Inefficiencies from Metropolitan Political and Fiscal Decentralization: Failures of Tiebout Competition

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1. Introduction.

The analogy between competition among firms in providing private goods and "Tiebout (1956) competition" among jurisdictions in providing local public goods is central to the economic study of local public finance. The basic idea is that household mobility will induce jurisdictions to provide efficient mixes of local public goods and taxes, or they will fail to attract residents. A large literature examines when the analogy is sufficiently compelling so that inter-jurisdictional competition is efficient and the nature of departures from efficiency when these conditions are not fulfilled.[1] For efficiency, essentially the tax system and housing-market price must control any externalities in residential choice with also efficient governmental choice of the levels of the local public goods. While standard models frequently fail to meet the conditions for efficiency, economic intuition suggests that some Tiebout competition is better overall than none: The alternative of centralized provision will do nothing to match heterogeneous preferences to provision of local public goods. This paper challenges this intuition by showing that Tiebout competition leads to aggregate welfare losses in calibrated and estimated models.

The model we consider is not contrived. A metropolitan area is made up of multiple jurisdictions with given boundaries. Households differ by income and a taste parameter with utility function over numeraire consumption, housing consumption, and the level of the local congested public good (e.g., per student educational expenditure). The local public good is financed by a property tax that is chosen by majority vote of residents of the jurisdiction. Households choose where to reside, and then vote in their

[1] See Epple and Nechyba (2004) and Scotchmer (2002) for recent surveys of this literature.

jurisdiction and consume. Our findings regard cases when an income-stratified equilibrium exists, i.e., when a Tiebout-type equilibrium arises.[2]

We show computationally that welfare in aggregate, measured by aggregate compensating variation, is lower than in the analogous centralized equilibrium with the same political process for realistically specified parameters. We go on to show the same holds in an estimated model of the Boston metropolitan area. We know a priori that the Tiebout equilibrium will not be Pareto Efficient. First, majority choice of the tax level satisfies a median resident's preference and will not generally satisfy the Samuelsonian condition for efficient provision of the local public good. Second, the property tax causes a distortion in the housing market, while a head tax would be non-distorting and efficient. Third, the latter distortions imply externalities in individual residential choice. With local head taxes chosen efficiently and equilibrium household choices of jurisdictions, the modified Tiebout allocation would generate substantial welfare gains. With the imperfect system, these potential welfare gains are not just lost, but are frequently reversed. We provide computational evidence that the most costly inefficiency is the externality in residential choices. Too many relatively poorer households move into richer jurisdictions. Efficient sorting would be more exclusive than arises in equilibrium. It is rather surprising that getting part way to an efficient Tiebout allocation is frequently less efficient than no sorting.

Section 2 presents the theoretical model and associated positive and normative properties. A calibrated computational model is analyzed in Section 3, where the welfare loss from Tiebout sorting is shown. The estimated model and welfare calculations are

[2] As described in detail below, such an equilibrium will arise for realistic parameter values when standard single-crossing conditions are satisfied.

presented in Section 4. Section 5 concludes. An appendix provides analysis of the robustness of the computational findings.

2. Theoretical Analysis.

a. Elements of the Model. Our intent is to examine an archetypical model of a metropolitan area with property taxation. Households have a utility function over numeraire consumption x, housing consumption h, and the level of the local public good g measured in dollars. Households differ by endowed income y and a taste parameter α, with the latter measuring taste for the local public good as clarified below. The joint distribution on household type (y,α) is continuous and given by F(y,α), with joint density function f(y,α) assumed positive on its support $S \equiv [\underline{\alpha}, \overline{\alpha}] \text{x} [\underline{y}, \overline{y}] \subset R^2_+$. Let U = U(x,h,g; α) denote the household utility function, strictly quasi-concave, increasing, and twice continuously differentiable in (x,h,g). Further restrictions on U are discussed below.

We compare a Tiebout-type equilibrium having the metropolitan area divided into jurisdictions to the counterpart single-jurisdiction centralized equilibrium. Focusing first on the former case, the metropolitan area is divided into J jurisdictions, each with non-decreasing housing supply function $H^j_s(p^j_s)$, where p^j_s denotes the net-of-tax or supplier price of housing, and j = 1,2,…,J henceforth unless indicated otherwise. We assume absentee housing owners that supply housing competitively, but will account for their rents in our welfare calculations.[3] We assume absentee housing owners simply because it is most standard.

[3] One interpretation is that the MA is divided into jurisdictions with fixed amounts of land, and land is combined with elastically supplied factors to produce units of housing. Then the "absentee housing owners" could just as well be absentee land owners. In Section 3, we provide a specific example of this.

Equilibrium is determined in three stages. First, households purchase a home in a jurisdiction. Second, they vote in their jurisdiction for a property tax that is used to finance the local public good. Last, the local public good is determined from local governmental budget balance, and households consume (although their housing consumption is determined in the first stage). Households have rational expectations, thus anticipate all continuation equilibrium values.

This specification conforms to the case sometimes called "myopic voting," because households take as given residences, housing consumption, and the supplier price of housing when voting, which are all established in the first stage.[4] We examine this case because it is historically the most standard case in the literature. We show in the robustness analysis in the appendix that the welfare loss we find from Tiebout sorting *increases* with other standard specifications of the timing of choices and thus voter beliefs that may be more appealing.

b. Positive Properties of Equilibrium. To provide a formal description of equilibrium, begin with the third stage. Let $f_j(y,\alpha)$ denote the density of household types living in jurisdiction j, t_j the property tax rate, and $h_j(y,\alpha)$ housing consumption of household (y,α), all of which are given in the third stage. The gross housing price (p_j), local public good level (g_j), and household numeraire consumption are determined in the third stage, satisfying respectively:

$$p_j = (1+t_j)p_s^j; \tag{1}$$

[4] The label "myopic voting" is potentially confusing since voters are fully rational given residence and housing consumption have been committed in the first stage. The "myopia" interpretation arises if households could move or otherwise adjust housing consumption after voting, but voters fail to recognize this. Equilibrium is the same with either interpretation because no such changes are made in equilibrium in either case.

$$g_j \int_S f_j(y, \alpha) dy d\alpha = t_j p_s^j H_s^j(p_s^j) \tag{2}$$

and

$$x = y - (1 + t_j) p_s^j h_j(y, \alpha); \tag{3}$$

where p_s^j is also given, established in the first stage.[5] The congestion assumption about the public good implicit in (2) is also fairly standard, as for public schooling, and avoids issues of economies in providing local public goods. Obviously, the third stage values exist and are unique for any input vector.

Now consider the second, voting stage. Substitute (1) into (3), and then (3) into the utility function and write indirect utility of household (y,α) as a function of (p_j, g_j):

$$V(p_j, g_j; y, \alpha) = U(y - p_j h_j(y, \alpha), h_j(y, \alpha), g_j; \alpha). \tag{4}$$

When voting on the property tax rate, households maximize V(·) while correctly anticipating that (p_j, g_j) will satisfy (1)-(2), taking as given $(f_j(y, \alpha), h_j(y, \alpha), p_s^j)$.

Suppress the j indicator and compute the slope of an indifference curve of V = constant in the (g,p) plane:

$$\left.\frac{dp}{dg}\right|_{V=\text{const.}} = -\frac{V_g}{V_p} = \frac{U_g / U_y}{h(y, \alpha)}; \tag{5}$$

where the arguments in the numerator of the right-hand side of (5) are the same as in the right-hand side of (4). We make the following “single-crossing assumptions:”

$$\frac{\partial\left((dp / dg)\big|_{V=\text{const.}} \right)}{\partial y} > 0; \tag{SRI}$$

and

[5] Because households will correctly anticipate all equilibrium values, a negative numeraire will never arise in equilibrium.

$$\frac{\partial\left(\left.(\mathrm{dp}/\mathrm{dg})\right|_{\mathrm{V=const.}}\right)}{\partial\alpha} > 0. \qquad \text{(SR}\alpha\text{)}$$

Assumption SRI, "slope rising in income," means that the willingness to trade an increase in housing price for higher g rises with income. Intuitively, from the right-hand side of (5), one can see that this corresponds to cases where the marginal value of g rises faster with income than does housing demand. We provide examples of and evidence supporting this assumption below. The intended nature of the taste parameter is embodied in Assumption SRα. For given income, higher-α households are also more willing to trade an increase in housing price for increased g.

Proposition 1 summarizes key properties of the voting stage.

Proposition 1: Assume that $V(p_j,g_j;y,\alpha)$ is twice continuously differentiable and strictly quasi-concave in (p_j,g_j) for $(p_j,g_j) > 0$. Assume also the Inada condition that $V_g \to \infty$ as $g \to 0$. Then:

a. Majority voting equilibrium exists and is unique.

b. The equilibrium is the preferred choice of households (y,α) on the downward sloping locus $y_j^m(\alpha)$ satisfying:

$$\int_{\underline{\alpha}}^{\overline{\alpha}}\int_{\underline{y}}^{y_j^m(\alpha)} f_j(y,\alpha)\,dy\,d\alpha = .5N_j; \qquad (6)$$

$$N_j \equiv \int_{\underline{\alpha}}^{\overline{\alpha}}\int_{\underline{y}}^{\overline{y}} f_j(y,\alpha)\,dy\,d\alpha. \qquad (7)$$

c. Households living in community j with (y,α) to the "northeast" ("southwest") of the $y_j^m(\alpha)$ locus in the (α,y) plane prefer a higher- (lower-) than equilibrium tax. (See Figure 1A.)[6]

Proposition 1 is a generalization to taste variation of well known results in the literature and is a variation on Propositions 1 and 2 in Epple and Platt (1998). We provide a proof here for completeness.

