Communication in Cournot Oligopoly*

PRELIMINARY AND INCOMPLETE

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Abstract

We study communication in a static oligopoly model with unverifiable private information. Contrary to the previous literature, we show that communication between firms in the static setting can be informative even when it is not substantiated by any commitment or costly actions. We exhibit a simple mechanism that ensures informative communication and show that any informative communication equilibrium interim Pareto dominates the uninformative equilibrium for the firms.

1 Introduction

It is well recognized in both the theoretical literature and the antitrust law that communication in oligopoly can have several effects (see, for example, Nalebuff and Zeckhauser (1986) and Kühn and Vives (1994)). On the one hand, more precise information about the market allows the economic agents to make more effective decisions. On the other hand, information sharing between the firms may facilitate collusion and increase barriers to entry, which reduce consumer surplus. Therefore, assessing the effects of communication on equilibrium prices and production is both interesting from a theoretical point of view and important for developing guidelines for competition policy. This paper contributes to the discussion by studying the possibility of informative communication in a Cournot oligopoly model where the firms have unverifiable private information about their costs.

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There is a large literature on information sharing in oligopoly with private information about costs that considers the case of verifiable information, where a firm can conceal its private information but cannot misrepresent it: examples include Fried (1984), Li (1985), Gal-Or (1986), Shapiro (1986), Okuno-Fujiwara, Postlewaite and Suzumura (1990), Raith (1996) and Amir, Jin and Troege (2010).[1] Most of these papers assume that each firm decides whether to share its information or not before it observes the cost realization; if it decides to share, it has to transmit the information precisely and truthfully. (An exception is the paper by Okuno-Fujiwara, Postlewaite and Suzumura (1990), which assumes that each firm decides whether to reveal its cost realization after observing it). The conclusion from this literature is that in a Cournot oligopoly with linear demand and constant marginal cost, each firm will find it profitable to commit to disclose its private information.

However, the assumption that private information is costlessly verifiable may be restrictive. For example, Ziv (1993) notes that the information about a firm's cost function "is part of an internal accounting system that is not subject to external audit and not disclosed in the firm's financial statements," which makes it potentially costly or impossible to verify. Therefore, one may wish to examine whether the conclusions of the literature on information sharing in oligopoly are robust to the assumption that information is verifiable. Ziv (1993) addresses this question in the framework of a Cournot oligopoly with private information about costs, where before choosing its output each firm can send a cheap-talk message to its competitors. He shows that if the information is unverifiable, the conclusion that each firm will be willing to share the information no longer holds. To understand this result, suppose that there exists an equilibrium where each firm announces its cost realization truthfully, the competitors take each announcement at face value, and the output of each type of each firm is positive. Then, regardless of the true cost realization, each firm would like to deviate and announce the lowest possible cost in order to appear more aggressive and thus make the competitors reduce their production.[2]

Various mechanisms to make unverifiable cost announcements credible have been considered in the literature. For instance, different announcements can be accompanied by appropriate levels of 'money burning' (Ziv, 1993), the announcements can determine the amount of side payments in a collusive contract (Cramton and Palfrey, 1990), or the level of future 'market-share favors' from the competitors in repeated settings (Chakrabarti, 2010).

In this paper, we take a different approach. We consider a Cournot duopoly with linear

[1] A related strand of literature (Novshek and Sonnenschein (1982), Vives (1984), Gal-Or (1985), Kirby (1988)) studies information sharing between firms having private information about demand; Raith (1996) and Amir, Jin and Troege (2010) cover both cost uncertainty and demand uncertainty.

[2] We take this reasoning one step further by showing that no informative communication through cheap talk is possible in this setting, unless one allows for the possibility of negative output.

demand and constant marginal cost, which is unverifiable private information. We assume that the game is played only once, the firms cannot commit to information disclosure ex ante, and the communication between the firms cannot be substantiated by any costly actions. However, the firms are allowed to use more complex communication protocols than simply sending one cheap-talk message. In particular, the firms have access to a neutral and trustworthy third party. Each firm privately reports its cost to the third party, which then makes announcements to the firms. The reports by the firms are assumed to be costless and unverifiable; the announcements may be either public or private, and may depend in a deterministic or stochastic way on the firms' reports.[3]

We show that for a range of parameters there exist communication protocols that can sustain informative communication. To understand how this is possible, consider the following simple protocol. The third party can announce one of two public messages, $High$ or Low. It announces the message $High$ if both firms' reported costs exceed a certain threshold c^*, and the message Low otherwise.

For an appropriately chosen value of c^*, both firms will find it optimal to report their costs truthfully. To see why, note that if a firm reports that its cost is below c^*, then it knows that the third party will announce Low, whereas if a firm reports that its cost is above c^*, then the third party's announcement Low reveals to it that the opponent's cost is low, while $High$ reveals that the opponent's cost is high. Hence the firm reporting that its cost is above c^* leads to additional information about the opponent (and thus to better informed decisions by the firm) than reporting that the cost is below c^*. On the other hand, the firm reporting that its cost is above c^* results in a higher expected output by the opponent than a report that its cost is below c^*. Thus there is a tradeoff between lower production by the opponent and more information. We show that the higher the firm's cost realization, the more willing it is to resolve this tradeoff in favor of additional information. This 'single-crossing property' allows to sustain truthful reporting by the firms under certain conditions.

In the above communication protocol, the third party plays a role of an information filter between the firms: a firm does not get to see the competitor's cost report, and the amount of information that it gets about the competitor's report depends on its own report to the third party.[4] More generally, we show that any incentive compatible communication mechanism has the property that higher cost report leads to higher expected quantity produced by the

[3]Liu (1996) considers communication protocols which make use of a third party (i.e. the correlated equilibria) in a Cournot oligopoly with complete information. He shows that the possibility of communication does not enlarge the set of possible outcomes: the only correlated equilibrium is the Bayesian-Nash equilibrium.

[4]The idea that introducing noise into communication in sender-receiver games can improve information transmission has been introduced by Myerson (1991) and analyzed in detail by Blume, Board and Kawamura (2007).

competitor, but at the same time more precise information about the competitor.

Our paper belongs to the literature on mechanism design without enforcement, where, unlike in the standard mechanism design approach, the principal cannot commit to an outcome rule contingent on the agents' messages, but can only suggest actions to the agents.[5] As a result, the firms are not doing as well as they could in a cartel with enforcement power.[6]

The idea that a sender may be induced to reveal information by making the amount of information he gets about his competitor contingent on his own message appears in Baliga and Sjöström (2004). They study a two-player arms race game where each player's cost of acquiring weapons is her private information. In their model, all types want to reduce the probability that the opponent arms, and all types have a preference for resolving the uncertainty about the opponent's action. Using the fact that different types trade off these two objectives at different rates, Baliga and Sjöström (2004) construct an equilibrium with informative pre-play cheap-talk communication.[7]

The notion that information aggregation by a third party may facilitate information transmission between the firms has interesting implications for competition policy. Currently, the competition policy treats the exchange of disaggregated data as more conducive to collusion than the exchange of aggregate statistics. For example, Kühn and Vives (1994) note that the European Commission "has no objection to the exchange of information on production or sales as long as the data does not go as far as to identify individual businesses." The existing theoretical literature suggests some reasons for why one might expect the sharing of individualized data to be more conducive to collusion than the sharing of aggregate data: the former can enable the firms to identify the deviators from the collusive agreement and devise individualized punishment strategies. In our model, however, informative communication may be impossible when the firms send direct cheap-talk messages to each other, or the third party fully discloses the firms' reports, but possible when the firms' reports are aggregated by the third party. We also show that information exchange between the firms lowers the ex ante consumer surplus. Therefore, from the viewpoint of a regulatory agency whose aim is to protect consumers, the exchange of aggregate data could be more harmful than the regime of full disclosure of individual data.

[5]Myerson (1982) provides a revelation principle for mechanism design problems without enforcement. This approach has been used to study sealed-bid double auctions (Matthews and Postlewaite, 1989), battle of the sexes (Banks and Calvert, 1992), bargaining in the shadow of war (Hörner, Morelli and Squintani, 2011).

[6]See Cramton and Palfrey (1990) for such cartels in a static setting. In the case of repeated interactions, enforcement can be achieved by threats of future punishment (Chakrabarti, 2010).

[7]In other respects, their model is very different from ours: each player has only two actions, and their assumptions allow to support nonmonotonic equilibria where low types pool on the same message with high types, which could never happen in our model.

The rest of the paper is organized as follows. In Section 2 we introduce the set-up, define the communication mechanisms, and discuss the incentive constraints. In Section 3 we derive some general properties of the feasible mechanisms, and focus on simple public deterministic mechanisms in Section 4. Concluding comments are in Section 5. All proofs are relegated to the Appendix unless stated otherwise.

