta c(q_h^*)$. Therefore, the maximal investment that a full-disclosure contract can induce satisfies the equation $\Delta c(q_h^*) = \psi'(e)$ and is lower than $e_{\max}$. Therefore, in order to create investment incentives, the optimal contract should include a signal with a high q_z and a low e_z (z_2 in Figure 2).

The next feature of the optimal contract that should be explained is why the mediator has to introduce noise in both states of the world, and, in particular, why a contract where one of the states is disclosed without noise, similarly to the optimal contract in Laffont and Tirole (1990), will not work. Indeed, suppose the contract in Figure 2 did not include the signal z_1, so that only z_2 would be sent in the high state. Then the buyer's probability of the high state conditional on z_2, denoted by e_2, would be higher than his prior probability e. Therefore, the marginal benefit of investment conditional on z_2, which cannot exceed $\Delta c\left(\hat{q}_h(e_2)\right)$, would be lower than $\Delta c(\hat{q}_h(e))$, and the resulting investment would be lower than $e_{\max}$.

Now, suppose that the contract in Figure 2 did not include z_0. The resulting contract would not incentive compatible for the seller in the high state. Indeed, the high-state seller prefers z_2 to z_1. Because $e_2 < e_1$, misreporting that the state is low would result in a higher probability of the mediator announcing z_2 than reporting truthfully. Thus z_0 should be included in the contract to prevent the seller from misreporting in the high state.

Achievable investment levels. Propositions 3 and 4 describe the contracts that are optimal given different levels of e_0. The next step toward solving for the optimal contract is to find what investment levels are achievable. From Lemma 2, we know that the set of investment levels that can be induced by trade contracts is the interval $[e_{\min}, e_{\max}]$. The proof of Lemma 2 also shows that investment higher than $e_{\max}$ cannot be induced by any no-disclosure contract.[12] So, if inducing investment above $e_{\max}$ is at all possible, it must be possible only with the help of a partial-disclosure contract.

Let

$$f_a(e_1, e) = \frac{e_1\left(\Delta c\left(\hat{q}_h\left(e_1\right)\right) - \Delta c\left(\hat{q}_h\left(e\right)\right)\right)}{\left(e - e_1\right)\Delta c\left(\hat{q}_h\left(e_1\right)\right)};$$
$$f_{a'}(e_1, e) = \frac{e_1\left(\Delta c\left(\hat{q}_h\left(e_1\right)\right) - \Delta c\left(\hat{q}_h\left(e\right)\right)\right)}{\left(e - e_1\right)\left(\Delta c\left(\hat{q}_h\left(e_1\right)\right) - \Delta c(q_l^*)\right)}$$

We will say that **Condition (a)** holds if there exist $e_1 < e_{\max},\ e_2 > e_{\max}$ such that

$$f_a(e_1, e_{\max}) > f_a(e_2, e_{\max})$$

and that **Condition (a')** holds if there exist $e_1 < e_{\max},\ e_2 > e_{\max}$ such that

$$f_{a'}(e_1, e_{\max}) > f_{a'}(e_2, e_{\max})$$

The next lemma shows that the set of investment levels above $e_{\max}$ that can be induced only with partial disclosure contracts is an interval with the lower boundary at $e_{\max}$. The lemma also provides a necessary and a sufficient condition for this interval to be nonempty.

[12]Later we show (Part (i) of Proposition 5) that it is never optimal to induce investment below $e_{\min}$, so we will not consider the question of how to achieve such investment levels.

Lemma 3. *(i) Condition (a) is necessary and Condition (a') sufficient for the set of achievable investment levels above* $e_{\max}$ *to be nonempty.*

(ii) If $e_0 > e_{\max}$ *is achievable, then any* $\tilde{e} \in (e_{\max}, e_0)$ *is achievable as well.*

Consider a three-signal contract of the type depicted in Figure 2, and suppose that it implements $e > e_{max}$. Conditions (INV) and (PR_h) uniquely determine $\zeta^h(z_1)$ and $\zeta^h(z_2)$. Condition (a') is the result of substituting these expressions into condition (IC_h) and noting that if the resulting inequality holds weakly for $e > e_{\max}$, then it must hold strictly for $e = e_{\max}$. Conversely, it is possible to prove that any implementable investment level above $e_{\max}$ can be implemented by a three-signal contract (Lemma 8 in the Appendix). Condition (a) follows from (IC_h) for such contracts.

Conditions (a) and (a') are restrictions on the shape of the function $\Delta c\left(\hat{q}_h(e)\right)$. To see informally how the shape of $\Delta c\left(\hat{q}_h(e)\right)$ relates to the implementability of $e > e_{\max}$, consider the three-signal contract depicted in Figure 2 that implements $e > e_{max}$. The seller's expected utility in the high state, $\zeta_1^h \Delta c(q_1) + \zeta_2^h \Delta c(q_2)$, is higher than $\Delta c\left(\hat{q}_h(e_{\max})\right)$. Because $\Delta c(q_2) > \Delta c\left(\hat{q}_h(e_{\max})\right) > \Delta c(q_1)$, this will hold if ζ_2^h is high enough. However, (IC_h) bounds ζ_2^h from above. Therefore, the function $\Delta c\left(\hat{q}_h(e)\right)$ has to be 'sufficiently convex' for $e > e_{\max}$ to be implementable. As shown below, $\Delta c\left(\hat{q}_h(e)\right)$ never satisfies this restriction in Example 1, whereas in Example 2 it does for θ large enough.

Example 1 continued *Here*

$$f_a(e_1, e) = \frac{e_1}{e - e_1} \frac{\left(\frac{e_1 \theta}{e_1(1+\alpha)-\alpha}\right)^2 - \left(\frac{e\theta}{e(1+\alpha)-\alpha}\right)^2}{\left(\frac{e_1 \theta}{e_1(1+\alpha)-\alpha}\right)^2}$$

This function is increasing in e_1*; therefore, Condition* (a) *cannot be satisfied, and it is impossible to achieve any investment level above* $e_{\max}$.

Example 2 continued *Here*

$$f_{a'}(e_1, e) = \frac{e_1}{e - e_1} \frac{\exp\left(\frac{1}{2}\left(\hat{q}_h(e_1)\right)^2\right) - \exp\left(\frac{1}{2}\left(\hat{q}_h(e)\right)^2\right)}{\exp\left(\frac{1}{2}\left(\hat{q}_h(e_1)\right)^2\right) - \exp\left(\frac{1}{2}\left(q_l^*\right)^2\right)},$$

where $\hat{q}_h(e)$ *is defined implicitly by equation (4). For example, suppose that* $\alpha = 1$ *and* $\beta = 5$ *(these parameter values satisfy Assumption 2 if and only if* $\theta < 11.358$*). Then* $\lim_{e_1 \to \frac{\alpha}{1+\alpha}} f_{a'}(e_1, e_{\max}) > f_{a'}(1, e_{\max})$ *if and only if* $\theta > 6.462$*. Therefore, Condition* (a') *holds when investment is cooperative enough.*

Optimal Investment. Finally, we can optimize over the investment level. Proposition 5 gives a necessary and sufficient condition for the optimal investment level to be above $e_{\max}$, which means that the optimal contract is a partial-disclosure contract.

We will say that **Condition (b)** holds if

$$V^{h}\left(\hat{q}_{h}\left(e\right)\right)-c^{h}\left(\hat{q}_{h}\left(e\right)\right)-\left(V^{l}(q_{l}^{*})-c^{l}(q_{l}^{*})\right)-\psi'(e)-(1-e)\,\psi''\left(e\right)\Big|_{e=e_{\max}}>0$$

Proposition 5. (*i*) *Any contract that implements investment $e<e_{\min}$ is suboptimal.*

(*ii*) *If the optimal contract implements $e>e_{\max}$, Condition (b) holds.*

(*iii*) *Suppose that there exist investment levels $e>e_{\max}$ that are achievable with partial-disclosure contracts, and Condition (b) holds. Then the optimal contract is a partial-disclosure contract.*

The idea behind the proof is the following. Let

$$g(e)=e\varphi^{h}\left(0,\Delta c^{-1}(\psi'\left(e\right))\right)+(1-e)\varphi^{l}\left(1,\Delta c^{-1}(\psi'\left(e\right))\right)-\psi(e)$$

Note that g is a concave function that equals the maximal total surplus given the investment level e if $e\in[e_{\min},e_{\max}]$ and exceeds this surplus if $e>e_{\max}$ or $e<e_{\min}$. By equation (7), the left-hand side of Condition (b) equals $g'\left(e_{\max}\right)$, which is the net marginal benefit of investment at $e_{\max}$. Therefore, Condition (b) is equivalent to $g'\left(e_{\max}\right)>0$.

The proof of part (i) consists of showing that $g'(e)>0$ for $e<e_{min}$, which implies that the maximal total surplus given $e<e_{\min}$ is lower than $g(e_{\min})$, and therefore investment below $e_{\min}$ is suboptimal. Because $g(e)$ is concave, it reaches its global maximum at $e>e_{max}$ if and only if $g'\left(e_{\max}\right)>0$. Therefore, if Condition (b) does not hold, the optimal investment level is below $e_{\max}$, hence part (ii). Conversely, if Condition (b) holds, then by the envelope theorem, the derivative of the total surplus with respect to investment at $e_{\max}$ is positive, and the optimal investment level is above $e_{\max}$ (part (iii)).

Example 2 continued *Here Condition (b) takes the form*

$$\theta\hat{q}_{h}(e_{\max})-\exp\left(\frac{1}{2}\left(\hat{q}_{h}(e_{\max})\right)^{2}\right)-q_{l}^{*}+(1+\alpha)\left(\exp\left(\frac{1}{2}\left(q_{l}^{*}\right)^{2}\right)\right)-\alpha-\beta>0$$

If $\alpha=1$ and $\beta=5$, this condition holds if and only if $\theta>5.066$. Combining this with Condition (a') and Assumption 2, we can conclude that a partial-disclosure contract is optimal if $\theta\in(6.462,11.358)$.

7. Alternative Renegotiation Scenarios

In this section, we relax the assumption that the renegotiation lasts for one round. Unlike the case of complete information, the optimal single-round renegotiation offer in our model can result in inefficient trade in one of the states. Therefore, after the seller has revealed the state by choosing from the menu that the buyer offered, the buyer has an incentive to renegotiate the quantity further to the efficient level. If the seller anticipates such renegotiation, he may no longer have incentives to reveal his type truthfully by choosing the menu item designed for him.

There is no single obvious way to model multi-stage renegotiation in our environment. We will consider two versions of the renegotiation game. In each case, the renegotiation game starts with the outside option determined by the contract and each round of the game includes the following moves:

(i) The buyer proposes a menu of contracts or ends the renegotiation. If he decides to end the renegotiation, the outside option is executed.

(ii) If the buyer has proposed a menu, the seller either accepts or rejects.

(iii) If the seller has accepted, he makes a choice from the menu, which becomes the new outside option.

Renegotiation game (1) ("renegotiation with frictions") There are $T \geq 2$ rounds of renegotiation, where T is either finite or infinite. If the seller rejects the offer at stage $t \leq T$, then renegotiation ends and the outside option is executed immediately.

Renegotiation game (2) ("renegotiation without frictions") There are $T \geq 2$ rounds of renegotiation, where T is either finite or infinite. Unlike game (1), if the seller rejects the offer at stage $t < T$, the buyer can make a new renegotiation offer, and the outside option remains the same as in the previous round.

The equilibrium concept that we will use is perfect Bayesian equilibrium.

Note that, if the buyer's beliefs are taken as given, the buyer cannot strictly gain from the possibility of multi-stage renegotiation in comparison to one-shot renegotiation: for any equilibrium of any game with multi-stage renegotiation, there exists an outcome-equivalent incentive compatible and feasible mechanism in the one-shot renegotiation game. The familiar conclusion that lack of commitment hurts the principal follows from the revelation principle (Myerson, 1982) and the fact that we allow the buyer to optimize over all possible mechanisms in the one-shot renegotiation scenario. Therefore, if there is an equilibrium of a multi-stage renegotiation game that gives the same payoff to the buyer in every state as the optimal offer in the one-shot renegotiation game, then this equilibrium is the best for the buyer.

