Life Cycle Portfolio Choice in Taxable and Tax-deferred Accounts

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(preliminary, incomplete)

Abstract

In this paper we study the optimal life cycle portfolio choice for households with both taxable accounts (TA) and tax-deferred accounts (TDA). According to the 2001 Survey of Consumer Finances (SCF), 46% of households have positive balances in both accounts and their investment in stock represents 87% of total stock holding of the entire SCF population. There are completely different patterns over life cycle in terms of stock market participation rate and stock share in TA and TDA. As a fraction of these households, the stockholders in TDA decrease considerably from 90% when households are young to less than 50% when they are old, while the stockholders in TA increase from 34% to 70%. For stock market participants, the mean (median) stock share in TDA is very high and varies little over life cycle, while the mean (median) stock share in TA is decreasing with age. We calibrate a life cycle model which generates patterns of stock share in TA and TDA for participants similar in the data. The model explicitly incorporates labor income risk, progressive tax, accessibility restrictions on TDA and other features.

Keywords: Portfolio Choice, Life Cycle, Participation, Tax-deferred Accounts.

JEL Classifications: G11

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1 Introduction

Recently there has been an increasing interest in studying household portfolio composition due to the availability and sophistication of micro-data sets.\(^1\) As more households rely on tax-deferred assets to finance their retirement expenditures, assets held in tax-deferred accounts\(^2\) are a large and growing component of household net wealth. The U.S. Flow of Funds Accounts show that assets held in tax-deferred accounts (IRA and various types of defined contribution pension plans) represented 16.8% of total financial assets at the end of 2001.\(^3\) According to the 2001 Survey of Consumer Finances (SCF), 45.54% of households have assets in both taxable (TA) and tax-deferred accounts (TDA).\(^4\) The total amount of assets held in TDA by these households accounted for about 30% of their total financial assets.

The question facing these households is how the different tax-treatments in taxable and tax-deferred accounts affect their portfolio choices between TA and TDA. What is the optimal life-cycle portfolio composition in both accounts? Should they hold the same proportion of stocks and bonds in both accounts or only hold highly-taxed assets (like bonds) in TDA first as suggested by tax-arbitrage argument or do the opposite?

The 2001 SCF shows that for households having assets in both TA and TDAAs, the fraction of stockholders in TDA has decreased considerably from 90% when households are young to less than 50% when they are old, while the fraction of stockholders in TA has increased from about 34% to about 70%. It is important to realize that a significant fraction of households do not participate in stock market in TA. Conditional on participation in stock market, the mean stock share in TDA is about 80%. This share is roughly stable over life cycle with a slightly lower share for households over 75. The mean (median) stock share in TA is decreasing with age. It drops from 75% (92%) when households are young to about 54% (53%) when they are old.

The goal of this paper is to contribute to the literature on portfolio choice in both taxable

\(^1\)These studies include Ameriks and Zeldes (2001), Vissing-Jorgensen (2002), Bergstresser and Poterba (2004) and papers in Guiso, Haliassos and Jappelli (2002).

\(^2\)In U.S., examples of tax-deferred accounts include IRA, KEOGH, 401(k), 403(b), etc. Usually the owners of these accounts have some control on how to invest the money in these accounts.

\(^3\)See table 1 in Bergstresser and Poterba (2004).

\(^4\)For TA, we exclude checking accounts. See the detailed definition in next section.
and tax-deferred accounts through a careful analysis of how households’ portfolio choices, and particularly investments in the stock market, vary with households’ age. We need to model and examine at least three choices: the decision to save in which account, the decision to participate in stock market or not in each account, and (conditional on participation) the choice of how much stock to hold in each account. We solve numerically for the savings decisions and portfolio choices in TA and TDA using a realistically calibrated life cycle model. Households in the model face stochastic labor income, borrowing and short-sale constraints, progressive tax system and different tax treatment and withdrawal policy in TA and TDA.

The portfolio allocation implied by the model (our numerical analysis) is in general consistent with the data. For stock market participants in TDA, they allocate most (even all for some households) of their TDA wealth to stocks in the model as in the data. The mechanisms are the following: First, since the average returns on stocks are higher, the balance in TDA grows faster if stocks are held in TDA as suggested by Shoven (1999). Second, given progressive tax system, tax difference between interest income and capital gain is lower for most households, which lowers the benefit of holding taxable bonds in TDA. Third, from precautionary point of view, households would like to hold riskless assets in TA since there is penalty on early withdrawal from TDA, thus stock will be held in TDA. For stock market participants in TA, the mean (median) stock share in TA is decreasing with age in the model. We observe a similar pattern in the data. Since future labor income (human capital) acts as a substitute for the riskless asset bonds\(^5\), young households allocate a larger fraction of their financial wealth in TA to stocks. As age increases, the present value of future labor income decreases and households respond by shifting more wealth out of stocks in order to offset the decline in the value of their human capital.

For stock market participation rate in both accounts, our benchmark shows that the participation rate is very high in TDA over life cycle (almost all households choose to participate in TDA) and it drops with age in TA. The data tell us that the participation rate is high in TDA when households are young and drops when households are old, while the participation rate is low in TA when households are young and increases with age in TA. To solve this disparity, we consider a fixed stock market entry cost in the paper. This

\(^5\)As in Bodie, Merton and Samuelson (1992) and Jogannathan and Kochemlakota (1996).
entry cost deters many young households from participating in stock market.\textsuperscript{6} Once the entry cost is added, we find that stock market participation rate is lower when households are young.