2 The Model

2.1 The Environment

We consider an industry where two firms, A and B, produce a homogeneous good. The inverse demand curve is given by $P(Q) = \max\{A - Q, 0\}$, where Q is the aggregate output and $A \geq 1$. For each $i \in \{A, B\}$, firm i's cost function is $c_i(q_i) = c_i q_i$, where q_i is the output of firm i. The marginal cost parameter c_i is private information of firm i; c_A and c_B are independently distributed on $C = [0, 1]$ according to a continuous distribution function F with density f, mean μ, and variance $\sigma^2 > 0$. Thus the profit of firm i with marginal cost c_i when it produces q_i and its competitor produces q_{-i} is

$$\pi_i(c_i, q_i, q_{-i}) = (A - q_i - q_{-i} - c_i)\, q_i \tag{1}$$

Note that the restrictions that the inverse demand curve has unit slope and that the marginal costs are distributed on $[0, 1]$ are without loss of generality, in the following sense: a model where the marginal costs c_i are distributed on $[\underline{c}, \overline{c}]$ according to a distribution function $\tilde{F}$ and the inverse demand curve is $\tilde{P}(\tilde{Q}) = \max\left\{\tilde{A} - \tilde{b}\tilde{Q}, 0\right\}$ can be 'translated' into the above model as follows:

$$F(c_i) = \tilde{F}\left((\overline{c} - \underline{c})\, c_i + \underline{c}\right), \;\; A = \frac{\tilde{A} - \underline{c}}{\overline{c} - \underline{c}}, \;\; q_i = \frac{\tilde{b}}{\overline{c} - \underline{c}}\tilde{q}_i \tag{2}$$

The firms compete by simultaneously choosing outputs. For comparison, let us first consider the case where there is no communication between the firms. Then a pure strategy for firm i is a function $q_i : C \to \mathbb{R}$. It is easily verified that the unique Bayesian-Nash equilibrium of this game is

$$q_i(c_i) = \frac{1}{3}A - \frac{1}{2}c_i + \frac{1}{6}\mu \tag{3}$$

and the ex ante expected equilibrium profit of firm i is

$$\Pi_i = \frac{1}{9}(A - \mu)^2 + \frac{1}{4}\sigma^2$$

2.2 Communication Mechanisms

We assume that, before choosing how much to produce, the firms can communicate with a neutral trustworthy third party (a mediator). Both firms, as well as the mediator, can send private or public messages according to a mediation rule, or mechanism, which specifies what messages the parties can send, in what sequence, and whether the messages are public or private. After the communication has ended, the firms simultaneously choose their outputs.

The set of possible mechanisms, as described above, is very large. However, if one's goal is to characterize the outcomes that can be achieved by communication mechanisms, then by the revelation principle one can restrict attention to direct revelation mechanisms (Myerson, 1982; Cotter, 1989). In such mechanisms each firm i first sends a single message $\hat{c}_i \in C$ to the mediator, which is interpreted as firm i's report about its marginal cost. The messages are private, i.e. neither of the firms gets to see the message sent by the opponent. Having received the messages, the mediator selects a pair of messages $(\hat{q}_A, \hat{q}_B) \in \mathbb{R}^2$. Message $\hat{q}_i$ is interpreted as the output that the mediator recommends firm i to choose, and is observed only by firm i.

Formally, a direct revelation mechanism is a family of probability measures $g(\cdot \mid \hat{c}_A, \hat{c}_B)$ over the set of the firms' quantity pairs ($\mathbb{R}^2$), indexed by the pair of cost reports submitted to the mediator ($(\hat{c}_A, \hat{c}_B) \in C^2$). Thus for every possible pair of cost reports $(\hat{c}_A, \hat{c}_B)$, the mechanism specifies a lottery over output recommendations $(\hat{q}_A, \hat{q}_B) \in \mathbb{R}^2$ that the mediator makes. Unless specified otherwise, in what follows we will restrict attention to direct revelation mechanisms.

Given a mechanism, a firm's pure strategy includes two components: the reporting strategy, which describes what cost level the firm should report as a function of its true cost, and the output strategy, which describes the firm's output as a function of its true cost level, its reported cost level, and the mediator's recommendation. Formally, a (pure) strategy of firm i is $(\hat{c}_i(c_i), q_i(c_i, \hat{c}_i, \hat{q}_i))$, where $\hat{c}_i : C \to C$ and $q_i : C^2 \times \mathbb{R} \to \mathbb{R}$. A truthful and obedient strategy is a strategy $(\hat{c}_i^*(c_i), q_i^*(c_i, \hat{c}_i, \hat{q}_i))$ such that for any $c_i \in C$, $\hat{c}_i^*(c_i) = c_i$ and for any $c_i \in C$ and $\hat{q}_i \in \mathbb{R}$, $q_i^*(c_i, c_i, \hat{q}_i) = \hat{q}_i$: that is, each firm always reports its cost level truthfully and follows the mediator's output recommendations. The revelation principle shows that, without loss of generality, we can restrict attention to mechanisms where in a Bayesian-Nash equilibrium all firms are playing truthful and obedient strategies. The mechanisms that satisfy this property will be called **feasible**.

Before moving on to studying feasible mechanisms, let us define some classes of mechanisms that will be used in this paper. A mechanism g will be called **symmetric** if it treats both firms equally ex ante.[8] A mechanism g will be called **deterministic** if for every $(c_A, c_B) \in C^2$, $g(c_A, c_B)$ is a degenerate probability distribution. A deterministic mechanism can thus be interpreted as a function from C^2 to $\mathbb{R}^2$. A mechanism will be called **public** if its equilibrium where both firms are truthful and obedient is outcome equivalent to an equilibrium of some mechanism where all of the mediator's messages are public. A mechanism will be called a **correlated equilibrium** if its equilibrium where both firms are truthful and obedient is outcome equivalent to an equilibrium of some mechanism where the firms send no reports to the mediator, but the mediator may send private messages to the firms.[9] For the most part of the paper we will focus on symmetric mechanisms. Correlated equilibria are characterized in Section 3.2. All mechanisms studied in Section 4 are deterministic and public.

2.3 Characterization of the Feasible Mechanisms

A mechanism has to satisfy two types of requirements in order to be feasible. First, each firm has to have an incentive to report its cost truthfully. Second, each firm that has reported its cost truthfully should find it optimal to follow the mediator's recommendations, assuming that the competitor is truthful and obedient. We begin with characterizing the implications of the latter requirement, and then talk about the former.

Consider firm i with cost c_i, which has reported cost $\hat{c}_i$ and received the output recommendation $\hat{q}_i$ from the mediator. Under the assumption that the opponent is truthful and obedient, the firm can figure out the probability distribution of the opponent's output conditional on report $\hat{c}_i$ and recommendation $\hat{q}_i$. Note that this probability distribution does not depend on the true cost c_i, because of the assumption that the firms' costs are independent. Denote by $E\left[q_{-i} \mid \hat{c}_i, \hat{q}_i\right]$ the expected opponent's output.[10]

Since the profit given in (1) is linear in the opponent's output, the firm's optimal output

[8] Formally, a mechanism g is symmetric if for every $(c, c') \in C^2$, and every measurable set $S \subseteq \mathbb{R}^2$ we have

$$g\left(S \mid c, c'\right) = g\left(S^T \mid c', c\right) \text{ where } S^T = \left\{(q', q) \in \mathbb{R}^2 : (q, q') \in S\right\}.$$

[9] Thus our notion of correlated equilibrium is equivalent to strategic form correlated equilibrium in Forges (1993).

[10] Formally,

$$E\left[q_{-i} \mid c_i, q_i\right] = \int_C \left(\int_{\mathbb{R}} q_{-i} g_{-i}\left(dq_{-i} \mid q_i; c_i, c_{-i}\right)\right) dF\left(c_{-i}\right)$$

where $g_{-i}\left(\cdot \mid q_i; c_i, c_{-i}\right)$ is the distribution of q_{-i} conditional on q_i, c_i, and c_{-i}., i.e.

$$g_{-i}\left(S_{-i} \mid q_i; c_i, c_{-i}\right) g\left(\{q_i\} \times \mathbb{R} \mid c_i, c_{-i}\right) = g\left(\{q_i\} \times S_{-i} \mid c_i, c_{-i}\right)$$

for every measurable set $S_{-i} \subseteq \mathbb{R}$, and every $q_i \in \mathbb{R}$, $c_i, c_{-i} \in C$.

choice depends only on the expectation of the opponent's output:

$$q_i = \frac{1}{2}\left(A - c_i - E\left[q_{-i} \mid \hat{c}_i, \hat{q}_i\right]\right),$$

and the maximized level of profit is

$$\frac{1}{4}\left(A - c_i - E\left[q_{-i} \mid \hat{c}_i, \hat{q}_i\right]\right)^2. \tag{4}$$

Using these preliminaries we can now state the conditions for the optimality of being obedient.