Proof of Proposition 1: We suppress the community j indicator in the proof.

a. Substituting (1) into (2), a voter's preferred choice of t corresponds to the choice of (p,g) that solves:

$$\underset{p,g}{Max}\ V(p,g;y,\alpha) \tag{8}$$

$$s.t.\ \ gN = (p - p_s)H_s(p_s); \tag{9}$$

where, recall, p_s and H_s are fixed in this stage, as well as N. Since V is strictly quasi-concave and the constraint (9) is linear, voter preferences are single peaked. Thus majority voting equilibrium exists and is the preference of a median-preference voter. Strict quasi-concavity of V and linearity of the constraint (along with Inada condition) imply every voter's preferred choice is unique and interior; thus equilibrium is unique.

b. and c. The first-order conditions for a voter's preferred choice are:

$$-\frac{V_g}{V_p} = \frac{N}{H_s} \tag{10}$$

and (9). Let $g^*(y,\alpha)$ denote the preferred choice of g by voter (y,α).[7] *Differentiating (9) and (10) one obtains:*

[6] The $y_b^{ji}(b)$ loci partition the (α,y) plane into jurisdictions and are discussed below.

$$\frac{\partial g^*}{\partial \alpha} = \frac{\partial\left(-V_g / V_p\right) / \partial \alpha}{-\left[\frac{\partial(-V_g / V_p)}{\partial p}\left(-\frac{V_g}{V_p}\right) + \frac{\partial(-V_g / V_p)}{\partial g}\right]} > 0 \tag{11}$$

and

$$\frac{\partial g^*}{\partial y} = \frac{\partial\left(-V_g / V_p\right) / \partial y}{-\left[\frac{\partial(-V_g / V_p)}{\partial p}\left(-\frac{V_g}{V_p}\right) + \frac{\partial(-V_g / V_p)}{\partial g}\right]} > 0. \tag{12}$$

The denominators in (11) and (12) are (with sign) positive by strict quasi-concavity of V. The numerators are positive by SRα and SRI (see (5)), implying the inequalities. Let g^e *denote equilibrium g, which satisfies* $g^e = g^*(y,\alpha)$ *for median preference voters. Let* $y^m(\alpha)$ *satisfy the latter equation, which is continuous and unique by (11) and (12). Differentiating* $g^e = g^*(y,\alpha)$ *one obtains:*

$$\frac{dy^m}{d\alpha} = -\frac{\partial g^* / \partial \alpha}{\partial g^* / \partial y} < 0; \tag{13}$$

the inequality by (11) and (12). Thus the locus of median-preference voters is downward sloping as illustrated in Figure 1A.[8]

Any voter in community j with (y,α) to the southwest of the $y^m(\alpha)$ *locus has flatter indifference curve through* (g^e,p^e) *in Figure 1B than any median preference voter by the single-crossing conditions. (Any median preference voter has indifference curve with the same slope through* (g^e,p^e)*.) Such voters: (i) prefer lower g and p than* (g^e,p^e)*; and (ii) would vote against any tax leading to higher g and p. The reverse is true for any voters with (y,α) to the northeast of the* $y^m(\alpha)$ *locus. Voting equilibrium then requires (6), which*

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[7] Any voter (y,α) with the same preference for g has the same preference for p since (2) must be satisfied. Of course, the same preference for t is implied.
[8] The proof does not require that $y^m(\alpha)$ is everywhere interior to the set of residents as in the example in Figure 1A.

completes the proof of Part b. By (1) and (2), preference for higher (lower) g and p corresponds to preference for higher (lower) t, implying Part c. ■

Remarks on Proposition 1:

1. An example that satisfies the conditions for Proposition 1 is the CES utility function: $U = [\beta_x x^\rho + \beta_h h^\rho + \beta_g(\alpha) g^\rho]^{1/\rho}$, with $\rho < 0$ and $\beta_g'(\alpha) > 0$. We examine a variant of the latter in detail in Section 3.

2. The analysis is much simpler without taste variation, in which case V need not be quasi-concave to obtain the analogue of the results of Proposition 1.[9] With taste variation, the quasi-concavity condition need not necessarily hold. See Epple and Platt (1998) for an alternative condition.

Now consider the first-stage household choices and the implications for the full (three-stage) equilibrium. Households choose jurisdictions and housing consumption in this stage. Since households correctly anticipate all equilibrium values, their housing consumption satisfies ordinary demand, which we denote by h_d. Thus a household that chooses to live in jurisdiction j consumes housing:

$$h = h_d(p_j, g_j, y, \alpha) \text{ for all } j \text{ and } (y, \alpha). \tag{14}$$

Given jurisdictional choices, housing market clearance in community j determines the supplier price of housing:

$$\int_S h_d(p_j, g_j, y, \alpha) f_j(y, \alpha) dy d\alpha = H_s^j(p_s^j); \tag{15}$$

where p_j satisfies (1) for correctly anticipated t_j.

To determine choice of jurisdiction, find indirect utility

[9] Existence of voting equilibrium then only requires SRI. And households with income higher (lower) than the median income in community j prefer higher (lower) tax than the equilibrium tax.

$$\tilde{V}(p_j, g_j; y, \alpha) = U(y - p_j h_d(p_j, g_j, y, \alpha), h_d(p_j, g_j, y, \alpha), g_j; \alpha). \tag{16}$$

Households choose among the J jurisdictions to maximize $\tilde{V}$, correctly anticipating equilibrium (p_j, g_j), j = 1,2,…,J. Applying the Envelope Theorem, the slope of $\tilde{V}$ = constant in the (g_j, p_j) plane is of the same form as the slope of V = constant:

$$\left.\frac{dp}{dg}\right|_{\tilde{V}=\text{const.}} = -\frac{\tilde{V}_g}{\tilde{V}_p} = \frac{U_g / U_y}{h_d}; \tag{17}$$

but evaluated as is utility on the right-hand side of (16). We make the analogous single-crossing assumptions on $\tilde{V}$ as SRI and SRα, which we reference as $\widetilde{\text{SRI}}$ and $\widetilde{\text{SR}\alpha}$. The two pairs of single-crossing assumptions are closely related, and, for example, are exactly the same in the CES example in Remark 1 to Proposition 1.

Summarizing, an equilibrium arises if the following conditions are satisfied: In each community j, (p_j, g_j) satisfy (1) and (2). Household numeraire consumption satisfies (3). The tax rate in each community is the majority choice, where households maximize V when voting. Housing consumption satisfies ordinary demand, (14), and the supplier price of housing in each community satisfies housing-market clearance (15). Residential choices maximize $\tilde{V}$.

There are two types of equilibria that can arise. Our interest is in Tiebout-type equilibria with differences among jurisdictions in levels of provision of the public good and with at least some households having strict preference for their choice of jurisdiction. Thus, assume for now that $g_i \neq g_j$, for all jurisdictions $i \neq j$. Proposition 2 summarizes key characteristics of such equilibria:

Proposition 2: Tiebout equilibria with jurisdictions numbered such that $g_1 < g_2 < \ldots < g_J$:

a. Have ascending bundles: $p_1 < p_2 < \ldots < p_J$.

b. Are stratified by income and the taste parameter: For given α, if household with income y_1 resides in higher-numbered jurisdiction than household with income y_2, then $y_1 \geq y_2$ with equality for at most one income level. For given y, if household with taste parameter α_1 resides in higher-numbered jurisdiction than household with taste parameter α_2, then $\alpha_1 \geq \alpha_2$ with equality for at most one value of α.

c. Exhibit boundary indifference and strict preference for non-boundary households: Households that exist with income level $y_b^{ji}(\alpha),\ i > j = 1, 2, ..., J-1,$ for whom:

$$\tilde{V}(p_j, g_j; y, \alpha) = \tilde{V}(p_i, g_i; y, \alpha) = \underset{k=1,2,...,J}{Max}\ \tilde{V}(p_k, g_k; y, \alpha) \tag{18}$$

form a boundary in the (α,y) plane that partitions residents between communities j and i (see Figure 1A). Households on a boundary are indifferent between their chosen residents while all other residents strictly prefer their residential choice.

Versions of these results are in the literature (see, e.g., Epple and Platt (1998)), and we just outline the logic here. Proposition 2a must hold to have anyone choose a lower numbered community. Proposition 2b follows from the single-crossing assumptions $\widetilde{\text{SRI}}$ and $\widetilde{\text{SR}\alpha}$. Proposition 3c is essentially definitional. Typically, a boundary will be between communities j and j+1, but we cannot rule out that for some α no types will choose a community (implying, e.g., a boundary might be between j and j+2 for some α). Note that Proposition 2b implies that boundaries will be downward sloping.

Existence of Tiebout equilibrium in the three-stage model is not guaranteed, but is not unusual.[10] We provide computed examples below. Multiplicity of Tiebout equilibria can arise if housing supplies differ across jurisdictions. For example, with two

[10] Restrictions on preferences and technology sufficient for existence in the model with no taste variation are developed in Epple, Romer, and Filimon (1993).

jurisdictions having different housing supplies, either might be the lower-g jurisdiction. Non-stratified equilibrium always exists in the model as well. Suppose, for example, that each jurisdiction has the same housing supply. Suppose, further, that households choose jurisdictions in the first stage such that $f_j = f/J$ for all y. Then the continuation equilibrium values are the same in each jurisdiction; the jurisdictions are clones. In turn, the initial residential choices are equilibrium ones since the households are indifferent to their community. These non-Tiebout equilibria do not require the same housing supplies; initial residence choices can be adjusted so that the same (p,g) values arise in each jurisdiction. There are also mixed equilibria generally where proper subsets of jurisdictions are clones, these acting like one jurisdiction in a fully stratified equilibrium. Such equilibria are unstable (see, e.g., Fernandez and Rogerson, 1996). We study here the (full) Tiebout equilibrium, obviously in cases where it exists.

The comparison centralized equilibrium assumes the metropolitan area is one jurisdiction, with housing supply that is the usual aggregation of the jurisdictional housing supplies in the non-centralized case. Equilibrium is determined analogously to above, but with no alternative jurisdictions to choose from in the first stage and with one vote of the entire population for the tax rate, followed by consumption and provision of the public good. From above, it follows that centralized equilibrium exists and is unique. Obviously, no matching of preferences to public goods arises in the centralized case. Our interest is in the welfare comparison of the centralized equilibrium to the Tiebout equilibrium, when the latter exists. We should note that the centralized equilibrium values correspond to those in the de-centralized non-stratified (clone) equilibrium discussed in the previous paragraph, so one can interpret the comparison this way as well.

c. Efficiency Considerations. The main finding in this paper is that Tiebout sorting may likely be inefficient. In this sub-section, we first examine the social welfare problem to provide a theoretical perspective on the causes of the inefficiency we find. We then go on to clarify how we measure efficiency when we calculate welfare effects.

