The Lock-In Effect and the Corporate Payout Puzzle

Chris Mitchell*
cmitch29@uwo.ca
Ph.D. Candidate, Economics Department
University of Western Ontario, London, Ont.

November 10, 2011

Abstract

Out of their after-tax profits, corporations pay dividends and often repurchase shares to generate capital gains for their shareholders. Capital gains typically attract lower effective rates of tax than dividends, which leads to the payout puzzle: Why do corporations continue to pay dividends in spite of their relative tax disadvantage? This paper develops a model of corporate payout policy to explain some aspects of the puzzle. The key element of the model is the lock-in effect caused by the fact that capital gains are taxed only upon realization, which implies that taxpayers can reduce the effective rate of tax by postponing realizations, i.e., by locking in accrued capital gains. Shareholders with an accrued capital gain, thus, would require a lock-in premium to compensate them for their higher tax liability resulting from the sale. This premium is an increasing function of accrued capital gains and the desired holding period for equity. In this setting firms pay dividends in equilibrium whenever the lock-in premium is high relative to the tax disadvantage of dividend payments. It follows that firms with a relatively high proportion of shareholders with sizable capital gains and/or long holding periods face higher repurchasing costs and are thus more likely to pay dividends. The model predicts that total payout increases as the capital gains tax and the profits tax increase, whereas the dividend tax has no effect. The model's predictions are examined empirically using data from a panel of Canadian corporations listed on the TSX over the period 1987-2008. The empirical results, being largely consistent with those of the model, are encouraging.

*First I would like to thank my advisor Kul Bhatia for his invaluable insights and support over the last three years. I would also like to thank Jim Davies for numerous comments and suggestions pertaining to this paper. Peter Streufert also deserves thanks for a number of helpful discussions regarding the theoretical section. Last I would like to thank Mike McCausland for providing detailed comments on an earlier draft.

1 Introduction

This paper proposes a new model of corporate payout policy, and presents estimates on empirical relationships between corporate payout policy and taxes at the personal and corporate level in Canada. The question of whether taxes affect payout choice - dividend payments vs. share repurchases - has received considerable attention in the economics literature. Authors arguing that the tax structure affects payout choice have the burden of explaining why dividends are paid in practice despite their tax disadvantage relative to share repurchases. The explanations provided include diverse factors such as the mitigation of agency costs, the signaling role of payout, the intrinsic value of dividend payments, and constraints on the ability to repurchase shares. The theoretical section of this paper offers an additional explanation, arguing that there is a dead-weight cost associated with share repurchases, which make dividend payments desirable in certain situations. In practice, firms likely pay dividends for a number of these reasons. Isolating and exploring each helps our understanding of firm payout decisions.

In their pioneering work, Modigliani & Miller (1961) show that firm value is independent of its payout mix in the absence of personal taxes; arbitrage opportunities ensure equity markets value dividend payments and share repurchases identically when capital markets are complete. The authors go on to show that management's influence over firm value is limited to its choice over investment level and total payout - the payout method is irrelevant. A justification for this view is provided by Miller & Scholes (1978), who argue that marginal investors are unaffected by personal taxes. Miller & Scholes (1978) point out that nuances in the US tax code allow for the avoidance of capital income taxes by some taxable individuals, and tax-exempt entities such as pension funds avoid personal taxes altogether.

When complete tax avoidance is not possible, the payout choice is likely to be relevant. Indeed, a tax differential on payout method can enable management to influence firm value through the payout mix. For example Black (1976) argues that relatively high dividend taxes vs. capital gains taxes[1] from 1965-1972 in the US made dividend payments an inferior form of payout. He claims the existence of dividend payments over this period, given their tax disadvantage, is a puzzle. The payment of dividends in light of their tax disadvantage is highlighted in a number of economic studies and seems to be prevalent in both the US and Canada. Poterba (2004) argues that dividend payments were tax disadvantaged in the US from 1929-2003, and we shall argue below that there was a similar tax disadvantage in Canada from 1987-2008. During these respective periods, a substantial fraction of total payout in the US and Canada was made in the form of dividends. Dividends account for the majority of total payout in Canada, and until recently, were also the majority payout method in the US (Brav et al. (2008)). This paper presents a model of firm payout which addresses the payout puzzle.

[1] Income generated from a share repurchase takes the form of a capital gain.

The model explores the possibility that shareholder lock-in effects (i.e., the incentive to postpone the sale of equity with an accrued capital gain) make repurchasing shares sufficiently expensive that dividend payments become the marginal form of payout for some firms. Capital gains are taxed upon realization in the model, consistent with capital gains taxation in both Canada and the US. Under this tax structure, investors can reduce the effective rate of tax on accrued capital gains by postponing realizations, thereby deferring the tax liability. It follows that when equity has an accrued capital gain the locked-in value exceeds the after-tax value of liquidating the position. Shareholders with an accrued capital gain will thus require a lock-in premium to sell equity back to the firm. This premium is an increasing function of accrued capital gains and the desired holding period for firm equity. When shareholders are heterogeneous with respect to these arguments they have heterogenous required lock-in premiums. This leads to upward sloping equity supply curves, and causes the marginal benefit of a repurchase to fall as more shares are repurchased. In equilibrium firms repurchase shares until the marginal benefit of doing so equals the marginal benefit of paying dividends. At this point firms start paying dividends. Empirical evidence supporting upward sloping equity supply curves, caused by heterogeneous shareholder lock-in effects, is found in Bagwell (1992) and Landsman & Shackelford (1995) for the US. McNally (2003) reports similar findings for Canada.

The equity supply curve depends in large part on the distribution over accrued capital gains and desired holding period specific to a firm's shareholders. In order to examine the effect of this distribution on dividend payments we solve the model and run comparative statics for a number of distributional forms and parameterizations. In general, when the average accrued capital gain is high, the average desired holding period is long, and/or the variance of each is low, firms face higher lock-in premiums and are more likely to pay large dividends.

We use the model to explore the consequences of corporate and personal tax policy for firm investment and payout decisions. We find that both the corporate tax and the capital gains tax reduce the level of corporate investment, whereas the dividend tax has no effect. Short run total payout increases in both the corporate tax and the capital gains tax, whereas long-run total payout decreases in both. The dividend tax has no effect on total payout in either the short-run or the long-run. All three taxes are shown to affect the payout mix. However, the effects depend on a number of factors including the marginal form of payout and the magnitude of the tax change. We discuss these effects in detail in the theoretical section.

Our theoretical results provide insight into empirical relationships between corporate payout policy and taxes at the personal and corporate level. These insights are explored further with data from a panel of Canadian corporations listed on the Toronto Stock Exchange (TSX) over the period 1987-2008. In our empirical analysis we estimate the short-run effects of tax changes on total payout, dividend payments and share repurchases. The results suggest that short-run total payout increases in both the corporate tax and the capital gains tax, but is unaffected by the dividend tax. This is consistent with the results of the model. The empirical results also suggest that dividend payments are negatively related to the dividend tax and positively related to the capital gains tax. This is intuitive. As the relative tax penalty on dividends decreases, either through a reduction in the dividend tax or an increase in the capital gains tax, firms increase dividend payments. Share repurchases are found to decrease in the capital gains tax, and, for the most part, to increase in the dividend tax. This is also intuitive: when the relative tax penalty on capital gains is low, firms spend more money repurchasing shares. To the best of our knowledge this analysis is the first to simultaneously estimate the effects of tax changes on dividend payments, share repurchases and total payout. Examining all three components of corporate payout simultaneously provides a more complete view of the empirical relationships between tax policy and payout policy. The results also support the position that taxes are not irrelevant for corporate payout.

1.1 Other Literature on the Payout Puzzle

The use of repurchase premiums to explain dividend payments is also explored in Chowdhry & Nanda (1994). In their signaling model of payout, a firm's true value is the private information of management. When equity is temporarily undervalued share repurchases are used to increase non-tendering shareholder wealth. In this way, share repurchases become a signal of undervaluation, and investors demand a premium to tender shares. This premium adds to the cost of a repurchase, and tax disadvantaged dividend payments are made when the premium is high relative to the undervaluation. Unlike this model, the current model assumes no informational asymmetries; the firm's market value is known by both management and investors. The repurchase premium in Chowdhry & Nanda (1994) stems from the information content of repurchases, whereas the repurchase premium in the current model is caused by shareholder-specific lock-in effects.

Signaling models also appeal to tax clienteles and their differential ability to monitor firms. For instance Allen et al. (2000) argue for a model of tax clienteles, in which non-taxed "institutional" investors have the ability and initiative to discover the true value of firms, while taxed "retail" investors do not. Both investor types own a positive fraction of all firms for purposes of diversification. To compensate retail investors for relatively high dividend taxes, the pre-tax yield on dividend paying stocks is higher. This higher yield attracts a larger proportion of institutional investors to dividend paying stocks, which is desirable for high quality firms seeking to signal their true value. In a separating equilibrium, high quality firms pay dividends and low quality firms fail to pay dividends.

A second class of payout model explains dividends by appealing to agency costs. Easterbrook (1984) proposes that dividends can mitigate two types of agency costs. The first is caused by management's incentive to protect job tenure. Reinvesting firm profits in low-risk low-return projects can reduce the chance of firm bankruptcy, and increase the probability that management remains employed. This investment strategy enhances the value of corporate debt at the expense of equity. Dividend payments are a way of reducing the amount of free cash flow available for this purpose. The second type of agency cost is related to monitoring firm management. When firms are widely held, the cost of monitoring by an individual investor is usually prohibitively high relative to the potential gain. Reducing cash reserves by way of dividends forces management to seek financing in capital markets more often. New investors provide capital only when compensated for existing agency costs, incentivising their reduction by management. Other authors, such as Busaba (2011), believe dividends are a result of agency costs but do not mitigate them. Busaba (2011) argues that the ability to secure debt with collateralizable assets can reduce the agency costs faced by creditors. Small firms with few collateralizable assets are unable to secure debt financing cheaply, and are forced to rely on internal funds at the cost of dividends. Larger firms with sizable collateralizable assets have fewer agency costs associated with debt and are able to obtain debt financing cheaply, allowing them to pay dividends.

Within the class of perfect information payout models, in which this paper fits, the "Traditional" view, discussed in Auerbach & Hassett (2003), and the "New" view, developed by Auerbach (1979), Bradford (1981), and King (1977), are prominent. In the Traditional view, investors derive an intrinsic value from dividend payments. Equity markets react positively to their payment, which reduces the cost of capital and increases market value. In the New view, firms face an exogenous binding repurchase constraint, and are unable to use share repurchases to satisfy the whole of total payout; any payout in excess of this amount is paid in the form of dividends. Both models have received criticism based on their assumptions. The intrinsic value of dividends has little theoretical or empirical justification as noted in Poterba & Summers (1985), and the repurchase constraint, assumed in the New view, may have been defensible in the US prior to the SEC's 1982 adoption of Rule 10b-18, which eased restrictions on share repurchases,[2] but has little justification at present in both the US and Canada.[3] In the theoretical and empirical sections below, we compare the results of these models with the current one.

The remainder of this paper is organized as follows. Section 2 presents Canadian summary data which supports the existence of a Canadian payout puzzle. Section 3 discusses the payout model and its predictions for corporate payout and investment. Section 4 uses numerical methods to solve the model and presents comparative statics and dynamic simulations. Section 5 presents the empirical findings, and Section 6 concludes.

[2]See Allen and Michaely (2003) for a discussion of this rule, and its implications for US share repurchases.

[3]In Canada firms are able to repurchase up to 10% of the public float or 5% of shares outstanding (of a particular class) annually - see Ikenberry et al. (2000) or TSX (2008) for more information on the regulations regarding share repurchases by TSX listed corporations. However, this constraint was found to be non-binding in practice.

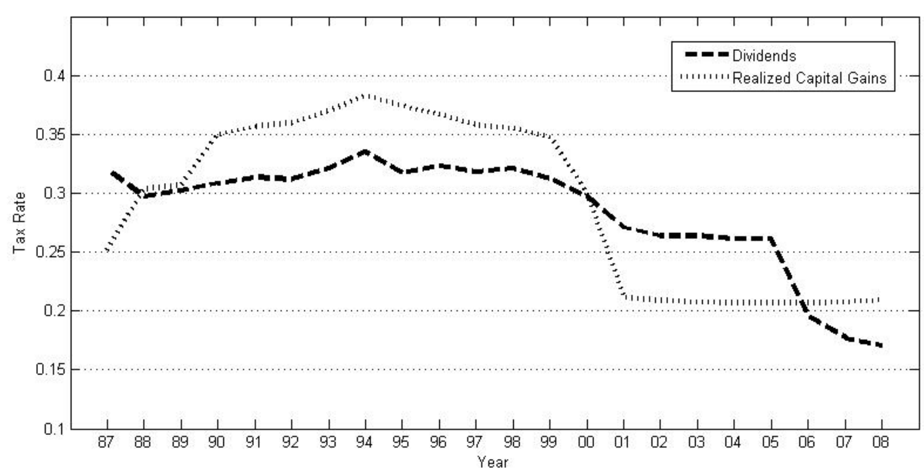
2 Summary Data

This section provides statistics on Canadian marginal tax rates and Canadian payout policy to illustrate the empirical basis for the payout puzzle in Canada. The marginal tax rates faced by individual shareholders on income from both dividends and realized capital gains are functions of individual tax brackets - determined by total income (along with deductions and tax credits), and province of residence - determining tax schedules. Average statutory rates are estimated using the methodology in Sialm (2009), which takes a weighted average of individual marginal rates, weighted by the individual's share of total income from dividends and realized capital gains. That is:

$$T_{b,t}^{Avg} = \sum_{l} \Theta_{l,b,t} \cdot T_{l,b,t} \quad b \, \epsilon \, \{d, g\},$$

where $T_{d,t}^{Avg}$ is the average marginal tax rate on dividend income in year t, $T_{g,t}^{Avg}$ is the average marginal tax rate on realized capital gains income in year t, $\Theta_{l,d,t}$ is the proportion of total dividend income earned by individual l in year t, $\Theta_{l,g,t}$ is the proportion of total realized capital gains income earned by individual l in year t, $T_{l,d,t}$ is individual l's marginal tax rate in year t on income from dividends, and $T_{l,g,t}$ is individual l's marginal tax rate in year t on income from realized capital gains. Figure 1 plots these average Canadian marginal tax rates from 1987 to 2008.[4]

Figure 1: Average Canadian Marginal Tax Rates: Dividends and Realized Capital Gains



Due to the postponement of capital gains taxes until realization, the effective tax rate on *accrued* capital gains is necessarily lower than the statutory rate on *realized* capital gains. Share repurchases generate income in the form of an accrued capital gain,[5] and therefore, the applicable tax liability is the effective rate, not the statutory rate. A number of economic studies have estimated the effective accrual capital gains tax rate as a fraction of the statutory rate. The estimate in Poterba (1987)

[4] Appendix A contains a detailed discussion of how these rates are calculated.

[5] They also generate income in the form of realized capital gains for tendering shareholders.

is approximately .25, whereas the estimates in Protapapadakis (1983) range from .56 to .59. The true ratio is a matter of some debate. The numerical section of this paper estimates the effective tax rate using a discrete-time extended version of the methodology in Davis & Glenday (1990), which also incorporates income from both dividends and accrued capital gains.[6] This methodology, known as the accrual equivalent capital gains tax rate (AECGTR), is the rate of taxation, that if applied to capital gains as they accrue, would result in the same future value of after-tax capital gains income as taxing capital gains upon realization.[7] We estimate the Canadian AECGTR using an estimate of the Canadian average dividend and capital gain yield provided by Campbell (2008) over the period 1982-2007,[8] the average dividend and capital gains tax rates estimated above (over the period 1987-2008) and a ten year average holding period. This produces a ratio of .76. Table 1 reports the relative tax burden (RTB) of dividends vs. capital gains over the period 1987-2008 using all three tax ratios provided above,[9] where the retaliative tax burden is defined as in Poterba (2004):

$$RTB_t = \frac{(1 - T_{d,t}^{Avg})}{(1 - T_{g,t}^{Avg} \cdot Ratio)},$$

where *Ratio* is the ratio of effective to statutory capital gains taxes.

Table 1: Dividend Tax Burden

Year	.25 Ratio	.575 Ratio	.76 Ratio	**Year**	.25 Ratio	.575 Ratio	.76 Ratio
1987	0.726	0.795	0.841	**1998**	0.745	0.853	0.930
1988	0.761	0.852	0.914	**1999**	0.753	0.860	0.935
1989	0.756	0.848	0.910	**2000**	0.760	0.849	0.910
1990	0.758	0.866	0.942	**2001**	0.770	0.830	0.869
1991	0.754	0.863	0.942	**2002**	0.777	0.837	0.875
1992	0.756	0.868	0.947	**2003**	0.777	0.836	0.874
1993	0.748	0.862	0.944	**2004**	0.779	0.838	0.876
1994	0.735	0.852	0.938	**2005**	0.779	0.838	0.877
1995	0.753	0.870	0.954	**2006**	0.848	0.913	0.955
1996	0.745	0.858	0.939	**2007**	0.868	0.934	0.977
1997	0.749	0.858	0.936	**2008**	0.875	0.942	0.986

In all three series the relative tax burden of dividends is below unity, meaning dividend payments yielded a lower after-tax return to shareholders than repurchasing firm equity. Due to this tax disadvantage, it would appear that Canadian firms could have maximized market value by choosing a payout policy excluding dividend payments altogether. This was not the case in Canada, however. Despite their tax disadvantage, dividend payments have persisted in Canada over the period

[6]Davis & Glenday (1990) use a continuous time methodology that omits income from dividends.

