Dynamic Female Labor Supply

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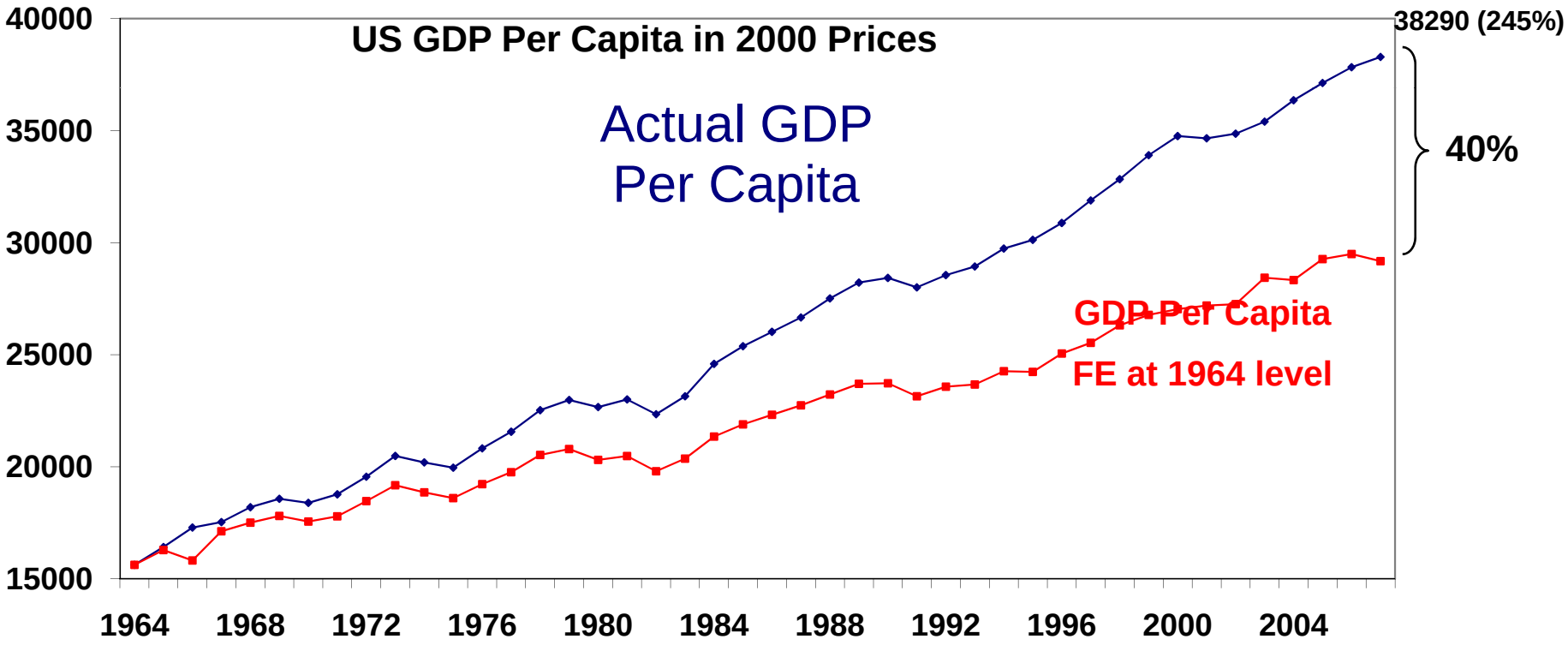
Why Do We Study Female Employment (FE)?

