

A Leverage-based Model of Speculative Bubbles*

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Abstract

This paper provides a framework for exploring the role of various policies in giving rise to or ruling out the possibility of speculative bubbles. As in previous work by Allen and Gorton (1993) and Allen and Gale (2000), a bubble can occur in my model because traders are assumed to purchase assets with borrowed funds. My model adds to this literature by allowing creditors and traders to enter into a general class of contracts, as well as allowing speculators to trade strategically. The paper examines whether it is possible to rule out speculative bubbles through restrictions on the types of contracts speculators can use, down-payment requirements, and increases in short-term rates, respectively.

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Introduction

The spectacular rise and fall of stock prices in the late 1990s and housing prices in the mid 2000s have been described by many pundits as examples of asset bubbles. Economists typically use the term “bubble” to mean that the price of an asset differs from its “fundamental” value, i.e. the present discounted value of dividends generated by the asset. Whether these episodes accord with this definition is difficult to ascertain. However, the mere notion that asset prices may have become unhinged from fundamentals during this period has affected discussions about macroeconomic policy. For example, there are some who have criticized the aggressive easing pursued by the Federal Reserve in response to the 2001 recession on the grounds that it may have led to the emergence of bubbles in asset markets. Others have faulted the Fed in its regulatory capacity for permitting the proliferation of exotic loan contracts that allegedly invited speculation, e.g. offers of low initial or “teaser” rates that are eventually reset to higher rates over the duration of the loan. Even setting aside the question of whether assets were truly overvalued during this period, it is hard to evaluate the merit of these critiques. This is because they are based on intuitive arguments rather than a clearly articulated channel for how these policies affect the possibility of asset bubbles.

The main difficulty with modelling such a channel is that in many standard economic models, bubbles cannot arise under any circumstances. This was demonstrated by Tirole (1982), who derived conditions for ruling out the possibility of bubbles in equilibrium. Although there are several models that violate these conditions and allow bubbles to emerge, many of these have been criticized as implausible or not conducive for policy analysis. One prominent example are overlapping generation models of money such as Samuelson (1958) and Diamond (1967), which Tirole (1985) emphasized could be viewed as models of bubbles. Bubbles can only emerge in these models if the economy grows at least as fast as the riskless rate of return on savings; yet Abel, Mankiw, Summers, and Zeckhauser (1989) show that a generalization of this prediction is rejected empirically. Santos and Woodford (1997) further argue that the bubbles that emerge in these models are theoretically fragile, since they would cease to exist as long as even some agents who own a non-vanishing share of the aggregate endowment had infinite horizons. Other theoretical examples assume agents have different prior beliefs over the fundamental value of the asset, e.g. Harrison and Kreps (1978), Allen, Morris, and Postlewaite (1993), and Scheinkman and Xiong (2003), or that some agents trade in a way that does not depend on fundamentals, e.g. DeLong, Shleifer, Summers, and Waldmann (1990). But without a model for why agents disagree about fundamentals or ignore them when trading, it is hard to predict how changes in policy will affect the possibility of speculative bubbles. Nor is it obvious how confident policymakers should be in their own beliefs when they know private agents disagree about fundamentals.

An alternative theory of bubbles, which inspires the present paper, was developed by Allen and Gorton (1993) and Allen and Gale (2000). These papers emphasize the role of agency problems as a source of bubbles. More specifically, they consider environments in which agents enter into contracts with financiers who cannot monitor what agents do with the funds they receive. Allen and Gorton (1993) show that this feature can give rise to a speculative bubble in which the price of an intrinsically worthless asset is

repeatedly bid up until some random point in time at which it collapses. They assume traders rely on profit-sharing contracts that entitle them to a fraction of any positive profits they earn. Unfortunately, this makes it difficult to use their model to analyze the effect of changing interest rates or the set of contracts agents can use. In fact, given the asset in their model is intrinsically worthless, one can show that financiers could screen out speculators if they coordinated to offer debt contracts rather than profit-sharing contracts. Allen and Gale (2000) develop a model in which a bubble emerges even though creditors can offer debt contracts. But theirs is not a model of “speculative” bubbles in the sense of Harrison and Kreps (1978), who define speculative behavior as a willingness to pay more for an asset for the option to resell it in the future. This distinction is important for thinking about the impact of policy on bubbles, since agents may behave differently if they plan to sell an asset than if they plan to hold on to it.

This paper builds on these last two models. More specifically, it examines the possibility of speculative bubbles when agents can design financial contracts in any way they see fit. I show that when agents expect the price of an asset to keep drifting away from its fundamental value for some time, financiers might be unable to design contracts that discourage motivated speculators from trading the asset, at least under certain conditions. The other novel feature of my model is that it allows speculators to trade strategically. By contrast, Allen and Gorton (1993) assume traders have a bliss point over consumption, and sell the asset when they no longer value the profits they could earn holding on to it. While this assumption greatly simplifies their analysis, it ignores the fact that creditors may seek to influence the trading strategies of the speculators they lend to. Indeed, one of the implications of the model is that financiers will offer speculators distinct contracts that encourage them to unload such assets quickly.

My model also offers new insights regarding the role of policies in either facilitating or curtailing the possibility of speculation, thus extending the work of Allen and Gale (2004) on the policy implications of such models of bubbles. First, I show that contracts with low initial rates that are reset later on over the course of the loan emerge endogenously when creditors expect some of their borrowers are speculating. These credit arrangements may thus be a response to, rather than a cause of, speculation. Precluding lenders from offering these financial products might therefore not only fail to curb speculation, but also expose creditors to greater risk. Another implication of the model is that the possibility of bubbles depends on the path of interest rates over the entire period during which a bubble could emerge and not just the level of the interest rate at any one point in time. Thus, a dramatic but *temporary* cut in real rates, like the one the Fed engineered in 2003, will not necessarily give rise to a bubble as is often argued by pundits.

The paper is organized as follows. Section 1 reviews the difficulties in modelling bubbles and outlines the features of my model that overcome these difficulties. Section 2 lays out the formal analysis. Section 3 solves the contracting problems between borrowers and financiers. Section 4 characterizes the equilibrium. Section 5 explores the effects of various policy prescriptions in the model. Section 6 concludes.

1 Overview

Before describing the model, it will be useful to review the difficulties in modelling speculative bubbles and how my model overcomes them. I consider a finite environment in which there is a known terminal date, agents have bounded amounts of resources, and assets trade hands only finitely many times. I focus on such an environment because, as pointed out in Tirole (1982), bubbles can emerge in infinite environments even without the frictions emphasized here.¹ Choosing an environment in which a bubble would not occur absent these frictions serves to highlight their role. Consider an asset that pays some dividend stream. Agents arrive at an exchange at random times prior to the terminal date and can buy the asset only once they arrive. The asset will be described as a bubble if it trades above the expected value of the dividends it has yet to pay. The question is whether this could ever occur in equilibrium. One reason agents might buy an overvalued asset is speculation, i.e. rather than hold on to collect its dividends, they hope to sell it for a higher price than they paid for it. But this intuition cannot sustain a bubble. Since agents have finite resources, the price of the asset must be bounded. As we approach the terminal date, there will be less scope for the asset to appreciate further in price, and the expected profits from selling it must tend to zero. By contrast, buying an overvalued asset and not reselling it incurs an expected loss bounded away from zero. If the asset trades hands finitely many times, the probability an agent will fail to sell it is bounded away from zero. Hence, at dates sufficiently close to the terminal date, no trader would expect to gain from buying a bubble. But agents would then refuse to buy the asset earlier, knowing they would have no one to sell it to later. The bubble unravels, and the asset never trades above its expected value.

As Allen and Gorton (1993) pointed out, this unravelling need not occur if agents speculate with borrowed funds rather than their own. In this case, traders no longer incur a loss if they don't sell the asset: the creditor is the one who puts up the funds to buy the asset but can at most collect back the dividend. As long as speculators can keep part of the positive profits they could realize from speculation, they would be willing to buy the asset close to the terminal date. Hence, a bubble could emerge if creditors agree to fund speculators and if the contracts they offer leave speculators with some share of the profits they might accrue. Since creditors expect to incur losses on speculators who purchase the asset close to the terminal date, they will never fund such speculators if they could readily identify them. We therefore need to assume there are some non-speculators who wish to borrow at similar times as speculators and can repay their debts, and that creditors cannot distinguish the two types. Allen and Gorton (1993) rely on a similar assumption.

In addition, we need to ensure that financiers offer speculators a contract that allows them to keep some of the profits they generate. Allen and Gorton (1993) assume financiers offer traders to keep a share of the profits if they sell the asset. While this contract is an equilibrium in their model, it will not survive as an equilibrium more generally. This is because the expected profits from reselling the asset tend to zero in their

¹For example, Tirole (1985) showed that in overlapping generation models with infinite horizons, infinitely many traders, and traders with arbitrarily large endowments, bubbles can emerge in some circumstances. Of these three features, the one that allows a bubble to emerge is the possibility of infinitely many traders.

model as we approach the terminal date. Creditors thus have an incentive to charge borrowers who show up close to the terminal date positive interest and render speculation unprofitable. But then the bubble in their model would unravel. To avoid this, I rely on a feature borrowed from Allen and Gale (2000), namely that dividends are stochastic and with some probability are large. Competition among creditors will drive the interest charged to non-speculators down until it just covers the expected losses from speculators. As long as the largest realization of dividends exceeds this rate, traders who buy the asset can guarantee themselves positive expected profits even if they don't sell the asset.

To recap, the essential elements of my model that allow a bubble to emerge are as follows:

1. Agents can buy assets using borrowed funds, and face limited liability.
2. Dividends on the asset are stochastic and with positive probability are large.
3. Credit markets are competitive.
4. There are some agents who are willing to borrow not for the purpose of speculation.
5. Creditors cannot distinguish between speculators and non-speculators.

Assumptions 1-3 ensure borrowers can profit from speculation in equilibrium. Assumptions 4-5 ensure creditors will finance speculators. These elements are a hybrid of Allen and Gorton (1993) and Allen and Gale (2000), and at different times agents behave in line with either one model or the other. Agents who purchase the asset late hold on to it in case it pays out large dividends, as in Allen and Gale (2000). Early agents purchase the asset in hope of selling it later at a higher price, as in Allen and Gorton (1993).

2 The Model

The model is characterized as follows. As in Allen and Gorton (1993), I assume trade occurs in continuous time, with a horizon that terminates at some finite date normalized to 1. Neither of these features is essential. In particular, one could obtain the same results in a discrete-time model with at least three periods, allowing speculators who buy the asset in the first period to choose not just whether to sell the asset but also when. Likewise, the horizon of agents could be infinite without affecting the key results.

There is a single, indivisible asset that cannot be sold short and is endowed to some agent, henceforth known as the original owner, at date 0. For simplicity, I assume the asset pays a single dividend d , where

$$d = \begin{cases} D & \text{with probability } \epsilon \\ 0 & \text{with probability } 1 - \epsilon \end{cases} \quad (1)$$

and D and ϵ are both positive. As will become clear below, a bubble can only occur while the value of d is unknown, so the relevant horizon is the life of the asset. Hence, I assume d is not revealed prior to date 1. I could alternatively allow the date at which d is determined to be distributed over some interval that includes date 1, but this would yield little new insight. Agents do not discount, so the fundamental value of the asset is just the expected payoff ϵD at date 1. I focus on the limiting case where $\epsilon \rightarrow 0$ and D is large,

in a sense made precise below. For example, the asset could represent real estate in a booming area that might be hit by a large migration wave, making land scarce and leading to temporarily high rents on even the least desirable properties. Alternatively, the asset could represent an equity stake in a firm that owns a patent which may or may not pan out, but will be enormously profitable if it does. Since a large value of D is essential for sustaining a bubble, the model predicts bubbles can only emerge under some circumstances, e.g. a booming region prone to a temporary shortage or an era of technical change that allows large profits.

Let $p(t)$ denote the price of this asset at time t . Traders take the path $p(t)$ as given and know it in advance. I define an asset as a *bubble* if (i) there exists some date t such that $p(t) > \epsilon D$, i.e. if the price of the asset exceeds its fundamental value, and (ii) the asset will trade at date t with positive probability. I further define an asset to be a *speculative bubble* if an agent who buys the asset at date t assigns positive value to the right to sell it, in line with the definition of speculation in Harrison and Kreps (1978). At a minimum, this requires that there exists some date $s > t$ such that $p(s) > p(t)$.

In what follows, I adopt the approach of Allen and Gorton (1993) of treating $p(t)$ as exogenous and focusing on the narrower question of whether optimizing agents want to and are able to trade the asset given this path. The determination of asset prices is a distraction for questions such as whether leveraged traders would like to speculate or whether creditors can design contracts to discourage speculation. Nevertheless, in a companion paper I show that speculative bubble paths can emerge as market clearing equilibria in the type of models I consider. In particular, as I discuss in the Conclusion, one can construct equilibrium speculative bubbles in which when the asset is traded, its price depends only on calendar time as I assume here. Given my focus on speculative bubbles, I restrict attention to paths that satisfy the following assumptions:

Assumption A1: $p(t)$ is continuous and increasing in t for $t \in [0, 1)$.

Assumption A2: $\epsilon D \leq p(t) \leq 1$ for all $t \in [0, 1)$.

Thus, agents believe the price of the asset keeps rising further above its fundamental value until date 1, yet is strictly bounded. The reason for this bound will become apparent below. I now describe the rest of the model, and in the next section I examine whether agents facing these prices end up trading the asset.

As noted in the previous section, traders who rely on their own funds would refuse to buy an overvalued asset near the terminal date. To prevent the bubble from unraveling, then, agents must be able to purchase the asset with borrowed funds near the terminal date. For simplicity, I assume none of the agents who can trade in the asset own resources, i.e. all must rely on creditors, although I relax this assumption later. There is a large number of creditors, each with vast wealth, and their total resources exceed what agents wish to borrow. Agents can contact any creditor, but can enter into a contract with only one. Since exclusivity imposes fewer constraints on what a contract can achieve, traders would be willing to commit in this way.

The asset market is organized as follows. Agents can buy the asset only at specific and exogenously set dates. This assumption allows me to avoid dealing with agents trying to strategically time their trades. To

be concrete, suppose agents reside on an island, and all economic activity takes place at the center of the island. Agents must set off for the center at date 0 if they wish to trade. They cannot contact a creditor or purchase the asset before they arrive. Once they arrive, they must immediately secure funds and purchase the asset, or else leave. A trader who sells the asset must also leave. If an agent manages to buy the asset, he stays at the center and must decide whether to sell the asset if and when a new buyer arrives.

To make the decisions of traders non-trivial, I assume the number of traders and the dates at which they arrive are random. Neither assumption is necessary for sustaining a bubble. However, if both $p(t)$ and the arrival times were deterministic, the original owner should wait to sell the asset when its price is highest, and the trader who buys it would not be able to profit from selling it. Not only is this type of market timing unrealistic, but it rules out speculation: traders who buy the asset would never end up selling it. By contrast, if the arrival of future traders were uncertain, a trader who bought the asset might subsequently resell it. Let N denote the number of potential traders, and t_1, \dots, t_N denote the times at which these traders arrive. To make the analysis tractable, it will be convenient to impose particular distributional assumptions. Let N be distributed as a Poisson(λ), i.e.

$$\Pr(N = n) = \frac{e^{-\lambda} \lambda^n}{n!} \text{ for } n = 0, 1, 2, \dots \quad (2)$$

and let the arrival times of the N individuals be independent and uniformly distributed over $[0, 1]$, i.e.

$$\Pr(t_n \leq u) = u \text{ for } u \in [0, 1] \quad (3)$$

The fact that the number of traders can be infinite is not essential. In fact, even though the potential number of traders is unbounded, the number of times the asset will be traded in equilibrium will be bounded. The virtue of this assumption lies in its implication that the number of traders who arrive before any date is independent of the number who arrive after it.² This independence implies traders can ignore the number of traders who already arrived and trade based solely on the time left until the terminal date.

Next, I turn to credit markets. Since traders own no resources, they would need to secure financing to be able to buy the asset. I assume agents incur a tiny utility cost to enter a financial contract. This ensures they will only purchase the asset if they expect to earn strictly positive profits. Note that as we near the terminal date, creditors would prefer not to finance speculators. This is because, as shown in the Appendix, the probability that at least one more agent arrives after date $s \in [0, 1)$ is

$$Q(s) = 1 - e^{-\lambda(1-s)}$$

This implies $\lim_{s \rightarrow 1} Q(s) = 0$, so a speculator who buys the asset close to the terminal date will face low odds of selling it. But since $p(s) > \epsilon D$ for all $s \in (0, 1)$, the creditor would expect to recoup less on

²To see this, note that these assumptions are equivalent to assuming independent arrivals with a constant arrival rate. That is, if individuals arrived at constant rate λ each instant, the number of arrivals in any period $[t_0, t_1]$ would be Poisson($\lambda(t_1 - t_0)$), and arrival times would be distributed uniformly over $[t_0, t_1]$.

average than he lent out to buy the asset, even if he required the trader to hand over all of his profits. Creditors would thus refuse to knowingly finance speculators close to the terminal date. Sustaining the bubble requires introducing additional agents whom it will be profitable for creditors to finance, together with the assumption that creditors cannot distinguish these from speculators.

The notion that speculators can blend in with other borrowers whom creditors would like to finance is plausible in certain contexts. For example, in the case of real estate, creditors cannot easily gauge whether an agent borrowing to buy property is doing so to speculate or because he really likes a certain house but has yet to earn enough income to buy it. If creditors can charge the latter type enough to offset the expected losses on speculators, they would agree to finance a random agent who asks to borrow to buy real estate. Similarly, it might be difficult to distinguish speculators from genuine investors who purchase equity as part of an arbitrage strategy or because they plan to take over a firm and improve its productivity.

It will be convenient to model the agents creditors want to finance as potential entrepreneurs who lack the funds to start production, although I could have equally modeled them as consumers who wish to make some purchase before they receive the income to pay for it. More precisely, entrepreneurs have access to a technology that requires commitment of one unit of resources prior to date 1 and yields $R > 1$ units of output at date 1. The essential aspect of this timing is that entrepreneurs cannot commit to make payments before the dividend d on the asset above is realized. If they could, creditors could punish speculators by penalizing any repayment made after the last date at which entrepreneurs are able to pay back, which would make it unprofitable to purchase the bubble at *any* date. Production by entrepreneurs in no way involves the asset; it is a separate activity whose only significance is that it creates additional demand for credit.

