Informational Control and Organizational Design*

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Abstract

This paper focuses on organizational issues of allocating authority between an uninformed principal and an informed expert. We show that the standard result that delegating decisions to a perfectly informed expert is better than communication is reversed if the principal can restrict the precision of the expert's information (without learning its content). We demonstrate that these organizational forms—informational control and delegation—can be either complements or substitutes, depending on the principal's ability to affect the expert's discretion about the set of allowed policies.

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1 Introduction

Situations in which principals do not have enough information and have to consult experts before implementing a policy can be found almost everywhere. Auctioneers consult experts about an object's value, managers consult analysts before making corporate decisions, and politicians consult advisors on special subjects. However, the benefits of communication are often impaired by a conflict of interest. If the parties' interests do not match exactly, the expert may want to strategically misrepresent information in an attempt to manipulate a principal's decision.

A potentially effective solution to this communication problem is to delegate authority to the expert herself and gain from her informational advantage. Then, even though the expert's decision is biased, the trade-off between the loss of authority in delegation and the loss of information in communication often favors the former (see Dessein, 2002). However, despite the informational benefits of delegation, many companies today still centralize

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authority at the upper level of the hierarchy.[1] To provide a possible explanation of this fact, this paper analyzes the benefits of another instrument, sometimes available to the principal—controlling the quality of the expert's information without learning its content. As shown by Fischer and Stocken (2001) for some special cases, restricting the amount of information available to the expert be better for the principal. It is natural to ask whether restricting the availability of information does better than full or partial delegation and how the two modes of modifying the communication game interact with each other. This paper addresses these questions. We demonstrate that informational control in communication is generally beneficial for the principal compared to both communication with the perfectly informed expert and delegation. Moreover, restricting the expert's information is beneficial even when combining these instruments; that is, when the principal restricts the expert's information and delegates decision making afterwards. This finding is counterintuitive, because the expert's superior information is a major factor that benefits the principal in delegation.

In many situations, the principal can directly affect the precision of the expert's information. For instance, governments (principals) usually collect reports from oil companies (experts) to estimate the amount of oil in the oilfields before a lease sale. Depending on the company's report, a government determines the auction rules: its type, a reserve price, etc. This behavior creates an incentive for companies to misrepresent information in their favor. For its part, the government can restrict the company's information by specifying estimating procedures: oil exploration techniques allowed, the number and location of test drills permitted, etc.[2]

In general, loss of information in communication implies that the expert possesses too much information relative to the amount that she is ready to reveal to the principal. Thus, some information is not relevant to the principal's decisions because it is never discovered. We show that properly restricting the expert's information fosters her incentives to reveal it truthfully. As a result, the principal faces another trade-off between the precision of the expert's primary information and her incentives to convey it in further communication. In other words, the principal prefers to restrict the expert's information to only decision-relevant information, which can be fully revealed, and to make this information as precise

[1] According to the Boston Consulting Group, centralization is still the most common type of organization. Moreover, companies with decentralized decision making and accountability have sometimes opted to centralize their structure. For example, Nestlé, a Swiss food and drinks group, initially was decentralized that "was seen as the best way to cater to local taste and to establish emotional links with clients in far-flung places". Nevertheless, it recently centralized control over specific businesses and consolidated the management of its factories in individual countries into regions, even though the company's performance strongly depended on local preferences of consumers, which are known better by local management (from *The Economist*, Aug. 5th 2004).

[2] The Mineral Management Service (MMS) of the U.S. Department of the Interior does not perform any direct data-collection activities. Instead, it issues permits to industry for collecting prelease geological and geophysical data. In general, companies wishing to collect data on the Outer Continental Shelf prior to a lease sale must obtain a permit from the MMS. The permits set forth the specific details for each data-gathering activity, including the area where the data are collected, the timing of the data-gathering activity, approved equipment and methods, and other similar detailed information relevant to each specific permit. After a permit is granted, the MMS monitors all field data collection activities to ensure compliance with the terms of the permit. It is empowered to select and obtain copies of the data that are collected by private firms. The MMS uses the obtained data for several purposes, including evaluation of tracts' market values, determination of bidding procedures, leasing, and post-lease operations.

as possible.[3]

We extend Crawford–Sobel's (1982) model of communication with a perfectly informed expert, by giving the principal control over the quality of the expert's information. The key point of this paper is a race between generalized versions of two instruments: controlling the expert's information (Fischer and Stocken's 2001 approach) and delegation (Dessein's 2002 approach). First, we extend the result of Fischer and Stocken (2001), who first recognized that the quality of information of the principal is not monotone in that of the expert. They, however, characterize the optimal quality of the expert's information in a leading uniform-quadratic case of the model only for specific values of the bias in the players' interests. In our extension, we characterize the optimal information structure for an arbitrary bias, and show that by properly restricting the quality of the expert's information, the principal can elicit more information from the expert than in the standard Crawford–Sobel (hereafter, CS) environment. Based on this result, our major contribution is that informational control results in a higher principal's payoff than optimal delegation whenever an equilibrium with the informative communication exists. This result directly addresses the question examined by Dessein (2002), who compares the performance of delegation versus CS communication. Dessein (2002) establishes that "the principal optimally delegates control as long as the divergence in preferences is not too large relative to the principal's uncertainty about the environment." We show that this result can be reversed as soon as the principal has the power to influence the precision of the expert's information. For a wider class of players' preferences and distribution functions, informational control is better than CS communication whenever informative communication is feasible, and better than delegation whenever the players' interests are sufficiently close relative to the principal's prior uncertainty about the state. Thus, controlling the precision of the expert's information generally is a more powerful tool than delegation—it allows the principal to gain from the expert's information without loss of control over decisions.

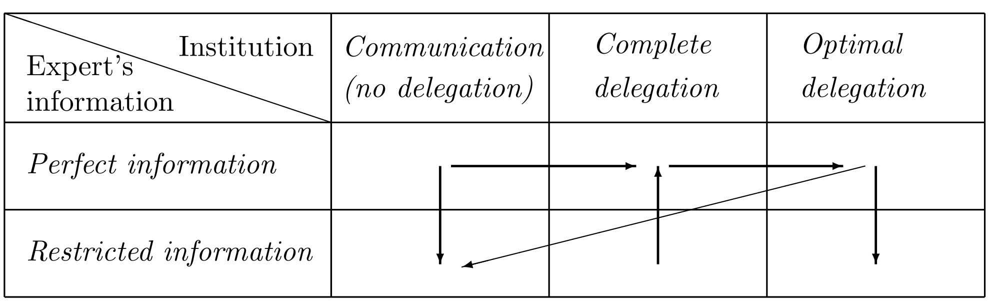

Figure 1: Payoff relations between institutions for small conflict in preferences

Figure 1 compares the performance of three organizational forms for different qualities of an expert's information (arrows represent the payoff dominance). The first form is pure communication, when the principal requests an expert's advice, but can use the obtained

[3]For example, top managers can restrict access to corporate information for those who are lower in the company's hierarchy. According to Charles Knight, the head of the electric and electronics business Emerson, communication in the company is kept to a minimum: "Our planning and control cycle provides ample opportunity to communicate the most important business issues... we don't burden our system with non-essential communications and information" (from *The Economist*, Jan. 21st 2006).

information to make an arbitrary decision. In the second case, the principal completely delegates authority to the expert or commits to comply with any expert's recommendation. Finally, the principal can delegate decision rights only partially by restricting policies that can be chosen by the expert to prevent her from implementing, for example, extreme actions. Optimal delegation imposes the policy restrictions that maximize expected payoff of the principal.[4]

As this figure shows, there is only one situation in which the principal prefers not to limit the precision of the expert's information. This is the case, when the principal cannot affect the expert's discretion about the set of delegated decisions, that is, the only possible delegation is complete. However, even in this scenario, the principal can gain from influencing the expert's information and purely communicating with her afterwards.

The paper proceeds as follows. Section 2 discusses the related literature. Section 3 highlights motivating examples, which illustrate that the optimal information structure can be coarse as well as that controlling information can perform better than delegation. Section 4 presents the formal model. The general analysis of the model is provided in Section 5. Section 6 compares the performance of our model versus delegation, and tests the robustness of the results to the model specifications. Section 7 consider a combination of controlling information and delegation. Section 8 concludes the paper.

2 Literature Review

Since our work contributes to the literature by comparing benefits of different organizational forms, it relates to two directions of the existing studies: that which deals with various aspects of endogenous information in communication, and that which focuses on delegation, when the principal delegates authority to the expert. With respect to the former topic, the first analysis of strategic communication is attributed to Crawford and Sobel (1982) in their seminal paper. They introduce a model of the interaction between a perfectly informed expert and an uninformed principal whose payoffs depend on a random state of nature. After a private observation of the state, the expert sends a costless message to the principal, who implements an action afterward. Crawford and Sobel show that full information revelation is never possible unless the players' interests are perfectly aligned. In addition, when a conflict of interest arises, the quality of the disclosed information falls, eventually resulting in the *babbling* equilibrium with no useful information conveyed.

The fact that the imperfect quality of an expert's information can be beneficial to the principal was first demonstrated by Fischer and Stocken (2001). They, however, restrict the set of possible biases in the players' preferences b, introduced by Crawford and Sobel (1982), to that of the discrete form $b = 1/2n$, where n is an integer, and analyze pure-strategy equilibria only. Their main result for the uniform-quadratic setting is that the optimal structure of the information partition is uniform of size n, that is, equally spaced. This is not a general feature of the model for other values of b. In this paper, we characterize the optimal information structure for all values of the bias. In general, non-uniform partitions can result in a higher expected payoff to the principal than delegation that cannot be achieved with uniform partitions.

[4] For a detailed discussion of the optimal restrictions on the delegation set, see Alonso and Matouschek (2005).

Austen-Smith (1994) consider strategic communication with costly information acquisition. Namely, the expert can observe the state at some privately known cost. In addition, the expert is able to prove the fact of her information acquisition, but not the fact that she is uninformed. Intuitively, positive costs of information acquisition decrease expert's incentives to acquire it and, as a result, the average quality of her information. However, introducing partial verifiability of the quality of the expert's information extends the range of biases, for which informative communication is possible.[5] In contrast, there are no such verifiability issues in our case, since the principal determines the expert's information structure directly.

Bester and Strausz (2001) and Krishna and Morgan (2005) analyze a different instrument to improve the quality of the conveyed information in communication—monetary transfers from the principal to the expert as the functions of messages. Bester and Strausz (2001) extend the revelation principle to the finite type environment in which the principal can commit only to some dimensions of the whole set of decisions. They show that any incentive efficient outcome (i.e., that which provides equilibrium payoffs on the Pareto frontier) is payoff-equivalent to the equilibrium outcome in some direct mechanism. Krishna and Morgan (2005) extend this result to the infinite type space and characterize the optimal contracts under both perfect and imperfect commitment. They demonstrate that the gains from contracting are the highest for moderate values of the bias in preferences. Similar to these studies, we establish a model-specific revelation principle, which narrows the set of the optimal information structures to only those structures, in which the expert reveals all available information.

The issue of the endogenous quality of information for the mechanism design is also studied by Bergemann and Pesendorfer (2001). They consider an auction in which the seller determines the precision of the bidders' valuations and to whom to sell at what price. Similar to our model, the seller specifies the information structure for each bidder without learning their private signals. In this case, the information structures in the optimal auction are coarse and represented by the finite number of monotone partitions.[6]

Alternatively, there is an established literature on delegation or communication with commitment, when the principal commits to rubber-stamp any agent's recommendations if they belong to the specified delegation set. Dessein (2002) studies the benefits of the special forms of delegation—complete delegation, communication with a biased intermediary, and delegation with a veto-power—and compares them with the benefits of pure communication. Holmström (1977), Melumad and Shibano (1991) , and Alonso and Matouschek (2005) investigate the optimal restrictions on the set of delegated policies, which maximize the principal's expected payoff. Whereas these studies consider the information structure of the expert as exogenous, this work connects the endogenous quality of information with delegating control over decisions to the expert.

[5]For the uniform-quadratic case, to $b < 1/2$.

[6]An interesting property of the optimal structure is that the partitions are asymmetric across bidders even for symmetric distributions of the object's values.

3 Examples

We start with the uniform-quadratic variant of the communication model introduced by Crawford and Sobel (1982). Two players, the uninformed receiver (R) and the better informed sender (S), communicate on some state of nature, which is represented by a random variable θ, uniformly distributed on the unit interval. We can treat the sender as an expert (she) and the receiver as a principal (he). The expert sends a costless message m to the principal, who then implements an action a, which affects the payoffs of both players. The players' state-relevant utility functions are quadratic:

$$U_R(a,\theta) = -(a-\theta)^2, \text{ and } U_S(a,b,\theta) = -(a-b-\theta)^2, \tag{1}$$

where a parameter $b > 0$ reflects the bias in the players' interests.

Suppose first that the expert is *perfectly* informed about the state. Crawford and Sobel demonstrate that all the equilibria are characterized by finite monotone partitions. That is, for any bias b there are at most $N^{CS}(b)$ intervals on the state space so that the expert sends one message for each interval $W_k = [w_k, w_{k+1}]$, which is associated with a corresponding action $a_k = E[\theta|\theta \in W_k]$.[7] Also, there are exactly $N^{CS}(b)$ equilibria with $1, 2, ..., N^{CS}(b)$ intervals, where the equilibrium with $N^{CS}(b)$ intervals is Pareto superior to all other equilibria.

Example 1. Let the bias $b = \frac{1}{5}$. In the most informative equilibrium, the expert sends a "low" message if the state less than $\frac{1}{10}$, and a "high" message otherwise. Thus, if the principal receives a higher message (which occurs with the probability $\frac{9}{10}$), his prior information is updated insignificantly. A lower message is more informative, but the probability of receiving it is just $\frac{1}{10}$. The reason is that the principal knows the expert's motives to exaggerate information and tries to correct his actions correspondingly. As a result, if the principal gets a lower message, he infers that the expert's type has to be very low, whereas a higher message is more expected and thus is not very informative. That is, communication can be effective only for low states that results in the principal's expected payoff $U_R^{CS} \simeq -\frac{1}{16}$, which only slightly exceeds his payoff $-\frac{1}{12}$ in the case of no communication.

However, if the principal controls the expert's information in a such way that the expert observes only whether θ is higher or lower than $\frac{1}{2}$, then there is an equilibrium, in which the expert truthfully reveals her information. This increases the principal's expected utility to $-\frac{1}{48}$. Moreover, there is an equilibrium with three messages, namely, for θ less than $\frac{1}{5}$, between $\frac{1}{5}$ and $\frac{4}{5}$, and higher than $\frac{4}{5}$, which provides the expected utility $U_R \simeq -\frac{1}{52}$. A finer information structure violates the sender's incentives to communicate truthfully, which results in the distortion of information and lower principal's payoffs.[8]

The intuition for this result is that the preferences of a less informed expert become closer to those of the principal. In the CS case, the partitional structure is determined by

[7] Formally, Crawford and Sobel (1982) define equilibrium strategies in a slightly different way to avoid probability zero events. They require $m(\theta)$ to be uniformly distributed on $[w_k, w_{k+1}]$, if $\theta \in (w_k, w_{k+1})$, and $a(m) = E[\theta|\theta \in W_k]$ for all $m \in (w_k, w_{k+1})$.

[8] Like Crawford and Sobel (1982), we use the term "finer" informally, implying a partition with a larger number of elements.

marginal types who are indifferent between two consequent actions (in the above example, it is type $\theta_1 = \frac{1}{10}$) that is relatively low. Technically, because of the expert's positive bias the next higher action has to be far from the marginal type. Given the principal's optimal behavior in response to received information (the conditional means of the state in the intervals), this is possible only if the next interval is sufficiently large. However, if the expert cannot distinguish among different states in a lower interval, this decreases her incentives to induce a higher action, since for all states in the interval (except the indifferent type on the upper bound of the interval) the lower action is strictly better. Thus, this particular form of information imperfection replaces the marginal CS type by the mean type in the lower interval. As a result, finer partitions can be supported as equilibria than in the CS case.

Moreover, the beneficial effect of controlling the expert's information is so powerful that this organizational form can bring higher payoffs to the principal than delegation, as demonstrated in the example below.

Example 2. Let the bias $b = \frac{1}{5}$. As shown above, the most informative equilibrium in the CS communication provides the principal's ex-ante payoff approximately $-\frac{1}{16}$. However, if the principal delegates authority completely, that is, without restrictions on the set of sender's feasible policies, then for any state θ, the sender implements her optimal policy $\theta + b$, which has a constant bias b relative to the receiver's optimal policy θ. That is, both ex-post and ex-ante utilities to the receiver are $-b^2 = -\frac{1}{25}$. The optimal delegation set $[0, \frac{4}{5}]$ brings the expected payoff $-b^2 + \frac{4}{3}b^3 \simeq -\frac{1}{34}$. Therefore, the principal's expected payoff in delegation is higher than that in CS communication. However, it is lower than his payoff $-\frac{1}{52}$ in the case of communication with an imperfectly informed expert.

