Optimal Auctions with Information Acquisition*
(Job Market Paper)

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February 2007

Abstract

This paper studies optimal auction design in a private value setting where a seller wants to sell a single object to one of several potential buyers who can each covertly acquire information about their valuations prior to participation. A simple but robust finding is that the buyers' incentives to acquire information increase as the reserve price moves toward the mean valuation. Thus, a seller who wants to encourage information acquisition should set the reserve price closer to the mean valuation than the standard reserve price in Myerson (1981). We present conditions under which the seller will prefer that the buyers acquire more information, conditions under which standard auctions with an adjusted reserve price are optimal, and conditions under which the buyers will acquire socially excessive information in standard auctions. These results are obtained in a general setting with rotation-ordered information structures and continuous information acquisition.

Keywords: optimal auctions, information acquisition, informational efficiency, rotation order, first order approach.

JEL Classification: C70, D44, D82, D86

*I am indebted to my advisor, Dirk Bergemann, and my committee members, Hanming Fang, Benny Moldovanu, and Stephen Morris, for their guidance and encouragement throughout the course of this project. I am grateful to Phil Haile, Stephanie Lau, and Ben Polak for invaluable discussions and feedback. I also wish to thank Luis Cabral, Rodney Chan, Jernej Copic, Eduardo Faingold, John Geanakoplos, Dino Gerardi, Itzhak Gilboa, Panle Jia, Rongzhu Ke, George Mailath, Yang Qiao, Colin Stewart, Joel Watson, Lixin Ye and seminar participants at Yale University for suggestions and comments. Financial support from Cowles Foundation under the Carl Arvid Anderson fellowship is gratefully acknowledged.

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1 Introduction

1.1 Overview

The mechanism design literature studies how a principal can design the rules of a game to achieve certain objectives given that agents will play strategically and may hold private information. A typical assumption in most of the existing literature is that the information held by market participants is exogenous. In many real world situations, however, the agents' information is acquired rather than endowed. For example, when a firm files bankruptcy under Chapter 7 and is offered for sale, potential buyers may not know how much they are willing to pay, and assessing the value of the firm may be costly. Moreover, the selling mechanism proposed by the seller affects the buyers' incentives to collect information about the goods and services being traded. The purpose of this paper is to study how the seller should design the selling mechanism when information acquisition is endogenous.

Specifically, we consider a model where a seller wants to sell an indivisible object to one of several potential buyers (or bidders). Buyers' valuations for the object are unknown ex-ante to both parties, but prior to participation, buyers can privately acquire costly information about their valuations. The buyers can improve the informativeness of their signals, but with an increasing convex cost. The timing of the game is as follows: first, the seller announces the selling mechanism; after observing the mechanism, buyers decide how much information to acquire, and based on the acquired information buyers determine whether to participate; each participating buyer then submits a report about their private information to the seller; and the outcome is realized.

If the seller chooses a mechanism that encourages information acquisition, the efficiency of allocation may increase because buyers with higher valuations will get the object more often, but the buyers' information rent is also higher. In contrast, if the seller chooses a mechanism that discourages information acquisition, the rent left to the buyers will be lower, but the allocation may be less efficient. The seller's task is therefore to choose a selling mechanism that balances these two forces. The optimal trade-off between surplus extraction and incentives to acquire information is the focus of this paper.

In order to study this problem, we adopt Myerson's (1981) symmetric independent private values framework.[1] Myerson shows that, under some regularity conditions, standard auctions with a reserve price are optimal if the buyers' information is exogenous.[2] We refer to the reserve price in Myerson's optimal auctions as the *standard reserve price*. If information is costly, however, the seller faces an additional constraint: the chosen mechanism must provide the buyers with incentive to acquire the level of information that she prefers.

A simple but robust finding of this paper is that a buyer's incentive to acquire information increases as the reserve price moves toward the mean valuation. To see this, consider the simple setting with one buyer and binary information acquisition. The seller first posts a price, and then

[1] In a private value setting, a buyer's valuation does not depend on the private information of his opponents.

[2] In this paper, we use standard auctions to denote the four commonly used auction formats: first price auctions, Vickery auctions, English auctions, and Dutch auctions.

the buyer decides whether to acquire information and whether to buy. If the reserve price is very high or very low, new information is unlikely to change the buyer's purchasing decision. In contrast, if the reserve price is close to the mean valuation, new information is valuable because it helps the buyer make the right decision: buy or not buy. This observation remains valid in a general setting.

It follows naturally from this observation that the optimal reserve price will be closer to the mean valuation than the standard reserve price if the seller wants to encourage information acquisition. We present conditions under which the seller benefits from more information, conditions under which standard auctions with an adjusted reserve price are optimal among the class of selling mechanisms considered in Myerson (1981),[3] and conditions under which bidders have socially excessive incentive to acquire information in standard auctions.

This paper contributes to the mechanism design literature with endogenous information acquisition and is complementary to the existing literature on optimal auctions with information acquisition. Most of the existing literature assumes that the seller can control either the information sources or the timing of the information acquisition (*centralized* information acquisition).[4] In contrast, information acquisition in our analysis is *decentralized*: buyers can choose to acquire information prior to participation. The information structure we study is quite general, and we allow buyers to choose the level of information acquisition continuously.

To illustrate the model, we first study optimal auctions with a single bidder and the Gaussian specification. Here the optimal selling mechanism is to post a (reserve) price. The buyer's true valuation is normally distributed, and is ex-ante unobservable to both parties. The buyer, however, can acquire a noisy signal, which is the sum of the true valuation and a normally distributed error. The buyer can increase the informativeness of his signal by reducing the variance of the error, but with an increasing cost.

Because the buyer pays the information cost but may have to share the gain from more information with the seller, the buyer and the seller may have conflicting interests in information choice: the one preferred by the buyer may be excessive or insufficient to the seller. Moreover, the buyer's information choice is not observable to the seller. Therefore, we can interpret it as a principle-agent problem in which the seller (principal) sets a reserve price to align the buyer's interest with her own. Taking into account the buyer's information decision, the ex-post optimal standard reserve price (or the monopoly price in this case) is not optimal ex-ante to the seller.

Since the buyer always prefers a low reserve price, it may seem, at first glance, that a lower reserve price always gives the buyer a higher incentive to gather information. This intuition is wrong, however, because the buyer's incentive to acquire information depends on his relative gain from information acquisition rather than on his absolute payoff. Indeed, as we pointed out earlier, the marginal value of information to the buyer increases as the reserve price moves towards the

[3]There are selling mechanisms more general than those considered here. For example, we do not consider sequential mechanisms, as in Cremer, Spiegel and Zheng (2003), and pre-play communication, as in Gerardi and Yariv (2006).

[4]For example, Bergemann and Pesendorfer (2001), and Eso and Szentes (2006) assume the seller controls the information sources. On the other hand, in Levin and Smith (1994), and Cremer, Spiegel and Zheng (2003), the seller controls the timing of the information acquisition. See next subsection for a detailed discussion.

mean valuation.

It turns out that the equilibrium reserve price is always adjusted downward compared to the standard reserve price in this simple setting. The reason is the following. If the standard reserve price is higher than the mean valuation, more information will benefit the seller because more information increases the probability of trade. In order to induce the buyer to acquire more information, the seller must adjust the standard reserve price downwards. On the other hand, if the standard reserve price is lower than the mean valuation, more information will hurt the seller because more information reduces the probability of trade. Again, the seller will set the equilibrium reserve price lower than the standard reserve price, but this time to induce the buyer to acquire less information.

To summarize, the simple one-bidder model has two main findings. First, the buyer's incentive to acquire information is higher when the reserve price is closer to the mean valuation. Second, with endogenous information acquisition by the buyer, the seller will set the optimal reserve price lower than the standard reserve price. The analysis of the general model is more subtle and complicated because the optimal selling mechanism no longer admits the simple form of a posted price. But the first observation remains valid The adjustment of reserve price, however, is not as simple as in the case with one bidder. With sufficiently many bidders, the optimal reserve price is adjusted toward the mean valuation compared to the standard reserve price.

In order to generalize the first result to general information structures, we need an information order to rank the informativeness of different signals. Motivated by the observation that two commonly used information technologies, the Gaussian specification and the "truth-or-noise" technology,[5] both generate a family of distributions that is rotation ordered, we adopt the *rotation order* as our information order.[6] This order has an intuitive interpretation: more informative signals lead to more spread out distributions of the buyers' posterior estimates. We show that the marginal value of information to a buyer increases when the reserve price moves toward the mean valuation if and only if the signals are rotation ordered.

In order to generalize the second result, we need to characterize the solution of the seller's optimization problem with many bidders. Since bidders are ex-ante symmetric, we focus, for tractability, on the symmetric equilibrium in which all bidders acquire the same level of information.[7] In addition, for tractability, we replace the information acquisition constraint with the first order condition of the buyers' maximization problem — this is the so-called *first order approach* in the principal-agent literature (Mirrlees (1999), Rogerson (1985)).[8]

[5]The truth-or-noise technology was introduced into the literature by Lewis and Sappington (1994). Under this specification, the signal sometimes perfectly reveals underlying value, but at other times is just noise.

[6]The rotation order was recently introduced by Johnson and Myatt (2006) in order to model how advertising, marketing and product design affect consumers' valuations.

[7]The symmetry restriction is not needed in identifying the rule of adjusting researve price. But we need this restriction to determine the sign of Lagrangian multiplier of the information acquisition constraint. This is an important restriction. Although the function of information cost is assumed to be convex, we cannot exclude the possibility that the seller might still become better off by implementing an asymmetric equilibrium rather than a symmetric one.

[8]A condition, analogous to CDFC in Rogerson (1985), is shown to be sufficient for the first order approach to be valid when the support of buyers' posterior estimates is invariant to buyers' information choices. This condition,

For the general model, we first provide sufficient conditions under which the seller benefits from a reduction of the marginal cost of information, i.e., the seller prefers more information. In many cases, more information is beneficial to the seller as long as there are sufficiently many buyers. The reason is that the seller's revenue is related to the second order statistic of buyers' posterior estimates which, for a large number of buyers, increases as buyers acquire more information and make the distribution of their posterior estimates more spread out.

Second, applying Myerson's (1981) technique, we show that the seller should set the optimal reserve price between the mean valuation and the standard reserve price if she benefits from more information acquired by the buyers. Otherwise, she should set the optimal reserve price away from the mean valuation compared to the standard reserve price. As shown in Appendix B, this simple rule for adjusting the reserve price is also robust to an alternative specification in which the information acquisition is discrete.

Third, for the Gaussian specification and the truth-or-noise technology with sufficiently many bidders, standard auctions with an adjusted reserve price are optimal. This result implies revenue equivalence, and can be generalized to other information structures with the property that the gain from information acquisition is higher for bidders with higher posterior estimates.

Finally, the information efficiency of the standard auctions is investigated. We show that the buyers' incentives to acquire equilibrium are socially excessive when the reserve price is lower than the mean valuation. The intuition is that when the reserve price is zero, the equilibrium information choice coincides with the social optimum. As the reserve price increases from zero to the mean valuation, the bidders' incentives to acquire information increase and exceed the social optimum.

1.2 Related Literature

This paper is related to the growing literature on information and mechanism design. First of all, our framework extends the principal-agent model with information acquisition to a multi-agent setting. Our analysis is also related to studies on information acquisition in given auction formats. Finally, this paper is close to the existing optimal auction literature where information acquisition is centralized. For a broad survey of the literature on information and mechanism design, see Bergemann and Valimaki (2006a).

The first strand of literature related to this work studies information acquisition in the principal-agent model. Cremer and Khalil (1992) and Cremer, Khalil, and Rochet (1998a, 1998b) introduce endogenous information acquisition into the Baron-Myerson (1982) regulation model. They illustrate how the standard Baron-Myerson contract has to be adjusted in order to give the agent incentives to acquire information.[9] Szalay (2005) extends their framework to a setting with continuous information acquisition, and demonstrates that their findings are robust. These models share with ours a similar information structure and a focus on the interim participation constraint, but their models lack the strategic interaction among bidders that we incorporate.

however, does not hold for the two leading information technologies. Different sufficient conditions are presented for these two information technologies in Appendix B.

[9] See also Lewis and Sappington (1993) for a principal-agent model with an ignorant agent.

Another strand studies information acquisition in auctions. Matthews (1984) studies information acquisition in a first price, common value auction, and investigates how the seller's revenue varies with the amount of information bidders acquire, and whether the equilibrium price fully reveals bidders' information. Persico (2000) shows that the incentive to acquire information is stronger in the first price auction than in the second price auction if bidders' valuations are affiliated. Ye (2006) studies information acquisition in two-stage auctions and shows that efficient entry is not guaranteed in the second stage. Compte and Jehiel (2006) make the important observation that bidders have the option to acquire information in the middle of dynamic auctions, and argue that ascending auctions or multi-round auctions perform better than static sealed-bid auctions.[10] In contrast, the current paper studies the optimal mechanism that maximizes the seller's revenue, rather than studying the given auction formats.[11]

A final related strand studies mechanism design problems where the seller controls either the information sources or the timing of information acquisition. Since Milgrom and Weber (1982), the seller's disclosure policy in the affiliated value setting has been extensively investigated. Recent studies in the independent private value setting, however, are more closely related to this paper. The information order used in the present paper, the rotation order, was first introduced by Johnson and Myatt (2006). They use it to show that a firm's profits are a U-shaped function of the dispersion of consumers' valuations, so a monopolist will pursue extreme positions, providing either a minimal or maximal amount of information. Eso and Szentes (2006) study optimal auctions in a setting where the seller controls the information sources. They show that the seller will fully reveal her information and can extract all of the benefit from the released information.[12] In these models, the seller, rather than the buyers, makes the information decision.

Several papers study the optimal selling mechanism in a setting where buyers make the information decision, but the seller controls the timing of the information acquisition. These models (hereafter refer to as "entry models") impose the ex-ante participation constraint, so the buyers' information decision is essentially an entry decision. The optimal selling mechanism typically consists of a participation fee followed by a second price auction with no reserve price, with the participation fee being equal to the bidders' expected rent from attending the auction. For example, Levin and Smith (1994) demonstrate that a second price auction with no reserve price and no entry fee maximizes the seller's revenue.[13] Similarly, with an ex-ante participation constraint, Cremer, Spiegel and Zheng (2003) construct a sequential selling mechanism in which the seller charges a positive entry fee and extracts the full surplus from buyers.

[10] Other related papers include Tan (1992) and Arozamena and Cantillon (2004), who study investment incentives before auctions.

[11] Bergemann and Valimaki (2002) also study information acquisition and mechanism design, but their focus is on efficient mechanisms.

[12] Bergemann and Pesendorfer (2001) characterize the optimal information structure in the optimal auctions, while Ganuza and Penalva (2006) study the seller's optimal disclosure policy when the information is costly.

[13] Ye (2004) extends their results to the setting where bidders can learn additional information after costly entry. Stegeman (1996) studies efficient auctions when the buyers' private information are endowed but the communication between the seller and buyers is costly.

In contrast to these papers, information acquisition in the present paper is decentralized: buyers make the information decision, and can acquire information prior to participation. Thus, we impose the interim rather than the ex-ante participation constraint, which makes our model different from and complementary to the existing literature.[14] The relationship between our model and the existing literature can be partially summarized in the following table.

	given auction formats	mechanism design approach
centralized info acquisition	optimal disclosure in auctions	entry models
decentralized info acquisition	info acquisition in auctions	our model

The remainder of the paper is organized as follows. Section 2 introduces the model, Section 3 studies optimal auctions with a single bidder and the Gaussian specification, Section 4 contains the analysis of optimal auctions with many bidders, and we conclude in Section 5. All proofs are relegated to Appendix A, unless otherwise noted. Appendix B contains discussions and extensions omitted in the text.

2 The Model

A seller wants to sell a single object to n ex-ante symmetric buyers (or bidders), indexed by $i \in \{1, 2, ..., n\}$.[15] Both the seller and buyers are risk neutral. The buyers' true valuations $\{\omega_i : i = 1, ..., n\}$, unknown ex-ante, are independently drawn from a common distribution F with support $[\underline{\omega}, \overline{\omega}]$. F has a strict positive and differentiable density f. The mean valuation μ is defined as:

$$\mu = \int_{\underline{\omega}}^{\overline{\omega}} \omega_i f(\omega_i)\, d\omega_i.$$

A buyer with valuation ω_i gets utility u_i if he wins the object and pays t_i:

$$u_i = \omega_i - t_i.$$

The seller's valuation for the object is normalized to be zero.

2.1 The Information Structure

Buyer i can acquire a costly signal s_i about ω_i, with $s_i \in [\underline{s}, \overline{s}] \subseteq \mathbb{R}$. Signals received by different buyers are independent. Buyer i acquires information by choosing a joint distribution of (s_i, ω_i) from a family of joint distributions $G_{\alpha_i} : \mathbb{R} \times [\underline{\omega}, \overline{\omega}] \to [0, 1]$, indexed by $\alpha_i \in [\underline{\alpha}, \overline{\alpha}]$. Each fixed α_i corresponds to a statistical experiment, and the signal with higher α_i is more informative in a sense to be defined later. We refer to the joint distribution G_{α_i}, or simply α_i, as the information

[14] Cremer, Spiegel and Zheng (2006) also analyze optimal auctions where buyers can acquire information prior to participation, but the seller, rather than the buyer, pays the information cost.

[15] It is straigthforward to extend the analysis to a multi-unit setting where each buyer has a unit demand.

structure. The cost of performing an experiment α_i is $C(\alpha_i)$, which is assumed to be convex in α_i. A buyer can conduct the experiment $\underline{\alpha}$ at no cost, so $\underline{\alpha}$ is interpreted as the endowed signal.

Let $G_{\alpha_i}(\cdot|\omega_i)$ denote the prior distribution of signal s_i conditional on ω_i, and $G_{\alpha_i}(\cdot|s_i)$ denote the posterior distribution of ω_i conditional on s_i. With a little abuse of notation, $G_{\alpha_i}(\omega_i)$ and $G_{\alpha_i}(s_i)$ are used to denote the marginal distributions of ω_i and s_i, respectively. They are defined in the usual way, that is, $G_{\alpha_i}(\omega_i) = \mathbb{E}_{s_i}[G_{\alpha_i}(\omega_i|s_i)]$ and $G_{\alpha_i}(s_i) = \mathbb{E}_{\omega_i}[G_{\alpha_i}(s_i|\omega_i)]$. Consistency requires that $G_{\alpha_i}(\omega_i) = F(\omega_i)$ for all α_i and i. We use $g_{\alpha_i}(s_i, \omega_i)$, $g_{\alpha_i}(\cdot|\omega_i)$, $g_{\alpha_i}(\cdot|s_i)$, $g_{\alpha_i}(\omega_i)$ and $g_{\alpha_i}(s_i)$ to denote the corresponding densities.