Since entrepreneurs own no resources, they too must borrow. I assume entrepreneurs approach creditors at random dates in a way that precludes creditors from deducing whether a borrower is an entrepreneur or a potential speculator. For tractability, I assume the number of entrepreneurs is a known constant M , that entrepreneurs arrive at the center at uniformly drawn times in $[0, 1]$, and that they must contact a creditor and start their project immediately upon arriving. These assumptions imply that the probability a random arrival is a non-entrepreneur is constant over time, specifically $\phi \equiv \frac{\lambda}{M + \lambda}$. Entrepreneurs require exactly one unit of resources. Since $p(t) < 1$ per Assumption A2, speculators require fewer resources than entrepreneurs, so creditors cannot screen out speculators by restricting the size of the loan. I allow entrepreneurs to trade in the asset instead of running the project if they wish. However, I assume they can only do one or the other, and impose restrictions on parameter values that ensure they prefer the project. That is, I assume the return to a project exceeds the maximum profit from buying and selling the asset:

$$R - 1 > \lim_{t \rightarrow 1} p(t) - p(0). \quad (4)$$

To ensure non-entrepreneurs want to speculate in equilibrium requires two additional parameter restrictions. First, I need D to be sufficiently large to make purchasing the asset profitable for non-entrepreneurs:

$$D \geq R. \quad (5)$$

Second, I need R to be large enough for lending to be profitable for creditors, i.e.

$$(1 - \phi)(R - 1) - \phi > 0. \tag{6}$$

The second condition implies that giving one unit resource to a random agent and collecting all of his output if he is an entrepreneur ensures strictly positive expected profits. Competition among creditors would then force creditors to charge entrepreneurs less than R . Speculators can thus guarantee themselves positive expected profits by pretending to be entrepreneurs and holding the asset to see if it pays D . Note that since $0 < p(t) < 1$, assumption (4) implies (6) when $\phi < \frac{1}{2}$.

Finally, I need to specify what creditors can observe after they extend credit to an agent. It is important that creditors not be able to independently learn an agent's type or actions after they extend credit. Otherwise, they could condition the contract on this information and punish speculators. Hence, creditors must not be able to observe an agent's exact wealth, which would reveal the actions he took. The fact that creditors cannot observe wealth implies they would be equally unable to tell if a fellow creditor approached them pretending to be an agent. This consideration imposes an important constraint on the contracts creditors can offer. In particular, it prevents them from offering to pay agents not to speculate, since if they did they would be flooded by other creditors posing as non-entrepreneurs and demanding payment.

However, creditors cannot be totally uninformed, or else they would refuse to extend credit. In the absence of any information, any agent could always claim he ran down his wealth and is unable to pay his obligations. I therefore endow creditors with the limited ability to observe if an agent has zero or positive wealth, but not his exact wealth, at date 1, but not beforehand. To motivate this assumption, note that in practice creditors can sue agents who claim to have exhausted their wealth (and thus unable to pay), but have no legal standing if agents makes no such claim. This feature allows creditors to threaten to seize the agent's wealth if he claims to have none, but not to make repayments contingent on wealth.

Lastly, because I assume there is only one unit of the asset, I need to assume creditors cannot observe the history of past loans. This is because the dates at which previous agents took out loans and the contracts they chose may reveal when the asset was last traded and whether its current owner would agree to sell it. More generally, if speculators buy different assets and creditors cannot observe what assets they buy, past lending would be irrelevant. For example, suppose there were many islands like the one above, each with its own specific asset, and the number of agents and their arrival times were independent across islands. Agents can only trade on their own island, but borrow from a common pool of creditors who supply all islands. In the limit as the number of islands becomes large, creditors who observe previous loans but not which islands they were directed to (i.e. which assets they involved) would attach no value to such data.

To summarize, agents in the model can engage in trade at random dates. For some, this involves operating some productive technology; others are only able to trade in some asset they believe is a speculative bubble. Either action requires the agent to first contact a creditor for financing. Creditors cannot tell what an agent

who approaches them intends to do. If an agent receives financing and opts to use his technology, he can do nothing until date 1 when it pays off. If an agent opts to purchase an asset, he must decide whether to sell it or wait whenever another trader offers to buy it. Whether an agent would find it profitable to buy the asset and whether and when he would choose to sell it depends on the price path $p(t)$; the distribution of the number of agents who might trade in the asset, as summarized by the parameter λ ; and the terms of the contract they sign. I therefore now turn to the contracting problem between creditors and agents.

3 Contracting

In analyzing the contracting problem between agents and creditors, I follow the customary route of modelling a contract as a direct revelation mechanism in which those who have private information (in this case, agents) disclose it to those who do not (in this case, creditors), and the parties take actions and transfer resources depending on what information is disclosed. Such a contract is said to be incentive compatible if those who have private information are willing to disclose it truthfully to other parties under the contract. Let X denote the set of all incentive-compatible contracts. An incentive compatible contract $x \in X$ is said to be an *equilibrium contract* if there exists no other contract $x' \in X$ that is strictly preferred to x by some agents and which yields strictly positive expected profits to the creditor who offers it.

To preview my results, I show that creditors cannot design a contract that attracts entrepreneurs but deters speculators. Under any equilibrium contract, non-entrepreneurs would like to buy the asset even though it is overvalued and even though – in fact, partly because – its price becomes more overvalued before it crashes. At best, creditors can minimize the cost of lending to speculators by offering them contracts that encourage them to sell the asset quickly, specifically contracts that backload interest payments.

Formally, the most general contract would require the agent to reveal his private information at every instant starting with his arrival date, denoted by t , and would stipulate transfers of resources between the creditor and agent given the history of these announcements. This private information amounts to the following: (1) whether he is an entrepreneur; (2) his actions since date t ; and (3) his cumulative income since date t . Let $\omega \in \{e, n\}$ denote whether an agent is an entrepreneur or not, respectively. The agent's actions since date t up to any $\tau \in [t, 1]$ can be summarized using a single variable $a_t(\tau)$ as follows:

$$a_t(\tau) = \begin{cases} \emptyset & \text{if the agent did nothing at date } t \\ t & \text{if the agent invested in the project at date } t \\ s \in (t, \tau] & \text{if the agent bought the asset at date } t \text{ and sold it at date } s \\ 1 & \text{if the agent bought the asset at date } t \text{ and has yet to sell it} \end{cases}$$

For notational convenience, define $a \equiv a_t(1)$. Finally, let $y_t(\tau)$ denote the cumulative income the agent earned between dates t and τ . For $\tau < 1$, $y_t(\tau)$ can be deduced from $a_t(\tau)$. At date 1, if $a = 1$, cumulative

income $y \equiv y_t(1)$ is equal to $-p(t)$ with probability $1 - \epsilon$ and $D - p(t)$ with probability ϵ . Otherwise,

$$y = \begin{cases} 0 & \text{if } a = \emptyset \\ R - 1 & \text{if } a = t \\ p(s) - p(t) & \text{if } a = s \in (t, 1) \end{cases}$$

The most general type of contract would require the agent to announce $\hat{\omega} \in \{e, n\}$ at date t , $\hat{a}_t(\tau)$ at each date $\tau \in [t, 1]$, and $\hat{y}_t(1)$ at date 1. Such a contract is rather cumbersome. To simplify the analysis, it will prove convenient to restrict attention to a reduced class of simple contracts in which the agent makes announcements and engages in transfers at only two dates, t and 1, as follows:

1. At date t , the agent announces a type $\hat{\omega} \in \{e, n\}$ and is given a transfer $x_t^0(\hat{\omega}) \geq 0$. At this point, the agent can choose $a_t(t)$ from the set of actions $A_t(\hat{\omega}, \omega)$, where

$$A_t(\hat{\omega}, \omega) = \begin{cases} \{\emptyset, t, 1\} & \text{if } \omega = e \text{ and } x_t^0(\hat{\omega}) \geq 1 \\ \{\emptyset, 1\} & \text{if } \{\omega = n \text{ and } p(t) \leq x_t^0(\hat{\omega})\} \text{ or} \\ & \{\omega = e \text{ and } p(t) \leq x_t^0(\hat{\omega}) < 1\} \\ \{\emptyset\} & \text{if } x_t^0(\hat{\omega}) < p(t) \end{cases}$$

2. At dates $\tau \in (t, 1)$, the agent makes no announcements, and chooses $a_t(\tau)$ from the set

$$A_\tau(\hat{\omega}, \omega) = \begin{cases} \{\tau, 1\} & \text{if } a_t(t') = 1 \forall t' < \tau \text{ and a buyer for the asset arrives at date } \tau \\ \{1\} & \text{if } a_t(t') = 1 \forall t' < \tau \text{ and no buyer for the asset arrives at date } \tau \\ \left\{ \inf_{t' \in (t, \tau)} a_t(t') \right\} & \text{if } \exists t' < \tau \text{ s.t. } a_t(t') \neq 1 \end{cases}$$

3. At date 1, the agent announces (\hat{a}, \hat{y}) from a set of reports $\Omega(\hat{\omega}, \omega, a, y)$ that an agent of type (ω, a, y) can report given he previously reported $\hat{\omega}$, and then transfers $x_t^1(\hat{\omega}, \hat{a}, \hat{y})$ to the creditor.

There are several reasons to restrict the set of reports (\hat{a}, \hat{y}) an agent can make at date 1. First, an agent cannot be forced to transfer resources he doesn't have, i.e. he cannot announce (\hat{a}, \hat{y}) such that

$$x_t^0(\hat{\omega}) + y < x_t^1(\hat{\omega}, \hat{a}, \hat{y})$$

Second, since creditors can verify if agents exhausted their wealth by date 1, an agent cannot misreport his income in a way that leaves him with zero wealth, i.e. if $x_t^1(\hat{\omega}, \hat{a}, \hat{y}) = x_t^0(\hat{\omega}) + y$, then \hat{y} must equal y . Finally, as I argue below, any general contract can be replicated with a simple contract in which the set of permissible reports Ω for each type is restricted in a particular way.

A simple contract will be defined as incentive compatible if it meets the following conditions:

IC-1: Agents prefer to report their type ω truthfully at date t :

$$\omega = \arg \max_{\hat{\omega}} E \left[\max_{a_t(\tau) \in A_\tau(\hat{\omega}, \omega)} \max_{(\hat{a}, \hat{y}) \in \Omega(\hat{\omega}, \omega, a, y)} \{x_t^0(\hat{\omega}) + y - x_t^1(\hat{\omega}, \hat{a}, \hat{y})\} \right] \quad (7)$$

IC-2: Given they reveal ω truthfully at date t , agents prefer to report their actions and income (a, y) truthfully at date 1:

$$(a, y) = \arg \max_{(\hat{a}, \hat{y}) \in \Omega(\omega, \omega, a, y)} E \left[\max_{a_t(\tau) \in A_\tau(\omega, \omega)} x_t^0(\omega) + y - x_t^1(\omega, \hat{a}, \hat{y}) \right] \quad (8)$$

IC-3: Creditors have no incentive to pretend to be agents and enter a contract with other creditors.

To better understand (IC-3), note that wealthy creditors can meet any obligations in a contract offered to agents with finite incomes. Hence, they can always pass themselves off as agents. Since a creditor can benefit by pretending to be an agent when the contract sets $x_t^1(\hat{\omega}, \hat{a}, \hat{y}) < x_t^0(\hat{\omega})$, (IC-3) requires a contract allow $x_t^1(\hat{\omega}, \hat{a}, \hat{y}) < x_t^0(\hat{\omega})$ only in the verifiable event that the agent has zero wealth, i.e. $x_t^0(\hat{\omega}) + \hat{y} - x_t^1(\hat{\omega}, \hat{a}, \hat{y}) = 0$.

I now argue that we can solve the more general contracting problem using these simple contracts. Recall that under a general contract, agents would announce their type ω upon their arrival at date t , then announce their actions $a_t(\tau)$ at all dates $\tau \in (t, 1]$, and at date 1 would announce their terminal income $\hat{y} = \hat{y}(1)$. At any date $\tau \in (t, 1)$, the contract would stipulate an amount x_t^τ to be transferred from the agent to the creditor given his announcements up to date τ , and at date 1 it would similarly stipulate an amount $x_t^1(\hat{\omega}, \{\hat{a}_t(t')\}_{t' \in [t, 1]}, \hat{y})$ to be transferred. The set of reports an agent can make at any date τ would depend not only on his type but also on all past announcements, i.e.

$$\hat{a}(\tau) \in \Omega(\hat{\omega}, \omega, \{\hat{a}_t(t')\}_{t' \in [t, \tau]}, a_t(\tau)).$$

To see that any outcome that can be achieved with a general contract can also be achieved with a simple contract, note that the set of actions $A_\tau(\hat{\omega}, \omega)$ an agent can take at date $\tau > t$ is unaffected by transfers after date t : the agent cannot purchase anything beyond date t , so his wealth at such dates is irrelevant. Transfers can, however, prevent an agent from pretending to be certain types, e.g. by asking an agent who claims to have sold the asset at date s to pay immediately, we prevent him from pretending he sold the asset earlier. But we can capture this using a simple contract with a restricted set $\Omega(\hat{\omega}, \omega, a, y)$ at date 1.

Formally, given a general contract, we can construct a related simple contract as follows. First, for each $(\hat{\omega}, \hat{a}, \hat{y})$, we set $x_t^1(\hat{\omega}, \hat{a}, \hat{y})$ to achieve the same terminal wealth as the under general contract, i.e.

$$x_t^1(\hat{\omega}, \{\hat{a}_t(t')\}_{t' \in [t, 1]}, \hat{y}) + \sum_{\{\tau \in (t, 1) \mid x_t^\tau \neq 0\}} x_t^\tau(\hat{\omega}, \{\hat{a}_t(t')\}_{t' \in [t, \tau]}). \quad (9)$$

This restriction ensures that announcing $(\hat{\omega}, \hat{a}, \hat{y})$ yields the same payoffs under the simple contract and the general contract. We then modify the set $\Omega(\hat{\omega}, \omega, a, y)$ in the simple contract to exclude any (\hat{a}, \hat{y}) for which there exists some date $\tau \in (t, 1]$ such that the implied pair $(\hat{a}(\tau), \hat{y}(\tau))$ does not belong to the set

$$\Omega(\hat{\omega}, \omega, \{\hat{a}_t(t')\}_{t' \in [t, \tau]}, a_t(\tau))$$

As long as we take into account the way in which the general contract limits what an agent can report, the incentives for the agent will be the same under the original general contract and under the modified

simple one. To find the best set of outcomes that could be achieved with general contracts, we simply pare down $\Omega(\widehat{\omega}, \omega, a, y)$ to the minimal set of reports an agent could be restricted to in state (ω, a, y) . Since this contract is subject to the fewest incentive constraints, it will weakly dominate all other contracts.

To construct this minimal set, note that an arriving agent has at most three options: operate a project, purchase the asset, or do nothing. Suppose first that he runs a project, i.e. $a_t(t) = t$. Would it be possible to use transfers between dates t and 1 to detect if this type misrepresented himself? Suppose we force agents who claim they are not running a project to make a temporary transfer that leaves them with the minimum they need for the action they claim. Thus, if they announce they bought the asset, so $\widehat{a}_t(t) = 1$, they must transfer $x_t^0(\widehat{\omega}) - p(t)$ to the creditor just after date t , and if they announce they did nothing, so $\widehat{a}_t(t) = \emptyset$, they must transfer $x_t^0(\widehat{\omega})$ just after date t . We then adjust $x_t^1(\widehat{\omega}, \{\widehat{a}_t(s)\}_{s \in [t, 1]}, \widehat{y})$ to keep cumulative transfers in (9) unchanged. Agents who run a project could not make these transfers, but agents who do not and are truthful could. Hence, transfers at intermediate dates can force entrepreneurs who run a project to report truthfully. The minimal set for an agent who operates a project is therefore

$$\Omega(\widehat{\omega}, \omega, t, R - 1) = \begin{cases} \{(t, R - 1)\} & \text{if } x_t^0(\widehat{\omega}) \geq 1 \text{ and } \omega = e \\ \emptyset & \text{else} \end{cases}$$

Next, suppose an agent purchases the asset at date t , so $a_t(t) = 1$. Can we use transfers between dates t and 1 to preclude this type from reporting he did not buy the asset? Once again, we can force agents who claim they did nothing, i.e. $\widehat{a}_t(t) = \emptyset$, to transfer $x_t^0(\widehat{\omega})$ to the creditor just after date t , and adjust $x_t^1(\widehat{\omega}, \{\widehat{a}_t(s)\}_{s \in [t, 1]}, \widehat{y})$ to keep cumulative transfers at date 1 unchanged. An agent who purchased the asset could not make this transfer, but an agent who did nothing could. However, there is no way to use transfers to prove an agent does *not* have resources. Thus, we cannot use transfers to catch an agent who falsely reports he ran a project. An agent who buys the asset must report truthfully if $x_t^0(\widehat{\omega}) < 1$, but otherwise he could claim he invested in the project.

Second, once the agent purchased the asset, can we use transfers between dates t and 1 to detect if he misreported the date s at which he sold it? Suppose we forced agents who announce they sold the asset at date $s \in (t, 1)$ to transfer $x_t^0(\widehat{\omega}) + p(s) - p(t)$ resources at date s , but adjust $x_t^1(\widehat{\omega}, \{\widehat{a}_t(s)\}_{s \in [t, 1]}, \widehat{y})$ to keep the cumulative transfers in (9) unchanged. This transfer would only be possible for an agent who truly sold at s . So an agent cannot falsely claim to sell the asset at a date he did not. However, since transfers cannot prove an absence of resources, the agent could still falsely report he did not sell the asset.

In sum, the minimal set of reports for an agent who bought the asset at date t and sold it at s is given by

$$\Omega(\widehat{\omega}, \omega, s, p(s) - p(t)) = \begin{cases} \left\{ \begin{array}{l} (s, p(s) - p(t)), (t, R - 1), \\ (1, -p(t)), (1, D - p(t)) \end{array} \right\} & \text{if } x_t^0(\widehat{\omega}) \geq 1 \\ \left\{ \begin{array}{l} (s, p(s) - p(t)), \\ (1, -p(t)), (1, D - p(t)) \end{array} \right\} & \text{if } p(t) \leq x_t^0(\widehat{\omega}) < 1 \\ \emptyset & \text{if } x_t^0(\widehat{\omega}) < p(t) \end{cases}$$

while the minimal set of reports for an agent who bought the asset and did not sell it by date 1 is the same whether $y = D - p(t)$ or $y = -p(t)$, and is given by

$$\Omega(\hat{\omega}, \omega, 1, y) = \begin{cases} \{(t, R-1), (1, -p(t)), (1, D-p(t))\} & \text{if } x_t^0(\hat{\omega}) \geq 1 \\ \{(1, -p(t)), (1, D-p(t))\} & \text{if } p(t) \leq x_t^0(\hat{\omega}) < 1 \\ \emptyset & \text{if } x_t^0(\hat{\omega}) < p(t) \end{cases}$$

Finally, suppose an agent does nothing, i.e. $a = \emptyset$. Would it be possible to use transfers between dates t and 1 to detect if this type misrepresents himself? By the same argument as above, we can use transfers to prevent the agent from pretending that he bought the asset at date t and sold it at some date s . However, there is nothing we could do between dates t and 1 to prevent the agent from reporting that he bought the asset but failed to sell it, or from reporting that he invested in the project. This implies that the minimal set of reports for an agent who does nothing is given by

$$\Omega(\hat{\omega}, \omega, \emptyset, 0) = \begin{cases} \{(\emptyset, 0), (t, R-1), (1, -p(t)), (1, D-p(t))\} & \text{if } x_t^0(\hat{\omega}) \geq 1 \\ \{(\emptyset, 0), (1, -p(t)), (1, D-p(t))\} & \text{if } p(t) \leq x_t^0(\hat{\omega}) < 1 \\ (\emptyset, 0) & \text{if } x_t^0(\hat{\omega}) < p(t) \end{cases}$$

Having constructed the minimal set Ω , I now derive some results that characterize the equilibrium contract. The proofs of the claims are delegated to an Appendix.

