Abstract

The paper studies a storable votes model where members of a committee voting sequentially on a known series of binary proposals are each granted a single extra bonus vote to cast as desired. Conditions guaranteeing that the storable votes scheme ex ante welfare dominates simple majority voting are derived. The simplicity of the model allows to compare strategies and expected outcomes when the order of the proposals is exogenously given to the case where a committee chair, having observed past votes, determines the order freely and sequentially. Granting the chair control over the agenda order has effects on ex ante expected welfare that are of ambiguous sign but always of small quantitative importance. The theoretical conclusions are confirmed by laboratory experiments.

1 Introduction

Consider a group of voters faced with a series of binary decisions, each of which can either pass or fail. Decisions are taken according to the majority of votes cast, but suppose that each voter is allowed to spend his budget of votes freely among the different decisions, casting more or less votes on each. This is the idea behind storable votes, a simple voting scheme designed to elicit truthfully voters’ intensity of preferences. By inducing voters to cast more votes on decisions they consider more important, storable votes typically increase ex ante welfare, relative to simple majority voting. In particular, storable votes allow the minority to win occasionally, but only when its preferences on average are strong and the majority’s on average weak. Especially in the presence of a minority group whose preferences are systematically opposed to the majority’s,

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the result is an increase in fairness and representation with little if any cost, in fact most often with a gain, in terms of aggregate efficiency.\footnote{For existing results on storable votes, see Casella (2005), Casella, Gelman and Palfrey (2006), Casella and Gelman (2007), and Casella, Palfrey and Riezman (forthcoming).}

One important concern that previous work has not addressed is the potential for agenda manipulation. Because storable votes allow voters to modify their "weight" in decision-making by cumulating or reducing the number of votes they cast, full knowledge and control of the agenda may be particularly important. Could a strategic agenda-setter deplete others’ votes before calling an unpopular proposal he can then carry on the strength of his own votes alone? Can the outcome then be quite inefficient? A priori it is not clear whether problems will arise: having multiple votes that can be shifted across proposals may make the order of the proposals more important, but also increase the ability to resist possible manipulations of this order.

This paper proposes a first enquiry into the impact of strategic agenda-setting on storable votes. Because agenda-setting problems are famously difficult, and the storable votes game, although built on an intuitive idea, quite complex, the paper exploits two simplifications. First, the storable votes mechanism is streamlined: in addition to their regular votes, agents are endowed with a single indivisible bonus vote to be cast freely over any of the proposals. The analysis shows that when the agenda is fixed and known, this simplified mechanism can be sufficient to achieve welfare gains over simple majority voting. More precisely, expected welfare gains obtain if the number of voters is either even or large enough, or if the differences in intensity of preferences across proposals are important enough, and the relative value of the bonus vote is chosen correctly (in practice, is not too large).

Second, suppose now that one of the voters assumes the role of committee chair. Knowing his own preferences over all decisions and having observed the votes that have been cast, the chair selects the proposal on which the next vote is called, among those left on the agenda. All proposals are eventually voted upon, but their order is chosen by the chair. The question is whether the chair can distort the mechanism in his favor and cause storable votes to become inefficient, relative to simple majority voting. The analysis shows that in this model controlling the order of proposals need not affect strategies and outcomes: the equilibrium with exogenous agenda order remains an equilibrium in the presence of a committee chair. However, when the order of proposals is endogenous, other equilibria exist too, and the reason for their existence is quite intuitive. Control of the agenda allows the chair to transmit information credibly about his own priorities and hence his own voting choices: the chair’s chosen order works as a public announcement of the proposal on which his bonus vote is going to be cast. Other voters refrain from competing on that proposal, and the chair sees his probability of being pivotal increase exactly when it matters to him most. As a consequence, his expected utility increases too. It is more difficult to evaluate the effect on the other voters’ expected utility, and on the ex ante utility of a random group member before the chair’s
position has been assigned. All numerical simulations show that the effect is quantitatively small. In the model studied in this paper, the impact of agenda control, in the limited sense of choosing the order of the votes, is either nihil or small.

The second part of the paper tests this conclusion experimentally.

2 The Model

Consider a committee of \( n \) voters, meeting to decide whether or not to implement \( T \) distinct proposals, \( \{P_1, \ldots, P_T\} \), each of which can either pass or fail. The \( T \) proposals constitute the known, fixed agenda of the meeting, and the committee votes over the proposals sequentially. Each member \( i \) of the committee has a preference over whether any specific proposal \( P_t \) should pass or fail and attaches some importance to having the proposal decided in his preferred direction. Both features of \( i \)'s preferences - direction and intensity - are summarized by a cardinal valuation \( v_{it} \). Valuations \( v_{it} \) are independently and identically distributed both across proposals and across individuals according to the distribution \( G(v) \) with support \([-1, 1]\), symmetric around 0. A negative valuation means that the voter is against the proposal; a positive valuation that he is in favor. For each proposal \( P_t \), a committee member \( i \) receives utility equal to the absolute value of his valuation \( |v_{it}| \equiv v_{it} \) if the decision goes in the preferred direction, and 0 otherwise. \(^2\) Thus the sign of the valuation indicates the preferred outcome for the proposal, while the absolute value indicates the intensity of preferences. The utility from the entire meeting is the sum of the utilities derived from each individual proposal.

In addition to a regular vote over every proposal, each committee member is given a single indivisible bonus vote. The bonus vote is worth \( B \) regular votes, where \( B \) can be smaller or larger than 1, and I will refer to \( B \) as its value. When the vote over \( P_1 \) is called, each individual decides whether to vote for or against, and whether to cast 1 or \( 1 + B \) votes. If a voter casts \( 1 + B \) votes over \( P_1 \), he will be able to cast only his regular vote over each of the successive proposals; if instead he casts 1 vote over \( P_1 \), he retains his bonus vote for one of the successive proposals. Voting continues in this manner until the last proposal, when any remaining bonus vote will be cast. Each proposal is decided in the direction that receives more votes; in case of a tie, a coin is flipped.

When voting over any proposal, a committee member knows his own valuations over all proposals and the set of voters still endowed with the bonus vote. I denote such a set by \( \Lambda_t \). The committee member does not know others’ valuations over any of the proposals, but the independence of the valuations and the distribution \( G \) from which they are drawn are common knowledge. The state of the game at \( t \) is summarized by \( (\Lambda_t, t) \).

\(^2\) Only the difference in utility between the two outcomes (passing or failing) matters.
3 Exogenous order of proposals

In this section, I suppose that the proposals are voted upon in some predetermined, publicly known order.

3.1 Equilibrium

The analysis focuses on symmetrical Perfect Bayesian equilibria in weakly undominated Markov strategies. With independent valuation draws and weakly undominated strategies, all individuals vote in the direction they sincerely prefer. The only question is when to cast the bonus vote. Because the direction of the vote is always chosen sincerely, $G$ is symmetric around $0$, and valuations are independent, the decision to cast the bonus vote will depend on the valuations’ intensity, not on their direction. Denote by $v_i$ the vector of realized intensities for voter $i$: $v_i = \{v_{i1}, ..., v_{iT}\}$, and by $F(v)$ the intensities’ probability distribution, defined over support $[0, 1]$. If we call $x_{it}$ the number of votes cast by voter $i$ over proposal $P_t$, then $x_{it} = x_{it}(v_i, \Lambda_t, F, t)$ where $x_{it} \in \{1, 1+B\}$ and $\sum_{t=1}^T x_{it} = T + B$. At first sight, identifying equilibrium strategies seems difficult: for a given vector $v_i$ the choice to cast the bonus vote on proposal $P_t$ in general may depend on how many bonus votes remain available to the other voters. But when valuations are independent, the answer in fact is very simple:

**Proposition 1.** (1) For all $T, n, G$, and $B$ there exists an equilibrium where each voter $i$ casts his bonus vote over $P_t$ if and only if $v_{it} = \max\{v_i\}$. (2) This is the unique symmetrical equilibrium in weakly undominated strategies holding for all $T, n, G$, and $B$.

The proof of lemma 1 is in the Appendix, but the intuition is immediate. If everyone else follows the strategy, at any point in the game the expected number of bonus votes cast over any future proposal is identical. There is no reason to postpone or anticipate casting the bonus vote - the only consideration is the intensity of preferences.

Notice that the game is then effectively static: the information about other voters’ use of the bonus vote is irrelevant. Even with independent valuations, the result would be different if the bonus vote were divisible, but for the specific mechanism discussed here the equilibrium strategy is as simple and as intuitive as possible.