[7]Appendix B presents a discussion of how this rate is derived.

[8]Campbell (2008) provides a range of estimates based on different modeling assumptions. We use a 6% total yield, split evenly between dividends and capital gains, which is consistent with his set of estimates and assumptions.

[9]For Protapapadakis (1983), we use the middle of his range of estimates.

1987-2008. They are the largest form of payout in Canada, and are used by more firms than are share repurchases.

To illustrate this, Figure 2 plots aggregate dividend payments made TSX listed Canadian corporations from 1987 to 2008, and Figure 3 plots aggregate share repurchases over the same period.[10] Aggregate dividend payments were higher in each of the 22 years reported. Dividend payments were 87 times higher than share repurchases in 1987, and 223 times higher in 1991. From 1991 onward the multiple has decreased substantially, reaching a low in 2007 of 2.8 times share repurchases.

Figure 2: Aggregate Dividend Payments: Canadian Firms ($2008)

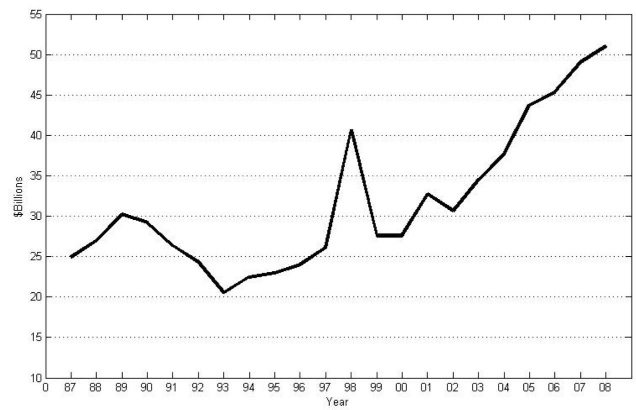
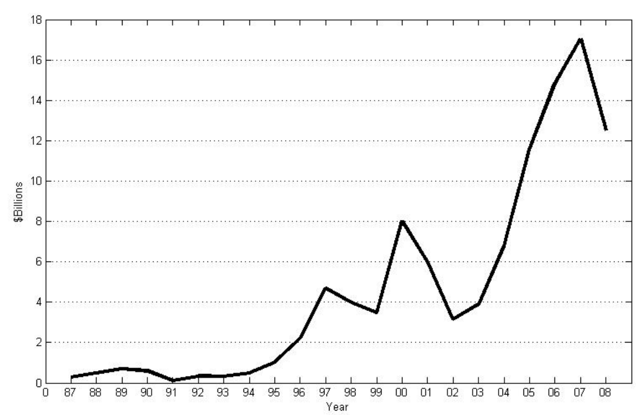


Figure 3: Aggregate Share Repurchases: Canadian Firms ($2008)



The average dividend payment is also higher, as Figure 4 illustrates. The average dividend payment among dividend paying firms, and the average amount spent repurchasing shares among repurchasing firms, have both increased over the period 1987-2008. The average amount spent repurchasing shares increased 21 fold over the period 1987-2006, later dropping to 9.5 times 1987

[10] Appendix A contains a detailed discussion of how these series are calculated.

levels in 2008. Average dividend payments rose more steadily over this period, increasing a little over twofold from 1987-2008. In spite of the higher growth rate, average share repurchases were lower than average dividend payments in each of the 22 years.

Figure 4: Average Payout by Paying Firms: Dividends and Share Repurchases ($2008)

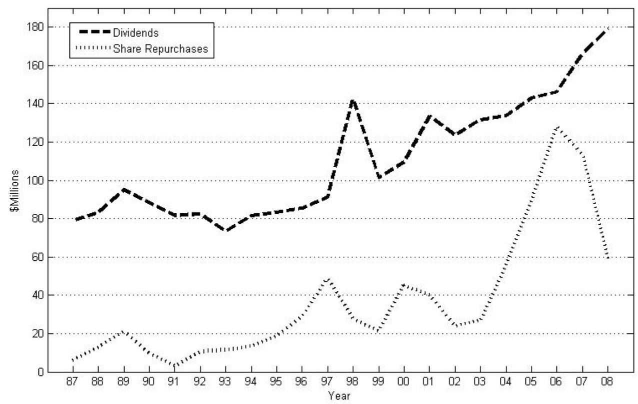
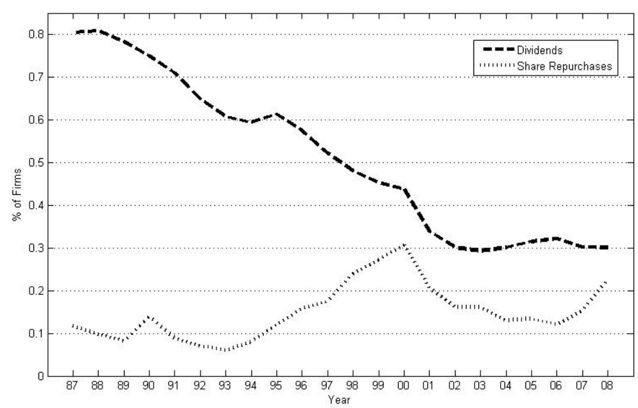


Figure 5: Fraction of Firms Paying Dividends and Repurchasing Shares



In addition to higher levels, a larger number of Canadian firms chose to pay dividends than repurchase shares, as Figure 5 illustrates. Dividend payments are chosen by more firms in each of the 22 years reported. However, the fraction of firms paying dividends has decreased over time. In 1987 80% of firms paid a dividend, whereas only 30% of firms paid one in 2008. The trend is opposite for repurchases; in 2008 23% of firms repurchased stock, whereas only 12% repurchased stock in 1987. This trend is similar to that found in the United States.

Share repurchases have become a more prominent form of payout as of late, but Canadian firms still have a preference for paying dividends. The model presented in the next section provides an explanation for why dividends are paid in practice.

3 The Lock-In Model

We assume the economy is populated by a number of corporations each producing a single good sold in markets with perfectly elastic demand. Profit functions have decreasing returns and only depend on non-depreciating capital. Firms are financed exclusively through equity via capital markets with perfectly elastic supply. There is no uncertainty about future output demand or the required after-tax rate of return on capital, which are both constant over time. Firms live forever and seek to maximize shareholder wealth by maximizing current market value. Profits generated by the firm are subject to corporate income tax, which is constant across firms; after-tax profits can be retained, used to pay dividends, or used to repurchase shares.

Each firm is owned by a large number (continuum) of shareholders requiring an after-tax rate of return equal to ρ. Shareholders are heterogeneous with respect to their desired holding periods and level of accrued capital gains on firm equity.[11] A joint realization over desired holding period (H) and tax base (α) (one minus the ratio of capital gains to asset value) characterizes an investor-type, and each firm faces a stationary distribution over investor-type. Denote the distribution over investor type by $F(\alpha, H)$ and the corresponding density by $f(\alpha, H)$.[12] Shareholders seek to maximize wealth and are indifferent between receiving after-tax income in the form of dividends and accrued/realized capital gains; all three sources of income are identical according to the Haig-Simons income definition.

At the personal level, dividend income results in an immediate tax liability, whereas capital gains are taxed upon realization. This system of taxation is used in both Canada and the United States. For a discussion of Canadian capital income tax policy see Kerr (forthcoming). To simplify the model we use a single dividend tax rate and a single tax rate on realized capital gains. This is standard in the literature (for example, see Poterba and Summers (1985)) and reduces the model's complexity. The after-tax value of dividend income is straightforward to calculate; it equals the gross dividend payment multiplied by one minus the dividend tax rate. The after-tax value of accrued capital gains is more complicated due to the postponement of capital gains taxes. As discussed in Section 2, we use an AECGTR to value after-tax accrued capital gains income. The AECGTR is shareholder specific, owing to differences among optimal holding periods.[13] For the current model we use a single AECGTR to value accrued capital gains income. This rate can be thought of as an average among the firm's shareholders.

[11] The potential processes that generate each source of heterogeneity are not modeled in the current paper. This would complicate the analysis while adding little to the main thesis. In practice, both sources of heterogeneity are witnessed. The first depends on factors such as retirement planning and portfolio management, while the second depends on the timing of past share purchases.

[12] Shareholders are distritbuted continuously over tax base, and discretely - in one year intervals - over optimal holding period.

[13] It also depends on shareholder tax brackets, via the realized capital gains tax rate and the dividend tax rate. Above we assumed these were constant across shareholders.

The timing of firm operations is as follows. Firms have a specific level of capital to start each period, which they use to produce output. At the end of each period firms sell output, generate profits, and pay corporate tax. The remaining net income can then be used to pay dividends, repurchase shares, and purchase capital. Firms purchase capital first, pay dividends next, and repurchase shares last. Firms may also sell capital at the end of each period to pay dividends and repurchase shares. The value of a firm in period t takes the following form:

$$V_t = \frac{D_t(1-\tau_d)}{(1+\rho)} + \frac{(1-\tau_a)[\frac{V_{t+1}}{(1-\delta_t)} - V_t]}{(1+\rho)} + \frac{V_t}{(1+\rho)}, \qquad (1)$$

where V_t is the value at time t, D_t is the dividend payment made at the end of period t, δ_t is the fraction of the firm repurchased at the end of period t, V_{t+1} is the value of the firm at the beginning of period $t+1$ - which occurs simultaneously with the end of period t - τ_d is the dividend tax rate, τ_a is the AECGTR, and ρ is the after-tax required rate of return on equity. Accrued capital gains ($[\frac{V_{t+1}}{(1-\delta_t)} - V_t]$) can be generated in one of two ways: by increasing the market value of the firm (V_{t+1}) through capital acquisition; and by repurchasing shares (δ_t). After a share repurchase, the remaining shareholders own a larger fraction of the firm. Provided the firm's value does not decrease by too large an amount the market value of each remaining share will increase, leading to an accrued capital gain.

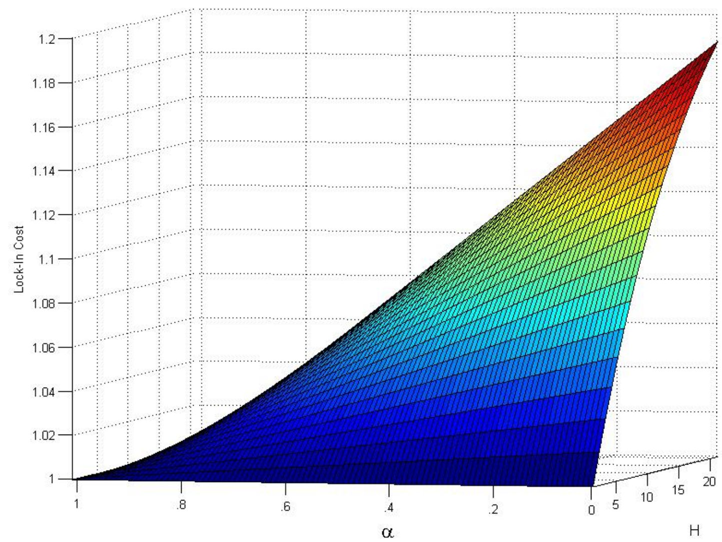
In both the New view and Traditional view models of corporate payout policy, all shares are repurchased at a price equal to the intrinsic value of equity. The current model differs in this respect by arguing that most shareholders will in fact require a premium over this price to sell their shares during a repurchase program. The origins of this premium are discussed next.

The ability to postpone taxes on income from accrued capital gains produces a lock-in effect, wherein shareholders have a financial incentive to postpone their realization. Once capital gains are realized, the resulting tax liability reduces an investor's gross wealth by the tax. That part of gross wealth used to pay the tax is no longer available for investment, and the returns that would have accrued on it are lost. A renumeration above the intrinsic value of equity must be offered for an investor to be indifferent between selling equity with an accrued capital gain and holding it for a desired number of years. We call this renumeration the lock-in cost, which is characterized by the following equation:

$$L(\alpha, H) = X\left\{\frac{(1-\alpha\tau_g)\{(1-\tau_g)(1+r+(1-\tau_d)d)^H + \tau_g(1-\tau_d)d\cdot\sum_{h=0}^{H-1}(1+r+(1-\tau_d)d)^h\} + \alpha\tau_g(1-\tau_g)}{\{(1-\tau_g)(1+r+(1-\tau_d)d)^H + \tau_g + \tau_g(1-\tau_d)d\cdot\sum_{h=0}^{H-1}(1+r+(1-\tau_d)d)^h\}(1-\tau_g)} - 1\right\},$$

where X is the intrinsic value of the stock, α is the ratio of tax base to market value, H is the desired holding period, τ_g is the realized capital gains tax rate, τ_d is the dividend tax rate, d is the rate of return from dividends, and r is the rate of return from capital gains. The lock-in cost is decreasing in the tax base and increasing in the desired holding period.[14] Recall that shareholders are heterogeneous with respect to these arguments, implying heterogeneity among shareholder-specific lock-in costs. Figure 6 below graphs a numerical example of the lock-in cost as a function of tax base (α) and holding period (H).

Figure 6: Lock-In Cost



For this example, the gross rate of return on dividends and capital gains are each set to 3.5%, the tax rate on dividends is set to 17.1%, and the tax rate on realized capital gains is set to 20.9% (which are the average rates for 2008 from Section 2).

The firm's equity supply curve can be derived by ranking shareholders according to their lock-in costs, from lowest to highest. The equity supply curve is an increasing function (possibly weakly over some intervals) of the amount repurchased. However, the exact shape depends on the shareholder distribution over accrued capital gains and desired holding period ($F(\alpha, H)$). If we denote the equity supply curve by S(q), where q is the number of shares repurchased, and S(q) is the reservation price of the marginal shareholder, i.e., the intrinsic value of equity (X) plus the marginal shareholder's lock-in cost, then it is characterized by the following equations:

$$S(q) = X + \widehat{L(q)}$$

$$where \quad q = \sum_{H=0}^{H_M} \int_{\{(\alpha,H)|L(\alpha,H)\leq\widehat{L(q)}\}} f(\alpha, H) d\alpha,$$

where H_M is the maximum desired holding period among shareholders of the firm, and $\{(\alpha, H)|L(\alpha, H) \leq \widehat{L(q)}\}$ is the set of shareholders with holding period H that have lock-in costs less than $\widehat{L(q)}$.

[14]See Appendix C for a proof of these results.

As we show below, optimal firm behavior dictates that shares are repurchased in the most inexpensive way possible. This ensures that the market value of equity - with respect to the remaining shareholders - is maximized. Due to shareholder-specific lock-in costs firms must pay a premium over the intrinsic value of equity to attract shareholders with an accrued capital gain and non-zero holding period. We call this the lock-in premium. The lock-in premium required by each shareholder constrains the number of shares that can be repurchased for a given amount spent. The extent to which this happens depends on the tendering behavior of shareholders, i.e., the relationship between a shareholder's lock-in cost and the lock-in premium they demand. We can derive upper and lower bounds on this premium as a function of: investor type, amount spent, and the distribution over investor type.

If $F(\cdot)$ is the distribution over investor type and A is the total amount spent on a repurchase, then the minimum lock-in premium demanded by investor type (α, H) is:

$$P((\alpha, H), F(\cdot), A) = L(\alpha, H),$$

where $P(\cdot)$ is the lock-in premium and $L(\alpha, H)$ is investor type (α, H)'s lock-in cost. Any lock-in premium below this amount decreases investor type (α, H)'s wealth. The maximum lock-in premium demanded by investor type (α, H) is:

$$P((\alpha, H), F(\cdot), A) = max_{\{(\alpha,H)\}^B}\{L(\alpha, H)\},$$

where $\{(\alpha, H)\}^B$ is the set of investor types from which shares are repurchased.[15] That is, the maximum lock-in premium demanded by investor type (α, H) is equal to the maximum lock-in cost among the set of shareholders that sell equity back to the firm. To see this, note that if an investor type $(\alpha', H')\epsilon\{(\alpha, H)\}^B$ demands a price above the intrinsic value of equity (X) plus this lock-in premium, e.g. $X + P((\alpha', H'), F(\cdot), A) + \beta$, $(\beta > 0)$, then given the continuity of α, and the continuity of the lock-in cost function with respect to α, there exists an $\eta > 0$ and investor type $(\alpha'', H'')\epsilon\{(\alpha, H)\}^{NB} = \{(\alpha, H)\}^U/\{(\alpha, H)\}^B$ such that $X + L(\alpha'', H'') + \eta < X + P((\alpha, H), F(\cdot), A) + \beta$[16]. If investor type (α'', H'') offers this price firms choose to repurchase from them and not (α', H') - they can repurchase shares more cheaply this way. Investor type (α', H') is worse off by $P((\alpha, H), F(\cdot), A) - L(\alpha', H') \geq 0$ and would therefore not choose to demand a higher price.