(i) The Planner's Problem. We first characterize Pareto Efficient allocations. Let $\omega(y,\alpha) > 0$ denote the weight on household (y,α)'s utility in the social welfare function and $\omega_R > 0$ the same for the absentee initial housing owners.[11] Let $r(y,\alpha)$ denote the planner's monetary transfer to household (y,α) and R the total transfer to the initial housing owners. The social planner is permitted to levy in community j both a head tax T_j and a property tax (t_j), the former necessary to obtain efficiency as we show. After solving this problem, we will then examine the constrained efficiency problem that does not allow head taxation, as this will provide further insight into inefficiencies that arise in Tiebout (property-tax) equilibria. It is again convenient to work with an indirect utility function. Let:

$$V^e(p_j, g_j, r(y) - T_j; y, \alpha) \equiv \underset{h}{\text{Max}}\ U(y + r(y) - T_j - p_j h, h, g_j, \alpha); \tag{19}$$

where the solution to the maximization problem in (19) is given by $h_d(p_j, y+r(y)-T_j, g_j, \alpha)$, recalling that $h_d(\cdot)$ denotes ordinary housing demand . Finally, let $a_j(y,\alpha) \in [0,1]$ denote the proportion of households (y,α) assigned by the planner to community j.

The social planner's problem is:

$$\underset{r(y,\alpha), a_i(y,\alpha), T_i, t_i, p_i, g_i}{\text{Max}} \sum_{i=1}^{J} \left\{ \int_S \omega(y,\alpha) V^e(p_i, r(y) - T_i, g_i; y, \alpha) a_i(y,\alpha) f(y,\alpha) dy d\alpha + \omega_R \left(R/J + \int_0^{p_i/(1+t_i)} H_s^i(z) dz\right) \right\} \tag{20}$$

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[11] We assume housing owners have quasi-linear utility functions and the social planner treats them all the same.

s.t. $$R + \int_S r(y,\alpha) f(y,\alpha) dy d\alpha = 0; \tag{21}$$

$$\int_S h_d(p_i, y + r(y) - T_i, g_i, \alpha) a_i(y,\alpha) f(y,\alpha) dy d\alpha = H_s^i(p_i/(1+t_i)),\ i = 1,2,...,J; \tag{22}$$

$$T_i \int_S a_i(y,\alpha) f(y,\alpha) dy d\alpha + \frac{t_i p_i}{1+t_i} H_s^i(p^i/(1+t_i)) = g_i \int_S a_i(y,\alpha) f(y,\alpha) dy d\alpha,\ i =,1,2,...,J; \tag{23}$$

$$a_i(y,\alpha) \in [0,1] \text{ and } \sum_{i=1}^{J} a_i(y,\alpha) = 1\ \forall (y,\alpha). \tag{24}$$

A solution to the problem is Pareto Efficient.[12] Since the problem is written requiring competitive provision of housing and also requiring jurisdictional balanced budgets, it may appear we have imposed some second-best requirements on the "efficient" allocation. However, as discussed below, these impositions are consistent with first-best Pareto Efficiency (but see the previous footnote). As the social weights $(\omega(y,\alpha),\omega_R)$ are varied alternative Pareto Efficient allocations are determined. If the utility possibilities set is convex, then all Pareto Efficient allocations are a solution to the problem for some set of weights.[13] Note, too, that r(y) = R = 0 will arise in the solution to the planner's problem for some weights $(\omega(y,\alpha),\omega_R)$, which is the case most naturally compared to the market equilibrium allocation.

To solve the problem, write the Lagrangian function:

$$\begin{aligned} L = & \sum_{i=1}^{J} \left\{ \int_S \omega V_i^e a_i f dy d\alpha + \omega_R (R/J + \int_0^{p_i/(1+t_i)} H_s^i dz) \right\} + \\ & \sum_{i=1}^{J} \lambda_i [(T_i - g_i) \int_S a_i f dy + \frac{t_i p_i}{1+t_i} H_s^i] + \sum_{i=1}^{J} \eta_i [\int_S h_d a_i f dy - H_s^i] + \Omega[R + \int_S r f dy d\alpha]; \end{aligned} \tag{25}$$

[12] We treat the housing supplies to jurisdictions as a technological constraint. That is, we do not allow jurisdictional lines to be redrawn, which would effectively permit trading of housing between jurisdictions.
[13] If the constrained utilities possibilities set is not convex, then one can still find all Pareto Efficient allocations as extrema of the planner's problem. Some solutions would be local minima of the problem but would satisfy the same (first-order) conditions we derive below.

where λ_i, η_i, and Ω are multipliers, we have suppressed arguments of functions, V_i^e is notation indicating that V^e has arguments corresponding to community i, and constraint (24) is taken account of below. The first-order condition on (r(y,α),R) can be written:

$$-\Omega = \sum_{i=1}^{J} \omega U_1^i a_i + \sum_{i=1}^{J} \eta_i \frac{\partial h_d^i}{\partial y} a_i = \omega_R \;\; \forall (y,\alpha); \tag{26}$$

where U_1^i is the partial derivative of U with respect to its first argument and the superscript indicates evaluation of the function at community i values. (We continue to use such notation below.) Let:

$$MSV_i(y,\alpha) \equiv L_{a_i f} = \omega V_i^e + \lambda_i [T_i - g_i] + \eta_i h_d^i \tag{27}$$

denote the marginal social value of assigning a measure $a_i f(y,\alpha)$ of household type (y,α) to community i, which equals the first variation in the Lagrangian with respect to type (y,α).[14] Using this notation and now taking account of (24), the optimal household assignment criterion can be written[15]:

$$a_i(y,\alpha) \begin{pmatrix} =0 \\ \in [0,1] \\ =1 \end{pmatrix} \text{ as } MSV_i(y,\alpha) \begin{pmatrix} < \\ = \\ > \end{pmatrix} \text{Max}_{j \neq i} MSV_j(y,\alpha) \;\; \forall (y,\alpha). \tag{28}$$

To write out the remaining first-order conditions, let:

$$N_i \equiv \int_S a_i(y,\alpha) f(y,\alpha) dy d\alpha \text{ and } \varepsilon_s^i \equiv \frac{H_s^{i\prime}}{H_s^i} \frac{p_i}{(1+t_i)} \tag{29}$$

denote respectively the number of residents of community i and the elasticity of housing supply. We have:

[14] This is scaled by f(y,α) just to be comparable across types.

[15] If the middle line of (28) characterizes the solution for a household y, then the summation constraint in (24) comes into play. However, we will focus on cases where this does not characterize the optimum as discussed below.

$$L_{t_i} = 0 \rightarrow -\omega_R + \lambda_i(1 - t_i\varepsilon_s^i) + \frac{1+t_i}{p^i}\eta_i\varepsilon_s^i = 0; \tag{30}$$

$$L_{T_i} = 0 \rightarrow -\int_S \omega U_1^i a_i f dy d\alpha + \lambda_i N_i - \eta_i \int_S \frac{\partial h_d^i}{\partial y} a_i f dy d\alpha = 0; \tag{31}$$

$$L_{g_i} = 0 \rightarrow \int_S \omega U_3^i a_i f dy d\alpha + \eta_i \int_S \frac{\partial h_d^i}{\partial g_i} a_i f dy d\alpha - \lambda_i N_i = 0; \tag{32}$$

and

$$L_{p_i} = 0 \rightarrow \frac{1+t_i}{H_s^i}\left[\eta_i \int_y \frac{\partial h_d^i}{\partial p^i} a_i f dy d\alpha - \int_y \omega U_1^i h_d^i a_i f dy d\alpha\right] + t_i\lambda_i(1+\varepsilon_s^i) - \frac{\eta_i(1+t_i)\varepsilon_s^i}{p_i} + \omega_R = 0. \tag{33}$$

We restrict attention to cases where it is efficient to have *differentiated communities* as in Tiebout allocations. This conforms to cases such that $a_i(y,\alpha) = 1$ for some community i for a.e. household (see (27) and (28)). The alternative has homogeneous communities. Whether differentiation is optimal depends on the utility weights in the social welfare function. Essentially we want to examine when equilibrium allocations *with differentiation* are associated with externalities in community choice.

First we confirm what is very intuitive: The social optimum will have no property taxation, just head taxes. More to our purposes, unilateral household choice of residence with an efficiently chosen head tax would be consistent with the efficient allocation. We will then go on to examine the second-best problem where only property taxes are allowed.

Proposition 3: In an efficient differentiated allocation: (a) $t_i = \eta_i = 0$ and $T_i = g_i$; *(b)* g_i *satisfies the Samuelsonian condition; and (c) households are assigned to the community where* V_i^e *is at a maximum.*

Proof of Proposition 3: (a) First we show that $t_i = \eta_i = 0$. *From (26) and that the allocation is differentiated:*

$$\omega U_1^i + \eta_i(\partial h_d^i / \partial y) = \omega_R \quad \textit{for all households (y,α) assigned to community i.} \tag{34}$$

Multiply through (34) by $\alpha_i f$ and integrate to obtain:

$$\int_S \omega U_1^i a_i f dy d\alpha + \eta_i \int_S \frac{\partial h_d^i}{\partial y} a_i f dy d\alpha = N_i \omega_R. \tag{35}$$

Then (35) and (31) imply:

$$\lambda_i = \omega_R. \tag{36}$$

Also (36) and (30) imply:

$$t_i \omega_R = \frac{\eta_i (1 + t_i)}{p_i}. \tag{37}$$

Since $\omega_R > 0$, if $t_i = 0$, then $\eta_i = 0$ and the reverse. Now we show that $t_i \neq 0$ implies a contradiction. Multiply through (34) by $h_d^i a_i f$ and integrate to obtain:

$$\int_S \omega U_1^i h_d^i a_i f dy d\alpha = \omega_R H_s^i - \eta_i \int_S \frac{\partial h_d^i}{\partial y} h_d^i a_i f dy d\alpha; \tag{38}$$

where we have substituted the housing market clearance condition ((22)). Now substitute from (36), (37), and (38) into (33) to get:

$$\eta_i \left\{ \frac{1+t_i}{H_s^i} \left(\int_S \frac{\partial h_d^i}{\partial p_i} a_i f dy d\alpha + \int_S \frac{\partial h_d^i}{\partial y} h_d^i a_i f dy d\alpha - \frac{1+t_i}{t_i p_i} H_s^i \right) + \frac{1+t_i}{p_i}(1 + \varepsilon_s^i) - \frac{1+t_i}{p_i} \varepsilon_s^i + \frac{1+t_i}{t_i p_i} \right\} = 0.$$

This simplifies to:

$$\frac{\eta_i}{H_s^i} \left\{ \int_S \left(\frac{\partial h_d^i}{\partial p_i} + \frac{\partial h_d^i}{\partial y} h_d^i \right) a_i f dy d\alpha \right\} = 0. \tag{39}$$

The term in parentheses in the integrand in (39) is the slope of the compensated demand for housing and is then negative. Hence, the integral term is negative, implying $\eta_i = 0$. This contradicts (37), so it must be that $t_i = \eta_i = 0$.