Claim 1: In equilibrium, an agent who does nothing or who holds on to an asset which pays no dividends at date 1 will have zero terminal wealth.

According to this claim, the equilibrium contract would require confiscating all of the resources of agents who show no positive income. This follows directly from (IC-3): otherwise, creditors could take out loans, claim they made no positive income, and then pocket resources left to them under the contract.

The next few claims show that the decisions of agents are uniquely determined in equilibrium.

Claim 2: Let $\epsilon \rightarrow 0$. Then agents will be able to buy the asset under the equilibrium contract if they wanted, i.e. there exists a $\hat{\omega} \in \{n, e\}$ such that $x_t^0(\hat{\omega}) \geq p(t)$.

Claim 3: Let $\epsilon \rightarrow 0$. Then non-entrepreneurs who have the chance to buy the asset will do so in equilibrium.

Claim 4: Let $\epsilon \rightarrow 0$. Then $x_t^0(e) \geq 1$ under the equilibrium contract and entrepreneurs will invest in the project in equilibrium.

Claim 5: Under the equilibrium contract, expected profits to the creditor must be zero.

These results can be understood as follows. Assumption (6) ensures that creditors will find it profitable to lend to an agent of unknown type if they could collect all of his output if he were an entrepreneur, even if they collect nothing from non-entrepreneurs. Hence, in equilibrium, creditors prefer lending to staying out of the credit market altogether. Since competition among creditors drives profits to zero, agents who claim to be entrepreneurs will be asked to repay less than R at date 1. Given $D \geq R$, a non-entrepreneur can ensure himself positive expected profits by pretending to be an entrepreneur, buying the asset, then holding it until date 1 to see if it pays out D and repay the amount demanded from entrepreneurs. Since creditors cannot pay non-entrepreneurs not to speculate, speculation must occur in equilibrium. All creditors can hope to do is minimize the costs of funding speculators by tailoring the terms of the contracts they offer.

In deriving the terms that will be offered to the different agents in equilibrium, I first normalize some features of the contract that are not uniquely determined but whose exact specification is irrelevant. First, I assume that the equilibrium contract stipulates any announcement $(\widehat{\omega}, \widehat{a}, \widehat{y})$ that is not feasible leaves the agent with zero wealth. Punishing patently untruthful reports to the maximum extent possible only serves to discourage misrepresentation, even if it is not always necessary. Second, I assume $x_t^0(e) = 1$ and $x_t^0(n) = p(t)$. From Claims 3 and 4 we know $x_t^0(e) \geq 1$ and $x_t^0(n) \geq p(t)$. If these inequalities were strict, we could always replace the original contract with a new contract \tilde{x} where

$$\begin{aligned} \tilde{x}_t^0(e) &= 1 \\ \tilde{x}_t^0(n) &= p(t) \\ \tilde{x}_t^1(\widehat{\omega}, \widehat{a}, \widehat{y}) &= \begin{cases} x_t^1(\widehat{\omega}, \widehat{a}, \widehat{y}) + 1 - x_t^0(e) & \text{if } \widehat{\omega} = e \\ x_t^1(\widehat{\omega}, \widehat{a}, \widehat{y}) + p(t) - x_t^0(n) & \text{if } \widehat{\omega} = n \end{cases} . \end{aligned}$$

This contract leaves all agents with the same expected utility as the original contract x . Under these normalizations, the terms of the contract for an agent who announces $\widehat{\omega} = e$ at date t reduce to the net transfer $r_t^e = x_t^1(e, t, R - 1) - x_t^0(e)$. Claim 5 implies $r_t^e < R - 1$. The next claim establishes $r_t^e > 0$:

Claim 6: Under the equilibrium contract, $r_t^e = x_t^1(e, t, R - 1) - x_t^0(e) > 0$.

Next, I turn to the terms for those who announce $\widehat{\omega} = n$. Let $V(n, \widehat{\omega})$ denote the expected utility for a non-entrepreneur under the equilibrium contract if he reports to be type $\widehat{\omega}$ at date t . (IC-2) implies

$$V(n, n) \geq V(n, e).$$

The next claim establishes that this constraint will hold with equality in equilibrium.

Claim 7: In equilibrium, the incentive constraint for type n will be binding, i.e. $V(n, n) = V(n, e)$

Hence, a non-entrepreneur expects to earn the same under the equilibrium contract as he could earn by pretending to be an entrepreneur, buying the asset with the funds he receives, and then trading optimally given he must either pay back what he borrowed plus r_t^e or else hand over all of his wealth. Denote the payoff

to this strategy by $V_0(r_t^\epsilon)$, i.e. $V_0(r_t^\epsilon) = V(n, e)$. The creditor will choose terms to maximize his expected profits subject to the non-entrepreneur achieving an expected utility of at least $V_0(r_t^\epsilon)$. Under certain conditions, these terms will differ from the simple debt contract offered to entrepreneurs in equilibrium. This is because under a simple debt contract traders hold on to the asset for longer than the creditor would like, raising the probability that the asset will not be sold and the creditor would have to absorb a loss. The creditor would prefer to offer speculators a different contract that induces them to sell the asset earlier.

One way to induce agents to sell the asset earlier is to backload interest payments and charge those who sell the asset early a lower rate than those who sell it late. In fact, creditors might wish to charge a negative rate to those who repay their debts early. However, (IC-3) requires agents to repay at least $x_t^0(n)$ if they sell the asset. Hence, the best creditors can do is charge a zero interest rate to those who sell the asset early and a high rate to those who sell late. Formally, the optimal contract is characterized by two parameters: a cutoff time $T_t \in (t, 1]$ and an amount R_t^n the agent must pay if he fails to sell the asset and it pays D . The terms of the contract are as follows: if an agent sells the asset, he must pay back only what he borrowed if he sells it before the cutoff date T_t , but hand over all of his wealth if he sells it beyond this date. If an agent does not sell the asset, he would have to pay R_t^n if $d = D$. Formally, we have

$$x_t^1(n, s, p(s) - p(t)) = \begin{cases} x_t^0(n) & \text{if } s < T_t \\ x_t^0(n) + p(s) - p(t) & \text{if } s \geq T_t \end{cases} \quad (10)$$

for any $s \in (t, 1)$, and

$$x_t^1(n, 1, D - p(t)) = R_t^n \quad (11)$$

Optimality further requires R_t^n to equal D if $T_t < 1$, i.e. if the agent ever hands over his wealth when he sells the asset, he must also do so if he keeps the asset and it pays out. If $T_t = 1$, however, R_t^n can potentially assume any value between $p(t) + r_t^\epsilon$ and D . Since $r_t^\epsilon < R - 1$, one can show there exists a unique (T_t, R_t^n) that leaves the agent with utility $V_0(r_t^\epsilon)$. The next claim establishes the above contract is optimal.

Claim 8: In equilibrium, the contract in (10) and (11) maximizes the expected profits to a creditor among all contracts that deliver utility $V_0(r_t^\epsilon)$ to a non-entrepreneur.

The terms of backloaded contract above may be indistinguishable from the terms offered to entrepreneurs. In particular, suppose that even when payments are backloaded all the way to date 1 (i.e. $T_t = 1$), agents refuse to sell the asset before date 1. In this case, keeping the agent's utility at $V_0(r_t^\epsilon)$ requires setting $R_t^n = r_t^\epsilon$, which is equivalent to offering him the same terms as entrepreneurs. Non-entrepreneurs who purchase the asset close to date 1 will behave precisely this way: the asset will not appreciate enough before date 1 to yield as much profit as waiting to see if the asset pays D . This is formalized in the next claim:

Claim 9: If $\epsilon > 0$, then there exists some date t^* such that $(T_t, R_t^n) = (1, x_t^0(n) + r_t^\epsilon)$ for all $t \in [t^*, 1]$.

The last claim reveals that speculators and entrepreneurs receive identical terms close to the terminal date. Would the terms appear distinct for agents who arrive at earlier dates? That depends on the path

$p(t)$. If the price $p(t)$ does not appreciate much over time, speculators would prefer to hold on to the asset, and all agents would be asked to pay back what they borrowed plus r_t^e at date 1. This includes the special case where $p(t) = \epsilon D$ for all t , i.e. where the price of the asset equals its fundamental. For this price path, non-entrepreneurs would want to buy the asset but not sell it, while the original owner would be indifferent about selling it. Hence, when the price equals fundamentals, the asset will trade hands at most once.

By contrast, if the price of the asset does appreciate significantly, agents might be willing to sell the asset. Creditors would then offer non-entrepreneurs distinct contracts in which they pay zero interest if they sell the asset early, as opposed to $r_t^e > 0$, but a high interest exceeding r_t^e if they sell it late or not at all. The fact that the equilibrium contract might be separating distinguishes this model from Allen and Gorton (1993) and Allen and Gale (2000), where all agents receive identical terms. The difference arises because agents in my model trade strategically, and creditors structure contracts to affect trading strategies. As a result, creditors in my model might know exactly which of the agents they fund engage in speculation given the terms they chose, in contrast with these previous models. However, creditors cannot use this information against speculators, or else speculators would blend in with entrepreneurs and hide their true intent.

4 Equilibrium

So far, I have characterized the terms entrepreneurs and non-entrepreneurs receive in equilibrium. This section shows how to explicitly solve for these terms, then highlights some features of the equilibrium.

4.1 Solving for Equilibrium

Recall that at any date t , the contract offered to non-entrepreneurs can be summarized with two variables, T_t and R_t^n . Since these variables are chosen to deliver a utility of $V_0(r_t^e)$ to speculators, we can express these variables as functions of the rate r_t^e charged to entrepreneurs, reducing the task of solving for an equilibrium to solving for the single variable r_t^e .

The first step in solving for r_t^e is to obtain expressions for T_t and R_t^n . This requires us to derive the expected utility of a speculator both under the backloaded contract given by (10) and (11) and under the simple debt contract offered to entrepreneurs. The latter expression is what I earlier referred to as $V_0(r_t^e)$. I begin by deriving this expression. As shown in the proof of Claim 4, the optimal trading strategy for a speculator facing a simple debt contract is to hold on to the asset until some date σ_t^* , then sell it to the first trader to arrive thereafter. According to the same proof, the probability another trader will arrive after some date s does not depend on the number of traders who arrived previously, and is given by

$$Q(s) = 1 - e^{-\lambda(1-s)}. \quad (12)$$

The distribution of the arrival of the time of the first trader beyond date s given at least one trader arrives

turns out to be similarly independent of how many traders arrived by date s , and the likelihood $f(x|s)$ that this arrival occurs at date $x > s$ is given by

$$\frac{\lambda e^{-\lambda(x-s)}}{1 - e^{-\lambda(1-s)}}. \quad (13)$$

Since the expected utility under the optimal trading strategy is just the value of waiting until date σ_t^* and then selling the asset to the next trader to arrive, we have

$$V_0(r_t^e) = Q(\sigma_t^*) \int_{\sigma_t^*}^1 (p(x) - p(t) - r_t^e) f(x|\sigma_t^*) dx + \epsilon [1 - Q(\sigma_t^*)] (D - p(t) - r_t^e) \quad (14)$$

To solve for σ_t^* , note that if $\sigma_t^* < 1$, the agent must be just indifferent at σ_t^* between selling the asset and waiting to sell it to the next trader to arrive after σ_t^* , i.e.

$$p(\sigma_t^*) - p(t) - r_t^e = Q(\sigma_t^*) \int_{\sigma_t^*}^1 (p(x) - p(t) - r_t^e) f(x|\sigma_t^*) dx + \epsilon [1 - Q(\sigma_t^*)] (D - p(t) - r_t^e) \quad (15)$$

We can therefore solve σ_t^* from (15) and use it to compute $V(r_t^e)$ in (14). If the value of σ_t^* that solves (15) exceeds 1, the agent would hold on to the asset until date 1, and (14) would reduce to $\epsilon(D - p(t) - r_t^e)$.

Next, I obtain an expression for the utility of an agent facing a backloaded contract characterized by (T_t, R_t^n) . The optimal trading strategy given this contract once again involves selling the asset from some cutoff date s_t^* on. Hence, the expected value under is given by

$$Q(s_t^*) \int_{s_t^*}^{T_t} (p(x) - p(t)) f(x|s_t^*) dx + \epsilon [1 - Q(s_t^*)] (D - R_t^n). \quad (16)$$

If the cutoff $s_t^* < 1$, then it must satisfy the indifference condition

$$p(s_t^*) - p(t) = Q(s_t^*) \int_{s_t^*}^{T_t} (p(x) - p(t)) f(x|s_t^*) dx + \epsilon [1 - Q(s_t^*)] (D - R_t^n). \quad (17)$$

Once again, if the value of s_t^* that solves (17) is larger than 1, the agent would not sell the asset before date 1, and (17) would reduce to $\epsilon(D - R_t^n)$.

To express (T_t, R_t^n) as functions of r_t^e , we equate (14) and (16). To do this, we use the following two-step procedure. First, for any value of r_t^e , we check if when $T_t = 1$ there exists an $R_t^n \in [p(t) + r_t^e, D]$ that equates (14) and (16). If not, we set $R_t^n = D$ and seek a value of $T_t \in (t, 1]$ that equates them. Once we obtain (T_t, R_t^n) as functions of r_t^e , we can also express the cutoff s_t^* in terms of r_t^e using (17).

To solve for the value of r_t^e in equilibrium, we use the fact that the expected profits of a creditor are equal to zero in equilibrium, as shown in Claim 5. Let ϕ_t denote the unconditional probability that an agent who wishes to borrow at date t is a non-entrepreneur. Creditors earn a profit of r_t^e per entrepreneur and all of the profits the speculator earns beyond date T_t . Expected profits from lending at date t are thus

$$\phi_t \left\{ Q(s_t^*) \int_{T_t}^1 [p(x) - p(t)] f(x|s_t^*) dx + [1 - Q(s_t^*)] (\epsilon R_t^n - p(t)) \right\} + (1 - \phi_t) r_t^e \quad (18)$$

Since we can express T_t , R_t^n , and s_t^* as functions of r_t^e , expected profits in (18) depends entirely on this variable. Solving for equilibrium thus amounts to finding the value of r_t^e that sets (18) to zero. However, we first need to know ϕ_t , the probability that an agent who wishes to borrow at date t is a non-entrepreneur.

In general, ϕ_t will differ from ϕ , the probability an arriving agent is a non-entrepreneur. This is because the agent who owns the asset at date t will not always agree to sell it, so an arriving non-entrepreneur might not be able to buy it and will thus not borrow. By contrast, entrepreneurs will always wish to borrow. Let $\pi(t)$ denote the probability that the agent who owns the asset at date t will be willing to sell it at that date. Then the probability an agent who wishes to borrow at date t is a non-entrepreneur is given by

$$\phi_t = \frac{\phi\pi(t)}{1 - \phi + \phi\pi(t)}$$

When $\pi(t) = 1$, this expression collapses to ϕ . But the probability that an agent will be able to buy an asset will generally not equal 1. First, as shown in the proof of Claim 8, the original owner of the asset follows a cutoff rule, and will agree to sell it only from some date s_0 on. Hence, $\pi(t) = 0$ for all $t \in [0, s_0)$, so that $\phi_t = 0$ as well. At $t = s_0$, a non-entrepreneur will necessarily be the first to have an opportunity to buy the asset, so $\pi(t) = 1$. Beyond date s_0 , an agent who shows up will be able to buy the asset with probability less than 1. The exact probability depends on the distribution of arrival times and the trading strategies of those who buy the asset before date t , i.e. on the values of s_τ^* for all $\tau \in [s_0, t)$.

To obtain a formula for how ϕ_t depends on variables up to date t , it will help to describe ϕ_t using order statistics. Consider a set of i.i.d. uniform random variables $\{T_1, \dots, T_N\}$, where N has a Poisson(λ) distribution and is independent of T_1 through T_N . If we name agents as 1 through N , then T_n corresponds to the arrival time of individual n . Let $\{T_{(1)}, \dots, T_{(N)}\}$ denote the ordered values in T_1, \dots, T_N , i.e. $T_{(n)}$ is the n -th smallest value in the set $\{T_1, \dots, T_N\}$. The variable $T_{(n)}$ corresponds to the time of the n -th arrival.

Next, consider a continuous function $S(t)$ where (i) $t < S(t) \leq 1$ for all $t \in [0, 1]$; and (ii) there exists a $t^* < 1$ such that $S(t) = 1$ for $t \in [t^*, 1]$. Let us use $S(\cdot)$ to construct a new sequence of random variables as follows. First, set $Y_0 \equiv 0$ and $L_0 = 0$. Next, for $j \geq 1$, define L_j and Y_j recursively as follows:

$$\begin{aligned} L_j &= \min \{n \leq N : T_{(n)} \geq S(Y_{j-1})\} \\ Y_j &= T_{(L_j)} \end{aligned}$$

Let $J+1$ denote the (random) number of variables Y_j thus constructed. Constructing the sequence $\{Y_j\}_{j=0}^J$ is analogous to compiling record values from a sequence of observations, where an observation is counted as a new record only if it exceeds the previous record value by a threshold that depends on the value of the previous record, i.e. Y_j must exceed $S(Y_{j-1})$ rather than Y_{j-1} .³ Define $S(0) = s_0$, the date at which the original owner would agree to sell the asset, and set $S(t) = s_t^*$ for all $t \in (0, 1)$, i.e. the date at which

³For a survey of the statistical literature that defines and studies the behavior of record values derived from random sequences, see Arnold, Balakrishnan, and Nagaraja (1998).

a speculator who bought the asset at date t would first be willing to sell it. With these assumptions, the sequence $\{Y_j\}_{j=1}^J$ corresponds to the list of dates at which the asset is traded, since a sale occurs only if a trader arrives after the cutoff set by the last trader who purchased the asset. The expression $\Pr(Y_j = t \text{ for some } j \mid T_{(n)} = t \text{ for some } n)$ would then correspond to the unconditional probability $\pi(t)$ that an agent would be able to buy the asset at date t . Appendix B derives an analytical expression for $\pi(t)$ that involves a finite sum of integrals involving the function s_τ^* for $t \in [s_0, t]$. That appendix also describes a numerical algorithm for solving ϕ_t for all $t \in [s_0, 1]$.