In this context it is important to note that full delegation is not necessarily optimal in the space of all delegation sets, that is, sets of actions that can be delegated to the sender. Since the expert has a positive bias, for high states she prefers actions that are out of the range of the principal's optimal actions. Excluding these extreme actions from the delegation set implies that the expert would implement the highest possible action for the high states, which is close to the principal's optimal policies for these states. Melumad and Shibano (1991) prove that the optimal delegation set for the uniform-quadratic settings is the interval $[0, y']$, where the upper bound $y' = \left[0, \max\left\{1 - b, \frac{1}{2}\right\}\right]$. However, even optimal delegation performs worse than communication with the imperfectly informed expert.

4 The Model

Consider a uniform-quadratic setup of the CS model, in which the principal takes control of the quality of the expert's information about the state without observing its content. We call this modification the CI (Controlled Information) model. The key modification of our model is a preliminary stage in which the receiver specifies the sender's information structure at zero cost.

Information structure. In particular, the receiver partitions the state space $\Theta = [0, 1]$ into a finite number n of intervals $W_k = (w_k, w_{k+1}]$, $k \in K = \{0, 1, ..., n-1\}$, where $w_0 = 0$, $w_n = 1$, and determines a message space M. Equivalently, an information partition

$\Omega = \{W_k\}_{k=0}^{n-1}$ can be described by a strictly increasing sequence $\{w_k\}_{k=0}^{n}$ of its boundary points, or a sequence of interval lengths $\{\Delta w_k\}_{k=0}^{n-1}$, where $\Delta w_k = w_{k+1} - w_k$. We call a partition **uniform** of size n if it consists of n intervals of the same lengths, that is, $\Delta w_k = \frac{1}{n}$, $\forall k$. As shown below, such partitions play an important role in our analysis.

Then, the state θ is realized, which is drawn from a twice differentiable distribution $F(\theta)$ with a density $f(\theta)$, supported on the unit interval. However, the sender privately observes only an element of the partition W_k, which contains the state, but not the state itself. Since the sender cannot distinguish among different states in W_k, her information is imperfect and determined by the distribution $F(\theta|W_k)$ of the state over the observed interval. Thus, a measure of the sender's imprecision about the state is $P(W_k) = \Pr(\theta \in W_k) = F(w_{k+1}) - F(w_k)$. We denote this sender as W_k-type.

Notice that the described information structure assumes monotonicity of partitions. That is, $\theta \in W_k$, $\theta' \in W_j$, $j > k$ implies that $\theta < \theta'$. Mainly, this form of information structure is supported by two arguments. The first argument is feasibility: it is difficult for the principal to implement an information system so that the expert's information has the form of "a true state is either high or low, but not intermediate". Second, all characterized equilibria in the CS model have the information structure of the monotone partitional form. Thus, the monotone partitioning is convenient for comparing, for example, the distribution of informational losses in the CS case and that in our model, through comparing the number and lengths of the intervals in the information partitions.

Preferences. We consider the class of the players' utility functions similar to that used by Dessein (2002). Namely, the receiver's utility function $U_R(a, \theta)$ has a unique maximum for $a = \theta$ and can be written as

$$U_R(a, \theta) = U(|a - \theta|), \tag{2}$$

where $U(.)$ is twice differentiable, and $U'(0) \leq 0$, $U''(x) < 0$.[9] If $U'(0) = 0$, we additionally require $U'''(.)$ to be continuous in the neighborhood of 0.

Similarly, the sender's utility function $U_S(a, b, \theta)$ has a maximum for $a = \theta + b$ and can be written as

$$U_S(a, b, \theta) = V(|a - b - \theta|), \tag{3}$$

where $V'(x) \leq 0$ and $V''(x) < 0$. For future references we will refer to (2) and (3) as **symmetric preferences**.[10]

The timing of the game. The game is played as follows. First, the receiver specifies an information structure. Second, a realization of the state occurs, and the sender privately observes an element of the partition, which contains the state. Then, the sender transmits a costless message to the receiver. In general, the sender may mix over messages. After receiving the message, the receiver updates his beliefs about the state and implements an action that determines the players' payoffs.

[9] Formally, Dessein (2002) also specifies normalization components in the players' utility functions, which do not affect the results, however.

[10] Also, Krishna and Morgan (2004) consider a special case of such preferences, namely, $U_R(a, \theta) = -|a - \theta|^{\rho}$ and $U_S(a, b, \theta) = -|a - b - \theta|^{\rho}$, where $\rho \geq 1$.

4.1 Equilibrium

Given information structure Ω, a perfect Bayesian equilibrium (hereafter, equilibrium) $(\sigma(m|W_k), a(m), \Omega)$ consists of a signaling strategy $\sigma : \Omega \to \Delta M$, which specifies a probability distribution $\sigma(m|W_k)$ over the space of messages for each type W_k, the principal's action's rule $a : M \to \mathbb{R}$, and a belief function $G : M \to \Delta\Theta$, which specifies a probability distribution over Θ for each message m, including messages that are not sent in equilibrium.

The action's rule $a(m)$ maximizes the receiver's utility $U_R(a|m) = E[U_R(a,\theta)|m]$ given his belief function $G(\theta|m)$.[11] The belief function is constructed on the basis of Bayes' rule where applicable.[12] Given the action's rule $a(m)$, the signaling strategy maximizes the sender's type-relevant utility function

$$U_S(a,b|W_k) = E_\theta[U_S(a,b,\theta)|\theta \in W_k] = \frac{1}{P(W_k)} \int_{w_k}^{w_{k+1}} U_S(a,b,\theta)\, dF(\theta).$$

That is, the signaling strategy $\sigma(m|W_k)$ must satisfy

$$\text{if } \bar{m} \in \text{supp } \sigma(.|W_k), \text{ then } \bar{m} \in \arg\max_{m \in M} U_S(a(m), b|W_k), \text{ and} \tag{4}$$

$$\int_M \sigma(m|W_k)\, dm = 1, \text{ for all } k \in K.$$

Let $\bar{M}(\bar{a}) = \{m : a(m) = \bar{a}\}$. We say that an action $\bar{a}$ is **induced** by a W_k-type, if $\int_{\bar{M}(\bar{a})} \sigma(m|W_k)\, dm > 0$, and is **purely induced** if $\int_{\bar{M}(\bar{a})} \sigma(m|W_k)\, dm = 1$.

The action's rule $a(m)$ maximizes the principal's utility $U_R(a|m) = \int_0^1 U_R(a,\theta)\, dG(\theta|m)$, where the density of the belief function $g(\theta|m)$ is constructed on the basis of Bayes' rule

$$g(\theta|m) = \sum_{k=0}^{n-1} \frac{\sigma(m|W_k)}{g(m)} f(\theta)\, 1_{W_k}(\theta),$$

where $1_{W_k}(\theta)$ is the indicator function and $g(m) = \sum_{k=0}^{n-1} P(W_k)\, \sigma(m|W_k)$.

Then, we can represent $U_R(a|m)$ as

$$U_R(a|m) = \sum_{k=0}^{n-1} g_k(m)\, U_R(a|W_k), \tag{5}$$

where $g_k(m) = \frac{P(W_k)\sigma(m|W_k)}{g(m)}$, and $U_R(a|W_k) = \frac{1}{P(W_k)} \int_{w_k}^{w_{k+1}} U_R(a,\theta)\, dF(\theta)$ is the principal's type-relevant utility function.

[11] Due to the strict concavity of the principal's utility function over actions, he never mixes between actions.

[12] For all messages $m \notin M$, we define the receiver's beliefs in a such way that he interprets them as some $m_0 \in M$.

The receiver's expected utility is

$$U_R = \int\limits_M U_R\left(a\left(m\right)|m\right) g\left(m\right) dm = \sum_{k=0}^{n-1} \int\limits_M \int\limits_{w_k}^{w_{k+1}} \sigma\left(m|W_k\right) U_R\left(a\left(m\right), \theta\right) dF\left(\theta\right) dm$$

$$= \sum_{k=0}^{n-1} \int\limits_M P\left(W_k\right) \sigma\left(m|W_k\right) U_R\left(a\left(m\right)|W_k\right) dm.$$

The following section provides the general analysis of the model.

5 Optimal Information Structure

This section focuses on characterizing the optimal information structure and its comparative statics. First, we are interested in the cardinality of the optimal partition and how it is affected by divergence in the players' interests. Second, we want to compare structures of the optimal information partitions in our model with information partitions endogenously determined in the case of the perfectly informed expert. We demonstrate below that the cardinality of the optimal partition and the distribution of the informational losses in it are both crucial factors that determine a payoff dominance of informational control over delegation. For the purpose of comparison of the endogenous CS and the optimal CI partitions, we use the characterization of Crawford and Sobel (1982) for the leading uniform-quadratic setup of the model. Thus, for this part, the analysis will be predicated upon the assumption of uniform-quadratic settings of the model. Then, to compare the performances of different organizational forms, we consider more general model settings.

Uniform-quadratic case. In this case, we completely characterize the optimal information partition and show that it is always bounded away from the full information as long as the player's interests are imperfectly matched. Further, we demonstrate that the structure of the optimal partition substantially differs from the CS partitions in a few aspects. First, the cardinality of the optimal partition grows much faster as the bias in preferences tends to zero. Second, the optimal information partition allocates informational losses more efficiently across the state space.

5.1 Equilibrium characterization

In this subsection, we outline the basic characteristics of equilibrium strategies. First, the sender's type-relevant utility function $U_S\left(a, b|W_k\right)$ can be written as

$$U_S\left(a, b|W_k\right) = U_S\left(a, b, \bar{w}_k\right) - D\left(W_k\right), \tag{6}$$

where $\bar{w}_k = \frac{w_k + w_{k+1}}{2}$ is a conditional mean of the state, and $D\left(W_k\right) = \frac{1}{12}\Delta w_k^2$ is a conditional residual variance. Since $U_S\left(a, b|W_k\right) \leq U_S\left(\bar{w}_k + b, b, \bar{w}_k\right) - D\left(W_k\right) = -D\left(W_k\right)$, the residual variance represents informational losses of the sender, which always exist whenever the sender does not know the state precisely.

Similarly, the receiver's type-relevant function $U_R\left(a|W_k\right)$ can be written as

$$U_R\left(a|W_k\right) = U_R\left(a, \bar{w}_k\right) - D\left(W_k\right). \tag{7}$$

From (6) and (7), it follows that given any sender's information, the players' preferences over actions are purely determined by the means $\bar{w}_k$. That is, $U_S(a, b|W_k) \geq U_S(a', b|W_k)$ if and only if $U_S(a, b, \bar{w}_k) \geq U_S(a', b, \bar{w}_k)$, and $U_R(a|W_k) \geq U_R(a'|W_k)$ if and only if $U_R(a, \bar{w}_k) \geq U_R(a', \bar{w}_k)$. Thus, type-relevant utility functions $U_S(a, b|W_k)$ and $U_R(a|W_k)$ inherit all important properties of state-relevant functions: strict concavity over actions, single-crossing, and symmetry with respect to optimal actions $a^S(\bar{w}_k) = \bar{w}_k + b$ and $a^R(\bar{w}_k) = \bar{w}_k$. This gives the no-crossing property: $a^S(\bar{w}_k) > a^R(\bar{w}_k)$, $k \in K$. Thus, using the same technique as that developed in Lemma 1 in Crawford and Sobel (1982), it follows that the number of induced actions in equilibrium is finite. All proofs can be found in the Appendix.

Lemma 1 *In any equilibrium, the number of induced actions is finite. Further, the distance between any two actions is not less than* $2b$.

Formally, the number of actions is finite, since the strict concavity of $U_S(a, b|W_k)$ guarantees that the sender of each type induces at most two actions. However, this lemma demonstrates that finiteness of the number of actions comes from the bias in the players' interests rather than from the cardinality of the type space. Thus, an increase in the fineness of the information structure does not eventually bring further informational benefits, since the sender chooses among a finite set of actions. Instead, this introduces additional incentive-compatibility constraints for each type. As a result, for a substantially fine partition, the sender's signaling strategy is no longer invertible, which leads to losses in conveyed information.

Thus, we may restrict the message space to a finite set $M' = \{m_i\}_{i=0}^{I-1}$, where $m_i \in \bar{M}(a_i)$, $i \in \mathcal{I} = 0, 1, ..., I-1$. Then, conditional distributions $\sigma(m|W_k)$ can be replaced by conditional probabilities $\{\sigma_{i,k}\}$, where $\sigma_{i,k} = \int_{\bar{M}(a_i)} \sigma(m|W_k)\, dm$ is the probability to send the message m_i, $i \in \mathcal{I}$, by the sender of the type W_k, $k \in K$.

The following lemma characterizes the sender's equilibrium strategies.

Lemma 2 *Any equilibrium signaling strategy* $\{\sigma_{i,k}\}$ *satisfies the following conditions:*
(A) $\sigma_{i,k} > 0$ *implies* $\sigma_{j,k} = 0$ *for all* $j < i - 1$ *and* $j > i + 1$,
(B) $\sigma_{I-1,n-1} = 1$, *and* $\sigma_{i,n-1} = 0$ *for all* $i < I - 1$,
(C) $\sigma_{i,k} > 0$ *implies* $\sigma_{j,s} = 0$ *for all* $s < k$, $j > i$, *and* $s > k$, $j < i$,
(D) $\sigma_{i,k} > 0$ *and* $\sigma_{i+1,k} > 0$ *imply* $\sigma_{i+1,k+1} > 0$ *for all* $k < n - 1$, *and*
(E) $\sigma_{i,k} > 0$ *and* $\sigma_{i,k'} > 0$ *imply* $\sigma_{i,s} = 1$ *for all* s *such that* $k < s < k'$.

The first condition states that mixing is possible only between messages that induce two adjacent actions. The second requires the highest-type sender to purely induce the highest action. The third condition is the monotonicity condition, which implies that if a sender of some type induces an action, then no sender of a higher type can induce a lower action, and vice versa. Condition (D) argues that if the W_k-type sender induces two actions, then the $W_{k+1}-$type must also induce the higher action. Finally, condition (E) states that if some action is induced by types W_k and $W_{k'}$, then this action is purely induced by all types between these two.

Although Lemma 2 characterizes equilibrium strategies, it still leaves much freedom in terms of the players' expected payoffs. To narrow the set of optimal payoffs, we need to formulate a model-specific revelation principle, which is described in the next section.

5.2 Equilibrium selection: revelation principle

The lack of the principal's ability to commit to actions results in the failure of the standard revelation principle, which restricts the set of all equilibria outcomes to that of truth-telling direct equilibria. Two examples of contracting with imperfect commitment are due to Bester and Strausz (2001) and Krishna and Morgan (2005). In both cases, the sender of a binary type transmits three messages in equilibria. No direct mechanism can replicate these equilibria in terms of induced actions and outcomes.[13]

Nevertheless, the following lemma states that we can restrict attention to direct equilibria only; that is, the cardinality of the message space can be chosen to be equal to that of the type space, or $I = n$.

Lemma 3 *Any equilibrium is payoff equivalent to some direct equilibrium.*

To show this result, we prove that the number of types in any equilibria is not less than that of induced actions. Technically, in any indirect equilibrium there must be a type who induces two actions so that the higher action is induced by this type only. However, this contradicts properties (B) and (D) of Lemma 2.[14]

Further, consider direct truth-telling, or **incentive-compatible equilibria**, in which each type W_k discloses all available information by sending a type-specific message m_k. Given this signaling strategy, the receiver's best-response is $a_k = E[\theta|\theta \in W_k] = \bar{w}_k$. Also, a partition Ω is called **incentive-compatible** if it sustains an incentive-compatible equilibrium.

Using the concavity and symmetry of the sender's type-relevant function $U_S(a, b|W_k)$, she prefers to induce an action a_k instead of a_{k+1} (and all $a > a_{k+1}$), if it is closer to his optimal policy $\bar{w}_k + b$:

$$|a_{k+1} - \bar{w}_k - b| \geq |a_k - \bar{w}_k - b|. \tag{8}$$

Given the receiver's best-response, (8) can be simplified to $w_{k+2} - w_k \geq 4b$. Similarly, the condition to induce a_k instead of a_{k-1} requires $w_{k+1} - w_{k-1} \geq 4b$. Therefore, a partition is incentive-compatible if and only if

$$w_{k+2} - w_k = \Delta w_{k+1} + \Delta w_k \geq 4b, \; k = 0, 1, ..., n-2. \tag{9}$$

The family of inequalities (9) determines the incentive-compatibility (IC) constraints. These conditions are an analogue of the CS no-arbitrage conditions $w_{k+1} = 2w_k - w_{k-1} + 4b$, which can be rewritten as $\Delta w_{k+1} = \Delta w_k + 4b$. Comparing these expressions, one can see that constraints (9) are less restrictive,[15] which implies that the principal can specify a finer information structure in the CI model than in the CS case.