Economists have long been interested in the effect of taxation on household saving and portfolio choice.\textsuperscript{7} The implications of tax-deferred accumulation opportunities for household portfolios in TA and TDA have attracted much research attention recently. This paper is closely related to the asset location\textsuperscript{8} puzzle in the literature – asset location decisions observed in practice deviate substantially from the prediction of models.

Standard theory suggests that tax-deferred investments should be specialized in higher-taxed assets (like bonds). Here tax efficiency is the dominant consideration.\textsuperscript{9} According to this idea, households are advised never to hold higher-taxed assets (like bonds) in TA, as long as there exists an opportunity to move such assets to TDA. Black (1980) and Tepper (1981) first made this tax arbitrage argument when they analyze the optimal investment policy for corporations having the choice between two accounts with different tax treatments. Some recent studies support the idea. Huang (2003) confirms the location result of preferring bonds in the TDA using the same argument in a multi-period setting. The result is based on the ability to replicate asset holding in TDA by a portfolio held in TA.

Dammon, Spatt and Zhang (2004) solve numerically the optimal location and allocation decisions and show that taxable bonds have a preferred location in TDA and stocks in TA. They assume that pre-tax nonfinancial (labor) income is a constant fraction (15\%) of the investor’s total beginning-of-period wealth prior to age 65 and zero thereafter. This type of income process simplifies the problem, but it overstates households’ wealth in earlier ages and understates the income in older ages (since there is no social security benefit). This will dramatically affect households’ portfolio choices in TA and TDA. Second, they do not match the stock market participation rate in their exercise. We notice that in the data stock market participation rate is very different in TA and TDA. Third, proportional tax rates

\textsuperscript{6}Previous studies have considered the entry costs, see Haliassos and Michaelides (2003) and Gomes and Michaelides (2005). Moreover, empirical work suggests that small entry costs can be consistent with the observed low stock market participation rates, see Monica Paiella (2001) and Vissing-Jorgenson (2002).

\textsuperscript{7}See Bernheim (2002) and Poterba (2002) for excellent surveys.

\textsuperscript{8}Asset location means households should decide where to hold assets, in TA or TDA.

\textsuperscript{9}An asset allocation is said to be tax-efficient if the highest-taxed asset is always located in the tax-deferred accounts before any lower-taxed asset can be placed there.
are used in their setting. Given the progressive tax system in the reality, tax difference between interest income and stock return is much lower for most households, which will potentially lower the tax benefit of holding taxable bonds in TDA. Zaman (2005) adds one more risky asset in the setting of Dammon, Spatt and Zhang (2004) and analyzes the impact of diversification concern on the asset location decision. He finds that stock can be held in TDA in some cases.

Empirical studies normally show that investors maintain higher stock positions in TDA, see Bodie and Crane (1997), Amromin (2003), Bergstresser and Poterba (2004). Some studies attempt to solve this disparity. Poterba, Shoven and Sialm (2001) and Shoven and Sialm (2004) show that actively-managed equity mutual funds should be held in TDA. Because actively-managed equity mutual funds impose substantial tax burden on their investors, this asset location pattern is broadly consistent with tax-minimizing behavior. Other research claims that it is possible for investors to find that holding stock in TDA is the preferred strategy. Shoven (1999) uses static examples to show that the conventional wisdom of first placing heavily taxed corporate bonds in the pension account (and holding equity mutual funds outside the account) is the wrong asset location strategy for most people and most circumstances. The reason for this conclusion is (p.10): “By placing the asset with the higher gross rate of return in the pension plan, this preferred environment grows more rapidly to the advantage of the retirement saver.” Mintz and Smart (2002) provide an explanation based on differences between tax-deferred and taxable assets in the tax treatment of capital losses. Amromin (2003) uses precautionary savings motive to explain the puzzle. The 3-period model explicitly incorporates both the uninsurable risk in labor income (labor income uncertainty is modeled exclusively as an unemployment shock, with 0.5% probability of losing 99% of labor income) and accessibility restrictions that are an institutional feature of TDA. The key message is that stricter accessibility restrictions and stronger precautionary motives are associated with lower equity shares in regular taxable accounts, but with higher equity shares in TDA.

Recently Gomes, Michaelides, and Polkovnichenko (2004) adopt Epstein-Zin preferences and heterogeneous preferences to match wealth accumulation profiles for households having assets in both accounts and assess the cost of deviations from the optimal TDA contribution. They exogenously divide the households into two groups: indirect stockholders (own stocks
only in TDA) and direct stockholders (hold stocks in TD and may also in TDA). Thus stock market participation choice in TA is omitted. We consider the stock market participation choice in both accounts. All papers mentioned above use proportional tax rates. We incorporate progressive tax system in our model.