Since $t_i = 0$, $T_i = g_i$ by local budget balance (i.e., (23)).

(b) Using $\eta_i = 0$, substitute from (31) into (32). Then use that ωU_1^i equals a constant from (34) to obtain the Samuelsonian condition for a congested public good:

$$\int_S \frac{U_3^i}{U_1^i} a_i f dy d\alpha = N_i. \tag{40}$$

(c) Using the results in part (a), (27) and (28) imply that a household is optimally assigned to the community where V_i^e is maximized. ■

Remarks:

1. It is straightforward to confirm that the same results obtain if the planner also assigns housing consumption to each household and if the government budget constraint is economy wide, rather than local. Regarding the former, households would, of course, be assigned the level of housing they demand. Regarding the latter, direct income transfers permit the government to accomplish the same set of utility levels as would also allowing transfers across jurisdictions. The reason we have specified the problem imposing competitive housing consumption and jurisdictional budget balance is because we want to impose these requirements in the second-best analysis that follows.

2. The key implication of Proposition 3 is that if a community were to use head taxation to provide the local public good optimally, then household choice of communities would be socially optimal. Unilateral choice of community would lead households to choose the community where V^e is at a maximum, which, by Proposition 3c, is efficient.

Likewise, competitive provision of housing is efficient. The non-distorted price of housing and the head tax efficiently price access to communities.[16] There are no externalities in community choice in this case.

To determine the character of jurisdictional choice externalities in the property tax equilibrium, we now examine the planner's problem assuming head taxation is not allowed. Set $T_i = 0$ everywhere above and drop the first-order condition describing the efficient choice of T_i, i.e., (31). With $T_i = 0$, the other first-order conditions remain valid.[17] Of course, t_i will be positive here and is optimally chosen by the planner, but will also discuss later the alternative where t_i is suboptimal. Household choice of a jurisdiction would now be associated with an externality, and its character is the focus. With reference to (27)-(28), the value of what we call the "jurisdictional choice externality (JCE)" by household (y,α) of jurisdiction i is given by:

$$\mathrm{JCE}_i(y,\alpha) \equiv -\lambda_i g_i + \eta_i h_d(p_i, y + r(y), g_i, \alpha). \tag{41}$$

$\mathrm{JCE}_i(y,\alpha)$ equals the social value of choice of community i by household (y,α) in excess of the household's own (weighted) utility. We assume here to simplify the analysis that housing demand is independent of g_i, as arises in the cases we analyze below.[18]

To convey the main results here, we introduce a bit more notation. Let $h_c(\cdot)$ denote a household's compensated demand function for housing. Let:

$$\tau_i(y,\alpha) \equiv \frac{t_i p_i h_d(p_i, y + r(y), \alpha)}{(1 + t_i)}; \tag{42}$$

[16] The fact that the multipliers (η_i) on the housing market clearance conditions (22) equal 0 is crucial to the proof and may not be intuitive. This may seem to suggest that exogenously increasing a community's housing stock would not be welfare improving. This seeming paradox is resolved by noting that such an increase in the housing stock in community i would show up three places in the Lagrangian, and in fact social welfare increases with the housing stock at rate $\omega_R p_i$ at the optimum. Related to this, the constraint (22) should be interpreted as a constraint on housing prices, not on the supply of housing.
[17] We continue to study cases with differentiated allocations.
[18] We also indicate what changes if housing demand does depend on g_i.

and

$$\theta_i \equiv \frac{(1+t_i)\varepsilon_s^i}{(1+t_i)\varepsilon_s^i - \frac{p_i}{H_s^i}\int_S \frac{\partial h_c^i}{\partial p_i} a_i f dy d\alpha}. \tag{43}$$

Observe that τ_i is household (y,α)'s tax payment in jurisdiction i, and $\theta_i \in [0,1]$ where the integral term in the denominator of θ_i is a weighted average of households' compensated demand elasticities. We have:

Proposition 4: (a) The jurisdictional choice externality in the planner's solution satisfies:

$$JCE_i(y,\alpha) = -\lambda_i[g_i - \tau_i(y)\theta_i]; \tag{44}$$

with

$$\lambda_i = \frac{\int_S \omega U_3^i a_i f dy d\alpha}{N_i} > 0. \tag{45}$$

(b) $JCE_i(y,\alpha) \to -\lambda_i g_i$ *as* $\varepsilon_s^i \to 0$; $JCE_i(y,\alpha) \to -\lambda_i(g_i - \tau_i(y,\alpha))$ *as* $\varepsilon_s^i \to \infty$.

(c) $JCE_i(y,\alpha)$ *is negative for all households in community i with housing demand below the mean.*

Proof of Proposition 4: (a) Substitute from (30) and (38) into (33) to obtain:

$$\eta_i = -\lambda_i \frac{t_i p_i \varepsilon_s^i}{\frac{p_i}{H_s^i}\int_S \frac{\partial h_c^i}{\partial p_i} a_i f dy d\alpha - (1+t_i)\varepsilon_s^i}. \tag{46}$$

Substituting (42), (43), and (46) into (41), yields (44). Expression (45) follows from (32) using our assumption that housing demand is independent of g_i, and the value of λ_i is obviously positive.[19]

(b) These results follow trivially from (44) and the definition of θ_i (i.e., (43)).

(c) This follows from (44) since $\theta_i \in [0,1]$ and g_i equals the tax payment of the household in community i with average housing consumption. ■

Remarks:

1. The main implication is that an equilibrium allocation with efficient property tax would have too many households choosing jurisdictions with high g's, especially poorer households (assuming housing demand is normal). The value of the externality for a household is highest, ironically, when housing supply elasticity equals 0. In this case, the entire tax is, of course, absorbed by the absentee housing owners; and there is no distortion in the housing market. But there is not efficient pricing of the congestion externality from consumption of the local public good.[20] While the level of the externality for a poorer household that chooses a richer community is higher with lower housing supply elasticity, capitalization of higher g in housing prices is elevated the lower is the housing supply elasticity. We find computationally that the increased capitalization acts as a substantial deterrent to poorer households choosing richer jurisdictions and welfare rises as the housing supply elasticity falls.[21] In any case, poorer households that consume less housing have an incentive to crowd richer jurisdictions. The equilibrium model in the paper does not have efficient choice of property tax due to

[19] If housing demand depends on g_i, then a sufficient condition for λ_i to be positive is that housing demand is non-increasing in g_i. The remaining results in Proposition 4 are as stated.
[20] Household community choice would be efficient if the local public good were not congested.
[21] This analysis is the appendix on robustness of the computational findings.

majority choice, but this distortion is typically minor. *The theoretical distortion identified here – poorer types crowding richer jurisdictions – we find below to be key to the welfare losses from Tiebout sorting that arise with property taxation.*

2. Unless housing supply elasticity equals 0, note from (46) that the multiplier (η) on the housing-market clearance condition is no longer 0, but positive. This is because the gross housing price inefficiently deters housing consumption and is not enough to deter poor households from moving into high-g communities. Requiring housing consumption in excess of demand could improve efficiency.[22] If the tax t_i is inefficient (i.e., is not chosen by the planner), one finds that[23]:

$$\eta_i = \frac{t_i\left[\int_S \omega U_1^i h_d^i a_i f dy d\alpha - (1+\varepsilon_s^i)\frac{H_s^i}{N_i}\int_S \omega U_3^i a_i f dy d\alpha\right]}{\int_S \frac{\partial h_c^i}{\partial p_i} a_i f dy d\alpha + t_i \int_y \frac{\partial h_d^i}{\partial p_i} a_i f dy d\alpha - \frac{(1+t_i)\varepsilon_s^i H_s^i}{p_i}}. \tag{47}$$

Now η_i can be positive or negative. This is because g_i might be over-provided (conditional on using property taxation) and limiting housing consumption would reduce this distortion.

(ii) Measuring Welfare. We treat the centralized equilibrium as the status quo and use (the negative of) aggregate compensating variation associated with the Tiebout equilibrium as our welfare measure. Let $U^c(y,\alpha)$ denote utility of household (y,α) in the centralized equilibrium and $U^T(y,\alpha)$ utility in Tiebout equilibrium.[24] Let $v(y,\alpha)$ denote compensating variation, defined in $U^c(y,\alpha) = U^T(y+v,\alpha)$. Let $CV = \int_S v(y,\alpha) f(y,\alpha) dy d\alpha$

[22] See Calabrese, Epple, and Romano (2007) on residential zoning that improves efficiency.
[23] This is found by solving the planner's problem with t_i exogenous, hence suppressing condition (30). We continue to assume that T_i must be 0.
[24] If Tiebout equilibrium does not exist, then the degenerate "Tiebout equilibrium" corresponds to the centralized equilibrium and $U^c(y,\alpha) \equiv U^T(y,\alpha)$.

denote aggregate household compensating variation. Let $R^c = \sum_{j=1}^{J} \int_0^{p_s^c} H_s^j(p)dp$ denote housing rents in the centralized equilibrium, where p_s^c denotes the net housing price. Let $R^T = \sum_{j=1}^{J} \int_0^{p_s^j} H_s^j(p)dp$ denote housing rents in the Tiebout equilibrium. Compensating variation of the absentee landlords is given by: $R^c - R^T$. We report $W^T = -[CV + R^c - R^T]$ as our welfare measure, while also reporting aggregate consumer welfare (-CV). The negative of compensating variation is reported just so a positive value indicates a gain from Tiebout sorting.