[13] The positive result of Bester and Strausz (2001) is that for a finite set of states any incentive-efficient mechanism (i.e., that which provides equilibrium payoffs on the Pareto frontier) is payoff-equivalent to some direct mechanism. Similarly, Krishna and Morgan (2005) demonstrate that in the case of a continuum of types, any equilibrium outcome of an indirect mechanism can be replicated in a direct mechanism.

[14] In Krishna and Morgan's (2005) example of an indirect equilibrium, the main incentive for a sender of the higher type to induce a lower action is a higher transfer for sending lower messages, which is sufficient compensation for a less desirable policy implemented afterwards. The lack of such transfers in our setup narrows the set of equilibria.

[15] Actually, any CS partition satisfies (9).

Using the following two lemmas, we can constitute a model-specific revelation principle, which states that any optimal equilibrium payoff can be replicated in an incentive-compatible equilibrium. To prove the first result, we show that the payoff-equivalent partition can be obtained from the initial one by the collapsing the partition's elements that induce identical actions. For the second lemma, the superior equilibrium is constructed in two steps. First, we derive all types that play mixed strategies and assign probability one to the lower actions. Second, given the modified signaling strategy, we collapse the partition elements that induce identical actions and adjust the receiver's beliefs and the best-response.

Lemma 4 *For any pure-strategy equilibrium, there exists an incentive-compatible equilibrium, which is payoff equivalent.*

Lemma 5 *For any mixed-strategy equilibrium, there exists an incentive-compatible equilibrium, which is payoff superior.*

Thus, the principal never wants to provide the expert with information that would still be her private knowledge after she sends a report to the principal. However, the cost of this is the expert's informational losses, which the principal needs to minimize without breaking her incentive-compatibility constraints. The next subsection addresses this issue.

5.3 Optimal information structure

To find the optimal incentive-compatible partition, we first determine the maximal size of the incentive-compatible partition $n(b)$. It can be shown that

$$n(b) = \begin{cases} 2\lfloor \frac{1}{4b} \rfloor + 1, \text{ if } b \neq \frac{1}{2m} \text{ for some even integer } m \\ m, \text{ otherwise} \end{cases} \tag{10}$$

where $\lfloor x \rfloor$ is the largest integer smaller than or equal to x. Notice that for $b > \frac{1}{4}$ informative communication is not feasible. However, for $b = \frac{1}{4}$, the finest partition has two elements, so communication is informative, in contrast to the CS model. The next proposition describes the structure of the optimal partition.

Proposition 1 *For any b, there exists $b^*(c) \in (\frac{1}{2c}, \frac{1}{2(c-1)})$, where $c = n(b)$, such that if $b > b^*(c)$, then the optimal partition is uniform of size $c-1$. For $b \leq b^*(c)$, the optimal partition is one of size c such that: 1) if $\frac{1}{2c} < b \leq b^*(c)$, then the IC constraints (9) are binding for all $\{w_k\}_{k=0}^{n(b)}$, and 2) if $\frac{1}{2(c+1)} < b \leq \frac{1}{2c}$, then the optimal partition is uniform.*

Corollary 1 *If the sender' bias is $b \leq \frac{1}{4}$, then there exists an equilibrium in the CI model, which is Pareto superior to all equilibria in the CS model.*

The first difference of the optimal information partition as compared to endogenous CS partitions is that the principal does not always prefer the partition with the highest number of elements. Basically, the optimal partition highlights a trade-off between two different structures. One of them is uniform, so it more efficiently shares informational losses of the risk-averse principal, whereas the other benefits him because of the possibility of better responding to messages through a higher number of actions. Notice that the latter structure is never optimal in the model of Fischer and Stocken (2001) due to a special choice of the bias in their model. The cut-off levels $b^*(c)$ are exactly the biases, at which these information structures are payoff equivalent.[16]

The second feature is the fineness of the information structure. The cardinality of the finest partition grows as $1/b$ in the CI model relative to $1/\sqrt{b}$ in the CS case. As a result, the principal is able to respond to changes in the state more sensitively.

The last feature is the variance in the lengths of the partition elements. From the CS arbitrage condition, the length of any interval in a CS partition must exceed that of the previous interval by $4b$. Thus, informational losses grow monotonically with the state of nature. In contrast, the IC constraints (9) impose fewer restrictions on the functional relationship between lengths of different intervals. As a result, informational losses can be shared more effectively across the state space.

6 Informational Control versus Delegation

Delegation is broadly considered an alternative to communication. Instead of relying on the expert's non-verifiable information, the principal can delegate his power to the expert and gain from her superior information.[17] However, the informational benefits are mitigated by losses because the expert's decisions are biased. Nevertheless, in a variety of situations, the aggregate effect leads to an ex-ante Pareto-improvement compared to communication (with a fully informed expert). Another useful feature of delegation is its ease of implementation: generally, there are no costs to empower the expert with a right to carry out policies. Due to these and other factors, many firms have pushed decision rights down in the hierarchy in recent years.[18]

Despite the seemingly obvious benefits of delegation, however, a surprising number of companies today still have the centralized structure. Actually, it remains the most popular organizational form. Moreover, companies that do decentralize decision making and accountability often centralize it again when they run into trouble.[19] In addition, the example of Emerson, mentioned above, illustrates that keeping control over decisions is generally not independent from keeping control over information. Thus, when comparing the performance of different organizational forms, we have to consider the possibility that the principal may restrict the expert's information.

Technically, delegation and informational control utilize different factors for payoff improvement. Delegation allows for the receiver to acquire benefits from the expert's informational advantage, whereas controlling the sender's information smooths misalignment

[16] For instance, for the bias $b = \frac{1}{5}$, we have $n(\frac{1}{5}) = 3$ and the cutoff level $b^*(3) \simeq 0.202$. The principal's expected payoffs under the three-element partition with the binding IC constraints $\{0, 0.2, 0.8, 1\}$ and the two-element uniform partition $\{0, \frac{1}{2}, 1\}$ are $-\frac{1}{52}$ and $-\frac{1}{48}$, respectively. However, as the bias grows to 0.22, the IC constraints make the finest incentive-compatible partition less uniform, so it becomes $\{0, 0.12, 0.88, 1\}$, which decreases the payoff to approximately $-\frac{1}{27}$. In contrast, this change in the bias has no effect on the uniform partition of a smaller size, which is still incentive-compatible.

[17] See, for example, Alonso and Matouschek (2005), Dessein (2002), Holmström (1997), and Melumad and Shibano (1991).

[18] See Dessein (2002).

[19] For example, Motorola had a decentralized structure by the mid-1990s. However, then the company's mobile-phone business was growing so fast that decentralization made it impossible to control. In 1998, the company repatriated control to the headquarters.

between the players' preferences due to marginal types. Thus, at first glance, there seems to be no clear intuition about which effect is generally stronger.

6.1 Uniform-quadratic case

We now show that if informative communication is feasible ($b \leq \frac{1}{4}$), then controlling the expert's information strictly dominates delegation in terms of the receiver's expected payoffs. This result is formalized by the following theorem.

Theorem 1 *If informative communication is feasible, then there exists an equilibrium in the CI model which provides a higher expected payoff to the principal than optimal delegation.*

The intuition behind this result requires some background. First, consider the case of small divergence in players' interests. Second, notice from Proposition 1 that the informational losses in the optimal information partition in the CI model are distributed more or less uniformly across the state space. Given these preliminaries, it is sufficient to compare the principal's expected losses $D(W_k)$ for an average partition element versus his expected losses b^2 in the case of *complete* delegation. (The effect of restricted delegation due to the upper bound $1-b$ on the delegation set disappears as b falls.) Then, the IC constraints (9) imply that the length of the average element in the optimal CI partition is of order $2b$. As a result, the principal's expected losses are of the order $\frac{(2b)^2}{12}$, or $\frac{b^2}{3}$, which is three times lower than his losses in delegation.[20] On the other hand, if the bias is close to $\frac{1}{4}$, there is some improvement in the performance of delegation due to the effect of the restricted delegation set. Moreover, controlled communication performs worse for large biases because the relative difference in the lengths of elements of the optimal non-uniform partition is increasing in the value of b. That is, the informational losses are shared less effectively. However, the influence of these factors is of second order and cannot reverse the main result.

This result is in contrast to the CS case. In the case of the perfectly informed expert, even though the lengths of all partition elements fall as the bias tends to zero, the size of the second smallest element in the CS partition is no less than $4b$, which results in a lower principal's payoff than in delegation.[21]

Thus, as soon as there is a scope for informative communication, the principal is better off from controlling the expert's information than delegating authority to the expert. Fig. 2 demonstrates the principal's expected payoff under the optimal partition in the CI model, optimal delegation, and the most informative equilibrium in the CS model.[22]

[20] Moreover, $\Delta_k \backsimeq 2b$ implies that the ex-post losses of the principal $-(\bar{w}_k - \theta)^2$ are comparable to those in delegation $-b^2$, only if θ is very close to the boundaries of the partition element W_k, and are strictly lower for all other states.

[21] Because of the no-arbitrage condition $\Delta w_{k+1} = \Delta w_k + 4b, \forall k$.

[22] A feature of our model is that the principal's expected payoff is discontinuous in the bias due to the "regime switching" effect. When the bias falls, this effect takes place at points $b = \frac{1}{2n}$, where n is an even integer. At these points, the maximal size of the incentive-compatible partitions changes from $n-1$ to n and the uniform partition of this size becomes incentive-compatible. As a result, the optimal partition switches from the uniform of size $n-1$ to the uniform partition of size n, and the expected payoff jumps from $-\frac{1}{12(n-1)^2}$ to $-\frac{1}{12n^2}$. In contrast, the incentive-compatibility of a partition of an odd size does not guarantees that the uniform partition of the same size is incentive-compatible. Thus, the optimal partition

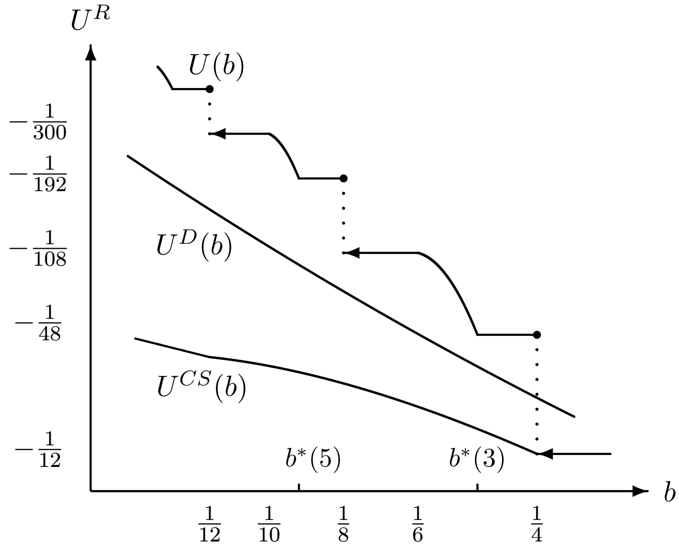

Figure 2: Payoffs in the CI model, optimal delegation, and the CS model

The above discussion raises a natural question—whether the result is driven by the specific uniform-quadratic setup of the model or it can be replicated in broader environments. The next subsection illustrates that extending the model's settings generally does not change the result.

6.2 General distributions and preferences

This section examines the robustness of the previous results to changes in the specifications of the model, namely the players' utility functions and distributions of the state. These results generally hold whenever the difference in players' interests is not too large relative to the principal's uncertainty about the environment. That is, as the bias in preferences falls, controlling the expert's information becomes more attractive than delegation. This result is in stark contrast to that of Dessein (2002), who demonstrates that delegation is more likely than CS communication when the players' preferences are close.

General preferences: the role of risk-aversion. First, we consider the symmetric form (2) and (3) of the players' preferences. In this case, the players' type-relevant utility functions $U_S(a, b|W_k)$ and $U_R(a|W_k)$ are concave in a and symmetric with respect to the optimal policies $\bar{w}_k + b$ and $\bar{w}_k$, respectively. This implies that the receiver's best-response to the truth-telling signaling strategy is $a(m_k) = \bar{w}_k$. Therefore, the IC constraints (9) also hold, and the optimal information structure is the same as that determined by Proposition 1 up to the values of the switching points $b^*(c)$ between uniform partitions of size $c-1$ to non-uniform partitions of size c. This implies that communication is informative only if $b \leq \frac{1}{4}$. Similarly, the arbitrage condition in the CS model is not affected, which implies that CS equilibria are invariant to this modification in preferences. Based on these observations, the Corollary 1 can be proved straightforwardly.

of an odd size can be non-uniform and provide an expected payoff, which is continuous in b. Hence, the switch between a uniform partition of an even size $n-1$ to a non-uniform partition of an odd size n at points $b^*(n)$ is not accompanied by a discontinuous change in payoffs.

Theorem 2 *If the state is uniformly distributed and preferences are symmetric, then for $b \leq \frac{1}{4}$, there exists an equilibrium in the CI model which is Pareto superior to all equilibria in the CS model.*

Since IC constraints are not affected by a change in preferences, the optimal information partition preserves all positive properties of that in the uniform-quadratic case, such as a finer structure and efficient sharing of informational losses. In particular, given any CS partition, the uniform partition of the same size is incentive-compatible in the CI model and provides higher expected utility.

Before we compare the principal's payoff in the CI model with that in delegation, notice that for the class of interval delegation sets (i.e., an expert's policy must belong to a single interval) and $b \leq \frac{1}{2}$, the optimal delegation set is still of a form $[0, 1-b]$. Then, we can generalize the result of Theorem 1—informational control performs better than the optimal delegation, when the bias in the players' interests is not too large.

Theorem 3 *If the state is uniformly distributed and the preferences are symmetric, then there is $b^* \leq \frac{1}{4}$ such that for all $b < b^*$, there exists an equilibrium in the CI model, which provides a superior payoff to the receiver compared to that in optimal delegation.*

This result is weaker than Theorem 1 for the case of the quadratic preferences since it does not guarantee that the controlling information performs better than delegation whenever informative communication is feasible. Basically, this result cannot be strengthened because of the risk-aversion of the principal.[23] In communication, an induced action is unbiased on average, but there is a high chance of making a mistaken action (if a state is close to a boundary of a partition element). This increases informational losses for highly concave utility functions. Delegation, however, provides a permanent bias in the expert's decision, which can be more preferable by the very risk-averse principal. Nevertheless, when the bias decreases, the optimal information structure becomes sufficiently fine to reduce the variance between optimal and induced actions, which results in better performance of the CI model over delegation.

Thus, if the bias is moderate, then the relationship between the principal's payoffs in different organizational forms strongly depends on the structure of the information partition. For instance, for $b = \frac{3}{17}$ and $U(|a-\theta|) = -|a-\theta|^4$, the optimal information structure is the three-element partition $\{0, \frac{5}{17}, \frac{12}{17}, 1\}$. It provides the expected payoff -2.03×10^{-4}, which exceeds that in optimal delegation -6.96×10^{-4}. However, restricting information structures to only uniform partitions gives a lower payoff -7.81×10^{-4}, because the three-element uniform partition is not incentive-compatible, whereas two-element partitions are too coarse.

General distributions. Another way to generalizing the model's settings is to consider general distributions of the state. In particular, we restrict attention to the class of distributions with a positive and differentiable density and supported on a unit

[23]Consider the principal's utility function $U_2(|a-\theta|) = -|a-\theta|^7$, and the bias $b = 0.126$. Then the optimal partition in the CI model is the uniform three-element one. It is informative and provides expected utility $U_R \simeq -4.5 \cdot 10^{-7}$. However, optimal delegation gives $U_R^D \simeq -3.9 \cdot 10^{-7}$, which is superior to that in the CI model.

interval. In this case, restricting the expert's information is always beneficial whenever CS communication is informative.

Theorem 4 *If the preferences are quadratic, then there exists an equilibrium in the CI model which is superior to all informative equilibria in the CS model.*

In general, any informative CS partition is characterized by larger informational losses for the high values of the state. Thus, they can be reduced if the principal locally modifies this partition without violating IC constraints in such a way that the variance in the intervals' lengths becomes smaller.