Given these lower and upper bounds on lock-in premiums, we model tendering behavior two ways. The first assumes shareholders are willing to tender shares at a price equal to the intrinsic value of equity plus their specific lock-in cost. This tendering behaviour is analogous to perfect

[15]The set of investor-types from which shares can be repurchased is constrained by the distribution over investor-types ($F(\cdot)$) and the amount spent (A).
[16]Where $\{(\alpha, H)\}^U$ is the set of all shareholders.

price discrimination, except here, the buyer pays different amounts for the same good. The second assumes shareholders are willing to tender shares at a price equal to the intrinsic value of equity plus the maximum lock-in cost among the set of tendering shareholders. This tendering behaviour is analogous to perfect competition, where a single market clearing price is paid for all goods sold. In both cases the most inexpensive way to repurchase shares is to repurchase from the pool of shareholders with the lowest lock-in costs, i.e. those with relatively low accrued capital gains and/or those wishing to sell relatively soon. The repurchase function - defined as the intrinsic value of equity repurchased, given an amount spent repurchasing shares - based on the first tendering assumption, denoted the weak repurchase function (WRF), is characterized as follows:

$$R(A) = max_{\{\alpha(H)\}} \left[\sum_{H=0}^{H_M} \int_{\alpha(H)}^{1} f(\alpha, H)\, d\alpha \right] X$$

$$S.T. \quad \sum_{H=0}^{H_M} \int_{\alpha(H)}^{1} [X + L(\alpha, H)] \cdot f(\alpha, H)\, d\alpha = A.$$

These equations state that for every H, firms choose an $\alpha(H)$ such that the intrinsic value of shares repurchased ($R(A)$) is maximized, while adhering to the constraint that the cost of repurchasing these shares is equal to the amount spent (A). Where the cost of each share repurchased is equal to the intrinsic value (X) plus the shareholder-specific lock-in cost ($L(\alpha, H)$). This repurchase function represents the cheapest possible way a firm can repurchase shares. Appendix D includes an alternate derivation of the WRF using the firm's equity supply curve. The repurchase function based on the second tendering assumption, denoted the strong repurchase function (SRF), is characterized as follows:

$$R(A) = max_{\{\alpha(H)\}} \left[\sum_{H=0}^{H_M} \int_{\alpha(H)}^{1} f(\alpha, H)\, d\alpha \right] X$$

$$S.T. \quad [max_{\{H\}}\, L(\alpha(H), H) + X] \cdot \sum_{H=0}^{H_M} \int_{\alpha(H)}^{1} f(\alpha, H)\, d\alpha = A.$$

The interpretation of these equations is similar to those of the WRF except the cost of each share is equal to the intrinsic value of equity plus the maximum lock-in cost among the repurchased shares ($max_{\{H\}}\, L(\alpha(H), H)$). This repurchase function represents the most expensive way to repurchase firm equity. Appendix D includes a derivation of the SRF using the firm's equity supply curve. For any distribution over investor-type, both the weak and strong repurchase functions are increasing in the amount spent and below (possibly weakly) the 45° line. As more shares are repurchased, the marginal shareholder's lock-in cost increases. From this, it follows that the WRF is concave in the amount spent.[17] The concavity of the SRF depends on the distribution over investor type. However, the SRF is bounded above by the WRF, i.e., for any amount spent, fewer shares are re-

[17]For a proof of this result see appendix D.

purchased using the SRF.[18] The derivative of each repurchase function is decreasing in the intrinsic value of the firm. As the intrinsic value of the firm declines more shares are repurchased for a given amount spent, and the marginal shareholder's lock-in cost becomes higher. At this point it is not important to choose among the two functional forms when discussing firm equilibrium behavior; the qualitative results are identical with both. A specific functional form is specified in the numeric section, but for now we refer to both as the "repurchase function".

3.1 Firm Equilibrium Behavior

The modeling environment is now characterized, we move on to the firm's equilibrium behaviour next. Recall that firms seek to maximize market value, represented by equation (1). This can be re-written as:

$$V_t = \frac{D_t(1-\tau_d)}{(1+\rho)} + \frac{(1-\tau_a)[V_{t+1} + R(A_t) - V_t]}{(1+\rho)} + \frac{V_t}{(1+\rho)},$$

since the fraction of the firm repurchased (δ_t,), equals $\frac{R(A_t)}{V_{t+1}+R(A_t)}$.[19] Solving for V_t gives us:

$$V_t = [1 + \frac{\rho}{(1-\tau_a)}]^{-1}\{D_t\frac{(1-\tau_d)}{(1-\tau_a)} + R(A_t) + V_{t+1}\}.$$

This equation can be rewritten further as the following infinite sum (with transversality condition: $lim_{t\to\infty}[(1+\frac{\rho}{(1-\tau_a)})^{-t} \cdot V_t] = 0$)

$$V_t = \sum_{t=1}^{\infty}[1 + \frac{\rho}{(1-\tau_a)}]^{-t}[D_t\frac{(1-\tau_d)}{(1-\tau_a)} + R(A_t)].$$

The per-period budget constraint is:

$$(1-\tau_c)\pi(K_t) = D_t + A_t + I_t,$$

where τ_c is the corporate tax rate, K_t is the capital stock available at the beginning of period t, $\pi(\cdot)$ is the decreasing returns profit function, A_t is the amount spent repurchasing shares at the end of period t, and I_t is the level of investment made at the end of period t.[20] The law of motion for capital is:

$$K_{t+1} = K_t + I_t,$$

since dividends cannot be negative, the following constraint also applies:

[18]For a proof of this result also see appendix D.

[19]The fraction of the firm repurchased equals the intrinsic value of repurchases divided by the market value of the firm, which equals the continuation value (value after shares are repurchased) plus the intrinsic value of repurchases.

[20]Given this budget constraint, it is clear that for any amount spent repurchasing shares (A_t), the firm's value is largest when shares are repurchased in the most inexpensive way, i.e., when $R(A_t)$ is maximized for a give A_t.

$$D_t \geq 0.$$

The Lagrangian for the infinite sum is:

$$\mathcal{L} = \sum_{t=1}^{\infty}[1+\frac{\rho}{(1-\tau_a)}]^{-t}\{[D_t\frac{(1-\tau_d)}{(1-\tau_a)}+R(A_t)] \quad - \quad \mu_t[\pi(K_t)(1-\tau_c)-D_t-A_t-I_t]$$

$$- \quad \lambda_t[K_{t+1}-K_t-I_t] \quad - \quad \xi_t[D_t]\},$$

where λ_t, μ_t, and ξ_t are Lagrange multipliers. The first order conditions for period t are:

$$D_t : \tfrac{(1-\tau_d)}{(1-\tau_a)} + \mu_t - \xi_t = 0 \qquad (2)$$

$$K_t : \lambda_t + \mu_t\pi'(K_t)(1-\tau_c) - (1+\tfrac{\rho}{(1-\tau_a)})^{-1}\lambda_{t+1} = 0 \qquad (3)$$

$$I_t : \mu_t + \lambda_{t+1} = 0 \qquad (4)$$

$$A_t : R'(A_t) + \mu_t = 0 \qquad (5)$$

3.2 Equilibrium Firm Payout and the Payout Puzzle

The shadow value of net profit is equal to $R'(A_t)$ from equation 5, i.e., the marginal value of total payout is always equal to the marginal value of share repurchases. When the dividend constraint binds (i.e., firms fail to pay dividends) the multiplier on the dividend constraint (ξ_t) is positive and $R'(A_t) > \frac{(1-\tau_d)}{(1-\tau_a)}$ from equation 2. In this case, a marginal share repurchase is more valuable than a dividend payment, and share repurchases are the marginal (and only) form of payout. When the dividend constraint fails to bind (i.e. firms wish to pay dividends) the lagrange multiplier on dividends (ξ_t) is zero. From equation 2 we know $R'(A_t) = \frac{(1-\tau_d)}{(1-\tau_a)}$, i.e. the marginal benefit of a share repurchase is equal to the marginal benefit of dividends. Since the repurchase function is concave, whenever dividends are paid they become the marginal form of payout.[21] This is intuitive since the marginal value of dividends is constant at $\frac{(1-\tau_d)}{(1-\tau_a)}$, while the marginal benefit of a repurchase is decreasing in the amount spent. Firms repurchase shares up to the point where the marginal benefit of doing so equals the marginal benefit of paying dividends. At this point firms stop repurchasing shares and switch to dividend payments. In equilibrium dividends are paid when the

[21] The WRF is everywhere concave. The SRF is concave locally whenever dividends become the marginal form of payout.

repurchase function is sufficiently concave, total payout is sufficiently large, and/or the relative tax burden is sufficiently high. The crucial assumption here is the existence of heterogeneous lock-in costs which cause the marginal benefit of repurchases ($R'(A_t)$) to fall as more share are repurchased.

3.3 The Value of Firm Capital

The marginal value of capital in the Traditional view - also known as marginal q - is equal to unity (see Poterba & Summers (1985)). This follows from share repurchases being the marginal form of payout, and the ability of firms to repurchase all shares for their intrinsic value. In the New view, dividends are the marginal form of payout, and the marginal value of capital is $\frac{(1-\tau_d)}{(1-\tau_a)} < 1$ (also see Poterba & Summers (1985)). That is, the value of capital within a firm is less than its replacement cost; capital is essentially "trapped" within a firm. The marginal value of capital in the Lock-In model (λ_t) can be derived using equations 4 & 5, which gives us $\lambda_t = R'(A_t)$. If firms pay dividends, marginal q equals $\frac{(1-\tau_d)}{(1-\tau_a)}$. As with the New view, when dividends are the marginal form of payout, capital is trapped within a firm. When share repurchases are the marginal form of payout, marginal q is greater than $\frac{(1-\tau_d)}{(1-\tau_a)}$ but also less than unity. Unlike the Traditional view, when share repurchases are the marginal form of payout the marginal value of capital within a firm is less than its replacement cost. In the Lock-In model, capital is trapped within a firm regardless of the marginal form of payout. Shareholders would like to reduce the amount of firm capital, since it's internal value is less than it's outside value, but have no way of doing so without incurring costs. In the case of dividends, the cost is a high dividend tax rate; in the case of share repurchases, the cost is a repurchase premium.

3.4 Investment Levels

The level of corporate investment and aggregate capital stock have real effects on the long-term aggregate welfare in an economy. This section discusses the effects of taxes on the level of corporate investment and capital stock. Optimal corporate investment requires that the rate of return on a marginal investment is equal to the user cost of capital. If we denote by i the gross return on a marginal investment, then undertaking this investment yields $i(1-\tau_c)$ in net-of-corporate-tax profit, one period hence. If $R(A_t)'$ is the marginal value of total payout in period t (discussed above), and $R(A_{t+1})'$ is the marginal value of total payout in period $t+1$, then the marginal investment generates an accrued capital gain of: $i(1-\tau_c)R(A_{t+1})' + [R(A_{t+1})' - R(A_t)']$. The first term of this expression is the after-corporate-tax value of profits generated by the investment, while the second is the intertemporal difference in the marginal value of capital. Since accrued capital gains generate a tax liability, captured by the AECGTR (τ_a), only $(1-\tau_a)\{i(1-\tau_c)R(A_{t+1})' + [R(A_{t+1})' - R(A_t)']\}$ of the investment's return is left after both taxes (corporate and AECGTR) are accounted for. The cost of this investment is the forgone marginal value of total payout in period t ($R(A_t)'$), leading to the following after tax return:

$$\frac{(1-\tau_a)\{i(1-\tau_c)R(A_{t+1})' + [R(A_{t+1})' - R(A_t)']\} + R(A_{t+1})'}{R(A_t)'} - 1.$$

Shareholders require a constant after-tax rate of return ρ. Equating the above return with ρ, and solving for i characterizes the user cost of capital:

$$i^* = \frac{R'(A_t)}{R'(A_{t+1})}\{\frac{\rho}{(1-\tau_c)(1-\tau_a)} + \frac{1}{(1-\tau_c)}[1 - \frac{R'(A_{t+1})}{R'(A_t)}]\}. \tag{6}$$

In an equilibrium, firms invest up to the point where a marginal investment yields a return equal to the user cost of capital, i.e. $\pi'(K_t) = i^*$. The user cost of capital is an increasing function of both the corporate tax rate and capital gains tax rate. Both taxes reduce the net return on investment, requiring a larger gross return to satisfy a shareholder's required after-tax return (ρ). Given the decreasing returns profit function, both taxes also reduce the level of corporate capital. Dividend taxes do not effect the user cost of capital, and like the New view, have no effect on capital levels. In a steady state $A_{t+1} = A_t$, and the user cost of capital is: $\frac{\rho}{(1-\tau_c)(1-\tau_a)}$, its long-run level. When firms are not in a steady state the user cost of capital depends on whether dividends are paid. When dividends are paid $R'(A_{t+1}) = R'(A_t) = \frac{(1-\tau_d)}{(1-\tau_a)}$, and the user cost of capital is equal to the steady state level. Here, investment is a function of exogenous variables only (τ_g, τ_c, ρ, and $\pi'(\cdot)$). When dividends are not paid A_{t+1} does not equal A_t, and total investment depends on the functional form of $R(\cdot)$. In this case, investment and total payout depend on endogenous variables, i.e., the amount spent on repurchases over time. In both the Traditional view and New view, a new steady state is reached immediately following a permanent corporate and/or capital gains tax change. In the Lock-In model, transition to a new steady state is gradual when share repurchases are the marginal form of payout.

3.5 The Effect of Tax Changes on Corporate Payout

3.5.1 Corporate Tax Rate

From the discussion above, we know the user cost of capital is an increasing function of the corporate tax rate, i.e., $\frac{\partial[i*]}{\partial\tau_c} = \frac{\rho}{(1-\tau_c)^2(1-\tau_a)} > 0$. Short-run investment moves in the opposite direction of a corporate tax change (given the decreasing returns profit function, and the condition $\pi'(K_t) = i^*$), causing short-run total payout to move in the same direction. In a long-run equilibrium, both the capital stock and total payout move in the opposite direction of a corporate tax change, whereas investment is unaffected due to the non-depreciating capital assumption. The value of dividends and share repurchases are unaffected in both the short-run and long-run. Whereas the marginal cost of a repurchase is increasing in the corporate tax rate; the marginal tendering shareholder's lock-in cost is a decreasing function of firm value, which is in turn a decreasing function of the corporate tax rate. The net effect of a corporate tax change on the level of dividends and share repurchases, in both the short-run and long-run, depends on the marginal form of payout before the tax change and the relative strength of two effects: the change in total payout and the change in the repurchase function. Both the level of dividends and share repurchases are increasing functions of total payout.[22] Dividends are an increasing function of the marginal cost of share repurchases, and share repurchases are a decreasing function of their own marginal cost. Table 2 reports the short-run and long-run effects of a corporate tax change on investment, total payout, capital stock, dividends, share repurchases, and the payout ratio - i.e., the ratio of dividends to share repurchases.

3.5.2 Realized Capital Gains Tax Rate

Payout is affected by the realized capital gains tax in three ways. First, it affects the user cost of capital in a fashion similar to the corporate tax rate, through the AECGTR. The user cost of capital is an increasing function of the capital gains tax rate, causing short-run total payout to be an increasing function as well. Both the long-run capital stock and long-run total payout move in the opposite direction of a capital gains tax change. The second payout effect is on the relative value of dividends and share repurchases. The marginal value of repurchases move in the opposite direction of the AECGTR, which in turn moves in the same direction as the capital gains tax. The marginal value of dividends is an increasing function of the capital gains tax rate, since dividend payments reduce the level of accrued capital gains and their associated tax liability. Third, the capital gains tax alters the slope of the repurchase function. The marginal cost of a repurchase is an increasing function of the capital gains tax rate for two reasons. First, firm value is a decreasing function of the capital gains tax, which causes the marginal tendering shareholder's lock-in cost to be an increasing function of the tax. Second, *every* shareholder's lock-in cost is an increasing function of the capital gains tax.[23] The net effect of a capital gains tax change on the level of dividends and share repurchases depends on whether dividends are paid before the tax change, the sign of the change, and the relative strength of two effects: the change in total payout and the change to the relative benefit of dividends and share repurchases. Both dividends and share repurchases are increasing functions of total payout. Dividends are a decreasing function of the relative marginal benefit of repurchases, and share repurchases are an increasing function of this benefit. Table 2 reports the short-run and long-run effects of a capital gains tax change on investment, total payout, capital stock, dividends, share repurchases, and the payout ratio.

3.5.3 Dividend Tax Rate

The dividend tax has no effect on the user cost of capital, and therefore no effect on the level of investment or total payout. It may, however, affect the marginal form of payout, depending on whether dividends are paid before and/or after a dividend tax change. To see why the dividend tax leaves the user cost of capital unaffected, first suppose dividends are the marginal form of payout.

[22] They may be weakly increasing depending on whether dividends are paid: when dividends are paid share repurchases are weakly increasing; when dividends are not paid, dividends are weakly increasing for small tax changes.

[23] For a proof of this result see Appendix D.