3.2 Efficiency

How desirable are the welfare properties of the mechanism? In this set-up there is no reason to treat individuals asymmetrically, and the natural measure is ex ante utility - expected utility before the realization of any valuation, but given $G(v)$ and the expectation that all voters will choose the equilibrium strategy. Call $p_B$ the probability of obtaining the desired outcome when casting $1+B$ votes, and $p_1$ the corresponding probability when casting only the regular vote, noting from Proposition 1 that $p_{Bt} = p_B$ and $p_{1t} = p_1$ for all $t$. $Ev$ is the
expected value of the absolute valuation, and $E v(k)$ the expected $k$th order statistic among each voter’s $T$ absolute valuations (hence the $T$th order statistics is the expected highest absolute draw). Then expected ex ante utility over the whole meeting, $E V$, is given by:

$$E V = p_B E v_T + p_1 \sum_{s=1}^{T-1} E v(s) = (p_B - p_1) E v_T + T p_1 E v.$$

(1)

To evaluate the mechanism’s performance, I compare $E V$ to ex ante utility in the absence of the bonus vote, when decisions are taken by simple majority, $E W$:

$$E W = T p E v.$$

(2)

The symbol $p$ indicates the probability of obtaining one’s desired outcome when every voter casts a single vote:

$$p = \sum_{z=(n+I-1)/2}^{n+I-1} \binom{n+I-1}{z} \left( \frac{1}{2} \right)^{n+I-1}$$

(3)

where:

$I = \begin{cases} 1 & \text{if } n \text{ is even} \\ 0 & \text{if } n \text{ is odd} \end{cases}$

The Appendix shows:

**Proposition 2.** For all $G$, $T$, and $n$ there exists a range of strictly positive values for $B, B(n)$, such that for all $B \in B(n)$: (i) If $n$ is even, $E V > E W$. (ii) If $n$ is odd, $E V > E W$ as long as $E v_T/E v > [T(n+1)]/[T(n-1) + 2]$.

If the number of voters is even it is always possible to choose a value of $B$ that leads to a welfare improvement relative to simple majority. Not surprisingly, welfare improvements relative to simple majority are more difficult to guarantee when $n$ is odd, for the simple reason that majority voting performs much better in that case. Whether the condition in the proposition is satisfied depends on the distribution $G$, on $n$, and on $T$. Three observations are immediate. First, notice that $[T(n+1)]/[T(n-1) + 2]$ is strictly declining in $n$. In the limit, when the number of voters becomes unboundedly large, the ratio approaches 1, and since $E v_T/E v > 1$ by definition, strict welfare gains can be obtained for all distributions $G$ and all $T$. At large $n$, the distinction between $n$ odd and $n$ even has to be irrelevant. Second, the effect of the number of proposals $T$ is less straightforward, because both $E v_T/E v$ and the ratio $[T(n+1)]/[T(n-1) + 2]$ increase with $T$. Whether the condition is more easily satisfied at small or large $T$ depends on the distribution $G$. Indeed, and this is the third observation, the shape of $G$ plays a central role. Suppose for example $G(v) = |v|^b = v^b$. Then $E v_T = Tb/(Tb + 1)$ and $E v = b/(b + 1)$. With $[T(n+1)]/[T(n-1) + 2]$ strictly declining in $n$, if the condition is satisfied at $n = 3$, it is satisfied at all

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3 Casella and Gelman (2007) analyze in detail the scope of storable votes in large elections.
At $n = 3$ the inequality is satisfied strictly by all $b < 1$, and weakly by $b = 1$ (the uniform distribution). More generally, for any $T$ and $n$, the inequality is satisfied for all $b \leq (n - 1)/2$.

What is the meaning of these conditions, and why is the inequality in the proposition required? With the extra bonus vote, the probability of obtaining the desired outcome rises when the bonus vote is cast, but falls in the case of the other proposals. A welfare improvement then requires a sufficient wedge between the highest valuation and the mean valuation. The example of the power distribution makes the point clearly. The ratio $Ev(T)/Ev$ is strictly decreasing in $b$. With $b < 1$, the density function is decreasing and convex, and the probability mass is concentrated around zero. Most draws will have small values, but a draw may come from the tail of the distribution: the wedge between the expected highest valuation and the mean valuation is relatively large. The higher probability of winning the vote over that one issue is valuable and overrides the lower probability of winning the low valuations proposals. With $b > 1$, on the other hand, the density function is increasing in $v$, and there is larger probability mass at higher valuations: most proposals are likely to be considered important, and the ratio $Ev(T)/Ev$ is relatively small. Increasing one’s influence on a single issue is less valuable because it comes at the cost of reduced influence over all others.

It is important to note that the critical range $B(n)$ identified in the proposition depends on $n$ only: although we are assuming that $G$ is common knowledge, the mechanism does not require the planner to know the distribution of valuations. The general lesson is that, although strictly positive, $B$ should not be too large.4

Example. Consider the example of $G$ Uniform, and $B = 1$. To evaluate more intuitively the welfare properties of storable votes and simple majority voting, express ex ante expected utility with either mechanism as share of expected first best efficiency. Expected first best efficiency $EV^*$ is a voter’s expected utility if each proposal is decided in favor of the side with higher total valuation (the criterion that maximizes ex ante utility). Its expression, together with the explicit equations for all relevant probabilities, is given in the Appendix. All three criteria, $EV$, $EW$, and $EV^*$, are scaled by a plausible lower bound: expected utility with random decision-making $ER = TEv(1/2)$. The normalized variables $(EV - ER)/(EV^* - ER)$ and $(EW - ER)/(EV^* - ER)$ express the share of available surplus over random decision-making that the mechanism is expected to appropriate. For the representative case $T = 3$ and $n \in \{2, .., 21\}$, they are plotted in Figure 1, where the black dots correspond to storable votes, and the grey dots to simple majority. In the case of simple majority, the grey dots trace two distinct curves: a lower curve, corresponding to $n$ even, and a higher curve, corresponding to $n$ odd.

4The proof of Proposition 1 identifies thresholds for $B$ that decline monotonically with $n$. The restriction is sufficient for welfare gains, but not necessary. All numerical simulations I have run suggest that it is much stronger than needed.
Figure 1.

$G$ Uniform, $T = 3$, $B = 1$. Share of available surplus that the mechanism is expected to appropriate, as function of $n$. The black dots correspond to storable votes, and the grey dots to simple majority.

At $n = 3$, the two mechanisms are equivalent ($n = 3$ corresponds to the highest point in the figure where the black and the grey dot are superimposed), but for all other $n$ storable votes outperform simple majority. The most striking aspect of the figure is the stability of the storable votes welfare measure, as opposed to the predictable sensitivity of simple majority to $n$ odd or even. As $n$ increases, the two variables stabilize to a 5 percent margin in favor of storable votes.\footnote{The absolute improvement over randomness decreases with $n$. With $G$ symmetric, for very large $n$, random decision-making, majority voting and storable votes all approach efficiency asymptotically, although the percentage gain in favor of storable votes is maintained.}

4 Endogenous order of proposals

Suppose now that the order of the different proposals is not predetermined. One of the voters has the role of committee chair and at any time $t \in \{0, ..., T - 1\}$, knowing the state of the game $(\Lambda_t, t)$, chooses which of the proposals still to be decided is voted upon next. All proposals on the agenda are voted on, but the order is decided, proposal by proposal, by the chair. A voter’s strategy is, for all voters, the choice of the proposal on which to cast the bonus vote, and, for the chair alone, the order in which the votes are called. The question is whether the possibility to manipulate the order of the proposals affects the efficiency properties of the mechanism. There is a plausible reason to worry about it. Could the chair choose the order so as to exhaust other voters’ bonus votes before presenting his own favorite proposal, and then carry it through the strength of his own bonus vote, even with a narrow support and an efficiency loss?
4.1 Equilibrium

A first answer to this question comes immediately from the results established so far. Allowing for an endogenous order of proposals need not reduce efficiency:

**Proposition 3.** For all $T, n, G,$ and $B$ there exists an equilibrium where at any $t$ the chair chooses proposal $P_t$ randomly among all remaining proposals and casts his bonus vote over $P_t$ if and only if $v_{it} = \max\{v_{it}\}$. Ex ante welfare $EV$ is identical to the case of exogenous order of proposals.

I discussed earlier how in this equilibrium the probability that a voter endowed with his bonus vote at $t$ will cast it over any of the remaining proposal is constant at $1/(T - t)$. From the point of view of a different committee member, then, the only criterion for casting the bonus vote is the intensity of his own preferences: there is no other source of gains from anticipating or postponing the use of the bonus votes. The identical reasoning applies when the order of proposals is endogenous. If the proposer chooses randomly, no information about his relative intensities is conveyed; casting the bonus vote on their highest priority proposal remains a best response for all other voters, guaranteeing that a random choice is itself a best response for the proposer. The endogeneity of the proposals’ order, and the identity of the proposer, have no impact on expected outcomes.

This reasoning, however, already suggests the possibility of different equilibria where the proposer’s choice of agenda order is informative. Consider the following example.