Similarly, the theorem below compares the principal's payoffs in the CI model with that in the complete delegation. It demonstrates that the result of Theorem 1 for general distributions holds, if the sender's bias is small.[24]

Theorem 5 *If the preferences are quadratic, then there is $\tilde{b}$ such that for all $b < \tilde{b}$, there exists an equilibrium in the CI model, which provides a superior payoff to the receiver compared to that in the complete delegation.*

When the sender's bias tends to zero, the size of the intervals in the finest incentive-compatible partition converges to $2b$ regardless of the distribution $F(\theta)$. Equivalently, the number of elements $n(b)$ in the finest incentive-compatible partition grows as $\frac{1}{2b}$, exactly as in the case of the uniform distribution. This implies that the principal's expected losses in the most informative equilibrium fall as $-\frac{1}{12\times n(b)^2}$ or $-\frac{b^2}{3}$, which is less than that $-b^2$ in delegation.

7 Delegation to the Imperfectly Informed Expert

In a variety of situations, the principal can use both of the analyzed instruments—delegating control over decisions and restricting the quality of the expert's information—if a combined effect from utilizing them is positive. For example, a top manager can restrict the employee's access to information and delegate a task afterwards. Moreover, he can determine the set of policies the employee is allowed to choose from.

The analysis above shows that delegation can outperform informational control, if the players' interests significantly diverge or the principal is highly risk-averse. However, even in this case, the principal can have incentives to deteriorate the expert's information. Our main finding is that the total effect from using both instruments purely depends on the principal's ability to restrict the set of delegated policies. To demonstrate this argument, consider an example.

Example 3. Consider the uniform-quadratic setup with the bias $b = \frac{1}{5}$. If the sender is perfectly informed, then his payoffs in the full and optimal delegations are $-b^2 = -\frac{1}{25}$ and

[24] The problem of optimal delegation for general distributions and quadratic preferences is solved by Alonso and Matouschek (2005). They provide necessary and sufficient conditions for delegation sets to be optimal for cases of complete delegation, centralization (the delegation set that contains only the principal's preferred actions given prior information), and interval delegation. However, when the players' preferences are sufficiently close, one can expect that the principal's incentives to restrict the sender's actions are small, and the optimal delegation set and the players' payoffs converge to that of a case of complete delegation.

$-b^2 + \frac{4}{3}b^3 = -\frac{1}{34}$, respectively. Thus, the principal's losses from choosing the delegation set optimally decrease by 36%. On the other hand, if the sender's information structure is a three-element partition $\{0, \frac{3}{10}, \frac{7}{10}, 1\}$, then complete delegation brings him a payoff approximately $-\frac{1}{20}$, whereas the three-action delegation set $\{0.17, 0.53, 0.87\}$ results in a payoff $-\frac{1}{96}$. That is, the principal's losses fall by almost four times.

Full delegation. The example above illustrates that in the case of full delegation, the perfectly informed expert performs better than the imperfectly informed one. If there are no restrictions on the delegation set, then controlling the expert's information is detrimental. Intuitively, given any information, the bias between an expert's decision and the principal's optimal policy will be, on average, the same as in the case of the perfectly informed expert. Then, decreasing the quality of expert's information only introduces additional informational losses. For the quadratic preferences, this logic is shaped formally into the following lemma.

Theorem 6 *Under complete delegation, the optimal information structure is complete information.*

Thus, if the principal has power to limit the expert's information, but not her discretion, he should never decrease the precision of the expert's information before delegating authority to her.

Restricted delegation. Therefore, controlling the expert's information before delegating policies can be beneficial only if the principal is able to affect the set of the expert's feasible decisions. Note that restricting both the information and the delegation set cannot perform worse than pure communication with the imperfectly informed expert, since the principal can always specify an information structure and a set of actions as in a communication equilibrium. As shown above, for small biases, communication with the imperfectly informed expert dominates optimal delegation with the fully informed expert. Thus, the imperfect expert's information is always beneficial for the principal in the case of restricted delegation, if the divergence in players' interests is small. In the uniform-quadratic case the cut-off bias is $b = \frac{1}{4}$. The following result for the uniform-quadratic case shows that an imprecise expert's information is beneficial even if the bias is large.

Theorem 7 *For $b < \frac{1}{2}$, there exists an information structure and a delegation set, which provide a superior payoff to the principal than that in the optimal delegation with the perfectly informed expert.*

Basically, the lack of the principal's commitment in communication requires him to react optimally to the expert's messages. This means that the set of induced actions is bounded from above by the highest sender's type. Given these restrictions, for a large bias in preferences and just two types, the sender of the lower type always benefits from a higher action. The finer information structure only decreases sender's incentives to communicate truthfully.[25] As a result, the principal will ignore any messages from the expert, because she transmits the same message unconditionally on her information. In contrast, if the

[25] Since it imposes a higher number of incentive-compatibility constraints.

principal restricts the information structure by the same two intervals (types) and commits to implementing two essentially different actions, then the expert of a lower type would prefer a lower action, if a higher action is far enough from her optimal policy. For a smaller bias, the principal can specify a finer information structure and delegation set, but the major argument is the same as in the above example. Namely, the principal faces a trade-off between providing the expert with more information to reduce informational losses, and creating incentives for the sender of each type to make an action sufficiently close to the principal's optimal policy.

The following lemma characterizes the properties of the optimal information structure and the delegation set.

Theorem 8 *The optimal information structure and delegation set satisfy the following properties:*

1) the information structure and delegation set are finite, and

2) given the sender's information that $\theta \in [w_k, w_{k+1}]$*, the induced action* $a_k \in [w_k, w_{k+1}]$.

Corollary 2 *For* $b < \frac{1}{2}$*, delegation to the imperfectly informed expert is payoff superior to pure communication with the imperfectly informed expert.*

In general, the optimal information structure inherits the major features of that in the communication game. First, it is finite. A very fine information structure requires a larger number of actions in the delegation set,[26] which creates difficulties with locating actions quite far from each other to satisfy the sender's incentive-compatibility constraints. Second, the informational losses of the sender are distributed more or less uniformly across the state space. Finally, if the sender knows that the state is in some interval, then an induced action belongs to this interval (the optimal delegation set for the fully informed expert does not have such property). In addition to these properties, an appropriate choice of the delegation set allows for determining a finer information structure than that in communication and/or allocating the sender's informational losses more efficiently.

8 Concluding Comments

The main contribution of this paper is as follows: if the principal is able to control the extent of the expert's informativeness (without knowing its content), he can do better than by optimally delegating decisions to the expert. This finding reverses the result about the payoff dominance of delegation over pure communication. This might be one of the factors that explains the fact that, despite seemingly clear benefits of delegation, many companies do not decentralize decision-making, and even often recentralise their structures again.

We deliberately did not address the case when the person who determines the quality of the expert's information is the expert herself, because the answer is straightforward. As demonstrated by Crawford and Sobel (1982) for the leading uniform-quadratic example, the principal's expected utility is equal to the residual variance of the state in any communication equilibrium, since the principal's decisions are on average unbiased. As

[26] Otherwise, if several types induce the same action, the principal can collapse them into one type.

a result, the expert's expected utility differs from that of the principal by a constant term b^2. This argument holds for any communication equilibrium unconditionally on the quality of the expert's information. Thus, if there is a credible mechanism of the expert's commitment to the precision of information, in which the expert commits "not to know too much" or her competence of the subject is verifiable, then the expert's choice of the optimal information partition will be the same.

Another issue, which we left behind is comparison of controlling information to other organizational forms such as delegating authority to a biased intermediary and delegation with a veto power, when the principal has a choice between only two decisions: recommended by the expert and some default option. These institutions are special cases of restricted delegation, which implies that they cannot perform more effectively than optimal delegation. Therefore, as soon as controlling information is preferred by the principal to optimal delegation, it is strictly preferred to all discussed institutions.

An important aspect of the considered model is the number of equilibria, significantly exceeding the number of equilibria in Crawford and Sobel. In addition to pure-strategy equilibria, there exist multiple mixed-strategy equilibria even with the same partition. Thus, we need to care about ranking equilibria in terms of the principal's expected payoff. However, despite the fact that all mixed-strategy equilibria are payoff inferior to pure-strategy ones, they can still be superior to equilibria in the CS model and delegation.

9 Appendix

In this section, we provide proofs of the lemmas and theorems.

For the uniform-quadratic case, the solution to the problem of a W_k-type sender is

$$\begin{gathered} m_i \in \underset{m \in M'}{\arg\max}\, U_S\left(a\left(m\right), b | W_k\right), \text{ if} \\ U_S\left(a_i, b, \bar{w}_k\right) \geq U_S\left(a_j, b, \bar{w}_k\right) \text{ for all } j \in \mathcal{I}. \end{gathered}$$

The family of inequalities $U_S\left(a_i, b, \bar{w}_k\right) \geq U_S\left(a_j, b, \bar{w}_k\right)$, $i, j \in \mathcal{I}$, $k \in K$, can be written as

$$\left(a_i - \bar{w}_k - b\right)^2 \leq \left(a_j - \bar{w}_k - b\right)^2$$

or

$$\begin{aligned} &1)\ a_i + a_j \geq w_k + w_{k+1} + 2b \text{ for all } a_j > a_i, \text{ and} \\ &2)\ a_i + a_j \leq w_k + w_{k+1} + 2b \text{ for all } a_j < a_i. \end{aligned} \tag{11}$$

The principal's best response $a_i = a\left(m_i\right)$, $i \in \mathcal{I}$, is

$$a_i = E\left[\theta | m_i\right] = E\left[\bar{w}_k | m_i\right] = \sum_{k=0}^{n-1} P\left(W_k | m_i\right) \bar{w}_k = \frac{1}{2} \frac{\sum\limits_{k=0}^{n-1} \sigma_{i,k}\left(w_{k+1}^2 - w_k^2\right)}{\sum\limits_{k=0}^{n-1} \sigma_{i,k}\left(w_{k+1} - w_k\right)}, \tag{12}$$

Proof of Lemma 1. Let a and a' be two induced actions, where $a' > a$. Consider types W_k and $W_{k'}$, which induce corresponding actions, that is, $U_S\left(a, b | W_k\right) \geq U_S\left(a', b | W_k\right)$ and $U_S\left(a', b | W_{k'}\right) \geq$

$U_S(a,b|W_k)$. Then, it follows from (6) that $U_S(a,b,\bar{w}_k) \geq U_S(a',b,\bar{w}_k)$ and $U_S(a',b,\bar{w}_{k'}) \geq U_S(a,b,\bar{w}_{k'})$.

The single-crossing property of the sender's state-relevant utility function $\frac{d^2}{dad\theta}U_S(a,b,\theta) > 0$ implies that there exists a state $\theta \in (\bar{w}_k, \bar{w}_{k'})$ such that $U_S(a',b,\theta) = U_S(a,b,\theta)$. Also, this property leads to (i) $a < a^S(\theta) < a'$, where $a^S(\theta) = \theta + b$, (ii) a is not induced by any type W_i such that $\bar{w}_i > \theta$, and (iii) a' is not induced by any type W_j such that $\bar{w}_j < \theta$. The last two properties along with the single-crossing property of $U_R(a,\theta)$ imply $a \leq a^R(\theta) = \theta \leq a'$.

In addition, the symmetry of $U_S(a,b,\theta)$ with respect to $a^S(\theta)$ implies that $a' - \theta - b = \theta + b - a$, or $a^S(\theta) = \theta + b = \frac{a+a'}{2}$. This means that both $a^S(\theta)$ and $a^R(\theta)$ belong to the interval $[a, \frac{a+a'}{2}]$. Since $a^S(\theta) - a^R(\theta) = b$, it follows that $\frac{a+a'}{2} - a \geq b$, or $a' - a \geq 2b$. To complete the proof, notice that the set of induced actions is bounded by $a^R(0)$ and $a^R(1)$. ■

Proof of Lemma 2. (A) This property follows from the strict concavity of $U_S(a,b|W_k)$ in a. By contradiction, let $\sigma_{i,k} > 0$ and $\sigma_{j,k} > 0$, where $j > i+1$, for some k. This implies that $U_S(a_i,b|W_k) = U_S(a_j,b|W_k) \geq U_S(a_l,b|W_k)$ for all $l \in \mathcal{I}$. Since a_{i+1} can be represented as a convex combination of a_i and a_j, $a_{i+1} = \lambda a_i + (1-\lambda)a_j$ for some $\lambda \in (0,1)$, this results in a contradiction: $U_S(a_{i+1},b|W_k) > \lambda U_S(a_i,b|W_k) + (1-\lambda)U_S(a_j,b|W_k) = U_S(a_i,b|W_k)$.

(B) From (12), the maximal induced action is $a_{I-1} \leq \bar{w}_{n-1} < \bar{w}_{n-1} + b$. Since $U_S(a,b|W_k)$ is strictly increasing in a, for all $a < \bar{w}_k + b$, the result follows immediately.

(C) Let $\sigma_{i,k} > 0$ and $\sigma_{j,s} > 0$ for some $s > k$ and $j < i$. $\sigma_{i,k} > 0$ implies $U_S(a_i,b|W_k) \geq U_S(a_j,b|W_k)$, and $\sigma_{j,s} > 0$ implies $U_S(a_j,b|W_s) \geq U_S(a_i,b|W_s)$. Combining these inequalities results in $U_S(a_i,b|W_s) - U_S(a_j,b|W_s) \leq 0 \leq U_S(a_i,b|W_k) - U_S(a_j,b|W_k)$, which contradicts the single-crossing property $U_S(a_i,b|W_s) - U_S(a_j,b|W_s) > U_S(a_i,b|W_k) - U_S(a_j,b|W_k)$.

(D) By contradiction, let $\sigma_{i,k} > 0$, $\sigma_{i+1,k} > 0$, and $\sigma_{i+1,k+1} = 0$ for some $k < n-1$. Condition (C) for $\sigma_{i,k} > 0$ implies $\sigma_{i+1,s} = 0$ for all $s < k$. Condition (C) for $\sigma_{i+1,k}$ implies $\sigma_{j,k+1} = 0$ for all $j < i+1$. Since $\sigma_{i+1,k+1} = 0$, then $\sigma_{j',k+1} > 0$ for some $j' > i+1$. Again, using condition (C) for $\sigma_{j',k+1}$, we have $\sigma_{i+1,s} = 0$ for all $s > k+1$. Hence, $\sigma_{i+1,s} = 0$ for all $s \neq k$. It follows from (12) that $a_{i+1} = \bar{w}_k$. Then, $\sigma_{i,k} > 0$ and $\sigma_{i+1,k} > 0$ imply $U_S(a_i,b|W_k) = U_S(a_{i+1},b|W_k)$, which results in a contradiction: $a_i < \bar{w}_k + b < a_{i+1} = \bar{w}_k$.

(E) This is a corollary of property (C). If $\sigma_{i,k} > 0$, then $\sigma_{j,s} = 0$ for all $j < i$ and $s > k$. Similarly, $\sigma_{i,k'} > 0$ implies $\sigma_{j,s} = 0$, $j > i$, $s < k'$. Thus, for all s such that $k < s < k'$, we have $\sigma_{j,s} = 0$, $j \neq i$, that gives the desired result. ■

Proof of Lemma 3. By contradiction, suppose that there exists an indirect equilibrium, in which the number of induced actions exceeds the number of types, that is, $I > n$. This implies that there exists a type W_k such that $\sigma_{i,k} > 0$, $\sigma_{i+1,k} > 0$, and $\sigma_{i+1,k+1} = 0$. Since the highest type W_{n-1} never mixes (by property (B) of Lemma **2)**, it follows that $k < n-1$. Then, property (D) of Lemma **2** is violated, which completes the proof. ■

Given an equilibrium signaling strategy $\{\sigma_{i,k}\}$, define a correspondence $p : \mathcal{I} \Longrightarrow K$ such that $p(i) = \{k \in K : \sigma_{i,k} > 0\}$. Thus, $p(i)$ determines the subset of types, which induce action a_i. Similarly, define a function $j : K \to \mathcal{I}$ such that $j(k) = \min\{i \in \mathcal{I} : \sigma_{i,k} > 0\}$, and a correspondence $\mu : \mathcal{I} \Longrightarrow K$ such that $\mu(i) = j^{-1}(i) = \{k \in K : j(k) = i\}$. Function $j(k)$ determines the minimal action induced by the sender of W_k-type. Conversely, given an action a_i, $\mu(i)$ determines the set of types, for which this action is minimal. That is, if $k \in \mu(i)$, then $\sigma_{l,k} = 0$

for all $l < i$. The next lemma describes properties of $p(i)$, $j(k)$, and $\mu(i)$.

Lemma 6 *$p(i)$, $j(k)$, and $\mu(i)$ satisfy the following properties.*

a) $j(k)$ is (weakly) increasing,

b) $\mu(i)$ is non-empty, strictly increasing, and convex-valued,

c) $p(i)$ is non-empty, (weakly) increasing, and convex-valued. In a pure-strategy equilibrium, $p(i)$ is strictly increasing, and

d) $\mu(i) \subset p(i)$ and $\max \mu(i) = \max p(i)$.