Table 2: Comparative Statics: Capital Tax Changes

	Dividends Are Paid						Dividends Are Not Paid					
	Corporate		**Capital Gains**		**Dividends**		**Corporate**		**Capital Gains**		**Dividends**	
	Increase	Decrease	Increase	Decrease	Increase	Decrease	Increase	Decrease	Increase	Decrease	Increase	Decrease
	Short-Run		Short-Run		Short-Run		Short-Run		Short-Run		Short-Run	
User Cost of Capital	↑	↓	↑	↓	NC	NC	↑	↓	↑	↓	NC	NC
Investment	↓	↑	↓	↑	NC	NC	↓	↑	↓	↑	NC	NC
Total Payout	↑	↓	↑	↓	NC	NC	↑	↓	↑	↓	NC	NC
Capital Stock	NC	NC	NC	NC	NC	NC	NC	NC	NC	NC	NC	NC
Dividends	↑	↓	↑	↓	↓	↑	NC/↑	NC	NC/↑	NC	NC	NC/↑
Share Repurchases	NC	↓/NC	↓	↓/NC/↑	↑	↓	↑	↓	↓/NC/↑	↓	NC	↓/NC
Payout Ratio	↑	↓	↑	↓	↓	↑	NC/↑	NC	NC/↑	NC	NC	NC/↑
	Long-Run		Long-Run		Long-Run		Long-Run		Long-Run		Long-Run	
User Cost of Capital	↑	↓	↑	↓	NC	NC	↑	↓	↑	↓	NC	NC
Investment	NC	NC	NC	NC	NC	NC	NC	NC	NC	NC	NC	NC
Total Payout	↓	↑	↓	↑	NC	NC	↓	↑	↓	↑	NC	NC
Capital Stock	↓	↑	↓	↑	NC	NC	↓	↑	↓	↑	NC	NC
Dividends	↓	↑	↓/NC/↑	↓/NC/↑	↓	↑	NC	NC/↑	NC/↑	NC	NC	NC/↑
Share Repurchases	↓/NC	NC	↓	↑	↑	↓	↓	↑	↓	↑	NC	↓/NC
Payout Ratio	↓	↑	↓/NC/↑	↓/NC/↑	↓	↑	NC	NC/↑	NC/↑	NC	NC	NC/↑

NC - No Change, ↑ - Increases, ↓ - Decreases, / - Both Effects Can Occur

Then a dividend tax change causes the marginal value of capital to change by $\frac{\partial \frac{(1-\tau_d)}{(1-\tau_a)}}{\partial m} = \frac{(-1)}{(1-\tau_a)}$ times the tax change, in both the current and subsequent period. From equation 6 we know that when both $R(A_t)'$ and $R(A_{t+1})'$ change by the same proportion the user cost of capital is unaffected. Therefore, a dividend tax change has no effect on the user cost of capital when dividends are the marginal form of payout. When share repurchases are the marginal form of payout the same result holds. When the dividend yield is positive the marginal value of a share repurchase is an increasing function of the dividend tax rate. However, when the dividend yield is zero (which must be the case when share repurchases are the marginal form of payout) a dividend tax change has no effect on the repurchase function,[24] and therefore no effect on the marginal value of capital in both the current and subsequent period. Given that $R(A_t)'$ and $R(A_{t+1})'$ are unaffected, the user cost of capital is also unaffected. In both cases the user cost of capital remains constant following a dividend tax change, leaving the level of investment and total payout unchanged as well.

The marginal form of payout may switch following a dividend tax change. If dividends are the marginal form of payout a dividend tax increase (decrease) necessarily decreases (increases) the level of dividends and increases (decreases) share repurchases. When a dividend tax increase is sufficiently large, the marginal form of payout switches from dividends to share repurchases. If share repurchases are the marginal form of payout, an increase in the dividend tax has no effect on payout, whereas a decrease may reduce share repurchases and increase dividend payments, depending on the size of the change. In the latter case, the marginal form of payout may switch from share repurchases to dividends. Table 2 reports the short-run and long-run effects of a capital gains tax change on investment, total payout, capital stock, dividends, share repurchases, and the payout ratio.

4 Numerical Solution to the Lock-In model

This section presents steady-state comparative statics of the Lock-In model using a numerical solution to the firm's problem. It also presents simulated dynamic responses to various tax changes. The results are intended to supplement the general results provided above and give us a better understanding of how modeling assumptions affect equilibrium outcomes. The appendix contains a discussion of the steps involved in deriving a steady state equilibrium, illustrating the model's underlying mechanism.[25]

An important building block of the Lock-In model is the multivariate distribution over accrued capital gains and holding period specific to a company's shareholders. This distribution characterizes the measure of shareholders having each possible lock-in cost and plays a significant role in determining the cost of a share repurchase program, and thus equilibrium outcomes. Ideally, a

[24] See the appendix for a proof of these results.

[25] A copy of this section of the appendix will be provided upon request.

number of company-specific empirical distributions could be estimated and used in our numerical exercises. However, lacking the necessary data on basis values and shareholder characteristics, this task is beyond the scope of the current paper. Instead, we parameterize the joint distribution, and run comparative statics when these parameters are changed. We do not claim the distributions used here reflect any true empirical distribution, but we do attempt to make clear how the chosen parameters affect equilibrium outcomes.

The first set of parameter values characterize the joint distribution's support. For the base-case distribution we assume investors hold assets between zero years (would like to sell immediately) and twenty years, and have between zero and one hundred percent capital gain. We initially ignore the case where shares are held with a capital loss, reflecting the work of Constantinides (1960), where it is shown that holding assets with a loss is never optimal when capital markets are complete and capital loss offsets exist.[26] Such positions are optimally sold immediately. Second, we impose bivariate normality (truncated and discretized over the support), and assume the mean is centered on the support, and that each standard deviation is 50% of the mean. Last, we assume the correlation coefficient between accrued capital gains and holding period is negative. The rational for this assumption is as follows: investors purchase stock with an ideal initial holding period, those with shorter current holding periods have thus, on average, held the stock for longer and have accumulated larger capital gains.[27] If we make the stronger assumption that investors have uniform initial holding periods, and equity returns are constant, the correlation coefficient would be negative one. However, investors have different initial holding periods and equity returns are stochastic in practice, so the correlation coefficient is likely greater than negative one. For our initial parametrization we choose a correlation coefficient of -.35, and allow it to change in our comparative statics section.

In addition to the joint distribution over investor type, the company's repurchase function also depends on the parameter values of the lock-in cost function, which includes the company's equity value (X), the statutory tax rate on income from dividends and realized capital gains (τ_d and τ_g respectively), and the firm's payout policy (the yield on dividends (d) and on capital gains (r)). Recall the lock-in cost function:

$$L(\alpha, H) = X\left\{\frac{(1-\alpha\tau_g)\{(1-\tau_g)(1+r+(1-\tau_d)d)^H+\tau_g(1-\tau_d)d\cdot\sum_{h=0}^{H-1}(1+r+(1-\tau_d)d)^h\}+\alpha\tau_g(1-\tau_g)}{\{(1-\tau_g)(1+r+(1-\tau_d)d)^H+\tau_g+\tau_g(1-\tau_d)d\cdot\sum_{h=0}^{H-1}(1+r+(1-\tau_d)d)^h\}(1-\tau_g)}-1\right\}.$$

In the base-case parametrization we use our most recent estimate of the average statutory tax rate paid by Canadian investors on income from dividends and realized capital gains, which are 17.1% and 20.9%, respectively in 2008. In a steady state, both firm value and the firm's payout

[26]The second requirement is a feature of the Canadian tax code. The first requirement is debatable.
[27]Given a positive return from accrued capital gains.

policy are equilibrium quantities that depend on the repurchase function itself, the profit function, statutory tax rates (both corporate and personal), the AECGTR, and the required after-tax rate of return ρ. In addition, the AECGTR is endogenous with respect to the yield on dividends and capital gains. This gives rise to a fixed point problem in four variables: firm value, dividend yield, capital gain yield, and the AECGTR. The appendix discusses the methodology used to solve these fixed point problems. In the base-case parametrization we use our 2008 estimate of the average national corporate tax rate paid by Canadian firms[28], which was 31.5%. A 6% after-tax required rate of return is used, which is consistent with the range of equity returns estimated by Campbell (2008) for Canadian firms over the period 1982-2007, which was discussed above. Without loss of generality a logarithmic profit function is used.[29]

Table 3 reports steady-state equilibrium values using the base-case model for both the weak and strong repurchase functions.

Table 3: Steady-State Equilibrium Values: Benchmark Parametrization

Repurchase Function	Firm Value	Capital Stock	Dividends	(% Yield)	Repurchases	(% Yield)	AECGTR
Strong	21.85	8.58	.86	(3.9)	.72	(3.3)	16.38
Weak	21.91	8.57	.13	(.6)	1.44	(6.6)	16.43

Equilibrium values depend on the type of repurchase function used. The SRF's first derivative is everywhere lower than that of the WRF, causing the marginal benefit of a repurchase to fall faster when the SRF is used. Firms pay dividends once the marginal benefit of doing so equals the marginal benefit of repurchasing shares, therefore firms pay more dividends with the SRF. The dividend yield is 3.9% when the SRF is used, which is 6.5 times higher than for the WRF (.6%). Due to the higher dividend yield, the AECGTR is lower for the SRF, which causes the steady-state capital stock to be higher. Total payout is higher for the SRF, but the average after-tax value of total payout is lower, since more payout is made in the form of tax disadvantaged dividends. The gross return using the SRF is 7.24% compared to only 7.21% using the WRF. If we subtract the intrinsic value of repurchases from the amount spent repurchasing shares we can calculate the repurchase premium. Both the absolute value of the repurchase premium and the per-share repurchase premium are higher for the SRF. The absolute value is \$.006 for the SRF and \$.0055 for the WRF. The per-share repurchase premium is \$.19 for the SRF, and only \$.09 for the WRF.

[28] The appendix includes a detailed discussion of how average corporate tax rates are calculated.

[29] To ensure profits are positive, the profit function is: ln(K+1), since K can be less than 1 in principle.

4.1 Comparative Statics

We now run comparative statics (CS) when the parameter values of the investor-type distribution are changed. The results presented here are derived using only the SRF. The results are qualitatively identical for both types. We maintain the distribution's truncated normality at first, and analyze dividend payouts when adjustments are made to the mean, covariance matrix, and support, while holding constant the AECGTR from the base-case model. The second set of CS exercises act on the class of investor-type distribution.

Table 4 reports the fraction of total payout going toward dividends when the mean of the investor-type distribution is adjusted. The table's columns correspond to changes in average holding period, which is varied from 7 to 13 years. The rows correspond to changes in average base, which is varied from 35% to 65% base. Dividend payments increase in average holding period and decrease in average base. For instance, no dividends are paid when the average investor has a base of 65% and holding period of 7 years, when the average holding period is increased to 13 years, and average base is 35%, dividend payments make up 80% of total payout. As average holding period increases and average base decreases a larger fraction of shareholders require higher lock-in premia to sell their shares, making large share repurchases less desirable, and increases the fraction of payout made with dividends.

Table 4: Fraction of Total Payout Going Toward Dividends: Mean Adjustments

		Average Holding Period						
		7	**8**	**9**	**10**	**11**	**12**	**13**
	35	0.469	0.569	0.644	0.702	0.743	0.776	0.801
	40	0.410	0.510	0.602	0.660	0.718	0.743	0.776
Avg.	**45**	0.319	0.435	0.544	0.619	0.660	0.702	0.735
Base	**50**	0.210	0.344	0.452	0.544	0.602	0.660	0.685
(%)	**55**	0.076	0.218	0.344	0.452	0.527	0.585	0.627
	60	0.000	0.076	0.218	0.327	0.419	0.494	0.544
	65	0.000	0.000	0.067	0.210	0.302	0.377	0.435

The next CS exercise acts on the covariance matrix. Table 5 reports the fraction of total payout going toward dividends when the standard deviation of both holding period and base, as well as the correlation coefficient between the two, are adjusted. The rows of Table 5 report adjustments to the standard deviation of both base and holding period, which are varied between 30% and 90% of the respective means. The columns correspond to adjustments to the correlation coefficient, which is varied between -.65 and -.05. The payment of dividends decreases in standard deviation and increases in the correlation coefficient. As the standard deviation increases the mass of shareholders requiring both low and high lock-in premia also increases. The fraction of equity that may be repurchased in a given period is limited by the inability to issue debt and the constant profit stream assumed by the model. Because repurchase programs target low-cost shares, it follows that all selling shareholders have below-average lock-in costs, making changes to the distribution of shareholders with above-average lock-in costs irrelevant for the repurchase decision. The net result of this is a higher standard deviation increases the number of cheap shares, making repurchase programs more desirable, and reduces the yield on dividends. A high correlation coefficient reduces the mass of shareholders with relatively short holding periods *and* few capital gains, and simultaneously increases both the mass of shareholders with relatively short holding periods *and* large capital gains, and shareholders with long holding periods *and* few capital gains.[30] The first effect increases the cost of a repurchase program whereas the last two decreases it. On net, the first effect dominates, and the cost of a repurchase program increases in the correlation coefficient.

Table 5: Fraction of Total Payout Going Toward Dividends: Covariance Matrix Adjustments

		Correlation Coefficient Between α and H						
		-0.65	**-0.55**	**-0.45**	**-0.35**	**-0.25**	**-0.15**	**-0.05**
Standard	**30**	0.801	0.801	0.818	0.818	0.835	0.843	0.859
Deviation	**40**	0.660	0.677	0.677	0.685	0.685	0.702	0.718
of α and H	**50**	0.510	0.510	0.527	0.544	0.560	0.569	0.585
(% of Mean)	**60**	0.360	0.360	0.377	0.394	0.410	0.419	0.435
	70	0.235	0.235	0.252	0.252	0.268	0.285	0.302
	80	0.143	0.143	0.143	0.159	0.159	0.176	0.193
	90	0.076	0.076	0.076	0.076	0.076	0.092	0.109

The next CS exercise acts on the support of the investor-type distribution. Table 6 reports the fraction of total payout going toward dividends when shareholders are allowed to maintain equity positions with capital losses, and the maximum holding period is varied between 17 and 23 years. The columns correspond to upper bounds on tax base, which are varied between 1 and 1.3 times the current market price.[31] The rows correspond to maximum holding periods. All shares with a capital loss are sold for a price at least as great as the intrinsic value of equity since capital markets have perfectly elastic supply. When the upper bound on tax base increases the fraction of total payout going toward dividends declines. This happens because the measure of cheap shares increases, decreasing the repurchase premium for any amount spent, and increasing the benefit of a repurchase. The fraction of total payout going towards dividends decreases in maximum holding period. As the maximum holding period increases the slope of the investor-type distribution falls everywhere. This decreases the absolute value of the second derivative of the repurchase function, enabling more shares to be repurchased before the marginal benefit of repurchasing shares equals the marginal benefit of paying dividends.

In the final CS exercise we change the class of investor-type distribution. Table 7 reports steady-

[30]It also decreases the mass of shareholders with long holding periods and large capital gains - those with a high lock-in cost - which, as argued above, is not relevant for repurchase decisions.

[31]A tax base ratio greater than 1 means the initial stock price is greater than the current market price: a capital loss.

Table 6: Fraction of Total Payout Going Toward Dividends: Support Adjustments

		Upper Bound of Base						
		1.3	**1.25**	**1.2**	**1.15**	**1.1**	**1.05**	**1**
	17	0.050	0.159	0.268	0.377	0.452	0.527	0.585
Upper	**18**	0.034	0.143	0.252	0.344	0.435	0.510	0.569
Bound of	**19**	0.000	0.126	0.218	0.327	0.419	0.494	0.560
Holding	**20**	0.000	0.092	0.210	0.302	0.394	0.469	0.544
Period	**21**	0.000	0.076	0.193	0.285	0.377	0.452	0.527
	22	0.000	0.067	0.176	0.268	0.360	0.435	0.510
	23	0.000	0.050	0.159	0.252	0.344	0.419	0.494

state equilibrium values when the distribution is: uniform; log-normal; and chi-squared. With a uniform distribution, a larger mass of shareholders have low lock-in costs, and the repurchase function is less concave due to a constant sloping CDF, as compared with the base-case normal distribution. More shares are repurchased, and the dividend yield is lower as a result. Both the log-normal and chi-squared distributions have higher sloping PDF's over the set of tendering shareholders, as compared with the benchmark normal distribution. This results in more concave repurchase functions, and less shares are repurchased.[32]

Table 7: Steady-State Equilibrium Values: Different Distribution Class'

Class of Distribution	Firm Value	Capital Stock	Dividends	(% Yield)	Repurchases	(% Yield)	AECGTR
Uniform	21.90	8.55	.03	(.1)	1.55	(7.1)	16.59
Chi-Squared	21.83	8.58	1.07	(4.9)	.51	(2.3)	16.36
Log-Normal	21.83	8.58	.95	(4.3)	.63	(2.9)	16.41

4.2 Dynamic Simulations

This section presents impulse response functions (IRF) for changes in the corporate tax, dividend tax, and realized capital gains tax. Initial tax rates and parameter values are taken from the base-case model above. The top two sub-plots of Figure 7 show IRF's for a change in the corporate tax rate, where the change happens in period t. The upper-left sub-plot shows the IRF for a 1% increase in the corporate tax rate. This tax change increases the user cost of capital, which reduces the optimal level of capital. Investment in period t decreases as a result, and total payout increases. The lower steady-state level of capital is realized in period t+1 and total payout is lower from that period forward. The higher tax liability on corporate profits, and lower total payout after period t,

[32]The mean of all three distributions are the same as the base-case model. The covariance of the log-normal distribution is also the same as the base-case model. The uniform and chi-squared distributions do not have covariance parameters.