A buyer who observes a signal s_i from experiment α_i will update his prior belief on ω_i according to Bayes' rule:

$$g_{\alpha_i}(\omega_i|s_i) = \frac{g_{\alpha_i}(s_i|\omega_i) f(\omega_i)}{\int_{\underline{\omega}}^{\overline{\omega}} g_{\alpha_i}(s_i|\omega_i) f(\omega_i) d\omega_i}$$

Let $v_i(s_i, \alpha_i)$ denote buyer i's revised estimate of ω_i after performing experiment α_i and observing s_i :

$$v_i(s_i, \alpha_i) \equiv \mathbb{E}[\omega_i|s_i, \alpha_i] = \int_{\underline{\omega}}^{\overline{\omega}} \omega_i g_{\alpha_i}(\omega_i|s_i) d\omega_i$$

To simplify notation, we use v_i to denote $v_i(s_i, \alpha_i)$, and use v to denote the n-vector $(v_1, ..., v_n)$. Occasionally, we also write v as (v_i, v_{-i}), where $v_{-i} = (v_1, ...v_{i-1}, v_{i+1}, ..., v_n)$. We assume $v_i(s_i, \alpha_i)$ is increasing in s_i, that is, a higher signal leads to a higher posterior estimate given the information choice. Let H_{α_i} denote the distribution of v_i with corresponding density h_{α_i}. Then

$$H_{\alpha_i}(x) \equiv \Pr\{\mathbb{E}[\omega_i|s_i, \alpha_i] \leq x\} = \int_{\underline{s}}^{v_i^{-1}(x, \alpha_i)} g_{\alpha_i}(s_i) ds_i.$$

The upper limit of the integral, $v_i^{-1}(x, \alpha_i)$, is well defined since $v_i(s_i, \alpha_i)$ is increasing in s_i. That is, $H_{\alpha_i}(x)$ is the probability that the buyer i's posterior estimate v_i is below x given his information choice α_i. The family of distributions $\{H_{\alpha_i}\}$ have the same mean because

$$\mathbb{E}_{s_i}[v_i(s_i, \alpha_i)] = \mathbb{E}\omega_i = \mu.$$

For bidder i, different information choices $\{\alpha_i\}$ lead to different distributions $\{H_{\alpha_i}\}$. So choosing α_i is equivalent to choosing an H_{α_i} from the family of distributions $\{H_{\alpha_i}\}$. In what follows, we will extensively work with the posterior estimate v_i and its distribution H_{α_i}.

2.2 Timing

The timing of the game is as follows: the seller first proposes a selling mechanism; then given the mechanism, each buyer decides how much information to acquire; after the signals are realized, each buyer decides whether to participate; each participating buyer submits a report about his private information; and finally, an outcome, consisting of an allocation of the object and payments, is realized. Figure 1 summarizes the timing of the game:

seller announces mechanism	buyer i chooses α_i	buyer i observes s_i and decides whether to participate	buyers report private information	outcome realized

Figure 1. The timing of the game

The payoff structure, the timing of the game, the information structure $\{G_{\alpha_i}\}$ and distribution F are assumed to be common knowledge.

2.3 Mechanisms

In our setting, the buyer's private information is two-dimensional: the information choice α_i and the realized signal s_i. This suggests that the design problem here is multi-dimensional and could potentially be very complicated. However, similar to Biais, Martimont and Rochet (2002) and Szalay (2005), one single variable, the posterior estimate $v_i(\alpha_i, s_i)$, completely captures the dependence of buyer i's valuation on the two-dimensional information. Furthermore, the seller cannot screen the two pieces of information separately. For example, suppose there are two buyers, i and j, with the same posterior estimate $(v_i = v_j)$, but $\alpha_i > \alpha_j$. If the seller wants to favor the buyer with α_i, then buyer j can always report to have α_i. Therefore, the posterior estimate v_i is the only variable that the seller can use to screen different buyers.

Thus, we can invoke the Revelation Principle to focus on the direct revelation mechanisms $\{q_i(v), t_i(v)\}_{i=1}^n$:

$$\begin{aligned} q_i &: [\underline{\omega}, \overline{\omega}]^n \to [0,1], \\ t_i &: [\underline{\omega}, \overline{\omega}]^n \to \mathbb{R}, \end{aligned}$$

where $q_i(v)$ denotes the probability of winning the object for bidder i when the vector of report is v, and $t_i(v)$ denotes bidder i's corresponding payment.

Define

$$\begin{aligned} Q_i(v_i) &= \mathbb{E}_{v_{-i}} q_i(v_i, v_{-i}), \\ T_i(v_i) &= \mathbb{E}_{v_{-i}} t_i(v_i, v_{-i}). \end{aligned}$$

$Q_i(v_i)$ and $T_i(v_i)$ are the expected probability of winning and the expected payment conditional on v_i, respectively. The interim utility of bidder i who has a posterior estimate v_i and reports v_i' is

$$U_i(v_i, v_i') = v_i Q_i(v_i') - T_i(v_i').$$

Define $u_i(v_i) = U_i(v_i, v_i)$, the payoff of bidder i who has a posterior estimate v_i and reports truthfully.

A feasible mechanism has to satisfy the individual rationality constraint (IR):

$$u_i(v_i) = U_i(v_i, v_i) \geq 0, \quad \forall v_i \in [\underline{\omega}, \overline{\omega}], \tag{IR}$$

and the incentive compatibility constraint (IC):

$$U_i\left(v_i, v_i\right) \geq U_i\left(v_i, v_i'\right), \quad \forall v_i, v_i' \in [\underline{\omega}, \overline{\omega}]. \tag{IC}$$

With endogenous information acquisition, a feasible mechanism also has to satisfy the information acquisition constraint (IA): no bidder has an incentive to deviate from the equilibrium choice α_i^* :

$$\alpha_i^* \in \arg\max_{\alpha_i} \ \mathbb{E}_{v,\alpha_{-i}^*}\left[u_i\left(v_i\left(s_i, \alpha_i\right)\right)\right] - C\left(\alpha_i\right). \tag{IA}$$

Note that $\mathbb{E}_{v,\alpha_{-i}^*}\left[u_i\left(v_i\left(s_i, \alpha_i\right)\right)\right]$ is bidder i's expected payoff by choosing α_i conditional on other bidders choosing α_j^*, $j \neq i$. The subscript α_{-i}^* is to emphasize that the expectation depends on the information choices of i's opponents.

Since bidders are ex-ante symmetric, we focus on the symmetric equilibrium where $\alpha_i^* = \alpha^*$ for all i. The seller chooses mechanism $\{q_i(v), t_i(v)\}_{i=1}^n$ and α^* to maximize her expected sum of payment from all bidders,

$$\pi_s = \mathbb{E}_{v,\alpha^*} \sum_{i=1}^n T_i\left(v_i\right),$$

subject to (IA), (IC), and (IR).

3 Optimal Auctions with One Bidder and Gaussian Specification

We start with a simple model with only one buyer. If the buyer's information is exogenous, Riley and Zeckhauser (1983) show that the optimal selling mechanism is to post a nonnegotiable price. With endogenous information, their logic still applies and a posted price is optimal.[16] Therefore, with a single buyer, designing the optimal auction is equivalent to choosing a reserve price.

This section will focus on a special but important information structure: the *Gaussian* specification. We first analyze the buyer's information decision problem, and show that the marginal value of information to the buyer increases as the reserve price moves toward the mean valuation. Then we formulate the mechanism design problem as a principal-agent problem and derive the seller's optimal pricing strategy. We show that the equilibrium reserve price is always lower than the standard reserve price. Finally, the informational efficiency of the optimal auction is investigated.

3.1 Gaussian Specification

The buyer's true valuations ω_i are drawn from a normal distribution with mean μ and precision β :

$$\omega_i \sim N\left(\mu, 1/\beta\right).$$

[16] The one-bidder model is a special case of the general model we study later. As shown in the next section, after incorporating the information acquisition constraint, the seller's objective function will be the Lagrangian specified in (11). If there is only one bidder, it reduces to a simple form similar to the one analyzed in Riley and Zechhauser (1983). Therefore, their proof of the optimality of the posted price mechanism still applies here.

Lowering β has the consequence that the prior distribution becomes more spread out, yielding more potential gains from information acquisition.

The buyer can observe a costly signal s_i:

$$s_i = \omega_i + \varepsilon_i,$$

where the additive error ε_i is independent of ω_i, and $\varepsilon_i \sim N\left(0, 1/\alpha_i\right)$. The higher the α_i, the more precise the signal is. Thus, we interpret α_i as the informativeness (precision) of buyer's signal. α_i is assumed to have two parts:

$$\alpha_i = \underline{\alpha} + \gamma_i.$$

The first part, $\underline{\alpha}$, is the endowed signal precision; the incremental term γ_i is the additional precision obtained by investing in information acquisition. For illustration purposes, the cost of information is assumed to be linear in the incremental precision. That is,

$$C\left(\alpha_i\right) = c\gamma_i = c\left(\alpha_i - \underline{\alpha}\right),$$

where c is the constant marginal cost of one additional unit of precision.

After observing a signal s_i with precision α_i, the buyer updates his belief of ω_i. By the standard normal updating technique, the posterior valuation distribution conditional on the signal s_i will be normal:

$$\omega_i | s_i \sim N\left(\frac{\beta\mu + \alpha_i s_i}{\alpha_i + \beta}, \frac{1}{\alpha_i + \beta}\right).$$

It immediately follows that

$$v_i\left(s_i, \alpha_i\right) \equiv \mathbb{E}\left(\omega_i | s_i, \alpha_i\right) = \frac{\beta\mu + \alpha_i s_i}{\alpha_i + \beta}.$$

Thus, the distribution of posterior estimate v_i, $H_{\alpha_i}\left(v_i\right)$, is normal:

$$v_i \sim N\left(\mu, \sigma^2\left(\alpha_i\right)\right), \text{ where } \sigma\left(\alpha_i\right) = \sqrt{\frac{\alpha_i}{\left(\alpha_i + \beta\right)\beta}}.$$

Note that the variance of v_i is increasing in the information choice α_i. So the distribution H_{α_i} will be more spread out for a more precise signal. The following two graphs capture the relationship between two distributions of the posterior estimate with different signals. The left graph in Figure 2 shows that the density of the posterior estimate with a more informative signal is more dispersed than the one with a less informative signal. The right graph shows that the distribution with a less informative signal crosses the distribution with a more informative one from below at the mean valuation. In fact, with some algebra, we can show

$$v_i \gtrless \mu \Leftrightarrow \frac{\partial H_{\alpha_i}\left(v_i\right)}{\partial \alpha_i} \lesseqgtr 0. \tag{1}$$

This property is critical to our analysis and will be used to motivate the rotation order used in the paper.

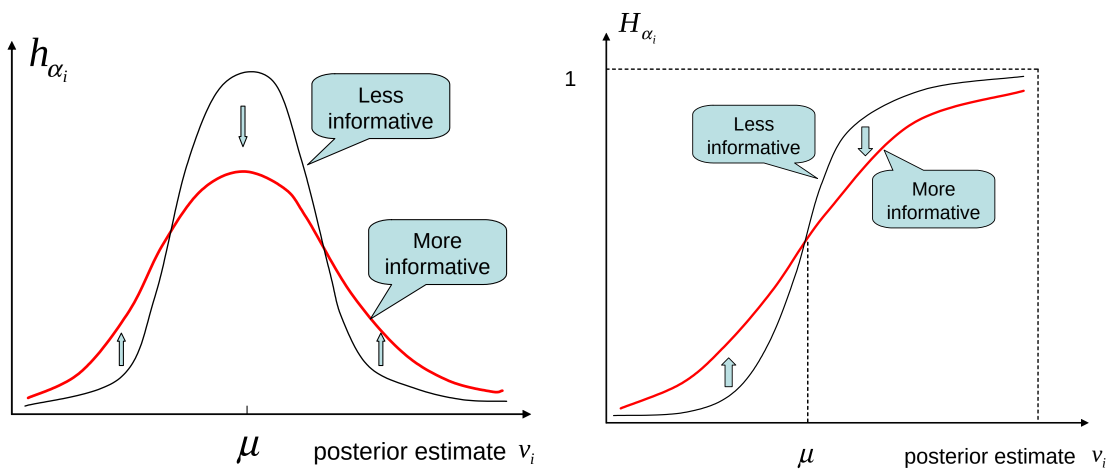

Figure 2. PDF and CDF of the posterior estimate with different signals

3.2 The Marginal Value of Information to the Buyer

Given the reserve price r, the buyer chooses α_i to maximize his expected payoff:

$$\begin{aligned} & \max_{\alpha_i} \int_r^{\infty} (v_i - r)\, h_{\alpha_i}(v_i)\, dv_i - c(\alpha_i - \underline{\alpha}) \\ = & \max_{\alpha_i} \int_r^{\infty} (1 - H_{\alpha_i}(v_i))\, dv_i - c(\alpha_i - \underline{\alpha}). \end{aligned}$$

The first order condition is

$$-\int_r^{\infty} \frac{\partial H_{\alpha_i}(v_i)}{\partial \alpha_i} dv_i = c. \tag{2}$$

The left hand side is the marginal value of information (MVI) to the buyer:

$$MVI \equiv -\int_r^{\infty} \frac{\partial H_{\alpha_i}(v_i)}{\partial \alpha_i} dv_i.$$

Thus, in equilibrium the marginal value of information to the buyer is equal to the marginal information cost.

The following proposition shows how the marginal value of information to the buyer varies with respect to the reserve price.

Proposition 1 (Marginal Value of Information to the Buyer)
The marginal value of information to the buyer increases as the reserve price r moves toward the mean valuation μ, and achieves maximum at $r = \mu$.

This finding is crucial in understanding other results obtained in this paper. To understand it better, let us consider a discrete version of the marginal value of information. Suppose there are two signals α_i and α_i', with $\alpha_i' > \alpha_i$. The buyer's gain from having signal α_i' rather than α_i is

$$\Delta VI = \int_r^{\infty} (H_{\alpha_i}(v_i) - H_{\alpha_i'}(v_i)) dv_i. \tag{3}$$

Since the two distributions have the same mean, we have

$$\mu = \int_{-\infty}^{\infty} (1 - H_{\alpha_i'}(v_i))dv_i = \int_{-\infty}^{\infty} (1 - H_{\alpha_i}(v_i))\, dv_i.$$

Therefore, we can also write the gain from more information as

$$\Delta VI = \int_{-\infty}^{r} (H_{\alpha_i'}(v_i) - H_{\alpha_i}(v_i))dv_i. \tag{4}$$

The following two graphs illustrate the buyer's gain from more information.[17]

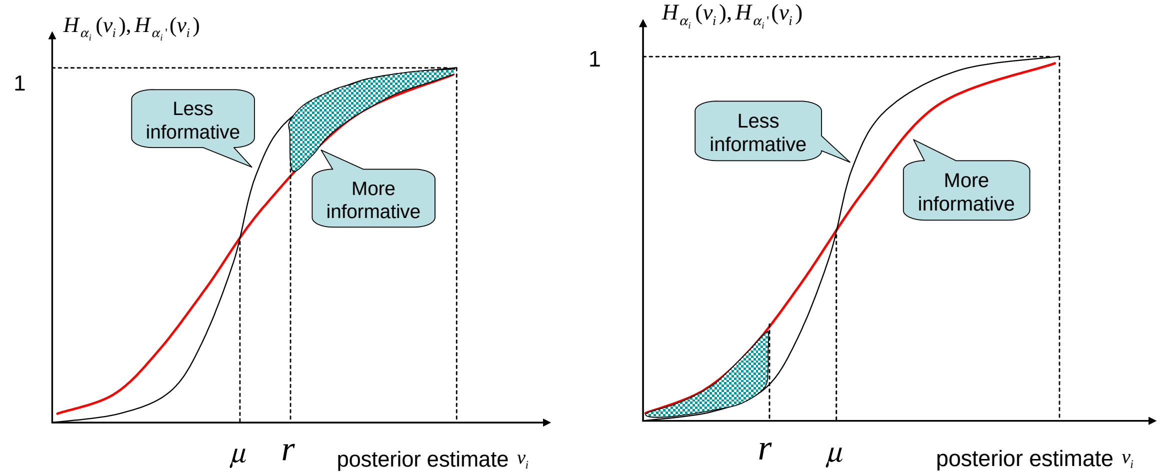

Figure 3: Buyer's gain from more information
Left ($r \geq \mu$): buyer's gain from more information (shaded area) decreases as r increases
Right ($r \leq \mu$): buyer's gain from more information (shaded area) increases as r increases

Given the reserve price r, the payoff of the buyer with signal α_i is the area above the distribution H_{α_i} but below one and to the right of reserve price r. When $r \geq \mu$, the buyer's relative gain with signal α_i' rather than α_i is the shaded area in the left graph (see also expression (3)). On the other hand, when $r \leq \mu$, according to expression (4), the buyer's gain from more information is the shaded area in the right graph. In both cases, the shaded area expands as r moves toward μ and achieves maximum at the mean valuation.

Another important observation obtained from Figure 3 is that the buyer's gain from a more informative signal is always positive. Under mild conditions, the buyer's expected payoff is an increasing concave function of α_i. Hence, the solution to the buyer's maximization problem will be unique, and the buyer's information choice will be decreasing in the information cost c (see Proposition 3 below).

[17] I would like to thank Ben Polak for suggesting these two graphs.

3.3 The Seller's Pricing Decision

For the seller, she chooses r and equilibrium α^* to maximize her revenue. That is

$$\begin{aligned} & \max_{r,\alpha^*} \; r\left(1 - H_{\alpha^*}(r)\right) \\ s.t. \quad : \quad & \alpha^* \in \arg\max_{\alpha_i} \int_r^\infty (v_i - r)\, h_{\alpha_i}(v_i)\, dv_i - c\left(\alpha_i - \underline{\alpha}\right). \end{aligned}$$

The buyer's (agent) information choice is unobservable to the seller (principal), and the seller sets r to align the buyer's incentive to her own. Thus, we can interpret it as a principal-agent problem. The standard way to solve this problem, the so-called *first order approach,* is to assume that the second order condition of the agent's maximization problem is satisfied, and use the first order condition to replace the incentive constraint. We will assume the second order condition is satisfied for now, and discuss it in detail at the end of this subsection.