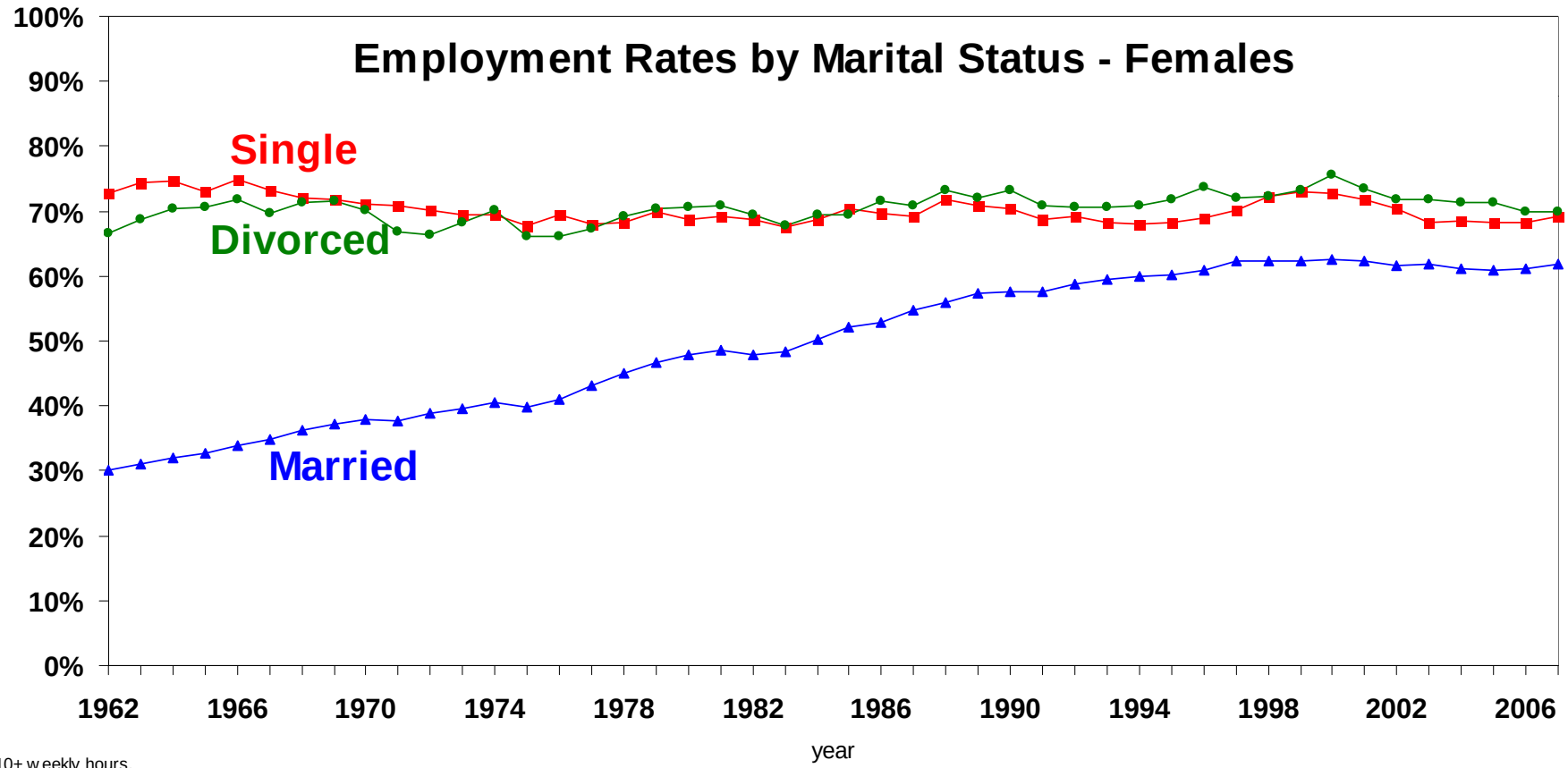
Because they contribute a lot to US GDP Per Capita…