As an illustration, consider the following numerical example. Set the average number of traders $\lambda = 7$, the fraction ϕ to 0.1, R and D to 2 to satisfy (4) and (6), and ϵ to 0.05. Lastly, suppose $p(t)$ is given by

$$p(t) = \max(\epsilon D, t)$$

in line with Assumptions A1 and A2. For these values, s_t^* is monotonically increasing in t , which greatly simplifies the numerical analysis. The exact details behind the calculations are in Appendix B.

Figure 1 plots s_t^* , $\pi(t)$, r_t^e , and R_t^n against t , the date at which the contract is signed ($T_t = 1$ for all t and so is not shown). Note that the difference $s_t^* - t$ decreases with t , implying that traders who buy the asset later will be willing to sell it more quickly. This is because the chance of meeting another agent falls as we near the terminal date, so it will be less valuable to wait and try to sell the asset at a higher price.

The probability $\pi(t)$ that an agent who arrives at date t can buy the asset is non-monotonic in t . Starting from date s_0 , when the original owner is first willing to sell it, $\pi(t)$ is decreasing in t . This is because as more time passes, the odds increase that another trader already swooped in to buy the asset from its original owner and is not yet willing to sell it himself. But at the first instant the asset could trade for a second time, i.e. when an agent who bought the asset at s_0 would first be willing to sell it, $\pi(t)$ begins to increase with t . This is because at later dates it is more likely that *if* someone bought the asset from the original owner he would now agree to sell it. The non-monotonicity of ϕ_t gives rise to non-monotonicity in the remaining terms r_t^e and R_t^n . However, r_t^e generally rises with t , so entrepreneurs who arrive closer to the terminal date will pay higher rates. This is because the expected losses per speculator rise with t , so entrepreneurs must be charged more to offset these losses. By contrast, the interest $R_t^n - p(t)$ charged to speculators who hold on to the asset tends to decline with t towards r_t^e . This is because there is a smaller likelihood that speculators who buy the asset late will be able to sell it, so inducing them to reveal their type requires a smaller repayment in the likely event they don't sell it. Creditors will thus appear to offer speculators increasingly more generous terms (less onerous repayment) as we move closer to the peak of the bubble.

4.2 Discussion

Before we turn to policy, it is worth pausing to reflect what the model has to say about speculative bubbles. For example, under what conditions would bubbles emerge? What should we expect as the bubble unfolds?

With respect to what conditions allow a speculative bubble to emerge, the model is similar to Allen and Gorton (1993) and Allen and Gale (2000) in laying the blame for bubbles on an agency problem in which traders gamble with other people’s money. In relating this back to Tirole (1982), it is worth noting that while my model violates two of his conditions for ruling out bubbles, only one of these allows bubbles to emerge. More precisely, contrary to Tirole, I assume both that there are infinitely many potential traders and that the initial allocation of resources is inefficient. Although the number of traders is potentially unbounded, the number of times the asset will trade in equilibrium is bounded by a finite number, as demonstrated in Appendix B. Tirole’s argument for ruling out bubbles continues to hold when the number of times the asset changes hands is bounded, regardless of the number of traders. Hence, the reason a bubble can arise is that an inefficient initial allocation creates gains from trade between entrepreneurs and creditors that speculators can cut into. In addition, the model suggests that an important element for sustaining a bubble is that there be some probability that the asset yields a large payoff. Thus, unlike the monetary models cited in the Introduction, the model here suggests speculative bubbles should be associated not with intrinsically worthless assets but with assets that with some probability could have large intrinsic worth.

Next, the model makes several predictions as to what should happen as a bubble unfolds. One implication is that as we near the date beyond which the bubble is not expected to survive, traders will shift from relying on contracts that reward early repayment to simple debt contracts. Thus, to the extent that housing in certain regions in the U.S. was indeed subject to a bubble during the early to mid 2000s, the model suggests we should observe the use of mortgages with built-in teaser rates largely confined to the regions where housing prices were inflated, and that in these regions use of these contracts should fall *before* the decline in housing prices. Another implication, which will become apparent in the next section, is that only highly leveraged agents will be willing to purchase the asset close to the terminal date. Agents who use some of their own funds might be willing to purchase a bubble, but only early on, when the odds of finding a buyer are still high. Hence, the model predicts that the fall in the use of teaser rate contracts should have coincided with a low fraction of buyers willing to put up a significant share of their own funds to purchase housing. Finally, as we move closer to the terminal date, the frequency of trades tends to rise as speculators agree to sell their asset more quickly after they buy it. In short, the model suggests that if we observe that traders are almost exclusively leveraged, that the use of teaser-rate contracts is waning, and that assets are turning over rapidly, we can infer traders do not expect the asset to continue to appreciate further.

5 Policy and Bubbles

So far, policy plays no role in my model. However, the whole motivation for developing the model is to explore whether certain policies can cause or rule out speculative bubbles. Accordingly, this section explores three types of policies in the context of the model. First, I consider restrictions on the type of contracts creditors can offer, in line with the claim that the use of exotic financial contracts encourages speculation. Next, I consider policies that force agents to use some of their own funds to buy assets, i.e. down-payment

or margin requirements. Lastly, I consider changes in the opportunity cost of funds for lenders that may influence the terms they offer, i.e. interest rate policy. In each case, I examine whether the equilibrium contracts that emerge continue to reward traders for engaging in speculation.

5.1 Restrictions on the Type of Contracts Creditors can Offer

The equilibrium terms for speculators derived in Section 3 bear a striking resemblance to financial contracts that have gained popularity in recent years, specifically loan contracts that offer low initial or “teaser” rates that are eventually reset to higher levels if the loan is not repaid after some period. Although I consider simple contracts where agents make transfers only at date 1, one can reinterpret these contracts as if traders repaid their obligations once they sell the asset. Under this interpretation, speculators who repay their debt early are charged little in interest, while speculators who repay their debt late will be asked to pay large amounts of interest that may leave them with zero wealth. These types of contracts have recently been the target of heated criticism. On the one hand, some have argued that these types of contracts encourage speculation by luring in speculators with low initial rates. Others have decried the fact that onerous payment requirements may bankrupt borrowers. This has led to the suggestion that regulators ought to restrict the use of such contracts or prevent lenders from resetting payments to higher rates.

Since these types of contracts emerge endogenously in the model, we can examine what the model implies about such a policy. A key insight from the model is that these sorts of contracts can evolve in response to speculative bubbles rather than cause them. The reason creditors backload payments in the model is to induce agents who are already speculating to unload the asset more quickly by rewarding them with a lower rate for selling the asset early. Not surprisingly, then, the model implies that requiring flat-rate contracts will not deter speculation. Formally, the flat rate charged to all agents must fall below $R - 1$ to ensure zero profits to creditors, as implied by (6). Since $D - p(t) \geq R - 1$, non-entrepreneurs would still be able to guarantee themselves positive expected profit by pretending to be entrepreneurs, buying the asset, and holding it to date 1. Restricting lenders to flat-rate contracts exposes them to greater risk by increasing the chance that the traders they fund end up not selling the asset. Creditors would then need to charge a rate above the original rate r_t^e to offset these losses. This makes entrepreneurs worse off, but it also hurts speculators, whose original contract left them indifferent to a flat rate contract with rate r_t^e . Since creditors continue to earn zero profits, they will be unaffected. Hence, restricting the type of contracts creditors are allowed to offer makes no agent better off, and instead merely increases the cost of providing credit.

To be sure, the model abstracts from various issues raised by detractors of such contracts. For example, one of the arguments against the use of such contracts is that onerous payments may force agents to prematurely liquidate their assets, driving down the price of similar assets and leading otherwise solvent agents to default on their loans. This model does not capture such coordination problems. But it does highlight that allowing creditors to backload interest payments can serve a useful role in responding to extant speculation, and restricting the contracts that creditors can offer may be socially costly.

5.2 Preventing Traders from Speculating with Borrowed Funds

An alternative to restricting the type of contracts lenders can offer is to limit the degree to which agents can purchase assets using borrowed funds, at least temporarily when policymakers are concerned about bubbles. Since borrowing plays a central role in allowing a speculative bubble to emerge in the model, this seems like a natural policy to explore. One example of such a policy is the initial margin requirement for investors who purchase stocks. The authority to set margin requirement was entrusted to the Federal Reserve under the Securities and Exchange Act of 1934, and the Fed varied this rate nearly two dozen times within the forty-year period following the passage of this act. Although the Fed opted not to change this rate from 1974 on, there have been occasional calls that it should return to using margin requirements as a policy tool.⁴ Another example of such a policy is a down-payment requirement on real estate purchases, although in the U.S. this requirement is not directly set by any regulatory agency.

Since agents in my model are endowed with no resources, requiring them to self-finance even a small part of their purchases would preclude non-entrepreneurs from buying the asset (and would equally preclude entrepreneurs from initiating projects that yield positive social surplus). Down payment requirements can thus discourage speculation by making assets unaffordable. But this implication relies on the extreme feature of the model that agents own no resources. More realistically, even if traders had to self-finance their asset purchases, some could still presumably afford to buy assets if they were so inclined. Whether margin requirements could curb speculation in practice depends more on whether these requirements make trading in speculative bubbles *unprofitable* rather than *unaffordable*.

As an illustration, suppose that agents each owned θ units of resources, where $0 < \theta < \lim_{t \rightarrow 1} p(t)$, and that they were required to use these resources towards any purchase they make. That is, agents can credibly demonstrate they made the payment required by regulators. Entrepreneurs would then need to borrow an additional $1 - \theta$ to initiate their projects, while non-entrepreneurs would need to borrow $p(t) - \theta$ to buy the asset. Would non-entrepreneurs find it profitable to buy the asset in this case? Since creditors lose at most one unit of resources per non-entrepreneur they finance, the zero profit condition implies entrepreneurs will never be asked to repay more than $\frac{1}{1-\phi}$ in equilibrium. Hence, by reporting he is an entrepreneur and holding on to the asset until date 1, a speculator can guarantee himself expected profits of at least

$$\epsilon \left(D - \frac{1}{1-\phi} \right) - \theta \tag{19}$$

From (5) and (6), the first term in (19) is strictly positive. Hence, agents could guarantee themselves positive expected profits if the amount θ they had to stake were sufficiently small. This argument does not carry over if θ were large. For example, if θ were close to $p(t)$, the fact that $p(t) > \epsilon D$ would imply (19) will

⁴For example, Cecchetti (2005) writes: “For equity bubbles, economists have suggested adjusting margin requirements. Margin trading accounts for approximately 20% of total trading in US equity markets. Increasing the cost of these transactions during periods when prices have been rising quickly has the potential to keep bubbles from growing large.” Kwan (2000) also discusses (and criticizes) the use of margin requirements for discouraging speculation.

be negative, and it would no longer necessarily follow that speculators can guarantee themselves positive expected profits. Below I argue that large down payment requirements can indeed rule out speculative bubbles as equilibria. However, such requirements must be set at a permanently high level rather than increased temporarily. This is because agents who can afford to buy the asset would be willing to speculate at any date in which θ were low enough to make (19) positive. Hence, lowering requirements would allow for speculative trade while requirements are low. More interestingly, lowering margin requirements at a certain date might also make it profitable for agents to buy the asset earlier, even if required to do so with their own funds. Lifting margin requirements could thus render earlier margin requirements ineffective. This result stands in contrast to Allen and Gorton (1993) and Allen and Gale (2000), where agents would always refuse to buy a bubble if required to do so using their own funds.

Demonstrating the above result requires me to modify the model to allow agents to shoulder part of the cost of the asset on their own. Rather than assume all agents own some resources, I once again assume agents have no resources of their own, but I allow creditors who have ample resources to also trade in the asset if they wanted. Focusing on traders with vast wealth provides a natural benchmark, since if these traders are willing to buy the asset, margin requirements couldn't possibly deter speculation: traders who borrow to finance part of their asset purchases gain more from speculation than those who self-finance.

Formally, let K denote the number of creditors who can trade in the asset. Suppose K is $\text{Poisson}(\mu)$ and is independent of the number of non-entrepreneurs N . The sum $N + K$ is thus $\text{Poisson}(\lambda + \mu)$. Creditors who wish to trade in the asset must also travel to the center of the island, and their arrival times are also uniformly distributed over $[0, 1]$. Travelling does not impede the ability of creditors to be contacted by agents who wish to borrow, and they can engage in lending while commuting. But since trade occurs at the center of the island, creditors must wait until they arrive at the center of the island to buy the asset or to approach other creditors for a loan, just as agents do. I assume μ is large, in line with my assumption that there is a large number of creditors. Hence, if contracts made it profitable for creditors to travel to the center of the island and apply for a loan pretending to be agents, the probability that a random arriving agent is another creditor will be large. Since the equilibrium contract satisfies (IC-3), creditors will not benefit from borrowing, so if they wish to buy the asset they would be willing to do so with their own funds.

If agents were required to pay for any part of their purchases at some date t , non-entrepreneurs would be unable to buy the asset at that date (and entrepreneurs would be unable to invest). But whenever this requirement is lifted, even temporarily, non-entrepreneurs strictly prefer to buy the asset: equilibrium contracts would always make it profitable to pretend to be an entrepreneur, buy the asset, and see if it yields D . Thus, as noted above, completely stamping out speculative trade necessitates margin requirements at all dates. To see that lifting margin requirements might in addition nullify the effect of margin requirements at previous dates, suppose there were a positive margin requirement between date 0 and some date $t^* \in [0, 1]$, but not between t^* and 1. The next claim characterizes the equilibrium outcome in this environment:

Claim 10: Suppose positive margin requirements are applied only at dates $t \in [0, t^*]$, and the price path $p(t)$ is consistent with A1 and A2. Then in equilibrium,

- i. Wealthy agents are willing to purchase the asset using their own funds up to some date $t^{**} \in [0, 1)$.
- ii. Non-entrepreneurs are willing to purchase the asset at any date, but can only do so from date t^* on.
- iii. If $t^* < 1$, then for λ sufficiently large, $t^{**} > t^*$. Hence, the probability that an arriving agent will be both willing and able to buy the asset is positive at all dates $t \in (0, 1)$.

Claim 10 reveals that traders might be willing to stake their own wealth in asset bubbles if they assigned a high probability (consistent with a large λ) to meeting buyers in the future. Unlike leveraged buyers who can profit from buying the asset and seeing if it pays a high dividend, those who stake their own wealth only benefit by “riding the bubble” in the sense of Abreu and Brunnermeier (2003) and Temin and Voth (2004), i.e. by holding on to an asset whose price is rising and letting go of it before its price “pops”. Riding the bubble will not be profitable unless the odds of meeting another buyer before the bubble pops are large. Hence, if margin requirements forced traders to always use their own funds to buy the asset, a speculative bubble could not occur. This is because if there were a last date $t^{**} > 0$ at which agents are willing to buy the asset with their own funds, it would be unprofitable to buy the asset with one’s own funds just before t^{**} given the odds of selling it are almost nil. But then t^{**} couldn’t be the last such date. But if margin requirements were lifted sometime after t^{**} , leveraged traders would be willing to buy the asset at that point, making it profitable to ride the bubble earlier and sell to these leveraged traders.

It is worth noting that Claim 10 only concerns whether traders are *willing* to buy the asset, not whether they actually buy them. As can be shown using the proof of Claim 8, the last date t^{**} at which creditors would be willing to buy the asset directly is also the date at which the original owners are first willing to sell it. Intuitively, opting not to sell the asset is equivalent to paying $p(t)$ to hold on to it. If an agent finds it profitable to buy the asset using his own funds, the original owner would find it profitable to keep it. Hence, creditors will only be able to buy the asset at the single date t^{**} when both they and the original owners are indifferent about holding the asset. However, if we relaxed Assumption A1 so that $p(t)$ could be weakly non-increasing, such trades could occur over an interval rather than a single point in time.

5.3 Changes in the Opportunity Cost of Funds

Although some economists have advocated using margin requirements to curb asset bubbles, most have focused on interest rate policy. To explore the effects of such a policy, it helps to view creditors in the model as bank intermediaries rather than individuals lending their own wealth. Under this view, creditors facilitate financial transactions, but even if they lack resources when an agent approaches them for financing, they can secure such resources in a market where they can credibly prove that they will be able to repay these

obligations out of their future reserves. This setup coincides with the model as I've described it if creditors are able to borrow and lend to one another at zero interest. The virtue of this interpretation is that it allows us to consider the effect of changes in the rate on short-term (overnight) loans that banks implicitly extend to one another in my model. The latter is precisely the rate set by the Fed, and what many have argued ought to be changed in response to the prospect of bubbles. As the proposed reinterpretation suggests, we can view the Fed as determining the opportunity cost of funds for creditors, which in turn affects the contract terms between creditors and the agents who may eventually speculate. Note that the overnight rate here represents a real rate, while in practice the Fed sets a nominal rate. I am therefore implicitly assuming that the Fed can affect real rates, at least over the relevant horizon.

Formally, let $r_t^{FF} \geq 0$ denote the instantaneous rate of return on a loan made at date t , i.e. r_t^{FF} represents the limit of the return per unit time on a loan due at date $t + \Delta$ as $\Delta \rightarrow 0$. Let $R_{t,s}^{FF}$ denote the compound return between dates t and s , i.e.

$$R_{t,s}^{FF} = \exp\left(\int_t^s r_x^{FF} dx\right)$$

As the notation suggests, r_t^{FF} is meant to capture the Federal Funds rate. I assume the Fed can set this rate at whatever level it chooses, and that agents and creditors treat its path as given. I also assume that any agent with funds can save them at the same rate r_t^{FF} . This is equivalent to assuming a competitive banking sector in which depositors earn the same rate of return that banks face in the Federal Funds market.

Before proceeding with the analysis, I first need to examine how this modification affects the contracting problem between creditors and agents. Before, I was able to simplify the contracting problem by focusing on simple contracts in which agents only had to make announcements and transfers at dates t and 1. Since agents can earn an instantaneous return of r_t^{FF} on their funds, we can continue to focus on this reduced class of contracts even when $r_t^{FF} > 0$. The reason is that under this assumption, it is immaterial who holds on to resources that are not committed to an asset or a project: either party would earn the return r_t^{FF} . The only reason not to wait until date 1 to make transfers is that earlier transfers can be used to detect if an agent misrepresented himself, and we can capture this by restricting the agent's set of reports Ω .