Then, we can replace the buyer's optimization problem with the first order condition, and rewrite the seller's optimization problem as[18]

$$\begin{aligned} & \max_{r,\alpha^*} \; r\left(1 - H_{\alpha^*}(r)\right) \\ s.t. \quad : \quad & -\int_r^\infty \frac{\partial H_{\alpha^*}(v_i)}{\partial \alpha^*} dv_i - c = 0. \end{aligned} \qquad [\lambda]$$

Let λ be the Lagrangian multiplier for the constraint. We write the Lagrangian in a way such that a positive value of λ means that the seller benefits from a reduction in the information cost; in other words, the seller prefers a more informed buyer.

Lemma 1
For a fixed reserve price r, the seller's revenue increases in α^* if and only if $r > \mu$, and the seller's revenue decreases in α^* if and only if $r < \mu$.

Proof. Immediate from the definition of the seller's revenue and property (1) of the Gaussian specification. ■

The intuition for this result is straightforward by looking at Figure 3. Suppose the buyer's information choice increases from α_i to α_i'. If $r > \mu$ (left figure), then more information increases the probability of trade from $(1 - H_{\alpha_i}(r))$ to $(1 - H_{\alpha_i'}(r))$. More information will therefore benefit the seller. In contrast, if $r < \mu$ (right figure), more information decreases the probability of trade from $(1 - H_{\alpha_i}(r))$ to $(1 - H_{\alpha_i'}(r))$, so more information will hurt the seller.

If we reinterpret our model as a monopoly pricing problem with a continuum of consumers, then this result is similar to one of the main findings in Johnson and Myatt (2006). To see this, we classify all markets into either *niche markets* or *mass markets* following Bergemann and Valimaki (2006b), and Johnson and Myatt (2006):

Definition 1 (Niche Market and Mass Market)
A market is said to be a niche (mass) market if the monopoly price is higher (lower) than the mean valuation μ.

[18] To simplify notation, in what follows, we will use $\frac{\partial H_{\alpha^*}(v_i)}{\partial \alpha^*}$ to denote $\frac{\partial H_{\alpha_i}(v_i)}{\partial \alpha_i}|_{\alpha_i = \alpha^*}$.

Therefore, the lemma states that the seller would prefer a more informed buyer if she is in a niche market. In contrast, the seller in a mass market will prefer a less informed buyer. This result immediately leads to the key insight in Johnson and Myatt (2006): if information is free, then a seller in the niche (mass) market will provide the maximal (minimal) amount information to consumers to maintain its niche (mass) position.

Before stating our results about the optimal reserve price, we need to define a benchmark: the *standard reserve price* when information is endowed rather than acquired.

Definition 2 (Standard Reserve Price)
The standard reserve price r_α is the optimal reserve price when the buyer's signal α is exogenous. That is

$$r_\alpha \in \arg\max_r r\left(1 - H_\alpha\left(r\right)\right) \Rightarrow r_\alpha - \frac{1 - H_\alpha\left(r_\alpha\right)}{h_\alpha\left(r_\alpha\right)} = 0.$$

In particular, we will denote $r_{\underline{\alpha}}$ as the standard reserve price when no additional information (other than the endowed signal $\underline{\alpha}$) is acquired, and denote $r_{\overline{\alpha}}$ as the standard reserve price when the buyer can observe his true valuation for free. Since normal distributions have an increasing hazard rate, r_α is uniquely defined for each H_α. The seller's optimal pricing rule can thus be stated as follows:

Proposition 2 (Optimal Reserve Price)
For a fixed β, there exists a $\widehat{\mu}$ such that

$$\begin{cases} \mu < r^* < r_{\alpha^*} & \text{if} \quad \mu < \widehat{\mu} \\ r^* = r_{\alpha^*} = \mu & \text{if} \quad \mu = \widehat{\mu} \\ r^* < r_{\alpha^*} < \mu & \text{if} \quad \mu > \widehat{\mu} \end{cases}.$$

Therefore, the optimal reserve price r^* with endogenous information is always (weakly) lower than the standard reserve price r_{α^*}.

In order to understand the seller's optimal pricing strategy, we can decompose the effect of a price increase on the seller's profits in three parts:

$$\frac{d\pi_s}{dr}|_{r=r^*} = \underbrace{1 - H_{\alpha^*}\left(r^*\right)}_{A} + \underbrace{\left[-r^* h_{\alpha^*}\left(r^*\right)\right]}_{B} + \underbrace{\left[-r^* \frac{\partial H_{\alpha^*}\left(r^*\right)}{\partial \alpha^*} \frac{\partial \alpha^*}{\partial r}\right]}_{C}.$$

First, the seller's profits increase given that a trade is made (term A). Second, for a fixed information choice, a price increase will reduce the probability of trade (term B). Third, with endogenous information acquisition, a price increase will affect the buyer's incentive to acquire information, thereby the probability of trade (term C). The first two terms are standard, while the last one is specific to the setting with endogenous information acquisition. If $r^* > \mu$, then an increase in r^* will discourage information acquisition. That is,

$$\frac{\partial \alpha^*}{\partial r} < 0.$$

In addition, by (1), for $r^* > \mu$,

$$\frac{\partial H_{\alpha^*}(r^*)}{\partial \alpha^*} < 0.$$

Therefore, term C is negative and the probability of trade decreases. Thus, the seller has less incentive to increase price compared to the case of exogenous information. As a result, $r^* < r_{\alpha^*}$. On the other hand, if $r^* < \mu$, then a price increase will encourage information acquisition, leading to

$$\frac{\partial H_{\alpha^*}(r^*)}{\partial \alpha^*} > 0, \text{ and } \frac{\partial \alpha^*}{\partial r} > 0.$$

Again, term C is negative and the seller is less willing to charge a high price compared to the case of exogenous information. Thus, $r^* < r_{\alpha^*}$. Finally, if $r^* = \mu$, a marginal increase in price does not affect buyer's incentive to acquire information. So $r^* = r_{\alpha^*}$.

We conclude this subsection by presenting sufficient conditions for the second order condition of the buyer's maximization problem to be satisfied. Under these conditions, the first order approach is valid and the buyer's expected payoff is globally concave in the information choice α_i.

Proposition 3 (Validity of the First Order Approach)
If $r \in [\mu - 2\sigma(\underline{\alpha}), \mu + 2\sigma(\underline{\alpha})]$ and $\underline{\alpha} \geq \beta$,the second order condition of the buyer's maximization problem is satisfied.

These conditions are stronger than necessary and are not very restrictive. Note that more than 95% of the normal density is within two standard deviations of the mean. Thus, the first condition is to ensure that the probability of trade under r will be higher than 2.5% but lower than 97.5%. In other words, the reserve price r is neither extremely high nor extremely low ensuring that the probability of trade is neither close to 1 nor close to 0. The second condition $\underline{\alpha} \geq \beta$ is to ensure $\alpha_i > \beta$ for all α_i.[19] It requires that signals be informative relative to the prior.

3.4 Informational Efficiency

In this subsection, we will investigate the informational efficiency of the single-bidder auction with a reserve price r. Since there is only one bidder, the individual cost of information is the same as the social cost of information. Thus, if the marginal value of information to a buyer in an auction is higher than the social marginal value of information, the equilibrium information acquisition will be socially excessive.

Recall that, at information level α_i, the marginal value of information to the buyer is

$$MVI(\alpha_i) = -\int_r^{\infty} \frac{\partial H_{\alpha_i}(v_i)}{\partial \alpha_i} dv_i. \tag{5}$$

From the social point of view, the social planner chooses α_i to solve the following maximization problem

$$\max_{\alpha_i} \int_0^{\infty} (1 - H_{\alpha_i}(v_i))\, dv_i - c(\alpha_i - \underline{\alpha}).$$

[19] Under this condition, the equilibrium information level is away from zero. Therefore, we can avoid the non-concavity of the value of information identified in Radner and Stiglitz (1984).

At information level α_i, the marginal value of information to the social planner is

$$MVI^{FB}(\alpha_i) = -\int_0^\infty \frac{\partial H_{\alpha_i}(v_i)}{\partial \alpha_i} dv_i. \tag{6}$$

Therefore, with the Gaussian specification, the difference between the individual and the social marginal value of information is

$$\begin{aligned}\Delta(\alpha_i) &= MVI^{FB}(\alpha_i) - MVI(\alpha_i) \\ &= \frac{1}{2\sqrt{2\pi}}\sqrt{\frac{\beta^3}{\alpha_i^3(\alpha_i+\beta)}}\sigma^2\left(\exp\left(-\frac{\mu^2}{2\sigma^2}\right) - \exp\left(-\frac{(r-\mu)^2}{2\sigma^2}\right)\right).\end{aligned}$$

This proves the following result:

Proposition 4 (Informational Efficiency)
If $r < (>) 2\mu$, information acquisition in auctions with a reserve price r is socially excessive (insufficient).

Note that when $r = 0$, the individual incentive to acquire information coincides with the social optimum. As r increases, the buyer's incentive to acquire information first increases then decreases after r exceeds μ. For Gaussian specification, the individual incentive to acquire information coincides with the social optimum again when $r = 2\mu$. Therefore, auctions with a single bidder and $r \in (0, 2\mu)$ lead to over-provision of information, while auctions with $r > 2\mu$ lead to under-provision of information.

4 Optimal Auctions with Many Bidders

The single-bidder model is simple because the strategic interaction among bidders is absent and because the Gaussian specification is special. This section studies the optimal auctions with many bidders and general information structures, and show that most of the insights from the previous section carry through as long as different signals are *rotation ordered*, a notion we will introduce below. Specifically, we show that: 1) A bidder's incentive to acquire information increases as the reserve price moves toward the mean valuation; 2) In the optimal auction, the seller who wants to encourage information acquisition sets the reserve price closer to the mean valuation than the standard reserve price; 3) Under some conditions, standard auctions with an adjusted reserve price are optimal; 4) The bidders's incentive to acquire information is socially excessive in standard auctions with a reserve price lower than the mean valuation.

One insight that cannot be carried over from the one-bidder case, however, concerns the seller's information preferences. If there are sufficiently many bidders, the seller will encourage information acquisition — even when the standard reserve price is lower than the mean valuation. We show that, in many cases, the seller will prefer that bidders acquire more information, as long as the number of bidders is large.

4.1 Information Order

In order to analyze a model with general information structures, we need an information order to compare the informativeness of different signals. As we showed before, the relevant variable for screening is the posterior estimate v_i, and there is one-to-one mapping between the information choice α_i and the distribution H_{α_i} of v_i. Thus, we would like to have an information order that directly ranks H_{α_i}. The rotation order, recently introduced by Johnson and Myatt (2006), meets this requirement.

Definition 3 (Rotation Order)
The family of distributions $\{H_{\alpha_i}\}$ is rotation-ordered if, for every α_i, there exists a rotation point $v^+_{\alpha_i}$, such that

$$v_i \gtrless v^+_{\alpha_i} \Leftrightarrow \frac{\partial H_{\alpha_i}(v_i)}{\partial \alpha_i} \lesseqgtr 0. \tag{7}$$

Two distributions ordered in terms of rotation cross only once: the distribution with lower α_i crosses the distribution with higher α_i from below. As shown below, the rotation order implies second order stochastic dominance. However, the reverse is not true, because two distributions ordered in terms of second order stochastic dominance can cross each other more than once.

Lemma 2 (Rotation Order Implies Second Order Stochastic Dominance)
If a family of distributions $\{H_{\alpha_i}\}$ is rotation-ordered and they all have the same mean, then they are also ordered in terms of second order stochastic dominance.

Proof. See Theorem 2.A.17 in Shaked and Shanthikumar (1994). ■

Following Blackwell (1951, 1953), we say that one signal is more informative than the other if a decision-maker can achieve a higher expected utility when basing a decision on the realization of the more informative signal. We extend Blackwell's information criterion to our multi-agent setting by applying his criterion to each bidder while fixing other bidders' information choices.

Proposition 5
Suppose that $\{H_{\alpha_i}\}$ is rotation-ordered and $\alpha'_i > \alpha''_i$. Then under any mechanism $\{q_i(v), t_i(v)\}$ that is incentive compatible, bidder i achieves a higher expected payoff with signal α'_i than signal α''_i.

The above result is intuitive. Because the bidder i's interim payoff $u(v_i)$ is convex in v_i under any incentive compatible mechanism (Rochet (1987)), and because $H_{\alpha'_i}$ second order stochastic dominates $H_{\alpha''_i}$ (by Lemma 2), the bidder i's expected payoff is higher under the more risky prospect $H_{\alpha'_i}$. Therefore, if $\{H_\alpha\}$ is rotation-ordered and $\alpha'_i > \alpha''_i$, then signal α'_i is indeed more informative than signal α''_i because α'_i corresponds to a higher expected payoff for bidder i.

4.2 Characterization of the Optimal Auctions

After introducing the rotation order, we are now ready to characterize the optimal auctions. Since the posterior estimate is the only relevant variable that the seller can contract on, by the Revelation Principle, we can restrict attention to the direct revelation mechanisms. The seller's optimization problem is to choose a menu $(q_i(v_i, v_{-i}), t_i(v_i, v_{-i}))$ and a vector of information choices $(\alpha_1^*, \alpha_2^*, ..., \alpha_n^*)$ to maximize her revenue subject to (IC) (IR) and (IA) constraints. We focus on the symmetric equilibrium with $\alpha_1^* = \cdots = \alpha_n^* = \alpha^*$. Before formally stating the seller's optimization problem, we first need to reformulate the three constraints.

It is well-known (Myerson (1981) and Rochet (1987)) that the incentive compatibility constraint (IC) is equivalent to the following two conditions:

$$u_i(v_i) = u_i(\underline{\omega}) + \int_{\underline{\omega}}^{v_i} Q_i(x)\, dx, \tag{8}$$

and

$$Q_i(v_i) \text{ is nondecreasing in } v_i. \tag{9}$$

With equation (8), we can write the individual rationality constraint (IR) simply as $u_i(\underline{\omega}) \geq 0$.

The information acquisition constraint (IA) requires that α^* be each bidder's best response given that other bidders choose α^*. That is, for all i,

$$\alpha^* \in \arg\max_{\alpha_i} \mathbb{E}_{v_{-i},\alpha^*} \left\{ \int_{\underline{\omega}_{\alpha_i}}^{\overline{\omega}_{\alpha_i}} [1 - H_{\alpha_i}(v_i)]\, q_i(v_i, v_{-i})\, dv_i - C(\alpha_i) \right\}.$$

As before, the subscript α^* of the expectation operator is to remind the readers that the expectation depends on the information choice α^* of bidder i's opponents. The subscript α_i in the lower and upper limits $(\underline{\omega}_{\alpha_i}, \overline{\omega}_{\alpha_i})$ is to emphasize the fact that the support of the posterior estimate may depend on the information choice α_i. The first order condition is

$$-\mathbb{E}_{v,\alpha^*} \left[\frac{\partial H_{\alpha^*}(v_i)}{\partial \alpha^*} \frac{1}{h_{\alpha^*}(v_i)} q_i(v_i, v_{-i}) \right] - C'(\alpha^*) = 0. \tag{10}$$

If the first order approach is valid, we can replace bidder i's optimization problem by (10). This approach is valid if the second order condition is satisfied, which we will assume for now, and discuss later in detail. In principle, there is a system of n first order conditions: one for each bidder. The restriction to the symmetric equilibrium helps us reduce the system of first order conditions to a single equation (10).[20]

Replacing the incentive constraint by equation (8) and (9), and replacing the (IA) constraint by (10), we can transform the seller's optimization problem from the allocation-transfer space into the allocation-utility space. That is,

[20] A sufficient condition for the existence of a symmetric equilibrium is that there exists a α^* satisfying both the first order condition and the second order condition of the buyer's maximization problem. If we assume $\lim_{\alpha \to \underline{\alpha}} C'(\alpha) = 0$, and $\lim_{\alpha \to \overline{\alpha}} C'(\alpha) = \kappa$ (where κ is a large positive number), then there must exist a α^* satisfies the first order condition (10). If the cost function is sufficiently convex, that is, $C''(\alpha)$ is sufficient large, then the second order condition is satisfied (See Appendix B for more detail). A quadradic cost function $C(\alpha) = \kappa_0 (\alpha - \underline{\alpha})^2$ with large κ_0 meets all the requirements.

$$\max_{q_i, u(\underline{\omega}), \alpha^*} \left\{ \mathbb{E}_{v,\alpha^*} \sum_{i=1}^{n} \left[\left(v_i - \frac{1 - H_{\alpha^*}(v_i)}{h_{\alpha^*}(v_i)} \right) q_i(v_i, v_{-i}) \right] - n u_i(\underline{\omega}) \right\}$$

subject to

$$0 \leq q_i(v_i, v_{-i}) \leq 1; \sum_{i=1}^{n} q_i(v_i, v_{-i}) \leq 1, \tag{Regularity}$$

$$Q_i(v_i) \text{ is nondecreasing in } v_i, \tag{Monotonicity}$$

$$u_i(\underline{\omega}) \geq 0, \tag{IR}$$

$$\mathbb{E}_{v,\alpha^*} \left[-\frac{\partial H_{\alpha^*}(v_i)}{\partial \alpha^*} \frac{1}{h_{\alpha^*}(v_i)} q_i(v_i, v_{-i}) \right] - C'(\alpha^*) = 0. \tag{IA}$$

It is easy to see that the (IR) constraint must be binding. For now we can ignore the regularity constraint and the monotonicity constraint and verify them later. Then the only remaining constraint is the (IA) constraint. Let λ denote the Lagrangian multiplier for the (IA) constraint, and write the Lagrangian for the seller's maximization problem as

$$\mathcal{L} = \mathbb{E}_{v,\alpha^*} \sum_{i=1}^{n} \left[\left(v_i - \frac{1 - H_{\alpha^*}(v_i)}{h_{\alpha^*}(v_i)} - \frac{\lambda}{n} \frac{\partial H_{\alpha^*}(v_i)}{\partial \alpha^*} \frac{1}{h_{\alpha^*}(v_i)} \right) q_i(v_i, v_{-i}) \right] - \lambda C'(\alpha^*). \tag{11}$$

Then a positive λ implies that the seller's revenue increases as the marginal cost of information decreases. The virtual surplus function $J^*(v_i)$ can be defined as

$$J^*(v_i) = v_i - \frac{1 - H_{\alpha^*}(v_i)}{h_{\alpha^*}(v_i)} - \frac{\lambda}{n} \frac{\partial H_{\alpha^*}(v_i)}{\partial \alpha^*} \frac{1}{h_{\alpha^*}(v_i)}. \tag{12}$$

In order to characterize the optimal solution to the seller's optimization problem, we make the following assumptions:

Assumption 1 (Rotation Order)
The family of distributions of the posterior estimate, $\{H_{\alpha_i}\}$, is rotation ordered and the rotation point is μ for all α_i.