Definition 1 *A mechanism is called* ***ex post incentive compatible*** *if every firm i which has reported its cost truthfully is willing to be obedient upon receiving the output recommendation (conditional on the opponent being truthful and obedient), i.e.*

$$q_i = \frac{1}{2}\left(A - c_i - E\left[q_{-i} \mid c_i, q_i\right]\right) \qquad \text{for every } c_i \in C,\ q_i \in R. \tag{5}$$

Next, we study the conditions for truthful reporting. Consider firm i with cost c_i, which has reported $\hat{c}_i$. Under the assumption that the opponent is truthful, the firm can figure out the probability distribution of the possible output recommendations from the mediator conditional on report $\hat{c}_i$. As before, the probability distribution does not depend on the true cost c_i, because of the assumption that the firms' costs are independent.

If the firm consequently chooses the output optimally given the recommendations, then its expected profit can be written as follows:

$$\Pi_i(c_i, \hat{c}_i) = \frac{1}{4} E_{\hat{q}_i}\left[\left(A - c_i - E\left[q_{-i} \mid \hat{c}_i, \hat{q}_i\right]\right)^2 \mid \hat{c}_i\right]$$

which is the expectation of the expression for the profit given in (4) with respect to the marginal probability distribution over recommendations $\hat{q}_i$ conditional on report $\hat{c}_i$.[11] Now we can state the conditions for the optimality of the truthtelling strategies.

Definition 2 *An ex post incentive compatible mechanism is called* ***interim incentive***

[11] Formally, let $g_i(\cdot|c_i)$ be the marginal probability measure of g on $\mathbb{R}$ conditional on c_i:

$$g_i(S_i|c_i) = \int_C g(S_i \times \mathbb{R}|c_i, c_{-i}) dF(c_{-i})$$

for every measurable set $S_i \subseteq \mathbb{R}$. To compute the expectation mentioned in the text we integrate over the set of possible q_i with respect to $g_i(\cdot|\hat{c}_i)$.

compatible *if every firm i finds it optimal to report its true cost, i.e.,*

$$\Pi_i(c_i) := \Pi_i(c_i, c_i) \geq \Pi_i(c_i, \hat{c}_i) \qquad \textit{for every } c_i, \hat{c}_i \in C.$$

The interim expected profit function can be rewritten in a more tractable way using the fact that the expression inside the expectation is quadratic in $E\left[q_{-i} \mid \hat{c}_i, \hat{q}_i\right]$. Let

$$Q_{-i}\left(c_i\right) = E_{\hat{q}_i}\left[E\left[q_{-i} \mid c_i, \hat{q}_i\right]\right]$$

be the expected output of the opponent when firm i reports c_i, and let

$$V_{-i}\left(c_i\right) = E_{\hat{q}_i}\left[\left(E\left[q_{-i} \mid c_i, \hat{q}_i\right] - Q_{-i}\left(c_i\right)\right)^2\right]$$

be the variance of the opponent's expected output when firm i reports c_i. Note that $V_{-i}\left(c_i\right)$ can be viewed as a degree of informativeness of the mediator's recommendations to firm i conditional on report c_i. Using this notation, the interim expected profit of firm i with cost c_i which has reported $\hat{c}_i$ can be rewritten as

$$\Pi_i(c_i, \hat{c}_i) = \frac{1}{4}\left(A - c_i - Q_{-i}(\hat{c}_i)\right)^2 + \frac{1}{4} V_{-i}(\hat{c}_i) \tag{6}$$

Thus the interim expected profit function is affine in V_{-i}. In particular, notice that in the above formula the variance V_{-i} does not interact with the cost c_i. Taking advantage of this "quasilinearity", we can show that the interim incentive compatibility is equivalent to two conditions: the expected output of the opponent is non-decreasing in the cost report, and the equilibrium interim expected profit is determined by the opponent's expected output function up to a constant. This result is analogous to a standard result in mechanism design for environments with quasilinear preferences.

Proposition 1 *An ex post incentive compatible mechanism is interim incentive compatible if and only if for every firm i*

(i) $Q_{-i}\left(c_i\right)$ *is nondecreasing;*

(ii) $\frac{1}{4} V_{-i}(c_i) = \Pi_i(c_i) - \frac{1}{4}\left(A - c_i - Q_{-i}(c_i)\right)^2$ *and* $\Pi_i(c_i) = \Pi_i(0) - \frac{1}{2}\int_0^{c_i}\left(A - \widetilde{c}_i - Q_{-i}(\widetilde{c}_i)\right) d\widetilde{c}_i$.

If Q_{-i} is differentiable at c_i, then $V_{-i}\left(c_i\right)$ and $Q_{-i}\left(c_i\right)$ are related as follows:

$$\frac{dV_{-i}\left(c_i\right)}{dc_i} = 2\left(A - c_i - Q_{-i}\left(c_i\right)\right)\frac{dQ_{-i}\left(c_i\right)}{dc_i} \tag{7}$$

Thus, if $A - c_i - Q_{-i}(c_i) > 0$ and Q_{-i} increases, then a firm of type c_i must be compensated with an increase in variance V_{-i} according to the above formula.[12]

Note that, other things being equal, the required increase in variance V_{-i} per unit of increase in Q_{-i} is smaller if c_i is high. Thus the high-cost firms value additional information relatively more than the low-cost firms. One possible interpretation is that the losses from the mismatch between the produced output and the residual demand are higher for the firms with high costs of production.

If A goes up, then the required increase in variance V_{-i} per unit of increase in Q_{-i} gets larger. As will be shown in Section 3.4, if A is high enough, then only mechanisms with constant Q_{-i} (uninformative mechanisms) are feasible, because the increase in the variance is prohibitively high.

3 Some General Properties

3.1 The Uniformly Worst Mechanism

In this section we derive a formula for the ex ante profits of the firms, and show that a mechanism that does not allow for any exchange of information results in the lowest possible profit among all feasible mechanisms. Starting from this section, we restrict attention to symmetric mechanisms.

An expression for the ex ante profit of firm i can be obtained by taking an expectation of the interim profits with respect to the prior probability distribution:

$$\Pi_i = \frac{1}{4} E_{c_i}\left[(A - c_i - Q_{-i}(c_i))^2\right] + \frac{1}{4} E_{c_i}\left[V_{-i}(c_i)\right].$$

By the law of iterated expectations, $E_{c_i}[Q_{-i}(c_i)] = E[q_{-i}]$, which we will denote as $E[q]$ since the mechanism is symmetric. Integration of condition (5) yields $E[q] = \frac{1}{3}(A - \mu)$. Hence, the ex ante profit of firm i can be rewritten as follows

$$\begin{aligned} \Pi_i &= \frac{1}{4}(A - \mu - E[q])^2 + \frac{1}{4} E_{c_i}\left[(c_i - \mu + Q_{-i}(c_i) - E[q])^2\right] + \frac{1}{4} E_{c_i}[V_{-i}(c_i)] \\ &= \frac{1}{9}(A - \mu)^2 + \frac{1}{4}\sigma^2 + \frac{1}{2} cov(c_i, Q_{-i}(c_i)) + \frac{1}{4} var(Q_{-i}(c_i)) + \frac{1}{4} E_{c_i}[V_{-i}(c_i)]. \end{aligned} \quad (8)$$

[12] If Q_{-i} has a jump at c_i, then it can be shown that

$$V_{-i}(c_i) - V_{-i}(c_i^-) = 2\left(A - c_i - \frac{Q_{-i}(c_i) + Q_{-i}(c_i^-)}{2}\right)(Q_{-i}(c_i) - Q_{-i}(c_i^-)).$$

The first two terms in the expression for Π_i are completely determined by the environment, and the last three terms depend on the mechanism. Since in every interim incentive compatible mechanism Q_{-i} is a nondecreasing function (Proposition 1), we have $cov\left(c_i, Q_{-i}\left(c_i\right)\right) \geq 0$. Combining this observation with the fact that $var\left(Q_{-i}\left(c_i\right)\right) \geq 0$ and $E_{c_i}\left[V_{-i}\left(c_i\right)\right] \geq 0$, we conclude that the ex ante profit in any feasible mechanism is at least $\frac{1}{9}\left(A-\mu\right)^2 + \frac{1}{4}\sigma^2$.

This lower bound is achieved by the following **uninformative mechanism**. Let $g^U\left(c_A, c_B\right) = \left(q_A(c_A), q_B(c_B)\right)$ for every (c_A, c_B), where $q_i(c_i)$ is the Bayesian-Nash equilibrium output of firm i with cost c_i defined by (3). We call such a mechanism "uninformative" because the report of the firm never influences the recommendation given to the opponent. Hence, in such a mechanism the interim incentive constraints are trivially satisfied. The ex post incentive constraints amount to the requirement that each firm's output choice is a best response to the strategy of the opponent given the prior beliefs, which is true by the definition of $q_i(c_i)$. Thus the uninformative mechanism is feasible for every A and for every distribution F. In this mechanism, $Q^U_{-i}\left(c_i\right) = E[q]$, and $V^U_{-i}\left(c_i\right) = 0$ for every c_i, which by (8) implies $\Pi^U_i = \frac{1}{9}\left(A-\mu\right)^2 + \frac{1}{4}\sigma^2$.[13]

Next we show that among all feasible mechanisms the uninformative mechanism not only yields the lowest possible ex ante profit, but it also results in the lowest interim profits for every cost type of every firm.