Proposition 6. *Fix the buyer's beliefs and the outside option, and let* $((p_h, q_h), (p_l, q_l))$ *be the buyer's optimal offer in the one-shot renegotiation game described in Lemma 1.*

(1) *Every equilibrium of the renegotiation game (1) results in the outcome* (p_i, q_i) *in state* $i \in \{h, l\}$.

(2) (a) *If* T *is finite, every equilibrium of the renegotiation game (2) results in the outcome* (p_i, q_i) *in state* $i \in \{h, l\}$.

(b) *If* T *is infinite, every equilibrium of the renegotiation game (2) results in efficient trade in both states.*

Proposition 6 implies that the possibility of multi-stage renegotiation by itself need not invalidate the results of the article. In the renegotiation game with frictions, the impossibility to renegotiate after the seller has rejected an offer serves as a commitment device for the buyer that enables him to replicate the one-shot renegotiation outcome; in the renegotiation game without frictions, a similar role is played by the fact that renegotiation is finite. In the infinite version of the renegotiation game without frictions, inefficiency is renegotiated away in equilibrium, and the mechanism for creating investment incentives described in the article no longer works.[13]

8. Discussion

This article considers a hold-up model where the investment level and the state of the world that depends on it are observable only by the investing party. We assume that investment has a cooperative element, i.e. it has a direct effect on the payoff functions of both parties. We find that under these conditions, the parties face a trade-off between high investment and efficient renegotiation. We also find how much information about investment should be transmitted by a contract. Under some conditions, contracts that transmit some information (partial-disclosure contracts) can induce higher investment than contracts that transmit no information, at the same time leading to less efficient renegotiation.

Our results make use of the assumption that the investment has a noncooperative element (i.e. affects the investing party's payoff from trade). The effect of investment on the investing party can be small relative to its effect on the other party, but it has to be present in order for the results to go through. If investment is purely cooperative, contracting is valueless in our model: any contract leads to zero investment. The reason is that in this case, both seller types have the same utility function, so it is impossible to discriminate between them and to create investment incentives. For the same reason, contracting is valueless in the perfect information model when the investment is purely cooperative, no matter how the bargaining power is distributed at the renegotiation stage.

Our result on the possible optimality of stochastic contracts relies on the assumption that renegotiation takes place after the contract is executed. If the parties were permitted to renegotiate earlier (for example, after the seller invests, but before he observes his type, as in Fudenberg and Tirole (1990)), then the optimal contract would be deterministic. However, one might argue that interim renegotiation might be infeasible. For example, suppose that the game takes place in continuous time. It is reasonable to assume that the buyer, who cannot observe the investment level and state realization, also does not know at what moment of time the investment takes place. If the buyer does not know whether the investment has taken place, he

[13] However, Proposition 4 of Beaudry and Poitevin (1995) implies that even in this case inefficiency could persist if we assumed that the seller was making the renegotiation offers.

might be unwilling to make a renegotiation offer to the seller before the contract is executed, fearing that such an offer will distort the seller's investment incentives.

9. Appendix

Proofs for Section 5.

Proof of Lemma 2. Suppose that a trade contract $(\bar{p}, \bar{q})$ is in place, and at the renegotiation stage the buyer places probability e on the state being high. Then the seller's problem at the investment stage is

$$\max_{\tilde{e}\in[0,1]} \tilde{e}U^h(e,\bar{p},\bar{q}) + (1-\tilde{e})\,U^l(e,\bar{p},\bar{q}) - \psi(\tilde{e})$$

Equations (6) imply that the first-order conditions are

$$\psi'(\tilde{e}) = U^h(e,\bar{p},\bar{q}) - U^l(e,\bar{p},\bar{q}) = \begin{cases} \Delta c(\hat{q}_l(e)), & \text{if } \bar{q} < \hat{q}_l(e); \\ \Delta c(\bar{q}), & \text{if } \bar{q} \in [\hat{q}_l(e), \hat{q}_h(e)]; \\ \Delta c(\hat{q}_h(e)), & \text{if } \bar{q} > \hat{q}_h(e) \end{cases}$$

In equilibrium, $\tilde{e} = e$. Therefore, there are three cases to consider:

(i) $\psi'(e) = \Delta c(\hat{q}_l(e))$ and $\bar{q} < \hat{q}_l(e)$. The first of these equations implies that $e = e_{\min}$. Therefore the second equation implies $\Delta c(\bar{q}) < \Delta c(\hat{q}_l(e_{\min})) = \psi'(e_{\min})$, so $(\psi')^{-1}(\Delta c(\bar{q})) < e_{\min}$.

(ii) $\psi'(e) = \Delta c(\bar{q})$ and $\bar{q} \in [\hat{q}_l(e), \hat{q}_h(e)]$. These equations imply $\psi'(e) \in [\Delta c(\hat{q}_l(e)), \Delta c(\hat{q}_h(e))]$, so $e \in [e_{\min}, e_{\max}]$.

(iii) $\psi'(e) = \Delta c(\hat{q}_h(e))$ and $\bar{q} > \hat{q}_h(e)$. In this case, $e = e_{\max}$ and $(\psi')^{-1}(\Delta c(\bar{q})) < e_{\min}$.

In all cases, $e \in [e_{\min}, e_{\max}]$, so investment below $e_{\min}$ or above $e_{\max}$ is not implementable by trade contracts. Conversely, for any $e \in [e_{\min}, e_{\max}]$, a trade contract with $\Delta c(\bar{q}) = \psi'(e)$ achieves investment e in equilibrium (case *(ii)*). □

Proofs for Section 6. A direct revelation mechanism is a tuple

$$C = (Z,\ B(Z),\ p: Z \to \mathbb{R},\ q: Z \to \mathbb{R}_+,\ \zeta: \{high, low\} \to \Delta(Z)),$$

where $B(Z)$ is a sigma-algebra on Z.

Let $e_z \equiv \Pr(h|z)$ denote the buyer's posterior belief that the state is high after receiving message $z \in Z$ when the investment level is e_0 and let

$$\gamma_z \equiv \frac{1-e_z}{e_z} \text{ and } \gamma_0 \equiv \frac{1-e_0}{e_0}.$$

By Bayes' rule,

$$\zeta_z^l = \zeta_z^h \frac{\gamma_z}{\gamma_0}, \text{ if } e_z > 0$$

Instead of $\left(\zeta_z^h,\ \zeta_z^l\right)_{z\in Z}$ a contract can specify $((\zeta_z^h)_{z\in Z},\ (e_z)_{z\in Z},\ \left(\zeta_z^l\right)_{z\in Z: e_z=0})$, because for $z \in Z$ such that $e_z > 0$, one can reconstruct ζ_z^l from ζ_z^h and e_z by Bayes' rule.

Using the new notation, Problem 1 can be reformulated as follows:

$$\max_{\left(\begin{array}{l} Z,\ (p_z,\ q_z,\ e_z)_{z\in Z}, \\ \zeta^h,\ \zeta^l,\ e_0 \end{array}\right)} e_0 \int\limits_{z\in Z} \varphi^h\left(e_z, q_z\right) d\zeta^h + (1-e_0)\left(\int\limits_{z\in Z: e_z>0} \frac{\gamma_z}{\gamma_0}\varphi^l\left(e_z, q_z\right) d\zeta^h \right.$$

$$\left. + \int\limits_{z\in Z: e_z=0} \left(V^l(q_l^*) - c^l(q_l^*)\right) d\zeta^l \right) - \psi\left(e_0\right) \quad \textbf{(Problem 1a)}$$

subject to

$$(IC_h) \qquad \int\limits_{z\in Z: e_z>0} \left(1-\frac{\gamma_z}{\gamma_0}\right) U_z^h d\zeta^h - \int\limits_{z\in Z: e_z=0} U_z^h d\zeta^l \geq 0;$$

$$(IC_l) \qquad \int\limits_{z\in Z: e_z>0} \left(\frac{\gamma_z}{\gamma_0}-1\right) U_z^l d\zeta^h + \int\limits_{z\in Z: e_z=0} \left(p_z - c^l\left(q_z\right)\right) d\zeta^l \geq 0;$$

$$(INV) \qquad \int\limits_{z\in Z: e_z>0} U_z^h d\zeta^h - \int\limits_{z\in Z: e_z>0} \frac{\gamma_z}{\gamma_0} U_z^l d\zeta^h - \int\limits_{z\in Z: e_z=0} \left(p_z - c^l\left(q_z\right)\right) d\zeta^l = \psi'\left(e_0\right);$$

$$(PR_h) \qquad \int\limits_{z\in Z} d\zeta^h = 1;$$

$$(PR_l) \qquad \int\limits_{z\in Z: e_z>0} \frac{\gamma_z}{\gamma_0} d\zeta^h + \int\limits_{z\in Z: e_z=0} d\zeta^l = 1;$$

$$(FEAS) \qquad q_z \geq 0;\ e_z \in [0,1];\ e_0 \in [0,1].$$

The following lemma allows us to restrict attention to disclosure policies with a finite number of signals.

Lemma 4. *If an optimal contract exists, then there exists an optimal contract with a finite Z and $B(Z)$ equal to the set of all subsets of Z.*

Proof. The proof will use a technique similar to that used in the proof of Lemma 3 of Bester and Strausz (2007). Suppose that $\left(Z^*,\ B\left(Z^*\right),\ p_z^*,\ q_z^*,\ \zeta^{*h},\ \zeta^{*l},\ e_z^*,\ e_0^*\right)$ is a solution to Problem 1a, and Z is infinite. Then $\left(Z^*,\ B\left(Z^*\right),\ p_z,\ q_z,\ \zeta^h,\ \zeta^l,\ e_z^*,\ e_0^*\right)$ is also a solution if

$$\int\limits_{z\in Z^*:e_z^*>0}\left(\varphi^h\left(e_z^*,q_z^*\right)+\frac{\gamma_z}{\gamma_0}\varphi^l\left(e_z^*,q_z^*\right)\right)d\zeta^{*h}=\int\limits_{z\in Z^*:e_z^*>0}\left(\varphi^h\left(e_z^*,q_z\right)+\frac{\gamma_z}{\gamma_0}\varphi^l\left(e_z^*,q_z\right)\right)d\zeta^h;$$

$$\int\limits_{z\in Z^*:e_z^*>0}\frac{\gamma_z}{\gamma_0}d\zeta^{*h}=\int\limits_{z\in Z^*:e_z^*>0}\frac{\gamma_z}{\gamma_0}d\zeta^h;$$

$$\int\limits_{z\in Z^*:e_z^*>0}\left(1-\frac{\gamma_z}{\gamma_0}\right)U_z^{*h}d\zeta^{*h}=\int\limits_{z\in Z^*:e_z^*>0}\left(1-\frac{\gamma_z}{\gamma_0}\right)U_z^h d\zeta^h;$$

$$\int\limits_{z\in Z^*:e_z^*>0}\left(\frac{\gamma_z}{\gamma_0}-1\right)U_z^{*l}d\zeta^{*h}=\int\limits_{z\in Z^*:e_z^*>0}\left(\frac{\gamma_z}{\gamma_0}-1\right)U_z^l d\zeta^h;$$

$$\int\limits_{z\in Z^*:e_z^*>0}\left(U_z^{*h}-\frac{\gamma_z}{\gamma_0}U_z^{*l}\right)d\zeta^{*h}=\int\limits_{z\in Z^*:e_z^*>0}\left(U_z^h-\frac{\gamma_z}{\gamma_0}U_z^l\right)d\zeta^h;$$

$$\int\limits_{z\in Z^*}d\zeta^{*h}=\int\limits_{z\in Z^*}d\zeta^h;$$

$$\int\limits_{z\in Z^*:e_z^*=0}U_z^{*h}d\zeta^{*l}=\int\limits_{z\in Z^*:e_z^*=0}U_z^h d\zeta^l;$$

$$\int\limits_{z\in Z^*:e_z^*=0}\left(p_z^*-c^l\left(q_z^*\right)\right)d\zeta^{*l}=\int\limits_{z\in Z^*:e_z^*=0}\left(p_z-c^l\left(q_z\right)\right)d\zeta^l;$$

$$\int\limits_{z\in Z^*:e_z^*=0}d\zeta^{*l}=\int\limits_{z\in Z^*:e_z^*=0}d\zeta^l.$$