Proof a) $j(k) = i$ implies $\sigma_{i,k} > 0$. Property (C) of Lemma **2** results in $\sigma_{i',k'} = 0$ for all $i' < i$ and $k' > k$, which leads to $j(k') \geq j(k)$.

b) For a given i, if $\sigma_{i,k} = 1$ for some k, then $j(k) = i$. Hence, $k \in \mu(i)$. If $\sigma_{i,k} > 0$ and $\sigma_{i+1,k} > 0$, then property (A) implies $\sigma_{i',k} = 0$ for all $i' < i$, thus, $j(k) = i$ and $k \in \mu(i)$. If $\sigma_{i-1,k} > 0$ and $\sigma_{i,k} > 0$, then property (D) implies $\sigma_{i,k+1} > 0$, and property (C) gives $\sigma_{i',k+1} = 0$ for all $i' < i$. Hence, $j(k+1) = i$ and $k+1 \in \mu(i)$. Thus, $\mu(i)$ is non-empty. To prove that $\mu(i)$ is strictly increasing, by contradiction, let $i' > i$ and $k' \leq k$, for some $k' \in \mu(i')$, $k \in \mu(i)$. Then, $k' \in \mu(i')$ implies $j(k') = i'$ and $\sigma_{i',k'} > 0$. Similarly, we have $j(k) = i$ and $\sigma_{i,k} > 0$. If $k' = k$, then $i' = j(k') = j(k) = i$ and we have a contradiction. If $k' < k$, then $\sigma_{i',k'} > 0$ and $\sigma_{i,k} > 0$ contradict condition (C). To show that $\mu(i)$ is convex-valued, let $k' \in \mu(i)$ and $k'' \in \mu(i)$. Thus, $\sigma_{i,k'} > 0$, $\sigma_{i,k''} > 0$. Then, property (E) implies $\sigma_{i,k} = 1$ for all k such that $k' < k < k''$.

c) The first part of the statement can be easily proved using the same techniques as those developed in the $\mu(i)$ context. The second part follows from the fact that $p(i) = \mu(i)$ in a pure-strategy equilibrium, hence, $p(i)$ is strictly increasing.

d) For any $k \in \mu(i)$, we have $j(k) = i$. This results in $\sigma_{i,k} > 0$, hence, $k \in p(i)$ and $\mu(i) \subset p(i)$. Now, for a given i' consider $k'' = \max \mu(i') = \max\{k \in K : j(k) = i'\}$. Then, $\sigma_{i',k''} > 0$ and $\sigma_{i',k} = 0$ for all $k > k''$. If not, that is, if $\sigma_{i',k} > 0$ for some $k > k''$, then condition (C) implies $\sigma_{i,k} = 0$ for all $i < i'$. This means $i' = \min(i : \sigma_{i,k} > 0) = j(k)$, which contradicts $k'' = \max \mu(i')$. Thus, $\sigma_{i',k} = 0$ for all $k > k''$, which results in $k \leq k''$ for all $k \in p(i')$. That is, we obtain $\max p(i) \leq \max \mu(i)$. ■

From the above lemma, there exist $k'(i) = \min \mu(i)$, $k''(i) = \max \mu(i)$, $k'_p(i) = \min p(i)$, and $k''_p(i) = \max p(i)$, $i \in \mathcal{I}$. That is, $k'_p(i)$ and $k''_p(i)$ are the smallest and the largest types, respectively, which induce action a_i. Similarly, $k'(i)$ and $k''(i)$ are the smallest and the largest types for which a_i is the minimal action. Property (d) of Lemma 6 implies $k''(i) = k''_p(i)$ and $k'_p(i) \leq k'(i)$.

Because $p(i)$ is convex-valued, the receiver's best-response is

$$a_i = \frac{1}{2} \frac{\sum_{k \in p(i)} \sigma_{i,k}\left(w_{k+1}^2 - w_k^2\right)}{\sum_{k \in p(i)} \sigma_{i,k}\left(w_{k+1} - w_k\right)} = \frac{1}{2} \frac{\sum_{k=k'_p(i)}^{k''_p(i)} \sigma_{i,k}(w_{k+1}^2 - w_k^2)}{\sum_{k=k'_p(i)}^{k''_p(i)} \sigma_{i,k}\left(w_{k+1} - w_k\right)},\ i \in \mathcal{I}. \tag{13}$$

Proof of Lemma 4. In a pure-strategy equilibrium, the receiver's best-response is

$$a_i = \frac{w_{k'_p(i)} + w_{k''_p(i)+1}}{2} = \frac{w_{k'_p(i)} + w_{k'_p(i+1)}}{2},\ i \in \mathcal{I},$$

and the expected payoff is

$$U_R = -\sum_{i=0}^{I-1} \int_{w_{k'_p(i)}}^{w_{k'_p(i+1)}} (a_i - \theta)^2 \, d\theta = -\frac{1}{12} \sum_{i=0}^{I-1} \left(w_{k'_p(i+1)} - w_{k'_p(i)} \right)^3 .$$

Consider the partition $\{W'_i\}_{i \in \mathcal{I}}$ such that $W'_i = \bigcup_{k \in p(i)} W_k = (w_{k'_p(i)}, w_{k'_p(i+1)}]$, $i \in \mathcal{I}$. Given the signaling strategy $m(i) = m_i$, $i \in \mathcal{I}$, the receiver's best-response is not affected by these transformations.

Since $\{\sigma_{i,k}\}$ is an equilibrium strategy, then for any $i \in \mathcal{I}$, we have

$$a_i + a_l \geq w_{k''_p(i)} + w_{k'_p(i+1)} + 2b \geq w_{k'_p(i)} + w_{k'_p(i+1)} + 2b \text{ for all } a_l > a_i.$$

By (11), this implies that $U_S(a_i, b|W'_i) \geq U_S(a_l, b|W'_i)$ for all $a_l > a_i$. Similarly,

$$a_i + a_l \leq w_{k'_p(i)} + w_{k'_p(i)+1} + 2b \leq w_{k'_p(i)} + w_{k'_p(i+1)} + 2b \text{ for all } a_l < a_i$$

implies $U_S(a_i, b|W'_i) \geq U_S(a_l, b|W'_i)$ for all $a_l < a_i$. Hence, the partition $\{W'_i\}_{i \in \mathcal{I}}$ is incentive-compatible. The payoff equivalence between the initial and the constructed equilibria follows straightforwardly. ■

Consider the sequence $\{\bar{a}_i\}_{i \in \mathcal{I}}$, such that $\bar{a}_i = \frac{1}{2} \sum_{k \in \mu(i)} \left(w_{k+1}^2 - w_k^2 \right) / \sum_{k \in \mu(i)} (w_{k+1} - w_k)$. Since $\mu(i)$ is convex-valued, it follows that

$$\bar{a}_i = \frac{1}{2} \sum_{k=k'(i)}^{k''(i)} (w_{k+1}^2 - w_k^2) / \sum_{k=k'(i)}^{k''(i)} (w_{k+1} - w_k) = \frac{w_{k'(i)} + w_{k''(i)+1}}{2} = \frac{w_{k'(i)} + w_{k'(i+1)}}{2}. \quad (14)$$

Lemma 7 *In any equilibrium* $(\{\sigma_{i,k}\}, \{a_i\}, \Omega)$*, we have* $\bar{a}_i \geq a_i$*,* $i \in \mathcal{I}$*.*

Proof For any $i \in \mathcal{I}$, if $\sigma_{i,k'_p(i)} = \sigma_{i,k''_p(i)} = 1$, then $\mu(i) = p(i)$ and

$$a_i = \frac{1}{2} \frac{\sum_{k \in p(i)} \sigma_{i,k} \left(w_{k+1}^2 - w_k^2 \right)}{\sum_{k \in p(i)} \sigma_{i,k} (w_{k+1} - w_k)} = \frac{1}{2} \frac{\sum_{k \in p(i)} \left(w_{k+1}^2 - w_k^2 \right)}{\sum_{k \in p(i)} (w_{k+1} - w_k)} = \frac{1}{2} \frac{\sum_{k \in \mu(i)} \left(w_{k+1}^2 - w_k^2 \right)}{\sum_{k \in \mu(i)} (w_{k+1} - w_k)} = \bar{a}_i.$$

If $\sigma_{i,k''_p(i)} < 1$, then property (d) of Lemma 6 implies $k''(i) = k''_p(i)$. If $\sigma_{i,k'_p(i)} < 1$, then by property (A) of Lemma 2, either $\sigma_{i+1,k'_p(i)} > 0$ or $\sigma_{i-1,k'_p(i)} > 0$. In the former case, condition (C) of the Lemma 2 implies $\sigma_{i,k} = 0$ for all $k > k'_p(i)$. Thus, $k''_p(i) = k'_p(i)$, and $p(i)$ is a singleton. Since $\mu(i)$ is non-empty and a subset of $p(i)$, we have $\mu(i) = p(i)$ and $a_i = \bar{a}_i$. If $\sigma_{i-1,k'_p(i)} > 0$, then condition (D) of Lemma 2 requires $\sigma_{i,k'_p(i)+1} > 0$. By condition (C) of Lemma 2, $\sigma_{l,k'_p(i)+1} = 0$ for all $l < i$. Thus, $j\left(k'_p(i) + 1\right) = i$ and $k'_p(i) + 1 \in \mu(i)$. Since $j(k)$ is increasing and $j\left(k'_p(i)\right) = i - 1$, there is no $k < k'_p(i) + 1$ such that $j(k) = i$. Therefore, $k'(i) = k'_p(i) + 1$. From (13), we obtain

$$a_i = \frac{1}{2} \frac{\sigma_{i,k'_p(i)}(w_{k'_p(i)+1}^2 - w_{k'_p(i)}^2) + \sum_{k=k'_p(i)+1}^{k''_p(i)-1} (w_{k+1}^2 - w_k^2) + \sigma_{i,k''_p(i)}(w_{k''_p(i)+1}^2 - w_{k''_p(i)}^2)}{\sigma_{i,k'_p(i)}(w_{k'_p(i)+1} - w_{k'_p(i)}) + \sum_{k=k'_p(i)+1}^{k''_p(i)} (w_{k+1} - w_k) + \sigma_{i,k''(i)}(w_{k''(i)+1} - w_{k''(i)})}.$$

The comparison of the last expression with (14) gives

$$\bar{a}_i = \frac{1}{2}\sum_{k=k'_p(i)+1}^{k''_p(i)} (w_{k+1}^2 - w_k^2) / \sum_{k=k'_p(i)+1}^{k''_p(i)} (w_{k+1} - w_k) = a_i(\sigma_{i,k'_p(i)} = 0, \sigma_{i,k''_p(i)} = 1).$$

Finally, we complete the proof by showing that a_i is decreasing in $\sigma_{i,k'_p(i)}$ and increasing in $\sigma_{i,k''_p(i)}$. A derivative of a_i with respect to $x = \sigma_{i,k'_p(i)}$ is

$$\frac{da_i}{dx} = \sum_{k=k'_p(i)+1}^{k''_p(i)} \sigma_{i,k} (w_{k+1} - w_k) (\bar{w}_{k'_p(i)} - \bar{w}_k) / \left(\sum_{k=k'_p(i)}^{k''_p(i)} \sigma_{i,k} (w_{k+1} - w_k) \right)^2 .$$

Since $\bar{w}_{k'_p(i)} - \bar{w}_k < 0$ for all $k > k'_p(i)$, then $\frac{da_i}{dx} < 0$. Similarly, for $\frac{da_i}{dy}$, where $y = \sigma_{i,k''_p(i)}$, we obtain $\frac{da_i}{dy} > 0$, which implies $\bar{a}_i \geq a_i$. ■

Proof of Lemma 5. Using property (A) of Lemma (2), we may represent the receiver's expected utility in an equilibrium $(\{a_i\}, \{\sigma_{i,k}\}, \Omega)$ as

$$U_R(\{a_i\}, \{\sigma_{i,k}\}, \Omega) = \sum_{k=0}^{n-1} P(W_k) \left(\sigma_{j(k),k} U_R\left(a_{j(k)}|W_k\right) + \sigma_{j(k)+1,k} U_R\left(a_{j(k)+1}|W_k\right)\right).$$

Modify the signaling strategy $\{\sigma_{i,k}\}$ as follows: derive all types $\bar{K}$ that induce two actions, and put $\sigma'_{j(k),k} = 1$ for all $k \in \bar{K}$. That is, if the sender of some type induced two actions in the initial equilibrium, now she purely induces a lower action.

Notice that $U_S\left(a_{j(k)}, b|W_k\right) = U_S\left(a_{j(k)+1}, b|W_k\right)$ for all $k \in \bar{K}$. The single-crossing property $\frac{d^2}{dadb} U_S(a, b|W_k) > 0$ implies

$$U_S\left(a_{j(k)+1}, b|W_k\right) - U_S\left(a_{j(k)}, b|W_k\right) > U_S\left(a_{j(k)+1}, 0|W_k\right) - U_S\left(a_{j(k)}, 0|W_k\right).$$

Since $U_S\left(a_{j(k)+1}, b|W_k\right) = U_S\left(a_{j(k)}, b|W_k\right)$ and $U_R(a|W_k) = U_S(a, 0|W_k)$, then $U_R\left(a_{j(k)}|W_k\right) > U_R\left(a_{j(k)+1}|W_k\right)$. Multiplying each term by $P(W_k)$ and summing across all $k \in K$ result in $U_R\left(\{a_i\}, \left\{\sigma'_{i,k}\right\}, \Omega\right) > U_R(\{a_i\}, \{\sigma_{i,k}\}, \Omega)$. Notice that $\{a_i\}$ is not the best-response to the signaling strategy $\left\{\sigma'_{i,k}\right\}$.

By construction, $\sigma'_{i,k} = 1$ if and only if $k \in \mu(i)$. Given the strategy $\left\{\sigma'_{i,k}\right\}$, the receiver's best response is $\{\bar{a}_i\}$, hence, $U_R(\bar{a}_i|m_i) \geq U_R(a_i|m_i)$, $i \in \mathcal{I}$. Multiplying each term by $P(m_i) = \sum_{k=0}^{n-1} P(W_k) \sigma'_{i,k} = w_{k'(i+1)} - w_{k'(i)}$ and summing across all $i \in \mathcal{I}$ result in $U_R\left(\{\bar{a}_i\}, \left\{\sigma'_{i,k}\right\}, \Omega\right) \geq U_R\left(\{a_i\}, \left\{\sigma'_{i,k}\right\}, \Omega\right)$.

Consider the partition $\bar{\Omega} = \left\{\bar{W}_i\right\}_{i \in \mathcal{I}}$ such that $\bar{W}_i = \bigcup_{k \in \mu(i)} W_k = (w_{k'(i)}, w_{k'(i+1)}]$, $i \in \mathcal{I}$, and the signaling strategy $\{\bar{\sigma}_{i,s}\}$, such that $m(i) = m_i$, $i \in \mathcal{I}$. A collapse of partition's elements does not affect the receiver's best-response, hence, it is $\{\bar{a}_i\}$. This implies $U_R\left(\{\bar{a}_i\}, \{\bar{\sigma}_{i,s}\}, \bar{\Omega}\right) = U_R\left(\{\bar{a}_i\}, \left\{\sigma'_{i,k}\right\}, \Omega\right)$, and

$$U_R\left(\{\bar{a}_i\}, \{\bar{\sigma}_{i,s}\}, \bar{\Omega}\right) \geq U_R\left(\{a_i\}, \left\{\sigma'_{i,k}\right\}, \Omega\right) > U_R(\{a_i\}, \{\sigma_{i,k}\}, \Omega).$$

We complete the proof by showing that $\{\bar{\sigma}_{i,s}\}$ is incentive-compatible. That is, $w_{k'(i+2)} - w_{k'(i)} \geq 4b$ for all $i = 0, ..., I-2$.