We know a priori that the Tiebout equilibrium is not Pareto Efficient. If households choose residences and housing followed by efficient public good provision satisfying the Samuelsonian condition financed by a head tax, then equilibrium would be efficient (Proposition 3). Such an allocation would maximize our welfare measure (i.e., the negative of aggregate compensating variation).[25] In the Tiebout equilibrium we study, the housing market distortion from use of a property tax to finance public provision is generally inefficient. Likewise, majority choice of the level of provision of the local public good is generally inefficient. These inefficiencies further imply that externalities arise in the individual choice of residences as we have shown. We know, then, that if we calculate welfare analogously in going from the centralized equilibrium to the efficient head-tax equilibrium, denoted by W^H, that $W^H > 0$ and $W^H > W^T$. In spite of the latter inequality, we perceive a strong belief among economists that $W^T > 0$ is to be expected: Some aggregate welfare gains will arise from the equilibrium matching of

[25] This is proved in an appendix available on request.

households to relatively desired public good levels.[26] In fact, we will see that this belief is not justified. We show below that $W^T < 0$ frequently when W^H is substantial.

3. Computational Analysis

To show that a welfare loss from Tiebout sorting is a real possibility, we first examine a calibrated computational model. Having shown a welfare loss does not appear to be pathological, we estimate a more general model using data from the Boston MA in the next section, and, again, find a welfare loss.

a. Calibration of the Model. The calibrated model abstracts from taste differences, with then households differing only by income. Household utility if assumed to be CES:

$$U = [\beta_x x^\rho + \beta_h h^\rho + \beta_g g^\rho]^{1/\rho}. \tag{48}$$

We must calibrate the MA income distribution, the number of jurisdictions, and the parameters of the utility function and housing supply functions. The distribution of MA income is calibrated using data from the 1999 American Housing Survey (AHS).[27] Median income reported by the AHS is \$36,942. Using data for the 14 income classes reported by the AHS, we estimate mean household income to be \$54,710. These values and our assumption that the income distribution is lognormal imply $\ln y \sim N(10.52,.785)$.

We assume constant elasticity housing supply function in each jurisdiction. Such a housing supply function arises if units of housing are produced competitively by combining a jurisdiction's inelastically supplied land L_j with an elastically supplied factor q according to constant-returns production function: $h = L^\gamma q^{1-\gamma}$, $\gamma \in (0,1)$. Specifically, then

[26] Our perception of the consensus belief is that $v(y,\alpha)$ will be positive for those that choose poorer (low g) communities in Tiebout equilibrium (i.e., there will be a welfare loss for them), but $v(y,\alpha)$ will be negative and offsetting for those that choose richer communities. We find that the offset does not typically occur.
[27] http://www.census.gov/hhes/www/housing/ahs/99dtchrt/tab2-12.html

$$H_s^j = L_j \left(p_s^j\right)^{\frac{1-\gamma}{\gamma}} \left(\frac{1-\gamma}{w}\right)^{\frac{1-\gamma}{\gamma}}, \tag{49}$$

where w is the given price of input q. The quantity of housing available at given housing price then varies across jurisdictions proportionately to their land endowment. In our baseline calibration, we assume five local jurisdictions in the MA – a large city and four smaller suburbs that have equal area. The total land supply in the MA is normalized to 1. The city is assumed to have 40% of the total land area and each of the suburbs 15%. We assume that the city is the poorest jurisdiction. The jurisdictions are numbered from poorest to richest: Hence, $L_1 = .4$, and $L_2 = L_3 = L_4 = L_5 = .15$, where L_j equals community j's land share. The parameter γ equals the share of land inputs in housing in our model. Based on the empirical evidence (see the discussion in Epple and Romer, 1991), we set $\gamma = ¼$. Note from (49) that this implies a housing supply elasticity equal to 3.[28]

The remaining parameter values are ρ, β_x, β_h, and β_g from the utility function (48), and w from the housing supply function (49). The calibrated parameters are summarized in Table 1. The remaining calibration is based on the single jurisdictional equilibrium for simplicity. First, we set $\beta_x = 1$, a normalization. While less obvious, w is also a "free parameter," which we also then set equal to 1. To see this, note from (49) that the housing supply function for the MA is: $H_s = (w)^{\frac{\gamma-1}{\gamma}} (p_s)^{\frac{1-\gamma}{\gamma}} (1-\gamma)^{\frac{1-\gamma}{\gamma}}$, and this is the only place that w appears in the model. For any γ, changing w is equivalent to

[28] This housing supply elasticity is within the range of estimates for new housing, though estimates vary substantially. See Dipasquale (1999), Blackley (1999), and Somerville (1999). Dipasquale and Wheaton (1992) estimate the long run rental housing supply elasticity to be 6.8. Other estimates also find a higher elasticity than 3 (see Mayer and Sommerville, 2000, and Epple, Gordon, and Sieg, 2007). In the appendix, we show that increasing the housing supply elasticity results in a higher welfare loss from Tiebout sorting under property taxation than in our baseline calculation.

changing the units of measurement of housing. No equilibrium values relevant to utilities then vary with w.

The values of β_g, β_h, and ρ are set so that in the single jurisdictional equilibrium the median voter chooses t = .35, the net-of-tax expenditure share on housing equals .20, and the price elasticity of housing is very close to -1. A t = .35 implies a tax rate on property value that is realistic, on the order of 2.5% to 3.0%.[29] The expenditure share on housing of .20 is in the range of values estimated in the literature (see Hanushek and Quigley (1980). Likewise, the housing market literature indicates a price elasticity close to -1.[30] The implied values of β_g and β_h are, respectively, 0.094 and .356. We set ρ = -.01, which implies a price elasticity of housing demand equal to -.993, while also implying SRI and existence of a Tiebout equilibrium when there are multiple jurisdictions.[31]

Table 1: Parameter Values Baseline Model

β_x	β_h	β_g	ρ	γ	w
1.00	.356	.094	-.010	.250	1.00

b. Findings. Table 2 summarizes the findings in this baseline specification, with positive results in the upper panel and normative results in the lower panel. Recall that we report the negative of compensating variation values so that gains from Tiebout

[29] Observed property tax rates are expressed as a percent of property value. In our model, rates are expressed as a percentage of annual implicit rent. Employing the approach of Poterba (1992), Calabrese and Epple (2006) conclude that tax rates on annualized implicit rents can be converted to rates on property values using a conversion rate on the order of 7% to 9%. Thus, our annualized rate of .35 translates to a tax rate on property value on the order of 2.5% to 3%, which is the order of magnitude of observed property tax rates.

[30] See Rosen (1979), Hoyt and Rosenthal (1990), and Rosenthal, Duca, and Gabriel (1991). Hanushek and Quigley (1980) obtain somewhat more inelastic estimates.

[31] A ρ = 0 implies a Cobb-Douglas utility function and a price elasticity of demand for housing exactly equal to -1. Here SRI fails and an equilibrium with Tiebout sorting does not arise in this case.

sorting correspond to positive values. Column 2 of the upper panel shows key values in the Tiebout equilibrium and column 1 corresponding values in the centralized equilibrium where the MA is one jurisdiction. Ignore the other columns for the moment. The Tiebout equilibrium is income stratified, supported by ascending housing prices, although the property tax rates vary little and are very close to that in the centralized equilibrium. Because these tax rates apply to substantially different housing expenditures, the public good levels vary substantially.

The lower panel shows the welfare effects. Only the poor and very rich are better off in the Tiebout equilibrium, with 95% worse off. On average consumers are worse off, with an average (minus) compensating variation of $41. The absentee land owners experience a negligible welfare loss. Column 5 reports values in the efficient allocation discussed above. Although 74% of households are worse off in this allocation, households experience an average welfare gain of $726 and land owners an average gain of $711. Hence, the environment is one where Tiebout sorting could lead to substantial welfare gains on average, yet these not only fail to be realized in property tax equilibrium but are reversed.

To parcel the welfare losses that arise in Tiebout equilibrium, we calculate two other allocations. As discussed above, three inefficiencies arise in Tiebout equilibrium: First, property taxation distorts housing consumption with the usual deadweight loss. Second, majority choice of the tax rate reflects the preference of the median-income household in a jurisdiction, which generally differs from the choice that would maximize average welfare.[32] Third, externalities arise in household choice of jurisdiction, which we show is the primary source of welfare loss.

The second, majority voting inefficiency, is generally believed to be small in these models. To verify this here, we compute multi-jurisdictional equilibrium with majority choice of a head tax. Equilibrium is determined precisely as in the property-tax model, but voting is over a local head tax that fully finances the local public good. Versions of Propositions 1 and 2 apply to this variation of the Tiebout sorting model.[33] Values for this equilibrium are shown in column 3 of Table 2. The head taxes are, of course, equal to the levels of public good provision.[34] Comparing column 3 to column 5, one sees that the allocation is very close to the efficient allocation. The welfare gain relative to the single-jurisdictional equilibrium is 99.8% of the potential welfare gain from sorting.

The welfare loss in the Tiebout equilibrium is then largely attributable to property taxation and household jurisdictional choice externalities rather than voting bias. To delineate these effects, we *assign* households to jurisdictions as arises in the efficient allocation, but then they vote for a local property tax to finance the public good. Hence, this allocation essentially removes externalities from household choice of jurisdiction, while retaining the property tax distortion (as well as the small voting bias). This is not an equilibrium allocation because some households would prefer to move. The

[32] If median income were equal to mean income and if the (indirect) utility function were linear, then the preference of the median-income household would maximize average welfare. But neither of these conditions is satisfied. These biases are well known.

[33] The ascending bundles property trivially regards the head tax, not the housing price. Since the head tax is a deterrent to moving into jurisdiction, it is theoretically possible that housing prices could decline with the level of the public good.

[34] Because the calibration of income begins at 0, some households in the poorest jurisdiction cannot afford to pay the $1691 head tax. The proportion of the population is only .000251, so we simply ignore this.

associated values are reported in column 4 of Table 2. We see that most of the potential welfare gain from efficient sorting arises in this allocation; about 80%.