First, consider $Z^0\equiv\left\{z\in Z^*:e_z^*=0\right\}$. By inspecting the last three equations, we can conclude that it is sufficient to have one signal z_0 such that $e_{z_0}^*=0$ and

$$U_{z_0}^h=\int\limits_{z\in Z^*:e_z^*=0}U_z^{*h}d\zeta^{*l},$$

$$p_{z_0}-c^l\left(q_{z_0}\right)=\int\limits_{z\in Z^*:e_z^*=0}\left(p_z^*-c^l\left(q_z^*\right)\right)d\zeta^{*l},$$

$$\zeta_{z_0}^l=\int\limits_{z\in Z^*:e_z^*=0}d\zeta^{*l}$$

Consider now $Z^+ \equiv \{z \in Z^* : e_z^* > 0\}$. Divide the first six equations by $\int\limits_{z\in Z^*} d\zeta^{*h} > 0$. The left-hand side of the resulting system lies in the convex hull of

$$\mathbf{Z}^+ \equiv \left\{ \left(\varphi^h\left(e_z^*, q_z^*\right) + \frac{\gamma_z}{\gamma_0}\varphi^l\left(e_z^*, q_z^*\right); \frac{\gamma_z}{\gamma_0}; \left(1-\frac{\gamma_z}{\gamma_0}\right)U_z^{*h}; \left(\frac{\gamma_z}{\gamma_0}-1\right)U_z^{*l}; U_z^{*h} - \frac{\gamma_z}{\gamma_0}U_z^{*l} \right),\ z\in Z^+ \right\},$$

so it can be represented as a convex combination of points in $\mathbf{Z}^+$. This convex combination defines a probability distribution concentrated on a finite number of points in $\mathbf{Z}^+$, so there exists an optimal disclosure policy that puts positive probability on a finite number of points in $\mathbf{Z}^+$. By Caratheodory's theorem, one can choose such a distribution so that it puts positive probability on not more than 6 points in $\mathbf{Z}^+$. □

Before proving Propositions 3 and 4, let us prove the following technical lemma.

Lemma 5. *Let* $\Phi^i(e,x) \equiv \varphi^i\left(e, \Delta c^{-1}(x)\right)$. *Then:*

(i) *If* $\left(e, \Delta c^{-1}(x)\right)$ *is in the interior of* B, *then* $\Phi_e^h(e,x) = \Phi_e^l(e,x) = 0$, $\Phi_x^h(e,x) = \Phi_{xx}^h(e,x) = 0$, $\Phi_x^l(e,x) > 0$, $\Phi_{xx}^l(e,x) \le 0$;

(ii) *If* $\left(e, \Delta c^{-1}(x)\right)$ *is in the interior of* C, *then* $\Phi_e^i(e,x) = \Phi_x^i(e,x) = \Phi_{xx}^i(e,x) = 0$, $i = h, l$;

(iii) *If* $\left(e, \Delta c^{-1}(x)\right)$ *is in the interior of* D, *then* $\Phi_e^h(e,x) = \Phi_e^l(e,x) = 0$, $\Phi_x^h(e,x) < 0$, $\Phi_{xx}^h(e,x) \le 0$, $\Phi_x^l(e,x) = \Phi_{xx}^l(e,x) = 0$;

(iv) *If* $\left(e, \Delta c^{-1}(x)\right)$ *is in the interior of* E, *then* $\Phi_e^h(e,x) > 0$, $\Phi_e^l(e,x) = 0$, $\Phi_x^i(e,x) = 0$, $i = h, l$.

Proof. **(i)** If $\left(e, \Delta c^{-1}(x)\right)$ is in the interior of B, then

$$\Phi_e^i(e,x) = \varphi_e^i(e, \Delta c^{-1}(x)) = 0,\ i = h, l$$

By equations (5) and the fact that $\frac{d\left(\Delta c^{-1}(x)\right)}{dx} = \frac{1}{\Delta c_q(\Delta c^{-1}(x))}$,

$$\begin{aligned}\Phi_x^h(e,x) &= \varphi_q^h(e, \Delta c^{-1}(x))\tfrac{1}{\Delta c_q(\Delta c^{-1}(x))} = 0;\\ \Phi_x^l(e,x) &= \varphi_q^l(e, \Delta c^{-1}(x))\tfrac{1}{\Delta c_q(\Delta c^{-1}(x))} > 0\end{aligned}$$

and

$$\begin{aligned}&\Phi_{xx}^h(e,x) = 0;\\ &\Phi_{xx}^l(e,x) = \left(V_{qq}^l\left(\Delta c^{-1}(x)\right) - c_{qq}^l\left(\Delta c^{-1}(x)\right)\right)\left(\tfrac{1}{\Delta c_q(\Delta c^{-1}(x))}\right)^2\\ &-\left(V_q^l\left(\Delta c^{-1}(x)\right) - c_q^l\left(\Delta c^{-1}(x)\right)\right)\tfrac{\Delta c_{qq}\left(\Delta c^{-1}(x)\right)}{\left(\Delta c_q(\Delta c^{-1}(x))\right)^3}\\ &= \tfrac{V_q^l\left(\Delta c^{-1}(x)\right) - c_q^l\left(\Delta c^{-1}(x)\right)}{\left(\Delta c_q(\Delta c^{-1}(x))\right)^2}\left(\tfrac{V_{qq}^l\left(\Delta c^{-1}(x)\right) - c_{qq}^l\left(\Delta c^{-1}(x)\right)}{V_q^l\left(\Delta c^{-1}(x)\right) - c_q^l\left(\Delta c^{-1}(x)\right)} - \tfrac{\Delta c_{qq}\left(\Delta c^{-1}(x)\right)}{\Delta c_q\left(\Delta c^{-1}(x)\right)}\right) \le 0,\end{aligned}$$

where the inequality uses Assumption 1 and the fact that $\left(e, \Delta c^{-1}(x)\right)$ is in the interior of B, so $\Delta c^{-1}(x) < q_l^*$.

(ii) Follows directly from equations (5).

(iii) If $\left(e, \Delta c^{-1}(x)\right)$ is in the interior of D, then

$$\begin{aligned}
\Phi_e^i(e,x) &= \varphi_e^i(e, \Delta c^{-1}(x)) = 0,\ i = h, l; \\
\Phi_x^h(e,x) &= \varphi_q^h(e, \Delta c^{-1}(x)) \tfrac{1}{\Delta c_q(\Delta c^{-1}(x))} < 0; \\
\Phi_x^l(e,x) &= \varphi_q^l(e, \Delta c^{-1}(x)) \tfrac{1}{\Delta c_q(\Delta c^{-1}(x))} = 0
\end{aligned}$$

and

$$\begin{aligned}
&\Phi_{xx}^l(e,x) = 0; \\
&\Phi_{xx}^h(e,x) = \left(V_{qq}^h\left(\Delta c^{-1}(x)\right) - c_{qq}^h\left(\Delta c^{-1}(x)\right)\right)\left(\tfrac{1}{\Delta c_q(\Delta c^{-1}(x))}\right)^2 \\
&- \left(V_q^h\left(\Delta c^{-1}(x)\right) - c_q^h\left(\Delta c^{-1}(x)\right)\right)\tfrac{\Delta c_{qq}\left(\Delta c^{-1}(x)\right)}{\left(\Delta c_q(\Delta c^{-1}(x))\right)^3} \\
&= \tfrac{V_q^h\left(\Delta c^{-1}(x)\right) - c_q^h\left(\Delta c^{-1}(x)\right)}{\left(\Delta c_q(\Delta c^{-1}(x))\right)^2}\left(\tfrac{V_{qq}^h\left(\Delta c^{-1}(x)\right) - c_{qq}^h\left(\Delta c^{-1}(x)\right)}{V_q^h\left(\Delta c^{-1}(x)\right) - c_q^h\left(\Delta c^{-1}(x)\right)} - \tfrac{\Delta c_{qq}\left(\Delta c^{-1}(x)\right)}{\Delta c_q\left(\Delta c^{-1}(x)\right)}\right) \leq 0,
\end{aligned}$$

where the inequality uses Assumption 1 and the fact that $\left(e, \Delta c^{-1}(x)\right)$ is in the interior of D, so $\Delta c^{-1}(x) > q_h^*$.

(iv) If $\left(e, \Delta c^{-1}(x)\right)$ is in the interior of E, then, by equations (5),

$$\begin{aligned}
\Phi_e^h(e,x) &= \varphi_e^h\left(e, \Delta c^{-1}(x)\right) > 0; \\
\Phi_e^l(e,x) &= \varphi_e^l\left(e, \Delta c^{-1}(x)\right) = 0; \\
\Phi_x^i(e,x) &= \varphi_q^i\left(e, \Delta c^{-1}(x)\right) \tfrac{1}{\Delta c_q(\Delta c^{-1}(x))} = 0 \ i = h, l
\end{aligned}$$

□

Lemma 6. *If there exists a contract that is optimal given* e_0*, then there exists an optimal contract such that:*

(1) $e_z = 0 \Rightarrow q_z = q_l^*$;

(2) $\forall z \in Z,\ (e_z, q_z) \in B \cup C \cup D$.

Proof. Suppose that for some $z \in Z,\ e_z = 0,\ \zeta_z^l > 0$, and $q_z \neq q_l^*$. Then $(e_z, q_z) \in A \cup C \cup D$, so $U_z^l = p_z - c^l(q_z)$. Consider a contract where signal z is replaced by z' such that $e_{z'} = 0,\ \zeta_z^l = \zeta_{z'}^l,\ q_{z'} = q_l^*,\ p_{z'} = p_z - c^l(q_z) + c^l(q_l^*)$. The new contract is feasible and results in (weakly) higher social surplus in the low state than the original contract. So we can restrict attention to contracts such that condition (1) is satisfied.

Now suppose that the optimal contract is such that $\exists z \in Z:\ (e_z, q_z) \in E$ and condition (1) holds. Consider a contract where any such z is replaced by z' such that $e_{z'} = e_z, q_{z'} = \hat{q}_h(e_z), p_{z'} = p_z - c^h(q_z) + c^h(\hat{q}_h(e_z))$, so that $(e_{z'}, q_{z'}) \in D$. It is easy to verify that $\varphi^i(e_{z'}, q_{z'}) = \varphi^i(e_z, q_z), U_{z'}^i = U_z^i,\ i = h, l$. As a result, the new contract is feasible and results in the same value of the objective function.

Similarly, suppose that the optimal contract is such that $\exists z \in Z:\ (e_z, q_z) \in A$ and condition (1) holds. Consider a contract where any such z is replaced by z' such that $e_{z'} = e_z, q_{z'} = \hat{q}_l(e_z), p_{z'} =$

$p_z - c^l(q_z) + c^l(\hat{q}_l(e_z))$, so that $(e_{z'}, q_{z'}) \in B$. It is easy to verify that $\varphi^i(e_{z'}, q_{z'}) = \varphi^i(e_z, q_z), U^i_{z'} = U^i_z$, $i = h, l$. As a result, the new contract is feasible and results in the same value of the objective function.

□

Proof of Proposition 3. Consider any contract G that achieves $e = e_0$. For this contract, (IC_h) and (INV) imply

$$\psi'(e_0) \geq \sum_{z \in Z} \Delta c(q_z) \zeta_z^l, \tag{A1}$$

whereas (IC_l) and (INV) imply

$$\psi'(e_0) \leq \sum_{z \in Z} \Delta c(q_z) \zeta_z^h \tag{A2}$$

By the definitions of $e_{\min}$ and $e_{\max}$, $e_0 \in [e_{\min}, e_{\max}]$ implies that $\Delta c(\hat{q}_l(e_0)) \leq \psi'(e_0)$ and either $\psi'(e_0) \leq \Delta c(\hat{q}_h(e_0))$, or $\hat{q}_h(e_0)$ does not exist. Therefore $\left(e_0, \Delta c^{-1}\left(\psi'\left(e_0\right)\right)\right) \in B \cup C \cup D$.