Central Question

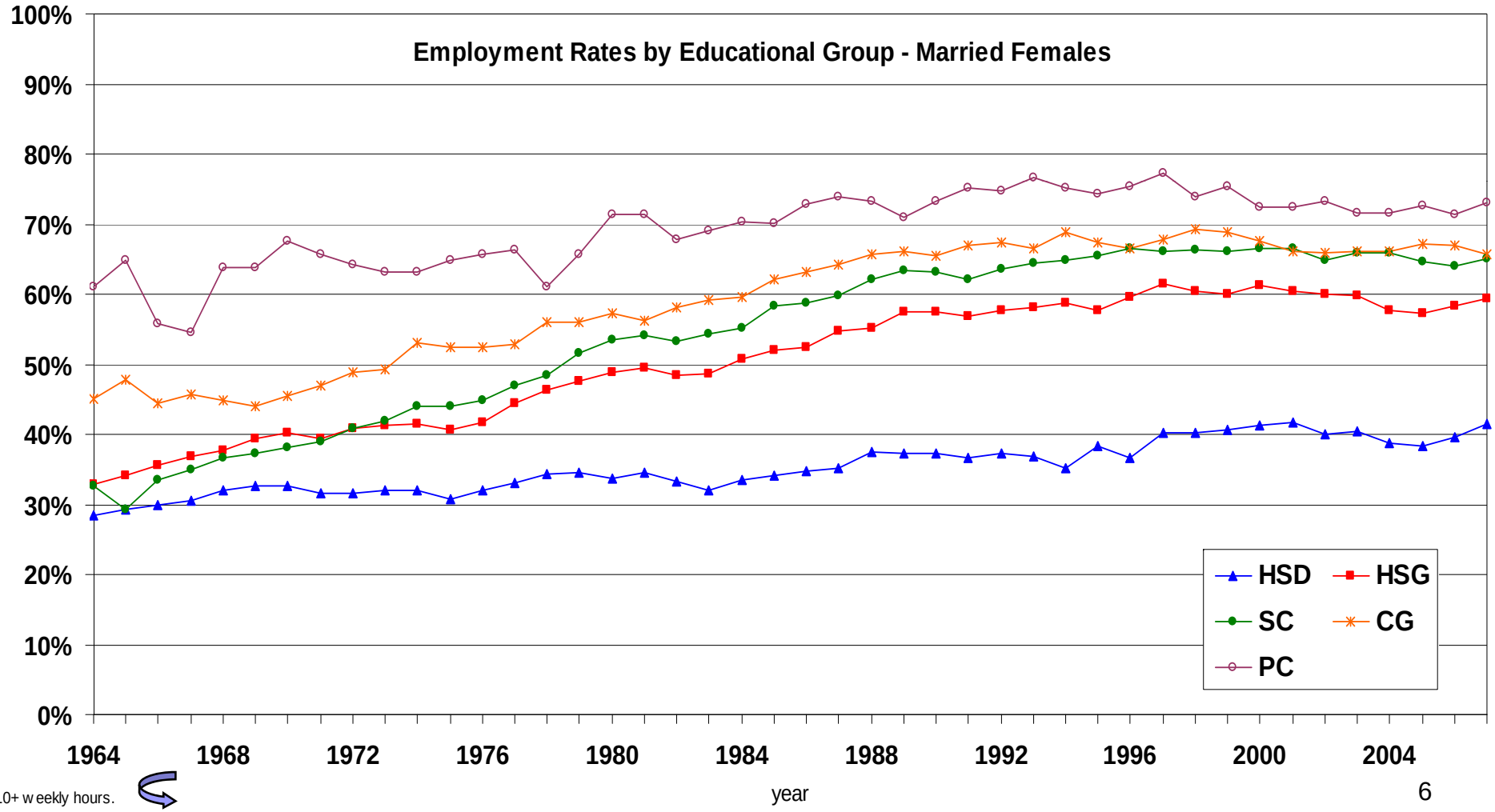
Why Did Female Employment (FE) Rise Dramatically?

Because Married FE Rose…..!



10+ weekly hours.

Who among the Married?
The Educated (HSG-CG) Females!



10+ weekly hours.

Why did ***Married*** Female Employment (FE) Rise Dramatically?