Assumption 2 (Monotonicity)

$$-\frac{\partial H_{\alpha_i}(v_i)}{\partial \alpha_i} \frac{1}{h_{\alpha_i}(v_i)} \text{ is nondecreasing in } v_i \text{ for all } \alpha_i \text{ and } v_i \in \left[\underline{\omega}_{\alpha_i}, \overline{\omega}_{\alpha_i}\right].$$

Assumption 3 (Regularity)

$$v_i - \frac{1 - H_{\alpha_i}(v_i)}{h_{\alpha_i}(v_i)} \text{ is nondecreasing in } v_i \text{ for all } \alpha_i \text{ and } v_i \in \left[\underline{\omega}_{\alpha_i}, \overline{\omega}_{\alpha_i}\right].$$

Assumption 1 assumes that the signals are rotation ordered and the rotation point $v^+_{\alpha_i}$ is μ for all α_i. The assumption $v^+_{\alpha_i} = \mu$ is not critical, but it greatly eases our presentation. We will discuss it later. Assumption 2 is stronger than the rotation order assumption, and it says that the expected gain from more information is higher for the buyers with higher v_i.[21] Finally, Assumption 3 is a regularity assumption.

Both the rotation order assumption and the regularity assumption are mild assumptions. The monotonicity assumption is relatively more restrictive, but two commonly used information technologies in the literature, the Gaussian specification and the truth-or-noise technology, satisfy all three assumptions.

Definition 4 (Truth-or-noise Technology)
The buyers' true valuations $\{\omega_i\}$ are independently drawn from a distribution F, and F has an increasing hazard rate. Buyer i can acquire a costly signal s_i about ω_i. With probability $\alpha_i \in [\underline{\alpha}, 1]$, the signal s_i perfectly matches the true valuation ω_i, and with probability $1 - \alpha_i$, s_i is a noise independently drawn from F.

Under the truth-or-noise specification, the signal s_i sometimes perfectly reveals buyer i's valuation ω_i, but is noise otherwise.

Lemma 3 (All Assumptions Hold for the Two Leading Examples)
Both the Gaussian specification and the truth-or-noise technology generate a family of distributions $\{H_{\alpha_i}\}$ that satisfies Assumptions 1, 2, and 3.

Note that Assumption 1 does not imply that the underlying distribution F is symmetric. For example, for the truth-or-noise technology, the underlying distribution F could be convex or concave, but the rotation point is still μ.

In the rest of this subsection, we first analyze the buyers' information decision and generalize Proposition 1. Then we investigate the seller's information preferences, and characterize the relationship between the optimal reserve price and the standard reserve price to generalize Proposition 2. Finally, we present conditions under which standard auctions with an adjusted reserve price are optimal.

Let r^* denote the reserve price in the optimal auction. If bidder i is allocated the object with positive probability, then his posterior estimate is at least r^*. That is,

$$q_i(v_i, v_{-i}) > 0 \Rightarrow v_i \geq r^*.$$

With reserve price r^*, the marginal value of information to bidder i under an incentive compatible mechanism $\{q_i(v), t_i(v)\}$ is

$$MVI = -E_{v_{-i},\alpha^*}\left[\int_{r^*}^{\overline{\omega}_{\alpha_i}} \frac{\partial H_{\alpha_i}(v_i)}{\partial \alpha_i} q_i(v_i, v_{-i})\, dv_i\right].$$

[21] Indeed, the monotonicity assumption, together with the mean-preserving property of our information structures, implies rotation order. To see this, first note that $\frac{\partial H_{\alpha_i}(v_i)}{\partial \alpha_i}$ cannot always be positive or negative, otherwise it will imply first order stochastic dominance which violates the fact that the family of distributions $\{H_{\alpha_i}\}$ have the same mean. Therefore, if monotonicity assumption holds, $\frac{\partial H_{\alpha_i}(v_i)}{\partial \alpha_i}$ must change sign from positive to negative only once. That is, $\{H_{\alpha_i}\}$ is rotation ordered.

Theorem 1 (Marginal Value of Information to a Bidder)
The marginal value of information to a bidder increases as r^* moves toward the mean valuation if and only if Assumption 1 is satisfied.

Theorem 1 generalizes Proposition 1 to a setting with many bidders and an information structure that is rotation-ordered. Therefore, if a seller wants to induce buyers to acquire more information, she should set a reserve price closer to the mean valuation.

But when will the seller want to encourage information acquisition? By definition, the Lagrangian multiplier λ for (IA) constraint is the seller's marginal gain from a deduction in marginal information cost. Therefore, the seller will encourage bidders to acquire more information if $\lambda > 0$. The following proposition provides sufficient conditions for $\lambda > 0$.

Lemma 4
The seller benefits from a reduction of marginal cost ($\lambda > 0$), when either one of the following two sets of conditions is satisfied:

(1) Assumptions 1, 2 and 3 hold and

$$\mu < \frac{1 - H_{\underline{\alpha}}(\mu)}{h_{\underline{\alpha}}(\mu)}.$$

(2) The Gaussian specification or the truth-or-noise technology, and large n.

The first condition implies $r^* > \mu$ which is sufficient for $\lambda > 0$. Recall that, in the case of one bidder, the seller prefers more information if $r^* > \mu$. An increase in the number of bidders only strengthens the seller's preference for more information. The second condition should be contrasted with Lemma 1 in the case of one bidder. The strategic interaction between buyers, which is absent in the one-bidder model, plays a crucial part here.[22] As shown by condition (2) in Lemma 4, as long as n is large, the seller will prefer that bidders acquire more information regardless of whether the optimal reserve price is higher or lower than the mean valuation. To see this, note that the seller's revenue is determined by the valuation of the marginal bidder (for example, the second highest bidder) and the reserve price. With many bidders, the valuation of the marginal bidder will be higher than the mean valuation. This valuation is likely to be higher when more information is acquired. In the case with one bidder, however, a seller will prefer a more informed buyer only when the optimal reserve price is higher than the mean valuation (niche market).

Remark. The next two theorems will characterize the optimal selling mechanism contingent on the sign of the endogenous Lagrangian multiplier λ. With Lemma 4, we can always restate the theorems by replacing the condition $\lambda > 0$ by the exogenous condition (1) or (2). However, since both condition (1) and (2) are not necessary for $\lambda > 0$, we state our theorems in terms of λ in order to be precise.

Now, we can present a simple rule for adjusting the reserve price in optimal auctions with information acquisition:

[22] In the discrete information acquisition setting, an important consequence of the strategic interaction is the possibility of the symmetric mixed strategy equilibrium. See Appendix B for an anlysis of the case of the discrete information acquisition.

Theorem 2 (Simple Rule for Adjusting the Reserve Price)
Suppose Assumptions 1 and 3 hold. If $\lambda > 0$, then the optimal reserve price r^* is closer to the mean valuation μ than the standard reserve price r_{α^*}. Specifically,

$$\begin{cases} \mu \leq r^* < r_{\alpha^*} & \text{if} \quad r_{\alpha^*} > \mu \\ r^* = \mu & \text{if} \quad r_{\alpha^*} = \mu \\ r_{\alpha^*} < r^* \leq \mu & \text{if} \quad r_{\alpha^*} < \mu \end{cases}.$$

If $\lambda < 0$, then $r^* < r_{\alpha^*} < \mu$.

Theorem 2 is conceptually a direct consequence of Theorem 1, and generalizes Proposition 2. It characterizes the relationship between the optimal reserve price in our setting and the standard reserve price in Myerson's optimal auctions. If the seller wants to encourage information acquisition, she has to set the optimal reserve price between the mean valuation and the standard reserve price because the bidders' incentives to acquire information are stronger when the reserve price is closer to the mean valuation.

This result is important in practice when the seller is concerned about bidders' incentives to acquire information. The reserve price is always the most important decision she has to make other than choosing the auction format. Theorem 2 identifies a simple rule to adjust the reserve price when endogenous information acquisition is important. The rule is simple and robust in the sense that it holds also in the discrete information acquisition specification (see Appendix B).

Furthermore, the empirical auction literature has attempted to evaluate the optimality of a seller's reserve price policy. Most of these studies assume exogenous information and do not consider the bidders' incentives to acquire information. They use observed bids and the equilibrium bidding behavior to recover the distribution of bidders' valuations, and then compare the actual reserve price with the standard reserve price calculated from the estimated distribution. Our results indicate that, in situations where information acquisition is important, the standard reserve price may not be an appropriate benchmark for comparison. The optimal reserve price in optimal auctions could be higher or lower than the standard reserve price when information is endogenous.

The next result shows that under the stronger Assumption 2, standard auctions with an appropriately chosen reserve price are optimal.

Theorem 3 (Optimal Auctions)
Suppose Assumptions 1, 2 and 3 hold, and $\lambda > 0$. Then standard auctions with the reserve price r^* adjusted according to Theorem 2 are optimal.

Assumption 2 is critical for the above theorem. It ensures that the bidders with higher posterior estimate gain more from information acquisition. Therefore, if the allocation rule assigns the object to the bidder with the highest posterior estimate (just as standard auctions do), then bidders' expected gain from information acquisition will be maximized, and bidders will have a strong incentive to acquire information. But this is exactly what the seller would like to see when $\lambda > 0$: an increase in information acquisition benefits the seller. An immediate consequence of Theorem 3 is the revenue equivalence among all standard auctions, because the allocation rule is the same across all standard auctions. Furthermore, since the bidders' expected gain from information acquisition is the same for all standard auctions, the equilibrium amount of information acquired is the same across standard auctions as well.

The restriction of symmetric equilibrium is important for the above result. If we allow different bidders to acquire different levels of information in equilibrium, then the revenue equivalence fails in general, and different auctions will induce different level of information acquisition. Moreover, this result may not be generalized to discrete information acquisition setting. With discrete information acquisition, in general, bidders will play mixed strategies in equilibrium, which will introduce asymmetry into the interim stage when all bidders have made their information decisions. Because the first price auction and the second price auction are not equivalent when bidders are asymmetric, revenue equivalence fails.

4.3 Informational Efficiency

Theorem 3 states that standard auctions with an adjusted reserve price are optimal under some conditions. In this subsection we will therefore focus on the informational efficiency of standard auctions to obtain a slightly more general results that apply to optimal auctions.

Since we restrict attention to the symmetric equilibrium in the optimal auctions, we need a symmetric benchmark as well. Thus, we assume that the social optimal information choice α^{FB} is the same for all bidders. That is, α^{FB} solves the following maximization problem for all i :

$$\alpha^{FB} \in \arg\max_{\alpha_i} \int_0^{\overline{\omega}_{\alpha_i}} \left(1 - H^n_{\alpha_i}(v_i)\right) dv_i - nC(\alpha_i).$$

At information level α_i, the marginal value of information to the social planner is

$$MVI^{FB}(\alpha_i) = -n \int_0^{\overline{\omega}_{\alpha_i}} \frac{\partial H_{\alpha_i}(v_i)}{\partial \alpha_i} H^{n-1}_{\alpha_i}(v_i)\, dv_i. \tag{13}$$

Recall that, at information level α_i, the marginal value of information to the bidder i is

$$MVI(\alpha_i) = -\int_r^{\overline{\omega}_{\alpha_i}} \frac{\partial H_{\alpha_i}(v_i)}{\partial \alpha_i} H^{n-1}_{\alpha_i}(v_i)\, dv_i. \tag{14}$$

Since the social planner has to pay n times the individual information cost, we normalize the social value of information by multiplying $1/n$. The difference between the social and individual gain from acquiring information is

$$\Delta(\alpha_i, n) = \frac{1}{n} MVI^{FB}(\alpha_i) - MVI(\alpha_i) = -\int_0^r \frac{\partial H_{\alpha_i}(v_i)}{\partial \alpha_i} H^{n-1}_{\alpha_i}(v_i)\, dv_i. \tag{15}$$

By definition of rotation order, if $r < \mu$, $\Delta(\alpha_i, n) < 0.$ That is, information acquisition in auctions with $r < \mu$ is socially excessive. Thus, we have proved the following result.

Proposition 6 (Informational Efficiency)
Suppose Assumption 1 holds. There exists $\delta > 0$ such that bidders have socially excessive incentives to acquire information in standard auctions if and only if $r < \mu + \delta$.

When $r = 0$, the bidders' incentive to acquire information coincides with the social optimum, which can be easily seen from equation (15).[23] As r increases, the buyers' incentive to acquire information increases, reaches maximum at $r = \mu$, and declines afterwards. Consequently, there exists a $\delta > 0$, such that the individual incentive to acquire information coincides with the social optimum when $r = \mu + \delta$. Therefore, the bidders' incentive to acquire information is socially excessive when $r \in (0, \mu + \delta)$. For the one-bidder model with the Gaussian specification, $\delta = \mu$, as shown in Proposition 4.

4.4 Discussion

In our model, the rotation order ranks different information structures by comparing the distributions of the posterior estimate. In contrast, most existing information orders (for example, Lehmann (1988)) impose restrictions on the prior or posterior distributions of underlying states and signals. One can show that a weaker version of Lehmann's order, the MIO-ND order in Athey and Levin (2001), generates a family of distributions $\{H_{\alpha_i}\}$ ordered in terms of second order stochastic dominance. The rotation order also implies second order stochastic dominance, but second order stochastic dominance is not strong enough for our analysis.

Assumption 1 restricts the rotation point to be the mean valuation. However, if the rotation point is different from the mean valuation, our results (Theorem 1-3 and Proposition 6-7) still hold as long as we replace μ in the statements of the results by the rotation point. If the rotation order assumption fails, so that two distributions of the posterior estimate cross each other more than once, then some of our results (for instance, Theorem 1) still hold locally around one of the crossing points.

The first order approach greatly simplifies our analysis and is valid if the second order condition of the bidder's maximization problem is satisfied. In Appendix B, we provides several sets of sufficient conditions for this condition to hold. First, it is satisfied if the cost function is sufficiently convex. Second, if the support of H_{α_i} is invariant with respect to α_i, then a condition analogous to the CDFC condition in the principal-agent literature (Mirrlees (1999), Rogerson (1985)) is sufficient. Third, we present sufficient conditions for the case of the Gaussian specification and the truth-or-noise technology, respectively. See Appendix B for further discussion of these conditions.

As pointed by Bolton and Dewatripont (2005), however, the requirement that the bidders' first-order condition be necessary and sufficient is too strong. All we need is that the replacement of the (IA) constraint by the first order condition can generate necessary conditions for the seller's original maximization problem. Thus, our analysis may remain valid even when the second order condition of the bidders' maximization problem fails.

In order to check whether our results are robust to alternative information specifications, we study discrete information acquisition in Appendix B. To ease comparison to the existing litera-

[23] Note that the standard auctions with zero reserve price are efficient mechanisms. Bergemann and Valimaki (2002) show that the individual incentives to acquire information coincide with the social optimum for efficient mechanisms in the private value setting. Thus, information acquisition in standard auctions with zero reserve price is also social optimal.

ture, we assume that information acquisition is binary and focus on the symmetric mixed strategy equilibrium. Under some technical assumptions, we show that the simple rule for adjusting the reserve price still holds. This result can also partially alleviate any concerns about the first order approach. With discrete information acquisition, however, standard auctions are no longer optimal because mixed strategy introduces asymmetry into the post-information game: in the bidding stage, bidders are no longer symmetric.

Finally, although our model focuses on the independent private value framework, it can also be immediately applied to a setting with a common component. For example, suppose buyer i's true valuation θ_i has two components:

$$\theta_i = \omega_i + y.$$

The first term ω_i represents the individual idiosyncratic valuation and is unknown ex-ante. Buyer i can acquire costly information about ω_i. The second term y is the common value component, and both the buyers and the seller learn it for free.[24] In this situation, all our analysis still applies as the common component only shifts the distribution but does not affect the buyers' incentives.

5 Conclusion

The mechanism design literature studies how carefully designed mechanisms can be used to elicit agents' private information in order to achieve a desired goal. Most of the papers in the literature, however, ignore the influence of the proposed mechanisms on agents' incentives to gather information. In particular, with endogenous information acquisition, the optimal selling mechanism should take into account the bidders' information decision as a response to the proposed mechanism. We show that under some conditions standard auctions with a reserve price remain optimal but the reserve price has to be adjusted in order to incorporate the buyers' incentives to acquire information.

Relative to the existing literature, our model has three distinctive features. First, we study the optimal mechanism that maximizes revenue in the presence of information acquisition. This distinguishes our model from papers studying information acquisition in fixed auction formats. Second, we study private and decentralized information acquisition, thus differing from previous studies on the seller's optimal disclosure policy and various entry models. Finally, the information structure required for our results is more general than most of the existing literature on mechanism design: we require only that the distributions of the posterior estimate be rotation-ordered.

[24] For example, a firm typically has two types of assets: liquid and illiquid. All potentiall buyers of the firm may value liquid assets in the same way, but they may value the illiquid assets differently. The value of liquid assets can be easily learned from financial statements.