**Example.** Suppose $T = 2$, $n = 3$, $B = 1$ and $G$ Uniform. Then there exists an informative equilibrium where the chair $c$ chooses $P_1$ such that $v_{1c} \geq v_{2c}$ and casts his bonus vote on $P_1$ with probability 1; each of the other two committee members, $j \neq c$, spends his bonus vote on $P_1$ if and only if $v_{1j} \geq (3/2)v_{2j}$, and thus is expected to cast the bonus vote on $P_1$ with probability $1/3$. There is also a second informative equilibrium, mirror-image of the first, where the chair chooses $P_2$ such that $v_{2c} \geq v_{1c}$ and always casts his bonus vote on $P_2$; each of the other two committee members spends his bonus vote on $P_1$ if and only if $v_{1j} \geq (2/3)v_{2j}$, and thus is expected to cast his bonus vote on $P_1$ with probability $2/3$.

The example considers the case of two proposals, but the insight it conveys is more general. When the order of proposals is exogenous, the game is effectively static: a committee member’s past votes do not provide information that allows to differentiate among his future votes: any future vote is just as likely to be in favor or against, or to include or not the bonus vote, as any other. The member’s equilibrium strategies and the final outcome are identical to what they would be if all votes were collected at the same time, as opposed to following a sequential order. But when the order of proposals is endogenous, the order itself can be used by the chair to provide information about his relative intensity of preferences, and thus about his voting strategies. The interesting result is that the chair’s transmission of information can be credible: knowing that the chair
Proposition 4. Suppose \( T = 2 \). Then for all \( G \) and \( n \) and \( B \in \mathcal{B}(n) \) there exist two payoff-equivalent informative equilibria. There exists a unique value \( \alpha(n,G) \geq 1 \) such that in one equilibrium: (i) the chair \( c \) chooses \( P_1 \) such that \( v_{1c} \geq v_{2c} \) and always casts his bonus vote on \( P_1 \); (ii) any other committee member \( i \) casts his bonus vote on \( P_1 \) if and only if \( v_{1i} \geq \alpha(n,G)v_{2i} \). The second equilibrium is the mirror image of the first: (i) the chair \( c \) chooses \( P_1 \) such that \( v_{1c} \leq v_{2c} \) and always casts his bonus vote on \( P_2 \); (ii) any other committee member \( i \) casts his bonus vote on \( P_2 \) if and only if \( v_{2i} \geq \alpha(n,G)v_{1i} \). These are the only two quasi-symmetric informative equilibria holding for all \( G \) and \( n \).

Proposition 4 generalizes the example discussed above, but may leave the incorrect impression that the existence of informative equilibria requires \( T = 2 \). With two proposals only, the static nature of the exogenous order game is obvious: any decision concerning the second proposal is simply residual, and each voter faces the single choice of casting, or not, the bonus vote on the first proposal. When the order is endogenous, if it matters at all, it can only be through the transmission of information. But in fact, as argued earlier, the intuition is more general, and the order of the agenda can be used to transmit information, and only to transmit information, for any number of proposals. With an eye to the parameter choices that will be made in the experimental part of the analysis, it is possible to show:

Proposition 4b. Suppose \( n \in \{3,4\} \) and \( B \in \mathcal{B}(n) \). Then for all \( G \) and \( T \) there exist \( T \) payoff-equivalent informative equilibria. Call \( P_s^* \) the proposal to which voter \( i \) attaches highest intensity: \( v_{is} = \max v_i \). In each equilibrium there exists a unique value \( \alpha(n,G,T) \geq 1 \) such that: (i) the chair \( c \) orders proposal \( P_s^* \) as the \( s \)th proposal and always casts his bonus vote on \( P_s \), with \( s \in \{1,2,..,T\} \); (ii) any other committee member \( j \neq c \) casts his bonus vote on \( P_j^* \) if either \( P_j^* \neq P_s \), or \( P_j^* = P_s \) but \( v_{js} \geq \alpha(n,G,T)v_{jt} \) for all \( t \neq s \); if \( P_j^* = P_s \) but \( v_{js} < \alpha(n,G,T)v_{jt} \) for some \( t \neq s \), then voter \( i \) casts his bonus vote on \( P_r^* \) such that \( v_{jr} \geq v_{jt} \) for all \( r,t \neq s \). (The proof is in the Appendix).

The proposition makes clear that in these equilibria the transmission of information is the only purpose of the order chosen by the chair. The chair’s
highest priority proposal can take any position in the order of the agenda, as long as such a position is understood as signaling the chair’s priority. All other voters reduce (at least weakly) their propension to cast their bonus votes on the chair’s priority proposal, supporting the chair’s strategy. Notice that the static nature of the game remains true here: although a proposal’s position in the order of the agenda matters, equilibrium strategies are not conditioned on the state of the game. The chair could announce the list of proposals at the start of the game and collect all votes simultaneously without any change in equilibrium strategies and outcomes. As is the case when the order of proposals is exogenous, once again voters’ past behavior conveys no information that allows to differentiate among future voting choices.

For future reference, note that with committees of 3 or 4 voters, $B = 1$ belongs to $B(n)$: the proposition applies to environments where the bonus vote is equivalent to a regular vote.6

4.2 Efficiency

From the perspective of the committee chair, the ability to fix the order of the proposals provides the opportunity to commit to a known voting strategy, reducing the competition for influence from the other voters and increasing the probability of being pivotal when it matters. Intuitively, informative equilibria should yield higher expected utility for the chair. The effect on other voters’ utility is less clear: in general better information should be welfare improving, but voting problems are riddled with externalities. Because all voters tend to shift their bonus votes away from the chair’s highest priority proposal, in informative equilibria the probability of being pivotal may well be lower for all voters who are not chair. I conjecture:

Conjecture. Suppose $n \in \{3, 4\}$ and $B \in B(n)$. Call $EV_i^I$ the ex ante expected utility of voter $i$ in the informative equilibria described in Proposition 4b, after the voter’s role in the committee is known, and $EV^I$ ex ante expected utility with equal probability of being chair. Then for all $G$ and $T$: (i) $EV_c^I \geq EV$; (ii) $EV_j^I \leq EV$ for all $j \neq c$; (iii) $EV^I \geq EV$, and (iv) $EV^I > EW$ if $n = 4$; $EV^I > EW$ if $n = 3$ and (condition).

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6Although not stated for $n = 2$, the proposition holds in such a case too, with $\alpha(G, T)|_{n=2} = 1$ for all $G, T$. The reason is that when $n = 2$ casting the bonus vote on one’s highest valuation is a dominant strategy, for any $G$ and for any $T$: the increase in the pivot probability associated with the use of the bonus vote is exactly identical, whether one’s opponent casts the bonus vote or not. Thus whether the agenda’s order is informative or not has no effect on the strategy of the voter who is not the chair. Similarly the chair can choose to transmit information through the agenda setting, or not to, with no effect on his own voting strategy.
Example.

Figure 2. $G$ Uniform, $n = 3$ and $n = 4$, $B = 1$.

Figure 2a: Expected utility with endogenous agenda, relative to expected utility with exogenous agenda, as function of $T$.

Figure 2b. Expected utility with endogenous agenda, relative to expected utility with simple majority voting, as function of $T$.

The upper black dots refer to the chair; the lower black dots to a representative voter who is not the chair and the red dots to ex ante expected welfare with equal probability of being chair.
5 Appendix

Proof of Proposition 1. (1) To verify that the strategies described are an equilibrium, consider $i$'s problem at any time $t$, where $i \in \Lambda_t$. Voter $i$ must decide whether to cast the bonus vote on $P_t$ or wait for a future proposal $P_{t+k}$, $k \in \{1, \ldots, T-t\}$. Call $l_t$ the number of voters other than $i$ left with the bonus vote at $t$; $p_{Bs}$ ($p_{1s}$) the probability of obtaining the desired outcome when casting $1 + B$ $(1)$ votes on $P_s$, and $\pi_s$ the probability that a voter $r \in \Lambda_t$ and other than $i$ casts the bonus vote over proposal $P_s$. Given the symmetry of $G$, at any $s$ the probability of obtaining the desired outcome when casting the bonus vote depends exclusively on the number of other voters who also cast the bonus vote. Thus:

$$p^t_{Bt} = \sum_{j=0}^{l_t} \binom{l_t}{j} (\pi^t_i)^j (1 - \pi^t_i)^{l_t-j} p_B(j)$$

where the superscript $t$ indicates the time at which the probability is evaluated, and $p_B(j)$ is the probability of obtaining the desired outcome when $i$ and $j$ other voters cast the bonus vote and all others do not. In the candidate equilibrium $\pi^t_i = 1/(T-t)$. Thus:

$$p^t_{Bt} = \sum_{j=0}^{l_t} \binom{l_t}{j} \left( \frac{1}{T-t} \right)^j \left( \frac{T-t-1}{T-t} \right)^{l_t-j} p_B(j) \quad (A.1)$$

Similarly, evaluated at $t$, the probability of obtaining the desired outcome when casting the bonus vote at $t+k$ is given by:

$$p^t_{Bt+k} = \sum_{j=0}^{l_t} \binom{l_t}{j} \left[ \prod_{s=0}^{k-1} (1 - \pi^t_{t+s}) \pi^t_{t+k} \right] \left[ 1 - \prod_{s=0}^{k-1} (1 - \pi^t_{t+s}) \pi^t_{t+k} \right]^{l_t-j} p_B(j) \quad (A.2)$$