Consider a contract G^* such that $Z = \{z^*\}$, $q_{z^*} = \Delta c^{-1}\left(\psi'\left(e_0\right)\right)$, $p_{z^*} = c^l(q_{z^*})$. This contract is feasible and achieves investment $e = e_0$. Let us show that it results in weakly higher expected surplus than G. By Lemma 6, without loss of generality G is such that $\forall z \in Z,\ (e_z, q_z) \in B \cup C \cup D$. The expected surplus resulting from G (not taking the investment cost into account) is

$$\begin{aligned}
e_0 \sum_{z \in Z} \varphi^h\left(e_z, q_z\right) \zeta_z^h + (1 - e_0) \sum_{z \in Z} \varphi^l\left(e_z, q_z\right) \zeta_z^h &= e_0 \sum_{z \in Z} \Phi^h\left(e_z, \Delta c\left(q_z\right)\right) \zeta_z^h + \\
(1 - e_0) \sum_{z \in Z} \Phi^l\left(e_z, \Delta c\left(q_z\right)\right) \zeta_z^l &= e_0 \sum_{z \in Z} \Phi^h\left(0, \Delta c\left(q_z\right)\right) \zeta_z^h + (1 - e_0) \sum_{z \in Z} \Phi^l\left(1, \Delta c\left(q_z\right)\right) \zeta_z^l \\
&\leq e_0 \Phi^h\left(0, \sum_{z \in Z} \Delta c\left(q_z\right) \zeta_z^h\right) + (1 - e_0) \Phi^l\left(1, \sum_{z \in Z} \Delta c\left(q_z\right) \zeta_z^l\right) \\
&\leq e_0 \Phi^h\left(0, \psi'\left(e_0\right)\right) + (1 - e_0) \Phi^l\left(1, \psi'\left(e_0\right)\right) = e_0 \Phi^h\left(e_0, \psi'\left(e_0\right)\right) + (1 - e_0) \Phi^l\left(e_0, \psi'\left(e_0\right)\right) \\
&= e_0 \Phi^h\left(e_0, \Delta c\left(q_{z^*}\right)\right) + (1 - e_0) \Phi^l\left(e_0, \Delta c\left(q_{z^*}\right)\right) = e_0 \varphi^h\left(e_0, q_{z^*}\right) + (1 - e_0) \varphi^l\left(e_0, q_{z^*}\right)
\end{aligned} \tag{A3}$$

The second and the third equalities follow from the fact that $\forall z \in Z,\ (e_z, q_z) \in B \cup C \cup D$ and from Lemma 5. The first inequality follows from the concavity of Φ^h and Φ^l in the second argument proven in Lemma 5. The second inequality follows from $(A1)$, $(A2)$ and the fact that Φ^h is weakly decreasing and Φ^l is weakly increasing in the second argument by Lemma 5. □

Lemma 7. *Suppose $e_0 > e_{\max}$ and e_0 can be achieved by a partial-disclosure contract. If there exists an optimal contract given e_0, then there exists a contract $G^* = (Z, p_z, q_z, \zeta_z^h, \zeta_z^l)$ that is optimal given e_0, satisfies conditions (1) and (2) of Lemma 6 and is such that $\forall z \in Z,\ p_z = c^l\left(q_z\right)$.*

Proof. Let $z_0 \in Z$ be the signal such that $e_{z_0} = 0$, $q_{z_0} = q_l^*$. Lemma 6 implies that without loss of generality, z_0 is the only signal with $e_z = 0$ and that we can restrict attention to contracts that contain only signals such that $(e_z, q_z) \in B \cup C \cup D$. The Lagrangian for Problem 1a with these additional constraints is

$$\begin{aligned}
L = {} & e_0 \sum_{z\in Z} \varphi^h(e_z,q_z)\zeta_z^h + (1-e_0)\left(\sum_{z\in Z: e_z>0} \tfrac{\gamma_z}{\gamma_0}\varphi^l(e_z,q_z)\zeta_z^h + \zeta_0^l\left(V^l(q_l^*) - c^l(q_l^*)\right)\right) \\
& -\psi(e_0) - \lambda^h\left(-\sum_{z\in Z:e_z>0}\left(1-\tfrac{\gamma_z}{\gamma_0}\right)\left(p_z - c^h(q_z)\right)\zeta_z^h + \zeta_0^l\left(p_{z_0} - c^h(q_l^*)\right)\right) \\
& -\lambda^l\left(-\sum_{z\in Z:e_z>0}\left(\tfrac{\gamma_z}{\gamma_0} - 1\right)\left(p_z - c^l(q_z)\right)\zeta_z^h - \zeta_0^l\left(p_{z_0} - c^l(q_l^*)\right)\right) \\
& -\mu\left(\sum_{z\in Z:e_z>0}\left[\left(p_z - c^h(q_z)\right) - \tfrac{\gamma_z}{\gamma_0}\left(p_z - c^l(q_z)\right)\right]\zeta_z^h - \zeta_0^l\left(p_{z_0} - c^l(q_l^*)\right) - \psi'(e_0)\right) \\
& - \textstyle\sum_{z\in Z}\nu_z^h\left(q_z - \hat{q}_h(e_z)\right) - \sum_{z\in Z}\nu_z^l\left(\hat{q}_l(e_z) - q_z\right) - \xi\left(\displaystyle\sum_{z\in Z}\zeta_z^h - 1\right) - \delta\left(\sum_{z\in Z:e_z>0}\zeta_z^h\tfrac{\gamma_z}{\gamma_0} + \zeta_0^l - 1\right),
\end{aligned}$$

where $\lambda_h, \lambda^l, \nu_z^h, \nu_z^l \geq 0$.

Consider a contract G that achieves e_0 and satisfies conditions (1) and (2) of Lemma 6. The proof will proceed through the following steps:

(i) G is such that there is at least one $z \in Z$ such that $(e_z, q_z) \in D$.

(ii) G satisfies (IC_l) with equality.

(iii) There exists a contract G^* that is optimal given e_0 and satisfies conditions (1), (2) of Lemma 6 and is such that $\forall z \in Z,\ p_z = c^l(q_z)$.

Proof of (i): Suppose not: i.e. $\forall z \in Z, q_z \leq q_h^*$. Then $(A2)$ implies

$$\psi'(e_0) \leq \Delta c(q_h^*)$$

This implies that, if $\hat{q}_h(e_0)$ is well-defined, then $\psi'(e_0) \leq \Delta c(\hat{q}_h(e_0))$. This is a contradiction with $e_0 > e_{\max}$.

Proof of (ii): For any $z \in Z$, the first-order condition for p_z implies

$$-\lambda^h + \lambda^l + \mu = 0 \tag{A4}$$

The first-order condition for q_z is:

$$e_0\frac{\partial\varphi^h(e_z,q_z)}{\partial q_z} + (1-e_0)\frac{\partial\varphi^l(e_z,q_z)}{\partial q_z} - \lambda^h c_q^h(q_z)\left(1-\frac{\gamma_z}{\gamma_0}\right) + \lambda^l c_q^l(q_z)\left(1-\frac{\gamma_z}{\gamma_0}\right) \tag{A5}$$
$$-\mu\left(-c_q^h(q_z) + \frac{\gamma_z}{\gamma_0}c_q^l(q_z)\right)\begin{cases} = 0, & \text{if } q_z \in \left(\hat{q}_l(e_z), \hat{q}_h(e_z)\right); \\ \geq 0, & \text{if } q_z = \hat{q}_h(e_z); \\ \leq 0, & \text{if } q_z = \hat{q}_l(e_z) \end{cases}$$

Substituting $(A4)$ into $(A5)$ results in

$$e_0 \frac{\partial \varphi^h(e_z,q_z)}{\partial q_z} + (1-e_0)\frac{\partial \varphi^l(e_z,q_z)}{\partial q_z} + \Delta c'(q_z)\left(\lambda^l - \lambda^h \frac{\gamma_z}{\gamma_0}\right)\begin{cases} = 0, & \text{if } q_z \in (\hat{q}_l(e_z), \hat{q}_h(e_z)); \\ \geq 0, & \text{if } q_z = \hat{q}_h(e_z); \\ \leq 0, & \text{if } q_z = \hat{q}_l(e_z) \end{cases} \quad (A6)$$

Suppose $z' \in Z$ is such that $(e_{z'}, q_{z'}) \in D$ (such a z' exists by (i)). Then, by equations (5), $\frac{\partial \varphi^h(e_{z'},q_{z'})}{\partial q_{z'}} < 0$, $\frac{\partial \varphi^l(e_{z'},q_{z'})}{\partial q_{z'}} = 0$. Therefore $(A6)$ implies that $\lambda^l > 0$ and (IC_l) holds with equality.

Proof of (iii) Suppose we have an optimal contract $G \equiv \left(Z,\ p_z,\ q_z,\ \zeta_z^h,\ \zeta_z^l\right)$ such that (IC_l) holds with equality and conditions (1) and (2) of Lemma 6 are satisfied. Now consider a contract $\tilde{G} \equiv \left(Z,\ \tilde{p}_z,\ \tilde{q}_z,\ \zeta_z^h,\ \zeta_z^l\right)$, where

$$(\tilde{p}_z, \tilde{q}_z) = \left(c^l(q_z), q_z\right)$$

The contract $\tilde{G}$ satisfies all the conditions required by the lemma. By construction, for every $z \in Z$, e_z, φ_z^h and φ_z^l are the same as in G. It remains to check that $\tilde{G}$ is feasible and that it induces the same investment e_0 as G. For $\tilde{G}$, constraint (IC_l) holds trivially, because $\tilde{p}_z - c^l(\tilde{q}_z) \equiv 0$; constraint (IC_h) takes the form

$$\sum_{z \in Z: e_z > 0}\left(1 - \frac{\gamma_z}{\gamma_0}\right)\Delta c(q_z)\zeta_z^h - \left(1 - \sum_{z \in Z: e_z > 0}\frac{\gamma_z}{\gamma_0}\zeta_z^h\right)\Delta c(q_l^*) \geq 0,$$

which is just the sum of (IC_l) and (IC_h) for G. It remains to check that $\tilde{G}$ induces investment e_0. Suppose that it induces investment e. Constraint (INV) for $\tilde{G}$ takes the form

$$\begin{aligned} &\psi'(e) = \sum_{z \in Z: e_z > 0}\left[\tilde{p}_z - c^h(\tilde{q}_z) - \tfrac{\gamma_z}{\gamma_0}\left(\tilde{p}_z - c^l(\tilde{q}_z)\right)\right]\zeta_z^h - \left(1 - \sum_{z \in Z: e_z > 0}\tfrac{\gamma_z}{\gamma_0}\zeta_z^h\right)\left(\tilde{p}_z - c^l(q_l^*)\right) \\ &= \sum_{z \in Z: e_z > 0}\Delta c(q_z)\zeta_z^h = \sum_{z \in Z: e_z > 0}\left(p_z - c^h(q_z) - \tfrac{\gamma_z}{\gamma_0}\left(p_z - c^l(q_z)\right)\right)\zeta_z^h \\ &- \left(1 - \sum_{z \in Z: e_z > 0}\tfrac{\gamma_z}{\gamma_0}\zeta_z^h\right)\left(p_{z_0} - c^l(q_l^*)\right) = \psi'(e_0), \end{aligned}$$

so $e = e_0$. The fourth equality follows from the fact that (IC_l) holds with equality for G. So $\tilde{G}$ is optimal and satisfies the required conditions. □

Proof of Proposition 4. By Lemma 7, without loss of generality we will restrict attention to contracts that satisfy conditions (1) and (2) of Lemma 6 and such that $p_z = c^l(q_z)$. This immediately implies (II).[14]

The proof will proceed through the following steps:

(a) If $q_z \geq q_h^*$, then without loss of generality $q_z = \hat{q}_h(e_z)$.

[14]Assumption 3 guarantees that if e_0 is achievable, then an optimal contract given e_0 exists. Intuitively, this assumption implies that it is possible to bound the default quantities q_z from above and thus compactify the feasible set of Problem 1a. The formal proof is available upon request.

(b) Any optimal contract that satisfies (a) must include at least one z such that $q_z < q_h^*$. For such z, either $(e_z, q_z) = (0, q_l^*)$ or $(e_z, q_z) \in B$.

(c) If $(e_z, q_z) \in B$ and $(e_{z'}, q_{z'}) \in D$, then $e_z < e_{z'}$.

(d) Any optimal contract that satisfies (a)-(c) must include at least two z such that $q_z \geq q_h^*$.

(e) There exists an optimal contract where $|Z|$ is either 3 or 4 (condition (I)).

Conditions (a)-(d) imply condition (III).