6 Appendix A

Proof of Proposition 1: With some algebra, we can show that the partial derivative of $H_{\alpha_i}(v_i)$ with respect to informativeness, α_i, is given by

$$\frac{\partial H_{\alpha_i}(v_i)}{\partial \alpha_i} = -\frac{(v_i - \mu)}{2\sqrt{2\pi}} \exp\left(-\frac{(v_i-\mu)^2}{2\sigma^2}\right)\sqrt{\frac{\beta^3}{\alpha_i^3(\alpha_i+\beta)}}. \tag{16}$$

Insert this into the expression of MVI, we have

$$\begin{aligned} MVI &= \int_r^\infty \frac{(v_i-\mu)}{2\sqrt{2\pi}} \exp\left(-\frac{(v_i-\mu)^2}{2\sigma^2}\right)\sqrt{\frac{\beta^3}{\alpha_i^3(\alpha_i+\beta)}} dv_i \\ &= \frac{1}{2\sqrt{2\pi}}\sqrt{\frac{\beta^3}{\alpha_i^3(\alpha_i+\beta)}}\sigma \int_r^\infty \left(\frac{v_i-\mu}{\sigma}\right)\exp\left(-\frac{(v_i-\mu)^2}{2\sigma^2}\right) dv_i \\ &= \frac{1}{2\sqrt{2\pi}}\sqrt{\frac{\beta^3}{\alpha_i^3(\alpha_i+\beta)}}\sigma^2 \exp\left(-\frac{(r-\mu)^2}{2\sigma^2}\right). \end{aligned}$$

Therefore, as $r \to \mu$, MVI increases. ■

Proof of Proposition 2: We can write the Lagrangian of the seller's optimization problem as follows:

$$\begin{aligned} \mathcal{L}(r,\alpha^*) &= r(1-H_{\alpha^*}(r)) + \lambda\left(-\int_r^\infty \frac{\partial H_{\alpha^*}(v_i)}{\partial \alpha^*} dv_i - c\right) \\ &= r(1-H_{\alpha^*}(r)) + \lambda\left(\frac{1}{2\sqrt{2\pi}}\sqrt{\frac{\beta^3}{\alpha^{*3}(\alpha^{*3}+\beta)}}\sigma^2 \exp\left(-\frac{(r-\mu)^2}{2\sigma^2}\right) - c\right). \end{aligned}$$

The last equality follows by substituting in expression (16). The first order conditions are

$$\frac{\partial \mathcal{L}}{\partial r} = 1 - H_{\alpha^*}(r^*) - rh_{\alpha^*}(r^*) - \lambda\frac{1}{2\sqrt{2\pi}}\sqrt{\frac{\beta^3}{\alpha^{*3}(\alpha^*+\beta)}}\exp\left(-\frac{(r^*-\mu)^2}{2\sigma^2}\right)(r^*-\mu) = 0 \tag{17}$$

$$\frac{\partial \mathcal{L}}{\partial \alpha^*} = -r^*\frac{\partial H_{\alpha^*}(r^*)}{\partial \alpha^*} + \lambda\left(-\int_{r^*}^\infty \frac{\partial^2 H_{\alpha^*}(v_i)}{\partial \alpha^{*2}} dv_i\right) = 0. \tag{18}$$

The second order condition of the buyer's maximization problem implies that

$$-\int_{r^*}^\infty \frac{\partial^2 H_{\alpha^*}(v_i)}{\partial \alpha^{*2}} dv_i < 0,$$

In addition, from (16) we can show

$$\frac{\partial H_{\alpha^*}(r^*)}{\partial \alpha^*} \lesseqgtr 0 \Leftrightarrow r^* \gtreqless \mu.$$

Therefore, condition (18) implies that

$$r^* \gtreqless \mu \Leftrightarrow \lambda \gtreqless 0. \tag{19}$$

Suppose $r_{\alpha^*} > \mu$. Then $r^* < r_{\alpha^*}$. To see this, suppose the opposite is true: $r^* \geq r_{\alpha^*}$. Then $r^* \geq \mu$ and $\lambda > 0$. Therefore, $\frac{\partial \mathcal{L}}{\partial r}|_{r=r^*} < 0$. A contradiction to the optimality of r^*. Next we argue that r^* cannot be less than μ. Suppose $r^* < \mu$ for contradiction. Then $\lambda < 0$ by (19). But $r^* < \mu$ and $\lambda < 0$ imply $\frac{\partial L}{\partial r} > 0$, a contradiction. In sum, $r^* \in [\mu, r_{\alpha^*})$. The other two cases can be proved analogously.

Therefore, we only need to prove that for a fixed β, there exists a $\widehat{\mu}$ such that $r^* > \mu$ if and only if $\mu < \widehat{\mu}$. Note that the first order condition for the buyer's maximization problem is

$$\frac{1}{2\sqrt{2\pi}}\sqrt{\frac{\beta}{\alpha\left(\alpha+\beta\right)^3}}\exp\left(-\frac{\left(r-\mu\right)^2}{2\sigma^2}\right)-c=0$$

With some algebra, we can show

$$\frac{\partial \alpha}{\partial r}\begin{cases} >0 & \text{if} \quad r<\mu \\ =0 & \text{if} \quad r=\mu \\ <0 & \text{if} \quad r>\mu \end{cases} \qquad \text{and } \frac{\partial^2 \alpha}{\partial r \partial \mu} > 0.$$

Note that we can also write the necessary first order condition the seller's maximization problem as

$$\frac{d\pi_s}{dr}|_{r=r^*} = 1 - H_{\alpha^*}\left(r^*\right) - r^* h_{\alpha^*}\left(r^*\right) - r^* \frac{\partial H_{\alpha}\left(r^*\right)}{\partial \alpha^*}\frac{\partial \alpha^*}{\partial r} = 0$$

Define

$$\Gamma\left(r^*, \mu\right) = 1 - H_{\alpha^*}\left(r^*\right) - r^* h_{\alpha^*}\left(r^*\right) - r^* \frac{\partial H_{\alpha^*}\left(r^*\right)}{\partial \alpha^*}\frac{\partial \alpha^*}{\partial r}.$$

Then

$$\frac{\partial \Gamma\left(r^*,\mu\right)}{\partial \mu} = \frac{\partial H_{\alpha^*}\left(r^*\right)}{\partial \alpha^*}\frac{\partial \alpha^*}{\partial r^*} + r^* \frac{\partial h_{\alpha^*}\left(r^*\right)}{\partial \alpha^*}\frac{\partial \alpha^*}{\partial r} + r \frac{\partial^2 H_{\alpha^*}\left(r\right)}{\partial \alpha^{*2}}\left(\frac{\partial \alpha^*}{\partial r^*}\right)^2 - r^* \frac{\partial H_{\alpha^*}\left(r^*\right)}{\partial \alpha^*}\frac{\partial^2 \alpha^*}{\partial r^* \partial \mu}$$

and

$$\frac{\partial \Gamma\left(r^*,\mu\right)}{\partial r^*} = -2h_{\alpha^*}\left(r^*\right) - 2\frac{\partial H_{\alpha^*}\left(r^*\right)}{\partial \alpha^*}\frac{\partial \alpha^*}{\partial r^*} - r^* \frac{\partial h_{\alpha^*}\left(r^*\right)}{\partial \alpha^*}\frac{\partial \alpha^*}{\partial r^*} - r^* \frac{\partial^2 H_{\alpha^*}\left(r^*\right)}{\partial \alpha^{*2}}\left(\frac{\partial \alpha^*}{\partial r^*}\right)^2 + r^* \frac{\partial H_{\alpha^*}\left(r^*\right)}{\partial \alpha^*}\frac{\partial^2 \alpha^*}{\partial r^* \partial \mu}$$

Therefore, if $r^* > \mu$

$$\frac{\partial \Gamma\left(r^*,\mu\right)}{\partial \mu} > 0, \text{ and } \frac{\partial \Gamma\left(r^*,\mu\right)}{\partial r} < 0$$

Furthermore,

$$\frac{\partial \Gamma\left(r^*,\mu\right)}{\partial \mu} < -\frac{\partial \Gamma\left(r^*,\mu\right)}{\partial r}$$

Therefore, for $r^* > \mu$,

$$\frac{dr^*}{d\mu} = -\frac{\frac{\partial \Gamma\left(r^*,\mu\right)}{\partial \mu}}{\frac{\partial \Gamma\left(r^*,\mu\right)}{\partial r}} \in \left(0, 1\right).$$

Furthermore, for $r^* = \mu$,

$$\frac{\partial \Gamma\left(r^*,\mu\right)}{\partial \mu} = 0, \ \frac{\partial \Gamma\left(r^*,\mu\right)}{\partial r} < 0, \text{ and } \frac{dr^*}{d\mu} = 0.$$

Note that $r^*(\mu) > \mu$ for $\mu < 0$. Therefore, there must exists a $\widehat{\mu}$ such that

$$r^*(\widehat{\mu}) = \widehat{\mu}.$$

Moreover, because $\frac{dr^*}{d\mu} \in (0,1)$ for $r^* > \mu$, and $\frac{dr^*}{d\mu} = 0$ for $r^* = \mu$, $\widehat{\mu}$ is unique and $\mu < \widehat{\mu} \Leftrightarrow r^* > \mu$. ■

Proof of Proposition 3: The first order condition for the buyer's optimization problem is

$$-\int_r^\infty \frac{\partial H_{\alpha_i}(v_i)}{\partial \alpha_i} dv_i - c = 0.$$

The second order condition is

$$-\int_r^\infty \frac{\partial^2 H_{\alpha_i}(v)}{\partial \alpha_i^2} dv_i < 0.$$

With some algebra, we can show

$$\frac{\partial^2 H_{\alpha_i}(v_i)}{\partial \alpha_i^2} = \frac{4\alpha_i + 3\beta}{2\alpha_i(\alpha_i+\beta)} \frac{(v_i-\mu)}{2\sqrt{2\pi}} \sqrt{\frac{\beta^3}{\alpha_i^3(\alpha_i+\beta)}} \exp\left(-\frac{(v_i-\mu)^2}{2\sigma^2}\right)\left(1 - \frac{\alpha_i+\beta}{4\alpha_i+3\beta}\frac{\beta^2}{\alpha_i}(v_i-\mu)^2\right).$$

Therefore, we can rewrite the second order condition as

$$\int_r^\infty \frac{(v_i-\mu)}{2\sqrt{2\pi}} \exp\left(-\frac{(v_i-\mu)^2}{2\sigma^2}\right)\left(1 - \frac{\alpha_i+\beta}{4\alpha_i+3\beta}\frac{\beta^2}{\alpha_i}(v_i-\mu)^2\right) dv_i > 0.$$

By a change of variable with $y = \frac{v_i-\mu}{\sigma}$, we can obtain

$$\int_x^\infty y \exp\left(-\frac{1}{2}y^2\right)\left(1-ky^2\right) dy > 0,$$

where

$$k = \frac{\alpha_i+\beta}{4\alpha_i+3\beta}\frac{\beta^2}{\alpha_i}\sigma^2 = \frac{\beta}{4\alpha_i+3\beta}, \quad \text{and} \quad x = \frac{r-\mu}{\sigma} = (r-\mu)/\sqrt{\frac{\alpha_i}{(\alpha_i+\beta)\beta}}.$$

The above inequality can be simplified into

$$-e^{-\frac{1}{2}x^2}\left(kx^2+2k-1\right) > 0 \Leftrightarrow k < \frac{1}{2+x^2}.$$

Substitute the expression of k and x and we can obtain

$$\frac{4\alpha_i+\beta}{\alpha_i+\beta}\frac{\alpha_i}{\beta^2} > (r-\mu)^2. \tag{20}$$

Now if $r \in [\mu - 2\sigma(\underline{\alpha}), \mu + 2\sigma(\underline{\alpha})]$, then $r \in [\mu - 2\sigma, \mu + 2\sigma]$ because $\sigma(\alpha_i) \geq \sigma(\underline{\alpha})$ for all α_i. Therefore, a sufficient condition for (20) is

$$\frac{4\alpha_i+\beta}{\alpha_i+\beta}\frac{\alpha_i}{\beta^2} > 4\sigma^2,$$

or equivalently,

$$\alpha_i > \frac{3}{4}\beta.$$

Since $\alpha_i > \underline{\alpha}$ for all i, the second order condition is satisfied when $\underline{\alpha} > \beta$. That is, the first order approach is valid when

$$r \in [\mu - 2\sigma(\underline{\alpha}), \mu + 2\sigma(\underline{\alpha})] \text{ and } \underline{\alpha} > \beta.$$

Thus, we conclude the proof. ■

Proof of Proposition 5: Under mechanism $\{q_i(v_i, v_{-i}), t_i(v_i, v_{-i})\}$, a bidder's expected payoffs (information rent) with information structure α_i' and α_i'' are, respectively[25]

$$\begin{aligned}
\mathbb{E}u(v_i; \alpha_i') &= \mathbb{E}_{v_{-i}}\left[\int_{\underline{\omega}}^{\overline{\omega}} \left(1 - H_{\alpha_i'}(v_i)\right) q_i(v_i, v_{-i})\, dv_i\right], \\
\mathbb{E}u(v_i; \alpha_i'') &= \mathbb{E}_{v_{-i}}\left[\int_{\underline{\omega}}^{\overline{\omega}} \left(1 - H_{\alpha_i''}(v_i)\right) q_i(v_i, v_{-i})\, dv_i\right].
\end{aligned}$$

Therefore,

$$\begin{aligned}
& \mathbb{E}u(v_i; \alpha_i') - \mathbb{E}u(v_i; \alpha_i'') \\
= & \ \mathbb{E}_{v_{-i}}\left[\int_{\underline{\omega}}^{\overline{\omega}} \left(H_{\alpha_i''}(v_i) - H_{\alpha_i'}(v_i)(v_i)\right) q_i(v_i, v_{-i})\, dv_i\right] \\
= & \ -\mathbb{E}_{v_{-i}}\left[\int_{\underline{\omega}}^{\overline{\omega}} \left(\int_{\underline{\omega}}^{\omega_i} q_i(x, v_{-i})\, dx\right)\left(h_{\alpha_i''}(v_i) - h_{\alpha_i'}(v_i)\right) dv_i\right] \quad \text{(integration by part)} \\
= & \ -\int_{\underline{\omega}}^{\overline{\omega}} \left(\int_{\underline{\omega}}^{\omega_i} Q_i(x)\, dx\right)\left(h_{\alpha_i''}(v_i) - h_{\alpha_i'}(v_i)\right) dv_i \ \ \left(\text{where } Q_i(x) = \mathbb{E}_{v_{-i}}[q_i(x, v_{-i})]\right).
\end{aligned}$$

Since $Q_i(x)$ is nondecreasing in x, $\int_{\underline{\omega}}^{\omega_i} Q_i(x)\, dx$ is convex. By Lemma 2 , $H_{\alpha_i'}$ SOSD $H_{\alpha_i''}$ and have the same mean. Therefore, $\mathbb{E}u(v_i; \alpha_i') - \mathbb{E}u(v_i; \alpha_i'') > 0$.■

Proof of Lemma 3: For the Gaussian specification, we know from the text that

$$H_{\alpha_i}(v_i) = \int_{-\infty}^{v_i} \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) dx \quad \text{where } \sigma^2 = \frac{\alpha_i}{(\alpha_i + \beta)\beta}.$$

Since H_α is normal, it has an increasing hazard rate and the regularity assumption is satisfied. Recall equation (16)

$$\frac{\partial H_{\alpha_i}(v_i)}{\partial \alpha_i} = -\frac{(v_i - \mu)}{2\sqrt{2\pi}} \exp\left(-\frac{(v_i - \mu)^2}{2\sigma^2}\right)\sqrt{\frac{\beta^3}{\alpha_i^3(\alpha_i + \beta)}}.$$

In addition,

$$\frac{\partial H_{\alpha_i}(v_i)}{\partial \alpha_i}\frac{1}{h_{\alpha_i}(v_i)} = -\frac{\beta(v_i - \mu)}{2\alpha_i(\alpha_i + \beta)}.$$

It is easy to see that the other two assumptions are satisfied as well.

[25] If the support of the posterior estimate varies with respect to signal informativeness, we need to redefine the distribution as follows. Suppose under information structure α_i, the support is $\left[\underline{\omega}_{\alpha_i}, \overline{\omega}_{\alpha_i}\right]$. Then define $H_{\alpha_i}(v_i) = 0$ if $v_i \in \left[\underline{\omega}, \underline{\omega}_{\alpha_i}\right]$ and $H_{\alpha_i}(v_i) = 1$ if $v_i \in \left[\overline{\omega}_{\alpha_i}, \overline{\omega}\right]$.

For the truth-or-noise technology, a buyer who observes a realization s_i with precision α_i will revise his posterior estimate as follows:

$$v_i(s_i, \alpha_i) = \mathbb{E}(\omega_i | s_i, \alpha_i) = \alpha_i s_i + (1 - \alpha_i)\mu.$$

The distribution and density of the posterior estimate are, respectively

$$H_{\alpha_i}(v_i) = F\left(\frac{v_i - (1-\alpha_i)\mu}{\alpha_i}\right); \quad h_{\alpha_i}(v_i) = \frac{1}{\alpha_i} f\left(\frac{v_i - (1-\alpha_i)\mu}{\alpha_i}\right).$$

Simple calculations lead to

$$\frac{\partial H_{\alpha_i}(v_i)}{\partial \alpha_i} = f\left(\frac{v_i - (1-\alpha_i)\mu}{\alpha_i}\right)\frac{(\mu - v_i)}{\alpha_i^2}, \tag{21}$$

$$\frac{\partial H_{\alpha_i}(v_i)}{\partial \alpha_i}\frac{1}{h_{\alpha_i}(v_i)} = -\frac{v_i - \mu}{\alpha_i}, \tag{22}$$

$$\frac{h_{\alpha_i}(v_i)}{1 - H_{\alpha_i}(v_i)} = \frac{1}{\alpha_i}\frac{f\left(\frac{v_i-(1-\alpha_i)\mu}{\alpha_i}\right)}{1 - F\left(\frac{v_i-(1-\alpha_i)\mu}{\alpha_i}\right)}. \tag{23}$$

Equation (21) shows that the family of distributions $\{H_{\alpha_i}(\cdot)\}$ is rotation-ordered with rotation point equal to μ. Equation (22) shows that

$$\frac{\partial H_{\alpha_i}(v_i)}{\partial \alpha_i} / h_{\alpha_i}(v_i)$$

is decreasing in v_i. Finally, $H_{\alpha_i}(\cdot)$ has an increasing hazard rate, because, by assumption, the underlying distribution $F(\cdot)$ has an increasing hazard rate. Therefore, the family of distributions $\{H_{\alpha_i}(\cdot)\}$ generated by the "truth-or-noise" technology satisfies all assumptions. ■

Proof of Theorem 1: Given mechanism $\{q, t\}$ and reserve price r^*, buyer i chooses α_i to maximize his expected payoff:

$$\max_{\alpha_i} \mathbb{E}_{v_{-i},\alpha^*}\left\{\int_{r^*}^{\overline{\omega}_{\alpha_i}} [1 - H_{\alpha_i}(v_i)]\, q_i(v_i, v_{-i})\, dv_i - C(\alpha_i)\right\}.$$

Therefore, the marginal value of information to buyer i is

$$MVI = -\int_{r^*}^{\overline{\omega}_{\alpha_i}} \frac{\partial H_{\alpha_i}(v_i)}{\partial \alpha_i} Q_{i,\alpha^*}(v_i)\, dv_i.$$

Therefore,

$$\frac{\partial [MVI]}{\partial r^*} = \frac{\partial H_{\alpha_i}(r^*)}{\partial \alpha_i} Q_{i,\alpha^*}(r^*). \tag{24}$$

Sufficiency: if different signals are rotation-ordered, the above equation shows that MVI is increasing in r^* if $r^* < \mu$ and is decreasing in r^* if $r^* > \mu$. In other words, MVI is increasing as r^* moves toward the mean valuation.