But in the candidate equilibrium $\pi^t_{t+s} = 1/(T-t-s)$. The probabilities in the square brackets of equation (A.2) simplify, and we obtain:

$$p^t_{Bt+k} = \sum_{j=0}^{l_t} \binom{l_t}{j} \left( \frac{1}{T-t} \right)^j \left( \frac{T-t-1}{T-t} \right)^{l_t-j} p_B(j) = p^t_{Bt} \quad (A.3)$$

Following the same logic, we can show $p^t_{1t} = p^t_{1t+k}$. At time $t$ voter $i$ decides whether to cast the bonus vote on $P_t$ or $P_{t+k}$ by evaluating the difference in expected utility from the two actions: $Eu_i(P_t) - Eu_i(P_{t+k}) = v_i(p^t_{Bt} - p^t_{1t}) - v_{t+k}(p^t_{Bt+k} - p^t_{1t+k}) = (p^t_B - p^t_1)(v_t - v_{t+k})$. Since $p^t_B \geq p^t_1$, casting the bonus vote on $P_t$ if $v_t > v_{t+k}$ is a best response response strategy for $i$ for all $k \in \{1, \ldots, T-t\}$ and all $t$. We can conclude that the candidate equilibrium is indeed an equilibrium.

(2) To establish part (2), note first that the strategy described in the lemma is a best response for all other voters’ strategies that result in $p^t_{Bs} = p^t_B$ and
\( p_{i,s} = p_i \) for all \( s \). But with iid valuations, all strategies that depend exclusively on relative valuations have this property. Thus no strategy of the type: cast the bonus vote on the \( k \)th higher valuation with probability \( \pi_k \) can be an equilibrium unless \( k = 1 \) and \( \pi_k = 1 \). If another equilibrium exists, it must involve strategies where voters differentiate among the proposals on the basis of their order. For our purposes it is sufficient to show that no such equilibrium exists for specific values of \( n \) and \( T \). Suppose for example \( n = 3 \) and \( T = 2 \), and consider a candidate equilibrium where \( \pi_1 = q \neq 1/2 \). With \( n = 3 \), the bonus vote can make a difference only if \( B \geq 1 \), an assumption we maintain here. In this case \( p_{i1} = p_{12} = 3/4 - (I[q(1-q)])/2 \) (where \( I = 1/2 \) if \( B = 1 \), and \( I = 1 \) if \( B > 1 \)), but \( p_{11} = 3/4 + I[(1-q)^2]/4 \) and \( p_{12} = 3/4 + I[q^2]/4 \). Voter \( i \)'s best response is to cast the bonus vote on \( P_1 \) if \( v_1/v_2 > (2q - q^2)/(1-q^2) \). If \( q > 1/2 \), \( (2q - q^2)/(1-q^2) > 1 \), and thus voter \( i \) casts the bonus vote on \( P_1 \) with probability \( \pi_1 < 1/2 \). But if \( q < 1/2 \), \( (2q - q^2)/(1-q^2) < 1 \), and voter \( i \) casts the bonus vote on \( P_1 \) with probability \( \pi_1 > 1/2 \): no symmetrical equilibrium exists for \( q \neq 1/2 \).  

**Proof of Proposition 2.** The probabilities \( p_B \) and \( p_1 \) in (1) depend on the value of the bonus vote. The proof proceeds by verifying the proposition at the smallest value of the bonus vote for which the voting mechanism differs from simple majority voting, and thus \( p_B \neq p \neq p_1 \).

(1) Consider first the case of \( n \) even. Any positive value of the bonus vote, no matter how small, affects the outcome with positive probability: there is always a positive probability that the voters are exactly split and that one more voter on one side than on the other casts the bonus vote. Consider then a value of the bonus vote small enough that as long as the two groups are not equally split, the bonus votes cannot modify the outcome. This threshold is defined by the condition that a minority of size \( n/2 - 1 \) casting all bonus votes does not win over a majority of size \( n/2 + 1 \) casting no bonus votes, or:

\[
\left( \frac{n}{2} - 1 \right)(1 + B) < \left( \frac{n}{2} + 1 \right) \iff B < \frac{4}{n - 2}
\]

We can now derive \( p_B \) and \( p_1 \) for all values of \( B \in (0, 4/(n - 2)) \). The only difference with respect to simple majority voting can arise when the voters are split into groups of equal size. In this case, simple majority yields a tie, but the existence of the bonus vote creates many possible configurations of voting choices for which one side strictly wins. Consider the problem from the point of view of voter \( i \). It turns out to be simpler to isolate a second voter \( j \) on the opposite side of \( i \). Whenever voter \( j \) casts the same number of votes as \( i \), the probability that \( i \)'s side wins equals the probability that \( j \)'s side wins, and the expected deviations from simple majority cancel out. The relevant cases are those where \( i \) and \( j \) cast a different number of votes. In calculating \( p_B \) then, the deviation from \( p \) occurs when \( j \) does not cast the bonus vote. Call \( m_i \) the number of voters on \( i \)'s side who vote \( 1 + B \), ignoring \( i \), and similarly \( m_j \) the number of voters on \( j \)'s side who vote \( 1 + B \), ignoring \( j \). It is immediate that \( i \)'s side wins if \( m_i > m_j - 1 \). Similarly, \( i \)'s side ties if \( m_i = m_j - 1 \) and loses if
Thus:

\[ p_B = p + \left( \frac{1}{2} \right)^{n-1} \binom{n-1}{n/2} \left( \frac{T-1}{T} \right) \left[ \frac{1}{2} \text{prob}(m_i > m_j - 1) - \frac{1}{2} \text{prob}(m_i < m_j - 1) \right] \]

where \((T - 1)/T\) is the probability that voter \(j\) casts a single vote, and the probabilities inside the square brackets are multiplied by \((1/2)\) because, absent the bonus vote, the relevant events would result in a tie. But \(\text{prob}(m_i > m_j - 1) = \text{prob}(m_i = m_j) + \text{prob}(m_i > m_j)\), and \(\text{prob}(m_i < m_j - 1) = \text{prob}(m_i < m_j) - \text{prob}(m_i = m_j)\). And since \(\text{prob}(m_i > m_j) = \text{prob}(m_i < m_j)\), we can write:

\[ p_B = p + \left( \frac{1}{2} \right)^n \binom{n-1}{n/2} \left( \frac{T-1}{T} \right) \left[ \text{prob}(m_i = m_j) + \text{prob}(m_i = m_j - 1) \right] \]

Similarly,

\[ p_1 = p - \left( \frac{1}{2} \right)^n \binom{n-1}{n/2} \left( \frac{1}{T} \right) \left[ \text{prob}(m_i = m_j) + \text{prob}(m_i = m_j + 1) \right] \]

But the problem is symmetrical, and thus \(\text{prob}(m_i = m_j - 1) = \text{prob}(m_j = m_i - 1)\): the expressions in square brackets in (A.4) and (A.5) are identical. Call them \(C\). We can now write:

\[ p_B - p_1 = \left( \frac{1}{2} \right)^n \binom{n-1}{n/2} C \]  

(A.6)

\[ p - p_1 = \left( \frac{1}{2} \right)^n \binom{n-1}{n/2} \left( \frac{1}{T} \right) C \]  

(A.7)

We know from (1) that:

\[ EV > EW \iff Ev_{(T)}(p_B - p_1) > TEv(p - p_1) \]  

(A.8)

Substituting (A.6) and (A.7), we obtain:

\[ EV > EW \iff Ev_{(T)} > TEv \left( \frac{1}{T} \right) \text{ or } Ev_{(T)} > Ev \]

which is always satisfied. We can conclude that if \(n\) is even, ex ante efficiency strictly improves over simple majority voting for all values of \(B \in (0, 4/(n-2))\).

Note that if \(n = 2\), the value of the bonus vote is unconstrained.