Since $\sigma_{i,k''(i)}$ belongs to the initial equilibrium profile for each $i \in \mathcal{I}$, we have $U_S\left(a_i, b|W_{k''(i)}\right) \geq$

$U_S\left(a_{i+1}, b|W_{k''(i)}\right)$. This implies

$$a_i + a_{i+1} \geq w_{k''(i)} + w_{k''(i)+1} + 2b = w_{k''(i)} + w_{k'(i+1)} + 2b.$$

It follows from Lemma 7 that $a_i \leq \bar{a}_i$ and $a_{i+1} \leq \bar{a}_{i+1}$. Combining these inequalities results in

$$w_{k'(i)} + w_{k'(i+1)} + 2b \leq w_{k''(i)} + w_{k'(i+1)} + 2b \ \leq a_i + a_{i+1} \leq \bar{a}_i + \bar{a}_{i+1}$$
$$= \frac{w_{k'(i)} + w_{k'(i+1)}}{2} + \frac{w_{k'(i+1)} + w_{k'(i+2)}}{2} = \frac{w_{k'(i)}}{2} + w_{k'(i+1)} + \frac{w_{k'(i+2)}}{2},$$

which gives $w_{k'(i+2)} - w_{k'(i)} \geq 4b$. ■

Lemma 8 *If the uniform partition of size n is incentive-compatible, then the incentive-compatible equilibrium under this partition is payoff superior to any incentive-compatible equilibrium under a partition of the same size.*

Proof The expected utility of the receiver in an incentive-compatible equilibrium is

$$U_R = -\sum_{k=0}^{n-1} \int_{w_k}^{w_{k+1}} (a_k - \theta)^2 d\theta = \sum_{k=0}^{n-1} P\left(W_k\right)\left(U_R\left(\bar{w}_k, \bar{w}_k\right) - D\left(W_k\right)\right) =$$
$$= -\sum_{k=0}^{n-1} P\left(W_k\right) D\left(W_k\right) = -\sum_{k=0}^{n-1} \frac{\Delta w_k^3}{12} = \sum_{k=0}^{n-1} f\left(\Delta w_k\right), \tag{15}$$

where $\Delta w_k = w_{k+1} - w_k > 0$ and $f\left(x\right) = -\frac{1}{12}x^3$.

Clearly, $f\left(x\right)$ is strictly concave for $x > 0$ and $\sum_{k=0}^{n-1} \Delta w_k = 1$. For the uniform partition of size n, we have $\Delta w'_k = \frac{1}{n}$ for all k. For any other partition of the same size, the Jensen's inequality results in

$$U_R = \sum_{k=0}^{n-1} f\left(\Delta w_k\right) < nf\left(\frac{1}{n}\sum_{k=0}^{n-1} \Delta w_k\right) = nf\left(\frac{1}{n}\right) = \sum_{k=0}^{n-1} f\left(\Delta w'_k\right) = U'_R.$$

■

Lemma 9 *If a partition of size n is incentive-compatible, then the uniform partition of size $n-1$ is incentive-compatible as well.*

Proof Since a partition $\{w_k\}_0^n$ is incentive-compatible, we have $w_n = 1 \geq w_{n-2} + 4b \geq ... \geq w_1 + \frac{n-1}{2}4b \geq \frac{n-1}{2}4b$ for odd n, and $w_n = 1 \geq w_{n-2} + 4b \geq ... \geq w_0 + \frac{n-1}{2}4b = \frac{n-1}{2}4b$ for even n. In both cases, we obtain $\frac{2}{n-1} \geq 4b$. Then, for the uniform partition $\{w'_k\}_0^{n-1}$, we have $w'_{j+2} - w'_j = \frac{j+2}{n-1} - \frac{j}{n-1} = \frac{2}{n-1} \geq 4b$. ■

Lemma 10 *Among all partitions of an odd size n such that $\frac{1}{2n} < b \leq \frac{1}{2(n-1)}$, the highest expected payoff in the incentive-compatible equilibrium is reached under the partition with all IC constraints (9) binding.*

Proof We prove the lemma using the Karamata's inequality.[27] Let sequences $\{x_k\}_1^n$ and $\{y_k\}_1^n$ be non-increasing, that is, $x_1 \geq x_2 \geq ... \geq x_n$ and $y_1 \geq y_2 \geq ... \geq y_n$. If all the following conditions

[27] See, for example, Hardy, Littlewood and Polya (1988).

satisfied: $x_1 \geq y_1$, $x_1 + x_2 \geq y_1 + y_2$, $x_1 + x_2 + x_3 \geq y_1 + y_2 + y_3$, ..., $x_1 + x_2 + ... + x_{n-1} \geq y_1 + y_2 + ... + y_{n-1}$, and $x_1 + x_2 + ... + x_n = y_1 + y_2 + ... + y_n$, then we say that $\{x_k\}_1^n$ **majorizes** $\{y_k\}_1^n$. The Karamata's inequality states that if $\{x_k\}_1^n$ majorizes $\{y_k\}_1^n$, and a function $f(x)$ is continuous and concave, then $\sum_{k=1}^{n} f(x_k) \leq \sum_{k=1}^{n} f(y_k)$.

From (15), the receiver's expected payoff in the incentive-compatible equilibrium is $U_R(\{w_k\}_0^n) = \sum_{k=0}^{n-1} f(\Delta w_k)$, where $\Delta w_k = w_{k+1} - w_k > 0$, and $f(x) = -\frac{1}{12}x^3$, which is continuous and strictly concave for $x > 0$.

Consider the partition $\{y_k\}_0^n$, for which the IC constraints are binding, hence, $y_k = 2kb$ for even k, and $y_k = 1 - 2b(n-k)$ for odd k. We need to show that if $\frac{1}{2n} < b \leq \frac{1}{2(n-1)}$, then $U_R(\{y_k\}_0^n) \geq U_R(\{w_k\}_0^n)$ for any partition $\{w_k\}_0^n$, which satisfies (9).

For the sequence $\{y_k\}_0^n$, we have $\Delta y_k = y_{k+1} - y_k = 1 - 2b(n-k-1) - 2bk = 1 - 2b(n-1)$ for even k. The condition $b < \frac{1}{2(n-1)}$ implies $\Delta y_k > 0$. Similarly, we have $\Delta y_k = 4b - \Delta y_{k-1} = 2b(n+1) - 1$ for odd k, and $b > \frac{1}{2n} > \frac{1}{2(n+1)}$ implies $\Delta y_k > 0$. In addition, for odd k, we obtain $\Delta y_k - \Delta y_{k-1} = 2(2bn-1) > 0$. Thus, by permuting $\{\Delta y_k\}_0^{n-1}$, we obtain a non-increasing sequence $\{Y_k\}_1^n = \{Y_1, Y_2, ..., Y_{\frac{n-1}{2}}, Y_{\frac{n+1}{2}}, ..., Y_n\}$, where $Y_k = 2b(n+1) - 1$ for $k \in S_1 = 1, 2, ..., \frac{n-1}{2}$, and $Y_k = 1 - 2b(n-1)$ for $k \in S_2 = \frac{n+1}{2}, ..., n$. Note that S_1 has one element less than S_2, since n is odd. Also, (9) implies $Y_k + Y_j = 4b$, $k \in S_1$, $j \in S_2$.

Now, consider a sequence $\{w_k\}_0^n$, which satisfies (9). We need to show that a non-increasing permutation $\{X_k\}_1^n$ of $\{\Delta w_k\}_0^{n-1}$ majorizes $\{Y_k\}_1^n$.

First, for even k, we have $w_k \geq w_{k-2} + 4b \geq ... \geq w_0 + \frac{k}{2}4b = 2kb = y_k$. Similarly, for odd k, we have $w_k \leq y_k$. Therefore, $\Delta w_k = w_{k+1} - w_k \geq y_{k+1} - y_k = \Delta y_k$ for odd k and $\Delta w_k \leq \Delta y_k$ for even k. Thus, a non-increasing permutation $\{X_k\}_1^n$ of $\{\Delta w_k\}_0^{n-1}$ can be represented as $\{X_k\}_1^n = \{X_1, X_2, ..., X_{\frac{n-1}{2}}, X_{\frac{n+1}{2}}, ..., X_n\}$, where $X_j \geq Y_k$ for all $j, k \in S_1$, and $X_j \leq Y_k$ for all $j, k \in S_2$. This means $\sum_{k \in S_1'} X_k \geq \sum_{k \in S_1'} Y_k$ for any $S_1' \subset S_1$ and $\sum_{k \in S_2'} X_k \leq \sum_{k \in S_2'} Y_k$ for any $S_2' \subset S_2$.

Also, the IC constraints require that for any $k \in \tilde{S}_2 = S_2 - \{n\} = \frac{n+1}{2}, ..., n-1$, there must exist $q(k) \in S_1$ such that $X_{q(k)} + X_k \geq 4b$, which we define as follows. Let i_n to be the index of the smallest element Δw_{i_n} of the sequence $\{\Delta w_k\}_0^{n-1}$, which implies $\Delta w_{i_n} = X_n$. Then, for all X_k, $k \in \tilde{S}_2$, if $X_k = \Delta w_i$, then $X_{q(k)} = \Delta w_{i+1}$ for $i < i_n$ and $X_{q(k)} = \Delta w_{i-1}$ for $i > i_n$. Note that $k \neq k'$ for $k, k' \in S_2$ implies $q(k) \neq q(k')$.

Clearly, $X_1 \geq Y_1$, $X_1 + X_2 \geq Y_1 + Y_2$, ..., $X_1 + ... + X_{\frac{n-1}{2}} \geq Y_1 + ... + Y_{\frac{n-1}{2}}$. Also, we obtain

$$X_1 + ... + X_{\frac{n-1}{2}} + X_{\frac{n+1}{2}} = \sum_{k \in S_1 - q\left(\frac{n+1}{2}\right)} X_k + X_{q\left(\frac{n+1}{2}\right)} + X_{\frac{n+1}{2}} \geq \sum_{k \in S_1 - q\left(\frac{n+1}{2}\right)} X_k + 4b$$

$$\geq \sum_{k \in S_1 - k\left(\frac{n+1}{2}\right)} Y_k + 4b = \sum_{k \in S_1 - q\left(\frac{n+1}{2}\right)} Y_k + Y_{q\left(\frac{n+1}{2}\right)} + Y_{\frac{n+1}{2}} = Y_1 + ... + Y_{\frac{n-1}{2}} + Y_{\frac{n+1}{2}}.$$

This argument can be reapplied iteratively for all $k \in \tilde{S}_2$. Since $\sum_{k=1}^{n} X_k = \sum_{k=1}^{n} Y_k = 1$, this completes the proof. ■

Proof of Theorem 1. We can rewrite $n(b)$ as follows: if $\frac{1}{2(c+1)} < b < \frac{1}{2(c-1)}$ for some odd c, then $n(b) = c$, otherwise, for $b = \frac{1}{2(c-1)}$, $n(b) = c - 1$. Then, by Lemma (9), the uniform partition of size $c - 1$ is incentive-compatible, and provides the expected payoff (in the incentive-compatible equilibrium)

$$U_R^{c-1} = -\sum_{k=0}^{c-2} \frac{(w_{k+1} - w_k)^3}{12} = -\sum_{k=0}^{c-2} \frac{1}{12(c-1)^3} = -\frac{1}{12(c-1)^2}.$$

From Lemma 8, this partition is payoff superior to all partitions of the same size. In addition, it is superior to partitions of a smaller size.

Now, consider two cases: $\frac{1}{2(c+1)} < b \leq \frac{1}{2c}$ and $\frac{1}{2c} < b \leq \frac{1}{2(c-1)}$. In the first case, the uniform partition of size c is incentive-compatible, thus, it is optimal and brings the expected payoff $U_R = -\frac{1}{12c^2}$. In the second case, Lemma (10) implies that among all partitions of size $c = n(b)$, the superior partition is that with the binding IC constraints (9). It provides the expected payoff

$$U_R^c = -\frac{1}{12}\left(4b^2\left(c^2-1\right)(4bc-3)+1\right). \tag{16}$$

For $b = \frac{1}{2c}$, we obtain $U_R^c = \frac{1}{12c^2}$, which is equal to the expected payoff under the uniform partition of size c. For $b = \frac{1}{2(c-1)}$, we obtain $U_R^c = \frac{1}{3(c-1)^2} = \frac{1}{12\left(\frac{c-1}{2}\right)^2}$, which is equal to the expected payoff under the uniform partition of size $\frac{c-1}{2}$.

Notice that $n(b) = c$ for all $b \in \left(\frac{1}{2c}, \frac{1}{2(c-1)}\right)$. Then, the derivative of (16) with respect to b is $\frac{d}{db}U_R^c(b) = -2b\left(c^2-1\right)(2bc-1)$, which is negative for $b > \frac{1}{2c}$. Finally, $U_R^c\left(\frac{1}{2c}\right) = \frac{1}{12c^2} > U_R^{c-1} = \frac{1}{12(c-1)^2} > \frac{1}{3(c-1)^2} = U_R^c\left(\frac{1}{2(c-1)}\right)$ implies that there exists a unique $b^* \in (\frac{1}{2c}, \frac{1}{2(c-1)})$, such that $U_R^c(b^*) = U_R^{c-1}$. ■

Proof of Corollary 1. Formally, it is straightforward to prove that for any equilibrium partition in the CS model, the uniform partition of the same size is incentive-compatible in the CI model and provides a superior expected payoff. However, Theorem 1 below proves that for $b \leq \frac{1}{4}$, there exists an equilibrium in the CI model, which provides a higher expected payoff to the principal than optimal delegation. Due to Dessein (2002), delegation performs better than CS communication for $b \leq \frac{1}{4}$, which completes the proof. ■

Proof of Theorem 1. Informative communication is feasible, if $b \leq \frac{1}{4}$. Melumad and Shibano (1991) prove that for $b \leq \frac{1}{2}$, the optimal delegation set is the interval $[0, 1-b]$. In this case, the expert's actions are

$$a^S(\theta) = \begin{cases} \theta + b \text{ if } \theta \leq 1-2b \\ 1-b \text{ if } \theta > 1-2b \end{cases}, \tag{17}$$

which results in

$$U_R^D(b) = \int_0^1 U_R\left(a^S(\theta), \theta\right) d\theta = -\int_0^{1-2b} (\theta+b-\theta)^2 d\theta - \int_{1-2b}^1 (1-b-\theta)^2 d\theta = -b^2 + \frac{4}{3}b^3. \tag{18}$$

By Lemma (9), a uniform partition of size $n(b) - 1 = 2\lfloor\frac{1}{4b}\rfloor$ is incentive-compatible and leads to $U_R(b) = -\frac{1}{12\times\left(2\lfloor\frac{1}{4b}\rfloor\right)^2} = -\frac{1}{48\times\lfloor\frac{1}{4b}\rfloor^2}$. Since $\lfloor\frac{1}{4b}\rfloor \geq \frac{1}{4b} - 1$, we have $U_R(b) \geq -\frac{1}{48\left(\frac{1}{4b}-1\right)^2} = -\frac{b^2}{3(1-4b)^2}$, and

$$U_R(b) - U_R^D(b) \geq -\frac{b^2}{3(1-4b)^2} + b^2 - \frac{4}{3}b^3 = \frac{2}{3}b^2\frac{1-14b+40b^2-32b^3}{(1-4b)^2}.$$

The function $A(b) = 1 - 14b + 40b^2 - 32b^3$ has three roots. Only one of them, namely, $b_0 = \frac{1}{8}\left(3-\sqrt{5}\right) \simeq \frac{1}{11}$ is in the interval $\left[0, \frac{1}{4}\right]$. Since $A(0) = 1$, it follows that $U_R - U_R^D > 0$ for all $b < b_0$. For $b \in [b_0, \frac{1}{4}]$, consider three cases. If $b \in [\frac{1}{6}, \frac{1}{4}]$, then the uniform partition of size 2 is incentive-compatible, and provides the expected payoff $U_R = -\frac{1}{48}$. Then, $D(b) = U_R(b) - U_R^D(b) = -\frac{1}{48} + b^2 - \frac{4}{3}b^3$. Since $D'(b) > 0$ for $b < \frac{1}{2}$, and $D\left(\frac{1}{6}\right) = \frac{1}{1296}$, this implies $U_R(b) - U_R^D(b) > 0$ for all $b \in [b_0, \frac{1}{4}]$. For $b \in [\frac{1}{8}, \frac{1}{6})$, the uniform three-element partition is incentive-compatible, and brings the expected payoff $-\frac{1}{108}$. Then, $D(b) = U_R(b) - U_R^D(b) = -\frac{1}{108} + b^2 - \frac{4}{3}b^3 > 0$ for all $b \in [\frac{1}{8}, \frac{1}{6})$, since $D\left(\frac{1}{8}\right) = \frac{13}{3456}$. Finally, for $b \in [b_0, \frac{1}{8})$, the uniform 4-element partition is incentive-compatible, which results in payoff $U_R = -\frac{1}{192}$. Using the same technique as for $b \geq \frac{1}{6}$, it gives $D(b) > D\left(\frac{1}{12}\right) = \frac{5}{5184}$, which completes the proof. ■

Proof of Theorem 2. For symmetric preferences, the CS arbitrage condition and the IC constraints (9) in the CI model are the same as in the case of the quadratic preferences. Hence, for any $b \leq \frac{1}{4}$, the most informative equilibrium in the CS-model has a partition of size $N^{CS}(b) = \lceil -\frac{1}{2} + \frac{1}{2}(1+\frac{2}{b})^{1/2} \rceil$, where $\lceil x \rceil$ is the smaller integer greater than or equal to x. Notice that

$$N^{CS}(b) = \lceil -\frac{1}{2} + \frac{1}{2}(1+\frac{2}{b})^{1/2} \rceil \leq \lfloor \frac{1}{2} + \frac{1}{2}(1+\frac{2}{b})^{1/2} \rfloor \leq \frac{1}{2} + \frac{1}{2}(1+\frac{2}{b})^{1/2},$$

where $\lfloor x \rfloor$ is the largest integer smaller than or equal to x. Then, $2N^{CS}(b)\, b \leq 2b(\frac{1}{2} + \frac{1}{2}(1+\frac{2}{b})^{1/2}) = b(1+(1+\frac{2}{b})^{1/2}) = v(b)$. Since $v(\frac{1}{4}) = 1$, and $v'(b) = 1 + \frac{1+b}{\sqrt{b(2+b)}} > 0$, then $2N^{CS}(b)\, b < 1$ and the uniform partition of size $n = N^{CS}(b)$ is incentive-compatible.