The paper is also related to the stock market non-participation puzzle and literature on life-cycle portfolio profile. Empirical studies show that there is significant non-stockownership and the age profile of participation in stock market is hump-shaped. The participation rate peaks in the 50-59 age bracket. Models on portfolio choices with market frictions (transaction costs and short sales constraints), stochastic income and stock returns predict that, given the equity premium and the assumption of CRRA utility, all households should participate in the stock market as long as saving takes place. These models include both infinite-horizon and finite-horizon, life cycle setting. Lots of papers try to solve the puzzle. All these paper did not consider the difference of TA and TDA. In the data we find that stock market participation rate in TA is low and it is generally increasing in TA, while the participation rate in TDA is high and it drops in older age. It is important to study the different pattern of participation rate in both accounts.

For the literature on life-cycle portfolio profile, traditional models show that retirement is irrelevant for portfolio decisions under some conditions (homothetic preferences, frictionless markets, constant investment opportunities and no labor income or non-tradable assets).

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Thus risk asset share of financial asset is fixed. Common financial advisors suggest that investors should invest more in stocks when they are young and shift their investments towards safe assets when they are getting old. The basic economic reason is that even though labor income is risky and non-tradable, it acts, on balance, as a substitute for the risk-free asset. For recent studies, Viceira (2001) supports the typical recommendations of financial advisors; Cocco, Gomes and Maenhout (2005) and Gomes and Michaelides (2005) solve numerically a life-cycle portfolio choice model and show that the optimal share of financial wealth in equities is generally decreasing with age. Once again, these papers are not interested in asset location and do not distinguish TA and TDA. We show that the decreasing stock share over life-cycle is optimal in TA, while it is optimal for households to maintain a higher stock share in TDA in their lifespan.

The main contribution of this paper is to solve a realistically calibrated life-cycle model with stochastic income to address optimal portfolio choice (including stock market participation choice) between TA and TDA. Moreover, we incorporate the progressive tax system in the model and examine its effect. Life cycle factors play a central role in both stock market participation and stock share behavior in our model.

The rest of the paper is organized as follows. Section 2 describes the empirical evidence on stock market participation rate and stock share for participants in TA and TDA. Section 3 discusses the model’s assumptions and set-up. The calibration and parameterization is presented in section 4. Section 5 gives the simulation results for the benchmark case and explores sensitivity analysis. Finally, section 6 concludes. For the construction of variables from SCF data, see Appendix A. For the numerical procedure used to solve the model, see Appendix B.

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2 Data

2.1 Data Description

The Surveys of Consumer Finances (SCF) are the most complete data on household balance sheets in the United States. The data used in this study is the latest SCF conducted in 2001. It includes data on assets both inside and outside of the tax-deferred accounts and also includes extensive demographic information. The population weights are provided so that the results can be aggregated to represent the entire population. The data include an over-sampling of high-income households in order to more accurately measure aggregate asset holdings.

We classify the numerous account into two broad categories: regular taxable account (TA) and tax-deferred account (TDA). TA includes savings account, certificates of deposit, money market account, mutual funds, savings bonds and other bonds, directly owned stock, brokerage account, annuities, trust and managed investment account, cash value of life insurance, and part of miscellaneous assets. We exclude checking accounts in TA because holdings of checking accounts are driven mainly by liquidity concerns rather than asset allocation or tax issues. For assets in TA, SCF respondents separately report the dollar values of direct stock holdings, stocks held in mutual funds and stocks held in other accounts. Aggregating these reported stock holdings provides a measure of stock held in TA.

We define TDA as retirement accounts in which the owners make pre-tax contributions and can choose asset allocation. This includes individual retirement accounts (IRA and KEOGH plans) and most of the defined contribution plans (401K/403B/SRA, Thrift Savings, and TIAA-CREF). Unfortunately, information on allocations to narrowly defined asset classes does not exist in TDA. Thus the composition of holdings in TDA has to be inferred from categorical response. For example, the question on allocation of employer-sponsored pension plan asks, “How is the money in this account invested? Is it mostly in stocks, mostly in interest earning assets, is it split between these, or what?” Following this question, there is a table of possible answers. We use this information to construct estimates of the asset composition of TDA. I make the following assumptions: (1) all of the account

\footnote{When money market account and miscellaneous financial assets are also excluded, the patterns reported below are similar.}

\footnote{Similar assumptions are made in Amromin (2003, p25) and Bergstresser and Poterba (2004, p1900).}
value is assigned to the category that is indicated to be the single category in which “most” holdings are invested; (2) the account value is divided equally if a combination of categories is reported. This may result in an additional source of measurement error in retirement account portfolio allocations. For the details of variables we used, see Appendix A.

Then, stock market participation in each account is determined by checking whether the value of stocks in the account is greater than zero. We also construct the stock share of wealth in each account conditional on participation.

### 2.2 Stock market participation and stock share

Unless otherwise noted, all statistics utilize population weights. According to the 2001 SCF, 45.54% of households have assets in both TA and TDA. Their stock holdings account for 86.55% of total stock holdings of the entire SCF population. In this paper we only focus on these households and examine their stock market participation choice and stock share profiles in both accounts. We construct age profiles for mean and median asset holding in the following age groups: under 25, 25-34, 35-44, 45-54, 55-64, 65-74, 75 and above. We do not consider cohort effect in this paper.\(^{17}\)

Figure 1 plots the age profile of stock market participation rates in TA and TDA for households having assets in both accounts. The proportion of households holding stocks in TDA is very high when households are young. As households age, stock market participation rates in TDA dropped from about 90% to less than 50%. On the other hand, it is a totally different story for the stock market participation rates in TA. The participation rates have increased from 34% when households are young to about 70% when they are old.