Proposition 2 *Any feasible mechanism interim Pareto dominates the uninformative mechanism for the firms:* $\Pi_i(c_i) \geq \Pi^U_i(c_i)$, $\forall c_i \in C$, $i \in \{A, B\}$.

Proof. Suppose there exists a mechanism such that the interim expected payoff of the firm of some type c_i is lower than that in the uninformative mechanism:

$$\Pi_i(c_i) = \frac{1}{4}\left(A - c_i - Q_{-i}(c_i)\right)^2 + \frac{1}{4}V_{-i}(c_i) < \Pi^U_i(c_i) = \frac{1}{4}\left(A - c_i - E[q]\right)^2$$

Consider the following deviation strategy: at the reporting stage the firm randomizes over all possible reports according to the prior distribution F; at the output choice stage, the firm behaves optimally given its beliefs. If the firm plays this deviation, then its expected opponent's quantity is $E_{c_i}\left[Q_{-i}\left(c_i\right)\right] = E[q]$, the variance is $var\left(Q_{-i}\left(c_i\right)\right) + E_{c_i}\left[V_{-i}(c_i)\right]$, and the payoff is

$$\frac{1}{4}\left(A - c_i - E[q]\right)^2 + \frac{1}{4}\left(var\left(Q_{-i}\left(c_i\right)\right) + E_{c_i}\left[V_{-i}(c_i)\right]\right) \geq \Pi^U_i\left(c_i\right)$$

[13] The outcome of the uninformative mechanism can be alternatively implemented without any pre-play communication, if the firms simply choose their outputs simultaneously given their prior beliefs.

Thus such a deviation results in a payoff at least as large as the payoff from the uninformative mechanism, which is a contradiction. ■

3.2 Bounds on the Ex Ante Profit

In this section we derive a number of properties of the ex ante profit. Let

$$\Delta\Pi_i = \Pi_i - \Pi_i^U = \frac{1}{2}cov\left(c_i, Q_{-i}\left(c_i\right)\right) + \frac{1}{4}var\left(Q_{-i}\left(c_i\right)\right) + \frac{1}{4}E_{c_i}\left[V_{-i}\left(c_i\right)\right]$$

and similarly denote $\Delta\Pi_i\left(c_i\right) = \Pi_i\left(c_i\right) - \Pi_i^U\left(c_i\right)$ for every c_i.

The components comprising the expression for $\Delta\Pi_i$ can be given an interpretation in terms of the amount of information revelation in a given mechanism. The last term, $\frac{1}{4}E_{c_i}\left[V_{-i}\left(c_i\right)\right]$, represents the additional profit of the firm from learning about the opponent's output and thus making more informed output choices. The first term, $\frac{1}{2}cov\left(c_i, Q_{-i}\left(c_i\right)\right)$, describes how well the production is coordinated between the firms: high value of the covariance means that the more efficient firm i is, the smaller the output of its opponent. Clearly, such coordination can be achieved only if the mechanism to some extent reveals to the firms the cost information reported by their opponents. The second term, $\frac{1}{4}var\left(Q_{-i}\left(c_i\right)\right)$, can be viewed as a companion to the first term: given the monotonicity of Q_{-i}, we have that $\frac{1}{2}cov\left(c_i, Q_{-i}\left(c_i\right)\right) \geq 0$ if and only if $\frac{1}{4}var\left(Q_{-i}\left(c_i\right)\right) \geq 0$.

Here are a few useful inequalities for the ex ante profit; they are all derived without imposing the interim incentive compatibility constraints.

Proposition 3 *In every symmetric ex post incentive compatible mechanism:*

(i) $\Delta\Pi_i \leq cov\left(c_i, Q_{-i}\left(c_i\right)\right)$;

(ii) $\Delta\Pi_i \leq \frac{1}{12}\sigma^2 + \frac{2}{3}cov\left(c_i, Q_{-i}\left(c_i\right)\right)$;

(iii) $\Delta\Pi_i \geq \frac{1}{2}cov\left(c_i, Q_{-i}\left(c_i\right)\right) + \frac{5}{4}\frac{1}{\sigma^2}cov^2\left(c_i, Q_{-i}\left(c_i\right)\right)$.

These inequalities demonstrate that the three components of the net ex ante profit $\Delta\Pi_i$ are closely linked to each other not only substantively, but quantitatively as well. For example, the inequality in part (i) of the result implies that no feasible mechanism can have high $E_{c_i}\left[V_{-i}\left(c_i\right)\right]$ and low $cov\left(c_i, Q_{-i}\left(c_i\right)\right)$ at the same time. On the other hand, by the inequality in part (iii), it is not possible to have high $cov\left(c_i, Q_{-i}\left(c_i\right)\right)$ and low $E_{c_i}\left[V_{-i}\left(c_i\right)\right]$.

Using inequality (i) we can show the following result.

Corollary 1 *Every correlated equilibrium is interim payoff equivalent to the uninformative mechanism.*

Proof. Since the firms are not allowed to exchange information in such mechanisms, it must be that $Q_{-i}(c_i)$ is constant and equal to $E[q]$ for every c_i. Hence, $cov(c_i, Q_{-i}(c_i)) = 0$. Part (i) of Proposition 3 then implies that $\Delta\Pi_i = 0$, and thus $E_{c_i}[V_{-i}(c_i)] = 0$. Since by Proposition 1 constant Q_{-i} implies that V_{-i} is constant, we have $V_{-i}(c_i) = 0$ for every c_i. ■

It is natural to expect that the highest possible profit $\Delta\Pi_i$ is realized when information is fully shared between the firms. To verify that this is the case, we introduce the following **full-revelation mechanism**: the mechanism where for any report profile (c_A, c_B), the mediator recommends each firm i the output it would choose if (c_A, c_B) was common knowledge. Thus the output of firm i is

$$q_i^{FR}(c_i, c_{-i}) = \frac{1}{3}A - \frac{2}{3}c_i + \frac{1}{3}c_{-i}$$

In this mechanism, $Q_{-i}^{FR}(c_i) = \frac{1}{3}A - \frac{2}{3}\mu + \frac{1}{3}c_i$, and $V_{-i}^{FR}(c_i) = \frac{4}{9}\sigma^2$ for every c_i. It is straightforward to verify that the net ex ante expected profit is $\Delta\Pi_i^{FR} = \frac{11}{36}\sigma^2$.

The full-revelation mechanism satisfies the ex post incentive constraints, but not the interim incentive constraints. Higher cost reports lead to higher expected output by the opponent (Q_{-i}^{FR} is strictly increasing), which is not compensated for by increased variance (V_{-i}^{FR} is constant). By Proposition 1, this is not interim incentive compatible.

Nonetheless the full-revelation mechanism is of interest because it results in the highest possible ex ante profit among all mechanisms that satisfy ex post incentive compatibility constraints.

Corollary 2 *The full-revelation mechanism ex ante Pareto dominates any feasible mechanism for the firms:* $\Delta\Pi_i \leq \Delta\Pi_i^{FR}$.

Proof. Combine the inequalities given in parts (ii) and (iii) of Proposition 3 to get

$$\frac{5}{4}\frac{1}{\sigma^2}\left(cov(c_i, Q_{-i}(c_i)) - \frac{1}{3}\sigma^2\right)\left(cov(c_i, Q_{-i}(c_i)) + \frac{1}{5}\sigma^2\right) \leq 0$$

Hence, $cov(c_i, Q_{-i}(c_i)) \leq \frac{1}{3}\sigma^2$, which, together with the upper bound on $\Delta\Pi_i$ from part (ii) of Proposition 3, yields the result. ■

3.3 Consumer Surplus and Total Welfare

In this section we discuss how consumer surplus and total welfare depend on the amount of information revelation.

It is straightforward to show that the ex ante consumer surplus is

$$CS = E\left[\frac{1}{2}(q_A + q_B)^2\right] = \frac{2}{9}(A-\mu)^2 + \frac{1}{4}\sigma^2 - \frac{1}{4}\left(var\left(Q_{-i}(c_i)\right) + E_{c_i}\left[V_{-i}(c_i)\right]\right)$$

As discussed earlier, the magnitudes of the terms $var(Q_{-i}(c_i))$ and $E_{c_i}[V_{-i}(c_i)]$ are related to the amount of information revelation in a given mechanism. The sum of these terms is maximized at the fully revealing mechanism and minimized at the uninformative mechanism. Hence, the consumer surplus is minimized at the fully revealing mechanism ($CS^{FR} = \frac{2}{9}(A-\mu)^2 + \frac{1}{9}\sigma^2$) and maximized at the uninformative mechanism ($CS^U = \frac{2}{9}(A-\mu)^2 + \frac{1}{4}\sigma^2$). Intuitively, information sharing makes oligopolists coordinate their outputs, which reduces the variability of aggregate output. Consumer surplus is diminished because it is a convex function of output.