Proof of (a): Suppose that for some $z \in Z$, $q_z \geq q_h^*$ and $q_z < \hat{q}_h(e_z)$. Then a contract where z is replaced by z' such that $(q_{z'}, p_{z'}) = (q_z, p_z)$, $e_{z'} = \hat{q}_h^{-1}(q_z)$, $\zeta_{z'}^h = \zeta_z^h$ satisfies all the constraints and results in the same value of the objective function as the original contract.

Proof of (b): Suppose that an optimal contract that satisfies (a) is such that $\forall z \in Z,\ q_z \geq q_h^*$, which means that $\forall z \in Z,\ q_z = \hat{q}_h(e_z)$. Let $Z = \{z_1, ..., z_n\}$ and $q_{z_1} \geq q_{z_2} \geq ... \geq q_{z_n}$ with at least one strict inequality. By (a), this implies that $e_{z_1} \leq e_{z_2} \leq ... \leq e_{z_n}$ and $\gamma_{z_1} \geq \gamma_{z_2} \geq ... \geq \gamma_{z_n}$. Constraints (PR_h) and (PR_l) of Problem 1a imply

$$\sum_{i=1}^{n} \zeta_{z_i}^h \left(1 - \frac{\gamma_{z_i}}{\gamma_0}\right) = 0 \tag{A7}$$

Expressing $\zeta_{z_1}^h \left(1 - \frac{\gamma_{z_1}}{\gamma_0}\right)$ from $(A7)$, using the fact that $p_z = c^l(q_z)$ and substituting into (IC_h) yields

$$\sum_{j=2}^{n} \zeta_{z_j}^h \left(1 - \frac{\gamma_{z_j}}{\gamma_0}\right) \left(\Delta c\left(q_{z_j}\right) - \Delta c\left(q_{z_1}\right)\right) \geq 0$$

Because $\Delta c\left(q_{z_j}\right) - \Delta c\left(q_{z_1}\right) \leq 0,\ j \geq 2$, and $\gamma_{z_2} \geq ... \geq \gamma_{z_n}$, we have $1 - \frac{\gamma_{z_2}}{\gamma_0} \leq 0$ and consequently $1 - \frac{\gamma_{z_1}}{\gamma_0} \leq 0$.

Expressing $\zeta_{z_2}^h \left(1 - \frac{\gamma_{z_2}}{\gamma_0}\right)$ from $(A7)$ and substituting it into (IC_h) yields

$$\sum_{j \neq 2} \zeta_{z_j}^h \left(1 - \frac{\gamma_{z_j}}{\gamma_0}\right) \left(\Delta c\left(q_{z_j}\right) - \Delta c\left(q_{z_2}\right)\right) \geq 0$$

Because $1 - \frac{\gamma_{z_1}}{\gamma_0} \leq 0,\ \Delta c\left(q_{z_1}\right) - \Delta c\left(q_{z_2}\right) \geq 0$ and $\Delta c\left(q_{z_j}\right) - \Delta c\left(q_{z_2}\right) \leq 0,\ j \geq 3,$ we have $1 - \frac{\gamma_{z_3}}{\gamma_0} \leq 0$.

Repeating this for every $j = 1...n$, we conclude that

$$1 - \frac{\gamma_{z_j}}{\gamma_0} \leq 0,\ j = 1...n$$

which together with (PR_l) implies $1 - \frac{\gamma_{z_j}}{\gamma_0} = 0,\ j = 1...n$. So $e_z \equiv e_0$ and G is a no-disclosure contract, contradicting the fact that $e_0 > e_{\max}$. Therefore there must exist a signal $z \in Z$ such that $q_z < q_h^*$.

Now suppose that $q_z \in (q_l^*, q_h^*)$, so that $(e_z, q_z) \in C$. Then without loss of generality we can suppose that $e_z = 1$, so that $\gamma_z = 0$. Indeed, it is easy to check that if $e_z < 1$, then increasing e_z to 1 will result in a contract that is feasible and achieves the same investment and welfare.

The first-order conditions for p_z and q_z are given by $(A4)$ and $(A5)$, respectively, and imply $(A6)$. Noting that $\frac{\partial \varphi^h(e_z,q_z)}{\partial q_z} = \frac{\partial \varphi^l(e_z,q_z)}{\partial q_z} = 0$ when $(e_z, q_z) \in C$ (equations (5)) results in

$$\lambda^l = \lambda^h \frac{\gamma_z}{\gamma_0} \qquad (A8)$$

Because $\gamma_z = 0$ and $\lambda_l \geq 0$, $(A8)$ implies $\lambda_l = 0$. However, the proof of part (ii) of Lemma 7 establishes that $\lambda_l > 0$ – contradiction.

Therefore, it must be the case that if $q_z < q_h^*$, then $q_z \leq q_l^*$. Part (b) now follows from conditions (1) and (2) of Lemma 6.

Proof of (c): Suppose that $(e_z, q_z) \in B$ and $(e_{z'}, q_{z'}) \in D$. Condition $(A6)$, together with equations (5), implies that

$$\lambda_l - \lambda_h \frac{\gamma_z}{\gamma_0} < 0 < \lambda_l - \lambda_h \frac{\gamma_{z'}}{\gamma_0}$$

Therefore, $\gamma_{z'} < \gamma_z$ and $e_z < e_{z'}$.

Proof of (d): Part (i) of Lemma 7 establishes that there must exist at least one z such that $(e_z, q_z) \in D$. Constraint (INV) implies that for at least one such z must satisfy $\Delta c(q_z) > \psi'(e_0)$. This, together with thee fact that $e_0 > e_{\max}$, implies that $\Delta c(q_z) > \Delta c(\hat{q}_h(e_0))$. By (a), $q_z = \hat{q}_h(e_z)$, so $e_z < e_0$.

However, there must also exist at least one $z \in Z$ such that $q_z \geq q_h^*$ and $e_z \geq e_0$. Indeed, $(A7)$ implies that there is at least one $z' \in Z$ such that $\gamma_{z'} \geq \gamma_0$, so that $e_{z'} \geq e_0$. Because there exists $z \in Z$ such that $(e_z, q_z) \in D$ and $e_z < e_0$, parts (b) and (c) imply that $(e_{z'}, q_{z'}) \in D$.

Proof of (e): Parts (b) and (d) immediately imply that in any optimal contract $|Z| \geq 3$. Let us prove that without loss of generality $|Z| \leq 4$. By Lemma 4, there exists an optimal contract that puts positive probability on at most one signal in $Z^0 \equiv \{z \in Z^* : e_z^* = 0\}$ and at most 6 signals in $Z^+ \equiv \{z \in Z^* : e_z^* > 0\}$. Let this optimal contract be $G^* = \left(Z^*,\ p_z^*,\ q_z^*,\ \zeta_z^{*h},\ \zeta_z^{*l}\right)$. Let $Z^* = \{z_0, ..., z_6\}$, $Z^+ = \{z_1, ..., z_6\}$ and $Z^0 = \{z_0\}$. Then $\left(\zeta_z^{*h},\ \zeta_z^{*l}\right)$ solve

$$\max_{\zeta_z^h,\ \zeta_z^l} \sum_{z \in Z} \varphi^h\left(e_z^*, q_z^*\right) \zeta_z^h$$

$$\text{s.t. } (PR_h)\,;\ (PR_l)\,;\ (IC_h)\,;\ (INV)\,;$$

$$\zeta_z^i \geq 0,\ \forall z \in Z,\ i = h, l$$

Constraint (IC_l) can be ignored, because $p_z^* - c^l(q_z^*) \equiv 0$. This is a linear programming problem, which has the following canonical form:

$$\max_{\alpha} c\alpha$$
$$\text{s.t. } A\alpha = b, \alpha \geq 0,$$

where

$$A = \begin{pmatrix} 1 & ... & 1 & 1 & 0 \\ \gamma_{z_1}^* & ... & \gamma_{z_6}^* & \gamma_0 & 0 \\ \left(1 - \frac{\gamma_{z_1}^*}{\gamma_0}\right)\Delta c\left(q_{z_1}^*\right) & ... & \left(1 - \frac{\gamma_{z_6}^*}{\gamma_0}\right)\Delta c\left(q_{z_6}^*\right) & -\Delta c\left(q_l^*\right) & -1 \\ \Delta c\left(q_{z_1}^*\right) & ... & \Delta c\left(q_{z_6}^*\right) & 0 & 0 \end{pmatrix}$$

$$b = \left(1, \gamma_0, 0, \psi'(e_0)\right)^T$$
$$c = \left(\varphi^h\left(e_{z_1}^*, q_{z_1}^*\right) + \frac{\gamma_{z_1}^*}{\gamma_0}\varphi^l(e_{z_1}^*, q_{z_1}^*), ..., \varphi^h\left(e_{z_6}^*, q_{z_6}^*\right) + \frac{\gamma_{z_6}^*}{\gamma_0}\varphi^l(e_{z_6}^*, q_{z_6}^*), 0, 0\right)$$
$$\alpha = \left(\zeta_{z_1}^h, ..., \zeta_{z_6}^h, \zeta_{z_0}^l, s\right)^T$$

and $s \geq 0$ is a slack variable associated with (IC_h). If a solution to this problem exists, then there exists an extreme point of the feasible region that is optimal. Because there are 4 equality constraints, at most 4 coordinates of α are strictly positive at any extreme point. So without loss of generality, $|Z| \leq 4$.

□

Before proving Lemma 3, we will prove the following lemma.

Lemma 8. *If $e_0 > e_{\max}$ is achievable, then it is achievable with a contract that satisfies properties $(I)-(III)$ of Proposition 4 and puts positive probability on three signals.*

Proof. If $e_0 > e_{\max}$ is achievable, then, by Proposition 4, there exists an optimal contract that achieves e_0, puts positive probability on not more than 4 signals, and satisfies properties $(I) - (III)$. We will present the proof only for the case where the optimal contract includes a signal z_0 such that $e_{z_0} = 0$ (the proof for the other case is similar). Let

$$Z^* = \left((z_0, z_1, z_2, z_3), \ (p_z^*, \ q_z^*, \ e_z^*)_{z \in Z^*}, \ \zeta_{z_1}^{*h}, \zeta_{z_2}^{*h}, \zeta_{z_3}^{*h}, \ \zeta_{z_0}^{*l}\right)$$

denote the optimal contract, where (z_1, z_2, z_3) are the signals for which $e^*_z > 0$, and z_0 is the signal for which $e_z = 0$. Let $q^*_{z_1} \geq q^*_{z_2} \geq q^*_{z_3}$. Let us show that there exists $\left(\zeta^h_{z_1}, \zeta^h_{z_2}, \zeta^h_{z_3},\ \zeta^l_{z_0}\right)$ such that the contract $\left(Z^*,\ (p^*_z,\ q^*_z,\ e^*_z)_{z\in Z^*},\ \zeta^h_{z_1}, \zeta^h_{z_2}, \zeta^h_{z_3},\ \zeta^l_{z_0}\right)$ is feasible and achieves e_0, and at most three elements of the set $\left\{\zeta^h_{z_1}, \zeta^h_{z_2}, \zeta^h_{z_3}, \zeta^l_{z_0}\right\}$ are positive.

Because $\left(Z^*,\ (p^*_z,\ q^*_z,\ e^*_z)_{z\in Z^*},\ \zeta^{*h}_{z_1}, \zeta^{*h}_{z_2}, \zeta^{*h}_{z_3},\ \zeta^{*l}_{z_0}\right)$ satisfies the conditions of Lemma 7, the constraints (PR_h), (IC_h),and (INV) for the contract $\left(Z^*,\ (p^*_z,\ q^*_z,\ e^*_z)_{z\in Z^*},\ \zeta^h_{z_1}, \zeta^h_{z_2}, \zeta^h_{z_3},\ \zeta^l_{z_0}\right)$ take the form

$$(PR_h) \qquad \sum_{i=1}^{3} \zeta^h_{z_i} = 1;$$

$$(PR_l) \qquad \sum_{i=1}^{3} \zeta^h_{z_i} \frac{\gamma^*_{z_i}}{\gamma_0} + \zeta^l_{z_0} = 1;$$

$$(IC_h) \qquad \sum_{i=1}^{3} \zeta^h_{z_i} \left(1 - \frac{\gamma^*_{z_i}}{\gamma_0}\right) \Delta c\left(q^*_{z_i}\right) - \zeta^l_{z_0} \Delta c(q^*_l) \geq 0;$$

$$(INV) \qquad \sum_{i=1}^{3} \zeta^h_{z_i} \Delta c\left(q^*_{z_i}\right) = \psi'\left(e_0\right).$$

Because the optimal contract is feasible, we know that the system of equations and inequalities above has a nonnegative solution.