Since a significant fraction of households do not participate in stock market in TA, we wonder why this happens. Table 1 shows non-financial income and financial wealth by age for both stock market participants and non-participants in TA. We only report median rather than mean value because the wealth and income distribution is heavily skewed. We notice that: (1) both income profiles are hump-shaped. The median non-financial income is higher for participants in all age groups; (2) wealth-income ratios increase with age. After 65, the ratios “jump up” since many people retire by that age and non-financial income declines substantially. Throughout the life cycle, participants in TA have considerably

\(^{17}\)Ameriks and Zeldes (2001) suggest a cohort effect.
higher taxable income and accumulate much more wealth compared to non-participants: accumulated financial wealth in TA differs by a factor of about 7-12 across the two types of households in various age groups.

Conditional on participation in stock market, figure 2 and 3 plot the mean and median stock share for participants in TA and participants in TDA. The notable finding is that for participants in TDA the mean stock share is about 80% and roughly stable over life cycle, with a slightly lower share when households are over 75. The median participant allocated almost or all his wealth in TDA into stocks. For participants in TA, the mean and median stock share declines as age increases (with median stock share slightly higher when households are very old).

Our findings are generally consistent with other empirical studies (see Bodie and Crane (1997), Amromin (2003), Bergstresser and Poterba (2004)) that document the fact that investors often maintain higher equity positions in TDA.

We summarize our findings for households having assets in both TA and TDA:

(1) There are different trends in terms of stock market participation for these households. Stock market participation rate in TDA is high and it decreases as households age while participation rate in TA is low and it increases as age goes up.

(2) For stock market participants, the stock share in TDA is higher than that in TA. The mean (median) stock share in TDA is roughly stable, while the mean (median) stock share in TA decreases with age.
3 Model

3.1 Preferences

Time is discrete and \( t \) denotes adult age. The investor is adult for \( T \) periods, of which he works in the first \( K \) periods. \( T \) and \( K \) are assumed to be exogenous and deterministic. The investor’s preferences are described by the standard time-separable expected utility form:

\[
E \sum_{t=1}^{T} \beta^{t-1} \frac{C_t^{1-\gamma}}{1-\gamma} + \beta^{T-1} b \frac{(W_T/b)^{1-\gamma}}{1-\gamma}
\]  

where \( \beta < 1 \) is the discount factor, \( \gamma \) is the coefficient of relative risk aversion, \( C_t \) denotes consumption level at period \( t \), \( W_T \) is the total amount of wealth the investor bequeaths to his descendants at death, and \( b \) controls the intensity of the bequest motive (loosely speaking, \( b \) is the number of periods for which the investor cares about his descendant’s welfare). For simplicity, we assume that the utility function applied to the bequest is identical to the one applied to the investor’s own consumption when alive.

3.2 Labor Income Process

At each working period \( 1 \leq t \leq K \), we investigate an income process with regression towards the mean in log income.\(^{18}\) Following the standard specification in the life-cycle literature\(^{19}\), we consider both persistent and transitory income shocks. Thus the income of investor \( i \) at period \( t \), \( Y_{it} \), is exogenously given by\(^{20}\):

\[
\log(Y_{it}) = \bar{y}_t + z_{it} + u_{it}
\]  

where \( \log(Y_{it}) \) is the log income of an investor \( i \) at age \( t \); \( \bar{y}_t \) is the mean log income of all age \( t \) investors; the transitory shocks, \( u_{it} \), are independent and identically normally distributed \( N(0, \sigma_u^2) \); and the persistent shocks, \( z_{it} \), follow a first-order auto-regression:

\[
z_{it} = \rho z_{it-1} + \xi_{it}
\]

\(^{18}\)This income process has been studied in Hubbard, Skinner and Zeldes (1994, 1995), Huggett (1996), Huggett and Ventura (2000). Atkinson et al. (1992) survey the literature.


\(^{20}\)By having exogenous labor income we rule out the labor supply choice. For the issue of labor supply and portfolio choice, see Bodie, Merton and Samuelson (1992).
where \( \xi_{it} \) is distributed as \( N(0, \sigma_{\xi}^2) \) and is uncorrelated with \( u_{it} \). In parameterization we use a Markov process (characterized by a transition matrix) to approximate the first-order auto-regression. Thus \( z_{it} \) will take on a finite number of possible values. We also assume

\[
\tilde{y}_t = log(G_t) + \tilde{y}_{t-1}, \quad \text{where } G_t \text{ govern the age-profile of } \tilde{y}_t.
\]

When \( t > K \), all investors will retire. The replacement rate \( (\lambda_i) \) depends on investor \( i \)'s persistent income shock in period \( K \) \( (z_{iK}) \). Retirement income is given by:

\[
log(Y_{it}) = log(\lambda_i) + \tilde{y} + z_{iK}
\]

where \( \tilde{y} \) is the mean of \( \tilde{y}_t \) for all \( 1 \leq t \leq K \). This specification simplifies the solution of the model.\(^{21}\)

### 3.3 Financial Assets, Accounts, and Taxation

There are two financial assets in the economy: a riskless asset (riskless saving technology, called bond) and a risky asset (called stock or stock index). No transaction costs are incurred for trading these assets. We do not allow for short sales and there is no inflation. The riskless asset yields a constant real return \( r^b \). The real return on stocks at period \( t \), \( r^s_t \), is given by

\[
r^s_t - r^b = \mu^s + \epsilon^s_t
\]

where \( \mu^s \) is the average, real, before-tax equity premium, and \( \epsilon^s_t \) follows an i.i.d. \( N(0, \sigma^2_{\epsilon}) \).