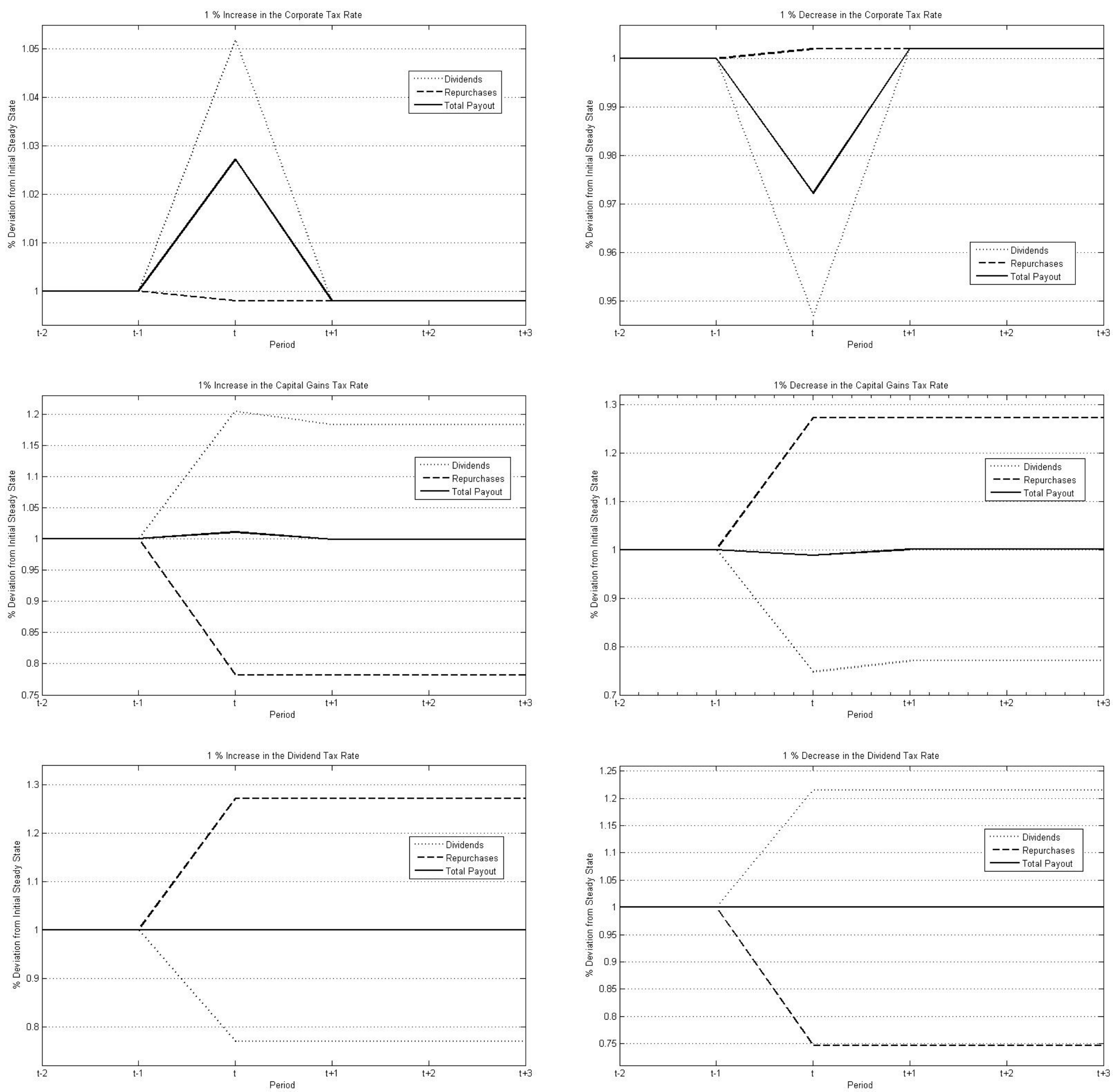
reduce total firm value. This reduces the marginal benefit of repurchasing shares for any amount spent as a larger fraction of the firm is repurchased, implying higher lock-in premia as more expensive shareholders are involved. As a result the amount spent repurchasing shares declines. Firm value in period t is slightly higher than in period t+1 onward, owing to the liquidation value of corporate capital and the corresponding higher total payout. Dividend payments increase in period t owing to higher total payout and lower repurchases. They decline in period t+1 onward due to lower total payout. The yield on dividends and capital gains are unchanged in the new steady state since corporate tax changes leave the relative value of dividends and share repurchases unaffected (for a given percentage of the firm repurchased). The upper-right sub-plot shows the IRF for a 1% decrease in the corporate tax rate. The effect on equilibrium quantities are opposite those for a 1% increase.

The middle two sub-plots show IRF's for a change in the capital gains tax. The middle-left sub-plot shows the IRF for a 1% increase in the tax during period t. By increasing the AECGTR, the increased capital gains tax raises the user cost of capital and lowers the steady-state capital stock. This reduces investment and increases total payout in period t. The new steady-state capital stock is realized in period t+1 and total payout is lower from that point onward. Share repurchases are lower for three reasons. First, firm value is reduced by the AECGTR in a similar fashion to the corporate tax, reducing the marginal benefit of a repurchase for any amount spent. Second, the increased AECGTR further reduces the marginal benefit of repurchases by strengthening the lock-in effect.[33] Third, dividend payments become more desirable as they reduce a now higher tax liability on capital gains. As a result, repurchases decline. Dividend payments are higher in period t for two reasons: first, the reduced level of share repurchases; and second, the higher level of total payout. From period t+1 onward dividends are higher owing to the first reason, which outweighs the lower level of total payout. The middle-right sub-plot shows the IRF for a 1% decrease in the capital gains tax rate, the effects of this on equilibrium quantities are opposite for the 1% increase.

The bottom two sub-plots show IRF's for dividend tax changes. The bottom-left sub-plot shows the IRF for a 1% increase in the dividend tax in period t. Unlike the previous two tax changes, a change in the dividend tax has no effect on the user cost of capital, and therefore no effect on the capital stock, investment levels, and total payout. A higher dividend tax reduces the benefit of paying dividends, by reducing the after-tax value of gross dividends. It also increases the marginal benefit of repurchasing shares by weakening the lock-in effect, but reduces it by decreasing firm value. On net, the weaker lock-in effect and decreased benefit of dividends dominates the firm value effect, and share repurchases increase. Total payout is constant and dividends decrease. The new steady state payout levels are achieved in period t+1. The bottom-right sub-plot shows the IRF for a 1% decrease in the dividend tax, again, the effects on equilibrium quantities are opposite those above.

[33] See Section 3 for a discussion of this result.

Figure 7: Dynamic Response to Tax Changes



5 Empirical Section

The Lock-In model predicts that a firm's choice over payout policy is made in accordance with statutory tax rates on corporate profits, dividend income, and capital gains income. Changes to these rates have effects on both the level of total payout and the payout mix. These effects were discussed in Section 3, and in Section 4 simulated dynamic responses to tax changes were presented. The question of whether firms actually adjust total payout, and the payout mix, in response to statutory tax changes is an empirical matter. This section adds to the empirical literature by presenting evidence supporting the hypothesis that Canadian firms choose payout policy in accordance with tax changes.

The existing literature has generally found significant relationships between payout policy and personal capital income taxation. Moser (2007), which studies US firms over the period 1986-2004 - during which a number of key tax regime changes took place - finds firms are more likely to distribute funds via share repurchases when the personal tax rate on dividends is high relative to capital gains. When the author separates firms according shareholder tax sensitivity (i.e. the degree of shareholder tax exposure) Moser (2007) finds firms with a high proportion of tax sensitive shareholders are more likely to substitute dividends for share repurchases when the dividend tax rate is relatively high. Chetty & Saez (2005) find the fraction of dividend paying firms and the average dividend payment among payers both increased following the 2003 US dividend tax cut. They also find total payout increased during this period, and reject the hypothesis that share repurchases were substituted for dividends. Further, they find dividend payments increased more when firms had tax sensitive shareholders, similar to Moser (2007). Brav et al. (2008) also find dividends increased following the 2003 dividend tax cut. However, they argue these were short-lived, and conclude the tax cut had no long-term effect. Contrary to these findings, the results in Poterba (2004) suggest the dividend tax penalty (defined in Section 2) has no short-run effect on dividend payout, but a significant long-run effect. The estimates of Poterba (2004) are based on US firms over the period 1935-2002, where an average long-run elasticity of 3.3 is found between dividend payments and the dividend tax penalty. Lie & Lie (1999) also find a positive relationship between share repurchases and the dividend tax rate using a sample of US firms over the period 1980-1994.

5.1 Regression Variables

The Lock-In model uses a single rate of tax on dividends and capital gains income when deriving optimal payout levels. In practice these rates differ across shareholder, and setting payout policy to maximize the wealth of each is not possible. In addition, shareholder-specific tax rates are unlikely to be known by the management of widely held corporations. In the empirical analysis we assume all firms choose optimal payout policy in accordance with the *average* marginal tax rate faced by its shareholders. We proxy for this using the average marginal tax rate series presented in Section

2. This assumption is used in Poterba and Summers (1984) when testing the predictions of the New and Traditional view models. The Lock-In model also predicts payout policy is a function of the corporate tax rate. Appendix A contains a discussion of the corporate tax rate variable we include, which estimates the average Canadian rate.

In an attempt to keep the theoretical analysis straightforward the Lock-In model uses a fairly stripped down representation of the firm. Many firm characteristics that likely affect firm payout were not included in the model. The literature has identified a number of these, which we included in our empirical analysis. Chetty & Saez (2005) include firm assets, as does Moser (2007), which also includes the ratio of debt to assets, the standard deviation of earnings, and market to book value. Canadian real GDP and the interest rate on 3-month government of Canada debt are also included.

5.2 Empirical Specification

In the Lock-In model total payout is a function of net income, which in turn depends on the firm's profit function and user cost of capital. In a steady state, net income completely explains total payout, and tax rates have no additional explanatory power. However, changes in taxes do partially explain total payout in the short-run (after conditioning on net income) through an effect on short-run investment levels, via the user cost of capital. In Sections 3 and 4 we discuss the short-run and long-run effects of tax changes. It is shown that dividend taxes have no effect on total payout, whereas the corporate tax and capital gains tax have a positive effect on short-run total payout and a negative effect on long-run total payout. Our estimation strategy is to identify the short-run effects of capital tax changes on total payout, after conditioning on net income.

More formally:

$$Total\ Payout = (1 - \tau_c)\pi(K_t) + [K_t - K^*(\pi(\cdot), \tau_d, \tau_g, \tau_c)],$$

where $K^*(\cdot)$ is the optimal capital stock. In a steady state $K_t = K^*(\cdot)$ and total payout is a function of net income ($(1 - \tau_c)\pi(K_t)$) only. When tax rates change, $K^*(\cdot)$ adjusts, and the second term of this expression has explanatory power for total payout. We hypothesis that:

$$\frac{\partial(Total\ Payout_t | Net\ Income_t)}{\partial[K_t - K^*(\cdot)]} > 0,$$

and use the following empirical expressions for short-run changes to investment caused by changes in personal capital income taxes and the corporate tax rate:

$$[K_t - K^*(\cdot)] = \beta_b[T^{Avg}_{b,t} - T^{Avg}_{b,(t-1)}] + \varepsilon_b \quad b \,\epsilon\, (d, g, c),$$

where $T^{Avg}_{d,t}$ is the average marginal tax rate on dividend income in year t from Section 2, $T^{Avg}_{g,t}$ is the average marginal tax rate on realized capital gains income in year t from Section 2, and $T_{c,t}^{Avg}$ is the average tax rate on corporate income in year t, which is explained in Appendix A. This leads to the following full empirical specification:

$$Total\ Payout_t = \sum_b \beta_b [T_{b,t}^{Avg} - T_{b,(t-1)}^{Avg}] + \gamma Z_t + \varepsilon,$$

where Z_t is a set of explanatory variables including net income, ε is a vector of coefficients, and ε is a compound error term. The Lock-In model predicts $\beta_d = 0$, $\beta_g > 0$, and $\beta_c > 0$.

5.3 Data Set

Firm level data consists of an unbalanced panel of TSX listed corporations spanning the years 1987-2008. The panel is unbalanced due to a changing composition of TSX listed firms over this period. Income trusts are omitted from the sample due to their special tax treatment. As flow-through entities, income trusts are exempt from paying Canadian corporate income tax. Canadian subsidiaries of foreign corporations are also omitted. The differential tax treatment of corporate profits across national lines is likely to have an impact on the financing and payout decisions of Canadian subsidiaries. We exclude Canadian subsidiaries to limit these confounding effects on our empirical estimates.[34] Our primary source for company financial data is Datastream, which is supplemented with Compustat data when observations are missing. We use Datastream and Compustat for annual dividend payments, net income, debt levels, total assets, book value, GDP, and interest rates. The share repurchase series is discussed in Appendix A. We use a Tobit regression model to correct for the limited dependent variables problem caused by a number of zero payout observations.[35] Table 9 reports coefficient estimates from the total payout regression.

5.4 Regression Results

Column two of Table 9 presents coefficient estimates when the first difference of taxes - i.e., the contemporary tax rate minus the one year lag - are included as regressors. All three coefficient estimates are not significantly different from zero, implying firms do not adjust total payout when differences between current and lagged taxes arise - contrary to the predictions of the Lock-In model. The third column of Table 9 reports coefficient estimates when lagged first differences are also included. Here, changes in taxes seem to matter. The coefficient estimates are positive and significant for both corporate tax changes and capital gains tax changes. These estimates suggest firms adjust total payout in response to tax changes, but with a lag. Possible reasons for this are the timing of tax change announcements or the speed with which investment can be adjusted. In the Lock-In model, investment is adjusted immediately, whereas in practice investment levels may

[34]Some of the firms in the sample are Canadian-based multinationals, so this problem is not eliminated completely.
[35]For a discussion of the Tobit model see Tobin (1958).

Table 8: Total Payout Regression (Tobit)

Variable	Model 1		Model 2		Model 3	
Net Income	0.16**	(.000)	0.16**	(.000)	0.154**	(.000)
Income Var	-4.39E-11*	(.061)	-4.47E-11*	(.056)	-4.55E-11*	(.056)
Assets	0.0048**	(.000)	0.0048**	(.000)	0.0049**	(.000)
Debt/Assets	4,680*	(.085)	4,673*	(.085)	4,685*	(.089)
Index/Book	8,413,998**	(.008)	4,151,934	(.235)	3,382,638	(.421)
GDP	-0.317**	(.000)	-0.223**	(.019)	-0.175	(.165)
Interest	10506**	(.000)	10,532**	(.000)	11,312**	(.001)
Div: Cur-Lag1	-45,803	(.734)	24,380	(.900)	229,300	(.395)
Div: Lag1-Lag2			-200,214	(.256)	-9,095	(.969)
Div: Lag2-Lag3					-273,077	(.130)
CapGain: Cur-Lag1	149543	(.235)	-42,102	(.720)	25,456	(.845)
CapGain: Lag1-Lag2			281,975**	(.003)	168,789	(.146)
CapGain: Lag2-Lag3					355,115**	(.000)
Corp: Cur-Lag1	-268230	(.341)	-191,591	(.535)	-9,653	(.979)
Corp: Lag1-Lag2			605,487**	(.041)	757,008**	(.046)
Corp: Lag2-Lag3					339,383	(.276)
Pseudo R^2	0.0291		0.0292		0.0294	

6761 left-censored observations, 7219 uncensored observations, ** - significant at the 5% level,
* - significant at the 10% level, P-values in parentheses. All regression variables are in thousands of 2008 Canadian dollars, other than interest and tax rates (which are represented as fractions).

take time to adjust. The positive coefficient estimates on corporate and capital gains tax changes are predicted by the Lock-In model, as discussed above. The coefficient estimates on the dividend tax are not significantly different from zero, also consistent with the Lock-In model. The above results are predicted by the New view as well. However, they are inconsistent with the Traditional view, where dividend tax changes are predicted to have an effect on short-run total payout. The last column of Table 9 includes the second lag of capital tax changes also. The results are similar to those of column three.

The next set of regressions analyze the effects of tax changes on the level of dividends and share repurchases using the empirical specification above. From the results of Section 3, the short-run effects of tax changes on the level of dividends and share repurchases depend (at times) on whether dividends are paid. We do not separate firms according to this criteria, and include all firms in these regressions. Therefore the empirical specifications used here are interpreted as reduced form. A Tobit regression model is also used for these regressions, reflecting a number of zero observations on both dividends and share repurchases.