Necessity: suppose signals are not rotation-ordered, then two distributions must cross at least twice. Without loss of generality, suppose one of crossing points is lower than μ. Then we can find a $r^* < \mu$ such that

$$\frac{\partial H_{\alpha_i}(r^*)}{\partial \alpha_i} < 0.$$

Then by equation (24), MVI decreases as r^* moves towards μ. Therefore, the rotation order is also necessary. ■

Proof of Lemma 4: Let α^* denote the equilibrium information choice of bidders in the symmetric equilibrium. We prove the lemma by establishing the following two claims.

Claim 1: The seller's revenue in standard auctions with reserve price r is increasing in α^* if (1) $r \geq \mu$; or (2) Gaussian specification or truth-or-noise information technology, and n is large.

Proof: Let $V_{k,n}$ denote the k-th order statistic from n random variables independently drawn from H_{α^*}. The seller's payoff in standard auctions with reserve price r is:

$$\begin{aligned}
\pi_s(\alpha^*, r) &= r \Pr(V_{n-1,n} < r \leq V_{n,n}) + \mathbb{E}[V_{n-1,n} | V_{n-1,n} \geq r] \Pr(V_{n-1,n} \geq r) \\
&= r[H_{n-1,n}(r) - H_{n,n}(r)] + \int_r^{\overline{\omega}_{\alpha^*}} v_i h_{n-1,n}(v_i)\, dv_i \\
&= r[1 - H_{\alpha^*}(r)^n] + \int_r^{\overline{\omega}_{\alpha^*}} \left[1 - nH_{\alpha^*}(v_i)^{n-1} + (n-1) H_{\alpha^*}(v_i)^n\right] dv_i.
\end{aligned}$$

Therefore,

$$\begin{aligned}
& \frac{\partial \pi_s(\alpha^*, r)}{\partial \alpha^*} \\
= \;& -rnH_{\alpha^*}(r)^{n-1} \frac{\partial H_{\alpha^*}(r)}{\partial \alpha^*} - \int_r^{\overline{\omega}_{\alpha^*}} \left[n(n-1) H_{\alpha^*}(v_i)^{n-2} - (n-1) nH_{\alpha^*}(v_i)^{n-1}\right] \frac{\partial H_{\alpha^*}(v_i)}{\partial \alpha^*} dv_i \\
& + [1 - n + (n-1)] \frac{\partial \overline{\omega}_{\alpha^*}}{\partial \alpha^*} \\
= \;& -rnH_{\alpha^*}(r)^{n-1} \frac{\partial H_{\alpha^*}(r)}{\partial \alpha^*} - n(n-1) \int_r^{\overline{\omega}_{\alpha^*}} H_{\alpha^*}(v_i)^{n-2} [1 - H_{\alpha^*}(v_i)] \frac{\partial H_{\alpha^*}(v_i)}{\partial \alpha^*} dv_i.
\end{aligned}$$

Case 1: $r \geq \mu$. Since $r \geq \mu$ and $\frac{\partial H_{\alpha^*}(v_i)}{\partial \alpha^*} \leq 0$ for all $v_i \geq \mu$, $\frac{\partial \pi_s(\alpha^*, r)}{\partial \alpha^*} > 0$.That is, seller's revenue is increasing in α^*.

Case 2: By the analyis of case 1, we only need to prove the case where $r < \mu$. For Gaussian specification, we have

$$\frac{\partial H_{\alpha^*}(v_i)}{\partial \alpha^*} = -\frac{\beta(v_i - \mu)}{2\alpha^*(\alpha^* + \beta)} h_{\alpha^*}(v_i).$$

Therefore,

$$
\begin{aligned}
& \frac{\partial \pi_s(\alpha^*, r)}{\partial \alpha^*} \\
= \;& -nrH_{\alpha^*}(r)^{n-1}\frac{\partial H_{\alpha^*}(r)}{\partial \alpha^*} - n(n-1)\int_r^{\infty} H_{\alpha^*}(v_i)^{n-2}\left[1-H_{\alpha^*}(v_i)\right]\frac{\partial H_{\alpha^*}(v_i)}{\partial \alpha^*}dv_i \\
= \;& nrH_{\alpha^*}(r)^{n-1}\frac{\beta(r-\mu)}{2\alpha^*(\alpha^*+\beta)}h_{\alpha^*}(r) + n(n-1)\int_r^{\infty}\frac{H_{\alpha^*}(v_i)^{n-2}\left[1-H_{\alpha^*}(v_i)\right]\beta(v_i-\mu)}{2\alpha^*(\alpha^*+\beta)}h_{\alpha^*}(v_i)\,dv_i \\
= \;& \frac{n\beta}{2\alpha^*(\alpha^*+\beta)}\left\{\begin{array}{c} r(r-\mu)H_{\alpha^*}(r)^{n-1}h_{\alpha^*}(r) + \left[(1-H_{\alpha^*}(v_i))(v_i-\mu)H_{\alpha^*}(v_i)^{n-1}\right]_r^{\infty} \\ -\int_r^{\infty}(1-H_{\alpha^*}(v_i)-h_{\alpha^*}(v_i)(v_i-\mu))H_{\alpha^*}(v_i)^{n-1}dv_i \end{array}\right\} \\
= \;& \frac{n\beta}{2\alpha^*(\alpha^*+\beta)}\left\{\begin{array}{c} (r-\mu)H_{\alpha^*}(r)^{n-1}h_{\alpha^*}(r)\left(r-\frac{1-H_{\alpha^*}(r)}{h_{\alpha^*}(r)}\right) \\ +\int_r^{\infty}\left(v_i-\frac{1-H_{\alpha^*}(v_i)}{h_{\alpha^*}(v_i)}\right)H_{\alpha^*}(v_i)^{n-1}h_{\alpha^*}(v_i)\,dv_i - \frac{1}{n}\mu\left(1-H_{\alpha^*}(r)^n\right) \end{array}\right\}
\end{aligned}
$$

If $r \le r_{\alpha^*}$, then $r - \frac{1-H_{\alpha^*}(r)}{h_{\alpha^*}(r)} \le 0$. Therefore, a sufficient condition for $\frac{\partial \pi_s(\alpha^*,r)}{\partial \alpha^*} > 0$ is

$$
\int_{-\infty}^{\infty}\left(v_i - \frac{1-H_{\alpha^*}(v_i)}{h_{\alpha^*}(v_i)}\right)nH_{\alpha^*}(v_i)^{n-1}h_{\alpha^*}(v_i)\,dv_i \ge \mu.
$$

This condition says that the second order statistic n independent random variables drawn from distribution $H_{\alpha^*}(\cdot)$ is higher than μ. When n is large, it holds in general.

If $r > r_{\alpha^*}$, then $r - \frac{1-H_{\alpha^*}(r)}{h_{\alpha^*}(r)} > 0$.

$$
\begin{aligned}
\frac{\partial \pi_s(\alpha^*, r)}{\partial \alpha^*} &= \frac{\beta}{2\alpha^*(\alpha^*+\beta)}\left\{\begin{array}{c} -n\int_r^{\mu}H_{\alpha^*}(r)^{n-1}h_{\alpha^*}(r)\left(r-\frac{1-H_{\alpha^*}(r)}{h_{\alpha^*}(r)}\right)dv_i \\ +n\int_r^{\infty}\left(v_i-\frac{1-H_{\alpha^*}(v_i)}{h_{\alpha^*}(v_i)}\right)H_{\alpha^*}(v_i)^{n-1}h_{\alpha^*}(v_i)\,dv_i - \mu\left(1-H_{\alpha^*}(r)^n\right) \end{array}\right\} \\
&> \frac{\beta}{2\alpha^*(\alpha^*+\beta)}\left[\int_{\mu}^{\infty}\left(v_i-\frac{1-H_{\alpha^*}(v_i)}{h_{\alpha^*}(v_i)}\right)d\left[H_{\alpha^*}^n(v_i)\right]-\mu\right].
\end{aligned}
$$

As n is large, the seller's revenue with n bidders and reserve price μ will be higher than μ. Therefore, $\frac{\partial \pi_s(\alpha^*,r)}{\partial \alpha^*} > 0$.

Similarly, for truth-or-noise technology, we have

$$
\frac{\partial H_{\alpha^*}(v_i)}{\partial \alpha^*} = -\frac{v_i-\mu}{\alpha^*}h_{\alpha^*}(v_i).
$$

Therefore,

$$
\begin{aligned}
& \frac{\partial \pi_s(\alpha^*, r)}{\partial \alpha^*} \\
= \;& -nrH_{\alpha^*}(r)^{n-1}\frac{\partial H_{\alpha^*}(r)}{\partial \alpha^*} - n(n-1)\int_r^{\overline{\omega}_{\alpha^*}} H_{\alpha^*}(v_i)^{n-2}\left[1-H_{\alpha^*}(v_i)\right]\frac{\partial H_{\alpha^*}(v_i)}{\partial \alpha^*}dv_i \\
= \;& nrH_{\alpha^*}(r)^{n-1}\frac{(r-\mu)}{\alpha^*}h_{\alpha^*}(r) + n(n-1)\int_r^{\overline{\omega}_{\alpha^*}}\frac{H_{\alpha^*}(v_i)^{n-2}\left[1-H_{\alpha^*}(v_i)\right](v_i-\mu)}{\alpha^*}h_{\alpha^*}(v_i)\,dv_i \\
= \;& \left\{\begin{array}{c} \frac{n}{\alpha^*}(r-\mu)H_{\alpha^*}(r)^{n-1}h_{\alpha^*}(r)\left(r-\frac{1-H_{\alpha^*}(r)}{h_{\alpha^*}(r)}\right) \\ +\frac{n}{\alpha^*}\int_r^{\overline{\omega}_{\alpha^*}}\left(v_i-\frac{1-H_{\alpha^*}(v_i)}{h_{\alpha^*}(v_i)}\right)H_{\alpha^*}(v_i)^{n-1}h_{\alpha^*}(v_i)\,dv_i - \frac{\beta}{2\alpha^*(\alpha^*+\beta)}\mu\left(1-H_{\alpha^*}(r)^n\right) \end{array}\right\}
\end{aligned}
$$

If $r \leq r_{\alpha^*}$, then $r - \frac{1-H_{\alpha^*}(r)}{h_{\alpha^*}(r)} \leq 0$. Thus, a sufficient condition for $\frac{\partial \pi_s(\alpha^*,r)}{\partial \alpha^*} > 0$ is

$$\int_{\underline{\omega}}^{\overline{\omega}} \left(v_i - \frac{1 - H_{\alpha^*}(v_i)}{h_{\alpha^*}(v_i)} \right) n H_{\alpha^*}(v_i)^{n-1} h_{\alpha^*}(v_i)\, dv_i \geq \mu.$$

This condition holds as long as n is large.

If $r > r_{\alpha^*}$, then $r - \frac{1-H_{\alpha^*}(r)}{h_{\alpha^*}(r)} > 0$. Therefore,

$$\begin{aligned} \frac{\partial \pi_s(\alpha^*, r)}{\partial \alpha^*} &= \frac{1}{\alpha^*} \left\{ \begin{array}{c} -n \int_r^{\mu} H_{\alpha^*}(r)^{n-1} h_{\alpha^*}(r) \left(r - \frac{1-H_{\alpha^*}(r)}{h_{\alpha^*}(r)} \right) dv_i \\ +n \int_r^{\overline{\omega}_{\alpha^*}} \left(v_i - \frac{1-H_{\alpha^*}(v_i)}{h_{\alpha^*}(v_i)} \right) H_{\alpha^*}(v_i)^{n-1} h_{\alpha^*}(v_i)\, dv_i - \mu \left(1 - H_{\alpha^*}(r)^n\right) \end{array} \right\} \\ &> \frac{1}{\alpha^*} \left[\int_{\mu}^{\overline{\omega}_{\alpha^*}} \left(v_i - \frac{1 - H_{\alpha^*}(v_i)}{h_{\alpha^*}(v_i)} \right) d\left[H^n_{\alpha^*}(v_i) \right] - \mu \right] \end{aligned}$$

Again, as n is large, the seller's revenue with n bidders and reserve price μ is higher than μ. So $\frac{\partial \pi_s(\alpha^*,r)}{\partial \alpha^*} > 0$.

Claim 2: If the seller's revenue is increasing in α^* in standard auctions with reserve price r, then $\lambda > 0$.

Proof: Recall the seller's maximization problem is

$$\begin{aligned} &\max_{q_i, u(\underline{\omega}), \alpha^*} \left\{ \mathbb{E}_{v,\alpha^*} \sum_{i=1}^{n} \left[\left(v_i - \frac{1 - H_{\alpha^*}(v_i)}{h_{\alpha^*}(v_i)} \right) q_i(v_i, v_{-i}) \right] - n u(\underline{\omega}) \right\} \\ s.t. \quad &: \quad 0 \leq q_i(v_i, v_{-i}) \leq 1; \sum_{i=1}^{n} q_i(v_i, v_{-i}) \leq 1, \qquad \text{(Regularity)} \\ &: \quad Q_i(v_i) \text{ is nondecreasing in } v_i, \qquad \text{(Monotonicity)} \\ &: \quad u(\underline{\omega}) \geq 0, \qquad \text{(IR)} \\ &: \quad -\mathbb{E}_{v,\alpha^*} \left[\frac{\partial H_{\alpha^*}(v_i)}{\partial \alpha^*} \frac{1}{h_{\alpha^*}(v_i)} q_i(v_i, v_{-i}) \right] - C'(\alpha^*) = 0. \qquad \text{(IA)} \end{aligned}$$

Note that the expectation term is independent of $u(\underline{\omega})$, and $u(\underline{\omega})$ is nonnegative, so the seller must set $u(\underline{\omega}) = 0$ to maximize revenue. Ignore the regularity constraint and monotonicity constraint for the moment.

We adopt the same strategy of Rogerson (1985) by weakening the equality (IA) constraint to the following inequality constraint.

$$-\mathbb{E}_{v,\alpha^*} \left[\frac{\partial H_{\alpha^*}(v_i)}{\partial \alpha^*} \frac{1}{h_{\alpha^*}(v_i)} q_i(v_i, v_{-i}) \right] - C'(\alpha^*) \geq 0.$$

With the inequality constraint, the corresponding Lagrangian multiplier δ is always nonnegative. If we can show that $\delta > 0$ at the optimal solution of the relaxed program, then the constraint is binding in equilibrium. Then, the optimal solution of relaxed program is also an optimal solution of the original program, and $\lambda > 0$.

Write the Lagrangian for the relaxed program as

$$\begin{aligned} L &= \mathbb{E}_{v,\alpha^*}\sum_{i=1}^{n}\left[\left(v_i - \frac{1-H_{\alpha^*}(v_i)}{h_{\alpha^*}(v_i)}\right)q_i(v_i,v_{-i})\right] + \delta\left[-\mathbb{E}_v\left[\frac{\partial H_{\alpha^*}(v_i)}{\partial\alpha^*}\frac{1}{h_{\alpha^*}(v_i)}q_i(v_i,v_{-i})\right] - C'(\alpha^*)\right] \\ &= \mathbb{E}_{v,\alpha^*}\sum_{i=1}^{n}\left[\left(v_i - \frac{1-H_{\alpha^*}(v_i)}{h_{\alpha^*}(v_i)} - \frac{\delta}{n}\frac{\partial H_{\alpha^*}(v_i)}{\partial\alpha^*}\frac{1}{h_{\alpha^*}(v_i)}\right)q_i(v_i,v_{-i})\right] - \delta C'(\alpha^*) \end{aligned}$$

The necessary first order condition is

$$0 = \frac{\partial L}{\partial\alpha^*} = \left\{ \begin{array}{c} \frac{\partial\left\{\mathbb{E}_{v,\alpha^*}\sum_{i=1}^{n}\left[\left(v_i - \frac{1-H_{\alpha^*}(v_i)}{h_{\alpha^*}(v_i)}\right)q_i(v_i,v_{-i})\right]\right\}}{\partial\alpha^*} \\ +\delta\frac{\partial\left[-\mathbb{E}_{v,\alpha^*}\left[\frac{\partial H_{\alpha^*}(v_i)}{\partial\alpha^*}\frac{1}{h_{\alpha^*}(v_i)}q_i(v_i,v_{-i})\right]-C'(\alpha^*)\right]}{\partial\alpha^*} \end{array} \right\}. \tag{25}$$

Since $\delta \geq 0$, Theorem 3 shows that a second price auction is optimal. Therefore, we can restrict attention to second price auctions.