We conclude that the jurisdictional choice externality is the main cause of the welfare loss we find. As already discussed, relatively poor households crowd richer jurisdictions to consume high levels of the public good while free riding on the initial housing owners and richer households that pay more in taxes. Referring to Table 2, we see that the equilibrium populations of the richer jurisdictions are substantially higher and the income levels substantially lower than in the efficient allocation. The fundamental explanation for the welfare loss in Tiebout property tax equilibrium is that the resulting sorting of households is inefficient; it's not stratified enough! While we are in a second-best economy so that "anything can happen," we, nevertheless, find this very surprising. Given property taxation, the model indicates that the degree of Tiebout sorting is crucial for welfare gains to be realized. While it is well known that Tiebout sorting is not good for poorer types, that it will sometimes be also bad on average makes it difficult to support.

The welfare loss we find is not contrived. To examine robustness, we vary the equilibrium concept with respect to the nature of assumed voter beliefs and the parameters of the model. The analysis is in the appendix, and we very briefly summarize here. Two alternative specifications of timing of choices and voter beliefs are analyzed. An alternative where voters anticipate changes in housing consumption, but not in jurisdictional choice, has negligible effects on our findings. An alternative where voters anticipate both changes in housing consumption and jurisdictional choice leads to substantially higher welfare losses from Tiebout sorting in property tax equilibrium.

Increasing of decreasing the number of jurisdictions by one has very minor effects. Reducing ρ, hence the elasticity of substitution in the CES utility function, leads to welfare gains from Tiebout sorting under property taxation.[35] Households consume more housing and are more reluctant to reduce their housing consumption. Property taxes decline and thus distortions are reduced. But the predicted property tax rates are unrealistically low.

Reducing γ increases housing supply elasticities and welfare losses from Tiebout sorting with property taxation increase rapidly. Housing prices rise more slowly as poor households move into richer jurisdictions, worsening the effect of the jurisdictional choice externality as more such movement takes place. Increasing γ has the reverse effects.

Increases in β_g, the weight on the public good in the utility function, increase demand for the local public good and exacerbates the inefficiencies and welfare losses in Tiebout property tax equilibrium. Increasing β_h, the weight on housing in the utility function, increases demand for housing. Property tax rates fall and households find it more difficult to substitute away from housing. As a consequence, inefficiencies in Tiebout property tax equilibrium are reduced.

While we do not always find losses from Tiebout sorting under property taxation, we find a loss in our preferred calibration and we find it is fairly persistent over a variety of parameter variations. Moreover, when gains arise, they are typically a small percentage of potential gains from efficient sorting. These computational findings

[35] We cannot consider higher values of ρ since any significant increases would violate SRI and preclude sorting of types in property tax equilibrium.

provide motivation for further pursuit of the efficiency question. The next section develops our main quantitative findings that are based on an estimated model.

4. Econometric Model and Findings

a. <u>The Econometric Model and Estimated Parameters</u>. The framework used in the econometric analysis is set forth in Epple and Sieg (1999) and Calabrese, Epple, Romer, and Sieg (2006).[36] We now summarize the econometric model and estimates. The specification of the local public good is generalized relative to the above model by inclusion of a jurisdictional peer effect to better fit the data. Suppressing the jurisdictional subscripts and letting q denote the quality of the local public good, the following indirect utility function is used:

$$\tilde{V}(q,p;y,\alpha) = \{\alpha\, q^{\tau} + [e^{\frac{y^{1-\nu}-1}{1-\nu}}\ e^{-\frac{Bp^{\eta+1}-1}{1+\eta}}]^{\tau}\}^{\frac{1}{\tau}}; \tag{50}$$

where

$$q = g \cdot \bar{y}^{\phi}; \tag{51}$$

and $\bar{y}$ is the mean income in the jurisdiction. Peer effects might operate through educational spillovers in the classroom, through parental monitoring of teachers and school administrators, or through other channels.[37] Note that the indirect utility function in (50) is the standard one that allows housing consumption to vary with prices, as specified in (16) above. This specification has the following useful properties.[38] It is separable in public- and private-good components, substantially simplifying estimation.

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[36] See also Epple, Romer, and Sieg (2001).

[37] Recent empirical analysis of peer effects in education include Angrist and Lang (2004), Arcidiacono and Nickolson (2005), Betts and Morell (1999), Cooley (2007), Dale and Krueger (2002), Ding and Lehrer (2007), Figlio (2003), Hanuushek, Kain, Markman, and Rivkin (2003), Hoxby and Weingarth (2205), Sacerdote (2001), Vigor and Nechyba (2005), Zimmer and Toma (1999), and Zimmerman (2000).

[38] There is not a closed form for the associated (direct) utility function. An appendix, available from the authors, provides detail on this specification, including demonstration that it satisfies all the standard properties (e.g., quasi-concavity in prices) of an indirect utility function.

The implied elasticity of substitution between the public good component and private good component is constant and equal to 1/(1-τ). Using Roy's identity, the implied housing demand function is $h_d = Bp^{\eta} y^{\nu}$. Thus housing demand has constant price and income elasticities, as is common in empirical analysis. The single-crossing conditions for stratified community choice, i.e., $\widetilde{SRI}$ and $\widetilde{SR\alpha}$, are satisfied if $\tau < 0$.[39] This condition is tested empirically.

The metropolitan population density function, f(y,α), is taken to be bi-variate lognormal:

$$\begin{pmatrix} \ln y \\ \ln \alpha \end{pmatrix} \sim N\left[\begin{pmatrix} \mu_y \\ \mu_\alpha \end{pmatrix}, \begin{pmatrix} \sigma_y^2 & \lambda\sigma_y\sigma_\alpha \\ \lambda\sigma_y\sigma_\alpha & \sigma_\alpha^2 \end{pmatrix}\right]. \tag{52}$$

The model then has ten parameters to be estimated: $\tau, \nu, \eta, B, \phi, \mu_{\ln y}, \sigma_{\ln y}, \mu_{\ln\alpha}, \sigma_{\ln\alpha}, \text{and}\, \lambda.$

Estimation is based on data for the 92 municipalities in the Boston Metropolitan Area. Data are used for 1980, which precedes the state imposition of property tax limits (Proposition 2½). The Boston metropolitan area is particularly well suited to estimation of the model. In Massachusetts school districts and municipalities are coterminous. Property taxes were the primary source of local revenues during this period, and residential property tax revenue tracks well educational expenditure per student in the 92 municipalities with a correlation coefficient of 0.73. Per student educational expenditure is then used for g in the estimation.

[39] This condition also ensures that voting equilibrium exists in the second stage. This is shown in the appendix discussed in the previous footnote.

Estimation proceeds in two stages. Table 3 reports the estimates. In stage one, the mean and standard deviation of ln(y) are estimated first, using the metropolitan income distribution. Three additional parameters are estimated by utilizing the stratification and boundary-indifference conditions. The boundary-indifference conditions and the indirect utility function imply the following expression for the boundary loci between jurisdictions j and j+1:

$$\ln(\alpha) + \frac{(y^{1-\nu} - 1)}{1-\nu} = \ln\left(\frac{Q_{j+1} - Q_j}{q_j^{\tau} - q_{j+1}^{\tau}}\right) \equiv K_j; \tag{53}$$

where:

$$Q_j = e^{-\tau \frac{Bp_j^{\eta+1} - 1}{\eta+1}}. \tag{54}$$

There are 91 such loci partitioning the metropolitan population into 92 municipalities. A minimum-distance estimator is then used to match quartiles of the 92 income distributions implied by the model to quartiles of the 92 income distributions that are estimated by the US Census. The additional parameters identified here are: τ/σ_{α}, ν, and λ. In addition to the 92 income distributions, the populations of the 92 municipalities are used in this stage. The K_j are solved out at each point in the parameter search. Hence, housing values and public good qualities are not needed at this stage. The asymptotics are with respect to the sample size taken by the Census so the parameters are estimated with a high degree of precision. The ratio, τ/σ_{α}, is negative and statistically significant, supporting the single-crossing assumptions. Note that this first-stage estimator invokes only necessary conditions for equilibrium, so uniqueness of equilibrium is not a concern.

Table 3: Parameter Estimates
(Standard errors are in parentheses)

$\mu_{\ln y}$	$\sigma_{\ln y}$	λ	$\tau / \sigma_{\ln \alpha}$	ν	B	ϕ	$\mu_{\ln \alpha}$	$\sigma_{\ln \alpha}$	η
9.790 (.002)	.755 (.004)	-.019 (.031)	-.283 (013)	.938 (.026)	.175 (.007)	2.623 (.147)	-2.643 (.017)	.1[a]	-.3[b]

[a]Set at minimum in the program. [b]Taken from literature on housing demand.

In the second stage (Calabrese, Epple, Romer, Sieg, 2007), the remaining parameters are estimated using maximum likelihood. Parameters identified in the first stage are fixed at the values estimated in that stage. In the second stage, at each step in the parameter search, the theoretical model with household sorting and myopic voting over property tax rates is solved for equilibrium values of property tax rates, expenditures on g, and aggregate housing values in each of the 92 communities. The observed values of these variables in the 92 communities are then presumed to be equal to the values implied by the model plus measurement error. Since estimation at this stage involves matching house values of the model to those in the data, the price elasticity of housing demand is only tenuously identified.[40] Hence, that parameter is fixed at η = -.3 using estimates from the literature, and remaining parameters are estimated. Variation in the value at which η is fixed confirms that estimates of the remaining parameters are not sensitive to its value. The best fit to the data is obtained with $\sigma_{\ln \alpha}$ equal to the minimum value the program permits, so this parameter is set there.[41] Calabrese, et. al. (2007) demonstrate that, conditional on the first stage results, the equilibrium in the second stage

[40] Data on housing values is only available at the jurisdictional level, which permits identification of key parameters, but does not provide good data for estimating the price elasticity.

[41] $\sigma_{\ln \alpha} = .1$ may give the impression that taste variation is then negligible, but that is incorrect because the magnitude of $\sigma_{\ln \alpha}$ depends on choice of scaling. For example, $\sigma_{\ln \alpha}$ can be rescaled by expressing the first argument in the indirect utility function (50) not as αq^{τ} but instead as $(\alpha q)^{\tau}$. The important point here is that second-stage estimates preserve the decomposition of the metropolitan distribution of income within and across communities that is captured in the first-stage estimates. As reported in Epple and Sieg (1999), 89% of the total variance in metropolitan income is within-community variance and 11% is across communities.

is unique. Thus, the estimation procedure is not vulnerable to concerns about multiple equilibria.