Expressing $\zeta^h_{z_1}$, $\zeta^h_{z_3}$ and $\zeta^l_{z_0}$ from (PR_h), (PR_l) and (INV) in terms of $\zeta^h_{z_2}$ and substituting into the nonnegativity constraints on $\zeta^h_{z_1}$, $\zeta^h_{z_3}$ and $\zeta^l_{z_0}$ results in the inequalities of the form

$$\zeta^h_{z_2} \in [A, B],$$

where

$$A = \begin{cases} 0, \text{ if } \left(\Delta c\left(q^*_{z_1}\right) - \Delta c\left(q^*_{z_2}\right)\right)\left(\gamma^*_{z_2} - \gamma^*_{z_3}\right) + \left(\Delta c\left(q^*_{z_2}\right) - \Delta c\left(q^*_{z_3}\right)\right)\left(\gamma^*_{z_2} - \gamma^*_{z_1}\right) \geq 0; \\ \frac{\left(\Delta c\left(q^*_{z_1}\right) - \psi'(e_0)\right)\left(\gamma_0 - \gamma^*_{z_3}\right) + \left(\psi'(e_0) - \Delta c\left(q^*_{z_3}\right)\right)\left(\gamma_0 - \gamma^*_{z_1}\right)}{\left(\Delta c\left(q^*_{z_1}\right) - \Delta c\left(q^*_{z_2}\right)\right)\left(\gamma^*_{z_2} - \gamma^*_{z_3}\right) + \left(\Delta c\left(q^*_{z_2}\right) - \Delta c\left(q^*_{z_3}\right)\right)\left(\gamma^*_{z_2} - \gamma^*_{z_1}\right)}, \\ \text{if } \left(\Delta c\left(q^*_{z_1}\right) - \Delta c\left(q^*_{z_2}\right)\right)\left(\gamma^*_{z_2} - \gamma^*_{z_3}\right) + \left(\Delta c\left(q^*_{z_2}\right) - \Delta c\left(q^*_{z_3}\right)\right)\left(\gamma^*_{z_2} - \gamma^*_{z_1}\right) < 0; \end{cases}$$

$$B = \begin{cases} \min\left\{ \frac{\psi'(e_0) - \Delta c\left(q^*_{z_3}\right)}{\Delta c\left(q^*_{z_2}\right) - \Delta c\left(q^*_{z_3}\right)}, \frac{-\psi'(e_0) + \Delta c\left(q^*_{z_1}\right)}{\Delta c\left(q^*_{z_1}\right) - \Delta c\left(q^*_{z_2}\right)}, \frac{\left(\Delta c\left(q^*_{z_1}\right) - \psi'(e_0)\right)\left(\gamma_0 - \gamma^*_{z_3}\right) + \left(\psi'(e_0) - \Delta c\left(q^*_{z_3}\right)\right)\left(\gamma_0 - \gamma^*_{z_1}\right)}{\left(\Delta c\left(q^*_{z_1}\right) - \Delta c\left(q^*_{z_2}\right)\right)\left(\gamma^*_{z_2} - \gamma^*_{z_3}\right) + \left(\Delta c\left(q^*_{z_2}\right) - \Delta c\left(q^*_{z_3}\right)\right)\left(\gamma^*_{z_2} - \gamma^*_{z_1}\right)} \right\}, \\ \text{if } \left(\Delta c\left(q^*_{z_1}\right) - \Delta c\left(q^*_{z_2}\right)\right)\left(\gamma^*_{z_2} - \gamma^*_{z_3}\right) + \left(\Delta c\left(q^*_{z_2}\right) - \Delta c\left(q^*_{z_3}\right)\right)\left(\gamma^*_{z_2} - \gamma^*_{z_1}\right) > 0; \\ \min\left\{ \frac{\psi'(e_0) - \Delta c\left(q^*_{z_3}\right)}{\Delta c\left(q^*_{z_2}\right) - \Delta c\left(q^*_{z_3}\right)}, \frac{-\psi'(e_0) + \Delta c\left(q^*_{z_1}\right)}{\Delta c\left(q^*_{z_1}\right) - \Delta c\left(q^*_{z_2}\right)}, \right\}, \\ \text{if } \left(\Delta c\left(q^*_{z_1}\right) - \Delta c\left(q^*_{z_2}\right)\right)\left(\gamma^*_{z_2} - \gamma^*_{z_3}\right) + \left(\Delta c\left(q^*_{z_2}\right) - \Delta c\left(q^*_{z_3}\right)\right)\left(\gamma^*_{z_2} - \gamma^*_{z_1}\right) \leq 0. \end{cases}$$

The fact that the original optimal contract is feasible implies that $\zeta^{*h}_{z_2} \in [A, B]$ and therefore the interval $[A, B]$ is nonempty. If $\zeta^h_{z_2} \in \{A, B\}$, then at least one of the probabilities $\zeta^h_{z_1}, \zeta^h_{z_3}, \zeta^l_{z_0}$ is zero.

We can now substitute the expressions for $\zeta^h_{z_1}$, $\zeta^h_{z_3}$ and $\zeta^l_{z_0}$ into (IC_h). The result is a linear inequality in one variable, $\zeta^h_{z_2}$. We know that this inequality is satisfied by $\zeta^{*h}_{z_2}$, so by linearity it is satisfied by at least one of the points in $\{A, B\}$. Therefore it is possible to find $\left(\zeta^h_{z_1}, \zeta^h_{z_2}, \zeta^h_{z_3}, \zeta^l_{z_0}\right)$ such that $\left((p^*_z,\ q^*_z,\ e^*_z)_{z\in Z^*},\ \zeta^h_{z_1}, \zeta^h_{z_2}, \zeta^h_{z_3}, \zeta^l_{z_0}\right)$ satisfies (IC_h), (PR_h), (PR_l) and (INV) and achieves e_0, and at most three elements of the set $\left\{\zeta^h_{z_1}, \zeta^h_{z_2}, \zeta^h_{z_3}, \zeta^l_{z_0}\right\}$ are positive.

The fact that the original optimal contract satisfies conditions $(I)-(III)$ of Proposition 4 immediately implies that the new contract satisfies conditions (I), (II) and parts $(ii)-(iv)$ of condition (III). Part (i) of condition (III) for the new contract follows from parts **(a)-(d)** of the proof of Proposition 4.

□

Proof of Lemma 3. Proof of Part (i): First, let us prove that if the set of achievable investment levels e_0 such that $e_0 > e_{\max}$ is nonempty, then Condition (a) has to hold. Suppose that $e_0 > e_{\max}$ is achievable. Then, by Lemma 8, there exists a contract that achieves e_0, puts positive probability on three signals, and satisfies properties $(I)-(III)$ of Proposition 4. There are two possible cases:

(1) There are two signals, z_1 and z_2, such that $e_{z_i} > 0,\ i = 1, 2$, and one signal z_0 such that $e_{z_0} = 0$. Let $q_{z_1} > q_{z_2}$. Then Proposition 4 and constraint (IC_h) imply that $\gamma_{z_1} > \gamma_0 > \gamma_{z_2}$.

(2) There are three signals, z_1, z_2 and z_3, such that $e_{z_i} > 0,\ i = 1, 2, 3$. Let $q_{z_1} > q_{z_2} > q_{z_3}$. Then Proposition 4 and constraint (IC_h) imply that $\gamma_{z_3} > \gamma_{z_1} > \gamma_0 > \gamma_{z_2}$.

Consider case (1) first. The probabilities $\zeta^h_{z_i},\ i = 1, 2$, can be expressed from (PR_h) and (INV) as follows:

$$\zeta^h_{z_i} = \left|\frac{\psi'(e_0) - \Delta c\left(q_{z_{3-i}}\right)}{\Delta c(q_{z_1}) - \Delta c(q_{z_2})}\right|,\ i = 1, 2 \tag{A9}$$

The contract has to satisfy the constraint (IC_h), which takes the form:

$$(\psi'(e_0) - \Delta c(q_{z_2}))(\gamma_0 - \gamma_{z_1})(\Delta c(q_{z_1}) - \Delta c(q^*_l)) + (\Delta c(q_{z_1}) - \psi'(e_0))(\gamma_0 - \gamma_{z_2})(\Delta c(q_{z_2}) - \Delta c(q^*_l)) \geq 0 \tag{A10}$$

Because $\Delta c(q_{z_1}) > \psi'(e_0) > \Delta c(q_{z_2}) > \Delta c(q^*_l)$ and $\gamma_{z_1} > \gamma_0 > \gamma_{z_2}$, condition $(A10)$ implies

$$(\psi'(e_0) - \Delta c(q_{z_2}))(\gamma_0 - \gamma_{z_1}) + (\Delta c(q_{z_1}) - \psi'(e_0))(\gamma_0 - \gamma_{z_2}) \geq 0 \tag{A11}$$

Therefore $(A10)$ implies

$$(\psi'(e_0) - \Delta c(q_{z_2}))(\gamma_0 - \gamma_{z_1})\Delta c(q_{z_1}) + (\Delta c(q_{z_1}) - \psi'(e_0))(\gamma_0 - \gamma_{z_2})\Delta c(q_{z_2}) \geq 0 \tag{A12}$$

Now consider case (2). Expressing the probabilities $\zeta_{z_i}^h,\ i = 1, 2, 3$ from (PR_h), (PR_l) and (INV) and substituting into (IC_h) results in

$$\left(\psi'\left(e_0\right) - \Delta c\left(q_{z_2}\right)\right)\left(\gamma_0 - \gamma_{z_1}\right)\left(\Delta c\left(q_{z_1}\right) - \Delta c(q_{z_3})\right) \\ + \left(\Delta c\left(q_{z_1}\right) - \psi'\left(e_0\right)\right)\left(\gamma_0 - \gamma_{z_2}\right)\left(\Delta c\left(q_{z_2}\right) - \Delta c(q_{z_3})\right)\frac{\gamma_{z_3} - \gamma_{z_1}}{\gamma_{z_3} - \gamma_{z_2}} \geq 0 \quad (A13)$$

The derivative of the left-hand side of $(A13)$ with respect to γ_3 is

$$\left(\Delta c\left(q_{z_1}\right) - \psi'\left(e_0\right)\right)\left(\gamma_0 - \gamma_{z_2}\right)\left(\Delta c\left(q_{z_2}\right) - \Delta c(q_{z_3})\right)\frac{\gamma_{z_1} - \gamma_{z_2}}{\left(\gamma_{z_3} - \gamma_{z_2}\right)^2} > 0$$

Therefore if $(A13)$ holds for a finite γ_3, it should also hold in the limit as $\gamma_3 \to \infty$:

$$\left(\psi'\left(e_0\right) - \Delta c\left(q_{z_2}\right)\right)\left(\gamma_0 - \gamma_{z_1}\right)\left(\Delta c\left(q_{z_1}\right) - \Delta c(q_{z_3})\right) \\ + \left(\Delta c\left(q_{z_1}\right) - \psi'\left(e_0\right)\right)\left(\gamma_0 - \gamma_{z_2}\right)\left(\Delta c\left(q_{z_2}\right) - \Delta c(q_{z_3})\right) \geq 0 \quad (A14)$$

As in case (1), the fact that $\Delta c\left(q_{z_1}\right) > \Delta c\left(q_{z_2}\right) > \Delta c\left(q_{z_3}\right)$ and $\gamma_{z_1} > \gamma_{z_0} > \gamma_{z_2}$ implies $(A11)$. Therefore, if $(A14)$ holds for a $q_{z_3} > 0$, then it also has to hold for $q_{z_3} = 0$, which is equivalent to $(A12)$.

The derivative of the left-hand side of $(A12)$ with respect to e_0 equals

$$\psi''\left(e_0\right)\left(\gamma_0 - \gamma_{z_1}\right)\Delta c\left(q_{z_1}\right) - e_0^{-2}\left(\psi'\left(e_0\right) - \Delta c\left(q_{z_2}\right)\right)\Delta c\left(q_{z_1}\right) - \left(\gamma_0 - \gamma_{z_2}\right)\Delta c\left(q_{z_2}\right)\psi''\left(e_0\right) \\ - \left(\Delta c\left(q_{z_1}\right) - \psi'\left(e_0\right)\right)e_0^{-2}\Delta c\left(q_{z_2}\right) < 0,$$

so if inequality $(A12)$ holds for some $e_0 > e_{\max}$, then it holds strictly for $e_{\max}$. Substituting e_i for e_{z_i} $(i = 1, 2)$ results in Condition (a).