The first term in the big bracket of (25) is positive by the assumption of Claim 2. In order to show $\delta > 0$, we need to show that the second term is negative. Note that in a second price auction with reserve price r

$$-\mathbb{E}_{v,\alpha^*}\left[\frac{\partial H_{\alpha^*}(v_i)}{\partial\alpha^*}\frac{1}{h_{\alpha^*}(v_i)}q_i(v_i,v_{-i})\right] - C'(\alpha^*) = \int_r^{\overline{\omega}_{\alpha^*}}(1-H_{\alpha^*}(v_i))\,H_{\alpha^*}(v_i)^{n-1}\,dv_i - C'(\alpha^*).$$

Thus,

$$\begin{aligned} &\frac{\partial\left[-\mathbb{E}_{v,\alpha^*}\left[\frac{\partial H_{\alpha^*}(v_i)}{\partial\alpha^*}\frac{1}{h_{\alpha^*}(v_i)}q_i(v_i,v_{-i})\right]-C'(\alpha^*)\right]}{\partial\alpha^*} \\ = &\underbrace{-\int_r^{\overline{\omega}_{\alpha^*}}\frac{\partial^2 H_{\alpha^*}(v_i)}{\partial\alpha^{*2}}H_{\alpha^*}(v_i)^{n-1}\,dv_i + \frac{\partial^2 H_{\alpha^*}(\overline{\omega}_{\alpha^*})}{\partial\alpha^{*2}}\frac{\partial\overline{\omega}_{\alpha^*}}{\partial\alpha^*} - C''(\alpha^*)}_{A} \\ &\underbrace{-\int_r^{\overline{\omega}_{\alpha^*}}\left(\frac{\partial H_{\alpha^*}(v_i)}{\partial\alpha^*}\right)^2(n-1)\,H_{\alpha^*}(v_i)^{n-2}\,dv_i}_{B} \end{aligned}$$

Since α^* maximize a bidder's expected payoff, the local second order condition of the bidder's maximization problem holds. As a result, term A is negative. Since term B is also negative, the partial derivative is negative.

By condition (25), it immediately follows $\delta > 0$ at the optimal solution (α^*, q^*). The relaxed program is the same as the original program, and the maximum of the relaxed program can be achieved by the original program. So $\lambda = \delta > 0$ if the seller's revenue is increasing in α_i in the optimal auctions.

Note that a sufficient condition for $r_{\alpha^*} > \mu$ is $r_{\underline{\alpha}} > \mu$. To see this, by definition of $r_{\underline{\alpha}}$ and Assumption 3, $r_{\underline{\alpha}} > \mu$ implies

$$\mu - \frac{1-H_{\underline{\alpha}}(\mu)}{h_{\underline{\alpha}}(\mu)} < 0.$$

By Assumption 1,

$$H_{\underline{\alpha}}(\mu) = H_{\alpha_i}(\mu) \text{ and } h_{\underline{\alpha}}(\mu) \geq h_{\alpha_i}(\mu) \quad \text{for all } \alpha_i \geq \underline{\alpha}$$

It follows that,

$$\mu - \frac{1 - H_{\alpha_i}(\mu)}{h_{\alpha_i}(\mu)} \leq \mu - \frac{1 - H_{\underline{\alpha}}(\mu)}{h_{\underline{\alpha}}(\mu)} < 0 \quad \text{for all } \alpha_i \geq \underline{\alpha}$$

Thus, $r_{\alpha_i} > \mu$. Therefore, a sufficient condition for $r_{\alpha^*} > \mu$ is

$$\mu < \frac{1 - H_{\underline{\alpha}}(\mu)}{h_{\underline{\alpha}}(\mu)}.$$

Finally, from Theorem 2, for $\lambda > 0$, $r_{\alpha^*} > \mu \Leftrightarrow r^* \geq \mu$.

The Lemma now follows from the results of Claim 1 and Claim 2. ■

Proof of Theorem 2: Recall the virtual surplus function is

$$J^*(v_i) = v_i - \frac{1 - H_{\alpha^*}(v_i)}{h_{\alpha^*}(v_i)} - \frac{\lambda}{n} \frac{\partial H_{\alpha^*}(v_i)}{\partial \alpha^*} \frac{1}{h_{\alpha^*}(v_i)}.$$

The optimal reserve price r^* has to satisfy

$$q_i(v_i, v_{-i}) > 0 \Rightarrow v_i \geq r^*,$$

and

$$r^* \leq \min\{r : J^*(v_i) \geq 0 \text{ for all } v_i \geq r\}. \tag{26}$$

The last condition says that the seller will sell the object as long as the marginal revenue is nonnegative.

Case 1: $\lambda > 0$ and $r_{\alpha^*} > \mu$. First we show $r^* < r_{\alpha^*}$. By definition of r_{α^*},

$$r_{\alpha^*} - \frac{1 - H_{\alpha^*}(v_i)}{h_{\alpha^*}(v_i)} = 0.$$

Then for all $v_i \geq r_{\alpha^*} > \mu$,

$$J^*(v_i) = v_i - \frac{1 - H_{\alpha^*}(v_i)}{h_{\alpha^*}(v_i)} - \frac{\lambda}{n} \frac{\partial H_{\alpha^*}(v_i)}{\partial \alpha^*} \frac{1}{h_{\alpha^*}(v_i)} = -\frac{\lambda}{n} \frac{\partial H_{\alpha^*}(v_i)}{\partial \alpha^*} \frac{1}{h_{\alpha^*}(v_i)} > 0.$$

The last inequality follows from the fact that $\{H_{\alpha^*}\}$ is rotation-ordered. Therefore, there exists $\varepsilon > 0$, such that

$$J^*(r_{\alpha^*} - \varepsilon) \geq 0.$$

Therefore, by (26), the optimal reserve price $r^* < r_{\alpha^*}$.

Next, we show $r^* \geq \mu$. Suppose $r^* < \mu$ by contradiction. Then

$$J^*(r^*) = r^* - \frac{1 - H_{\alpha^*}(r^*)}{h_{\alpha^*}(r^*)} - \frac{\lambda}{n} \frac{\partial H_{\alpha^*}(r^*)}{\partial \alpha^*} \frac{1}{h_{\alpha^*}(r^*)} < -\frac{\lambda}{n} \frac{\partial H_{\alpha^*}(r^*)}{\partial \alpha^*} \frac{1}{h_{\alpha^*}(r^*)} < 0.$$

The first inequality follows because $r^* < r_{\alpha^*}$, and the second inequality follows from the rotation order. This contradicts the fact the $J^*(r^*) \geq 0$. Thus, we have shown $\mu \leq r^* < r_{\alpha^*}$

Case 2: $\lambda > 0$ and $r_{\alpha^*} = \mu$. Then for all $v_i > \mu$,

$$J^*(v_i) = v_i - \frac{1 - H_{\alpha^*}(v_i)}{h_{\alpha^*}(v_i)} - \frac{\lambda}{n}\frac{\partial H_{\alpha^*}(v_i)}{\partial \alpha^*}\frac{1}{h_{\alpha^*}(v_i)} \geq -\frac{\lambda}{n}\frac{\partial H_{\alpha^*}(v_i)}{\partial \alpha^*}\frac{1}{h_{\alpha^*}(v_i)} > 0.$$

Therefore, r^* cannot be higher than μ. On the other hand, for all $v_i < \mu$

$$J^*(v_i) = v_i - \frac{1 - H_{\alpha^*}(v_i)}{h_{\alpha^*}(v_i)} - \frac{\lambda}{n}\frac{\partial H_{\alpha^*}(v_i)}{\partial \alpha^*}\frac{1}{h_{\alpha^*}(v_i)} \leq -\frac{\lambda}{n}\frac{\partial H_{\alpha^*}(v_i)}{\partial \alpha^*}\frac{1}{h_{\alpha^*}(v_i)} < 0.$$

Therefore, r^* cannot be lower than μ. Therefore, $r^* = r_{\alpha^*} = \mu$.

Case 3: $\lambda > 0$ and $r_{\alpha^*} < \mu$. Note that for all $v_i < r_{\alpha^*}$,

$$J^*(v_i) = v_i - \frac{1 - H_{\alpha^*}(v_i)}{h_{\alpha^*}(v_i)} - \frac{\lambda}{n}\frac{\partial H_{\alpha^*}(v_i)}{\partial \alpha^*}\frac{1}{h_{\alpha^*}(v_i)} \leq -\frac{\lambda}{n}\frac{\partial H_{\alpha^*}(v_i)}{\partial \alpha^*}\frac{1}{h_{\alpha^*}(v_i)} < 0.$$

Therefore, $r^* > r_{\alpha^*}$. Furthermore, for all $v_i > \mu$,

$$J^*(v_i) = v_i - \frac{1 - H_{\alpha^*}(v_i)}{h_{\alpha^*}(v_i)} - \frac{\lambda}{n}\frac{\partial H_{\alpha^*}(v_i)}{\partial \alpha^*}\frac{1}{h_{\alpha^*}(v_i)} \geq -\frac{\lambda}{n}\frac{\partial H_{\alpha^*}(v_i)}{\partial \alpha^*}\frac{1}{h_{\alpha^*}(v_i)} > 0.$$

Thus, $r^* \leq \mu$. As a result, $r_{\alpha^*} < r \leq \mu$.

Case 4: $\lambda < 0$ and $r_{\alpha^*} < \mu$. Note that for all $v_i \in [r_{\alpha^*}, \mu]$

$$J^*(v_i) = v_i - \frac{1 - H_{\alpha^*}(v_i)}{h_{\alpha^*}(v_i)} - \frac{\lambda}{n}\frac{\partial H_{\alpha^*}(v_i)}{\partial \alpha^*}\frac{1}{h_{\alpha^*}(v_i)} \geq -\frac{\lambda}{n}\frac{\partial H_{\alpha^*}(v_i)}{\partial \alpha^*}\frac{1}{h_{\alpha^*}(v_i)} > 0.$$

In addition, r^* cannot be higher than μ, otherwise $\lambda > 0$. Therefore, $r^* < r_{\alpha^*} < \mu$.

Since $r_{\alpha^*} \geq \mu$ implies $\lambda \geq 0$, the above four cases include all possible cases, and our proof is complete.■

Proof of Theorem 3: Under Assumption 2 and 3,

$$J^*(v_i) = v_i - \frac{1 - H_{\alpha^*}(v_i)}{h_{\alpha^*}(v_i)} - \frac{\lambda}{n}\frac{\partial H_{\alpha^*}(v_i)}{\partial \alpha^*}\frac{1}{h_{\alpha^*}(v_i)}$$

is increasing in v_i. In this case, we can define the reserve price as

$$r^* = \inf\{r : J^*(r) \geq 0\}.$$

Therefore the optimal auctions will assign the object to the bidder with highest posterior estimate provided his estimate is higher than r^*. So standard auctions with reserve price r^* are optimal.■

7 Appendix B

7.1 Sufficient Conditions for Validity of the First Order Approach

Here we will provide several sets of sufficient conditions to ensure the validity of the first order approach. Recall that bidder i chooses α_i to maximize his payoff given other bidders choose $\alpha_j\ (j \neq i)$. Bidder i's payoff under mechanism $\{q, t\}$ is,

$$\pi_b(\alpha_i) = \mathbb{E}_{v_{-i}} \left\{ \int_{\underline{\omega}_{\alpha_i}}^{\overline{\omega}_{\alpha_i}} [1 - H_{\alpha_i}(v_i)]\, q_i(v_i, v_{-i})\, dv_i - C(\alpha_i) \right\}.$$

The first partial derivative is

$$\frac{\partial \pi_b}{\partial \alpha_i} = \mathbb{E}_{v_{-i}} \left\{ -q_i\left(\underline{\omega}_{\alpha_i}, v_{-i}\right) \frac{\partial \underline{\omega}_{\alpha_i}}{\partial \alpha_i} - \int_{\underline{\omega}_{\alpha_i}}^{\overline{\omega}_{\alpha_i}} \frac{\partial H_{\alpha_i}(v_i)}{\partial \alpha_i} q_i(v_i, v_{-i})\, dv_i \right\} - C'(\alpha_i),$$

and the second partial derivative is

$$\frac{\partial^2 \pi_b}{\partial \alpha_i^2} = -\mathbb{E}_{v_{-i}} \left\{ \begin{array}{c} \frac{\partial q_i\left(\underline{\omega}_{\alpha_i}, v_{-i}\right)}{\partial v_i} \left(\frac{\partial \underline{\omega}_{\alpha_i}}{\partial \alpha_i}\right)^2 + q_i\left(\underline{\omega}_{\alpha_i}, v_{-i}\right) \frac{\partial^2 \underline{\omega}_{\alpha_i}}{\partial \alpha_i^2} - \frac{\partial H_{\alpha_i}\left(\underline{\omega}_{\alpha_i}\right)}{\partial \alpha_i} q_i\left(\underline{\omega}_{\alpha_i}, v_{-i}\right) \frac{\partial \underline{\omega}_{\alpha_i}}{\partial \alpha_i} \\ + \frac{\partial H_{\alpha_i}\left(\overline{\omega}_{\alpha_i}\right)}{\partial \alpha_i} q_i\left(\overline{\omega}_{\alpha_i}, v_{-i}\right) \frac{\partial \overline{\omega}_{\alpha_i}}{\partial \alpha_i} + \int_{\underline{\omega}_{\alpha_i}}^{\overline{\omega}_{\alpha_i}} \frac{\partial^2 H_{\alpha_i}(v_i)}{\partial \alpha_i^2} q_i(v_i, v_{-i})\, dv_i \end{array} \right\} - C''(\alpha_i).$$

The first order approach is valid if

$$\frac{\partial^2 \pi_b}{\partial \alpha_i^2} < 0. \tag{27}$$

It is easy to see that the above condition holds if the cost function is sufficient convex.[26]

If the support of the posterior estimate is independent of information choice α_i, all terms except the last one in the expectation is zero. Therefore, if the last term in the expectation is positive, together with the convex cost function, the first order approach is valid. A sufficient condition for the last term to be nonnegative is

$$\frac{\partial^2 H_{\alpha_i}(v_i)}{\partial \alpha_i^2} \geq 0 \text{ for all } v_i. \tag{28}$$

Condition (28) says the distribution of the posterior estimate is convex in the bidder's information choice. This condition is analogous to the CDFC (convexity of the distribution function condition) in the principal-agent literature, which requires that the distribution function of output be convex in the action the agent takes (Mirrlees 1999, Rogerson 1985).[27]

For a general information structure, it is difficult to verify whether condition (28) is satisfied. For specific information technologies, however, we are able to provide sufficient conditions to guarantee the validity of the first order approach.

[26] Persico (2000) makes such a assumption in his example of information acquisition.

[27] See also Jewitt (1988).

Proposition 7

For the truth-or-noise technology, if $C''(\alpha_i)\,\alpha_i \geq f(\overline{\omega})(\overline{\omega}-\mu)^2$ for all α_i, the second order condition is satisfied either (1) $F(x)$ is convex, or (2) $F(x) = x^b\,(b>0)$ with support $[0,1]$. For the Gaussian specification, the second order condition is satisfied if, for all α_i,

$$\sqrt{\frac{\beta^3}{\alpha_i^3(\alpha_i+\beta)^5}} < 2\sqrt{2\pi}C''(\alpha_i).$$

Proof: For the truth-or-noise technology,

$$\begin{aligned}
\frac{\partial^2\pi_b}{\partial\alpha_i^2} &= -\mathbb{E}_{v_{-i}}\left\{\begin{array}{c} \frac{\partial q_i\left(\underline{\omega}_{\alpha_i},v_{-i}\right)}{\partial v_i}\left(\frac{\partial\underline{\omega}_{\alpha_i}}{\partial\alpha_i}\right)^2 + q_i\left(\underline{\omega}_{\alpha_i},v_{-i}\right)\frac{\partial^2\underline{\omega}_{\alpha_i}}{\partial\alpha_i^2} - \frac{\partial H_{\alpha_i}\left(\underline{\omega}_{\alpha_i}\right)}{\partial\alpha_i}q_i\left(\underline{\omega}_{\alpha_i},v_{-i}\right)\frac{\partial\underline{\omega}_{\alpha_i}}{\partial\alpha_i} \\ +\frac{\partial H_{\alpha_i}(\overline{\omega}_{\alpha_i})}{\partial\alpha_i}q_i\left(\overline{\omega}_{\alpha_i},v_{-i}\right)\frac{\partial\overline{\omega}_{\alpha_i}}{\partial\alpha_i} + \int_{\underline{\omega}_{\alpha_i}}^{\overline{\omega}_{\alpha_i}}\frac{\partial^2 H_{\alpha_i}(v_i)}{\partial\alpha_i^2}q_i(v_i,v_{-i})\,dv_i \end{array}\right\} - C''(\alpha_i) \\
&< \left\{\begin{array}{c} -\frac{\partial H_{\alpha_i}(\overline{\omega}_{\alpha_i})}{\partial\alpha_i}Q_i\left(\overline{\omega}_{\alpha_i}\right)\frac{\partial\overline{\omega}_{\alpha_i}}{\partial\alpha_i} + \frac{\partial H_{\alpha_i}\left(\underline{\omega}_{\alpha_i}\right)}{\partial\alpha_i}Q_i\left(\underline{\omega}_{\alpha_i}\right)\frac{\partial\underline{\omega}_{\alpha_i}}{\partial\alpha_i} \\ -C''(\alpha_i) \end{array}\right\} - \int_{\underline{\omega}_{\alpha_i}}^{\overline{\omega}_{\alpha_i}}\frac{\partial^2 H_{\alpha_i}(v_i)}{\partial\alpha_i^2}Q_i(v_i,v_{-i})\,dv_i \\
&= \left\{\begin{array}{c} f(\overline{\omega})\frac{(\overline{\omega}-\mu)^2}{\alpha_i}Q_i\left(\overline{\omega}_{\alpha_i}\right) - f(\underline{\omega})\frac{(\mu-\underline{\omega})^2}{\alpha_i}Q_i\left(\underline{\omega}_{\alpha_i}\right) \\ -C''(\alpha_i) \end{array}\right\} - \int_{\underline{\omega}_{\alpha_i}}^{\overline{\omega}_{\alpha_i}}\frac{\partial^2 H_{\alpha_i}(v_i)}{\partial\alpha_i^2}Q_i(v_i,v_{-i})\,dv_i \\
&\leq f(\overline{\omega})\frac{(\overline{\omega}-\mu)^2}{\alpha_i} - C''(\alpha_i) - \int_{\underline{\omega}_{\alpha_i}}^{\overline{\omega}_{\alpha_i}}\frac{\partial^2 H_{\alpha_i}(v_i)}{\partial\alpha_i^2}Q_i(v_i,v_{-i})\,dv_i
\end{aligned}$$