(2) Consider now the case of \(n\) odd. With \(n\) odd, there are strictly positive values of the bonus vote that never alter the voting outcome from what it would be with simple majority. These are values such that when the voters are divided into two groups with opposite preferences of sizes as similar as possible \(( (n-1)/2 \) on one side, and \((n-1)/2 + 1 \) on the other), the minority loses even when all
voters on the minority side cast their bonus vote, and none of the voters on the majority side does so. Or:

\[
\left(\frac{n-1}{2}\right)(1 + B) < \left(\frac{n-1}{2} + 1\right) \iff B < \frac{2}{n-1}
\]

For all \( B < 2/(n-1) \) and \( n \) odd, then trivially \( EV \equiv EW \).\(^7\) Suppose then \( B \geq 2/(n-1) \), but suppose also that the bonus vote is small enough not to matter in any other possible configuration of votes. The next smallest threshold is given by a value of \( B \) such that when the two groups with opposite preferences are of sizes \((n-1)/2\) and \((n-1)/2 + 1\) and none of the majority voters uses the bonus vote while all but one of the minority voters do so, the result is a tie. I.e.:

\[
\left(\frac{n-1}{2}\right)(1 + B) - B = \frac{n+1}{2} \iff B = \frac{2}{n-1}
\]

Suppose then \( B \in [2/(n-1), 2/(n-3)) \). From the point of view of voter \( i \), when casting the bonus vote the probability of obtaining the desired outcome differs from the simple majority case only if he finds himself in a minority of size \((n-1)/2\) and all of the voters on his side cast the bonus vote, while none of the majority voters do so. Or:

\[
p_B = p + \left(\frac{1}{2}\right)^{n-1+I_{tie}} \left(\frac{n-1}{(n+1)/2}\right) \left(\frac{1}{T}\right)^{(n-3)/2} \left(\frac{T-1}{T}\right)^{(n+1)/2}
\]

where \( I_{tie} = 1 \) if \( B = 2/(n-1) \), and 0 if \( B \in (2/(n-1), 2/(n-3)) \).

Similarly, when casting a single vote the difference with respect to simple majority comes from the possibility of losing, even when belonging to the majority, if all minority voters cast the bonus vote and none of the majority voters do so:

\[
p_1 = p - \left(\frac{1}{2}\right)^{n-1+I_{tie}} \left(\frac{n-1}{(n-1)/2}\right) \left(\frac{1}{T}\right)^{(n-1)/2} \left(\frac{T-1}{T}\right)^{(n-1)/2}
\]

where the difference in the binomial term reflects the different size of the two groups and the fact that \( p_B \) in (A.9) is calculated supposing that voter \( i \) is in the minority group, and \( p_1 \) in (A.10) supposing \( i \) is in the majority. Hence:

\[
p_B - p_1 = \left(\frac{1}{2}\right)^{n-1+I_{tie}} \left(\frac{1}{T}\right)^{(n-1)/2} \left(\frac{T-1}{T}\right)^{(n-1)/2} \left[\left(\frac{n-1}{(n+1)/2}\right)(T-1) + \left(\frac{n-1}{(n-1)/2}\right)\right]
\]

Substituting (A.10) and (A.11) in (A.8):

\[
EV > EW \iff D \left[ Ev(T) \left(\frac{T(n-1)+2}{n+1}\right) - TEv\right] > 0
\]

\(^7\)A simple example makes the point clearly. Suppose \( n = 3 \). Then for all \( B < 1 \), a voter can win only if at least one of the other two voters agrees with him, exactly as in the case of simple majority voting.
where:

\[ D \equiv \left( \frac{1}{2} \right)^{n-1} \left( \frac{1}{T} \right)^{(n-1)/2} \left( \frac{T-1}{T} \right)^{(n-1)/2} \left( \frac{n-1}{(n-1)/2} \right) > 0 \]

Or:

\[ EV - EW > 0 \iff \frac{Ev(T)}{Ev} > \frac{T(n+1)}{T(n-1) + 2} \]

This is the condition in Proposition 2.

**Derivation of** \( p_1, p_B \) **for arbitrary** \( B \).

Consider the problem from the point of view of voter \( i \). Begin from the simpler case of exogenous (or equivalently of endogenous agenda but uninformative equilibrium). The expected voting behavior of the other voters is identical over all proposals. Consider then a proposal \( P \), and call \( \pi \) the ex ante probability that any voter casts the bonus vote on that proposal (where in equilibrium \( \pi = (1/T) \)). Call \( J \) the number of other voters who cast the bonus vote. Excluding \( i \), the total number of votes cast in the election is \( n - 1 + BJ \). Call M (\( m \)) the number of voters on \( i \)'s side who vote \( 1 + B \) (1) over \( P \), and thus \( m + (1 + B)M \) the total number of votes on \( i \)'s side, ignoring \( i \), and \( n - 1 + BJ - m - (1 + B)M \) the total number of votes on the opposite side. If voter \( i \) casts only his regular vote, for any \( M \) and \( J \), voter \( i \)'s side wins strictly if: \( m + (1 + B)M + 1 > n - 1 + BJ - m - (1 + B)M \), or \( m > \lfloor n + B(J - 2M) \rfloor /2 - 1 - M \), and ties if \( m = \lfloor n + B(J - 2M) \rfloor /2 - 1 - M \).

Taking into account that for generic \( n, J \), and \( B \lfloor n + B(J - 2M) \rfloor /2 - 1 - M \) need not be an integer, we can write:

\[ p_1 = \sum_{J=0}^{n-1} \binom{n-1}{J} \sum_{M=0}^{J} \binom{J}{M} \left[ \sum_{m=k}^{n-1-J} \binom{n-1-J}{m} + \frac{I_I}{2} \binom{n-1-J}{\bar{k}} \right] \pi^J (1-\pi)^{n-1-J}(1/2)^{n-1} \]

with:

\[ k \equiv I\left[ \frac{n + B(J - 2M)}{2} \right] - M \]

where \( I[x] \) is the largest integer smaller than \( x \);

\[ \bar{k} \equiv \frac{n + B(J - 2M)}{2} - 1 - M, \]

\[ I_I = \begin{cases} 1 & \text{if } \bar{k} \text{ is an integer} \\ 0 & \text{otherwise} \end{cases} \]

If voter \( i \) casts his bonus vote, then for any \( M \) and \( J \), voter \( i \)'s side wins strictly if: \( m + (1 + B)M + 1 + B > n - 1 + BJ - m - (1 + B)M \), or \( m > \lfloor n + B(J - 1 - 2M) \rfloor /2 - 1 - M \); and ties if \( m = \lfloor n + B(J - 1 - 2M) \rfloor /2 - 1 - M \).

We can write:

\[ p_B = \sum_{J=0}^{n-1} \binom{n-1}{J} \sum_{M=0}^{J} \binom{J}{M} \left[ \sum_{m=\bar{k}}^{n-1-J} \binom{n-1-J}{m} + \frac{I_{I_B}}{2} \binom{n-1-J}{\bar{k}_B} \right] \pi^J (1-\pi)^{n-1-J}(1/2)^{n-1} \]

\[ \bar{k}_B = \frac{n + B(J - 2M)}{2} - 1 - M, \]

\[ I_{I_B} = \begin{cases} 1 & \text{if } \bar{k}_B \text{ is an integer} \\ 0 & \text{otherwise} \end{cases} \]
with:

\[ k_B \equiv I\left[\frac{n + B(J - 1 - 2M)}{2}\right] - M \]

where \( I[x] \) is the largest integer smaller than \( x \);

\[ \tilde{k}_B \equiv \frac{n + B(J - 1 - 2M)}{2} - 1 - M; \]

\[ I_l = \begin{cases} 
1 & \text{if } \tilde{k}_B \text{ is an integer} \\
0 & \text{otherwise}
\end{cases} \]

When the agenda is endogenous, the informative equilibrium strategies are characterized by Proposition 4. Consider an equilibrium where the chair always casts the bonus vote on the first proposal, and call \( \pi_t \) the probability that a voter who is not the chair casts the bonus vote on the \( t \)th proposal. By Proposition 4, \( \pi_{t'} = \pi_t \) and \( \pi_{t'} \neq \pi_t \) for all \( t, t' \neq 1 \). The probabilities derived above must now be indexed by \( t \) and by the role of the voter. I will use the index \( c \) to distinguish the chair. With the correct \( \pi_t \), the probabilities \( p_{Bct} \) and \( p_{1ct} \) are still given by the expressions above. The appropriate expressions for \( p_{Bt} \) and \( p_{1t} \) are also derived easily from above, but must take into account that one of the other voters - the chair - behaves asymmetrically: \( \pi_{1c} = 1 \) if \( \pi_{1c} \neq 1 \), and \( \pi_{tc} = 0 \) for all \( t \neq 1 \).

**Efficiency**

Expected first best efficiency is given by:

\[
EV^* = (1/2)^{n-1} \left[ \frac{1}{2} + \sum_{k=0}^{n-2} \binom{n-1}{k} \left( \int_0^k (1/2)Pdx + \int_k^{k+1} \int_{x-k}^1 sdsPdx \right) \right]
\]

where \( P \) is the density function of a sum of \( n - 1 \) independent variables. The formula for \( P \) is relatively simple when the \( n - 1 \) variables are each distributed according to a Uniform over the interval \([0, 1]\):

\[
P = \sum_{k=0}^{n} (-1)^k \binom{n-1}{k} (n-k)^{n-2} \text{Sign}[x-k] \frac{1}{2(n-2)!}
\]

**Proof of Proposition 4.** I begin by studying the first candidate equilibrium. Recall that each voter \( i \), including the chair, decides whether to cast the bonus vote on \( P_1 \) or \( P_2 \) by evaluating the difference in expected utility from the two actions: \( Eu_i(P_1) - Eu_i(P_2) = v_{i1}(p_{iB1} - p_{i11}) - v_{i2}(p_{iB2} - p_{i12}) \), where all probabilities are evaluated at the time of the first decision. Any voter’s best response strategy is to cast the bonus vote on \( P_1 \) if and only if:

\[
\frac{v_{i1}}{v_{i2}} \geq \frac{p_{iB2} - p_{i12}}{p_{iB1} - p_{i11}} \quad (A.12)
\]

where \( i = c \) if the voter is the chair, and I ignore the subscript otherwise.
Restricting the value of the bonus vote to \( B \in B(n) \) allows us to calculate \( (A.12) \) and establish its properties.