Both assets can be accumulated in two accounts: a regular taxable account (TA) and a tax-deferred account (TDA). In TA, all taxes are assumed to be paid at the source. The labor income is taxed at a rate, \( \tau^b \), which is also the tax rate on interest income. Let \( \tau^c \) denote the tax rate on stock return. We do not model dividends directly for simplicity, and instead assume that all returns on stock come from capital gains.\(^{22}\) In order to abstract from questions of the timing of capital gains, we assume that all such gains are realized automatically. We will consider a lower tax rate on capital gains in section 5, which partially captures the timing of capital gains.

The TDA defers taxation on returns that accumulate on pre-tax contributions. Throughout working life \( t \leq K \), the investor contributes to the TDA a maximum fraction \( \bar{q} \) of

\(^{21}\)Under the US Social Security system, the average earnings measure used to calculate retirement benefits is an average of the 35 highest indexed earnings years rather than all years.

\(^{22}\)There is no tax on capital loss.
before-tax labor income. We assume that borrowing is not allowed in either account. But assets in TDA can be accessed prior to retirement (subject to a withdrawal limit) at an additional penalty $pen$. Thus the tax rate on early withdrawals is $\tau^b + pen$. During retirement, contribution to the TDA is not allowed and the investor starts withdrawing from TDA. There is minimum required distribution.\(^{23}\) The investor pays tax on the withdrawals at the income tax rate $\tau^b$.

We incorporate progressive income tax code in the model. As in Ventura (1999), the income tax is comprised of different brackets, defined by different thresholds with corresponding different marginal tax rates. An investor’s income subject to income taxation is defined to be labor income (net of contributions), interest income and withdrawals (there is additional penalty for early withdrawals). The tax rate on capital gains will depend on the marginal income tax rate the investor faces. We specify the details of the tax system in the calibration section.

### 3.4 Wealth Dynamics and Investor’s Optimization Problem

In each period investors will choose contribution to TDA, withdrawal from TDA, consumption, and stock share in each account. For investor $i$, let $\alpha^\tau_i$ and $\alpha^D_i$ denote the share of wealth invested in stocks in TA and TDA in period $t$, respectively. Let $W^\tau_{it}$ be the financial wealth in TA at the beginning of period $t$ (before the realization of current labor income). Similarly, $W^D_{it}$ is the wealth in TDA at the beginning of period $t$. We first consider $t \leq K$ (working periods). The wealth dynamics are given by (we drop $i$):

\[
W^\tau_{t+1} = \left[ \alpha^\tau_t (1 + r^{s}_{t+1} (1 - \tau^c)) + (1 - \alpha^\tau_t) (1 + r^b (1 - \tau^b)) \right] \times \left[ W^\tau_t + (1 - q) Y_t (1 - \tau^b) + X_t (1 - \tau^b - pen) - C_t \right] \tag{6}
\]

\[
W^D_{t+1} = \left[ \alpha^D_t (1 + r^{s}_{t+1}) + (1 - \alpha^D_t) (1 + r^b) \right] (W^D_t + q Y_t - X_t) \tag{7}
\]

where the term in the first bracket of equation (6) is the gross return on the portfolio held in TA from period $t$ to period $t+1$, $Y_t$ is labor income in period $t$, $q \in [0, \bar{q}]$ is the contribution rate, $X_t$ is the amount of withdrawal from TDA in period $t$, and $C_t$ is consumption. The term in the first bracket of equation (7) denotes the gross return on the portfolio held in TDA from period $t$ to period $t+1$.

\(^{23}\)According to the current regulations in U.S., individuals must begin to take withdrawals by age 70\(\frac{1}{2}\).
When \( t > K \) (retirement periods), the equations become:

\[
W_{t+1}^\tau = \left[ \alpha_t^\tau (1 + r^s_{t+1} (1 - \tau^c)) + (1 - \alpha_t^\tau)(1 + r^b(1 - \tau^b)) \right] \times \left[ W_t^\tau + Y_t(1 - \tau^b) + X_t(1 - \tau^b) - C_t \right]
\]

(8)

\[
W_{t+1}^D = \left[ \alpha_t^D (1 + r^s_{t+1}) + (1 - \alpha_t^D)(1 + r^b) \right] (W_t^D - X_t)
\]

(9)

We also impose the following short sale and borrowing constraints for all \( t \):

\[
\alpha_t^\tau \in [0, 1], \quad \alpha_t^D \in [0, 1]
\]