While it is possible to continue restricting attention to simple contracts in which transfers occur only at two dates, the contracting problem will change a little when the cost of funds is positive. In particular, we need to modify the definition of income y to include interest income. Thus, we have

$$y = \begin{cases} (R_{t,1}^{FF} - 1) x_t^0(\hat{\omega}) & \text{if } a = \emptyset \\ (R_{t,1}^{FF} - 1) [x_t^0(\hat{\omega}) - 1] + R - 1 & \text{if } a = t \\ (R_{t,1}^{FF} - 1) [x_t^0(\hat{\omega}) - p(t)] + R_{s,1}^{FF} [p(s) - p(t)] & \text{if } a = s \in (t, 1) \\ (R_{t,1}^{FF} - 1) [x_t^0(\hat{\omega}) - p(t)] + d - p(t) & \text{if } a = 1, \text{ where } d \in \{0, D\} \end{cases}$$

Given this measure of income, I again define a contract to be incentive compatible if it provides incentives for agents to reveal their information truthfully and discourages creditors from pretending to be agents. These

conditions imply the same (IC-1) and (IC-2) as before, provided we define y as above, but will slightly alter the formulation of (IC-3). Since any borrower can earn a return of $R_{t,1}^{FF}$ on the funds he borrows by assumption, for a creditor not to benefit from pretending to be an agent, the contract must stipulate

$$x_t^1(\hat{\omega}, \hat{a}, \hat{y}) \geq R_{t,1}^{FF} x_t^0(\hat{\omega}) \quad (20)$$

for any report $(\hat{\omega}, \hat{a}, \hat{y})$ which if truthful would leave the agent with positive terminal wealth, i.e.

$$x_t^0(\hat{\omega}) + \hat{y} > x_t^1(\hat{\omega}, \hat{a}, \hat{y})$$

Finally, expected profits to creditors must be nonnegative, i.e.

$$E [x_t^1(\omega, a, y) - R_{t,1}^{FF} x_t^0(\omega)] \geq 0.$$

Note that profits must be nonnegative in expectation rather than for each realization of ω , since the contract is designed without knowing the agent's type.

Constraint (20) is key for understanding why an increase in the opportunity cost of funds can discourage speculation. It implies increasing the cost of funds leads creditors to demand higher repayments from borrowers. Agents can thus be forced to hand over any profits they earn from speculation, rendering this activity unprofitable. Intuitively, raising the cost of funds creates an alternative that is more profitable for creditors than getting involved in speculation: the return to lending out reserves at the Federal Funds rate exceeds the expected return they could achieve either speculating themselves or extracting the gains others earn from speculating. By raising the real opportunity cost of funds, the Fed can siphon off the credit that is essential for speculation. Of course, such a policy might also siphon off funds that would have gone to entrepreneurs who need it for socially useful production, and may thus be inherently undesirable. But the model provides a clear link between the Federal Funds rate and the possibility of bubbles.

One implication of the model is that even though a sufficiently high Federal Funds rate will deter speculative trades while rates are high, the converse is not true: setting a low rate will not necessarily encourage speculative trading while rates are low. This is because the possibility of a bubble depends on the entire future path of the Federal Funds rate rather than its value at a particular point in time. To some extent, this should not be surprising, since speculation is inherently forward looking: agents are willing to buy an overvalued asset to see either if it pays out a large dividend or if they can sell it later at a higher price. Whether these bets pay off depends on what agents will have to repay their lender if and when the asset pays a large dividend, if and when they sell the asset, and whether future traders will agree to buy the asset. All of these depend on the path of the Federal Funds rate in the future. If r_t^{FF} is expected to be high at future values of t , a low value today may not suffice to make speculation profitable.

To show this result formally, I now argue that if r_t^{FF} were set to sufficiently high levels close to the terminal date, a speculative bubble will not emerge even if rates earlier were set arbitrarily close to zero. One problem with demonstrating this result in my model is that, for technical reasons that will become clear

below, relying exclusively on the opportunity cost of funds to discourage agents from buying the asset at dates that are arbitrarily close to the terminal date requires r_t^{FF} to shoot off to infinity as $t \rightarrow 1$. To avoid having a path in which the cost of funds is allowed to grow without bound, I assume policymakers impose a positive margin requirement from some date $t^* \in [0, 1)$ until date 1. This will preclude non-entrepreneurs from buying the asset beyond date t^* regardless of the path of r_t^{FF} over this period.⁵ However, they could buy the asset prior to date t^* if they wanted, when the margin requirement is not in effect. The next claim argues that if we set r_t^{FF} to high but finite levels over the interval $[t^*, 1]$, a speculative bubble will never trade at any date regardless of how we set r_t^{FF} prior to t^* .

Claim 12: Suppose $r_t^{FF} = r^*$ for some $r^* > 0$ for all $t \in [0, 1 - t^*)$, while for $t \in [t^*, 1]$, r_t^{FF} is such that

$$R_{1-t^*, 1}^{FF} \equiv \exp\left(\int_{1-t^*}^1 r_t^{FF} dt\right) \geq \frac{D}{p(0)} \quad (21)$$

Under A1 and A2, the probability of trade must be zero, i.e. an asset whose price implies a speculative bubble will not be traded.

Note that if we let $t^* \rightarrow 1$, the only way to satisfy condition (21) would be to let $r_t^{FF} \rightarrow \infty$ as $t \rightarrow 1$. Thus, in the absence of margin requirements, ruling out bubbles requires the opportunity cost of funds to be infinite near the terminal date. The reason for this is that the expected rate of return per unit time from buying the asset and holding it to maturity becomes infinite near the terminal date, since the expected profit to this strategy remains bounded away from zero regardless of how close to the terminal date the asset is purchased. A finite cost of funds would thus fail to discourage borrowers from speculating.

Claim 12 demonstrates that a temporary rate cut need not automatically give rise to bubbles: as long as rates are eventually raised to a high enough level, a bubble would not emerge even if the Federal Funds rate were set to nearly zero at earlier dates. This finding raises an important caveat to the claim that the dramatic cuts in the Federal Funds rate (and in the effective real rate) following the 2001 recession and the slow pace at which they were reversed were responsible for the bubble that emerged in the housing market. As long as the target rate the Fed eventually settled on when it began raising rates was high enough, lowering rates earlier or raising rates towards this target slowly would not have made a bubble possible. The fact that housing prices kept increasing even after the Fed raised the Federal Funds rate to a level considered by most observers to be either neutral or slightly contractionary suggests this level was not enough to turn speculation unprofitable. This would suggest that low interest rates during this period made speculation more *profitable*, but they were not what necessarily made them *possible*, since they continued after rates were raised. Preventing a bubble from emerging may have hinged more on whether the Fed was willing to adopt an extremely high Federal Funds rate once it stopped easing than on whether it eased in

⁵It will also preclude entrepreneurs in my model from investing. However, if agents did own some resources, unlike in my model, a large down-payment requirement would still deter non-entrepreneurs from borrowing to speculate, but it might not deter entrepreneurs who need to borrow resources temporarily and fully expect to repay them.

the first place. Indeed, one of the implications of the model is that if traders expect the most favorable realization for d to be large, only very high levels of r_t^{FF} would discourage them from speculating.

An interesting consequence of Claim 12 is that raising the opportunity cost of funds can rule out speculative bubbles without necessitating constant intervention as with margin requirements alone. So long as the Fed is willing to raise rates to high levels in the final stages of the bubble, perhaps in combination with temporarily high margin requirements, it need not take any action beforehand. However, concentrating this intervention over a short period may require setting the cost of funds to very high levels. In particular, discouraging non-entrepreneurs from buying the asset at all dates $t < t^*$ requires setting $R_{t^*,1}^{FF}$ high enough to exceed some threshold as in (21). The later is t^* and the lower is r^* , the higher the values of r_t^{FF} must be for $t \in [t^*, 1]$ to meet a given threshold. Hence, the longer the Fed allows the cost of funds to be lower, the higher rates must be when it eventually clamps down. In this sense, keeping rates low for an extended period may indeed allow a bubble to occur that wouldn't have otherwise. The notion that Fed policy allowed a bubble to emerge is not entirely groundless, although it is not obviously true either. It is also worth noting that a concentrated intervention only works if traders are forward-looking enough to understand that the bubble unravels because it will not trade in the future. By contrast, a persistent intervention does not hinge on such arguably subtle reasoning, and thus might be a more robust policy to eliminating bubbles.

6 Conclusion

The dramatic rise and fall of certain asset prices over the past decade has focused attention on bubbles, i.e. assets that trade at prices which exceed their fundamental value. The prospect of bubbles is often viewed as a source of concern, and recent events have generated considerable debate as to the role of policy both in causing and preventing bubbles. Some have argued that easing of credit conditions by the Fed in the wake of the 2001 recession and the slow pace at which they were tightened led to a housing bubble during the subsequent years. Others have faulted the Fed in its regulatory capacity for not preventing financing arrangements that supposedly lured in speculators and drove up asset prices. To explore these claims, this paper constructed a model in which credit plays an essential role in allowing for speculative bubbles. Yet it suggests easing credit conditions temporarily need not allow a bubble to emerge, and exotic financing arrangements may be a response to an emerging bubble rather than a cause of it.

One implication of the model is that speculative bubbles may be hard to avoid in some circumstances, e.g. periods of technological breakthroughs where those who figure out how to exploit new technologies stand to earn great profits. As long as speculators believe there is some chance that an asset can pay dividends that exceed the interest they are charged, and can hide their actions from creditors, such an asset could trade above its fundamental value. While much of the policy discussion has focused on whether the Fed lowered rates too much in the early 2000s, these conditions suggest the possibility of bubbles hinges more on whether the Fed is willing to raise rates to such high levels that even the most optimistic trader would

be discouraged from speculating. As evident from the model, relying on down-payment requirement that force traders to stake some of their own wealth might allow us to prevent bubbles from emerging without requiring such high rates, but these requirements may need to be put in place for extended periods.

While the model reveals new insights, it also abstracts from several important issues. Chief among these is that it deliberately ignores how the price of an asset bubble is determined. This approach allows me to better focus on how traders and creditors respond to the possibility that an asset became overvalued. However, it leaves open the question of whether an asset could become overvalued in the first place. In a separate paper, I show that a price path along the lines assumed here can indeed emerge in a slightly richer model. To sketch out the argument, suppose that rather than a single asset as I assumed here, there was a continuum of identical assets available in fixed supply. A random (possibly zero) flow of buyers arrives each instant that could purchase the asset assuming they obtain financing. The price of an asset must equate demand from arriving buyers with supply from current owners. As long as the cumulative flows through the terminal date can exceed the stock of assets with positive probability, the price of the asset will exceed its fundamental value. By choosing the distribution of flow arrivals appropriately, we can generate a price path in which whenever a positive flow arrives, the price of the asset rises from when it last traded, and the price can be expressed as a function of calendar time. Each new cohort of buyers sets a reservation price at which they would agree to sell the asset. In general, the reservation price of each cohort will differ from other traders, since their contract terms will be different. Thus, traders may hold on to the asset even as others are willing to sell it. Creditors will then have incentive to influence the trading strategy of speculators by rewarding them for selling the asset early, as I showed here taking the price as exogenous.

The model also abstracts from uncertainty in terms of when the bubble will (most likely) collapse. One could modify the model so that the date when the dividend is revealed is random. The main difference is that the bubble could burst before creditors switch to simple debt contracts or self-financed traders refuse to buy the asset. However, since the price of the asset is bounded above by the amount entrepreneurs borrow, speculation cannot continue indefinitely. Eventually the asset would cease to appreciate. Conditional on the dividend remaining unknown for long enough, the asset would eventually be bought up by traders who hold on to it to see if it pays out large dividends and would not sell it.

Finally, the model does not address whether bursting a bubble is desirable. Exploring welfare turns out to be somewhat complicated. First, even if we could eliminate bubble at no cost, the welfare effects of bursting a bubble are subtle. As the model is specified, a bubble merely redistributes income from some agents to others, and bursting it would not generate a Pareto improvement. By contrast, in some models of bubbles mentioned in the Introduction, bursting bubbles may be Pareto worsening. Even if we enriched the model so that bursting a bubble could be associated with a Pareto improvement, these gains might be more than offset by the costs of intervening, such as precluding entrepreneurs from generating social surplus. Exploring these questions is best left for future work.

Appendix A: Proofs

Claim 1: In equilibrium, an agent who does nothing or who holds on to an asset which pays no dividends at date 1 will have zero terminal wealth, i.e. $x_t^1(\hat{\omega}, \hat{a}, \hat{y}) = x_t^0(\hat{\omega}) + y$ if $y \leq 0$.

Proof of Claim 1: Suppose the agent does nothing. Since agents reveal their types truthfully in equilibrium, he would reveal himself to be type $(\omega, \emptyset, 0)$ at date 1. Suppose an agent who made this announcement were left with positive terminal wealth. Since $y = 0$, this would imply $x_t^0(\omega) > x_t^1(\omega, \emptyset, 0)$. But this contradicts (IC-3), which holds that $x_t^0(\hat{\omega}) \leq x_t^1(\hat{\omega}, \hat{a}, \hat{y})$ if terminal wealth is positive. Next, suppose an agent holds on to an asset which pays no dividend. In equilibrium, the agent would truthfully reveal his type $(\omega, 1, -p(t))$. Since his wealth is nonnegative, $x_t^0(\omega) - p(t) \geq x_t^1(\omega, 1, -p(t))$. But since $p(t) > 0$, then $x_t^1(\omega, 1, -p(t)) < x_t^0(\omega)$. (IC-3) then implies $x_t^1(\hat{\omega}, \hat{a}, \hat{y}) = x_t^0(\omega) + y$, as claimed. ■

Claim 2: Let $\epsilon \rightarrow 0$. Then agents will be able to buy the asset under the equilibrium contract if they wanted, i.e. there exists a $\hat{\omega} \in \{n, e\}$ such that $x_t^0(\hat{\omega}) \geq p(t)$.

Proof: Suppose not, i.e. $x_t^0(\hat{\omega}) < p(t)$ for all $\hat{\omega}$. Agents would then be unable to purchase the asset or invest, and it follows from Claim 1 that agents have zero terminal wealth. Suppose one of the creditors were to offer the following contract \tilde{x} :

$$\begin{aligned} \tilde{x}_t^0(e) &= \tilde{x}_t^0(n) = 1 \\ \tilde{x}_t^1(\hat{\omega}, \hat{a}, \hat{y}) &= \min\left(\frac{1+\epsilon}{1-\phi}, 1+y\right) \text{ for all } (\hat{\omega}, \hat{a}, \hat{y}) \end{aligned} \quad (\text{A.1})$$

where $\epsilon > 0$ is arbitrarily small. Since $\tilde{x}_t^0(\hat{\omega}) \geq 1$, an entrepreneur who accepts this contract will be able to invest in the project or purchase the asset. If he invests, his profits will equal

$$\tilde{x}_t^0(e) + y - \tilde{x}_t^1(e, t, R-1) = R - \frac{1+\epsilon}{1-\phi}$$

Given our maintained hypothesis that $(1-\phi)(R-1) - \phi > 0$, profits will be positive for ϵ sufficiently small. Hence, an entrepreneur would strictly prefer this contract to the original equilibrium contract. In addition, for sufficiently small ϵ , the entrepreneur will strictly prefer investing in the project to buying the asset. To see this, suppose he bought the asset. If he held it until date 1, his expected profit would equal

$$\epsilon \left(D + (1-p(t)) - \frac{1+\epsilon}{1-\phi} \right)$$

which goes to zero as $\epsilon \rightarrow 0$. If he instead sold the asset prior to date 1, the most he could earn is

$$1 + \lim_{s \rightarrow 1} p(s) - p(t) - \frac{1+\epsilon}{1-\phi}$$

Given assumption (4), it follows that $R-1 \geq \lim_{s \rightarrow 1} p(s) - p(t)$, so investing in the project is more profitable than purchasing the asset and selling it could ever be. Next, I argue that non-entrepreneurs will also opt

for this contract. In particular, they could always guarantee themselves positive expected profits by buying the asset and holding it to maturity, which would net them

$$\begin{aligned} \epsilon \left(D + (1 - p(t)) - \frac{1 + \epsilon}{1 - \phi} \right) &\geq \epsilon \left(R + (1 - p(t)) - \frac{1 + \epsilon}{1 - \phi} \right) \\ &\geq \epsilon \left(R - \frac{1 + \epsilon}{1 - \phi} \right) \end{aligned}$$

where recall that the last expression will be positive for small ϵ . Since the creditor loses at most 1 unit to non-entrepreneurs who engage in speculation, and since the fraction of agents who are non-entrepreneurs is at most ϕ , expected profits to the creditor are bounded below by

$$(1 - \phi) \frac{\phi + \epsilon}{1 - \phi} - \phi = \epsilon > 0$$

Hence, there exists a contract that agents strictly prefer and which delivers positive expected profits to creditors. The original contract must therefore not have been an equilibrium. ■

Claim 3: Let $\epsilon \rightarrow 0$. Then non-entrepreneurs will buy the asset under the equilibrium contract.

Proof: Suppose not. Then it follows that non-entrepreneurs do nothing, in which case by Claim 1 we know they will have zero wealth. Since by Claim 2 they must be able to buy the asset, incentive compatibility requires that buying the asset should yield them zero profits under the equilibrium contract. Consider any $\widehat{\omega}$ for which $x_t^0(\widehat{\omega}) \geq p(t)$. If the agent announces $\widehat{\omega}$, then, he could afford to buy the asset. We already established that an agent who buys the asset will be found out if he announced $a = \emptyset$, but he can announce other actions without being caught. We therefore need to make sure that he would earn zero profits regardless of what he announces in period 1, i.e.

$$\begin{aligned} x_t^1(\widehat{\omega}, 1, D - p(t)) - x_t^0(\widehat{\omega}) &\geq D - p(t) \\ x_t^1(\widehat{\omega}, 1, -p(t)) - x_t^0(\widehat{\omega}) &\geq D - p(t) \\ x_t^1(\widehat{\omega}, t, R - 1) - x_t^0(\widehat{\omega}) &\geq D - p(t) \\ x_t^1(\widehat{\omega}, s, p(s) - p(t)) - x_t^0(\widehat{\omega}) &\geq p(s) - p(t) \end{aligned}$$

These conditions are sufficient, since $D \geq R \geq 1 \geq \lim_{s \rightarrow 1} p(s)$ implies that if an agent earns zero profits from holding on to the asset he will also earn zero profits from selling it at some date s . Since an agent who announces that he sold the asset can be identified, for that announcement the only requirement is that his required transfer must exceed $p(s) - p(t)$. Given these restrictions on transfers, an entrepreneur who invests in the project must also earn zero profits, whether he invests in the project, buys the asset, or does nothing. But if all agents earn zero profits, both types would strictly prefer the alternative contract (A.1) introduced in the proof of Claim 2, and it would yield positive profits to the creditor who offers it. So the original contract could not have been an equilibrium. ■

Claim 4: Let $\epsilon \rightarrow 0$. Then $x_t^0(e) \geq 1$ under the equilibrium contract and entrepreneurs will invest in the project in equilibrium.