(1) Suppose first that \( n \) is even. The derivation of the relevant probabilities proceeds as in the proof of Proposition 2. Consider first the problem for a voter \( i \) who is not the chair, and begin by deriving \( p_{B1} \), the probability of winning \( P_1 \) when casting the bonus vote. Recall that \( B < 4/(n - 2) \), implying that the bonus vote can make a difference only if the two groups on opposite side of any proposal are of equal size. With probability \( 1/2 \), the chair is against \( i \), the chair’s bonus vote cancels \( i \)'s, and \( p_{B1} = p \) (where \( p \) is the probability of winning \( P_1 \) under simple majority). With probability \( 1/2 \), the chair is on \( i \)'s side. In this case, consider two voters on the opposite side, one of whom I call \( j \). If both cast their bonus votes, again \( p_{B1} = p \). However, if either one or both do not cast their bonus vote, then \( p_{B1} > p \). Working through the different scenarios, we can derive:

\[
p_{B1} = p + \left( \frac{1}{2} \right)^{n-1} \binom{n-2}{n/2} \left[ (1 - \pi_1)A_{11} + \frac{(1 + \pi_1)}{2}A_{01} + \frac{(1 - \pi_1)}{2}A_{21} \right]
\]

where \( A_{0t}, A_{1t}, \) and \( A_{2t} \) are defined as:

\[
A_{0t} \equiv \text{prob}(m_{it} = m_{jt}) = \sum_{m=0}^{n/2-2} \binom{n/2-2}{2m} \binom{n/2-2}{m} \pi_t^{2m}(1 - \pi_t)^{n-4-2m}
\]

\[
A_{1t} \equiv \text{prob}(m_{it} = m_{jt} + 1) = \sum_{m=0}^{n/2-3} \binom{n/2-2}{m} \binom{n/2-2}{m+1} \pi_t^{2m+1}(1 - \pi_t)^{n-5-2m}
\]

\[
A_{2t} \equiv \text{prob}(m_{it} = m_{jt} + 2) = \sum_{m=0}^{n/2-4} \binom{n/2-2}{m} \binom{n/2-2}{m+2} \pi_t^{2m+2}(1 - \pi_t)^{n-6-2m},
\]

and \( m_{it} (m_{jt}) \) is defined as the number of voters on \( i \)'s (\( j \)'s) side, up to a maximum of \( n/2 - 2 \) who cast the bonus vote on \( P_t \). The maximum is \( n/2 - 2 \) because we always exclude voter \( i \) and account separately for the chair, and, to keep the two sides balanced, for \( j \) and one other voter on \( j \)'s side. The details are cumbersome but not difficult, and we obtain\(^8\):

\[
p_{B1} - p_{11} = \left( \frac{1}{2} \right)^{n-1} \left\{ \binom{n-2}{n/2} \left[ (A_{11} + A_{01}) + (A_{21} - A_{01})(1 - \pi_1) \right] \right\}^2
\]

\[
+ \left( \frac{n-2}{n/2-1} \right) [(A_{11} + A_{01}) + (A_{21} - A_{01}) \pi_1 (1 - \pi_1)]
\]

When deriving \( p_{B2} - p_{12} \), on the other hand, voter \( i \) knows that the chair will not cast the bonus vote, and thus the expression will not be exactly symmetrical to \( (A.13) \). Again the details are somewhat cumbersome, but the logic.

\(^8\)The detailed derivation is available upon request.
straightforward. The expression is:

\[
p_{B_2} - p_{12} = \left(\frac{1}{2}\right)^{n-1} \left\{ \frac{n-2}{n/2} \left( (A_{12} + A_{02}) + (A_{22} - A_{02}) \pi_2^2 \right) + \frac{n-2}{n/2 - 1} \left( (A_{12} + A_{02}) + (A_{22} - A_{02}) \pi_2 \right) \right\} \]

With \( T = 2 \), \( \pi_1 = 1 - \pi_2 \), and simple manipulations show that for all \( \pi_1 \), \( A_{11} = A_{12} \), \( A_{01} = A_{02} \), and \( A_{21} = A_{22} \). It follows that \( p_{B_1} - p_{11} = p_{B_2} - p_{12} \). A voter \( i \) who is not the chair should cast the bonus vote on \( P_1 \) if and only if \( v_{i1} \geq v_{i2} \). Hence \( \alpha(n, G) = 1 \), and \( \pi_1 = 1/2 \).

Consider now the chair’s problem. Given \( \pi_1 = 1/2 = \pi_2 \), the chair’s best response is simply to cast his bonus vote on the proposal to which he attaches highest valuation. Thus if \( v_{c1} \geq v_{c2} \), always casting the bonus vote on \( P_1 \) is indeed supported as an equilibrium strategy. But we also know from Proposition 3 that with \( \pi_1 = 1/2 = \pi_2 \) from the point of view of the chair the order of proposals is irrelevant, and choosing \( P_1 \) such that \( v_{c1} \geq v_{c2} \) is consistent with the chair’s best response.

We can conclude that if \( n \) is even and \( T = 2 \), there is indeed an equilibrium where \( v_{c1} \geq v_{c2} \), \( x_{c1} = 1 + B \), and \( x_1 = 1 + B \) if and only if \( v_{i1} \geq v_{i2} \), or equivalently \( \alpha(n, G) = 1 \) for all \( G \). The equilibrium is informative because it is common knowledge that the chair always casts his bonus vote on the first proposal; however if \( n \) is even and \( T = 2 \), given individual valuations all voting strategies are identical to the non-informative equilibrium, and so therefore are expected payoffs.

(2) Suppose now that \( n \) is odd. Again consider first the problem for a voter \( i \) who is not the chair. Recall that \( B \in B(n) = [2/(n-1), 2/(n-3)] \), implying that the bonus vote makes a difference over simple majority only when the groups on opposite sides of a proposal differ in size by a single voter; all minority voters cast the bonus vote, and none of the majority does. In the candidate equilibrium, the chair always casts his bonus vote on \( P_1 \). Thus \( p_{B_1} \neq p \) only if the chair is on \( i \)'s side and so are \( (n - 1)/2 - 2 \) other voters, all of which cast their bonus vote, while none of the others do. That is:

\[
p_{B_1} = p + \left(\frac{1}{2}\right)^{n-1+I_{tie}} \left( \frac{n-2}{(n+1)/2} \pi_1^{(n-5)/2}(1 - \pi_1)^{(n+1)/2} \right) \]

\[
(A.15)
\]

The same logic allows us to derive \( p_{11} \), \( p_{B_2} \), and \( p_{12} \), and we obtain:

\[
p_{B_1} - p_{11} = \left(\frac{1}{2}\right)^{n-1+I_{tie}} \pi_1^{(n-5)/2}(1 - \pi_1)^{(n-1)/2} \left[ \left( \frac{n-2}{(n+1)/2} \right)(1 - \pi_1) + \left( \frac{n-2}{(n-1)/2} \right) \pi_1 \right] \]

\[
(A.16)
\]

and:

\[
p_{B_2} - p_{12} = \left(\frac{1}{2}\right)^{n-1+I_{tie}} \left( \frac{n-2}{(n-1)/2} \right) [\pi_2(1 - \pi_2)]^{(n-3)/2} \]

\[
(A.17)
\]
Similarly, whose members cast their bonus vote, while none of the others do. That is:

\[ \frac{p_{B2} - p_{12}}{p_{B1} - p_{11}} = \left( \frac{\pi_2(1 - \pi_2)}{\pi_1(1 - \pi_1)} \right)^{(n-3)/2} \left( \frac{\pi_1}{1 - \pi_1} \right) \left( \frac{n + 1}{n - 3 + 4\pi_1} \right) \]  
(A.18)

Taking into account that \( \pi_1 = 1 - \pi_2 \), (A.18) simplifies to:

\[ \frac{p_{B2} - p_{12}}{p_{B1} - p_{11}} = \left( \frac{\pi_1}{1 - \pi_1} \right) \left( \frac{n + 1}{n - 3 + 4\pi_1} \right) \equiv \beta(\pi_1) \]

Voter \( i \)'s best response strategy is to cast the bonus vote on \( P_1 \) if \( v_{i1} \geq \beta(\pi_1)v_{i2} \). Given \( \pi_{c1} = 1 \), a symmetrical best response by all voters who are not the chair amounts to finding a value of \( \pi_1 \) that solves \( \pi_1 = \text{prob}(v_1 \geq \beta(\pi_1)v_2) \), or:

\[ \pi_1 = \int_0^1 \left[ 1 - F(\min[1, \beta(\pi_1)v_2]) \right] dF(v_2) \]  
(A.19)