The total welfare is

$$W = CS + \Pi_A + \Pi_B = \frac{4}{9}(A-\mu)^2 + \frac{3}{4}\sigma^2 + cov\left(c_i, Q_{-i}(c_i)\right) + \frac{1}{4}\left(var\left(Q_{-i}(c_i)\right) + E_{c_i}\left[V_{-i}(c_i)\right]\right)$$

Thus the total welfare is maximized at the fully revealing mechanism ($W^{FR} = \frac{4}{9}(A-\mu)^2 + \frac{11}{9}\sigma^2$) and minimized at the uninformative mechanism ($W^U = \frac{4}{9}(A-\mu)^2 + \frac{3}{4}\sigma^2$).

3.4 The Case Where No Communication is Possible

In this section we show that there are no feasible mechanisms that result in informative communication if the demand intercept A is high enough.[14] By the interim incentive compatibility, an increase in A raises the required compensation in variance V_{-i} per unit of increase in Q_{-i}.[15] Since the range of V_{-i} is bounded, this implies that the range of Q_{-i} must go to zero. On the other hand, part (i) of Proposition 3 shows that the ex ante level of profit from any mechanism is limited by the amount of variability in Q_{-i}, and thus the profits from any mechanism go to zero as A increases.

Proposition 4 *If $A > \overline{A} := 3 - 2\mu$, then all feasible mechanisms are interim payoff equivalent to the uninformative mechanism.*

We do not think that the bound presented in the above proposition is tight. Our conjecture is that informative mechanisms cease to exist as long the demand intercept A is above

[14]By equation (2), high A can be the result of the support of the cost distribution ($\overline{c} - \underline{c}$) being small, or the profit margin for the lowest cost type ($\tilde{A} - \underline{c}$) being high.

[15]See Section 2.3.

$2-\mu$, which is the level of A above which the "min" mechanism studied in the next section ceases to exist.

4 Some Simple Mechanisms

4.1 Cheap Talk

In this section we study a simple class of public deterministic mechanisms, which we call cheap-talk mechanisms. These mechanisms do not require a mediator and work as follows. First, each firm i chooses a message m_i from a set of available messages M_i. The messages chosen by the firms are publicly observed. Finally, the firms choose their outputs.[16]

Having observed a pair of messages (m_A, m_B), the firms update their beliefs and play according to the Bayesian-Nash equilibrium with the updated beliefs. Denote by $\mu_i(m_i)$ the expected cost of firm i conditional on sending the message m_i. Then the optimal output of firm i with cost c_i that has observed a pair of messages (m_i, m_{-i}) is

$$q_i(c_i; m_i, m_{-i}) = \frac{1}{3}A - \frac{1}{2}c_i + \frac{1}{3}\mu_{-i}(m_{-i}) - \frac{1}{6}\mu_i(m_i) \tag{9}$$

Note that $q_i(c_i; m_i, m_{-i})$ is separable in c_i, m_i, and m_{-i}. Because of this, one can prove that different messages carry the same information about the opponent's quantity, i.e. $V_i(c_{-i})$ does not depend on c_{-i}, and correspondingly $V_{-i}(c_i)$ does not depend on c_i. If all types of firm i produce positive quantities, then constant V_{-i} is incompatible with nonconstant Q_{-i}, since firm i prefers to send a message that induces the lowest Q_{-i} for all cost realizations. It is possible to construct cheap-talk mechanisms with constant V_{-i} and nonconstant Q_{-i}, but one has to allow for negative quantities to be chosen with positive probability.

These observations are summarized and formally proved by the following proposition.

Proposition 5 *Without loss of generality, each firm sends at most two messages in any feasible cheap talk mechanism. An informative cheap-talk mechanism is feasible if and only if* $A < \frac{7}{4} - \frac{3}{4}\mu$. *In any such mechanism, the output of at least one of the firms is negative with positive probability.*

An unappealing feature of the cheap-talk equilibria described above is that they require the output to be negative for some cost values. If a nonnegativity constraint is imposed on the

[16] By the revelation principle, it is possible to replicate the equilibrium outcomes of such indirect mechanisms using direct mechanisms where the mediator makes private recommendations of what output to choose to the firms. However, we find direct mechanisms less convenient in this context.

output, then the formula in footnote 12 implies that no informative cheap-talk equilibrium can exist.

4.2 The "1-step min" Mechanism

In this section we study a particularly simple class of public symmetric deterministic mechanisms, which in many cases allow for informative communication between the firms.

Suppose that after the firms report their costs to the mediator, the mediator publicly announces whether the minimum of the reported costs exceeds some particular threshold $c^* \in (0,1)$, after that the firms choose their outputs. Let us call such a mechanism the **"min" mechanism with threshold** c^*.

To understand how it works, first consider the case when it is publicly announced that $\min\{c_i, c_{-i}\}$ is above c^* (message m^1). Each firm learns that the opponent's cost is above c^*, and they choose outputs according to the Nash equilibrium strategies described in (3) with the updated beliefs:

$$q_i\left(c_i, m^1\right) = \frac{1}{3}A - \frac{1}{2}c_i + \frac{1}{6}E[c|c > c^*].$$

Next consider the case when it is publicly announced that $\min\{c_i, c_{-i}\}$ does not exceed c^* (message m^0). If firm i has reported that its cost is above c^*, then it learns that $c_{-i} \leq c^*$, and, expecting an aggressive opponent, reduces its output relative to the Nash equilibrium strategy with prior beliefs given in (3). If firm i has reported that its cost does not exceed c^*, then it does not learn anything about the opponent. Such a firm will produce more that in the Nash equilibrium with prior beliefs, because, as argued above, it expects the opponents with costs above c^* to behave less aggressively. Here are the firms' equilibrium strategies in this case:

$$q_i\left(c_i, m^0\right) = \begin{cases} \frac{1}{3}A - \frac{1}{2}c_i + \frac{1}{6}\mu + \frac{1}{6}\frac{1-F(c^*)}{1+F(c^*)}\left(\mu - E[c|c \leq c^*]\right), & \text{if } c_i \leq c^*; \\ \frac{1}{3}A - \frac{1}{2}c_i + \frac{1}{6}\mu - \frac{1}{6}\frac{2+F(c^*)}{1+F(c^*)}\left(\mu - E[c|c \leq c^*]\right), & \text{if } c_i > c^*. \end{cases}$$

Using the above, it can be calculated that if firm i reports $c_i \leq c^*$, then it receives

$$\begin{aligned} &Q_{-i}(c_i) = \underline{Q} := E[q] - \tfrac{1}{3}\tfrac{1-F(c^*)}{1+F(c^*)}\left(\mu - E[c|c \leq c^*]\right) \\ &V_{-i}(c_i) = 0 \end{aligned}$$

If it reports $c_i > c^*$, then

$$\begin{aligned} Q_{-i}(c_i) &= \overline{Q} := E[q] + \frac{1}{3}\frac{F(c^*)}{1+F(c^*)}\left(\mu - E[c|c \le c^*]\right) \\ V_{-i}(c_i) &= \overline{V} := \frac{4}{9}\frac{F(c^*)}{(1-F(c^*))(1+F(c^*))^2}\left(\mu - E[c|c \le c^*]\right)^2 \end{aligned}$$

Thus reporting the cost above c^* leads to higher opponent's expected quantity ($\overline{Q} > \underline{Q}$). However, reporting the cost above c^* provides the firm with the information whether the opponent's cost is above or below c^*, which is formally captured by the positive variance ($\overline{V} > 0$). To ensure that this mechanism is interim incentive compatible it is enough to choose threshold c^* to be the type of the firm that is indifferent between either report. No firm of type $c \neq c^*$ can gain from misreporting, since the high cost firms value information relatively more than the low cost firms.[17]

Suppose for a given demand intercept A there exists a feasible "min" mechanism. We can show that, under the standard hazard rate conditions on the distribution F, the feasible "min" mechanism is unique, and thus can be characterized by a unique threshold $c^*(A)$. If we increase the demand intercept to $A' > A$, then the "min" mechanism with threshold $c^*(A)$ will cease to be incentive compatible. In particular, the threshold type $c^*(A)$ will now strictly prefer to send a message that results in $\underline{Q}$ and zero variance.[18] Thus, in order to restore incentive compatibility, the threshold has to go up: $c^*(A') > c^*(A)$.