Proof: Suppose not. Then an entrepreneur will either do nothing or purchase the asset. From Claim 3, we know that in equilibrium non-entrepreneurs earn positive expected profits from buying the asset. So, if there is an equilibrium in which type e agents do not invest in the project, they will buy the asset when given the opportunity. It then follows from Claim 3 that all agents will purchase the asset.

The proof relies on showing that there exists a contract \hat{x} that only entrepreneurs would prefer to the original contract and which yields positive expected profits to creditors. The idea is to offer a contract that allows entrepreneurs to invest in the project but demand a higher transfer payment in return. Since entrepreneurs earn more from investing in a project than from buying the asset, they would prefer this new contract. Non-entrepreneurs, by contrast, avoid this contract because it demands a higher transfer. Since creditors earn positive profits from lending exclusively to entrepreneurs, the original contract could not have been an equilibrium. Formally, I first argue that there exists a contract \tilde{x} of the form

$$\begin{aligned}\tilde{x}_t^0(e) &= \tilde{x}_t^0(n) = p(t) \\ \tilde{x}_t^1(\hat{\omega}, \hat{a}, \hat{y}) &= \min(p(t) + r^*, p(t) + y) \text{ for all } (\hat{\omega}, \hat{a}, \hat{y}) \in \Omega(\hat{\omega}, \omega, a, y)\end{aligned}\tag{A.2}$$

that yields the same expected profits to agents as the original contract, denoted V , where

$$V = E \left[\max_{a_t(\tau) \in A_\tau(\omega, \omega)} \{x_t^0(\omega) + y - x_t^1(\omega, a, y)\} \mid a_t(t) = 1 \right]$$

I then argue that we can find a sufficiently small ε such that a new contract \hat{x} of the form

$$\begin{aligned}\hat{x}_t^0(e) &= x_t^0(n) = 1 \\ \hat{x}_t^1(\hat{\omega}, \hat{a}, \hat{y}) &= \min(1 + r^* + \varepsilon, 1 + y) \text{ for all } (\hat{\omega}, \hat{a}, \hat{y}) \in \Omega(\hat{\omega}, \omega, a, y)\end{aligned}\tag{A.3}$$

will be strictly preferred by entrepreneurs to \tilde{x} , and thus to the original contract x , but non-entrepreneurs prefer the original contract. Lastly, I show the creditor offering contract \hat{x} will earn positive expected profits.

To prove there exists an r^* such that (A.2) yields an expected profit of V to a trader, I first derive bounds for V , and then show we can choose r^* to achieve any value within the derived bounds. Consider the expression $x_t^0(\omega) + y - x_t^1(\omega, a, y)$. If $y \leq 0$, we know from Claim 1 that $x_t^0(\omega) + y - x_t^1(\omega, a, y) = 0$. If $y > 0$, (IC-3) implies that $x_t^1(\omega, a, y) \geq x_t^0(\omega)$, so $x_t^0(\omega) + y - x_t^1(\omega, a, y) \leq y$. In the opposite direction, agents cannot be left with negative wealth, i.e. $x_t^0(\omega) + y - x_t^1(\omega, a, y) \geq 0$. Hence, for $y \geq 0$, the contract must stipulate

$$0 \leq x_t^0(\omega) + y - x_t^1(\omega, a, y) \leq y$$

The first inequality implies $V \geq 0$. To derive an upper bound on V , I use the fact that a contract that maximizes $x_t^0(\omega) + y - x_t^1(\omega, a, y)$ at each $y \geq 0$ must also maximize the expected payoff to the agent who buys the asset. Hence, the maximum expected payoff occurs if $x_t^0(\omega) + y - x_t^1(\omega, a, y) = y$ for all $y \geq 0$. To compute a value for expected utility in this case, we need the optimal action of the agent. From Claim 3, we know that in equilibrium, agents will prefer to purchase the asset in equilibrium over doing nothing.

Since entrepreneurs and non-entrepreneurs alike invest in the asset, the number of potential buyers on an island will be distributed over $\{M, M+1, \dots\}$, and the probability that the total number is equal to some m in this set will be given by $\frac{\lambda^{m-M} e^{-\lambda}}{(m-M)!}$. So long as fewer than M traders have arrived, the original owner of the asset will be better off waiting to sell it at a higher price. Define $m(s)$ as the number of traders who arrived by date s , and let $Q(s, n)$ denote the probability that at least one trader arrives if n traders already showed up by date s , i.e. $Q(s, n) = \Pr(m(1) \geq n+1 \mid m(s) = n)$. Then we have

$$\begin{aligned}
Q(s, n) &= \Pr(m(1) \geq n+1 \mid m(s) = n) \\
&= \frac{\Pr(m(1) \geq n+1 \cap m(s) = n)}{\Pr(m(s) = n)} \\
&= \frac{\sum_{k=n+1}^{\infty} \frac{\lambda^{k-M} e^{-\lambda}}{(k-M)!} \frac{(k-M)!}{n!(k-M-n)!} s^n (1-s)^{k-n}}{\sum_{k=n}^{\infty} \frac{\lambda^{k-M} e^{-\lambda}}{(k-M)!} \frac{(k-M)!}{n!(k-M-n)!} s^n (1-s)^{k-n}} \\
&= \frac{\lambda^n e^{-\lambda} s^n / n! \cdot e^{\lambda(1-s)} - 1}{\lambda^n e^{-\lambda} s^n / n! \cdot e^{\lambda(1-s)}} \\
&= 1 - e^{-\lambda(1-s)}
\end{aligned}$$

Hence, $Q(s, n)$ is independent of n , i.e. $Q(s, n) = Q(s)$. Next, let $F(x|n, s)$ denote the probability that the first arrival time after date s given n traders arrived by date s and there are at least $n+1$ traders occurs before date x . Then we have

$$\begin{aligned}
1 - F(x|s, n) &= \Pr(t_{n+1} \geq x \mid m(s) = n \cap m(1) \geq n+1) \\
&= \frac{\Pr(t_{n+1} \geq x \cap m(s) = n)}{\Pr(m(s) = n \cap m(1) \geq n+1)} \\
&= \frac{\sum_{k=n+1}^{\infty} \frac{e^{-\lambda} \lambda^{k-M}}{(k-M)!} \frac{(k-M)!}{n!(k-M-n)!} s^n (1-x)^{m-n}}{\sum_{k=n+1}^{\infty} \frac{e^{-\lambda} \lambda^{k-M}}{(k-M)!} \frac{(k-M)!}{n!(k-M-n)!} s^n (1-s)^{m-n}} \\
&= \frac{e^{\lambda(1-x)} - 1}{e^{\lambda(1-s)} - 1}
\end{aligned}$$

which again is independent of n . Differentiating with respect to x yields the conditional probability density function of the first arrival time beyond s :

$$f(x|s) = \frac{\lambda e^{-\lambda(x-s)}}{1 - e^{-\lambda(1-s)}}$$

Now, suppose $x_t^0(\omega) + y - x_t^1(\omega, a, y) = y$ for all $y \geq 0$. Define $W(t, s)$ as the expected profits under this contract for an agent who bought the asset at date t , did not sell the asset up to date s , and chooses what to do with the asset optimally thereafter. Then $W(t, s)$ solves the following integral equation:

$$W(t, s) = Q(s) \int_s^1 \max(W(t, \tau), p(\tau) - p(t)) f(\tau|s) d\tau + (1 - Q(s)) \epsilon(D - p(t))$$

I next argue that the optimal strategy for the agent will be a cutoff rule, i.e. the agent will sell the asset from some date s_t^* on, where $s_t^* \in (t, 1]$. Showing that the trader will choose to follow a cutoff rule is equivalent to showing that if $W(t, s) \geq p(s) - p(t)$, then $W(t, s') \geq p(s') - p(t)$ for all $s' < s$. Suppose instead that $W(t, s') < p(s') - p(t)$ for some $s' < s$. At date s' , the agent always has the option of holding on to the asset until date s and proceeding optimally thereafter. This implies $W(t, s') \geq W(t, s)$. But then

$$p(s') - p(t) > W(t, s') \geq W(t, s) \geq p(s) - p(t).$$

This contradicts the fact that $p(s)$ is non-decreasing. Hence, agents who buy the asset at date t will hold on to it until some cutoff date s_t^* , and will sell it to the first trader who arrives on or after s_t^* . Hence, the expected value from buying the asset if $x_t^0(\omega) = x_t^1(\omega, a, y)$ for all $y \geq 0$ is just

$$\bar{V} = Q(s_t^*) \int_{s_t^*}^1 [p(\tau) - p(t)] f(\tau | s_t^*) d\tau + [1 - Q(s_t^*)] \epsilon (D - p(t))$$

Since this is the most favorable contract the agent can receive, \bar{V} forms an upper bound for V .

I now argue that for any $V \in [0, \bar{V}]$, there exists an r^* for which the contract \tilde{x} defined by (A.2) yields an expected utility equal to V . First, note that if we set $r^* = D - p(t)$, the contract will always leave agents with zero terminal wealth. Next, if we set $r^* = 0$, the contract will be identical to the one I just argued yields a value of \bar{V} to the trader. If I can show that expected profits to the trader are continuous in r^* , it would follow by the intermediate value theorem that for any $V \in [0, \bar{V}]$ there exists an $r^* \in (0, D - p(t))$ such that the expected profits to the agent equal V . Let $W(t, s; r^*)$ denote the value of waiting at date s for an agent who bought the asset at date t and who faces the contract (A.2). Once again, $W(t, s; r^*)$ must satisfy the integral equation

$$W(t, s; r^*) = Q(s) \int_s^1 \max(W(t, \tau; r^*), p(\tau) - p(t) - r^*, 0) f(\tau | s) d\tau + (1 - Q(s)) \epsilon (D - p(t) - r^*)$$

It follows that $W(t, s; r^*)$ is continuous (and even differentiable) in r^* . Hence, we can always find a contract of the form in (A.2) that yields the same value to an agent as the equilibrium contract.

Finally, suppose a creditor were to offer the contract \hat{x} defined by (A.3). In the limit as $\epsilon \rightarrow 0$, the amount an entrepreneur could earn under contract \tilde{x} defined by (A.2) is bounded above by $\lim_{s \rightarrow 1} p(s) - p(t) - r^*$. By contrast, under contract \hat{x} defined by (A.3) he could earn $R - 1 - r^* - \epsilon$. Under the maintained hypothesis that $R - 1 > 1 \geq \lim_{s \rightarrow 1} p(s) - p(t)$, there exists an ϵ small enough such that the entrepreneur will strictly prefer (A.3) to (A.2) and hence to the original equilibrium contract. Non-entrepreneurs, however, will strictly prefer the original contract, since the expected profits under a contract of type (A.2) are decreasing in r^* . Hence, the expected profits to a creditor who offers contract (A.3) are $r^* + \epsilon > 0$, suggesting the original contract could not have been an equilibrium. ■

Claim 5: Under the equilibrium contract, expected profits to the creditor must be zero.

Proof: Suppose not, i.e. a creditor expects to earn strictly positive profits in equilibrium. From Claims 3 and 4, we know that in equilibrium entrepreneurs will invest in the project and non-entrepreneurs will purchase the asset if it is up for sale. Let $\Sigma(t) \equiv \{s \geq t \mid W(t, s) \leq p(s) - p(t) + x_t^0(n) - x_t^1(n, s, p(s) - p(t))\}$ denote the set of dates at which a non-entrepreneur would (weakly) prefer to sell the asset under the equilibrium contract. Suppose first that the net expected non-negative transfers from those who announce themselves to be non-entrepreneurs is strictly positive, i.e. either

$$x_t^1(n, 1, D - p(t)) > x_t^0(n)$$

or else

$$E[x_t^1(n, s, p(s) - p(t)) \mid s \in \Sigma(t)] > x_t^0(n).$$

Consider an alternative contract \tilde{x} where $\tilde{x}_t^0(n) = x_t^0(n)$ but which offered slightly more favorable terms to non-entrepreneurs, i.e. either

$$\tilde{x}_t^1(n, 1, D - p(t)) = x_t^1(n, 1, D - p(t)) - \varepsilon/\epsilon$$

or, if $x_t^1(n, 1, D - p(t)) = x_t^0(n)$, then

$$E[\tilde{x}_t^1(n, s, p(s) - p(t)) \mid s \in \Sigma(t)] = E[x_t^1(n, s, p(s) - p(t)) \mid s \in \Sigma(t)] - \varepsilon$$

for some small $\varepsilon > 0$. Since the original contract must have been incentive compatible, it follows that the net transfer of entrepreneurs is strictly positive, i.e.

$$x_t^1(e, t, R - 1) - x_t^0(e) > 0$$

If it were not positive, non-entrepreneurs would prefer to pass themselves off as entrepreneurs and take on a zero interest contract. As shown in Claim 4, this yields a payoff of \bar{V} , which is the upper bound on how much an agent can earn, whereas if expected non-negative transfers are positive, the utility to the agent from acting optimally will be below \bar{V} . Hence, for ε small enough, we could reduce $x_t^1(e, t, R - 1)$ by ε and still exceed $x_t^0(e)$. Thus, we set

$$\begin{aligned} \tilde{x}_t^0(e) &= x_t^0(e) \\ \tilde{x}_t^1(e, t, R - 1) &= x_t^1(e, t, R - 1) - \varepsilon \end{aligned}$$

Both parties will strictly prefer accepting contract \tilde{x} and telling the truth to accepting contract x and telling the truth. Moreover, since the original equilibrium contract must be incentive compatible and I subtract the same amount from the expected payoff of both types, non-entrepreneurs would continue to prefer telling the truth under contract \tilde{x} than passing themselves off as entrepreneurs. Finally, since entrepreneurs prefer to invest under the original contract, but expected payoffs to both investing and purchasing the asset are ε higher under contract \tilde{x} , they would continue to prefer investing in the project under contract \tilde{x} . Since profits to the creditor under the original contract were strictly positive, they will remain positive under the new contract for ε small enough. But then original contract could not have been an equilibrium.

Next, suppose $x_t^1(n, 1, D - p(t)) = E[x_t^1(n, s, p(s) - p(t)) \mid s \in \Sigma(t)] = x_t^0(n)$. This is equivalent to giving them a contract of type (A.2) with $r^* = 0$. Since there will be some non-entrepreneurs who fail to sell an asset which proves to be worthless, the only way for the creditor to earn nonzero expected profits is if $x_t^1(e, t, R - 1) > x_t^0(e)$, i.e. if entrepreneurs transferred positive resources to the creditor. Suppose a creditor offers the same contract but sets $\tilde{x}_t^1(e, t, R - 1) = x_t^1(e, t, R - 1) - \varepsilon$ for some arbitrarily small ε that ensures $\tilde{x}_t^1(e, t, R - 1)$ is positive. Non-entrepreneurs will prefer their original contract, since pretending to be an entrepreneur under this contract is equivalent to a contract of type (A.2) with $r^* = x_t^1(e, t, R - 1) - \varepsilon > 0$, while their original contract was equivalent to a contract of type (A.2) with $r^* = 0$. Entrepreneurs will prefer this alternative contract to the original equilibrium contract, and the expected profits to the creditor are $x_t^1(e, t, R - 1) - \varepsilon > 0$, implying the original contract could not have been an equilibrium. ■

Claim 6: Under the equilibrium contract, $r_t^e = x_t^1(e, t, R - 1) - x_t^0(e) > 0$.

Proof: Suppose not. Recall that an agent who reports an outcome that is not possible is assumed to be left with zero wealth. Given this, if a non-entrepreneur pretended to be an entrepreneur, the fact that $r_t^e = 0$ implies their expected utility would be the same as under a contract of type (A.2) with $r^* = 0$. Incentive compatibility requires that non-entrepreneurs prefer to disclose their information truthfully. But if the equilibrium contract set $x_t^1(n, \hat{a}, \hat{y}) > x_t^0(n)$ for any (a, y) , the agent would be better off pretending to be an entrepreneur. Thus, if $r_t^e = 0$, then $x_t^1(n, \hat{a}, \hat{y}) = x_t^0(n)$ for any $\hat{y} > 0$. Expected profits for the creditor are then equal to

$$-(1 - \epsilon)[1 - Q(s_t^*)]p(t)$$

where s_t^* denotes the date at which an agent who bought the asset at date t would first sell the asset if facing a contract of type (A.2) with $r^* = 0$. Since $s_t^* > t \geq 0$ and $Q(x) < 1$ for $x > 0$, this expression is negative. But creditors must earn non-negative profits in equilibrium, so the original contract could not have been an equilibrium. ■

Claim 7: In equilibrium, the incentive constraint for type n will be binding, i.e. $V(n, n) = V(n, e)$

Proof: Suppose not, i.e. an agent strictly prefers to announce n than to announce e . From Claim 6, we know $x_t^1(e, t, R - 1) - x_t^0(e) > 0$. Consider a contract \tilde{x} which offers slightly better terms to those who announce themselves to be entrepreneurs, i.e.

$$\tilde{x}_t^1(e, t, R - 1) - \tilde{x}_t^0(e) = x_t^1(e, t, R - 1) - x_t^0(e) - \varepsilon.$$

Since $V(n, n) > V(e, e)$, we can choose ε small enough so that non-entrepreneurs still prefer to reveal themselves truthfully under the original contract than to take the new contract and misrepresent themselves as entrepreneurs. In addition, suppose \tilde{x} offers identical terms to non-entrepreneurs or treats them worse by demanding a higher net transfer. Thus, we can assume that only entrepreneurs will be attracted to the new contract \tilde{x} , and for ε small enough the expected profits to the creditor who offers it will be $x_t^1(e, t, R - 1) - x_t^0(e) - \varepsilon$ which is strictly positive. Hence, the original contract could not have been an equilibrium. ■

Claim 8: In equilibrium, the contract in (10) and (11) maximizes the expected profits to a creditor among all contracts that deliver utility $V_0(r_t^e)$ to a non-entrepreneur.