Figure 2 relates observed jurisdictional values to those predicted by the estimated model. Jurisdictions are ordered by increasing median household income. Following Poterba (1992), the observed property tax rate is converted from that on home value to that on the implied net rental rate per unit of housing to correspond to our model of housing services and prices. Note that the dollar values for educational expenditure per household and housing value are in 1980 dollars in Figure 2. The figure at once illustrates the substantial Tiebout sorting and the predictive power of the estimated model.

b. Welfare Effects of Tiebout Sorting. The econometric analysis summarized above does not generate an estimate of the elasticity of housing supply, which is needed to perform our welfare computations. We assume constant elasticity housing supply and use the same housing supply elasticity as in the baseline computational model above (i.e., equal to 3). The equilibrium with 92 communities is computed as part of the estimation. To calculate the counter-factual equilibrium with a single metropolitan government, land area for housing must be obtained. Given the housing supply elasticity, the implied land area used for housing in each municipality can be calculated. These values are aggregated to obtain land area for housing with a single metropolitan government. This then permits calculation of the equilibrium with a single metropolitan government and no sorting of households.

In the computed single jurisdictional equilibrium, we obtain $g = \$1100.42$, $t = 0.43$, and $p = 2.07$. By way of comparison, the population weighted means in the 92-

community equilibrium are: $\overline{g} = \$1127$; $\overline{t} = 0.41$; and $\overline{p} = 2.35$. We then compute compensating variation for households and initial land owners as above. The household average compensating variation in going to the multi-jurisdictional equilibrium is \$478 and the per household CV for land owners equals -\$162. Hence, the Tiebout equilibrium implies a welfare loss equal to \$316 per household. This equals 1.3% of 1980 per household income or, if expressed using year 2000 values to be comparable to the calibrated model in Section 3, the loss equals \$711.

5. Concluding Remarks

Inequalities in the local public finance of schooling have lead to a revolution in education policy in much of the U.S. Few economists would challenge the notion that such Tiebout sorting had lowered the welfare of many, especially the poor. However, distributional issues aside, we perceive that most economists understand the Tiebout process to be efficiency enhancing. While the presence of inefficiencies in local property tax equilibria is understood, we know of no research that quantifies the net effects of such allocations. In pursuing such an analysis here, we have found that these inefficiencies are substantial and overturn potential average welfare gains in both a standard model calibrated model and the state-of-the-art estimated model. It is a bit shocking that the analysis here runs counter to our basic intuition concerning the Tiebout process.

The finding that average welfare losses arise in a reasonable model has led us to investigate the main source of the inefficiency. We find that the externality in choice of residence is the primary source of loss. It is almost as surprising as the finding of a net loss that the mobility central to the Tiebout argument underlies the inefficiency.

Our finding might help to explain the prevalence of residential zoning restrictions. In Calabrese, Epple, and Romano (2007), we pursue a theoretical and quantitative analysis of residential zoning that supports Hamilton's (1975) argument: Zoning serves as a substitute for head taxation. We show that local public choice of a zoning restriction on housing quality combined with a property tax closely mimics head taxation and almost all potential Tiebout welfare gains are realized.

This paper does not, of course, refute Tiebout's argument. Rather, it tells a cautionary tale about applying first-best arguments in a second-best environment. Moreover, our model is very Spartan. We think that it is of much interest to explore further the quantification of the welfare effects of local public goods equilibria.

References

Angrist, Joshua and Kevin Lang (2004), “Does School Integration Generate Peer Effects? Evidence from Boston’s Metco Program,” *American Economic Review*, 94(5), 1613-1634.

Arcidiacono, P. and S. Nickolson (2005), “Peer Effects in Medical Schools,” *Journal of Public Economics*, 89, 327-350.

Betts, J. and D. Morell (1999), “The Determinants of Undergraduate GPA: The Relative Importance of Family Background, High School Resources, and Peer Group Effects,” *Journal of Human Resources*, 107, 797-817.

Blackley, Dixie (1999), “The Long Run Elasticity of New Housing Supply in the United States for 1950 to 1994,” *The Journal of Real Estate Finance and Economics*, 18(1), 25-42.

Calabrese, Steve, Epple, Dennis, and Richard Romano (2007), “On the Political Economy of Zoning,” *Journal of Public Economics*, 91, 25-49.

Calabrese, Steve, Epple, Dennis, Romer, Thomas, and Holger Sieg (2006), “Local Public Good Provision: voting, Peer Effects, and Mobility,” *Journal of Public Economics*, 90, 959-981.

Cooley, Jane (2007), “Desegregation and the Achievement Gap: Do Diverse Peer Help?” working paper, University of Wisconsin-Madison.

Dale, S. and A. Krueger (2002), “Estimating the Payoff to Attending a More Selective College: An application of Selection on Observables and Unobservables,” *quarterly Journal of Economics*, 117(4), 1491-1527.

Ding, Weili and Steven Lehrer (2007), “Do Peers Affect Student Achievement in China’s Secondary Schools?” *The Review of Economics and Statistics*, 89(2), 300-312.

Dispasquale, Denise (1999), “Why Don’t We Know More About Housing Supply?” *The Journal of Real Estate Finance and Economics*, 18(1), 9-23.

Dispasquale, Denise and William Wheaton (1992), “The cost of Capital, Tax Reform, and the Future of the Rental Housing Market,” *Journal of Urban Economics,* 31(3), 337-359.

Epple, Dennis, Gordon, Brett, and Holger Sieg (2007), “A New Apporach to Estimating the Production Function for Housing,” working paper, Carnegie Mellon University.

Epple, Dennis, Filimon, R. and Thomas Romer (1993), "Existence of Voting and Housing Equilibrium in a System with of Communities with Property Taxes," *Regional Science and Urban Economics*, 23, 281-304.

Epple, Dennis and Thomas Nechyba (2004), "Fiscal Decentralization," Chapter 55 in *Handbook of Regional and Urban Economics*, J. Henderson and J. Thisse, eds., Elsevier Science.

Epple, Dennis and Glenn Platt (1998), "Equilibrium and Local Redistribution in an Urban Economy when Households Differ in both Preferences and Incomes," *Journal of Urban Economics*, 43(1), 23-51.

Epple, Dennis and Thomas Romer (1991), "Mobility and Redistribution," *Journal of Political Economy*, 99(4), 665-898.

Epple, Dennis, and Holger Sieg (1999), "Estimating Equilibrium Models of Local Jurisdictions," *Journal of Political Economy*, 107(4), 645-681.

Epple, Dennis, Romer, Thomas, and Holger Sieg (2001), "Interjurisdictional Sorting and Majority Rule: An empirical Analysis," *Econometrica*, 69, 1437-1465.

Fernandez, Raquel and Richard Rogerson (1996), "Income Distribution, Communities, and the quality of Public Education," *Quarterly Journal of Economics*, 111(1), 135-164.

Figlio, David (2003), "Boys Named Sue: Disruptive Children and their Peers," working paper, University of Florida.

Hamilton, Bruce (1975), "Zoning and Property Taxation in a System of Local Governments," *Urban Studies*, 12, 205-211.

Hanuschek, E. A., Kain, J.F., Markman, J.M., and S.G. Rivkin (2003), "Does Peer Ability Affect Student Achievement?" *Journal of Applied Econometrics*, 18(5), 527-544.

Hanuschek, Eric and John Quigley (1980), "What is the Price Elasticity of Housing Demand?" *The Review of Economics and Statistics*, 62(3), 449-454.

Hoxby, Caroline and G. Weingarth (2005), "Taking Race Out of the Equation: School Reassignment and the Structure of Peer Effects," working paper, Harvard Univerisity.

Hoyt, William and Stuart Rosenthal (1990), "Capital Gains Taxation and the Demand for Owner-Occupied Housing," *The Review of Economics and Statistics,* 72(1), 45-54.

Mayer, Christopher, and C. Tsuriel Sommerville (2000), "Residential Construction: Using the Urban Growth Model to Estimate Housing Supply," *Journal of Urban Economics*, 48, 85-109.

Poterba, J. (1992), “Taxation and Housing: Old Questions, New Answers,” *American Economic Review*, 82(2), 237-242.

Rosen, Harvey (1979), “Housing Decisions and the U.S. Income Tax,” *Journal of Public Economics*, 11, 1-24.

Rosenthal, Stuart, Duca, John, and Stuart Gabriel (1991), “Creidt Rationing and the Demand for Owner-Occupied Housing,” *Journal of Urban Economics*, 30, 48-63.

Sacerdote, B. (2002), “Peer Effects with Random Assignment: Results from Dartmouth Roommates,” *Quarterly Journal of Economics*. 116(2), 681-704.

Scotchmer, Suzanne (2002), “Local Public Goods and Clubs,” in *Handbook of Public Economics – Volume 4*, A. Auerbach and M. Feldstein, eds. Elsevier Science, 1997-2042.

Sommerville, C. Tsuriel, “Residential Construction Costs and the Supply of New Housing: Endogeneity and Bias in Construction Cost Indexes,” *The Journal of Real Estate Finance and Economics*, 18(1), 43-62.

Tiebout, C. M. (1956), “A Pure Theory of Local Expenditures,” *Journal of Political Economy*, 64, 416-424.

Vigdor, Jacob and Thomas Nechyba (2005), Peer Effects in North Carolina Public Schools,” working paper, Duke University

Zimmer, R.W. and E.F. Toma (1999), “Peer Effects in Public Schools Across Countries,” *Journal of Policy Analysis and Management*,” 19, 75-92.

Zimmerman, D. (2000), “Peer Effects in Private and Public Schools: A Cross Country Analysis,” working paper.