Proposition 6 *Suppose* $\frac{F(c)}{f(c)}$ *is nondecreasing and* $\frac{1-F(c)}{f(c)}$ *is nonincreasing. For every* $A < A^1 = 2 - \mu$ *there exists a unique feasible "min" mechanism with threshold* $c^*(A)$*, which is continuous and increasing in* A*.*

In contrast to the cheap-talk mechanism, the possibility of negative output is not crucial for the existence of an informative "min" mechanism. It is possible, although tedious, to show that if the non-negativity constraint on output is imposed, an informative "min" mechanism is feasible under the same set of conditions as without the constraint. If A is high enough, the non-negativity constraint is not binding at any $c_i \in [0, 1]$.

4.3 The "N-step min" Mechanism

The "N-step min" mechanism is a straightforward generalization of the "min" mechanism. Let $0 = c^0 < c^1 < ... < c^N < c^{N+1} = 1$. The **"$N$-step min" mechanism with thresholds** $(c^1, ..., c^N)$ works as follows. After the firms report their costs to the mediator, the mediator

[17] See discussion in Section 2.3.

[18] As discussed in Section 2.3, an increase in A changes the firm's preferences, causing it to place more value on reduction in the opponent's output relative to the information about the opponent's output.

makes a public announcement m^k ($k = 0, ..., N$) whenever the minimum of the reported costs is between c^k and c^{k+1}, and after that the firms choose their outputs.

When m^N is publicly announced, then it means that $\min\{c_i, c_{-i}\} > c^N$. Each firm learns that the opponent's cost is above c^N, and thus firm i strategy is

$$q_i\left(c_i, m^N\right) = \frac{1}{3}A - \frac{1}{2}c_i + \frac{1}{6}E[c|c > c^N].$$

When m^k with $k < N$ is publicly announced, then it means that $\min\{c_i, c_{-i}\} \in \left(c^k, c^{k+1}\right]$. If firm i has reported that $c_i > c^{k+1}$, then it learns that $c_{-i} \in \left(c^k, c^{k+1}\right]$; if firm i has reported that $c_i \in \left(c^k, c^{k+1}\right]$, then it learns only that $c_{-i} \in \left(c^k, 1\right]$. The firms' Nash equilibrium strategies in this case are

$$q_i\left(c_i, m^k\right) = \begin{cases} \frac{1}{3}A - \frac{1}{2}c_i + \frac{1}{6}\mu_{k,N+1} + \frac{1}{6}\frac{1-F\left(c^{k+1}|c>c^k\right)}{1+F\left(c^{k+1}|c>c^k\right)}\left(\mu_{k,N+1} - \mu_{k,k+1}\right) \text{ if } c_i \in \left(c^k, c^{k+1}\right]; \\ \frac{1}{3}A - \frac{1}{2}c_i + \frac{1}{6}\mu_{k,N+1} - \frac{1}{6}\frac{2+F\left(c^{k+1}|c>c^k\right)}{1+F\left(c^{k+1}|c>c^k\right)}\left(\mu_{k,N+1} - \mu_{k,k+1}\right) \text{ if } c_i \in \left(c^k, 1\right]; \end{cases}$$

where $\mu_{l,m} = E\left[c|c \in \left(c^l, c^m\right]\right]$ for every l and m such that $0 \leq l < m \leq N + 1$.

When F is uniform on $[0, 1]$, then it is possible to show that a feasible "2-step min" mechanism exists if and only if $A < A^2 \approx 1.1188$, while by Proposition 6 we know that a "min" mechanism exists whenever $A < A^1 = 1.5$. In general, we conjecture that the following is true for well-behaved distributions: for every $A < A^1$, there exists a nonnegative integer $N(A)$ such that, for every N with $1 \leq N \leq N(A)$, there exists a unique "N-step min" mechanism.[19] At the moment, however, we have the following result.

Proposition 7 *Suppose that* $\frac{F(c)-F(c')}{f(c)}$ *is nondecreasing in* c *for every* $c' < c$, *and* $\frac{1-F(c)}{f(c)}$ *is nonincreasing. If there exists a feasible "N-step min" mechanism, then there exists a feasible "min" mechanism.*

It is possible to show that for the uniform distribution the "2-step min" mechanism (whenever it exists) results in a higher ex ante profit than the "min" mechanism. We conjecture that for well-behaved distributions it is possible to show that the larger the number of steps in a "min" mechanism, the higher the ex ante profits.

[19] This conjecture is based on the similarity of the incentive compatibility conditions of the "N-step min" mechanisms to the equilibrium conditions in the model of Crawford and Sobel (1982).

5 Conclusion

We have shown that informative communication is possible in a static oligopoly model with unverifiable private information without any commitment or costly actions. The main idea is to link the precision of the informational feedback the sender receives to the message she sends. Generally, regardless of the firm's cost, a smaller output by the opponent is preferred to a larger output, and more information about the opponent's output choice is preferred to less information. However, different cost types put different relative weights on these two objectives, and thus may be given incentives to send different messages. We have confirmed that informative communication is possible by constructing a simple class of public symmetric deterministic mechanisms ("min" mechanisms). Though so far we have not solved for the ex ante profit maximizing communication mechanism, we have been able to provide a partial characterization of the feasible mechanisms.

In this paper, we have made a number of specific assumptions: two firms, symmetry, linear demand, constant marginal costs. However, it seems to us that in many related environments either the mechanisms presented here, or mechanisms that rely on similar ideas, can be used for informative communication. For example, consider a situation where only firm A has private information about costs, and firm B's cost is commonly known. In such a case public or deterministic mechanisms cannot support informative communication: firm A can precisely anticipate firm B's output choice, and thus there is no residual uncertainty about firm B's output (which is essential for sustaining information revelation by firm A). Nonetheless, it can be shown that the following stochastic mechanism with private signals works. After receiving the cost report from firm A, a mediator sends a noisy (but informative) private signal to firm B, and, in addition, a blind carbon copy of this signal is sent to firm A if and only if its reported costs are high. As a result, the types of firm A that report high costs expect on average a higher output by firm B, but are compensated by the information useful for predicting firm B's output.

6 Appendix

Proof of Proposition 1. *(Only if)* Interim incentive compatibility for every $c_i, c_i' \in C$ and (6) implies

$$\frac{1}{4}\left(A - c_i - Q_{-i}(c_i)\right)^2 + \frac{1}{4}V_{-i}(c_i) \geq \frac{1}{4}\left(A - c_i - Q_{-i}(c_i')\right)^2 + \frac{1}{4}V_{-i}(c_i');$$
$$\frac{1}{4}\left(A - c_i' - Q_{-i}(c_i')\right)^2 + \frac{1}{4}V_{-i}(c_i') \geq \frac{1}{4}\left(A - c_i' - Q_{-i}(c_i)\right)^2 + \frac{1}{4}V_{-i}(c_i)$$

Adding these two inequalities results in

$$(c_i' - c_i)\left(Q_{-i}(c_i') - Q_{-i}(c_i)\right) \geq 0,$$

which implies monotonicity. The envelope formula follows from (6) and the generalized Envelope Theorem (Corollary 1 in Milgrom and Segal, 2002).

(*If*) For every $c_i, c_i' \in C$,

$$\Pi_i\left(c_i, c_i'\right) = \frac{1}{4}\left(A - c_i - Q_{-i}(c_i')\right)^2 + \frac{1}{4}V_{-i}(c_i') = \frac{1}{4}\left(A - c_i' - Q_{-i}(c_i')\right)^2 + \frac{1}{4}V_{-i}(c_i')$$
$$+ \frac{1}{2}\int_{c_i}^{c_i'}\left(A - c - Q_{-i}(c_i')\right)dc = \Pi_i(c_i') + \frac{1}{2}\int_{c_i}^{c_i'}\left(A - c - Q_{-i}(c_i')\right)dc$$

Therefore

$$\Pi_i\left(c_i\right) - \Pi_i\left(c_i, c_i'\right) = \Pi_i(c_i) - \Pi_i(c_i') - \frac{1}{2}\int_{c_i}^{c_i'}\left(A - c - Q_{-i}(c_i')\right)dc$$
$$= \frac{1}{2}\int_{c_i}^{c_i'}\left(Q_{-i}(c_i') - Q_{-i}(c)\right)dc \geq 0.$$

■

Proof of Proposition 3. (i) Notice that

$$cov\left(E\left[q_{-i} \mid c_i, q_i\right], E\left[q_i \mid c_{-i}, q_{-i}\right]\right) = cov\left(2q_i + c_i, 2q_j + c_j\right) = 4cov\left(q_i, q_j\right) + 4cov\left(c_i, Q_{-i}\left(c_i\right)\right)$$
$$= 4\left(-\frac{1}{2}cov\left(c_i, Q_{-i}\left(c_i\right)\right) - \frac{1}{2}var\left(E\left[q_{-i} \mid c_i, q_i\right]\right)\right) + 4cov\left(c_i, Q_{-i}\left(c_i\right)\right)$$
$$= 2cov\left(c_i, Q_{-i}\left(c_i\right)\right) - 2var\left(E\left[q_{-i} \mid c_i, q_i\right]\right)$$

Also note that

$$cov\left(E\left[q_{-i} \mid c_i, q_i\right], E\left[q_i \mid c_{-i}, q_{-i}\right]\right) \geq -var\left(E\left[q_{-i} \mid c_i, q_i\right]\right)$$

This implies that

$$var\left(E\left[q_{-i} \mid c_i, q_i\right]\right) \leq 2cov\left(c_i, Q_{-i}\left(c_i\right)\right)$$

Applying this inequality to the expression for the ex ante profit given in (8) and using the fact that $var\left(E\left[q_{-i} \mid c_i, q_i\right]\right) = var\left(Q_{-i}\left(c_i\right)\right) + E_{c_i}\left[V_{-i}\left(c_i\right)\right]$ gives the result.