Table 9: Repurchase Regression (Tobit)

Variable	Model 1		Model 2		Model 3	
Net Income	0.012**	(.005)	0.012**	(.006)	0.0129**	(.005)
Income Var	5.47E-13	(.972)	3.35E-13	(.983)	-1.22E-12	(.939)
Assets	0.0011**	(.000)	0.0011**	(.000)	0.0011**	(.000)
Debt/Assets	-11,650	(.370)	-13,305	(.311)	-13,529	(.315)
Index/Book	16,900,000**	(.000)	11,400,000**	(.001)	12,000,000**	(.002)
GDP	-0.206**	(.006)	0.0008	(.993)	-0.0117	(.923)
Interest	-111	(.963)	8,098**	(.007)	12,314**	(.001)
Div: Cur-Lag1	169,420	(.245)	700,804**	(.000)	697,570**	(.010)
Div: Lag1-Lag2			315,366*	(.091)	223,172	(.335)
Div: Lag2-Lag3					-590,488**	(.001)
CapGain: Cur-Lag1	-472,609**	(.000)	-638,185**	(.000)	-599,643**	(.000)
CapGain: Lag1-Lag2			-165,550	(.102)	-200,488*	(.095)
CapGain: Lag2-Lag3					-30,838	(.763)
Corp: Cur-Lag1	-480,744	(.110)	-68,792	(.835)	311,248	(.390)
Corp: Lag1-Lag2			1,008,852**	(.001)	724,422*	(.065)
Corp: Lag2-Lag3					936,105**	(.004)
Pseudo R^2	0.008		0.0083		0.0086	

11567 left-censored observations, 2152 uncensored observations, ** - significant at the 5% level, * - significant at the 10% level, P-values in parentheses. All regression variables are in thousands of 2008 Canadian dollars, other than interest and tax rates (which are represented as fractions).

Table 10 reports coefficient estimates when the dependent variable is share repurchases. Column two reports estimates when the current first difference is included, column three includes a lagged first difference, and column four includes a second lagged first difference. In all three specifications, the capital gains tax is found to have a negative effect on the level of share repurchases. The dividend tax is found to have a positive effect on repurchases in both the second and third model, but no effect when only the current first difference is included. However, the second lagged first difference seems to have a negative effect on share repurchases. With the exception of this estimate, all other estimates seem to have the correct sign. When the tax penalty on capital gains increases, firms appear to reduce share repurchases in an apparent attempt to reduce the now higher tax liability. When the dividend tax increases, firms increase share repurchases, which are now more lightly taxed relative to dividends, except in the case of the second first difference, which appears to reduce share repurchases. The corporate tax rate has a positive effect on share repurchases in both the second and third models, but no effect in the first model. The positive coefficient estimates may be a result of higher total payout following a corporate tax increase (which was found in the total payout regressions) where part of the higher payout seems to flow into share repurchases.

Table 10: Dividend Regression (Tobit)

Variable	Model 1		Model 2		Model 3	
Net Income	0.16**	(.000)	0.16**	(.000)	0.154**	(.000)
Income Var	-5.23E-11*	(.076)	-5.30E-11*	(.072)	-5.49E-11*	(.075)
Assets	0.0043**	(.000)	0.0043**	(.000)	0.0044**	(.000)
Debt/Assets	5,796**	(.031)	5,828**	(.030)	5,855**	(.032)
Index/Book	5,175,817	(.114)	1,823,797	(.615)	610,915	(.890)
GDP	-0.336**	(.000)	-0.284**	(.004)	-0.2128	(.106)
Interest	11,435**	(.000)	9,143**	(.001)	9,060**	(.010)
Div: Cur-Lag1	-98,690	(.471)	-191,234	(.338)	82,343	(.767)
Div: Lag1-Lag2			-289,398	(.109)	18,687	(.938)
Div: Lag2-Lag3					-48,991	(.794)
CapGain: Cur-Lag1	333,402**	(.011)	170,267	(.160)	248,449*	(.067)
CapGain: Lag1-Lag2			352,131**	(.000)	227,183*	(.058)
CapGain: Lag2-Lag3					376,507**	(.000)
Corp: Cur-Lag1	20,709	(.943)	43,428	(.891)	143,484	(.702)
Corp: Lag1-Lag2			309,062	(.311)	704,025*	(.074)
Corp: Lag2-Lag3					-6,945	(.983)
Pseudo R^2	0.0307		0.0308		0.0309	

7556 left-censored observations, 6031 uncensored observations, ** - significant at the 5% level,
* - significant at the 10% level, P-values in parentheses. All regression variables are in thousands of 2008 Canadian dollars, other than interest and tax rates (which are represented as fractions).

Table 11 reports coefficient estimates when dividends are the dependent variable using the same three regression models outlined above. In all three models the capital gains tax has a positive effect on dividend payout; when the capital gains tax increases, dividends become less tax disadvantaged relative to share repurchases, and firms seem to increase dividend payments as a result. Dividend taxes are found to affect dividend levels in the second model only at an 11% significance level, although the sign of this estimate seems correct. Changes in corporate tax rates do not appear to have much of an effect on the level of dividends. The only significant coefficient estimate is the second first difference from model three, and this estimate is only significant at the 5% level.

In summary, the capital gains tax has a significant effect on: total payout, dividends, and share repurchases. The effect is positive for total payout and dividends, and negative for share repurchases. The dividend tax has, for the most part, a positive effect on share repurchases and a negative effect on dividends at the 11% level. The corporate tax has a positive effect on total payout, share repurchases and dividends.

6 Conclusion

Economic studies have shown that dividends are tax disadvantaged in the United States relative to share repurchases. This paper has shown the same is true for Canada. Despite this, dividend payments constitute a significant fraction of total corporate payout in both countries, giving rise to a payout puzzle: persistent dividend payments appear to be an inferior form of payout from a firm value maximization perspective. This paper has proposed a model of corporate payout policy which offers an explanation for the puzzle by appealing to the lock-in effect caused by the postponement of taxes on capital gains. It is argued that shareholder lock-in effects require firms to pay a premium over the intrinsic value of equity when repurchasing shares. This premium adds to the cost of share repurchases, and tax disadvantaged dividends are paid whenever this premium is sufficiently high. The premium depends on the shareholder distribution over accrued capital gains and desired holding period. It also depends on the tendering behaviour of shareholders, i.e., the relationship between a shareholder's lock-in cost and the premium demanded. We derive repurchase functions that reflect lower and upper bounds on these premiums, and solve the model using a number of shareholder distributions. In general, dividend payments are highest when shareholders demand the maximum lock-in premium, when the average accrued capital gain is high, the average holding period is long, and when the variance of these arguments is low.

The model is used to explore the consequences of tax policy for corporate investment and payout policy. It is found that investment is a decreasing function of both the corporate tax and the capital gains tax, and is unaffected by the dividend tax. Short-run total payout increases in both the corporate tax and the capital gains tax, whereas long-run total payout decreases in both. The dividend tax has no effect on total payout in either the short-run or the long-run. The model is also used to derive the marginal value of corporate capital. We find that the outside value of corporate capital, i.e., its replacement cost, is higher than its marginal value within a firm. That is to say, corporations are overcapitalized, shareholders would like to reduce the amount of capital within a firm but are unable to do so without incurring a cost. When dividends are the marginal form of payout the cost is a relatively high dividend tax rate, when share repurchases are the marginal form of payout the cost is a repurchase premium.

The paper also presented a set of empirical estimates on the relationship between corporate payout policy and taxes at the personal and corporate level in Canada. The results suggest that short-run total payout is unaffected by the dividend tax but increases in both the corporate tax and the capital gains tax. These results confirm the predictions of the Lock-In model. The empirical results also suggest that the method of payout is sensitive to both the dividend tax and the capital gains tax. When the dividend tax increases short-run dividend payments decline, for the most part, and share repurchases increase. When the capital gains tax increases, the opposite occurs: repurchases decline and dividend payments increase. The Lock-In model also performs well when its predictions are compared to these results. From Section 3 we know the short-run effects of personal tax changes on the payout mix are at times ambiguous; they are shown to depend on the marginal form of payout and other factors. However, the model unambiguously predicts that short-run dividend payments are weakly increasing in the capital gains tax and weakly decreasing in the dividend tax, which is found to be the case empirically.[36] The effects of personal taxes on share repurchases, predicted by the Lock-In model, are more unclear; the sign of these effects may change depending on a number of factors. However, the empirical results fit within the set of *possible* effects predicted by the model.

References

Allen, F., Michaelly, R., 2003. "Payout Policy," In H*andbook of the Economics of Finance Volume 1a Corporate Finance*, ed. G. M. Constantinides, M. Harris, and R. Stulz, 337-429. Amsterdam: Elsevier Science, North-Holland.

Allen, F., Bernardo, E., Welch, I., 2000. "A Theory of Dividends Based on Tax Clienteles," *The Journal of Finance*, Vol. 55, No. 6, 2499 - 2536.

Auerbach, A., 1979."Wealth Maximization and the Cost of Capital," *The Quarterly Journal of Economics*, Vol. 93, 433-46.

Auerbach, A., Hassett, K., 2002. "On the Marginal Source of Investment Funds," *Journal of Public Economics*, Vol. 87, 2005-232.

Bagwell, L., 1992. "Dutch Auction Repurchases: An Analysis of Shareholder Heterogeneity," *The Journal of Finance*, Vol. 47, No. 1, 71-105.

Barclay, M., Smith, C., 1988. "Corporate Payout Policy: Cash Dividends Versus Open Market Share Repurchases," *Journal of Financial Economics*, Vol. 22, 61-82.

Black, F., 1976. "The Dividend Puzzle," The Journal of Portfolio Management, Vol. 2, No. 2, 5-8.

Bradford, D., 1981."The Incidence and Allocation Effects of a Tax on Corporate Distributions," *Journal of Public Economics*, Vol. 15, 1-22.

Brav, A., Graham, J., Campbell, H., Michaely, R., 2008. "The Effect of the May 2003 Dividend Tax Cut on Corporate Dividend Policy: Empirical and Survey Evidence," *National Tax Journal*, Vol. 61, No. 3, 381-396.

[36] When dividends are not the marginal form of payout, both the dividend tax and the capital gains tax may have no effect on dividends. When share repurchases are the marginal form of payout dividends are strictly increasing in the capital gains tax and strictly decreasing in the dividend tax.

Busaba, W., 2011. "The Dividend Policy Puzzle," *Richard Ivey School of Business Working Paper.*

Chetty, R., Saez, E., 2005. "Dividend Taxes and Corporate Behavior: Evidence from the 2003 Dividend Tax Cut," *The Quarterly Journal of Economics*, Vol. 120, No. 3,791-833.

Chowdhry, B., Nanda, V., 1994. "Repurchase Premia as a Reason for Dividends: A Dynamic Model of Corporate Payout Policies," *The Review of Financial Studies*, Vol. 7, No. 2, 321-350.

Constantinides, G., 1983. "Capital Market Equilibrium with Personal Tax," *Econometrica*, Vol. 51, No. 3, 611-636.

Davis, J., Glenday, G. 1990. "Accrual Equivalent Marginal Tax Rates for Personal Financial Assets," *The Canadian Journal of Economics*, Vol. 23, No. 1, 189-209.

Easterbrook, F., 1984. "Two Agency-Cost Explanations of Dividends," *The American Economic Review*, Vol. 74, No.4, 650-659.

Ikenberry, D., Lakonishok, J., Vermaelen, T., 2000. "Stock Repurchases in Canada: Performance and Strategic Trading," Journal of Finance, Vol. 55, No. 5 2373 - 2398.

Kerr, H., McKenzie, K., Mintz, J., (forthcoming). "Tax Policy in Canada," *Canadian Tax Foundation.*

King, M., 1977. "Public Policy and the Corporation," London: *Chapman and Hall.*

Landsman, W., Shackelford, D., 1995. "The Lock-In Effect of Capital Gains Taxes: Evidence from the RJR Nabisco Leveraged Buyout," *National Tax Journal*, Vol. 48, No. 2, 245-259.

Lie, E., Lie, E., 1999. "The Role of Personal Taxes in corporate Decisions: An Empirical Analysis of Share Repurchases and Dividends," *The Journal of Financial and Quantitative Analysis*, Vol. 34, No. 4, 533-552.

McNally, W., Smith, B., Barnes, T., 2003. "Does Supply Curve Inelasticity Explain Abnormal Long-Run Returns Following Open Market Share Repurchases?," *Wilfrid Laurier Business & Economics Working Paper.*

Miller, M., Modigliani, F., 1961. "Dividend Policy, Growth, and the Valuation of Shares."

Journal of Business, Vol. 34, 411-433.

Miller, M., Scholes, M., 1978. "Dividends and Taxes," *Journal of Financial Economics*, Vol. 6, 333-364.

Moser, W., 2007. "The Effect of Shareholder Taxes on Corporate Payout Choice," *Journal of Financial and Quantitative Analysis*, Vol. 42, No. 4, 991-1020.

Poterba, J., 2004. "Taxation and Corporate Payout Policy," *The American Economic Review Papers and Proceedings*, Vol. 94, No. 2, 171-175.

Poterba, J., 1987. "Tax Policy and Corporate Saving," B*rookings Papers on Economic Activity*, No. 2, 454-504.

Poterba, J., Summers, L., 1984. "The Economic Effects of Dividend Taxation," *NBER Working paper* No. 1353.

Protopapadakis, A., 1983. "Some Indirect Evidence on Effective Capital Gains Tax Rates," *The Journal of Business*, Vol. 56, No. 2, 127-138.

Sialm, C., 2009. "Tax Changes and Asset Pricing," *American Economic Review*, Vol. 99, No. 4, 1356-1383.

Tobin, J., 1958. "Estimation of Relationships for Limited Dependent Variables," *Econometrica*, Vol. 24, 24-36.

TSX 2008. "Policy 5.6: Normal Course Issuer Bids," *TSX Venture Exchange Publication.*

Appendix: A

A.1 Average Marginal Tax Rate Construction

The marginal tax rate on income from dividends and realized capital gains faced by taxable Canadian shareholders depends on their level of taxable income (total income less deductions), tax credits and province of residence. Canada uses a bracket system of income taxation, where statutory marginal rates increase as taxable incomes rise. In addition, during the last 25 years, Canada has imposed a number of surtaxes and claw backs that also depend on income. These factors lead to non-linear tax schedules (a function relating marginal tax rates to income levels) at both the federal and provincial levels. All Canadians face the same federal tax schedule, but provincial tax schedules often differ by province; both federal and provincial tax schedules often differ by year as well. Using tax forms provided by the Canadian Revenue Agency (CRA) federal and provincial tax schedules for dividend and capital gains income are calculated over the years 1985-2008.[37] Lacking information on individual tax credits all schedules were derived by setting tax credits, other than the basic personal amount for individuals, to zero.

Every year the CRA publishes the aggregate amount of taxable income claimed by individual's across income brackets, broken down by source of income. From these statistics the proportion of dividend and capital gains income claimed by individuals within each bracket can be calculated. The marginal tax rate on each source of income within each income bracket is calculated using the tax schedules derived above.[38] Average marginal tax rates are computed by scaling the average marginal tax rate of each income bracket by the proportion of total income claimed by individuals within each bracket. This is done for every year, and every province.

The CRA also publishes the aggregate amount of income claimed by residents of each province broken down by income type. A weighted average of provincial averages is derived, where the weights are the proportion of total income (from dividend and realized capital gains) claimed by residents of each province. This provides the Canadian national average marginal tax rate series for dividends and realized capital gains income.[39]

A.2 Dividend and Realized Capital Gains Taxes in Canada

In an attempt to reduce the double taxation if dividend income, the CRA has instituted a rather complicated system of dividend taxation. Dividend income is initially "grossed up" by a factor intended to reflect the value of gross corporate income from which dividends are paid. This factor is currently 1.44 for "eligible" dividends.[40] Shareholders pay regular tax on this amount, which is akin to paying tax on the amount of gross corporate income. In this way corporate income is approximately treated as flow-through income, as would be the case for non-corporate business income in Canada. Next, shareholders are allowed to claim a tax credit on the grossed up amount of dividend income to offset the corporate taxes already paid by the firm. The federal tax credit is currently 17.97% for eligible dividends. The provincial tax credit varies by province. This system

[37]The marginal tax rate on dividend and capital gains income is different from other forms of income. See section A.2 for an explanation of how these rates are calculated.

[38]At times an income bracket has multiple marginal tax rates. The average marginal tax rate within these brackets is calculated by taking a weighted average of the individual rates, weighted by the proportion of income within each bracket subject to each rate.

[39]These series do not factor in the Lifetime Capital Gains Exemption (LCGE) introduced in 1985. In 1985 the LCGE allowed for an individual to exempt up to $500,000 of capital gains income over his/her lifetime. The exemption was reduced to $100,000 in 1987 except for farm property and shares held in qualifying small businesses. In 1994 the exemption was eliminated, except in the case of capital gains originating from farm property and small business. This exemption served to reduce the tax rate on capital gains.

