Essays on Multi-Object Pricing and Auctions

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Optimally Selling Heterogeneous Objects by Quantity

Job Market Paper

I study optimal selling of K heterogeneous objects to a buyer with private values through a quantity menu: the seller posts a price for each number of objects, and the buyer chooses a quantity, pays its price, and selects any objects up to that quantity. I propose a pricing method that aggregates critical prices for second differences of quantity values—the highest values attainable at each quantity. Critical prices can be determined separately when each second difference's conditional inverse-hazard ratio depends only on that difference. For additive values, under ex ante symmetry, I prove the resulting menu optimal among all incentive-compatible and individually rational mechanisms under explicit distributional and support conditions. A separate result applies the same pricing method to obtain an optimal menu for nonadditive bundle values without symmetry across object labels, assuming free disposal, independence between the quantity-value vector and the vector of bundle shortfalls from their respective quantity values, and corresponding conditions. Applications yield explicit optimal menus that involve quantity discounts, sparse quantity options, or two-part tariffs. These results differ from those establishing the optimality of pure bundling in Haghpanah and Hartline (2021), nested bundling in Yang (2025), separate sales in Kwak (2026), and core-peripheral bundling in Zhong (2026).

Optimal Multi-Object Auctions for Geographic Concessions under a Cardinality Constraint

This study considers multi-object auctions in which bidders have additive private values and geographic advantages, motivated by timber sales and mineral exploration concessions. Governments may limit the number of awarded concessions to protect the environment or control the scale of new policies and technologies. I study auctions implemented through bidder-specific quantity menus. Each menu sets prices for different numbers of concessions based only on rivals' reports. Bidders choose a quantity and then select their preferred available concessions in a randomly determined order. The total cap creates competition because an award to one bidder can displace an award to another. I derive sufficient conditions for the menus to maximize expected revenue among all feasible, dominant-strategy incentive-compatible and ex-post individually rational mechanisms. These conditions ensure that the menu choices respect the cap, attain the joint revenue upper bound, and leave no bidder surplus at the relevant lower boundaries of the value distribution. I provide a family of continuous, nondegenerate distributions satisfying these conditions.

Optimal Randomized Bundling

This study considers optimal bundle pricing for a buyer with private, nonadditive values. The buyer chooses how many draws to purchase before outcomes are revealed, with each object allocated at most once. Under uniform drawing, bundles of the same size are equally likely. The seller sets a marginal charge for each draw and sums these charges to obtain the total price. Uniform drawing cannot distinguish buyers with the same average bundle values at each size but different bundle preferences. I perturb the drawing probabilities and adjust prices so that preferences over particular bundles affect quantity choices. I derive sufficient conditions, including restrictions on the support of valuations and a boundary balance requirement involving conditional mean differences in bundle values. Under these conditions, the perturbed mechanism maximizes expected revenue among all feasible incentive-compatible and individually rational mechanisms.

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