

Numerical and Simulation Methods in Option Pricing and Portfolio Optimization

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Essay 1: Empirical Performance of Efficient Monte Carlo Simulation Strategies for Option Pricing in Incomplete Markets (Job Market Paper)

This paper compares different efficient Monte Carlo simulation methods for the purpose of pricing derivatives under the incomplete market environments. Option prices are simulated based on three incomplete option price models: stochastic volatility model, jump diffusion model, and stochastic volatility with concurrent jumps in the stock price and variance process model. Using the simulated option prices as well as the option prices based on S&P 500 index returns, I test and compare the performance of the standard Monte Carlo simulation and five other efficient simulation methods including Antithetic Variables, Control Variates, Stratified Sampling (SS), Importance Sampling, and Quasi-Monte Carlo (QMC). The comparison is based on different moneyness and maturity times. According to Root Mean Squared Error, QMC is the best choice for the out-of-the-money options. For in-the-money options, there is no clear winners as the performance of the methods changes with the option pricing model. Considering the standard error, QMC and SS do the best and much better than the other methods. The study may serve to improve the speed and accuracy of Monte Carlo methods for option pricing under incomplete environments.

Essay 2: A Numerical Analysis of the Properties of Optimal Portfolio under Inequality Constraints

When equality constraints hold, the exact distributions of optimal investment can be obtained under multivariate normality. However, considering the realistic optimization problem in which inequality constraints exist, the problem may become complex. The corner portfolios for the constrained frontier may become stochastic and the exact distributions may be unavailable. This paper uses a numerical approach to calculate the moments of optimal portfolios under inequality constraints. It takes the optimal portfolio as a linear combination of any two adjacent corner portfolios and the fund is divided between them. Also, the efficient frontier depends on the binding constraints. This analysis may serve to bring more understanding of the portfolio optimization problem with potential practical value.

Essay 3: The Effects of the Use of Realized Volatility on Volatility Trading Strategies

The implied stochastic volatility regression (ISVR) method has commonly been used to predict the conditional volatility of stock prices. However, implied volatility has proven to be a biased predictor of the realized volatility (RV) across asset markets. With the increasing emphasis on computer-assisted techniques, high frequency data can be applied to processing RV. This paper investigates the Delta Neutral strategy, with the realized volatility forecasting based on high frequency data. A comparison between the effectiveness of applying realized volatility to the trading strategies and that of the implied volatility is conducted. This study shows balanced research results between the performance of using the two types of volatility.