With \( v_1 \) and \( v_2 \) independent and distributed according to \( F \), the strategy described in Proposition 4 is a symmetrical best response if and only if, for all \( n \) and \( G \), there exists a solution to (A.19) at some \( \bar{\pi}_1 \in (0, 1/2) \). For given \( n \) and \( G \), call the right-hand side of (A.19) \( \varphi(\pi_1) \). The function \( \varphi(\pi_1) \) is continuous in \( \pi_1 \) and for all \( n \geq 3 \), \( \partial\beta(\pi_1)/\partial\pi_1 > 0 \), implying that \( \partial\varphi(\pi_1)/\partial\pi_1 \leq 0 \). But \( \lim_{\pi_1 \to 0} \beta(\pi_1) = 0 \), and hence \( \lim_{\pi_1 \to 0} \varphi(\pi_1) = 1 \), while \( \beta(1/2) = (n + 1)/(n - 1) > 1 \) for all finite \( n \), and hence \( \varphi(1/2) < 1/2 \). Thus there exists a unique \( \bar{\pi}_1 \in (0, 1/2) \) such that \( \bar{\pi}_1 = \varphi(\bar{\pi}_1) \), or equivalently a unique \( \alpha(n, G) > 1 \): the statement in the Proposition is verified.

Consider now the chair’s problem. The logic is unchanged. From the chair’s point of view, \( p_{cBt} \neq p \) only if the chair is in a minority of size \( (n - 1)/2 \) all of whose members cast their bonus vote, while none of the others do. That is:

\[ p_{cBt} = p + \left( \frac{1}{2} \right)^{n-1} \left( \frac{n - 1}{n + 1/2} \right) \pi_t^{(n-3)/2} (1 - \pi_t)^{(n+1)/2} \quad t = 1, 2. \]  
(A.20)

Similarly, \( p_{c1t} \neq p \) only if the chair is in a majority of size \( (n + 1)/2 \) none of whose members cast their bonus vote, while all of the others do:

\[ p_{c1t} = p - \left( \frac{1}{2} \right)^{n-1} \left( \frac{n - 1}{n - 1/2} \right) \pi_t^{(n-1)/2} (1 - \pi_t)^{(n-1)/2} \quad t = 1, 2. \]  
(A.21)

Thus:

\[ p_{cBt} - p_{c1t} = \left( \frac{1}{2} \right)^{n-1} \pi_t^{(n-1)/2} (1 - \pi_t)^{(n-1)/2} \left[ \left( \frac{n - 1}{n + 1/2} \right) \left( \frac{1 - \pi_t}{\pi_t} \right) + \left( \frac{n - 1}{n - 1/2} \right) \right] \quad t = 1, 2. \]

If \( \pi_1 \in (0, 1) \) we can write:

\[ \frac{p_{cB2} - p_{c12}}{p_{cB1} - p_{c11}} = \left( \frac{\pi_2(1 - \pi_2)}{\pi_1(1 - \pi_1)} \right)^{(n-1)/2} \left( \frac{\pi_t}{\pi_2} \right) \left( \frac{n - 1 + 2\pi_2}{n - 1 + 2\pi_1} \right) \]
or, again taking into account that \( \pi_1 = 1 - \pi_2 \) and simplifying:

\[
\frac{p_{B2} - p_{c12}}{p_{B1} - p_{c11}} = \left( \frac{\pi_1}{1 - \pi_1} \right) \left( \frac{n + 1 - 2\pi_1}{n - 1 + 2\pi_1} \right)
\]

(A.22)

The right-hand side of (A.22) is strictly increasing in \( \pi_1 \), and equals 1 at \( \pi_1 = 1/2 \); thus \( (p_{B2} - p_{c12})/(p_{B1} - p_{c11}) < 1 \) for all \( \pi_1 < 1/2 \). It follows that if \( v_{c1} \geq v_{c2} \), \( x_{c1} = 1 + B \) is indeed the chair’s best response strategy.

Finally, given \( \pi_1 < 1/2 \), could the chair gain by changing the order of proposals? Two deviations need to be ruled out. The chair could choose \( P_1 \) such that \( v_{c1} < v_{c2} \) and (i) cast the bonus vote on \( P_2 \), or (ii) still cast the bonus vote on \( P_1 \). Call \( V_c \equiv \max\{v_{c1}, v_{c2}\} \) and \( v_c \equiv \min\{v_{c1}, v_{c2}\} \). The first deviation is ruled out if: \( p_{cB1}V_c + p_{c12}v_c \geq p_{c11}v_c + p_{cB2}V_c \), or \( V_c( p_{cB1} - p_{cB2} ) \geq v_c( p_{c11} - p_{c12} ) \). We can verify immediately that \( p_{c11} - p_{c12} = 0 \), while \( p_{cB1} - p_{cB2} > 0 \) if \( \pi_1 < 1/2 \); the deviation cannot be profitable. The second deviation is ruled out if: \( p_{cB1}V_c + p_{c12}v_c \geq p_{cB1}v_c + p_{cB2}V_c \), or \( V_c( p_{cB1} - p_{cB2} ) \geq v_c( p_{cB1} - p_{cB2} ) \). This can be verified immediately from (A.20) and (A.21) above: again the deviation cannot be profitable.

We can conclude that for both \( n \) odd and \( n \) even, the strategies described in the first half of the Proposition are indeed equilibrium strategies. But with \( T = 2 \), the two proposals are completely symmetrical, and the proof can be applied identically to proposal 2. The two equilibria are mirror-images of each other, and are payoff-equivalent.

For given \( n \) and \( G \), the proof shows that \( \alpha(n, G) \) is unique. Nevertheless, as in the case of exogeneous agenda order, other quasi-symmetric informative equilibria may exist where voters who are not the chair condition their strategy on the order of the proposals alone, irrespective of their valuations. For example, it is not difficult to see from the equations above that \( x_1 = 1 + B \) for all voters (and thus \( \pi_1 = \pi_{c1} = 1 \) is an equilibrium for \( n \) odd and larger or equal to 5. But such equilibria do not exist for all \( n \), and an example is sufficient to establish the Proposition. Suppose \( n = 3 \). Consider candidate informative equilibria where \( \pi_{c1} \in (1/2, 1] \). Taking into account \( \pi_2 = 1 - \pi_1 \) and \( \pi_{c2} = 1 - \pi_{c1} \):

\[
p_{B1} - p_{c11} = (1 - \pi_1\pi_{c1})/8
\]

\[
p_{B2} - p_{c12} = (\pi_{c1} + \pi_1 - \pi_1\pi_{c1})/8
\]

If \( \pi_{c1} + \pi_1 > 1 \), then \( p_{B2} - p_{c12} > p_{B1} - p_{c11} \), implying \( \pi_1 < 1/2 \). But we showed above that if \( \pi_1 < 1/2 \), the chair’s best response is to choose \( P_1 \) such that \( v_{c1} \geq v_{c2} \) and always cast the bonus vote on \( P_1 \). The only symmetric best response for the other voters then is to cast the bonus vote on \( P_1 \) if and only if \( v_1 \geq \alpha(3, G)v_2 \), an event that occurs with probability \( \pi_1 \in (0, 1/2) \) - the equilibrium strategy characterized earlier. If \( \pi_{c1} + \pi_1 \leq 1 \), then \( p_{B2} - p_{c12} \leq p_{B1} - p_{c11} \), implying \( \pi_1 \geq 1/2 \); but \( \pi_{c1} > 1/2 \), and thus \( \pi_{c1} + \pi_1 > 1 \), a contradiction. Identical reasoning rules out any informative equilibrium with \( \pi_{c2} \in (1/2, 1] \) that would differ from the second equilibrium characterized in the Proposition. ■

**Proof of Proposition 4b.** Note first that with \( n \in \{3, 4\}, B = 1 \in B(n) \). For \( T = 2 \), the equilibria described fall under Proposition 4. I prove
here Proposition 4b for $T > 2$. The proof is logically identical to the proof of Proposition 4. Begin by analyzing the candidate equilibrium where the chair’s highest priority proposal is ordered first: $s = 1$ in the Proposition.