Proof: Consider the problem of choosing transfers $x_t^1(n, a, y)$ for $y \geq 0$ in order to maximize the expected profits of the creditor while keeping the expected utility of the non-entrepreneur equal to $V_0(r_t^e)$. Define $\Sigma(t)$ as the set of dates under the contract x_t at which an optimizing agent would opt to sell the asset. Let $Z(t)$ denote the probability that $\exists t_n \in \Sigma(t)$, i.e. that a buyer arrives when the agent is willing to sell the asset. Finally, let $h(s)$ denote the distribution of the arrival time of the first buyer that falls in the set $\Sigma(t)$, conditional on at least one such arrival. Then the creditor would choose $x_t^1(n, a, y)$ for $y \geq 0$ to maximize his expected payoff

$$\max_{x_t^1(n, a, y)} (1 - \phi_t) r_t^e + \phi_t Z(t) \int_{\Sigma(t)} [x_t^1(n, s, p(s) - p(t)) - p(t)] h(s) ds + \phi_t (1 - Z(t)) (\epsilon x_t^1(n, 1, D - p(t)) - p(t)) \quad (\text{A.4})$$

subject to the constraints

1. $Z(t) \int_{\Sigma(t)} [p(s) - x_t^1(n, s, p(s) - p(t))] h(s) ds + (1 - Z(t)) \epsilon (D - x_t^1(n, 1, D - p(t))) = V_0(r_t^e)$
2. $x_t^0(n) \leq x_t^1(n, a, y) \leq x_t^0(n) + y$ for all $a \in (t, 1]$ and the relevant values of y

If we substitute the first constraint into the objective function, we can rewrite this problem as

$$\max_{x_t^1(n, a, y) \in [x_t^0(n), x_t^0(n) + y]} (1 - \phi_t) r_t^e + \phi_t \left\{ Z(t) \int_{\Sigma(t)} [p(s) - p(t)] h(s) ds + (1 - Z(t)) (\epsilon D - p(t)) - V_0(r_t^e) \right\}$$

Since the choice of $x_t^1(n, a, y)$ has no effect on $V_0(r_t^e)$ and r_t^e , this problem identical to solving

$$\max_{x_t^0(n) \leq x_t^1(n, a, y) \leq x_t^0(n) + y} Z(t) \int_{\Sigma(t)} [p(s) - p(t)] h(s) ds + (1 - Z(t)) (\epsilon D - p(t)) \quad (\text{A.5})$$

The choices for $x_t^1(n, a, y)$ thus do not enter the objective function in (A.5) directly, but through their effect on the set $\Sigma(t)$, which in turn determines $Z(t) = \Pr(\exists t_n \in \Sigma(t))$ and $h(s)$. But choosing $\Sigma(t)$ is equivalent to the problem of an agent who buys the asset with his own funds and must decide when to sell it, subject to the relevant constraints.

Define $\Pi(t, s)$ as the value for an agent who bought the asset at date t with his own funds, opts to wait at date s , and acts optimally thereafter. $\Pi(t, s)$ is thus analogous to the value $W(t, s)$ for an agent who purchases the asset with borrowed funds as opposed to his own. The function $\Pi(t, s)$ satisfies an analogous integral equation:

$$\Pi(t, s) = Q(s) \int_s^1 \max(\Pi(t, \tau), p(\tau) - p(t)) f(\tau|s) d\tau + (1 - Q(s)) (\epsilon D - p(t))$$

Note the similarity to the problem of an agent who borrows funds but faces a zero-interest contract; the equation differs only in the last term, which involves $\epsilon D - p(t)$ rather than $\epsilon[D - p(t)]$. Note that setting $p(t) = 0$ yields a value for $\Pi(t, s)$ that is equal to the expected profits for the original owner who must choose optimally when to sell the asset he was endowed with.

By a similar argument as before, we can show that the optimal trading strategy would dictate selling the asset from some date σ_t on. To show this, it will again suffice to show that if $\Pi(t, s) \geq p(s) - p(t)$, then $\Pi(t, s') \geq p(s') - p(t)$ for all $s' < s$. Suppose instead that $\Pi(t, s') < p(s') - p(t)$ for some $s' < s$. At date s' , the agent always has the option of holding on to the asset until date s and proceeding optimally thereafter. This implies $\Pi(t, s') \geq \Pi(t, s)$. But then

$$p(s') - p(t) > \Pi(t, s') \geq \Pi(t, s) \geq p(s) - p(t)$$

which contradicts the fact that $p(s)$ is assumed to be increasing.

The above remark allows us to rewrite (A.5) as choosing a value for σ_t in order to maximize the expected payoff

$$Q(\sigma_t) \int_{\sigma_t}^1 p(x) f(x|\sigma_t) dx + (1 - Q(\sigma_t)) \epsilon D - p(t) \quad (22)$$

subject to the constraints on x_t . Note that since $p(t)$ appears as a constant in the above expression, it has no effect on the choice of σ_t . Thus, the optimal strategy would involve selling the asset from some fixed date σ^* on regardless of $p(t)$. If $t > \sigma^*$, the optimal strategy would be to induce the trader to sell the asset to the first buyer he meets.

Since (A.5) coincides with the problem the original owner faces as to when to sell the asset, it follows that the original owner would sell the asset from date σ^* on. Hence, non-entrepreneurs can only buy the asset from date σ^* on. This implies that in equilibrium, the contract should be designed to encourage the trader to sell the asset to the first arriving buyer. But the constraint that $x_t^1(n, a, y) \geq y$ for $y \geq 0$ turn out to binding. I now argue that the equilibrium contract that emerges will be the maximally backloaded contract that yields a utility of $V_0(r_t^e)$ to the agent, i.e. the contract that solves

$$\max_{x_t^0(n) \leq x_t(n, a, y) \leq x_t^0(n) + y} \sup \{s : x_t^1(n, s, p(s) - p(t)) = x_t^0(n)\}$$

subject to providing the agent with a utility of $V_0(r_t^e)$. Define $T_t \equiv \sup \{s : x_t^1(n, s, p(s) - p(t)) = x_t^0(n)\}$ under this contract. If $T_t < 1$, the contract will specify

$$x_t^1(n, a, y) = \begin{cases} x_t^0(n) + y & \text{if } a \geq T_t \\ x_t^0(n) & \text{else} \end{cases}$$

whereas if $T_t = 1$, the contract will specify

$$x_t^1(n, a, y) = \begin{cases} \in [x_t^0(n) + r_t^e, D] & \text{if } a = 1 \\ x_t^0(n) & \text{else} \end{cases}$$

For expositional convenience, let us refer to the maximally backloaded contract as x^* , and the set of dates during which an agent facing this contract would be willing to sell the asset by $\Sigma^*(t)$. It is easy to show that the optimal strategy given x^* is a cutoff rule, and so $\Sigma^*(t) = (s_t^*, 1)$ for some $s_t^* \leq T_t$. I now argue the contract x^* satisfies the following property. Pick any contract $x \neq x^*$ that yields the agent an expected utility of $V_0(r_t^e)$, and let $\Sigma(t; x)$ denote the set of dates at which an agent would choose to sell the asset.

Then $\Sigma(t; x) \subset \Sigma^*(t)$. Thus, any date s that an agent would agree to sell the asset under some contract that gives the agent utility $V_0(r_t^e)$ is a date that he would agree to sell it if he faced the contract x^* .

To see this, pick any $s \in \Sigma(t; x)$ where x is a contract that yields the relevant utility. If $s \geq T_t$, the backloaded contract will guarantee zero terminal wealth no matter what the agent does from time s on, and so an agent facing x^* will be willing to sell the asset. Suppose then that $s < T_t$. By definition, at this date it must be the case that

$$p(s) - p(t) + x_t^0(n) - x_t^1(n, s, p(s) - p(t)) \geq W(t, s; x)$$

where $W(t, s; x)$ denotes the value of waiting given the contract x . We wish to compare $W(t, s; x)$ and the value to continuing under the weakly backloaded contract, $W^*(t, s)$. Using the expressions for $Q(s)$ and $f(x|s)$, we have

$$\begin{aligned} W(t, s; x) &= Q(s) \int_s^1 \max [W(t, \tau; x), p(\tau) - p(t) + x_t^0(n) - x_t^1(n, \tau, p(\tau) - p(t))] f(\tau|s) d\tau \\ &\quad + (1 - Q(s)) \epsilon(D + x_1) \\ &= \int_s^1 \max [W(t, \tau; x), p(\tau) - p(t) + x_t^0(n) - x_t^1(n, \tau, p(\tau) - p(t))] \lambda e^{-\lambda(\tau-s)} d\tau \\ &\quad + e^{-\lambda(1-s)} \epsilon(D - p(t)) \\ &= e^{\lambda s} \left[\int_s^1 \max [W(t, \tau; x), p(\tau) - p(t) + x_t^0(n) - x_t^1(n, \tau, p(\tau) - p(t))] \lambda e^{-\lambda\tau} d\tau + e^{-\lambda} \epsilon(D - p(t)) \right] \end{aligned}$$

If we differentiate this expression with respect to s , we get

$$\frac{\partial W(t, s; x)}{\partial s} = \lambda W(t, s; x) - \lambda \max [W(t, s; x), p(s) - p(t) + x_t^0(n) - x_t^1(n, s, p(s) - p(t))]$$

Now, consider any contract $x \neq x^*$. Suppose $W(t, s; x) = W^*(t, s)$ for some $s < T_t$. Since any contract must satisfy $x_t^1(n, s, p(s) - p(t)) \geq x_t^0(n)$ by (IC-3), it follows that

$$\begin{aligned} \max [W^*(t, s; x), p(s) - p(t)] &\geq \max [W^*(t, s; x), p(s) - p(t) + x_t^0(n) - x_t^1(n, s, p(s) - p(t))] \\ &= \max [W(t, s; x), p(s) - p(t) + x_t^0(n) - x_t^1(n, s, p(s) - p(t))] \end{aligned}$$

Under contract x^* , the payoff from selling the asset at date $s < T_t$ is exactly $p(s) - p(t)$. Hence, $\max [W^*(t, s), p(s) - p(t)]$ corresponds to the second term in $\frac{\partial W(t, s; x)}{\partial s}$ when $x = x^*$. This implies that whenever $W(t, s; x) = W^*(t, s)$, then

$$\frac{\partial W^*(t, s)}{\partial s} \leq \frac{\partial W(t, s; x)}{\partial s}$$

Since $W^*(t, t) = W(t, t; x) = V_0(r_t^e)$, this is enough to ensure that

$$W^*(t, s) \leq W(t, s; x)$$

for all $s < T_t$. Hence, we have

$$\begin{aligned} p(s) - p(t) &\geq p(s) - p(t) + x_t^0(n) - x_t^1(n, s, p(s) - p(t)) \\ &\geq W(t, s; x) \\ &\geq W^*(t, s) \end{aligned}$$

These inequalities imply the agent would be willing to sell the asset and contract x^* .

Finally, since the creditor would like to have the agent to sell the asset at all dates, it follows that given two contracts x and x' that give the agent the same expected utility $V_0(r_t^e)$ such that $\Sigma(t; x) \subset \Sigma'(t; x')$, the creditor will prefer contract x' over contract x . The maximally backloaded contract thus solves the planner's problem. ■

Claim 9: If $\epsilon > 0$, then there exists some date t^* such that $(T_t, R_t^n) = (1, x_t^0(n) + r_t^e)$ for all $t \in [t^*, 1]$.

Proof: Consider any date t such that

$$p(t) > \lim_{s \rightarrow 1} p(s) - \epsilon(D - R)$$

Under Assumption A2, this will be true for all t that are sufficiently close to 1. Since the rate $r_t^e < R - 1$ in equilibrium, the capital gain from selling the asset will not exceed the expected profits from holding on to the asset. Since the most the agent can expect to get from selling the asset is the capital gain, it follows that for these dates non-entrepreneurs will hold on to the asset until date 1. In that case, the only way to achieve a utility of $V_0(r_t^e)$ is to require the agent to repay r_t^e if the asset pays a dividend of D . ■

Claim 10: Suppose positive margin requirements are applied only at dates $t \in [0, t^*]$, and the price path $p(t)$ is consistent with A1 and A2. Then in equilibrium,

- i. Creditors will be willing to purchase the asset using their own wealth up to some date $t^{**} \in [0, 1]$.
- ii. Non-entrepreneurs are willing to purchase the asset at any date, but can only do so from date t^* on.
- iii. If $t^* < 1$, then for λ sufficiently large, $t^{**} > t^*$. In this case, at any date t , the probability that an arriving agent will be both willing and able to buy the asset is positive.

Proof: I first derive the value to a creditor from purchasing an asset. Note that if a creditor owns an asset, then by a similar argument to Claim 4, his optimal strategy will be a cutoff rule. That is, if we define $W_0(t, s)$ as the expected utility of a creditor who purchased the asset at date t , has held on to it until and including date s , and acts optimally thereafter, then once again $p(s) - p(t) \geq W_0(t, s)$ implies $p(s') - p(t) \geq W_0(t, s')$ for all $s' \leq s$. A creditor who purchased the asset at date t will therefore sell it from some date σ_t^* on. The expected profits to a creditor who purchased an asset at date t are given by

$$V_0(t) = \max_{\sigma_t^*} Q(\sigma_t^*) \int_{\sigma_t^*}^1 [p(x) - p(t)] f(x|\sigma_t^*) dx + (1 - Q(\sigma_t^*)) [\epsilon D - p(t)]$$

where $Q(s)$ denotes the probability that at least one more agent who is willing and able to buy the asset will arrive after date s , and $f(x|s)$ denotes the density of the first such arrival conditional on there being

such an arrival. A creditor will be willing to purchase the asset if maximizing the above expression over all σ_t^* yields a positive expression. Applying the envelope theorem, we have that

$$\frac{dV_0(t)}{dt} = -p'(t).$$

Hence, if there exists a t^{**} such that $V_0(t^{**}) = 0$, creditors will not find it profitable to purchase the asset beyond date t^{**} . A contradiction argument shows that $t^{**} < 1$.

Beyond date t^{**} , the equilibrium will be identical to the case where creditors are unable to purchase the asset. Now, suppose $t^* < t^{**}$. We need to show that traders will be willing to buy the asset at any date $t \in [t^*, t^{**}]$. By the same logic as in Claims 1 through 4, we can show that in equilibrium entrepreneurs will invest in the project. Since a trader could always pretend to be an entrepreneur and hold the asset until maturity, his profits are bounded below by

$$\epsilon(D - p(t) - r_t^e)$$

Creditors will earn r_t^e per entrepreneur. Since the fraction of entrepreneurs is at most ϕ , expected profits to a creditor are at least

$$(1 - \phi)r_t^e - \phi$$

Using the same logic as in Claim 5, we know that expected profits will equal zero. But if $r_t^e \geq D - p(t) \geq R - 1$, then

$$(1 - \phi)r_t^e - \phi \geq (1 - \phi)(R - 1) - \phi > 0$$

i.e. profits would be strictly positive, so this could not be an equilibrium. Hence, $r_t^e < D - p(t)$, and non-entrepreneurs will wish to purchase the asset from date t^* on.

Lastly, we wish to determine whether it is possible for $t^{**} > t^*$. A sufficient condition for this is that the expression maximized to generate $V_0(t)$ is positive when we evaluate it at $t = t^*$, i.e.

$$Q(\sigma_t^*) \int_{\sigma_t^*}^1 p(x) f(x|\sigma_t^*) dx + (1 - Q(\sigma_t^*)) \epsilon D - p(t^*) > 0 \quad (\text{A.6})$$

This is because $V_0(t)$ will be at least as large as the LHS of (A.6). Condition (A.6) will hold provided $Q(t^*)$ is sufficiently close to 1, since under assumption A2 we know that $\int_{t^*}^1 p(x) f(x|t^*) dx \geq p(t^*)$. Hence, a sufficient condition that ensures $t^{**} > t^*$ is if $Q(t^*) \rightarrow 1$. To exploit this condition requires an analytical expression for $Q(s)$. Using a similar argument to the one in Appendix B, one can show that the number of individuals who arrive at any interval is independent of the number of arrivals at other intervals.

The value of $Q(s)$ will depend on how t^{**} compares with t^* . Consider first the case where $t^{**} > t^*$. Then we can partition time into three intervals: in the interval $[0, t^*]$ only creditors would be willing and able to buy the asset; in the interval (t^*, t^{**}) , both creditors and non-entrepreneurs would be willing and able to buy the asset; and in the interval $[t^{**}, 1]$, only non-entrepreneurs would be willing and able to buy the

asset. The probability that at least one trader arrives beyond date s is equal to one minus the probability that there are zero arrivals in the interval $(s, 1]$. Hence, if $t^{**} > t^*$, we have

$$\begin{aligned} Q(s) &= 1 - e^{-\lambda(1-\max(t^{**},s))} e^{-(\lambda+\mu)(\max(t^{**},s)-\max(t^*,s))} e^{-\mu(\max(t^*,s)-s)} \\ &= 1 - e^{-\lambda(1-\max(t^*,s))-\mu(\max(t^{**},s)-s)} \end{aligned} \quad (\text{A.7})$$

Next, consider the case where $t^* \geq t^{**}$. In this case, we can partition time into three intervals: in the interval $[0, t^{**}]$ only creditors would be willing and able to buy the asset; in the interval (t^{**}, t^*) , neither creditors and non-entrepreneurs would be both willing and able to buy the asset; and in the interval $[t^*, 1]$, only non-entrepreneurs would be willing and able to buy the asset. The probability that at least one trader arrives beyond date s is equal to one minus the probability that there are zero arrivals in the interval $(s, 1]$. Hence, if $t^* \geq t^{**}$, we have

$$Q(s) = 1 - e^{-\lambda(1-\max(t^*,s))} e^{-\mu(\max(t^{**},s)-s)} \quad (\text{A.8})$$

To show that for λ large enough it must be the case that $t^{**} > t^*$, suppose to the contrary $t^* \geq t^{**}$. From (A.8), we know that in this case, $Q(t^*) = 1 - e^{-\lambda(1-t^*)}$. But as $\lambda \rightarrow \infty$, this expression converges to 1, implying (A.6) will be satisfied. But then $V_0(t^*) > 0$, which implies $t^{**} > t^*$, a contradiction. ■

Claim 11: Suppose $r_t^{FF} > 0$ for $t \in [0, t^*]$ and $r_t^{FF} = 0$ for $t \in (t^*, 1]$. Then in equilibrium, we have

- i. Non-entrepreneurs would be willing to purchase the asset at any date beyond t^*
- ii. For any date $\tau < t^*$, there exists a path for r_t^{FF} such that no agent would be willing to buy the asset on or before date τ , regardless of the value of λ .