[40]The CRA makes a distinction between "eligible" dividends and "non-eligible" dividends. The former constitute the vast majority of dividend payments made by Canadian corporations. The gross up rate on non-eligible dividends is 1.25.

does not eliminate double taxation altogether, but reduces it. The degree to which this happens is a function of the individual's marginal tax rate on dividends. In general the higher is the individual's marginal tax rate the less effective is the elimination. For a more detailed discussion of the tax treatment of dividend income see (CRA publication), and for a discussion of how this system affects individuals with different income levels see (textbook). The marginal tax rate on realized capital gains is equal to the marginal tax rate on regular income multiplied by the inclusion rate. The inclusion rate has changed over time, and is currently equal to 50%.

A.3 Aggregate Dividend and Share Repurchase Series Construction

The Toronto Stock Exchange (TSX) reports the number of securities repurchased each month by TSX listed companies in its publication "The Daily Record". Back-dated issues of this publication are used to create a data set containing all common stock repurchases made from 2001 to 2008, and data from McNally & Smith (2007) is used for common stock repurchases spanning 1987 to 2000. These are combined to produce a unique data set containing all monthly repurchases of common stock made by TSX listed companies from 1987 to 2008. The Daily Record also publishes monthly repurchase data on preferred shares and income trust units. However, these data were omitted, the former due to the debt qualities of preferred shares, and the latter due to the tax treatment of income trusts, as discussed in Section 5. To estimate the amount spent by each repurchasing firm, in each month, the number of shares are multiplied by the average monthly mid-day stock price reported on Datastream. In some cases, especially in early years, price data is unavailable. These firms are omitted from the data set. This underestimates aggregate repurchase figures, and may also bias average numbers and repurchase frequencies, although the sign and degree of the bias is unknown. Dividend numbers are taken from Datastream, and when unavailable from Compustat. If firms had missing dividend data they were also omitted from the data set. This may also affect aggregate numbers, averages and frequencies as the omissions above.

A.4 Average Corporate Tax Rate Construction

In Canada, there are three rates of corporate tax, one for small business, one for large manufacturing firms and one for large non-manufacturing firms. The firms used in our empirical analysis are sufficiently large that none would qualify for the first rate. Each Canadian corporation pays the same federal corporate tax rate, but pays a unique provincial tax. Ideally we would be able to calculate the actual corporate tax rate faced by each firm in our sample, however we have limited firm-sector data and no data on province of incorporation. Therefore we estimate the national average corporate tax rate as follows. Using back-dated issues of "Finances of the Nation"[41] we construct a time series of corporate tax rates (for both manufacturing and non-manufacturing firms) for each province. We calculate the national average corporate tax rate faced by manufacturing and non-manufacturing firms by taking a weighted average of the province-specific rates (plus the federal

[41] Published by the Canadian Tax Foundation.

rate), weighted by the province's share of national corporate income, provided by Datastream. To estimate the average Canadian corporate tax rate we take a weighted average of the rates above, weighted by the proportion of corporate income generated by manufacturing and non-manufacturing firms provided by Datastream. This is done for every year between 1985-2008.

Appendix: B

B.1 Accrual Equivalent Capital Gains Tax Rate

The AECGTR is calculated based on the assumption that after-tax dividend income is reinvested in the firm. If we denote the dividend tax rate by τ_d, the tax rate on realized capital gains by τ_g, the AECGTR by τ_a, the rate of return from dividends as d, and the rate of return from capital gains as r, then Table 1 reports the after-tax value of an investment with market value X after a number of years (from zero to three years) using an realization based capital gains tax system (this illustrates how the general equation is derived).

Table 11: After-Tax Value of Equity Positions Using a Realizations Based Tax System

Year	After-Tax Value (ATV) Using τ_g	Market Value (MV)	Base (B)
0	$ATV_0 = X$	$MV_0 = X$	$B_0 = X$
1	$ATV_1^{\tau_g} = MV_0[(1-\tau_g)(1+r)+(1-\tau_d)d]+\tau_g B_0$	$MV_1 = MV_0(1+r+(1-\tau_d)d)$	$B_1 = B_0 + MV_0((1-\tau_d)d)$
2	$ATV_2^{\tau_g} = MV_1[(1-\tau_g)(1+r)+(1-\tau_d)d]+\tau_g B_1$	$MV_2 = MV_1(1+r+(1-\tau_d)d)^2$	$B_2 = B_1 + MV_1((1-\tau_d)d)$
3	$ATV_3^{\tau_g} = MV_2[(1-\tau_g)(1+r)+(1-\tau_d)d]+\tau_g B_2$	$MV_3 = MV_2(1+r+(1-\tau_d)d)^3$	$B_3 = B_2 + MV_2((1-\tau_d)d)$

In general, the after-tax value of equity using a realization based tax system (τ_g) is:

$$ATV_H^{\tau_g} = X\{[(1-\tau_g)(1+r)+(1-\tau_d)d](1+r+(1-\tau_d)d)^{(H-1)}+\tau_g(1-\tau_d)d\cdot\sum_{h=1}^{H}(1+r+(1-\tau_d)d)^{h-1}\}$$

and in general, the after-tax value (ATV) of equity using an accrual based tax system (τ_a) is:

$$ATV_H^{\tau_a} = X(1+(1-\tau_\alpha)r+(1-\tau_d)d)^H$$

Equating the after-tax value from both taxation schemes, and solving for τ_a yields:

$$\tau_a(\tau_m,\tau_g,r,d,H) = \frac{(1+r+(1-\tau_d)d)}{r} - \frac{((1-\tau_g)(1+r+(1-\tau_d)d)^H+\tau_g(1+(1-\tau_d)d\cdot SUM))^{\frac{1}{H}}}{r}$$

Where

$$SUM = \sum_{h=1}^{H}(1 + r + (1 - \tau_d)d)^{h-1}$$

Appendix: C

The lock-in cost function is:

$$L(\alpha, H) = X\{\frac{(1 - \alpha\tau_g)\{(1 - \tau_g)(1 + r + (1 - \tau_d)d)^H + \tau_g(1 - \tau_d)d \cdot SUM\} + \alpha\tau_g(1 - \tau_g)}{\{(1 - \tau_g)(1 + r + (1 - \tau_d)d)^H + \tau_g + g(1 - \tau_d)d \cdot SUM\}(1 - \tau_g)} - 1\}$$

Where

$$SUM = \sum_{h=1}^{H}(1 + r + (1 - \tau_d)d)^{(h-1)}$$

C.1 The derivative of the lock-in cost function with respect to α:

$$\frac{\partial L(\alpha, H)}{\partial \alpha} = \frac{\gamma \cdot \theta - \eta \cdot \lambda}{\theta^2}$$

where,

$$\lambda = (1 - \alpha\tau_g)\{(1 - \tau_g)(1 + r + (1 - \tau_d)d)^H + \tau_g(1 - \tau_d)d \cdot SUM\} + \alpha\tau_g(1 - \tau_g)$$

$$\gamma = -\tau_g\{(1 - \tau_g)(1 + r + (1 - \tau_d)d)^H + \tau_g(1 - \tau_d)d \cdot SUM\} + \tau_g(1 - \tau_g)$$

$$\theta = (1 - \tau_g)\{(1 - \tau_g)(1 + r + (1 - \tau_d)d)^H + \tau_g(1 - \tau_d)d \cdot SUM\} + \tau_g(1 - \tau_g)$$

$$\eta = 0$$

The denominator of this expression is positive; to show $\frac{\partial L(\alpha,H)}{\partial \alpha}$ is negative we must show the numerator is also negative.

The numerator is:

$$[-\tau_g(\psi) + \tau_g(1 - \tau_g)] \cdot [(1 - \tau_g)(\psi) + \tau_g(1 - \tau_g)]$$

Where $\psi = (1-\tau_g)(1+r+(1-\tau_d)d)^H + \tau_g(1-\tau_d)d \cdot SUM$.

Expanding this expression and collecting like terms gives us the following:

$$\tau_g(1-\tau_g)(1-2\tau_g)(\psi) - \tau_g(1-\tau_g)(\psi)^2 + \tau_g^2(1-\tau_g)^2$$

$$\Rightarrow \tau_g(1-\tau_g)(\psi)[(1-2\tau_g) - \psi] + \tau_g^2(1-\tau_g)^2$$

$$Note: (1-2\tau_g) - \psi = (1-2\tau_g) - (1-\tau_g)(1+r+(1-\tau_d)d)^H - \tau_g(1-\tau_d)d \cdot SUM <$$

$$< (1-2\tau_g) - (1-\tau_g)(1+r+(1-\tau_d)d)^H \leq (1-2\tau_g) - (1-\tau_g) = -\tau_g$$

Therefore:

$$\tau_g(1-\tau_g)(\psi)[(1-2\tau_g) - \psi] < -\tau_g^2(1-\tau_g)(\psi) =$$

$$= -\tau_g^2(1-\tau_g)[(1-\tau_g)(1+r+(1-\tau_d)d)^H + \tau_g(1-\tau_d)d \cdot SUM] < -\tau_g^2(1-\tau_g)^2$$

Which shows that:

$$\tau_g(1-\tau_g)(1-2\tau_g)(\psi) - \tau_g(1-\tau_g)(\psi)^2 + \tau_g^2(1-\tau_g)^2 < 0$$

and thus, $\frac{\partial L(\alpha,H)}{\partial \alpha}$ is negative. QED

C.2 The change in the lock-in cost function with respect to H:

$$\frac{L(\alpha,H) - L(\alpha,H-1)}{X} = \frac{(1-\alpha\tau_g)\vartheta(H) + \alpha\tau_g(1-\tau_g)}{(1-\tau_g)\vartheta(H) + \tau_g(1-\tau_g)} - \frac{(1-\alpha\tau_g)\vartheta(H-1) + \alpha\tau_g(1-\tau_g)}{(1-\tau_g)\vartheta(H-1) + \tau_g(1-\tau_g)}$$

Where

$$\vartheta(H) = (1-\tau_g)(1+r+(1-\tau_d)d)^H + \tau_g(1-\tau_d)d \cdot SUM(H)$$

and

$$\vartheta(H-1) = (1-\tau_g)(1+r+(1-\tau_d)d)^{(H-1)} + \tau_g(1-\tau_d)d \cdot SUM(H-1)$$

Note that $\vartheta(H) > \vartheta(H-1)$.

Finding a common denominator and canceling like terms gives us:

$$\frac{[\alpha\tau_g(1-\tau_g)^2\vartheta(H-1)+(1-\alpha\tau_g)\tau_g(1-\tau_g)\vartheta(H)]-[\alpha\tau_g(1-\tau_g)^2\vartheta(H)+(1-\alpha\tau_g)\tau_g(1-\tau_g)\vartheta(H-1)]}{[(1-\tau_g)\vartheta(H)+\tau_g(1-\tau_g)]\cdot[(1-\tau_g)\vartheta(H-1)+\tau_g(1-\tau_g)]}$$

The denominator of this expression is the product of two positive terms and is therefore positive. To show the lock-in cost function increases in H, we must show the numerator is also positive. The numerator can be rearranged as follows:

$$[\alpha\tau_g(1-\tau_g)^2-(1-\alpha\tau_g)\tau_g(1-\tau_g)]\cdot[\vartheta(H-1)-\vartheta(H)]$$

Since $(1-\alpha\tau_g)>(1-\tau_g)$, and $\tau_g>\alpha\tau_g$, both terms in the numerator are negative, and therefore the lock-in cost function is increasing in H. QED

C.3 The derivative of the lock-in cost function with respect to τ_d:

$$\frac{\partial L(\alpha,H)}{\partial\tau_d}=\frac{\gamma\cdot\theta-\eta\cdot\lambda}{\theta^2}$$

Where

$$\lambda=(1-\alpha\tau_g)\{(1-\tau_g)(1+r+(1-\tau_d)d)^H+\tau_g(1-\tau_d)d\cdot SUM\}+\alpha\tau_g(1-\tau_g)$$

$$\gamma=(1-\alpha\tau_g)\{-(1-\tau_g)\cdot H\cdot(1+r+(1-\tau_d)d)^{H-1}\cdot d-\tau_g\cdot d\cdot SUM+\tau_g\cdot d\cdot(1-\tau_d)\cdot SUM'\}$$

$$\theta=(1-\tau_g)\{(1-\tau_g)(1+r+(1-\tau_d)d)^H+\tau_g(1-\tau_d)d\cdot SUM\}+\tau_g(1-\tau_g)$$

$$\eta=(1-\tau_g)\{-(1-\tau_g)\cdot H\cdot(1+r+(1-\tau_d)d)^{H-1}\cdot d-\tau_g\cdot d\cdot SUM+\tau_g\cdot d\cdot(1-\tau_d)\cdot SUM'\}$$

$$SUM'=\sum_{h=1}^{H}-d\cdot(t-1)(1+r+(1-\tau_d)d)^{(h-2)}$$

When t = 1, then $SUM'=0$

When t > 1, then $SUM'<0$

The denominator of this expression is positive; to show $\frac{\partial L(\alpha,H)}{\partial \tau_d}$ is negative we must show the numerator is also negative.

The numerator is:

$$(1-\alpha\tau_g)(\delta)\cdot\{(1-\tau_g)(\psi)+\tau_g(1-\tau_g)\}-(1-\tau_g)(\delta)\cdot\{(1-\alpha\tau_g)(\psi)+\alpha\tau_g(1-\tau_g)\}$$

Where, $\psi=(1-\tau_g)(1+r+(1-\tau_d)d)^H+\tau_g(1-\tau_d)d\cdot SUM$

and $\delta=-d(1-\tau_g)H(1+r+(1-\tau_d)d)^{(H-1)}-\tau_g d\cdot SUM+\tau_g d(1-\tau_d)\cdot SUM'$.

After expanding this expression and canceling terms we are left with:

$$(1-\alpha)(1-\tau_g)\tau_g(\delta)$$

Each of these terms is positive other than δ, which is negative. To see this note that:

$$\delta=-d(1-\tau_g)H(1+r+(1-\tau_d)d)^{(H-1)}-\tau_g d\cdot SUM+\tau_g d(1-\tau_d)\cdot SUM'$$

Each term in the first and second part are positive, and all terms in the third part are positive other than SUM', which is negative when $H>1$ and zero when $H=1$. Therefore δ is the summation of either two or three negative terms and is therefore negative. QED

C.4 The derivative of the lock-in cost function with respect to τ_g:

$$\frac{\partial L(\alpha,H)}{\partial \tau_g}=\frac{\gamma\cdot\theta-\eta\cdot\lambda}{\theta^2}$$

Where

$$\lambda=(1-\alpha\tau_g)\{(1-\tau_g)(1+r+(1-\tau_d)d)^H+\tau_g(1-\tau_d)d\cdot SUM\}+\alpha\tau_g(1-\tau_g)$$

$$\gamma=-\alpha[(1-\tau_g)(1+r+(1-\tau_d)d)^H+\tau_g(1-\tau_d)dSUM]+(1-\alpha\tau_g)[-(1+r+(1-\tau_d)d)^H+(1-\tau_d)d\cdot SUM]+\alpha-2\alpha\tau_g$$

$$\theta=(1-\tau_g)\{(1-\tau_g)(1+r+(1-\tau_d)d)^H+\tau_g(1-\tau_d)d\cdot SUM\}+\tau_g(1-\tau_g)$$

$$\eta = -[(1-\tau_g)(1+r+(1-\tau_d)d)^H + \tau_g(1-\tau_d)d \cdot SUM] + (1-\tau_g)[-(1+r+(1-\tau_d)d)^H + (1-\tau_d)d \cdot SUM] + (1-\tau_g) - \tau_g$$

The denominator of this expression is positive; to show $\frac{\partial L(\alpha,H)}{\partial \tau_g}$ is positive we need to show the numerator is also positive.