(1) Suppose first $n = 4$. Consider the best response strategy for voter $i$, who is not the chair. With $n = 4$ in the candidate equilibrium the probabilities derived in the proof of Proposition 4 yield:

$$\frac{p_{B_s} - p_{1s}}{p_{B1} - p_{11}} = \frac{3 - 2\pi_s + \pi_s^2}{2 + \pi_1^2}$$  \hspace{1cm} (A.23)

where $\pi_s = (1 - \pi_1)/(T - 1)$. We can write the right-hand side of (A.23) as some function $\gamma(\pi_1)$. Equation (A.12) implies that $i$’s best response is to cast his bonus vote on $P_1$ if and only if $v_{i1} \geq \gamma(\pi_1)v_{is}$ for all $s \neq 1$. In a quasi-symmetrical equilibrium, $\pi_1$ must solve:

$$\pi_1 = \int_0^1 F\left(\frac{v}{\gamma(\pi_1)}\right)^{T-1} dF(v) \equiv \psi(\pi_1)$$  \hspace{1cm} (A.24)

Both $\gamma$ and $\psi$ are continuous in $\pi_1$. Note that $\lim_{\pi_1 \to 0} \gamma(\pi_1) \in (1, 3/2)$ for all $T > 2$, and hence $\lim_{\pi_1 \to 0} \psi(\pi_1) \in (0, 1/T)$; while $\gamma(\pi_1)|_{\pi_1=1/T} > 1$ for all $T > 2$, and hence $\psi(\pi_1)|_{\pi_1=1/T} < 1/T$. It follows that (A.24) must have a solution at some $\pi_1 < 1/T$. It is then easy to verify that $\gamma(\pi_1)|_{\pi_1=\pi_1^*} = \alpha(4, G) > 1$.

Finally, notice that for all $s, t \neq 1$ $p_{B_s} - p_{1s} = p_{Bt} - p_{1t}$. Voter $i$’s best response strategy then is to cast the bonus vote on $P_1$ if and only if $v_{i1} \geq \alpha(4, G)v_{it}$ for all $t \neq 1$, and cast the bonus vote on the second highest intensity proposal, if the inequality does not hold. For voters who are not the chair, the claim in Proposition 4b is confirmed.

Consider now the chair’s problem. With $B = 1$, in the candidate equilibrium:

$$\frac{p_{cB_s} - p_{c1s}}{p_{cB1} - p_{c11}} = \frac{1 - \pi(1 - \pi)}{1 - \pi_1(1 - \pi_1)}$$

an expression smaller than 1 for all $\pi > \pi_1$. Thus if $v_{c1} \geq v_{cs}$ for all $s \neq 1$, it is indeed optimal for the chair to cast his bonus vote on $P_1$. Can the chair do better by deviating in the order of the proposals? Again, two possible deviations must be considered. First, the chair could call the first vote on a proposal that is not his highest priority and refrain from casting the bonus vote until the highest priority proposal is on the table. Second, he could again call the first vote on a proposal that is not his highest priority, but still use his bonus vote on $P_1$. Using the earlier notation, call $V_c$ the chair’s highest intensity, and $v_{cr}$ the chair’s $r$th highest ($r \neq 1$). The first deviation is ruled out if $p_{cB1}V_c + p_{c1s}v_{cr} + p_{c1s} \sum_{v'_{cr} \neq V_c, v_{cr}} v_{cr} \geq p_{c11}v_{c1} + p_{cBs}V_c + p_{c1s} \sum_{v'_{cr} \neq V_c, v_{cr}} v_{cr}$, or $V_c(p_{cB1} - p_{cBs}) \geq v_{cr}(p_{c11} - p_{c1s})$. A little work shows that the relevant probabilities are given by:

$$p_{cB1} - p_{cBs} = 3(1/2)^4 \left[(1 - \pi_1)(1 - \pi_1 + \pi_1^2) - (1 - \pi)(1 - \pi + \pi^2)\right]$$

$$p_{c1s} - p_{c11} = 3(1/2)^4 \left[\pi(1 - \pi + \pi^2) - \pi_1(1 - \pi_1 + \pi_1^2)\right]$$
and thus \( p_{cB1} - p_{cBs} \geq (p_{c11} - p_{c1s}) \) for all \( \pi > \pi_1 \): the first deviation cannot be profitable. The second deviation is ruled out if 
\[
p_{cB1}v_c + p_{c1s}v_c' + p_{c1s}\sum_{v_c' \neq v_c} v_c' \geq p_{cB1}v_c' + p_{c1s}v_c + p_{c1s}\sum_{v_c' \neq v_c} v_c',
\]
or 
\[
V_c(p_{cB1} - p_{c1s}) \geq v_c'(p_{cB1} - p_{c1s}).
\]
A sufficient condition then is \( p_{cB1} > p_{c1s} \). It is not difficult to verify that \( p_{cB1} > p \), while \( p_{c1s} < p \), implying that the second deviation cannot be profitable either. The strategy described in Proposition 4b with \( t = 1 \) is indeed a best response for the chair. But we have established above that the same conclusion applies to the strategy attributed to the voters who are not chair, hence for \( n = 4 \) and \( T > 2 \) this is indeed an equilibrium.

(2) Suppose now \( n = 3 \). The logic proceeds identically. Again, consider the best response strategy for voter \( i \), who is not the chair. With \( n = 3 \), it is easy to derive that in the candidate equilibrium:
\[
\frac{p_{Bs} - p_{1s}}{p_{B1} - p_{11}} = \frac{1}{1 - \pi_1}. \tag{A.25}
\]
Call the right-hand side of (A.25) \( \delta(\pi_1) \). Equation (A.12) implies that \( i \)’s best response is to cast his bonus vote on \( P_1 \) if and only if \( v_{11} \geq \delta(\pi_1)v_{1s} \) for all \( s \neq 1 \). In a quasi-symmetrical equilibrium, \( \pi_1 \) must solve:
\[
\pi_1 = \int_0^1 \left[ F \left( \frac{v}{\delta(\pi_1)} \right) \right]^{T-1} dF(v) \equiv \zeta(\pi_1). \tag{A.26}
\]
Both \( \delta \) and \( \zeta \) are continuous in \( \pi_1 \). Note that \( \lim_{\pi_1 \to 0} \delta(\pi_1) = 1 \), and hence \( \lim_{\pi_1 \to 0} \zeta(\pi_1) = 1/T > 0 \); while \( \delta(\pi_1)|_{\pi_1=1/T} > 1 \) for all finite \( T \), and hence \( \zeta(\pi_1)|_{\pi_1=1/T} < 1/T \). It follows that (A.26) must have a solution at some \( \pi_1 \in (0, 1/T) \). Thus \( \delta(\pi_1)|_{\pi_1=\pi_1} \equiv \alpha(3, G) > 1 \). Finally, notice that for all \( s, t \neq 1 \)
\[p_{Bs} - p_{1s} = p_{Bt} - p_{1t} \] Voter \( i \)’s best response strategy then is to cast the bonus vote on \( P_1 \) if and only if \( v_{11} \geq \alpha(3, G)v_{1t} \) for all \( t \neq 1 \), and cast the bonus vote on the second highest intensity proposal, if the inequality does not hold. For voters who are not the chair, the claim in Proposition 4b is confirmed.

Consider now the chair’s problem. With \( B = 1 \) and \( n = 3 \), in the candidate equilibrium:
\[
\frac{p_{cBs} - p_{c1s}}{p_{cB1} - p_{c11}} = \left( \frac{1 - \pi_1^2}{1 - \pi_1^2} \right),
\]
an expression smaller than 1 for all \( \pi > \pi_1 \). Thus if \( v_{c1} \geq v_{cs} \) for all \( s \neq 1 \), it is indeed optimal for the chair to cast his bonus vote on \( P_1 \). Can the chair do better by deviating in the order of the proposals? Again, two possible deviations must be considered. First, the chair could call the first vote on a proposal that is not his highest priority and refrain from casting the bonus vote until the highest priority proposal is on the table. Second, he could again call the first vote on a proposal that is not his highest priority, but still use his bonus vote on \( P_1 \). The first deviation is ruled out if
\[
V_c(p_{cB1} - p_{cBs}) \geq v_c'(p_{c11} - p_{c1s}).
\]
It is easy to verify that:
\[
\begin{align*}
p_{cB1} - p_{cBs} &= (1/2)^3 \left[ (1 - \pi_1)^2 - (1 - \pi)^2 \right] \\
p_{c11} - p_{c1s} &= 2(1/2)^3 [\pi_1(1 - \pi_1) - \pi(1 - \pi)]
\end{align*}
\]
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and thus \((p_{cB1} - p_{cB3}) > 0\) while \((p_{c11} - p_{c1s}) < 0\) for all \(\pi > \pi_1\), \(T > 2\).

The first deviation cannot be profitable. The second deviation is ruled out if

\[ V_c( p_{cB1} - p_{c1s} ) \geq V_c( p_{cB1} - p_{c1s} ) \]

A sufficient condition then is \(p_{cB1} > p_{c1s}\).

As in the case of \(n = 4\), \(p_{cB1} > p\), while \(p_{c1s} < p\), implying that the second deviation cannot be profitable either.

The strategy described in Proposition 4b for \(t = 1\) is indeed a best response for the chair. But we have established above that the same conclusion applies to the strategy attributed to the voters who are not chair, hence for \(n = 3\) and \(T > 2\) this is indeed an equilibrium.

The problem is fully symmetrical across proposals. If the equilibrium holds for \(t = 1\), it holds identically for any \(t \in \{1, \ldots, T\}\). The Proposition is established.