Proof: Part (i) follows directly from the fact that beyond date t^* we are back to the benchmark version of the model in which by Claims 1-4 we know that non-entrepreneurs are both willing and able to purchase the asset. For part (ii), suppose we set the path of r_t^{FF} for $t \in [0, t^*]$ so that $r_t^{FF} > \dot{p}_t/p_t$, and for $t \in (\tau, t^*]$ we further require that

$$R_{\tau, t^*}^{FF} \equiv \exp\left(\int_{\tau}^{t^*} r_t^{FF} dt\right) > \frac{D}{p(0)} \geq \frac{1}{p(0)}$$

Note that these conditions do not depend on the value of λ . We need to show that no agent would wish to buy the asset before date τ . Suppose an agent purchases the asset and holds it until maturity. By (20) if the agent keeps positive wealth he would have to pay at least $R_{\tau, t^*}^{FF} p(t)$. But then

$$R_{\tau, t^*}^{FF} p(t) \geq R_{\tau, t^*}^{FF} p(0) > D$$

which contradicts the assumption that the agent has positive wealth. So holding the asset until maturity will yield zero wealth. Next, suppose an agent sells the asset. The maximal profits he could earn from selling the asset are given by

$$\max_s \{p(s) - R_{t,s}^{FF} p(t)\}$$

For all $s < t^*$, using the fact that $\dot{p}_t/p_t = \frac{d}{dt} \ln p_t$, we have

$$\begin{aligned} \exp\left(\int_t^s r_x^{FF} dx\right) &> \exp\left[\int_t^s \left(\frac{d}{dx} \ln p_x\right) dx\right] \\ &= \exp[\ln p(s) - \ln p(t)] \\ &= p(s)/p(t) \end{aligned}$$

and so

$$R_{t,s}^{FF} p(t) > p(s)$$

so that selling the asset before date τ yields zero profits. For $s \geq t^*$,

$$p(s) - R_{t,s}^{FF} p(t) = p(s) - R_{t,t^*}^{FF} p(t)$$

which is increasing in s . But then

$$R_{t,t^*}^{FF} p(t) \geq R_{\tau,t^*}^{FF} p(t) \geq R_{\tau,t^*}^{FF} p(0) \geq 1 \geq \lim_{s \rightarrow 1} p(s)$$

Thus, a non-entrepreneur could never earn positive profits from buying the asset, and given there is a tiny cost of entering into a financial contract, they would never wish to buy it. For an agent who does not have to borrow, the opportunity cost of buying the asset at date t and holding it until date s is $R_{t,s}^{FF} p(t)$. The same calculations then imply buying the asset yields less value than this opportunity cost. ■

Claim 12: Suppose $r_t^{FF} = r^*$ for some $r^* > 0$ for all $t \in [0, 1 - t^*)$, while for $t \in [t^*, 1]$, r_t^{FF} is such that

$$R_{1-t^*,1}^{FF} \equiv \exp\left(\int_{1-t^*}^1 r_t^{FF} dt\right) \geq \frac{D}{p(0)} \quad (23)$$

Under A1 and A2, the probability of trade must be zero, i.e. an asset whose price implies a speculative bubble will not be traded.

Proof: Define $T = \sup\{t \mid \text{Prob}(\text{asset sells at date } t) > 0\}$ as the supremum over all dates at which the asset might trade. We need to show that $T = 0$. Suppose that $T > 0$. Consider the expected profit from purchasing the asset at date $T - \varepsilon$. From Lemma B1 in Appendix B, the probability that at least one trader arrives in this interval is given by $Q = 1 - e^{-\lambda\varepsilon}$, which tends to 0 as $\varepsilon \rightarrow 0$. The expected payoff to an agent from buying the asset at date t is at most

$$Q \max\{p(T) - p(T - \varepsilon) - \delta, 0\} + \varepsilon(1 - Q) \max\{D - R_{1-t^*,1}^{FF} p(t), 0\}$$

Since $p(t) > p(0)$, the second term is equal to zero. Since $p(t)$ is assumed to be continuous, there exists an $\varepsilon^* > 0$ such that for all $t \in (T - \varepsilon^*, T)$, we have $p(T) - p(t) < r^*$. But then we have an open interval in which no non-entrepreneur would purchase the asset given the tiny cost of a financial transaction. But then T would not be the supremum for the set of dates at which the asset trades. ■

Appendix B: Solving for ϕ_t

I begin with the following two lemmas:

Lemma B1: In any interval $[t, t + \Delta]$, the number of arrivals is Poisson with parameter $\lambda\Delta$ and is independent of the number of arrivals in any non-overlapping interval.

Proof: Pick any two intervals $[t_1, t_1 + \Delta_1]$ and $[t_2, t_2 + \Delta_2]$ such that $[t_1, t_1 + \Delta_1] \cap [t_2, t_2 + \Delta_2] = \emptyset$. Define $N([a, b])$ as the cardinality of the set $\{n : a \leq T_{(n)} \leq b\}$. Proving the lemma requires showing that

$$\mathbb{P} = \Pr(N([t_1, t_1 + \Delta_1]) = n_1, N([t_2, t_2 + \Delta_2]) = n_2) = \frac{e^{-\lambda\Delta_1} (\lambda\Delta_1)^{n_1}}{n_1!} \frac{e^{-\lambda\Delta_2} (\lambda\Delta_2)^{n_2}}{n_2!}$$

We can compute \mathbb{P} by conditioning on the total number of arrivals N , as follows:

$$\begin{aligned} \mathbb{P} &= \sum_{n=n_1+n_2}^{\infty} \Pr(N([t_1, t_1 + \Delta_1]) = n_1, N([t_2, t_2 + \Delta_2]) = n_2 | N = n) \Pr(N = n) \\ &= \sum_{n=n_1+n_2}^{\infty} \frac{n!}{n_1!n_2!(n-n_1-n_2)!} \Delta_1^{n_1} \Delta_2^{n_2} (1 - \Delta_1 - \Delta_2)^{n-n_1-n_2} \frac{e^{-\lambda} \lambda^n}{n!} \\ &= e^{-\lambda} \frac{(\lambda\Delta_1)^{n_1}}{n_1!} \frac{(\lambda\Delta_2)^{n_2}}{n_2!} \sum_{n=n_1+n_2}^{\infty} \frac{(\lambda(1 - \Delta_1 - \Delta_2))^{n-n_1-n_2}}{(n-n_1-n_2)!} \\ &= e^{-\lambda} \frac{(\lambda\Delta_1)^{n_1}}{n_1!} \frac{(\lambda\Delta_2)^{n_2}}{n_2!} e^{\lambda(1-\Delta_1-\Delta_2)} \\ &= \frac{e^{-\lambda\Delta_1} (\lambda\Delta_1)^{n_1}}{n_1!} \frac{e^{-\lambda\Delta_2} (\lambda\Delta_2)^{n_2}}{n_2!} \end{aligned}$$

This establishes the claim. ■

Lemma B2: The probability density for the event that an agent arrives at date t is given by

$$f(T_{(n)} = t \text{ for some } n) = \lim_{\Delta \rightarrow 0} \frac{\Pr(N[t, t + \Delta] > 0)}{\Delta} = \lambda$$

and is independent of what happens at any other interval.

Proof: From Lemma B1, we know that the number of arrivals in $[t, t + \Delta]$ is independent of what happens at any other interval. The probability that there are zero arrivals in this interval are given by $\Pr(N([t, t + \Delta]) = 0) = e^{-\lambda\Delta}$. Hence, the probability that there is at least one arrival is given by $1 - e^{-\lambda\Delta}$. Dividing by Δ , taking the limit as Δ goes to zero, and applying L'Hopital's rule shows this expression is equal to λ . ■

Next, consider the particular event $\{Y_1 = y_1 \cap Y_2 = y_2 \cap \dots \cap Y_k = t\}$, i.e. there are at least k values in $\{Y_j\}_{j=1}^J$, and the first k realizations are given by y_1, y_2, \dots , and t . This event implies there exists some integers $n_1 < n_2 < \dots < n_k$ such that

$$T_{(n_1)} = y_1, T_{(n_2)} = y_2, \dots, T_{(n_k)} = t$$

and, in addition, that

$$N((S(0), y_1)) = N((S(y_1), y_2)) = N((S(y_2), y_3)) = \dots = N((S(y_{k-1}), t)) = 0$$

Define $y_0 = 0$ and $y_k = t$. Using the two lemmas above, the probability density associated with the event $\{Y_1 = y_1 \cap Y_2 = y_2 \cap \dots \cap Y_k = t\}$ is given by

$$\lambda^k \exp\left(-\lambda \sum_{m=1}^k (y_m - S(y_{m-1}))\right)$$

Hence, the probability that $Y_1 = t$, implying that t is the first arrival after $S(y_0)$, is equal to $\lambda e^{-\lambda(t-S(y_0))}$, while the probability that $Y_k = t$ for $k > 1$ is given by the integral

$$\int \dots \int_{(y_1, \dots, y_{k-1}) \in \mathbb{Y}_m(t)} \Pr(Y_1 = y_1 \cap Y_2 = y_2 \cap \dots \cap Y_k = t) dy_{k-1} \dots dy_1$$

where $\mathbb{Y}_k(t) = \{(y_1, \dots, y_{k-1}) \mid y_1 \geq S(0), y_2 \geq S(y_1), \dots, y_{k-1} \geq S(y_{k-2}), t \geq S(y_{k-1})\}$.

The probability density for the event that $\exists Y_j = t$ can be obtained by adding up over all possible realizations in which $Y_k = t$ for some t , i.e.

$$\begin{aligned} \Pr(\exists Y_j = t) &= \sum_{k=1}^{\infty} \Pr(Y_1 = y_1 \cap Y_2 = y_2 \cap \dots \cap Y_k = t) \\ &= \Pr(Y_1 = t) + \sum_{k=2}^{\infty} \int \dots \int_{(y_1, \dots, y_{k-1}) \in \mathbb{Y}_m(t)} \Pr(Y_1 = y_1 \cap Y_2 = y_2 \cap \dots \cap Y_k = t) dy_{k-1} \dots dy_1 \\ &= \lambda e^{-\lambda(t-S(y_0))} + \sum_{k=2}^{\infty} \int \dots \int_{(y_1, \dots, y_{k-1}) \in \mathbb{Y}_k(t)} \lambda^k \exp\left(-\lambda \sum_{m=1}^k [y_m - S(y_{m-1})]\right) dy_{k-1} \dots dy_1 \end{aligned}$$

To further simplify this expression, I now establish the following result:

Lemma B3: If $Y_k = t$ for some $t \in [0, 1]$, then $k < K_t$ where K_t is finite.

Proof: Recall that $S(t) > t$ and $S(t) = 1$ for $t \in [t^*, 1]$ for some $t^* < 1$. Define

$$\Delta = \inf_{t \in [0, t^*]} \{S(t) - t\}$$

Since $S(t)$ is continuous, $S(t) > t$ for all $t \in [0, t^*]$ and the interval $[0, t^*]$ is compact, the infimum is achieved at some $t \in [0, t^*]$. It then follows that $\Delta > 0$. Define $K = \frac{t^*}{\Delta}$. Suppose we apply $S(\cdot)$ recursively at some point $t \in [0, 1]$. At each point, the lowest value we could achieve is $t + K\Delta$. By construction, then, after K iterations we would eventually exceed t^* , and one final application of S would reach 1. For any value of $k > K + 1$, then, it would not be possible for $S(y_{k-1}) < t$ when $t \leq 1$. ■

We can therefore express $\pi(t) = \Pr(\exists Y_k = t \mid \exists T_{(n)} = t)$ as a finite sum of integrals:

$$\begin{aligned} \Pr(\exists Y_k = t \mid \exists T_{(n)} = t) &= \frac{\Pr(\exists Y_k = t \cap \exists T_{(n)} = t)}{\Pr(\exists T_{(n)} = t)} \\ &= \frac{\Pr(\exists Y_k = t)}{\Pr(\exists T_{(n)} = t)} \\ &= e^{-\lambda(t-S(y_0))} + \sum_{k=2}^{K_t} \int_{(y_1, \dots, y_{k-1}) \in \mathbb{Y}_k(t)} \dots \int \lambda^{k-1} \exp\left(-\lambda \sum_{j=1}^k [y_j - S(y_{j-1})]\right) dy_{k-1} \dots dy_1 \end{aligned}$$

where we use the fact that $\Pr(\exists T_{(n)} = t) = \lambda$ from Lemma B2.

When $S(t)$ is monotonically increasing, this expression can be further simplified to

$$\pi(t) = e^{-\lambda(t-S(y_0))} + \sum_{k=2}^{K_t} \int_{S(0)}^{[S^{-1}]^{k-1}(t)} \int_{S(y_1)}^{[S^{-1}]^{k-2}(t)} \dots \int_{S(y_{k-2})}^{S^{-1}(t)} \lambda^{k-1} \exp\left(-\lambda \sum_{j=1}^k [y_j - S(y_{j-1})]\right) dy_{k-1} \dots dy_1 \quad (24)$$

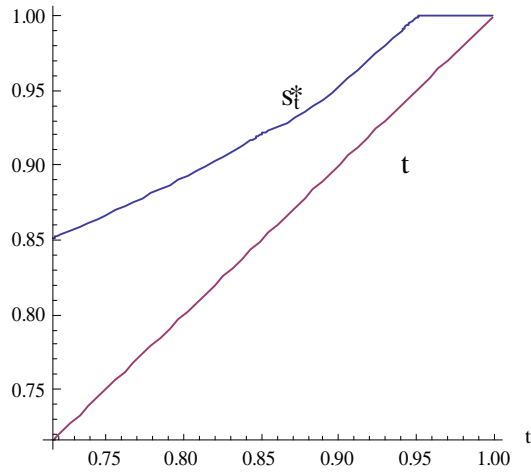
In addition, when $S(t)$ is monotonically increasing, the set $[0, 1]$ can be partitioned into a finite set of intervals $[0, t_0], (t_0, t_1], \dots, (t_K, 1]$ where $t_k = S^{k+1}(0)$, so that if $t \in (t_k, t_{k+1}]$, then $\pi(t)$ is the sum of k terms, and depends on the values of $S(t)$ for $t \leq t_k$. This greatly simplifies the task of solving for $\pi(t)$ numerically: for $t \in (t_0, t_1]$, we have

$$\pi(t) = e^{-\lambda(t-S(y_0))} = e^{-\lambda(t-t_0)}$$

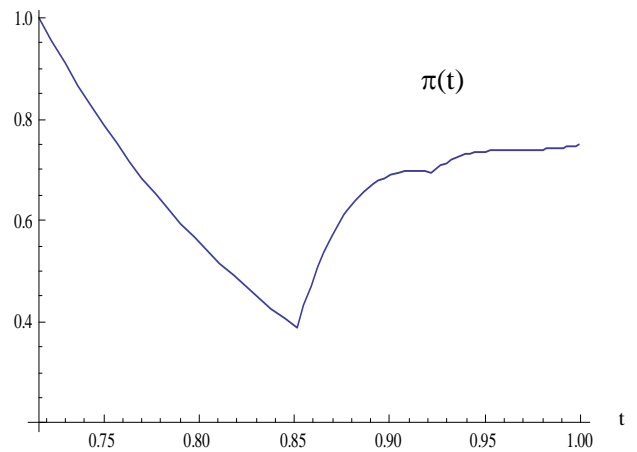
From $\pi(t)$, we can recover ϕ_t , r_t^e , and s_t^* . Hence, we can use a grid to interpolate the function $S(t)$ for all $t \in (t_0, t_1]$. We can then use this interpolated function to solve for $\pi(t)$ for any $t \in (t_1, t_2]$, and so on, until we recover the value of $\pi(t)$ at each interval. Note that $t_k = S^{k+1}(0)$ corresponds to the first date at which the asset could be traded for the $k+1$ -th time, which occurs if the original owners sell at date $s_0 = S(0)$, those who buy from them sell at the s_t^* associated with $t = s_0$, i.e. $S^2(0)$, and so on. In calculating the numerical example reported in the text, I computed $\pi(t)$ as defined in (24) for a grid of 20 equally spaced points within each interval, starting first with $(t_0, t_1]$. Given $\pi(t)$, I could then solve for s_t^* at each of these points, and interpolate the function s_t^* over the entire interval. This interpolation was done with Mathematica using a linear spline. Once I have an interpolated function for all intervals $(t_0, t_1]$ through $(t_k, t_{k+1}]$, I can proceed to use it to calculate $\pi(t)$ over the interval $(t_{k+1}, t_{k+2}]$ using (24).

Figure 1: Equilibrium contract for a particular numerical example

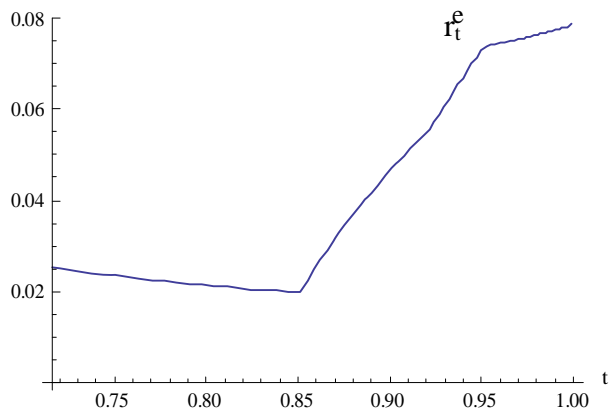
Numerical values used to generate the example: $\lambda = 7$; $\phi = .1$; $R = R_1 = 2$; $\epsilon = .05$; and $p(t) = \min(t, \epsilon R_1)$.



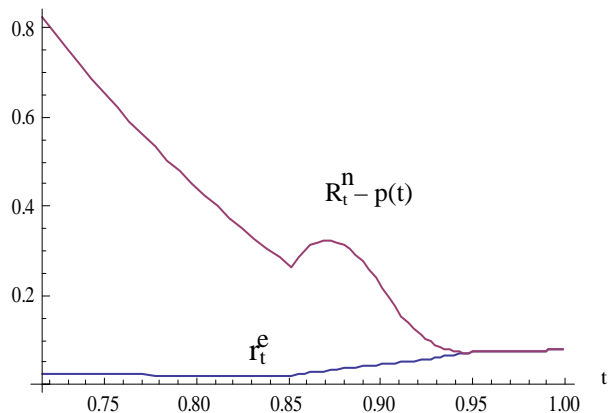
a. Optimal trading strategy: if buy asset at t , sell it from date s_t^* on



b. Probability that non-entrepreneurs arriving at date t will be able to buy the asset



c. Equilibrium rate r_t^e charged to entrepreneur as a function of the date t in which he arrived



d. Amount $R_t^e - p(t)$ a speculator arriving at date t would pay if he failed to sell the asset

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