Table 2*

Baseline Model Positive Properties	Property Tax One Jurisdiction	Property Tax Multiple Jurisdictions	Head Tax Multiple Jurisdictions	Property Tax / Fixed Boundaries Multiple Jurisdictions	Efficient Allocation Multiple Jurisdictions
P1 =	$17.13	$12.69	$12.93	$16.62	$13.02
P2 =		$15.21	$13.76	$17.53	$13.75
P3 =		$16.62	$13.77	$17.53	$13.74
P4 =		$18.43	$13.78	$17.50	$13.72
P5 =		$22.61	$13.78	$17.34	$13.61
Y1 =		$28,589	$56,409	$57,950	$57,950
Y2 =		$39,990	$82,073	$84,067	$84,067
Y3 =		$56,931	$119,902	$122,768	$122,768
Y4 =		$89,598	$192,823	$197,811	$197,811
Median Income J1 =	$36,942	$17,140	$25,741	$26,076	
Median Income J2 =		$33,866	$67,131	$68,840	
Median Income J3 =		$47,475	$97,029	$99,320	
Median Income J4 =		$69,929	$144,849	$148,279	
Median Income J5 =		$128,816	$249,542	$255,332	
N1 =		39%	68%	69%	69%
N2 =		15%	13%	13%	13%
N3 =		15%	9%	9%	9%
N4 =		15%	6%	6%	6%
N5 =		16%	3%	3%	3%
t1 =	35%	35.33%		35.17%	
t2 =		35.24%		35.17%	
t3 =		35.20%		35.17%	
t4 =		35.14%		35.16%	
t5 =		34.96%		35.11%	
g1 =	$3,830	$1,195	$1,691	$1,952	$1,829
g2 =		$2,390	$4,410	$4,887	$4,569
g3 =		$3,359	$6,374	$7,071	$6,612
g4 =		$4,988	$9,516	$10,679	$9,987
g5 =		$10,987	$16,393	$20,665	$17,922
Distributional and Welfare Results					
Interval of income made worse off					
	Low bound	$8,500	$0	$0	$0
	High bound	$349,500	$66,500	$57,500	$65,500
% of pop. made worse off		95%	75%	69%	74%
(-) Per Capita CV		-41	714	1158	726
(-) Δ Housing Rents		-0.22	721	-3.17	711
(-) [CV + Δ House Rents]		-41.68	1434.82	1154.37	1437.13

*The P's are net housing prices; the Y's minimum incomes in the jurisdictions; the N's are the percentage populations; the t's are the property tax rates; and the g's are public good expenditures.

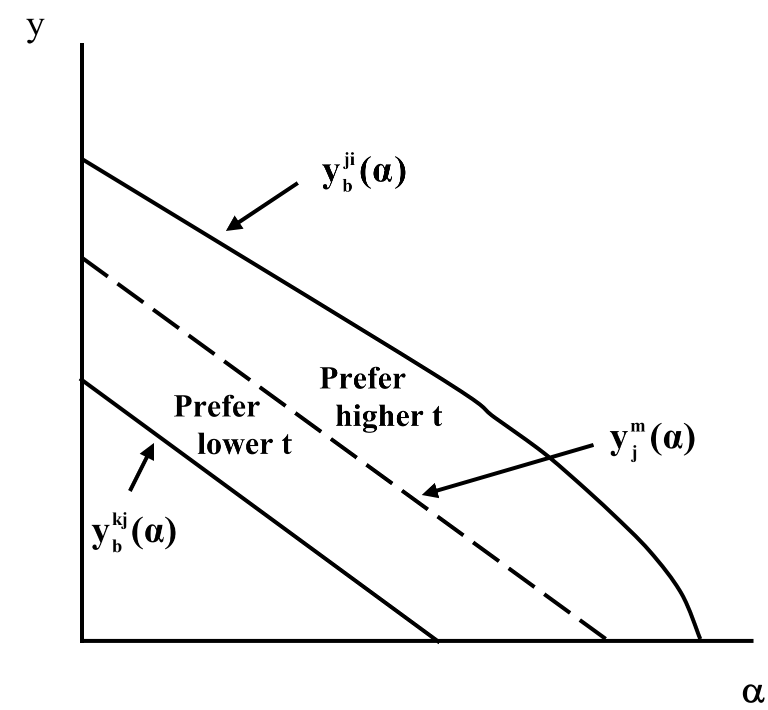

Figure 1A

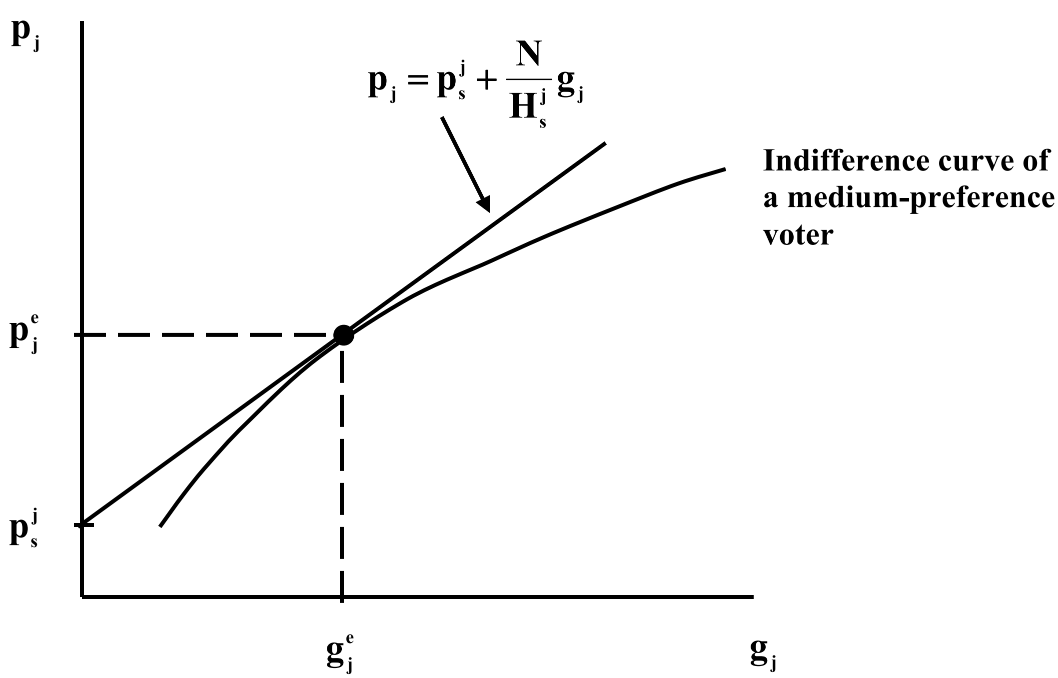

Figure 1B

Actual and Predicted Values for Tax Rates, Government Spending, and House Values in Boston Metropolitan Area (Calabrese, Epple, Romer, Sieg, JPubE, 2006)

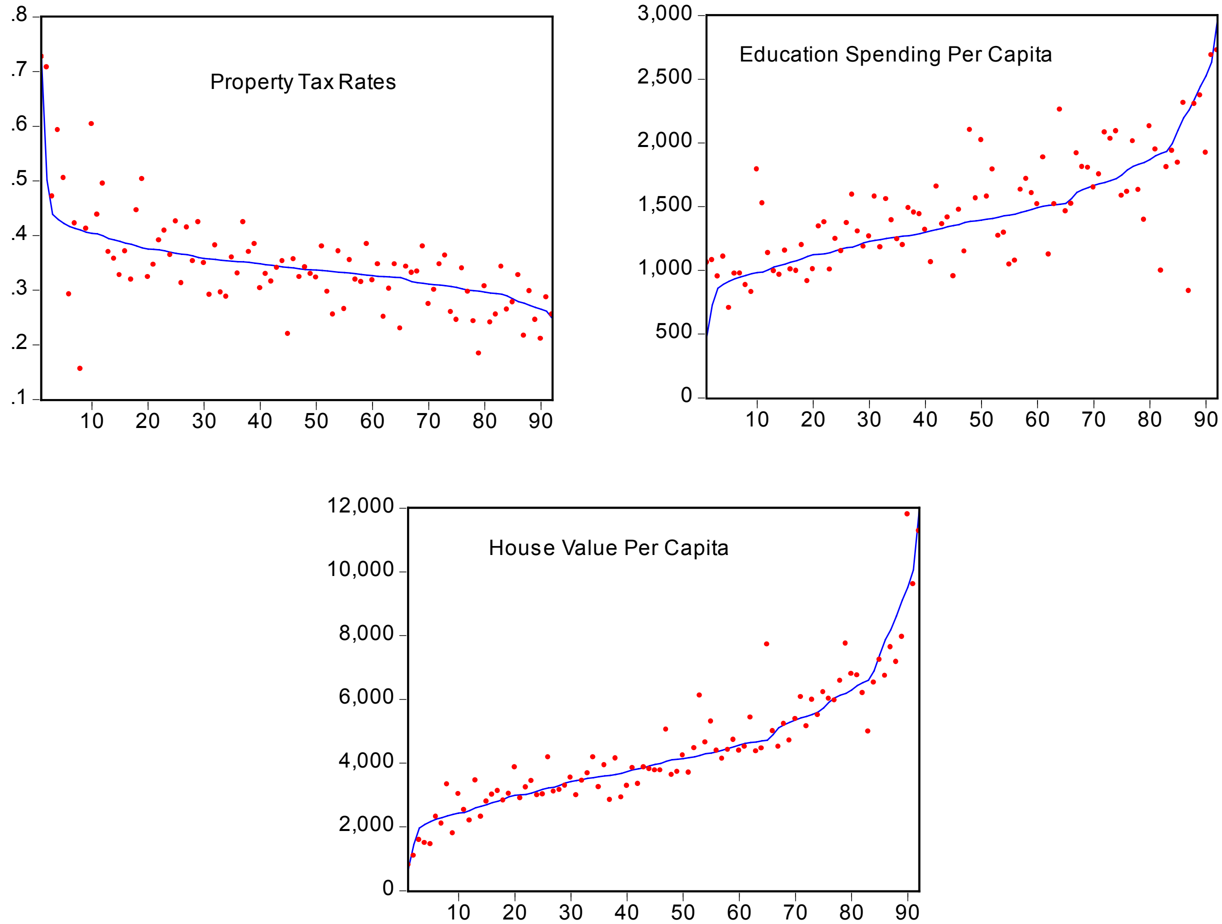

Figure 2

(Solid line is predicted value.)