If $C''(\alpha_i)\,\alpha_i \geq f(\overline{\omega})(\overline{\omega}-\mu)^2$, then

$$\begin{aligned}
\frac{\partial^2\pi_b}{\partial\alpha_i^2} &< -\int_{\underline{\omega}_{\alpha_i}}^{\overline{\omega}_{\alpha_i}}\frac{\partial^2 H_{\alpha_i}(v_i)}{\partial\alpha_i^2}Q_i(v_i)\,dv_i \\
&= -\int_{\underline{\omega}_{\alpha_i}}^{\overline{\omega}_{\alpha_i}}\left\{f'\left(\frac{v_i-(1-\alpha_i)\mu}{\alpha_i}\right)\frac{(\mu-v_i)^2}{\alpha_i^4} - f\left(\frac{v_i-(1-\alpha_i)\mu}{\alpha_i}\right)\frac{2(\mu-v_i)}{\alpha_i^3}\right\}Q_i(v_i)\,dv_i \\
&= -\mathbb{E}_{v_{-i}}\int_{\underline{\omega}}^{\overline{\omega}}\left\{f'(s_i)\frac{(s_i-\mu)^2}{\alpha_i} + f(s_i)\frac{2(s_i-\mu)}{\alpha_i}\right\}Q_i(\alpha_i s_i + (1-\alpha_i)\mu)\,ds_i
\end{aligned}$$

If $F(\cdot)$ is convex, then

$$\begin{aligned}
\frac{\partial^2\pi_b}{\partial\alpha_i^2} &< -\frac{2}{\alpha_i}\int_{\underline{\omega}}^{\overline{\omega}}(s_i-\mu)f(s_i)Q_i[\alpha_i s_i + (1-\alpha_i)\mu]\,ds_i \\
&< -\frac{2}{\alpha_i}\int_{\underline{\omega}}^{\mu}(s_i-\mu)f(s_i)Q_i(\mu)\,ds_i - \frac{2}{\alpha_i}\int_{\mu}^{\overline{\omega}}(s_i-\mu)f(s_i)Q_i(\mu)\,ds_i \\
&= -\frac{2}{\alpha_i}Q_i(\mu)\int_{\underline{\omega}}^{\overline{\omega}}(s_i-\mu)f(s_i)\,ds_i \\
&= 0.
\end{aligned}$$

But the convexity of $F(\cdot)$ is not necessary. Suppose $F(x) = x^b\,(0<b\leq 1)$ with support $[0,1]$,

then

$$\begin{aligned}
\frac{\partial^2 \pi_b}{\partial \alpha_i^2} &< -\int_{\underline{\omega}}^{\overline{\omega}} \left\{ f'(s_i) \frac{(s_i-\mu)^2}{\alpha_i} + f(s_i) \frac{2(s_i-\mu)}{\alpha_i} \right\} Q_i(\alpha_i s_i + (1-\alpha_i)\mu)\, ds_i \\
&= -\frac{1}{\alpha_i} \int_0^1 \left\{ b(b-1) s^{b-2} (s-\mu)^2 + 2b s^{b-1}(s-\mu) \right\} Q_i(\alpha_i s + (1-\alpha_i)\mu)\, ds \\
&= -\frac{1}{\alpha_i} \int_0^1 [(b+1)s + (1-b)\mu]\, b s^{b-2} (s-\mu)\, Q_i(\alpha_i s + (1-\alpha_i)\mu)\, ds \\
&< -\frac{1}{\alpha_i} Q_i(\mu) \int_0^1 ((b+1)s + (1-b)\mu)\, b s^{b-2}(s-\mu)\, ds \\
&= -\frac{1}{\alpha_i} Q_i(\mu)(b+1) \int_0^1 b s^{b-1}(s-\mu)\, ds - \frac{1}{\alpha_i} Q_i(\mu)(1-b)\mu b \int_0^1 s^{b-2}(s-\mu)\, ds \\
&= -\frac{1}{\alpha_i} Q_i(\mu) \frac{b}{(1+b)^2} \\
&< 0.
\end{aligned}$$

For the Gaussian specification, the second derivative is

$$\frac{\partial^2 \pi_b}{\partial \alpha_i^2} = -\mathbb{E}_{v_{-i}} \left\{ \int_{-\infty}^{\infty} \frac{\partial^2 H_{\alpha_i}(v_i)}{\partial \alpha_i^2} q_i(v_i, v_{-i})\, dv_i \right\} - C''(\alpha_i).$$

With some algebra, we can obtain

$$\frac{\partial^2 H_{\alpha_i}(v_i)}{\partial \alpha_i^2} = \frac{4\alpha_i + 3\beta}{2\alpha_i(\alpha_i+\beta)} \frac{(v_i-\mu)}{2\sqrt{2\pi}} \sqrt{\frac{\beta^3}{\alpha_i^3(\alpha_i+\beta)}} \exp\left(-\frac{(v_i-\mu)^2}{2\sigma^2}\right) \left(1 - \frac{\alpha_i+\beta}{4\alpha_i+3\beta} \frac{\beta^2}{\alpha_i} (v_i-\mu)^2\right).$$

So we can write the second derivative as

$$\begin{aligned}
\frac{\partial^2 \pi_b}{\partial \alpha_i^2} &= \begin{pmatrix} -\mathbb{E}_{v_{-i}} \left\{ \int_{-\infty}^{\infty} \frac{4\alpha_i+3\beta}{2\alpha_i(\alpha_i+\beta)} \frac{(v_i-\mu)}{2\sqrt{2\pi}} \sqrt{\frac{\beta^3}{\alpha_i(\alpha_i+\beta)}} \exp\left(-\frac{(v_i-\mu)^2}{2\sigma^2}\right) q_i(v_i, v_{-i})\, dv_i \right\} \\ +\mathbb{E}_{v_{-i}} \left\{ \int_{-\infty}^{\infty} \frac{\beta^2}{4\alpha_i^2} \frac{(v_i-\mu)^3}{\sqrt{2\pi}} \sqrt{\frac{\beta^3}{\alpha_i^3(\alpha_i+\beta)}} \exp\left(-\frac{(v_i-\mu)^2}{2\sigma^2}\right) q_i(v_i, v_{-i})\, dv_i \right\} \end{pmatrix} - C''(\alpha_i) \\
&= \begin{pmatrix} -\mathbb{E}_{v_{-i}} \left\{ \frac{4\alpha_i+3\beta}{2\alpha_i(\alpha_i+\beta)} \int_{-\infty}^{\infty} \left(-\frac{\partial H_{\alpha_i}(v_i)}{\partial \alpha_i}\right) q_i(v_i, v_{-i})\, dv_i \right\} \\ +\mathbb{E}_{v_{-i}} \left\{ \int_{-\infty}^{\infty} \frac{\beta^2}{4\alpha_i^2} \frac{(v_i-\mu)^3}{\sqrt{2\pi}\sigma} \frac{\beta}{\alpha_i(\alpha_i+\beta)} \exp\left(-\frac{(v_i-\mu)^2}{2\sigma^2}\right) q_i(v_i, v_{-i})\, dv_i \right\} \end{pmatrix} - C''(\alpha_i).
\end{aligned}$$

By Proposition 5, bidders always prefer higher α_i, which implies

$$\mathbb{E}_{v_{-i}} \left\{ \int_{-\infty}^{\infty} \left(-\frac{\partial H_{\alpha_i}(v_i)}{\partial \alpha_i}\right) q_i(v_i, v_{-i})\, dv_i \right\} > 0.$$

Thus, a sufficient condition for the second order condition is

$$\begin{aligned}
\mathbb{E}_{v_{-i}} \left\{ \int_{-\infty}^{\infty} \frac{\beta^2}{4\alpha_i^2} \frac{\beta}{\alpha_i(\alpha_i+\beta)} \frac{(v_i-\mu)^3}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(v_i-\mu)^2}{2\sigma^2}\right) q_i(v_i, v_{-i})\, dv_i \right\} &< C''(\alpha_i) \Leftrightarrow \\
\frac{\beta^3}{4\alpha_i^3(\alpha_i+\beta)} \int_{-\infty}^{\infty} \frac{(v_i-\mu)^3}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(v_i-\mu)^2}{2\sigma^2}\right) Q_i(v_i)\, dv_i &< C''(\alpha_i).
\end{aligned}$$

A sufficient condition for the above inequality is,

$$\begin{aligned}\frac{\beta^3}{4\alpha_i^3(\alpha_i+\beta)}\int_{\mu}^{\infty}\frac{(v_i-\mu)^3}{\sqrt{2\pi}\sigma}\exp\left(-\frac{(v_i-\mu)^2}{2\sigma^2}\right)dv_i &< C''(\alpha_i)\Leftrightarrow\\ \frac{1}{2\sqrt{2\pi}}\sqrt{\frac{\beta^3}{\alpha_i^3(\alpha_i+\beta)^5}} &< C''(\alpha_i).\end{aligned}$$

Note that if β/α_i is small, the above sufficient condition is easy to be satisfied. Therefore, if $\underline{\alpha}/\beta$ is sufficiently large, the second order condition is satisfied. ■

For the truth-or-noise technology, the condition, $C''(\alpha_i)\alpha_i \geq f(\overline{\omega})(\overline{\omega}-\mu)^2$, is to ensure that the relative gain from information acquisition is not too high so that bidders will not pursue extreme information choice $\overline{\alpha}$. The convex distribution means that if bidders acquire information they have better chance to get extreme values; the information acquisition is productive. But the convexity of F is not necessary. For example, $F(x)=x^b$ may not be convex but the second order condition is still satisfied.

For the Gaussian specification, if β is small and $\underline{\alpha}$ is large relative to β, then the second order condition is satisfied. This is quite intuitive. Small β implies the prior distribution is quite spread out, so the potential gain from information acquisition is high. If $\underline{\alpha}$ is large relative to β, then signal will be informative, which again implies information acquisition is profitable.

7.2 Discrete Information Acquisition

If the investment in information is lumpy, information acquisition may be discrete. A study on discrete information acquisition can help check the robustness of our results and partially relieve the concern about the first order approach. In order to be comparable to the existing literature, information acquisition is assumed to be binary. If a buyer acquires information, he observes his true valuation; otherwise, he does not know his type. We will refer to bidders acquiring information as informed bidders, and bidders not acquiring information as uninformed bidders. Since bidders are risk neutral, the expected valuation of uninformed bidders is the mean valuation μ. That is

$$v_i=\begin{cases}\omega_i & \text{if bidder } i \text{ becomes informed}\\ \mu & \text{if bidder } i \text{ stays uninformed}\end{cases}.$$

We focus on the symmetric equilibrium and the direct revelation mechanisms $\{q_i(v), t_i(v)\}_{i=1}^n$. The timing is the same as before. The seller first announces the mechanism. Each bidder chooses to become informed with probability $p\in[0,1]$. The seller compares revenue under different p's, and chooses $\{q_i(v), t_i(v)\}$ to implement the optimal p.

If information cost c is very low, then the optimal $p=1$, and the standard Myerson auction is optimal. However, if c is very high, then the optimal $p=0$, and the posted price $r=\mu$ is optimal. Thus, there exists a cost region $c\in[\underline{c},\overline{c}]$ such that bidders play mixed strategies with $p\in(0,1)$.[28] We will focus on this case.

[28] With some algebra, one can actually identify the cost thresholds $\underline{c}$ and $\overline{c}$.

If the seller chooses to implement p, the resulting distribution of the bidders' valuation is

$$H(v_i) = pF(v_i) + (1-p) \cdot 1_{\{v_i \geq \mu\}}.$$

Because a fraction of $(1-p)$ bidders choose to stay uninformed with μ as their expected valuation, there is a mass point at the mean valuation.

Since $\{q_i(v), t_i(v)\}$ is incentive compatible, we have

$$u(v_i) = u(\underline{\omega}) + \int_{\underline{\omega}}^{v_i} Q_i(s)\, ds.$$

The expected payment for bidder i is

$$\begin{aligned}
& \mathbb{E}_{v_i}[T_i(v_i)] \\
= \; & \mathbb{E}_{v_i}\left[v_i Q_i(v_i) - u(\underline{\omega}) - \int_{\underline{\omega}}^{v_i} Q_i(s)\, ds\right] \\
= \; & p\int_{\underline{\omega}}^{\overline{\omega}} \left[v_i Q_i(v_i) - \int_{\underline{\omega}}^{v_i} Q_i(s)\, ds\right] f(v_i)\, dv_i + (1-p)\left[\mu Q_i(\mu) - \int_{\underline{\omega}}^{\mu} Q_i(s)\, ds\right] - u(\underline{\omega}) \\
= \; & p\int_{\underline{\omega}}^{\overline{\omega}} \left[v_i - \frac{1-F(v_i)}{f(v_i)}\right] Q_i(v_i) f(v_i)\, dv_i + (1-p)\left[\mu Q_i(\mu) - \int_{\underline{\omega}}^{\mu} Q_i(s)\, ds\right] - u(\underline{\omega}) \\
= \; & \int_{\underline{\omega}}^{\overline{\omega}} \left[pv_i - p\frac{1-F(v_i)}{f(v_i)} - (1-p)\frac{1_{\{v_i \leq \mu\}}}{f(v_i)}\right] Q_i(v_i) f(v_i)\, dv_i + (1-p)\mu Q_i(\mu) - u(\underline{\omega}) \\
= \; & \int_{\underline{\omega}}^{\mu} \left[v_i - \frac{1-pF(v_i)}{pf(v_i)}\right] Q_i(v_i) g(v_i)\, dv_i + \int_{\mu}^{\overline{\omega}} \left[v_i - \frac{1-F(v_i)}{f(v_i)}\right] Q_i(v_i) g(v_i)\, dv_i \\
& + (1-p)\mu Q_i(\mu) - u(\underline{\omega}) \\
= \; & \mathbb{E}_{\theta}[J(v_i) q_i(v_i, v_{-i})] - u(\underline{\omega}),
\end{aligned}$$

where

$$J(v_i) = \begin{cases} v_i - \frac{1-pF(v_i)}{pf(v_i)} & \text{if} \quad v_i < \mu \\ \mu & \text{if} \quad v_i = \mu \\ v_i - \frac{1-F(v_i)}{f(v_i)} & \text{if} \quad v_i > \mu \end{cases}.$$

Therefore, the seller's revenue is

$$\mathbb{E}[\pi_s] = \sum_{i=1}^{n} \mathbb{E}_v[J(v_i) q_i(v_i, v_{-i})] - nu(\underline{\omega}).$$

The information acquisition constraint in this setting is:

$$\mathbb{E}_F[u(v_i)] - u(\mu) \geq c, \;\; \forall v_i \in [\underline{\omega}, \overline{\omega}],$$

where the expectation is taken with respect to $F(\cdot)$. We can calculate the payoff of both informed and uninformed bidders and rewrite the information acquisition constraint as

$$\int_{\underline{\omega}}^{\overline{\omega}} \left[\frac{1-F(v_i)}{f(v_i)} - \frac{1_{\{v_i \leq \mu\}}}{f(v_i)}\right] Q_i(v_i) f(v_i)\, dv_i \geq c.$$

Again, the (IR) constraint is binding: $u(\underline{\omega}) = 0$. With the reformulated (IC) and (IA) constraints, we can rewrite the seller's optimization problem as

$$\begin{aligned}
& \max_{q_i(v)} \sum_{i=1}^{n} \mathbb{E}_v \left[J(v_i)\, q_i(v_i, v_{-i}) \right] \\
s.t. \quad (1) \quad &: \quad 0 \leq q_i(v_i, v_{-i}) \leq 1; \sum_{i=1}^{n} q_i(v_i, v_{-i}) \leq 1 \\
(2) \quad &: \quad Q_i(v_i) \text{ is nondecreasing in } v_i \\
(3) \quad &: \quad \int_{\underline{\omega}}^{\overline{\omega}} \left[\frac{1 - F(v_i)}{f(v_i)} - \frac{1_{\{v_i \leq \mu\}}}{f(v_i)} \right] Q_i(v_i)\, f(v_i)\, dv_i \geq c. \qquad (29)
\end{aligned}$$

Let λ denote the Lagrangian multiplier for the information acquisition constraint (29). With some algebra, we can simplify the Lagrangian into

$$L = \sum_{i=1}^{n} \mathbb{E}_v \left[J^*(v_i)\, q_i(v_i, v_{-i}) \right] - \lambda c,$$

where the modified virtual surplus function

$$J^*(v_i) = \begin{cases} v_i - \frac{1 - pF(v_i)}{pf(v_i)} - \frac{\lambda}{n} \frac{F(v_i)}{pf(v_i)} & \text{if} \quad v_i < \mu \\ \mu & \text{if} \quad v_i = \mu \\ v_i - \frac{1 - F(v_i)}{f(v_i)} + \frac{\lambda}{n} \frac{1 - F(v_i)}{pf(v_i)} & \text{if} \quad v_i > \mu \end{cases}. \qquad (30)$$

One can show that as n is large, $\lambda > 0$. That is, the seller's revenue is higher when more bidders become informed.

The following proposition shows that the optimal auction adjusts the reserve price toward mean valuation to provide bidders with incentive to acquire information.

Proposition 8

If F has an increasing hazard rate, $c \in [\underline{c}, \overline{c}]$, and n is large, then the optimal reserve price is adjusted toward the mean valuation μ. That is, (1) if $r_{\overline{\alpha}} > \mu$, then the optimal reserve price $\mu \leq r^* < r_{\overline{\alpha}}$; (2) if $r_{\overline{\alpha}} = \mu$, then $r^* = \mu$; (3) if $r_{\overline{\alpha}} < \mu$, then the optimal reserve price $r_{\overline{\alpha}} < r^* \leq \mu$.

Proof: Given $J^*(\cdot)$ is single-crossing zero, the optimal reserve price is the smallest r such that $J^*(r) \geq 0$. Consider the case $r_{\overline{\alpha}} > \mu$. Suppose $r^* \notin [\mu, r_{\overline{\alpha}}]$. Then either $r^* < \mu$ or $r^* > r_{\overline{\alpha}}$. If $r^* < \mu$,

$$r^* - \frac{1 - F(r^*)}{pf(r^*)} - \frac{\lambda}{n} \frac{F(r^*)}{pf(r^*)} < 0,$$

a contradiction to the assumption that $J(r^*) \geq 0$. If $r^* > r_{\overline{\alpha}}$, notice that

$$r_{\overline{\alpha}} - \frac{1 - F(r_{\overline{\alpha}})}{f(r_{\overline{\alpha}})} + \frac{\lambda}{n} \frac{1 - F(r_{\overline{\alpha}})}{pf(r_{\overline{\alpha}})} > 0,$$

contradicting to the fact that r^* is smallest r such that $J^*(r) \geq 0$. The other two cases can be proved analogously. ■

Proposition 8 shows that the simple rule for adjusting the reserve price is robust to the discrete information acquisition specification.

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