Now let us prove that Condition (a') implies that the set of investment levels above $e_{\max}$ that are implementable by partial-disclosure contracts is nonempty. By continuity, if $e_{\max}$ satisfies Condition (a') for some e_1, e_2, then there exists a $e_0 > e_{\max}$ satisfies that satisfies Condition (a') for the same e_1, e_2. Consider the following contract:

$$Z = \{z_0, z_1, z_2\};$$

$$e_{z_0} = 0,\ e_{z_1} = e_1,\ e_{z_2} = e_2;$$

$$(q_{z_0}, p_{z_0}) = \left(q_l^*, c^l(q_l^*)\right),\ (q_{z_i}, p_{z_i}) = \left(\hat{q}_h(e_i), c^l(\hat{q}_h(e_i))\right),\ i = 1, 2;$$

$$\zeta_{z_i}^h = \left|\frac{\psi'(e_0) - \Delta c(q_{z_i})}{\Delta c(q_{z_1}) - \Delta c(q_{z_2})}\right|,\ i = 1, 2;$$

$$\zeta_{z_0}^l = 1 - \sum_{i=1}^{2} \zeta_{z_i}^h \frac{\gamma_{z_i}}{\gamma_0}.$$

By construction, this contract satisfies (PR_h), (PR_l) and (INV); simple calculations show that (IC_h) is implied by Condition (a'). It remains to be checked that $\zeta_{z_0}^l \geq 0$. Suppose not:

$$\zeta_{z_1}^h \frac{\gamma_{z_1}}{\gamma_0} + \zeta_{z_3}^h \frac{\gamma_{z_3}}{\gamma_0} > 1$$

Then (PR_h) implies that $\zeta_{z_1}^h \left(\frac{\gamma_{z_1}}{\gamma_0} - 1\right) + \zeta_{z_3}^h \left(\frac{\gamma_{z_3}}{\gamma_0} - 1\right) > 0$. Combined with (IC_h), this implies

$$\zeta_{z_1}^h \left(1 - \frac{\gamma_{z_1}}{\gamma_0}\right) (\Delta c(q_{z_1}) - \Delta c(q_{z_3})) > 0$$

However, $\gamma_{z_1} > \gamma_0$ and $\Delta c(q_{z_1}) > \Delta c(q_{z_3})$ – contradiction. So $\zeta_{z_0}^l \geq 0$. Therefore the proposed contract is feasible and achieves e_0 by construction.

Proof of Part (ii): Suppose $e_0 > e_{\max}$ is achievable; then either case (1) or case (2) above holds.

In case (1), Proposition 4 and constraint (INV) imply that $e_{z_1} < e_{\max} < e_{z_2}$ and $\Delta c(q_{z_1}) > \psi'(e_{\max}) > \Delta c(q_{z_2})$. As noted above, the same inequalities are satisfied with e_0 in place of $e_{\max}$; therefore, they are satisfied for any $\tilde{e} \in (e_{\max}, e_0)$.

Substituting $\tilde{e}$ instead of e_0 into $(A9)$ and plugging the resulting expressions for $\zeta_{z_i}^h$ into (IC_h) results in the inequality

$$(\psi'(\tilde{e}) - \Delta c(q_{z_2})) \left(\frac{1-\tilde{e}}{\tilde{e}} - \gamma_{z_1}\right) (\Delta c(q_{z_1}) - \Delta c(q_l^*)) \\ + (\Delta c(q_{z_1}) - \psi'(\tilde{e})) \left(\frac{1-\tilde{e}}{\tilde{e}} - \gamma_{z_2}\right) (\Delta c(q_{z_2}) - \Delta c(q_l^*)) \geq 0$$

Because $e_{z_1} < \tilde{e} < e_{z_2}$ and $\Delta c(q_{z_1}) > \psi'(\tilde{e}) > \Delta c(q_{z_2})$ for any $\tilde{e} \in (e_{\max}, e_0)$, the left-hand side of the above inequality decreases in $\tilde{e}$. By $(A10)$, this inequality holds for $\tilde{e} = e_0$; therefore it also holds for any $\tilde{e} \in (e_{\max}, e_0)$. Therefore a contract which has the same set of signals as the one that implements e_0, but where $\zeta_{z_i}^h$ are found by substituting $\tilde{e}$ instead of e_0 into $(A9)$, is feasible and implements $\tilde{e}$.

The reasoning for case (2) is identical, but uses $(A13)$ instead of $(A9)$.

□

Proof of Proposition 5. Let $g(e) = e\Phi^h(0, \psi'(e)) + (1-e)\Phi^l(1, \psi'(e)) - \psi(e)$. If $e \in [e_{\min}, e_{\max}]$, then $g(e) = e\Phi^h(e, \psi'(e)) + (1-e)\Phi^l(e, \psi'(e)) - \psi(e)$, which is the surplus achieved by the contract that is optimal given e. By the definition of Φ^h and Φ^l (see Lemma 5), if $(e, \Delta c^{-1}(\psi'(e))) \in A \cup B$, then

$$\begin{aligned} g'(e) &= \Phi^h(0, \psi'(e)) - \Phi^l(1, \psi'(e)) + (1-e)\frac{d\Phi^l\left(1, \psi'(e)\right)}{de} - \psi'(e) \\ &= (1-e)\frac{\partial\varphi^l(1,q)}{\partial q}\frac{\partial \Delta c^{-1}(\psi'(e))}{\partial e}\Big|_{q=\Delta c^{-1}(\psi'(e))} + \Phi^h(0, \psi'(e)) - \Phi^l(1, \psi'(e)) - \psi'(e) \\ &= \left((1-e)\ \frac{\partial\varphi^l(1,q)}{\partial q}\frac{\psi''(e)}{\Delta c'(q)} - V^l(q) + c^l(q)\right)\Big|_{q=\Delta c^{-1}(\psi'(e))} + V^h(q_h^*) - c^h(q_h^*) - \psi'(e) \end{aligned}$$

Similarly, if $(e, \Delta c^{-1}(\psi'(e))) \in D \cup E$, then

$$g'(e) = \left(e\ \frac{\partial\varphi^h(0,q)}{\partial q}\frac{\psi''(e)}{\Delta c'(q)} + V^h(q) - c^h(q)\right)\Big|_{q=\Delta c^{-1}(\psi'(e))} - \left(V^l(q_l^*) - c^l(q_l^*)\right) - \psi'(e)$$

If $(e, \Delta c^{-1}(\psi'(e))) \in C$, then

$$g'(e) = V^h(q_h^*) - c^h(q_h^*) - \left(V^l(q_l^*) - c^l(q_l^*)\right) - \psi'(e)$$

Note that if $e = e_{\max}$, then $\Delta c^{-1}(\psi'(e)) = \hat{q}_h(e)$, and by definition (equation (3)),

$$g'(e_{\max}) = -(1-e_{\max})\psi''(e_{\max}) + V^h(\hat{q}_h(e_{\max})) - c^h(\hat{q}_h(e_{\max})) - \left(V^l(q_l^*) - c^l(q_l^*)\right) - \psi'(e_{\max})$$

So Condition (b) is equivalent to $g'(e_{\max}) > 0$.

Note also that if $(e, \Delta c^{-1}(\psi'(e))) \in C$, then

$$g''(e) = -\psi''(e) < 0$$

If $(e, \Delta c^{-1}(\psi'(e))) \in D \cup E$, then

$$\begin{aligned} g''(e) &= e\left(\frac{d^2(V^h(q)-c^h(q))}{dq^2}\left(\frac{\psi''(e)}{\Delta c'(q)}\right)^2 + \frac{d(V^h(q)-c^h(q))}{dq}\frac{\psi'''(e)(\Delta c'(q))^2-(\psi''(e))^2\Delta c''(q)}{(\Delta c'(q))^3}\right)\Big|_{\Delta c(q)=\psi'(e)} \\ &+2\ \frac{\left(V^h(q)-c^h(q)\right)}{dq}\frac{\psi''(e)}{\Delta c'(q)}\Big|_{\Delta c(q)=\psi'(e)} - \psi''(e) \\ &= \frac{e}{(\Delta c'(q))^2}\left((\psi''(e))^2\left(\frac{d^2\left(V^h(q)-c^h(q)\right)}{dq^2} - \frac{\frac{d\left(V^h(q)-c^h(q)\right)}{dq}\Delta c''(q)}{\Delta c'(q)}\right)\right. \\ &\left.+\frac{d\left(V^h(q)-c^h(q)\right)}{dq}\psi'''(e)\,\Delta c'(q)\right)\Big|_{\Delta c(q)=\psi'(e)} + 2\ \frac{d\left(V^h(q)-c^h(q)\right)}{dq}\frac{\psi''(e)}{\Delta c'(q)}\Big|_{\Delta c(q)=\psi'(e)} - \psi''(e) < 0, \end{aligned}$$

where the inequality follows from the assumptions on the derivatives of V^h, c^h, Δc and ψ, the fact that $q \geq q_h^*$ and Assumption 1.

Proof of Part (i): Suppose that a contract $\left(Z,\ p_z,\ q_z,\ \zeta_z^h,\ \zeta_z^l\right)$ is optimal given $e_0 < e_{\min}$. By Lemma 6, we can assume without loss of generality that $\forall z \in Z, (e_z, q_z) \in B \cup C \cup D$. By $(A3)$, the welfare resulting from the contract equals

$$e_0 \sum_{z\in Z} \Phi^h\left(e_z, \Delta c\left(q_z\right)\right)\zeta_z^h + (1-e_0)\sum_{z\in Z}\Phi^l\left(e_z, \Delta c\left(q_z\right)\right)\zeta_z^l - \psi\left(e_0\right) \leq g(e_0) \tag{A15}$$

For any e such that $(e, \Delta c^{-1}(\psi'(e))) \in A$,

$$\begin{aligned} g'(e) &= \left((1-e)\ \tfrac{\partial\varphi^l(1,q)}{\partial q}\tfrac{\psi''(e)}{\Delta c'(q)} - V^l\left(q\right) + c^l\left(q\right)\right)\Big|_{q=\Delta c^{-1}(\psi'(e))} + V^h(q_h^*) - c^h(q_h^*) - \psi'\left(e\right) \\ &> V^h(q_h^*) - c^h(q_h^*) - \left(V^l(q) - c^l(q)\right)\big|_{q=\Delta c^{-1}(\psi'(e_{\min}))} - \psi'(e_{\min}) \\ &\geq V^h(q_h^*) - c^h(q_h^*) - \left(V^l(q_l^*) - c^l(q_l^*)\right) - \psi'(e^*) = 0, \end{aligned}$$

where the first inequality follows from $e < e_{\min}$ and the second one from $e_{\min} \leq e^*$. Therefore, $g(e) < g(e_{\min})$ whenever $(e, \Delta c^{-1}(\psi'(e))) \in A$, which, together with $(A15)$, implies

$$e_0 \sum_{z\in Z} \Phi^h\left(e_z, \Delta c\left(q_z\right)\right)\zeta_z^h + (1-e_0)\sum_{z\in Z}\Phi^l\left(e_z, \Delta c\left(q_z\right)\right)\zeta_z^l - \psi\left(e_0\right) < g(e_{\min})$$

Because $g(e_{\min})$ equals the surplus achieved by the optimal contract given $e_{\min}$, this means that the contract $\left(Z,\ p_z,\ q_z,\ \zeta_z^h,\ \zeta_z^l\right)$ is suboptimal.