The numerator is:

$$\{-\alpha(\psi) + (1-\alpha\tau_g)(\delta) + \alpha(1-\tau_g) - \alpha\tau_g\} \cdot \{(1-\tau_g)(\psi) + \tau_g(1-\tau_g)\} -$$

$$-\{-(\psi) + (1-\tau_g)(\delta) + (1-\tau_g) - \tau_g\} \cdot \{(1-\alpha\tau_g)(\psi) + \alpha\tau_g(1-\tau_g)\}$$

where $\psi = (1-\tau_g)(1+r+(1-\tau_d)d)^H + \tau_g(1-\tau_d)d \cdot SUM$, and $\delta = -(1+r+(1-\tau_d)d)^H + (1-\tau_d)d \cdot SUM$

After expanding this expression and canceling terms we are left with:

$$(1-\alpha)(\psi)^2 - (1-\alpha)(1-2\tau_g)(\psi) + (1-\alpha)\tau_g(1-\tau_g)(\delta)$$

$$\Rightarrow (1-\alpha)[(\psi)^2 - (1-2\tau_g)(\psi) + \tau_g(1-\tau_g)(\delta)]$$

$$\Rightarrow (1-\alpha)(\psi)\{(1-\tau_g)[(1+r+(1-\tau_d))^H - 1] + \tau_g(1-\tau_d)d \cdot SUM\} + (1-\alpha)\tau_g(1-\tau_d) \cdot SUM$$

It can be verified that every term in this expression is positive, so the numerator, and $\frac{\partial L(\alpha,H)}{\partial \tau_g}$ are also positive. QED

Appendix: D

D.1 Derivation of Each Repurchase Function Using the Equity Supply Curve

Recall the equity supply curve:

$$S(q) = X + \widehat{L(q)}$$

$$where \quad q = \sum_{H=0}^{H_M} \int_{\{(\alpha,H)|L(\alpha,H) \leq \widehat{L(q)}\}} f(\alpha, H) d\alpha$$

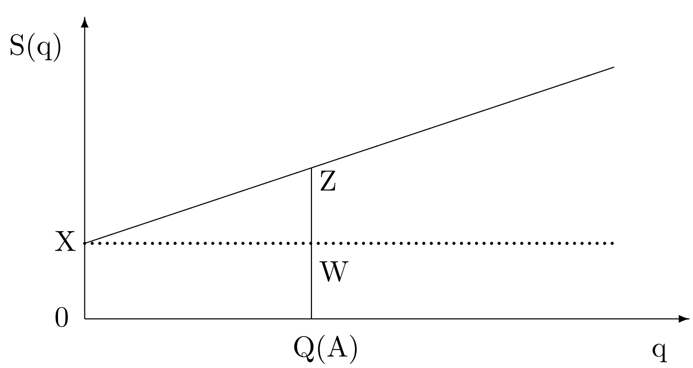
D.1.1 The Weak Repurchase Function

The weak repurchase function is characterized as follows:

$$R(A) = X \cdot Q(A)$$

$$where \quad A = \int_0^{Q(A)} S(q)dq$$

That is, the repurchase function is equal to the intrinsic value of the shares repurchased ($X \cdot Q(A)$), where the number of shares repurchased ($Q(A)$) is such that the integral below the supply curve from zero to the number of shares repurchased equals the amount spent. Graphically, this repurchase function can be represented as follows (note: S(q) is not necessarily linear):



Where A is equal to the area (0, X, Z, Q(A)), and R(A) is equal to $X \cdot Q(A)$. The aggregate lock-in premium in this case is area (X, Z, W).

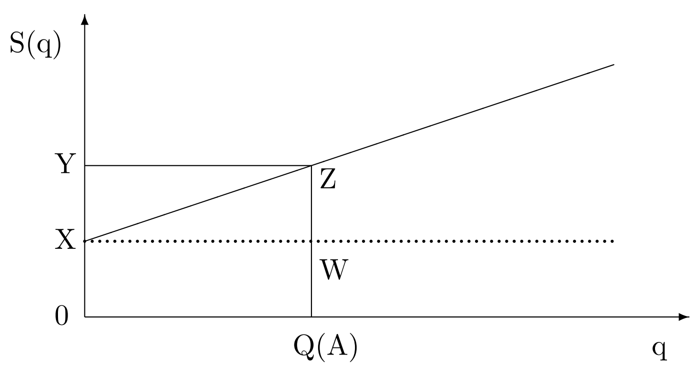
D.1.2 The Strong Repurchase Function

The strong repurchase function is characterized as follows:

$$R(A) = X \cdot Q(A)$$

$$where \quad A = S(Q(A)) \cdot Q(A)$$

That is, the repurchase function is equal to the intrinsic value of the shares repurchased ($X \cdot Q(A)$), such that the amount spent repurchasing shares (A) is equal to the product of the quantity of shares repurchased ($Q(A)$) and the largest lock-in cost among the repurchased shares ($S(Q(A))$). Graphically this repurchase function can be represented as follows:



Where A is equal to the area (0, Y, Z, Q(A)), and R(A) is equal to $X \cdot Q(A)$. The aggregate lock-in premium in this case is the area (X, Y, Z, W).

D.2 Concavity of the Weak Repurchase Function

The weak repurchase function is:

$$R(A) = max_{\{\alpha(H)\}} \left[\sum_{H=0}^{H_M} \int_{\alpha(H)}^{1} f(\alpha, H)\, d\alpha \right] X$$

$$S.T. \quad \sum_{H=0}^{H_M} \int_{\alpha(H)}^{1} [X + L(\alpha, H)] \cdot f(\alpha, H)\, d\alpha = A.$$

We can take the inverse of this function (amount spent given an amount repurchased):

$$A(R) = min_{\{\alpha(H)\}} \sum_{H=0}^{H_M} \int_{\alpha(H)}^{1} [L(\alpha, H) + X] \cdot f(\alpha, H)\, d\alpha$$

$$S.T. \quad R = \left[\sum_{H=0}^{H_M} \int_{\alpha(H)}^{1} f(\alpha, H)\, d\alpha \right] X$$

$$\Rightarrow R = \sum_{H=0}^{H_M} [F(1, H) - F(\alpha(H), H)]$$

$$\Rightarrow \frac{\partial R}{\partial \alpha(H)} = -f(\alpha(H), H)$$

As R increases, at least one of the $\alpha(H) > 0$ must decrease. Without loss of generality, suppose an increase in R is made through $\alpha(H^*) > 0$.

$$\Rightarrow \frac{\partial \alpha(H^*)}{\partial R} = \frac{-1}{f(\alpha(H), H)}$$

The change in A(R), given a change in $\alpha(H^*)$ is:

$$\frac{\partial A(R)}{\alpha(H^*)} - L(\alpha(H^*), H) \cdot f(\alpha(H^*), H)$$

$$\Rightarrow \frac{\partial A(R)}{\partial R} = L(\alpha(H^*), H)$$

This implies that A(R) is an increasing function of R (when the increase is made through $\alpha(H^*)$). The derivative of this function with respect to $\alpha(H^*)$ is:

$$\frac{\partial \partial A(R)}{\partial R \partial \alpha(H^*)} = L'(\alpha(H^*), H) < 0$$

(See the appendix for a proof that $L'(\alpha(H^*), H) < 0$)

$$\Rightarrow \frac{\partial \partial A(R)}{\partial R \partial R} = -\frac{L'(\alpha(H^*), H)}{f(\alpha(H^*), H)} > 0$$

Therefore, the function A(R) is convex in R using any $\alpha(H) > 0$ to expand the set of repurchased shares. Therefore the inverse of this function ((R(A)) is concave. QED

D.3 The Strong Repurchase Function is Everywhere Below the Weak Repurchase Function

The strong repurchase function is:

$$R(A) = max_{\{\alpha(H)\}} \left[\sum_{H=0}^{H_M} \int_{\alpha(H)}^{1} f(\alpha, H)\, d\alpha \right] X$$

$$S.T. \quad [max_{\{H\}}\, L(\alpha(H), H) + X] \cdot \sum_{H=0}^{H_M} \int_{\alpha(H)}^{1} f(\alpha, H)\, d\alpha = A.$$

We can take the inverse of this function (amount spent given an amount repurchased):

$$A(R) = min_{\{\alpha(H)\}} \{[max_{\{H\}}\, L(\alpha(H), H)] \cdot \sum_{H=0}^{H_M} \int_{\alpha(H)}^{1} f(\alpha, H)\, d\alpha\}$$

$$S.T. \quad R = \sum_{H=0}^{H_M} \int_{\alpha(H)}^{1} f(\alpha, H)\, d\alpha$$

Recall from the main text that firms repurchase from the pool of shareholders with the lowest lock-in effects regardless of shareholder tendering behavior. Therefore, for any amount repurchased (R), the $\alpha(H)s$ are the same for both the weak amount spent function and the strong amount spent function. Subtracting the weak amount spent function from the strong amount spent function, for a given R, results in:

$$A(R)^{strong} - A(R)^{weak} = \sum_{H=0}^{H_M} \int_{\alpha(H)}^{1} \{[max_{\{H\}}\, L(\alpha(H), H)] - L(\alpha, H)\} \cdot f(\alpha, H)\, d\alpha$$

Where the $\alpha(H)s$ are the solutions to both minimization problems.

Since $[max_{\{H\}}\, L(\alpha(H), H)] - L(\alpha, H) > 0$ for all $\alpha > \alpha(H)$ and all H, the above expression is an integral over two positive functions, which is itself positive. This shows that for any amount repurchased the amount spent must be higher, or conversely, for any amount spent the amount repurchased must be lower. This proves the strong repurchase function is everywhere below the weak repurchase function. QED

D.4 Repurchase Function Derivatives with respect to Changes in τ_d and τ_a

The weak repurchase function is:

$$R(A) = max_{\{\alpha(H)\}} \left[\sum_{H=0}^{H_M} \int_{\alpha(H)}^{1} f(\alpha, H)\, d\alpha \right] X$$

$$S.T. \quad \sum_{H=0}^{H_M} \int_{\alpha(H)}^{1} [X + L(\alpha, H)] \cdot f(\alpha, H)\, d\alpha = A.$$

The strong repurchase function is:

$$R(A) = max_{\{\alpha(H)\}} \left[\sum_{H=0}^{H_M} \int_{\alpha(H)}^{1} f(\alpha, H)\, d\alpha \right] X$$

$$S.T. \quad [max_{\{H\}}\, L(\alpha(H), H) + X] \cdot \sum_{H=0}^{H_M} \int_{\alpha(H)}^{1} f(\alpha, H)\, d\alpha = A.$$

NOTE: for a given A, the $\alpha(H)s$ are weakly lower for the weak repurchase function vs. the strong repurchase function.

Derivative of the weak repurchase function: when A increases at least one of the $\alpha(H)s$ must decrease. Suppose that $\alpha(H^*)$ decreases. Then:

$$\frac{\partial A}{\partial \alpha(H^*)} = -L(\alpha(H^*), H^*) \cdot f(\alpha(H^*), H^*)$$

$$\frac{\partial R(A)^{weak}}{\partial \alpha(H^*)} = -f(\alpha(H^*), H^*)$$

$$\Rightarrow \frac{\partial R(A)^{weak}}{\partial A} = \frac{1}{L(\alpha(H^*), H^*)}$$

Derivative of the strong repurchase function: when A increases at least one of the $\alpha(H)s$ must decrease. Suppose that $\alpha(H^*)$ decreases. Then:

$$\frac{\partial A}{\partial \alpha(H^*)} = \frac{\partial L(\alpha(H^*), H^*)}{\partial \alpha(H^*)} \cdot \sum_{H=0}^{H_M} \int_{\alpha(H)}^{1} f(\alpha, H) d\alpha - [L(\alpha(H^*), H^*) \cdot f(\alpha(H^*), H^*)]$$

$$\frac{\partial R(A)^{strong}}{\partial \alpha(H^*)} = -f(\alpha(H^*), H^*)$$

$$\Rightarrow \frac{\partial R(A)^{strong}}{\partial A} = \frac{-f(\alpha(H^*), H^*)}{\frac{\partial L(\alpha(H^*), H^*)}{\partial \alpha(H^*)} \cdot \sum_{H=0}^{H_M} \int_{\alpha(H)}^{1} f(\alpha, H) d\alpha - [L(\alpha(H^*), H^*) \cdot f(\alpha(H^*), H^*)]}$$

Now we look at the change in $\frac{\partial R(A)}{\partial A}$ for both the weak and strong repurchase functions when τ_d & τ_g change:

Change in τ_d:

Weak repurchase function:

$$\frac{\partial \partial R(A)^{weak}}{\partial A \partial \tau_d} = \frac{-\frac{\partial L(\alpha(H^*), H^*)}{\tau_d}}{L(\alpha(H^*), H^*)^2}$$

It has been shown that $\frac{\partial L(\alpha(H^*),H^*)}{\tau_d}$ is negative, therefore the numerator is positive. Since $L(\alpha(H^*), H^*)^2$ is positive $\frac{\partial\partial R(A)}{\partial A \partial \tau_d}$ is also positive. QED

Strong repurchase function:

$$\frac{\partial\partial R(A)^{strong}}{\partial A \partial \tau_d} = \frac{f(\alpha(H^*), H^*) \cdot \{\frac{\partial\partial L(\alpha(H^*),H^*)}{\partial\alpha(H^*)\partial\tau_d} \cdot \sum_{H=0}^{H_M} \int_{\alpha(H)}^{1} f(\alpha, H) d\alpha - \frac{\partial L(\alpha(H^*),H^*)}{\partial\tau_d} \cdot f(\alpha(H^*), H^*)\}}{DEN^2}$$

Where $DEN = \frac{\partial L(\alpha(H^*),H^*)}{\partial\alpha(H^*)} \cdot \sum_{H=0}^{H_M} \int_{\alpha(H)}^{1} f(\alpha, H) d\alpha - L(\alpha(H^*), H^*) \cdot f(\alpha(H^*), H^*)$.

With regard to the numerator of this expression, $f(\alpha(H^*), H^*)$ is positive, $\frac{\partial\partial L(\alpha(H^*),H^*)}{\partial\alpha(H^*)\partial\tau_d}$ is positive (as shown below), $\sum_{H=0}^{H_M} \int_{\alpha(H)}^{1} f(\alpha, H) d\alpha$ is positive, $\frac{\partial L(\alpha(H^*),H^*)}{\partial\tau_d}$ is negative, and $f(\alpha(H^*), H^*)$ is positive. Therefore the numerator is positive. The denominator (DEN^2) is positive, and therefore $\frac{\partial\partial R(A)^{strong}}{\partial A \partial \tau_d}$ is also positive.

Proof: that $\frac{\partial\partial L(\alpha(H^*),H^*)}{\partial\alpha(H^*)\partial\tau_d}$ is positive.

$$\frac{\partial\partial L(\alpha(H^*), H^*)}{\partial\alpha(H^*)} = \frac{\tau_g(1-\tau_g)(1-2\tau_g)\psi - \tau_g(1-\tau_g)\psi^2 + \tau_g^2(1-\tau_g)^2}{((1-\tau_g)\{(1-\tau_g)(1+r+(1-\tau_d)d)^H + \tau_g(1-\tau_d)d \cdot SUM\} + \tau_g(1-\tau_g))^2}$$

Where:

$\psi = (1-\tau_g)(1+r+(1-\tau_d)d)^H + \tau_g(1-\tau_d)d \cdot SUM$

$SUM = \sum_{h=0}^{H}(1+r+(1-\tau_d)d)^{(h-1)}$

Thus: $\frac{\partial\partial L(\alpha(H^*),H^*)}{\partial\alpha(H^*)\partial\tau_d} = \frac{NUM' \cdot DEN - 2 \cdot DEN' \cdot NUM}{DEN^3}$

Where:

$NUM = \tau_g(1-\tau_g)(1-2\tau_g)\psi - \tau_g(1-\tau_g)\psi^2 + \tau_g^2(1-\tau_g)^2$

$DEN = (1-\tau_g)\{(1-\tau_g)(1+r+(1-\tau_d)d)^H + \tau_g(1-\tau_d)d \cdot SUM\} + \tau_g(1-\tau_g)$

$NUM' = \tau_g(1-\tau_g)(1-2\tau_g)\psi(\tau_d)' - 2\tau_g(1-\tau_g)\psi\psi(\tau_d)'$

$DEN' = (1-\tau_g)\{-d(1-\tau_g)H(1+r+(1-\tau_d)d)^{(H-1)} - d\tau_g \cdot SUM + \tau_g(1-\tau_d)d \cdot SUM(\tau_d)'\}$

$\psi(\tau_d)' = -d(1-\tau_g)H(1+r+(1-\tau_d)d)^{(H-1)} - \tau_g dSUM + \tau_g(1-\tau_d)d \cdot SUM(\tau_d)'$

$SUM(\tau_d)' = \sum_{h=0}^{H} -d(t-1)(1+r+(1-\tau_d)d)^{(t-2)}$

Expanding the numerator of $\frac{\partial\partial L(\alpha(H^*),H^*)}{\partial\alpha(H^*)\partial\tau_d}$ and canceling like terms results in:

$$\tau_g^2(1-\tau_g)^2(1-2\tau_g)\psi(\tau_d)' - 2\tau_g^2(1-\tau_g)^2\psi\cdot\psi(\tau_d)' - \tau_g(1-\tau_g)^2(1-2\tau_g)\psi\cdot\psi(\tau_d)' - 2\tau_g^2(1-\tau_g)^3\psi(\tau_d)'$$

Reducing this expression further, leads to:

$$-\tau_g(1-\tau_g)^2\psi(\tau_d)'(\tau_g+\psi)$$

This expression is positive since $\tau_g(1-\tau_g)$ is positive, $\psi(\tau_d)'$ is negative, and $(\tau_g+\psi)$ is positive. Since DEN is always positive, $\frac{\partial\partial L(\alpha(H^*),H^*)}{\partial\alpha(H^*)\partial\tau_d}$ is the quotient of two positive numbers and is itself positive, completing the proof. QED

Change in τ_g :

Weak repurchase function:

$$\frac{\partial\partial R(A)^{weak}}{\partial A\partial\tau_g} = \frac{-\frac{\partial L(\alpha(H^*),H^*)}{\tau_g}}{L(\alpha(H^*),H^*)^2}$$

It has been shown that $\frac{\partial L(\alpha(H^*),H^*)}{\tau_g}$ is positive, therefore the numerator is negative. Since $L(\alpha(H^*),H^*)^2$ is positive $\frac{\partial\partial R(A)}{\partial A\partial\tau_g}$ is negative.