(ii) First note that condition (5) in the definition of ex post incentive compatible mechanisms implies that

$$var\left(q_i\right)=\frac{1}{4}\sigma^2+\frac{1}{2}cov\left(c_i,Q_{-i}\left(c_i\right)\right)+\frac{1}{4}var\left(E\left[q_{-i}\mid c_i,q_i\right]\right).$$

Using expression (8) we have

$$\Delta\Pi_i=var\left(q_i\right)-\frac{1}{4}\sigma^2.$$

Using the fact that $var\left(E\left[q_{-i}\mid c_i,q_i\right]\right)\le var\left(q_i\right)$ we get

$$\Delta\Pi_i\le\frac{1}{2}cov\left(c_i,Q_{-i}\left(c_i\right)\right)+\frac{1}{4}var\left(q_i\right)=\frac{1}{2}cov\left(c_i,Q_{-i}\left(c_i\right)\right)+\frac{1}{4}\Delta\Pi_i+\frac{1}{16}\sigma^2$$

The result follows from rearranging the terms in the above expression.

(iii) Note that

$$\begin{aligned}
0 &\le var\left(q_i+\frac{1}{2}\left(1+\beta\right)c_i-\beta c_j\right)\\
&= var\left(q_i\right)+\left(\frac{1}{4}\left(1+\beta\right)^2+\beta^2\right)\sigma^2+\left(1+\beta\right)cov\left(c_i,q_i\right)-2\beta cov\left(c_j,q_i\right)\\
&= var\left(q_i\right)+\left(\frac{1}{4}\left(1+\beta\right)^2+\beta^2\right)\sigma^2+\left(1+\beta\right)\left(-\frac{1}{2}\sigma^2-\frac{1}{2}cov\left(c_i,Q_{-i}\left(c_i\right)\right)\right)-2\beta cov\left(c_i,Q_{-i}\left(c_i\right)\right)\\
&= \Delta\Pi_i+\frac{5}{4}\beta^2\sigma^2-\left(\frac{1}{2}+\frac{5}{2}\beta\right)cov\left(c_i,Q_{-i}\left(c_i\right)\right).
\end{aligned}$$

Take $\beta=\frac{1}{\sigma^2}cov\left(c_i,Q_{-i}\left(c_i\right)\right)$ and rearrange to get the result. ■

Proof of Proposition 4. Using the Envelope formula (see Proposition 1) and integration by parts:

$$\Pi_i=\Pi_i\left(0\right)+\int_0^1\left(\int_0^{c_i}\frac{d}{dc_i}\Pi_i\left(\widetilde{c}_i\right)d\widetilde{c}_i\right)dF\left(c_i\right)=\Pi_i\left(0\right)-\frac{1}{2}\int_0^1\left(A-c-Q_{-i}(c)\right)\left(1-F\left(c_i\right)\right)dc_i$$

Hence

$$\Delta\Pi_i=\Delta\Pi_i\left(0\right)-\frac{1}{2}\int_0^1\left(E\left[q\right]-Q_{-i}(c)\right)\left(1-F\left(c_i\right)\right)dc_i=\Delta\Pi_i\left(0\right)+\frac{1}{2}cov\left(\frac{1-F\left(c_i\right)}{f\left(c_i\right)},Q_{-i}\left(c_i\right)\right)$$

Combining this formula with the upper bound on $\Delta\Pi_i$ from part (i) of Proposition 3 yields

$$0\le cov\left(c_i-\frac{1}{2}\frac{1-F\left(c_i\right)}{f\left(c_i\right)},Q_{-i}\left(c_i\right)\right)-\Delta\Pi_i\left(0\right)$$

Denote $\psi(c_i) = c_i - \frac{1}{2}\frac{1-F(c_i)}{f(c_i)}$. Notice that

$$\begin{aligned} cov(\psi(c_i), Q_{-i}(c_i)) &= \int_0^1 \psi(c_i) Q_{-i}(c_i) dF(c_i) - E[\psi(c_i)] E[q] \\ &= \int_0^1 \psi(c_i)(Q_{-i}(c_i) - Q_{-i}(0)) dF(c_i) - E[\psi(c_i)](E[q] - Q_{-i}(0)) \end{aligned}$$

Since $\psi(c_i) \leq \psi(1) = 1$, and $E[\psi(c_i)] = \frac{1}{2}\mu$, we have

$$\begin{aligned} cov(\psi(c_i), Q_{-i}(c_i)) &\leq \int_0^1 (Q_{-i}(c_i) - Q_{-i}(0)) dF(c_i) - E[\psi(c_i)](E[q] - Q_{-i}(0)) \\ &= \left(1 - \frac{1}{2}\mu\right)(E[q] - Q_{-i}(0)) \end{aligned}$$

Next note that

$$\begin{aligned} \Delta\Pi_i(0) &= \frac{1}{4}(A - Q_{-i}(0))^2 + \frac{1}{4}V_i(0) - \frac{1}{4}(A - E[q])^2 \\ &= \frac{1}{2}(A - E[q])(E[q] - Q_{-i}(0)) + \frac{1}{4}(E[q] - Q_{-i}(0))^2 + \frac{1}{4}V_{-i}(0) \\ &\geq \frac{1}{2}(A - E[q])(E[q] - Q_{-i}(0)) = \left(\frac{1}{3}A + \frac{1}{6}\mu\right)(E[q] - Q_{-i}(0)) \end{aligned}$$

where the last equality uses the definition of $E[q]$. Combining the above observations we get

$$\begin{aligned} 0 &\leq cov\left(c_i - \frac{1}{2}\frac{1-F(c_i)}{f(c_i)}, Q_{-i}(c_i)\right) - \Delta\Pi_i(0) \\ &\leq \left(\left(1 - \frac{1}{2}\mu\right) - \left(\frac{1}{3}A + \frac{1}{6}\mu\right)\right)(E[q] - Q_{-i}(0)) = \frac{1}{3}(3 - 2\mu - A)(E[q] - Q_{-i}(0)) \end{aligned}$$

Note that $\Delta\Pi_i > 0$ implies that $E[q] > Q_{-i}(0)$. Hence, $A \leq 3 - 2\mu$. ■

Proof of Proposition 5. Given a cheap-talk mechanism, let $\widetilde{Q}_{-i}(m_i)$ and $\widetilde{V}_{-i}(m_i)$ be the expectation of the opponent's output and the variance of the opponent's expected output when firm i sends message m_i. Using equation (9), we have

$$\widetilde{Q}_{-i}(m_i) = E_{c_{-i},m_{-i}}[q_{-i}(c_{-i}, m_{-i}, m_i) \mid m_i] = \frac{1}{3}A - \frac{2}{3}\mu + \frac{1}{3}\mu_i(m_i) \tag{10}$$

and

$$\widetilde{V}_{-i}(m_i) = E_{m_{-i}}\left[\left(E_{c_{-i}}[q_{-i}(c_{-i}, m_{-i}, m_i) \mid m_i, m_{-i}] - \widetilde{Q}_{-i}(m_i)\right)^2\right] = \frac{4}{9}E_{m_{-i}}\left[(\mu_{-i}(m_{-i}) - \mu)^2\right]$$

Hence, $\widetilde{V}_{-i}(m_i)$ is constant across m_i and can be denoted by $\widetilde{V}_{-i}$.

Using the arguments similar to the ones in Section 2.3, the expected profit of firm i with cost c_i which has sent message m_i, and consequently chooses the output optimally given the observed messages, is

$$\Pi_i(c_i; m_i) = \frac{1}{4}\left(A - c_i - \widetilde{Q}_{-i}(m_i)\right)^2 + \frac{1}{4}\widetilde{V}_{-i}$$

Next we argue that without loss of generality we can restrict attention to cheap talk equilibria such that for each Q_{-i} which can be induced in equilibrium, there is a unique message that induces it. Introduce a new message $m_i(Q_{-i})$ for every value of Q_{-i} that is induced in equilibrium. In the equilibrium strategy of firm i, replace each message $\widetilde{m}_i$ by message $m_i(Q_{-i})$ such that $\widetilde{Q}_{-i}(\widetilde{m}_i) = Q_{-i}$. Note that by equation (10), $\mu_i(\widetilde{m}_i) = \mu_i(m_i(Q_{-i}))$. Thus by equation (9) the new strategy profile leads to the same output choices as the original equilibrium.