(10)

\[
W_t^\tau \geq 0, \quad W_t^D \geq 0
\]

(11)

We assume that all funds are withdrawn from the TDA in the last period. We also assume that when a retired investor dies, it does so after that period’s consumption has taken place. In case of bequest, the amount of bequest is given by:

\[
W_T = W_T^\tau + W_T^D (1 - \tau^b) - C_T
\]

(12)

The problem that the investor faces is to maximize (1) subject to constraints (2) through (12), in addition to the non-negativity constraint on consumption. The control variables of the problem are: \( q, X_t, C_t, \alpha_t^\tau, \) and \( \alpha_t^D \). There are three state variables in addition to period \( t \): \( W_t^\tau, W_t^D \) and persistent income shock \( z_t \). The Bellman equation for this problem is given by:

\[
V_t(W_t^\tau, W_t^D, z_t) = \max_{q, X_t, C_t, \alpha_t^\tau, \alpha_t^D} \frac{C_t^{1-\gamma}}{1-\gamma} + \beta E_t \left[ V_{t+1}(W_{t+1}^\tau, W_{t+1}^D, z_{t+1}) \right]
\]

(13)

The problem cannot be solved analytically. Given the finite nature of the problem, a solution exists and can be obtained by backward induction. See the numerical solution in Appendix B.
4 Calibration

4.1 Preference Parameters

The preference parameters are set using a model period of one year. We choose the annual discount factor $\beta$ equal to 0.97 and the coefficient of relative risk aversion $\gamma$ equal to 3. The parameter $b$ is set to zero in the benchmark analysis. In section 5 we report results for higher value of $b$ to investigate the sensitivity of our benchmark results.

4.2 Labor Income Process

Investors are born at a real-life age of 25 (model period 1) and live up to a real-life age of 85 (model period 61). They receive retirement benefits at a real-life age of 65 (model period 41). Thus, we set $T = 61$ and $K = 40$.

Recall that $\bar{y}_1$ is the mean log income of all period 1 investors. Let $\bar{Y}_1 = \exp(\bar{y}_1)$. We choose $\bar{Y}_1$ to be $38000$. This number is higher than the median income of all households at age 24 to 26 in the 2001 SCF, but lower than that of households with both accounts at the same age. Our choice is based on two considerations. First, households that have access to TDA tend to have higher income compared to households without TDA. Second, there is income growth due to technological improvement over time.

$G_t$ reflect the age-earnings profile ($\bar{y}_1, \cdots, \bar{y}_K$ or $\bar{Y}_1, \cdots, \bar{Y}_K$). We set $G_t = 1.03$ for $2 \leq t \leq 30$ and $G_t = 1.00$ for $31 \leq t \leq 40$, which implies that the median income increases at a rate of 3% in period 2 to period 30 and is constant in period 31 to period 40. We choose this pattern of age-earnings profile based on the following sources of information: First, according to the 2001 SCF the annual growth rate of median household income is 1.23% from age 25 to 55 for households with both taxable and tax-deferred accounts. Given that SCF is cross-sectional data, it is not surprising to notice that the median household income drops after 55; Second, the Economic Report of the President (2002) shows that the annual

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24If income is log normally distributed, then mean log income and median income are related as follows: median income = $\exp$ (mean log income). Thus $\bar{Y}_1$ is the median income of all period 1 investors in the model.

25We calculate the median non-financial income for those households at age 24 to 26 and households at age 54 to 56. The non-financial income equals total income minus income from interests, dividends, capital gains, and other investment.
growth rate of median real family income between 1982 and 2000 is 1.31%.

The remaining parameters of the earning process are $\rho$, $\sigma_\xi^2$, and $\sigma_u^2$. Hubbard, et al. (1994) estimate that $\rho$ is about 0.95, $\sigma_\xi^2$ and $\sigma_u^2$ range from 0.016 and 0.014 for households with a college education to 0.033 and 0.040 for households with less than high school education. We set $\rho = 0.95$. Since on average the education level for households with both accounts is higher than the population mean\textsuperscript{26}, we choose $\sigma_\xi^2 = 0.02$, which implies that a one standard deviation shock increases or decreases earnings by about 14%.\textsuperscript{27} For the transitory shocks, we set $\sigma_u^2 = 0.04$ in the benchmark case. We examine sensitivity by also considering a smaller variance ($\sigma_u^2 = 0.01$) and a larger variance ($\sigma_u^2 = 0.09$) in section 5 (seems not important).

For computational reasons, the earnings process just specified must be approximated with a finite number of states. We use 3 states for the persistent shocks. These states range from $-3\sigma_\xi$ to $3\sigma_\xi$. Shocks are evenly spaced over these intervals. Transition probabilities are calculated by integrating the area under the normal distribution, conditional on the value of the state. This follows Tauchen (1986). The transitory shocks also take on 3 values between $-3\sigma_u$ to $3\sigma_u$.

### 4.3 Social Security Benefit

The social security benefits an investor receives equal to the replacement rate ($\lambda_i$) times the investor’s working-life average earnings, which depend on his persistent income level in the last working period prior to retirement for simplicity. There are 3 persistent income states. We set the replacement rates equal to 55%, 46%, and 38% respectively, with lower income investors having a higher replacement rate.\textsuperscript{28}

### 4.4 Asset Return

The constant real bond return $r^b$ is set at 3%, while for the stock return process we consider a mean equity premium ($\mu_s$) equal to 3% and a standard deviation ($\sigma_s$) of 16%.