The receiver's expected utility in the most informative CS equilibrium is

$$U_R^{CS} = \sum_{k=0}^{n-1} \int_{w_k}^{w_{k+1}} U\left(\left|\frac{w_k + w_{k+1}}{2} - \theta\right|\right) d\theta = 2\sum_{k=0}^{n-1} \int_0^{\frac{w_{k+1}-w_k}{2}} U(t)\, dt = \sum_{k=0}^{n-1} f(\Delta w_k),$$

where $\Delta w_k = w_{k+1} - w_k$, and $f(\Delta w_k) = 2 \int_0^{\frac{\Delta w_k}{2}} U(t)\, dt$. Then, for $x > 0$, we have $f'(x) = \frac{d}{dx} \int_0^{\frac{x}{2}} 2U(t)\, dt = U\left(\frac{x}{2}\right)$, and $f''(x) = \frac{1}{2}U'\left(\frac{x}{2}\right) < 0$. The receiver's ex-ante utility in the CI model under the uniform partition of size n is

$$\begin{aligned} U_R &= \sum_{k=0}^{n-1} \int_{w_k}^{w_{k+1}} U\left(\left|\frac{w_k + w_{k+1}}{2} - \theta\right|\right) d\theta = 2\sum_{k=0}^{n-1} \int_0^{\frac{w_{k+1}-w_k}{2}} U(t)\, dt \\ &= 2n \int_0^{\frac{1}{2n}} U(t)\, dt = nf\left(\frac{1}{n}\right) = nf\left(\frac{1}{n}\sum_{k=0}^{n-1} \Delta w_k\right). \end{aligned} \tag{19}$$

Since $f(x)$ is strictly concave and $\Delta w_{k+1} = \Delta w_k + 4b \neq \Delta w_k$ from the CS arbitrage condition, then the Jensen's inequality implies $f\left(\frac{1}{n}\sum_{k=0}^{n-1} \Delta w_k\right) > \frac{1}{n}\sum_{k=0}^{n-1} f(\Delta w_k)$ or $U_R > U_R^{CS}$. ■

Proof of Theorem 3. If preferences are symmetric, we use Proposition 4 from Alonso and Matouschek (2005), which states that the optimal delegation set is the same as for quadratic preferences, hence, it is the interval $[0, 1-b]$. Similarly, the sender's policy is determined by (17). This results in the receiver's ex-ante utility

$$\begin{aligned} U_R^D &= \int_0^1 U\left(\left|a^S(\theta) - \theta\right|\right) d\theta = \int_0^{1-2b} U(b)\, d\theta + \int_{1-2b}^1 U(|1-b-\theta|)\, d\theta \\ &= U(b)(1-2b) + 2\int_0^b U(\theta)\, d\theta. \end{aligned}$$

Now, consider the CI model. If $b \neq \frac{1}{2n}$ for any integer n, then the partition of size $n(b) = 2\lfloor \frac{1}{4b} \rfloor + 1$ is incentive-compatible. From Lemma 9, the uniform partition of size $c = n(b) - 1 = 2\lfloor \frac{1}{4b} \rfloor \geq \frac{1}{2b} - 1$ is incentive-compatible as well. If $b = \frac{1}{2n}$ for some integer n, then the uniform partition of size $n = \frac{1}{2b}$ is incentive-compatible, and so is the uniform partition of size $\frac{1}{2b} - 1$. From (19), the receiver's ex-ante utility under the uniform partition of size c is

$$U_R(c) = 2c \int_0^{\frac{1}{2c}} U(\theta)\, d\theta = E\left[U(\theta) | \theta < \frac{1}{2c}\right].$$

Since $U(.)$ is decreasing, it follows that U_R is increasing in c. Then,

$$U_R(c) \geq U_R\left(\frac{1}{2b} - 1\right) = 2\left(\frac{1}{2b} - 1\right) \int_0^{\frac{1}{2\left(\frac{1}{2b}-1\right)}} U(\theta)\, d\theta = +\frac{1-2b}{b} \int_0^{\frac{b}{1-2b}} U(\theta)\, d\theta.$$

Thus, $\left(U_R - U_R^D\right) \frac{b}{1-2b} \geq \int_0^{\frac{b}{1-2b}} U(\theta)\, d\theta - U(b)\, b - \frac{2b}{1-2b} \int_0^{b} U(\theta)\, d\theta = \phi(b)$. Clearly, $\phi(0) = 0$. Taking a derivative of $\phi(b)$ with respect to b gives

$$\phi'(b) = U\left(\frac{b}{1-2b}\right) \frac{1}{(1-2b)^2} - U'(b)\, b - U(b) - \frac{2}{(1-2b)^2} \int_0^{b} U(\theta)\, d\theta - \frac{2b}{1-2b} U(b).$$

From the last expression, $\phi'(0) = 0$. Taking the second derivative results in $\phi''(0) = -U'(0) \geq 0$. If $U'(0) < 0$, then by Taylor's formula $\phi(b) = \phi(0) + \phi'(0)\, b + \frac{1}{2}\phi''\left(\tilde{b}\right) b^2 = \frac{1}{2}\phi''\left(\tilde{b}\right) b^2$, where $\tilde{b} \in [0, b]$. Since $\phi''(0) > 0$ and $\phi''(b)$ is continuous, then there exists b^* such that $\phi''(b) > 0$, and hence, $\phi(b) > 0$ for all $b \in (0, b^*)$. If $U'(0) = 0$, then $\phi''(0) = 0$. Taking the third derivative gives $\phi'''(0) = -2U''(0) > 0$. By Taylor's formula, $\phi(b) = \phi(0) + \phi'(0)\, b + \frac{1}{2}\phi''(0)\, b^2 + \frac{1}{6}\phi'''(b^*)\, b^3 = \frac{1}{6}\phi'''(b^*)\, b^3$, where $b^* \in [0, b]$. Since $\phi'''(0) > 0$ and $\phi'''(b)$ is continuous, then $\phi(b) > 0$ for all b in the neighborhood of 0. ■

Proof of Theorem 4. The arbitrage condition in the CS model is

$$w_{k+1} + b - a_k = a_{k+1} - w_{k+1} - b, \tag{20}$$

where

$$a_k = E[\theta | \theta \in (w_k, w_{k+1}]] = \frac{1}{F(w_{k+1}) - F(w_k)} \int_{w_k}^{w_{k+1}} \theta dF(\theta). \tag{21}$$

In the CI model, the sender's type-relevant utility function is

$$U_S(a, b | W_k) = -\frac{1}{F(\theta_{k+1}) - F(\theta_k)} \int_{w_k}^{w_{k+1}} (a - b - \theta)^2\, dF(\theta).$$

This function is concave and symmetric with respect to $a_k^S = a_k + b$. Thus, the IC constraints $a_k^S - a_k \geq a_{k+1} - a_k^S$ can be written as

$$a_{k+1} - a_k \geq 2b,\ k = 0, ..., n-2. \tag{22}$$

The condition (20) can be expressed as $a_{k+1} - a_k = 2(w_{k+1} - a_k) + 2b > 2b$, since $w_{k+1} > a_k =$

$E[\theta|\theta \in (w_k, w_{k+1}]]$ for $f(\theta) > 0$. Thus, any CS partition $\{w_k\}_{k=0}^{n}$ is incentive-compatible in the CI model. Moreover, the IC constraints (22) are satisfied for all w_k' in some neighborhood of w_k, $k = 1, ..., n-1$, since a_k, $k = 0, ..., n-1$, are continuous in all w_k.

The receiver's ex-ante utility in the incentive-compatible equilibrium is

$$U_R = -\sum_{k=0}^{n-1} \int_{w_k}^{w_{k+1}} (a_k - \theta)^2 \, dF(\theta). \tag{23}$$

Then,

$$\frac{dU_R}{dw_1} = -(a_0 - w_1)^2 f(w_1) - \frac{da_0}{dw_1} \int_0^{w_1} (a_0 - \theta) \, dF(\theta) + (a_1 - w_1)^2 f(w_1) - \frac{da_1}{dw_1} \int_{w_1}^{w_2} (a_1 - \theta) \, dF(\theta).$$

From (21), the second and the last terms in the expression above are equal to 0, which implies

$$\frac{dU_R}{dw_1} = f(w_1)(a_1 - a_0)(a_0 + a_1 - 2w_1) = f(w_1)(a_1 - a_0)\, 2b > 0.$$

Thus, the partition $(0, w_1', w_2, ..., 1)$, where $w_1' \downarrow w_1$, is incentive-compatible and provides a higher expected payoff.[28] ■

Proof of Theorem 5. If $\Delta_k = w_{k+1} - w_k$ is the length of a partition's element W_k, then the receiver's optimal action (21) can be represented by Taylor's formula around w_k as

$$a_k = w_k + \frac{1}{2}\Delta_k + \frac{1}{12}\frac{f'(\tilde{w}_k)}{f(\tilde{w}_k)}\Delta_k^2, \tag{24}$$

where $\tilde{w}_k \in [w_k, w_{k+1}]$. Similarly, $a_{k-1} = w_k - \frac{1}{2}\Delta_{k-1} + \frac{1}{12}\frac{f'(\tilde{w}_{k-1})}{f(\tilde{w}_{k-1})}\Delta_{k-1}^2$, where $\tilde{w}_{k-1} \in [w_{k-1}, w_k]$. Then, the IC constraints (22) become

$$\Delta_{k-1} + \Delta_k + \frac{1}{6}\frac{f'(\tilde{w}_k)}{f(\tilde{w}_k)}\Delta_k^2 - \frac{1}{6}\frac{f'(\tilde{w}_{k-1})}{f(\tilde{w}_{k-1})}\Delta_{k-1}^2 \geq 4b. \tag{25}$$

Similarly, expanding the density $f(\theta)$ by Taylor's formula around w_k results in

$$f(\theta) = f(w_k) + f'(\hat{w}_k)(\theta - w_k), \tag{26}$$

where $\hat{w}_k \in [0, \theta]$. Using (24) and (26), the sum's element $U_R^k = -\int_{w_k}^{w_{k+1}} (a_k - \theta)^2 f(\theta)\, d\theta$ in the principal's ex-ante utility (23) can be estimated as

$$U_R^k = -\frac{1}{12} f(w_k)\,\Delta_k^3 + O\left(\Delta_k^4\right), \tag{27}$$

where $O\left(\Delta_k^4\right)$ has an order Δ_k^4. Then, taking the length of the uniform partition's element $\Delta_k = cb$, where $c \in \left(2, 2\sqrt{3}\right)$ is chosen to satisfy $cbN = 1$ for some integer N, transforms (25) into

$$(2c - 4)\, b + b^2 \left(\frac{1}{6}\frac{f'(\tilde{w}_k)}{f(\tilde{w}_k)}c^2 - \frac{1}{6}\frac{f'(\tilde{w}_{k-1})}{f(\tilde{w}_{k-1})}c^2\right) \geq 0,$$

[28] This argument can be reapplied to all boundary points w_k, $0 < k < n-1$.

which is satisfied for a sufficiently small b. Also, (27) can be written as

$$U_R^k = -\frac{1}{12} f(w_k) c^3 b^3 + O(b^4).$$

The principal's ex-ante utility in the case of complete delegation is

$$U_R^D = -b^2 = \sum_{k=0}^{N-1} U^k,$$

where

$$U^k = -b^2 (F(w_{k+1}) - F(w_k)) = -b^2 \left(f(w_k) \Delta_k + O\left(\Delta_k^2\right)\right) = -f(w_k) cb^3 + O\left(b^4\right).$$

This implies that for sufficiently small b,

$$U_R^k - U^k = f(w_k)\left(c - \frac{c^3}{12}\right) b^3 + O\left(b^4\right) = f(w_k)\left(1 - \frac{c^2}{12}\right) cb^3 + O\left(b^4\right) > 0,$$

and summing across all $k = 0, ..., N-1$ results in $U_R > U_R^D$. ■

Proof of Theorem 6. If the sender knows that $\theta \in W$, where $P(W) = \int_W dF(\theta) > 0$, then she implements an action

$$a' = \arg\max_a - \int_W (a - b - \theta)^2 dF(\theta) = E[\theta|W] + b.$$

The expected utility of the principal, given that $\theta \in W$, is

$$U_R(W) = -\frac{1}{P(W)} \int_W (a' - \theta)^2 dF(\theta).$$

By Jensen's inequality,

$$U_R(W) = E_\theta\left[(a' - \theta)^2 |W\right] < (a' - E[\theta|W])^2 = (\theta + b - \theta)^2 = b^2,$$

where the right part is the expected utility of the principal in the case of the perfectly informed expert. To complete the proof, it is sufficient to take an average payoff across all sets W. ■

For the information structure $\Omega = \{W_k\}_{k=0}^{n-1}$ and the delegation set $\mathcal{A}$, the incentive-compatible choice of a sender of the type W_k to make an action $a_i \in \mathcal{A}$ is determined by (11).

Note first that for a finite set of types, the set of actions is also finite, since the expert of any type makes no more than two actions because of the concave preferences. Second, to find the optimal delegation set, we can restrict the expert's action rule $\sigma : \Omega \to \Delta\mathcal{A}$ to pure strategies only, since for any mixed-strategy action rule there exists the pure-strategy rule, which provides the higher payoff to the principal. Namely, if W_k-type is indifferent between actions a' and a'', that is, $U_S(a'', b|W_k) = U_S(a', b|W_k)$, her new action rule is to induce a lower action. Then, the single-crossing property $\frac{d^2}{dadb} U_S(a, b|W_k) > 0$ implies

$$U_S(a'', b|W_k) - U_S(a', b|W_k) > U_S(a'', 0|W_k) - U_S(a', 0|W_k) = U_R(a''|W_k) - U_R(a'|W_k),$$

which results in $U_R(a'|W_k) > U_R(a''|W_k)$. Reapplying this argument to all mixing types, we obtain the pure-strategy rule, which is payoff superior to the initial one.

Thus, the cardinality of the delegation set in optimal delegation does not exceed that of the type set. In addition, if the number of types strictly exceeds that of actions, then Lemma 4 implies that the principal can collapse all types that induce identical actions, without affecting the sender's choice for the modified types. Hence, in the optimal delegation, the number of types coincides with that of actions.

Then, property (C) of Lemma 2 guarantees that induced actions are monotone in types, that is, the delegation set $\{a_k\}_{k=0}^{n-1} = \{a(W_k)\}_{k=0}^{n-1}$ is a strictly increasing sequence. Thus, the incentive-compatibility constraints (11) can be written as

$$a_k + a_{k+1} \geq w_k + w_{k+1} + 2b \text{ and } a_k + a_{k-1} \leq w_k + w_{k+1} + 2b, \ \forall k. \tag{28}$$

Now, consider two sequences, $\{\Delta_k\}_{k=0}^{n-1} = \{a_k - w_k\}_{k=0}^{n-1}$ and $\{\xi_k\}_{k=0}^{n-1} = \{w_{k+1} - a_k\}_{k=0}^{n-1}$. Note that $w_{k+1} - w_k > 0$ results in

$$\Delta_k + \xi_k > 0, \tag{29}$$

and $a_{k+1} - a_k > 0$ results in

$$\Delta_{k+1} + \xi_k > 0. \tag{30}$$

In addition, the condition $\sum\limits_{k=0}^{n-1}(w_{k+1} - w_k) = 1$ implies

$$\sum_{k=0}^{n-1}\Delta_k + \sum_{k=0}^{n-1}\xi_k = 1. \tag{31}$$

Conversely, any sequences, that satisfy the properties above, determine the information structure $\{w_k\}_{k=0}^{n-1}$ and the delegation set $\{a_k\}_{k=0}^{n-1}$ as $w_{k+1} = w_k + \Delta_k + \xi_k$ and $a_k = w_k + \Delta_k$, where $w_0 = 0$.