Next we show that without loss of generality we can restrict attention to equilibria where each firm sends at most two messages. Suppose there is an equilibrium where three distinct values of Q_{-i} (call them Q'_{-i}, Q''_{-i} and Q'''_{-i}) are each induced by a positive measure of types of firm i. Without loss of generality, suppose that $Q'_{-i} < Q''_{-i} < Q'''_{-i}$. The profit of the firm is convex in Q_{-i}, and thus

$$\frac{1}{4}\left(A - c_i - Q''_{-i}\right)^2 < \max\left\{\frac{1}{4}\left(A - c_i - Q'_{-i}\right)^2, \frac{1}{4}\left(A - c_i - Q'''_{-i}\right)^2\right\}$$

Hence there are no equilibria where more than two distinct values of Q_{-i} are induced with positive probability, and thus we can restrict attention to equilibria where each firm sends at most two messages.

Now suppose there is an informative cheap-talk mechanism, and firm i with positive probability sends two messages that lead to the opponent's expected outputs $\underline{Q}_{-i}$ and $\overline{Q}_{-i}$ such that $\underline{Q}_{-i} < \overline{Q}_{-i}$. By the monotonicity property of interim incentive compatible mechanisms (Proposition 1), there exists $c_i^* \in (0,1)$ such that all types below c_i^* induce $\underline{Q}_{-i}$, and above c_i^* induce $\overline{Q}_{-i}$. Hence

$$\begin{aligned}
\underline{Q}_{-i} &= \frac{1}{3}A - \frac{2}{3}\mu + \frac{1}{3}E\left[c|c < c_i^*\right]; \\
\overline{Q}_{-i} &= \frac{1}{3}A - \frac{2}{3}\mu + \frac{1}{3}E[c|c > c_i^*].
\end{aligned}$$

The condition that type c_i^* is indifferent between inducing $\underline{Q}_{-i}$ and $\overline{Q}_{-i}$ can be rewritten as

$$A = \frac{3}{2}c_i^* - \mu + \frac{1}{4}\left(E[c|c > c_i^*] + E[c|c < c_i^*]\right) \tag{11}$$

It is easy to see that the right-hand side of the above expression increases in c_i^*, equals $-\frac{3}{4}\mu$ when $c_i^* = 0$ and equals $\frac{7}{4} - \frac{3}{4}\mu$ when $c_i^* = 1$. Hence an informative cheap-talk mechanism is feasible if and only if $A < \frac{7}{4} - \frac{3}{4}\mu$.

Finally, note that in the above mechanism the expected quantity of firm i of type $c_i > c_i^*$ is

$$\begin{aligned} E_{m_{-i}}\left[q_i(c_i; m_i, m_{-i})\right] &= \frac{1}{2}\left(A - c_i - \overline{Q}_{-i}\right) = \frac{1}{3}\left(A - \frac{3}{2}c_i + \mu - \frac{1}{2}E[c|c > c_i^*]\right) \\ &= \frac{1}{12}\left(E[c|c < c_i^*] - E[c|c > c_i^*]\right) < 0 \end{aligned}$$

where the last equality is by equation (11). ■

Proof of Proposition 6. Suppose for a given A there a exists a "min" mechanism with threshold c^*. The indifference condition for c^* is

$$\left(A - c^* - \underline{Q}\right)^2 = \left(A - c^* - \overline{Q}\right)^2 + \overline{V}$$

Substituting the expressions for $\underline{Q}$, $\overline{Q}$, $\overline{V}$ and $E[q]$ and rearranging results in

$$A = \frac{3}{2}c^* - \frac{1}{2}\mu + \frac{1}{4}G\left(F\left(c^*\right)\right)\left(E[\widetilde{c}|\widetilde{c} \geq c^*] - E[\widetilde{c}|\widetilde{c} \leq c^*]\right)$$

where $G\left(x\right) = \frac{-2x^2+7x-1}{1+x}$. Denote the right-hand side of the above expression by $\Phi\left(c^*\right)$. Note that $\Phi\left(1\right) = 2 - \mu$, and $\Phi\left(0\right) = -\frac{3}{4}\mu$. Hence, to prove the proposition it is enough to show that $\Phi'\left(c\right) > 0$ for every c:

$$\begin{aligned} \Phi'\left(c\right) &= \frac{3}{2} + \frac{1}{4}G'\left(F\left(c^*\right)\right)f\left(c^*\right)\left(E[\widetilde{c}|\widetilde{c} \geq c] - E[\widetilde{c}|\widetilde{c} \leq c]\right) \\ &\quad + \frac{1}{4}G\left(F\left(c^*\right)\right)\left(\frac{d}{dc}\left(E[\widetilde{c}|\widetilde{c} \geq c] - E[\widetilde{c}|\widetilde{c} \leq c]\right)\right) \end{aligned}$$

The second term in the above expression is nonnegative, since $G'\left(x\right) > 0$ for every $x \in [0, 1]$.

To evaluate the third term, note that

$$\begin{aligned}\frac{d}{dc}\left(E[\widetilde{c}|\widetilde{c}\geq c]-E[\widetilde{c}|\widetilde{c}\leq c]\right) &= \frac{d}{dc}\left(\int_c^1 \frac{1-F(\widetilde{c})}{1-F(c)}d\widetilde{c}+\int_0^c \frac{F(\widetilde{c})}{F(c)}d\widetilde{c}\right)\\ &= \left(\int_c^1 (1-F(\widetilde{c}))\,d\widetilde{c}\right)\frac{f(c)}{(1-F(c))^2}-\left(\int_0^c F(\widetilde{c})\,d\widetilde{c}\right)\frac{f(c)}{F^2(c)}\end{aligned}$$

where the first equality uses integration by parts. Using the conditions on the hazard rates:

$$\begin{aligned}\int_c^1 (1-F(\widetilde{c}))\,d\widetilde{c} &= \int_c^1 \frac{1-F(\widetilde{c})}{f(\widetilde{c})}dF(\widetilde{c})\leq \int_c^1 \frac{1-F(c)}{f(c)}dF(\widetilde{c})=\frac{(1-F(c))^2}{f(c)}, \text{ and}\\ \int_0^c F(\widetilde{c})\,d\widetilde{c} &= \int_0^c \frac{F(\widetilde{c})}{f(\widetilde{c})}dF(\widetilde{c})\leq \int_0^c \frac{F(c)}{f(c)}dF(\widetilde{c})=\frac{F^2(c)}{f(c)}\end{aligned}$$

This implies that

$$\frac{d}{dc}\left(E[\widetilde{c}|\widetilde{c}\geq c]-E[\widetilde{c}|\widetilde{c}\leq c]\right)\in[-1,1].$$

Thus if $G(F(c))\geq 0$, then

$$\Phi'(c)\geq \frac{3}{2}-\frac{1}{4}G(F(c))\geq \frac{3}{2}-\frac{1}{2}>0$$

where the second inequality follows from $G(F(c))\leq G(1)=2$. If $G(F(c))<0$, then

$$\Phi'(c)\geq \frac{3}{2}+\frac{1}{4}G(F(c))\geq \frac{3}{2}-\frac{1}{4}>0$$

where the second inequality follows from $G(F(c))\geq G(0)=-1$. ■

Proof of Proposition 7. Suppose for a given A there exists a "N-step min" mechanism with thresholds $(c^1,...,c^N)$. The indifference condition for c^N can be written as follows:

$$A=\frac{3}{2}c^N-\frac{1}{2}\mu_{N-1,N+1}+\frac{1}{4}G\left(F\left(c^N\mid c>c^{N-1}\right)\right)(\mu_{N,N+1}-\mu_{N-1,N})$$

where $G(x)=\frac{-2x^2+7x-1}{1+x}$. Denote the right-hand side of the above expression by $\Phi\left(c^{N-1},c^N\right)$. Using an argument identical to the one in Proposition 6, it is straightforward to show that Φ is increasing in c^N. Thus

$$\begin{aligned}A &= \Phi\left(c^{N-1},c^N\right)\leq \Phi\left(c^{N-1},1\right)=\frac{3}{2}-\frac{1}{2}\mu_{N-1,N+1}+\frac{1}{4}G(1)(1-\mu_{N-1,N+1})\\ &= 2-\mu_{N-1,N+1}\leq 2-\mu\end{aligned}$$

where the last equality follows from $G(1)=2$. Hence, by Proposition 6, we know that a feasible “min” mechanism exists for such A. ■

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