Proof of Part (ii): Suppose that the optimal contract is a partial disclosure contract; then, by Proposition 3, $e > e_{\max}$. Consider a contract $\left(Z,\ p_z,\ q_z,\ \zeta_z^h,\ \zeta_z^l\right)$ that is optimal given e. By Lemmas 6 and 7, we can assume that $\forall z \in Z,\ (e_z, q_z) \in B \cup C \cup D$ and $p_z = c^l\left(q_z\right)$. This contract must achieve strictly higher surplus than the contract that is optimal given $e_{\max}$:

$$\begin{aligned} &e\sum_{z\in Z}\Phi^h\left(e_z, \Delta c\left(q_z\right)\right)\zeta_z^h + (1-e)\sum_{z\in Z}\Phi^l\left(e_z, \Delta c\left(q_z\right)\right)\zeta_z^l - \psi\left(e\right) \\ &\qquad > e_{\max}\Phi^h\left(e_{\max}, \psi'\left(e_{\max}\right)\right) + \left(1-e_{\max}\right)\Phi^l\left(e_{\max}, \psi'\left(e_{\max}\right)\right) - \psi\left(e_{\max}\right) \end{aligned} \tag{A16}$$

From $(A15)$ and $(A16)$ it follows that

$$\begin{aligned} &e\Phi^h\left(0, \psi'\left(e\right)\right) + (1-e)\Phi^l\left(1, \psi'\left(e\right)\right) - \psi\left(e\right) \\ &> e_{\max}\Phi^h\left(e_{\max}, \psi'\left(e_{\max}\right)\right) + \left(1-e_{\max}\right)\Phi^l\left(e_{\max}, \psi'\left(e_{\max}\right)\right) - \psi\left(e_{\max}\right) \\ &= e_{\max}\Phi^h\left(0, \psi'\left(e_{\max}\right)\right) + \left(1-e_{\max}\right)\Phi^l\left(1, \psi'\left(e_{\max}\right)\right) - \psi\left(e_{\max}\right), \end{aligned}$$

i.e. $g\left(e\right) > g\left(e_{\max}\right)$. This, together with the fact that $e > e_{\max}$ and the fact that $g(e)$ is concave whenever $(e, \Delta c^{-1}(\psi'(e))) \in C \cup D \cup E$, implies that $g'\left(e_{\max}\right) > 0$, which is Condition (b).

Proof of Part (iii): Suppose that Condition (b) holds, i.e. $g'(e_{\max}) > 0$. We will prove that a contract with no disclosure cannot be optimal.

First, note that if $g'(e_{\max}) > 0$, then for all e such that $(e, \Delta c^{-1}(\psi'(e))) \in C \cup D$, $g'(e) > 0$, so no such e can be optimal. Also, suppose that $(e, \Delta c^{-1}(\psi'(e))) \in B$ and $(\tilde{e}, \Delta c^{-1}(\psi'(\tilde{e}))) \in C$. Then

$$\begin{aligned} g'(e) &= \left((1-e) \frac{\partial \varphi^l(1,q)}{\partial q} \frac{\psi''(e)}{\Delta c'(q)} - V^l(q) + c^l(q) \right)\Big|_{q=\Delta c^{-1}(\psi'(e))} + \Phi^h(0, \psi'(e)) - \psi'(e) \\ &> \Phi^h(0, \psi'(\tilde{e})) - \Phi^l(1, \psi'(\tilde{e})) - \psi'(\tilde{e}) \end{aligned}$$

Therefore the fact that $g'(\tilde{e}) > 0$ implies that $g'(e) > 0$ and e is not optimal. It remains only to prove that $e = e_{\max}$ is not optimal.

For any e_0 s.t. $\left(e_0, \Delta c^{-1}(\psi'(e_0))\right) \in D \cup E$, consider the following problem (call it Problem 2):

$$\max_{Z,\ (q_z,\ p_z,\ \zeta_z^h,\ e_z)_{z\in Z},\ (\zeta_z^l)_{\substack{z\in Z\\ e_z=0}}} e_0 \int_{z\in Z} \varphi^h(e_z, q_z)\, \zeta_z^h dz + (1-e_0) \int_{z\in Z} \varphi^l(e_z, q_z)\, \zeta_z^l dz - \psi(e_0)$$

s.t. (PR_h), (PR_l), (IC_h), (IC_l), $(FEAS)$ and

$$(INV') \qquad \int_{z\in Z: e_z>0} \left[p_z - c^h(q_z) - \frac{\gamma_z}{\gamma_0} U_z^l \right] \zeta_z^h dz - \int_{z\in Z: e_z=0} U_z^l \zeta_z^l dz \geq \psi'(e_0)$$

The proofs of Lemmas 4 and 6 go through for this problem, so there exists a solution with a finite Z such that $\forall z \in Z$, $(e_z, \Delta c(q_z)) \in B \cup C \cup D$. The first-order conditions $(A4)$ and $(A6)$, for $z \in Z$ such that $(e_z, q_z) \in B$ and $\gamma_z > \gamma_0$ (such z must exist by (PR_h) and (INV')) imply that the Lagrange multiplier μ that corresponds to (INV) in the original problem is negative; therefore (INV') binds in Problem 2. So Problem 2 has the same value as the problem of finding an optimal contract that achieves e_0. Call this value $W(e_0)$. By Corollary 5 of Milgrom and Segal (2002), $W(e_0)$ is differentiable at $e_0 = e_{\max}$, so $W'(e_{\max}) = g'(e_{\max})$. It follows that $W'(e_{\max}) > 0$, so, for a small $\varepsilon > 0$, a contract that achieves $e_{\max} + \varepsilon$ is feasible (by assumption) and results in higher welfare than the optimal contract given $e_{\max}$. □

Proofs for Section 7.

Proof of Proposition 6. Let $e \in (0,1)$ be the probability that the buyer places on the high state before renegotiation starts, $(\bar{p}, \bar{q})$ the outside option, and $((p_h, q_h), (p_l, q_l))$ the buyer's optimal offer in the one-shot renegotiation game given $(e, \bar{p}, \bar{q})$. As Lemma 1 demonstrates, either q_h or q_l may be inefficient (depending on the outside option and the buyer's beliefs), but never both. Without loss of generality, suppose that $q_l = q_l^*$, which implies that the constraint $(\overline{IC})$ does not bind in the one-shot renegotiation game. Let $\overline{V} = e\left(p_h - c^h(q_h)\right) + (1-e)\left(p_l - c^l(q_l)\right)$ be the buyer's equilibrium expected payoff in the one-shot renegotiation game.

(1) Let us first show that in any equilibrium, the buyer's expected payoff equals $\overline{V}$. By the revelation principle, this payoff cannot exceed $\overline{V}$. We will show that the buyer's payoff cannot be less than $\overline{V}$ either: if this is the case in a candidate equilibrium, then the strategy of one of the players is not sequentially rational.

Sequential rationality for the seller implies that in any equilibrium, if the state is i and the current outside option is (p, q), then at any time t the seller will accept any menu that contains an item $(p'q')$ such that $p' - c^i(q') > p - c^i(q)$. Indeed, accepting the menu and choosing such an item is strictly better for the seller than getting the outside option, no matter how the renegotiation is expected to proceed.

Suppose that T is finite, and there is a candidate equilibrium where the buyer's expected payoff is strictly less than $\overline{V}$. Consider the deviation strategy for the buyer whereby at every t, regardless of the history, the buyer offers the menu $((p_h + t\varepsilon_h, q_h), (p_l + t\varepsilon_l, q_l))$, where $\varepsilon_h, \varepsilon_l > 0$ and the menu $((p_h + T\varepsilon_h, q_h), (p_l + T\varepsilon_l, q_l))$ satisfies the incentive compatibility constraints in the one-shot game as strict inequalities: i.e. $\varepsilon_l - \varepsilon_h \in \left(0, T^{-1}(p_h - c^h(q_h) - p_l + c^h(q_l))\right)$. Because the menu $((p_h, q_h), (p_l, q_l))$ satisfies the individual rationality constraints in the one-shot game, the seller will accept this menu at every t; at $t = T$, the seller will choose $(p_i + T\varepsilon_i, q_i)$ in state i by construction of ε_i. As ε_i can be chosen arbitrarily small, the buyer's payoff from the deviation can be made arbitrarily close to $\overline{V}$, making the deviation profitable.

Therefore, when T is finite, in every equilibrium the buyer's expected payoff equals $\overline{V}$. This implies that the equilibrium outcome in every state i is (p_i, q_i), because every equilibrium outcome is feasible in the one-shot renegotiation game, and the allocation $((p_h, q_h), (p_l, q_l))$ is the unique solution to the buyer's problem in that game.

Now suppose that T is infinite, and there is a candidate equilibrium where the buyer's expected payoff equals $V < \overline{V}$. Let M be a positive integer such that

$$M > \max_{i \in \{h,l\}} \frac{V - (V_i(q_i) - p_i)}{\Delta c(q_h) - \Delta c(q_l)}$$

and consider the following deviation strategy for the buyer. At every $t \in \{1, ..., 2M + 1\}$, regardless of the history, the buyer offers the menu $((p_h^t, q_h), (p_l^t, q_l))$, where

$$\begin{aligned} p_i^1 &= p_i + \varepsilon_i \ (i = h, l); \\ p_h^{t+1} &= p_l^t + c^h(q_h) - c^h(q_l) + \delta; \\ p_l^{t+1} &= p_h^t + c^l(q_l) - c^l(q_h) + \delta \end{aligned}$$

where $\varepsilon_h, \varepsilon_l, \delta > 0$ are small and satisfy $\varepsilon_h + \delta < \varepsilon_l$. The definition implies that $p_i^{t+1} < p_i^t$ if $\varepsilon_h, \varepsilon_l, \delta$ are small enough; in particular, $p_i^{2M+1} = p_i - M\left(\Delta c(q_h) - \Delta c(q_l)\right) + \varepsilon_i + 2M\delta$. At $t = 2M + 2$, the buyer ends the renegotiation regardless of the history.

This deviation guarantees the expected payoff of at least $\overline{V} - (e\varepsilon_h + (1-e)\varepsilon_l)$ to the buyer. Indeed, if the buyer follows this strategy, the seller will accept at $t = 1$, because the menu $((p_h, q_h), (p_l, q_l))$ is individually rational in the one-shot game and $\varepsilon_h, \varepsilon_l > 0$. Suppose now that $t \in \{1, ..., 2M+1\}$, the state is i, the current outside option is (p_j^{t-1}, q_j), and the buyer has offered $((p_h^t, q_h), (p_l^t, q_l))$. Then the only case when the seller could reject the offer is when $i = j$, because if $i \neq j$, then by construction $p_i^t - c^i(q_i) > p_j^{t-1} - c^i(q_j)$ and thus the seller will accept. Therefore, if the seller rejects the offer at $t \in \{2, ..., 2M+1\}$ in favor of the outside option (p_j^{t-1}, q_j), it must be that the state is j, and the buyer's payoff in this case is $V_j(q_j) - p_j^{t-1} \geq V_j(q_j) - p_j - \varepsilon_j$ (the equality holds only for $t = 2$). Otherwise renegotiation ends at $t = 2M+2$, and the buyer's payoff is at least $\min\left\{V_h(q_h) - p_h^{2M+1}, V_l(q_l) - p_l^{2M+1}\right\} > V$ by construction of M. As $\varepsilon_h, \varepsilon_l$ can be chosen arbitrarily small, the buyer's payoff from the deviation can be made arbitrarily close to $\overline{V}$. Therefore, like in the case of a finite T, in every equilibrium the buyer's expected payoff equals $\overline{V}$, and the equilibrium outcome in every state i is (p_i, q_i).

(2) (a) Similarly to part (1), the buyer's expected payoff cannot exceed $\overline{V}$. Suppose that in a candidate equilibrium the buyer's payoff is strictly less than $\overline{V}$, and consider the following deviation for the buyer. At every $t < T$, the seller offers the point contract $(\bar{p}, \bar{q})$, regardless of the history; at $t = T$, he offers the menu $((p_h + \varepsilon_h, q_h), (p_l + \varepsilon_l, q_l))$, where $\varepsilon_h, \varepsilon_l > 0$ are chosen so that the menu $((p_h + \varepsilon_h, q_h), (p_l + \varepsilon_l, q_l))$ is incentive compatible in the one-shot game (such ε_h and ε_l always exist and can be chosen arbitrarily close to 0). Sequential rationality for the seller implies that at $t = T$ the seller will choose $(p_i + \varepsilon_i, q_i)$ in state i. Thus, by making ε_h and ε_l arbitrarily small, the buyer's payoff can be made arbitrarily close to $\overline{V}$. Similarly to (1), this implies that the buyer's equilibrium payoff equals $\overline{V}$, and the equilibrium outcome in every state i is (p_i, q_i).

(b) Follows from Proposition 5 of Beaudry and Poitevin (1995).

□

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