\textsuperscript{26}Gale and Orszag (2003) report that pensions covered almost two-thirds of workers with a college degree and fewer than one-fifth of workers with less than a high school degree in 1999.

\textsuperscript{27}The same variance is also used in Huggett and Ventura (2000).

\textsuperscript{28}For U.S. Social Security benefit formula, see figure 1 in Huggett and Parra (2004).
The mean risk premium is lower than the historical value but is close to the expected equity risk premium reported in Fama and French (2002). Their estimates of expected rate of capital gains for 1951 to 2000 are 2.55% and 4.32%.

4.5 Tax

For the income tax, our strategy is to mimic the income tax code in the U.S., prevailing after 1995 and before Bush’s tax cut. Table 2 describes the actual federal income tax for married couples filing jointly:

<table>
<thead>
<tr>
<th>Marginal Tax Rate (%)</th>
<th>1996</th>
<th>2000</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>0 - 40100</td>
<td>0 - 43850</td>
</tr>
<tr>
<td>28</td>
<td>40100 - 96900</td>
<td>43850 - 105950</td>
</tr>
<tr>
<td>31</td>
<td>96900 - 147700</td>
<td>105950 - 161450</td>
</tr>
<tr>
<td>36</td>
<td>147700 - 263750</td>
<td>161450 - 288350</td>
</tr>
<tr>
<td>39.6</td>
<td>263750 +</td>
<td>288350 +</td>
</tr>
</tbody>
</table>

Table 2: federal income tax

Thus we choose the thresholds as $40000, $100000, $150000 and $260000 respectively. In order to calibrate the corresponding marginal tax rates, we need to approximate the complex exemptions and deductions present in the actual tax code first. We take the case of a household comprised of a couple filing jointly. In 1996 the standard deduction is $6700. Since there also exists personal exemption, we let the sum of standard deduction and personal exemptions be $11500, which is equal to 30% of \( \bar{Y}_1 \). Table 3 reports the marginal tax rates we use:

\[29\] See Mehra and Prescott (1985). Also see a reexamination of the equity premium puzzle in McGrattan and Prescott (2003).
<table>
<thead>
<tr>
<th>Income</th>
<th>Marginal Tax Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0, 0.3]</td>
<td>0%</td>
</tr>
<tr>
<td>(0.3, 1.3553]</td>
<td>15%</td>
</tr>
<tr>
<td>(1.3553, 2.9342]</td>
<td>28%</td>
</tr>
<tr>
<td>(2.9342, 4.25]</td>
<td>31%</td>
</tr>
<tr>
<td>(4.25, 7.1447]</td>
<td>36%</td>
</tr>
<tr>
<td>(7.1447 +</td>
<td>39.60%</td>
</tr>
</tbody>
</table>

Table 3: tax code used in model

The tax rate on capital gains depends on the marginal income tax rate the investor faces. For taxpayer in the 15% bracket or lower, the tax rate is 10%. For higher bracket taxpayers the tax rate is 20%. We also consider a lower tax rate on capital gains in section 5.
5 Results and Other Issues

5.1 Benchmark Results

See the attached figures.

5.2 Sensitivity Analysis

1. Add another transitory income shock: income is 5% with probability of 0.5%.
2. Increase the coefficient of relative risk aversion $\gamma$ and let it be 5.
3. Consider fixed stock market entry cost.

5.3 Other Issues

We set the correlation between labor income shocks and stock returns at zero, consistent with the empirical (lack of) correlation between labor income and stock market returns at the occupational level, as documented in Cocco, Gomes and Maenhout (2004) and Davis and Willen (2000).

Other concerns (housing, lifespan uncertainty).

6 Conclusion

To be written.
Appendix A: The Survey of Consumer Finances Data

The Survey of Consumer Finances (SCF) is probably the most comprehensive source of data on U.S. household assets. We use the latest 2001 survey to construct household portfolio composition in TA and TDA. The specific variables in the codebook used are given below.

Financial assets in TA ($W^T$) include savings accounts (x3804, x3807, x3810, x3813, x3816, x3818), certificates of deposit (x3721), money market accounts (x3706, x3711, x3716, x3718), mutual funds (x3822, x3824, x3826, x3828, x3830), bonds (x3902, x3906, x3908, x3910, x7633, x7634), directly held publicly traded stock (x3915), brokerage accounts (x3930), annuities, trusts and managed investment accounts (x6820, x6835), cash value of life insurance (x4006), and part of miscellaneous assets (x4018, x4022 if $61 \leq x4020 \leq 74$, $x4026$ if $61 \leq x4024 \leq 74$, $x4030$ if $61 \leq x4028 \leq 74$).

Financial assets in TDA ($W^D$) include IRA/KEOGH accounts (x3610, x3620, x3630) and pension from current main job (values of 401k/403b/SRA, Thrift or savings, defined-contribution plan, TIAA-CREF).