Then, we can represent (28) as

$$\Delta_k + \Delta_{k+1} \geq 2b, \text{ and} \tag{32}$$

$$\xi_{k-1} + \xi_k + 2b \geq 0. \tag{33}$$

Similarly, the expected utility of the principal can be expressed as

$$U_R = -\sum_{k=0}^{n-1}\int_{w_k}^{w_{k+1}}(a_k - \theta)^2 d\theta = -\frac{1}{3}\sum_{k=0}^{n-1}\left[(a_k - w_k)^3 + (w_{k+1} - a_k)^3\right] = -\frac{1}{3}\sum_{k=0}^{n-1}\left(\Delta_k^3 + \xi_k^3\right).$$

Proof of Theorem 7. By Theorem 1, it is sufficient to show that there exists the information structure and the delegation set, which provide the superior payoff for $b \in \left(\frac{1}{4}, \frac{1}{2}\right)$.

In the case of the perfectly informed expert, the optimal delegation set $[0, 1-b]$ provides the payoff $U_R^{PI} = -b^3 + \frac{4}{3}b^3$.

Consider the two-element information structure and the delegation set, such that $\Delta_0 = \Delta_1 = b$ and $\xi_0 = \xi_1 = \frac{1}{2} - b$. The sequences $\{\Delta_0, \Delta_1\}$ and $\{\xi_0, \xi_0\}$ satisfy (29)-(33) and provide the expected payoff

$$U_R = -\frac{1}{3}\left[2b^3 + 2\left(\frac{1-2b}{2}\right)^3\right] = -b^2 + \frac{1}{2}b - \frac{1}{12}.$$

Then, $U_R - U_R^{PI} = \frac{1}{2}b - \frac{4}{3}b^3 - \frac{1}{12} = \frac{1}{12}(1-2b)\left(8b^2 + 4b - 1\right)$. The last term is increasing in b and is equal to $\frac{1}{2}$ for $b = \frac{1}{4}$. Thus, $U_R - U_R^{PI} > 0$ for $b \in \left(\frac{1}{4}, \frac{1}{2}\right)$. ■

The optimal communication structure and the delegation set are the solution to the problem:

$$\max_{\{\Delta_k\}_{k=0}^{n-1},\{\xi_k\}_{k=0}^{n-1}} U_R\left(\{\Delta_k\}_{k=0}^{n-1},\{\xi_k\}_{k=0}^{n-1}\right)=\max_{\{\Delta_k\}_{k=0}^{n-1},\{\xi_k\}_{k=0}^{n-1}} -\frac{1}{3}\sum_{k=0}^{n-1}\left(\Delta_k^3+\xi_k^3\right), \tag{34}$$

given constraints (29)-(33).

Proof of Theorem 8. First, we prove the following results about the optimal information structure and the delegation set.

a) Constraints (33) are never binding. By contradiction, let $\xi_k+\xi_{k+1}+2b=0$ for some k. First, $\Delta_{k+1}+\xi_k>0$, $\Delta_{k+1}+\xi_{k+1}>0$ and $\xi_k+\xi_{k+1}=-2b<0$ imply that at least one of ξ_k, ξ_{k+1} is negative and $\Delta_{k+1}>|\xi_k|$, $\Delta_{k+1}>|\xi_{k+1}|$.

If $\xi_{k+1}<0$, put $\tilde{\Delta}_{k+1}=\Delta_{k+1}-\delta$, $\tilde{\xi}_{k+1}=\xi_{k+1}+\delta$, where $\delta\downarrow 0$. This results in the higher expected payoff of the principal, since

$$\begin{aligned}3\Delta U&=3U_R\left(\left\{\tilde{\Delta}_k\right\}_{k=0}^{n-1},\left\{\tilde{\xi}_k\right\}_{k=0}^{n-1}\right)-3U_R\left(\{\Delta_k\}_{k=0}^{n-1},\{\xi_k\}_{k=0}^{n-1}\right)\\&=-\tilde{\Delta}_{k+1}^3-\tilde{\xi}_{k+1}^3+\Delta_{k+1}^3+\xi_{k+1}^3=-\left(\Delta_{k+1}-\delta\right)^3-\left(\xi_{k+1}+\delta\right)^3+\Delta_{k+1}^3+\xi_{k+1}^3\\&=3\delta\left(\Delta_{k+1}^2-\xi_{k+1}^2-\delta\Delta_{k+1}-\delta\xi_{k+1}\right)>0.\end{aligned}$$

Since $\tilde{\xi}_{k+1}>\xi_{k+1}$ and $\tilde{\xi}_{k+1}+\tilde{\Delta}_{k+1}=\xi_{k+1}+\Delta_{k+1}$, the constraints (31) and (33) hold.

If $\xi_k<0$, then put $\tilde{\Delta}_{k+1}=\Delta_{k+1}-\delta$, $\tilde{\xi}_k=\xi_k+\delta$, where $\delta\downarrow 0$. This results in

$$3\Delta U=-\tilde{\Delta}_{k+1}^3-\tilde{\xi}_k^3+\Delta_{k+1}^3+\xi_k^3=3\delta\left(\Delta_{k+1}^2-\xi_k^2-\delta\Delta_{k+1}-\delta\xi_k\right)>0.$$

Then, $\tilde{\xi}_k>\xi_k$ and $\tilde{\xi}_k+\tilde{\Delta}_{k+1}=\xi_k+\Delta_{k+1}$ mean that (31) and (33) hold.

Also, $\Delta_k+\Delta_{k+1}>2b$. If $\Delta_k+\Delta_{k+1}=2b$, then $\xi_k+\xi_{k+1}+\Delta_k+\Delta_{k+1}=(\xi_k+\Delta_k)+(\xi_{k+1}+\Delta_{k+1})=0$, which contradicts (29). In addition, $\Delta_{k+2}+\xi_{k+1}>0$, $\Delta_{k+1}+\xi_k>0$, and $\xi_k+\xi_{k+1}+2b=0$ lead to $\Delta_{k+2}+\Delta_{k+1}+\xi_k+\xi_{k+1}+2b>2b$ or $\Delta_{k+2}+\Delta_{k+1}>2b$. Thus, (29), (32), and (30) are not affected by small perturbations in Δ_{k+1}, ξ_k, and ξ_{k+1}.

b) $\Delta_k\geq 0$, $\forall k$. By contradiction, let $\Delta_k<0$ for some k. Then, $\Delta_k+\xi_k>0$ implies $\xi_k>|\Delta_k|$. Put $\tilde{\Delta}_k=\Delta_k+\delta$, $\tilde{\xi}_k=\xi_k-\delta$, where $\delta\downarrow 0$. Then,

$$3\Delta U=-\tilde{\Delta}_k^3-\tilde{\xi}_k^3+\Delta_k^3+\xi_k^3=\delta\left(\xi_k^2-\Delta_k^2-\delta\Delta_k-\delta\xi_k\right)>0.$$

c) $\xi_k=\xi\geq 0$, $\forall k$. First, if there is ξ_j such that $|\xi_j|\neq|\xi_k|$, put $\tilde{\xi}_k=\xi_k+\delta$ and $\tilde{\xi}_j=\xi_j-\delta$, where $\delta\downarrow 0$ if $|\xi_j|>|\xi_k|$, and $\delta\uparrow 0$ if $|\xi_j|<|\xi_k|$. Then,

$$3\Delta U=-\tilde{\xi}_k^3-\tilde{\xi}_j^3+\xi_k^3+\xi_j^3=\delta\left(\xi_j^2-\xi_k^2-\delta\xi_j-\delta\xi_k\right)>0,$$

Thus, we must have $|\xi_k|=\xi$, $\forall k$.

If $\xi_k=-\xi<0$ for all k, then consider the sequences $\left\{\tilde{\Delta}_k\right\}_{k=0}^{n-2},\left\{\tilde{\xi}_k\right\}_{k=0}^{n-2}$, which are constructed from the initial ones by eliminating the elements Δ_s and ξ_s, where $\Delta_s=\min\limits_k\{\Delta_k\}$, and putting $\tilde{\xi}_j=\xi_j+\Delta_s+\xi_s=-2\xi+\Delta_s$ for some $j\neq s$. Since $\Delta_s+\xi_s>0$, it follows that $\tilde{\xi}_j>\xi_j$. This implies that (29) and (33) hold. Also, $\Delta_{s+1}\geq\Delta_s$ results in $\Delta_{s+1}+\tilde{\xi}_{s-1}>\Delta_s+\xi_{s-1}>0$ and $\Delta_{s-1}+\Delta_{s+1}\geq\Delta_{s-1}+\Delta_s\geq 2b$. Thus, (30) and (32) hold. Finally, (31) holds by construction. Then,

$$3\Delta U=-\tilde{\xi}_j^3+\Delta_s^3+\xi_s^3+\xi_j^3=-\left(-2\xi+\Delta_s\right)^3+\Delta_s^3-2\xi^3=6\xi\left(\Delta_s+\xi\right)^2>0.$$

Thus, there must exist $\xi_j = \xi > 0$. Moreover, if $\xi_j > 0$ and $\xi_k < 0$, there must be the type i between j and k, such that either $\xi_i = \xi = -\xi_{i+1}$ or $\xi_i = -\xi = -\xi_{i+1}$.

If $\Delta_{i+1} > \Delta_i$, consider the sequences $\left\{\tilde{\Delta}_k\right\}_{k=0}^{n-2}, \left\{\tilde{\xi}_k\right\}_{k=0}^{n-2}$, constructed from the initial ones by eliminating the elements Δ_i and ξ_i, and putting $\tilde{\xi}_{i+1} = \xi_{i+1}+\Delta_i + \xi_i = \Delta_i$. Since $\Delta_i + \xi_i > 0$, it follows that $\tilde{\xi}_{i+1} > \xi_{i+1}$. This implies that (29) holds. Also, $\xi_{i-1}+\tilde{\xi}_{i+1}+2b = \xi_{i-1}+\Delta_i+2b \geq 2b > 0$, which means that (33) holds. In addition, we have $\Delta_{i+2} + \tilde{\xi}_{i+1} = \Delta_{i+2} + \Delta_i > 0$, $\Delta_{i+1} + \xi_{i-1} > \Delta_i + \xi_{i-1} > 0$, and $\Delta_{i-1} + \Delta_{i+1} > \Delta_{i-1} + \Delta_i \geq 2b$. Thus, (30) and (32) hold. Finally, (31) holds by construction. Thus, we obtain

$$3\Delta U = -\tilde{\xi}_{i-1}^3 + \Delta_i^3 + \xi_i^3 + \xi_{i-1}^3 = -\Delta_i^3 + \Delta_i^3 - \xi^3 + \xi^3 = 0.$$

If $\xi_{i+1} = \xi$, then $\Delta_i + \xi_i > 0$ implies $\tilde{\xi}_{i+1} > \xi$, which contradicts the condition $|\xi_k| = \xi$, $\forall k$. If $\xi_{i+1} = -\xi$, $\xi_i = \xi$, and $\Delta_i > \xi$, we again obtain $\tilde{\xi}_{i+1} > \xi$. If $\Delta_i = \xi$, then $\Delta_{i+2} + \xi_{i+1} > 0$ requires $\Delta_{i+2} > \xi$, or $\Delta_{i+2} > \Delta_i$. Then, the conditions $\Delta_{i+1} + \Delta_i \geq 2b$, $\Delta_{i+1} + \Delta_{i+2} \geq 2b$, and $\Delta_{i+2} > \Delta_i$ imply that $\Delta_{i+1} + \Delta_{i+2} > 2b$. Finally, modifying the sequences $\left\{\tilde{\Delta}_k\right\}_{k=0}^{n-2}, \left\{\tilde{\xi}_k\right\}_{k=0}^{n-2}$ by putting $\Delta'_{i+1} = \Delta_{i+1} - \delta$, $\xi'_{i+1} = \tilde{\xi}_{i+1} + \delta = \Delta_i + \delta$, where $\delta \downarrow 0$, and using the fact that $\Delta_{i+1} > \Delta_i$, we obtain the sequences $\{\Delta'_k\}_{k=0}^{n-2}, \{\xi'_k\}_{k=0}^{n-2}$ that satisfy all the above constraints and provide the higher payoff to the principal.

Similarly, if $\Delta_{i+1} \leq \Delta_i$, consider the sequences $\left\{\tilde{\Delta}_k\right\}_{k=0}^{n-2}, \left\{\tilde{\xi}_k\right\}_{k=0}^{n-2}$, constructed from the initial ones by eliminating the elements Δ_{i+1} and ξ_{i+1}, and putting $\tilde{\xi}_i = \xi_i+\Delta_{i+1} + \xi_{i+1} = \Delta_{i+1}$, which results in $\Delta U = 0$. Since $\Delta_i + \xi_i > 0$, it follows that $\tilde{\xi}_{i+1} > \xi_{i+1}$. This implies that (29) holds. Also, $\tilde{\xi}_i + \xi_{i+2} + 2b = \Delta_{i+1} + \xi_{i+2} + 2b$. If $\xi_{i+2} > 0$, then $\Delta_{i+1} + \xi_{i+2} + 2b > 0$. If $\xi_{i+2} = -\xi < 0$ and $\xi_{i+1} = -\xi$, it follows from $\xi_{i+1} + \xi_{i+2} + 2b > 0$ that $-2\xi + 2b > 0$, or $\xi < b$. Hence, $\Delta_{i+1}+\xi_{i+2}+2b = \Delta_{i+1}-\xi+2b > \Delta_{i+1}+b > 0$. Finally, if $\xi_{i+2} = -\xi < 0$ and $\xi_{i+1} = \xi > 0$, it follows that $\xi_i = -\xi_{i+1} = -\xi$ and $\Delta_{i+1} + \xi_{i+2} + 2b = \Delta_{i+1} + \xi_i + 2b > 2b$. Thus, (33) holds. In addition, we have $\Delta_{i+2} + \tilde{\xi}_i = \Delta_{i+2} + \Delta_{i+1} > 0$ and $\Delta_{i+2} + \Delta_i \geq \Delta_{i+2} + \Delta_{i+1} \geq 2b$. Thus, (30) and (32) holds. Finally, (31) holds by construction. However, $\left|\tilde{\xi}_i\right| \neq \xi$, since $\Delta_{i+1} + \xi_{i+1} > 0$ and $\Delta_{i+1} + \xi_i > 0$ implies $\Delta_{i+1} > \xi$. This, however, contradicts the condition $|\xi_k| = \xi$, $\forall k$.

Given these results, the proof of statements is straightforward. First, $a_k \in [w_k, w_{k+1}]$ because of b) and c). Second, the finiteness of the information structure and the delegation set follows from (31), (32), b), and c). ■

Proof of Corollary 2. Notice that in any communication equilibrium under the optimal partition, we have $\Delta_k = \xi_k$, $\forall k$ from the receiver's best-response condition. Then, any non-uniform partition is not optimal, since $\{\xi_k\}$differ for odd and even k. Thus, to prove the statement, it is sufficient to show that any uniform partition in the communication game is not optimal.

Note that (9) along with $\Delta w_k = \frac{1}{n}$, $\forall k$ implies that $2bn \leq 1$. Among all uniform partitions that satisfy these conditions, the highest payoff $U_R = -\frac{1}{12n^2}$ is provided by the partition with the largest number of elements $n = \lfloor \frac{1}{2b} \rfloor$. Then, $2b(n+1) > 1$, and n is the same for all $b \in (\frac{1}{2(n+1)}, \frac{1}{2n}]$. Consider the information structure and the delegation set of size $n+1$, such that $\Delta_k = b$, $\xi_k = \frac{1-b(n+1)}{n+1}$, $\forall k$, which satisfies (29)-(33), and provides the payoff

$$3U_R^D = -(n+1)b^3 - (n+1)\left(\frac{1-b(n+1)}{n+1}\right)^3 = \frac{1}{(n+1)^2}\left(1 - 3b(n+1)(1-b(n+1))\right).$$

The difference in payoffs is

$$\Delta U = 3\left(U_R^D - U_R\right) = \frac{1}{4n^2} - \frac{1}{(n+1)^2}\left(1 - 3b\left(n+1\right)\left(1 - b\left(n+1\right)\right)\right).$$

For $b = \frac{1}{2n}$, $\Delta U = \frac{1}{2}\frac{n-1}{n^2(n+1)^2} > 0$. Since $\frac{d}{db}\Delta U = -\frac{3}{n+1}\left(2b\left(n+1\right) - 1\right) < 0$, we have $\Delta U > 0$ for all $b \in (\frac{1}{2(n+1)}, \frac{1}{2n}]$, which completes the proof. ■

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