Next we construct measures of stocks held in TA and TDA. Stocks held in TA consist of directly held stocks (x3915), stocks held in mutual funds (x3822 plus $\frac{1}{2}x3830$), and stocks held in annuities (x6820 if x6826=1, or $\frac{1}{2}x6820$ if x6826=5, or $\frac{1}{3}x6820$ if x6826=6), trust and managed investment accounts (x6835 if x6841=1, or $\frac{1}{2}x6835$ if x6841=5, or $\frac{1}{3}x6835$ if x6841=6). Stocks in TDA consist of stocks held in IRA/KEOGH accounts (total account value if x3631=2, or half of account value if x3631=5 or 6, or third of account value if x3631=4) and stocks held in current job pension plan (if the answer to the question how the money in this account is invested is “mostly or all stock”, then all of the account value is assigned to stock; if the answer is “split; between stock and interest earning assets”, then half of the account value goes to stock; otherwise stock value is zero.).

We only distinguish two types of assets in each account: risky asset (stock) and riskless asset (bond). Once we have the measure of stock holding in each account, we subtract it from the total financial wealth and obtain the bond holding in the account.

For non-financial income we adopt a broad definition. It is defined as the sum of total reported labor income, unemployment or worker’s compensation, social security, child support and other welfare and transfers. In practice, we use the following measure: $x5729$.

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30We exclude checking accounts in TA.
x5706-x5708-x5710-x5712-x5714.
Appendix B: Numerical Solution

We use numerical dynamic programming techniques to approximate the decision rules as well as the value function. The dynamic program has three state variables in addition to age: $W^\tau_t$, $W^D_t$, and $z_t$ (if there is stock market entry cost, we have one more state variable: stock market entry cost paid or not). We need to solve for five control variables in each period: contribution rate, the withdrawal amount from TDA, consumption, stock share in TA, and stock share in TDA. We exploit the scale-independence of the maximization problem and rewrite the level variables as ratios to $\bar{Y}_1$ (where $\bar{Y}_1 = exp(\bar{y}_1)$). We use lowercase letters to denote them: $w^\tau_t = \frac{W^\tau_t}{\bar{Y}_1}$, $w^D_t = \frac{W^D_t}{\bar{Y}_1}$, $x_t = \frac{X_t}{\bar{Y}_1}$, $c_t = \frac{C_t}{\bar{Y}_1}$.

We discretize the state-space along the two continuous state variables, $W^\tau_t$ and $W^D_t$. The model was solved using backward induction. We optimize using grid search.\footnote{We use equally spaced grid. Since we allow the maximum wealth differs in different periods, the grids are finer for low values of wealth.} In the last period ($t = T$) the policy functions are trivial.\footnote{Households withdraw all funds from TDA. No portfolio choice. If there is no bequest, households consume all available wealth. If there is bequest, households decide how much to consume and how much to bequest.} In periods prior to $T$, we calculate optimal decision rules for each possible combination of nodes, using stored information about the subsequent period’s decision rules and value function. We perform all numerical integrations using Gaussian quadrature to approximate the distributions of the innovations to the labor income process and the stock returns. For points which do not lie on the state-space grid, we evaluate the value function using a bi-cubic spline interpolation along the two wealth dimensions. After computing the values of all the alternatives, we pick the maximum, thus obtaining the decision rules for the current period. If there is stock market entry cost, then, at each point of the state space the decision to participate in stock market or not is made by comparing the value function conditional on having paid the fixed cost with the value function conditional on non payment. This process is iterated until $t = 1$.

Once we determine the optimal decision rules for all possible nodes in each period, it is straightforward to simulate a history for each of a large number of households (each cohort consists of 9000 households). By applying the optimal decision rules we have, we can compute the (mean and median) asset allocation for stock market participants and stock market participation rate in each account for different cohorts having assets in both
TA and TDA. We then compare the average of 50 times of simulation to the real data.

As you can see we need a huge amount of computation time to solve our model. All programs are parallelized and run on SHARCNET.
References


Stock market participation rates for households with both accounts, U.S., 2001 SCF

Figure 1
Table 1: Wealth and income for stock market participants and non-participants in TA

<table>
<thead>
<tr>
<th>Age</th>
<th>Stock market participants in TA</th>
<th>Stock market non-participants in TA</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Non-financial income</td>
<td>financial assets in TA</td>
</tr>
<tr>
<td>&lt;25</td>
<td>median</td>
<td>36790</td>
</tr>
<tr>
<td>25-34</td>
<td>68000</td>
<td>22800</td>
</tr>
<tr>
<td>35-44</td>
<td>85000</td>
<td>60000</td>
</tr>
<tr>
<td>45-54</td>
<td>88600</td>
<td>100000</td>
</tr>
<tr>
<td>55-64</td>
<td>66500</td>
<td>151800</td>
</tr>
<tr>
<td>65-74</td>
<td>38400</td>
<td>222000</td>
</tr>
<tr>
<td>75+</td>
<td>28000</td>
<td>273000</td>
</tr>
</tbody>
</table>

Note: (1) We consider households with both accounts. Checking accounts are excluded.
(2) Non-financial income: total income minus income from interests, dividends, capital gains, and other investment.
Figure 2

Mean stock share for participants, U.S., 2001 SCF

Figure 3

Median stock share for participants, U.S., 2001 SCF