Main Empirical Hypotheses

- Education increase (Becker)
- Wage increase/Gender Gap decline

  Heckman and McCurdy(1980), Goldin(1990), Galor and Weil(1996), Blau and Kahn(2000), Jones, Manuelli and McGrattan(2003), Gayle and Golan(2007)

- Fertility decline

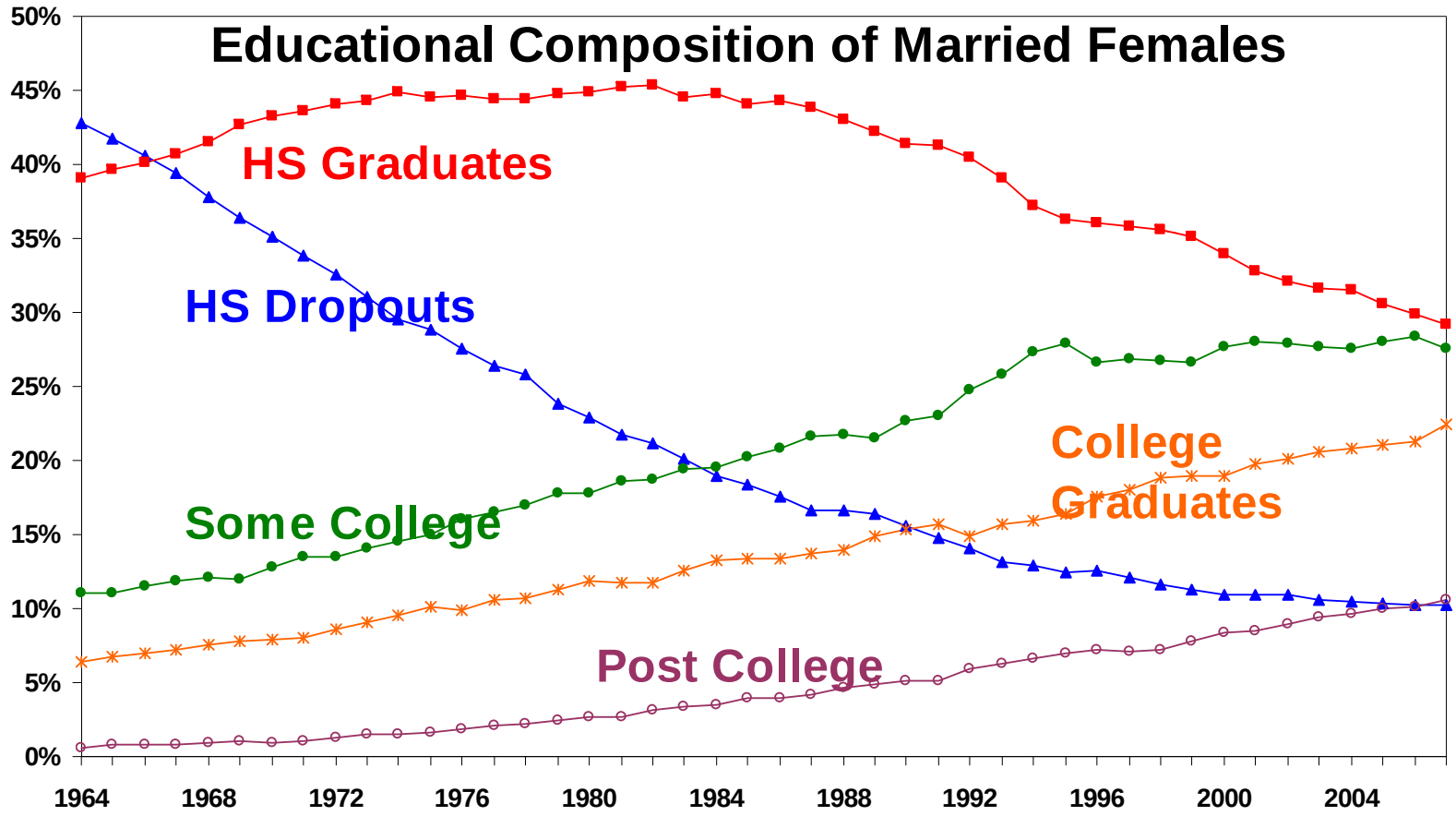
  Gronau(1973), Heckman(1974), Rosensweig and Wolpin(1980), Heckman and Willis(1977), Albanesi and Olivetti(2007) Attanasio at.al.(2008)

- Marriage decline/Divorce increase

  Weiss and Willis(1985,1997), Weiss and Chiappori(2006)

- Other

Education Increase

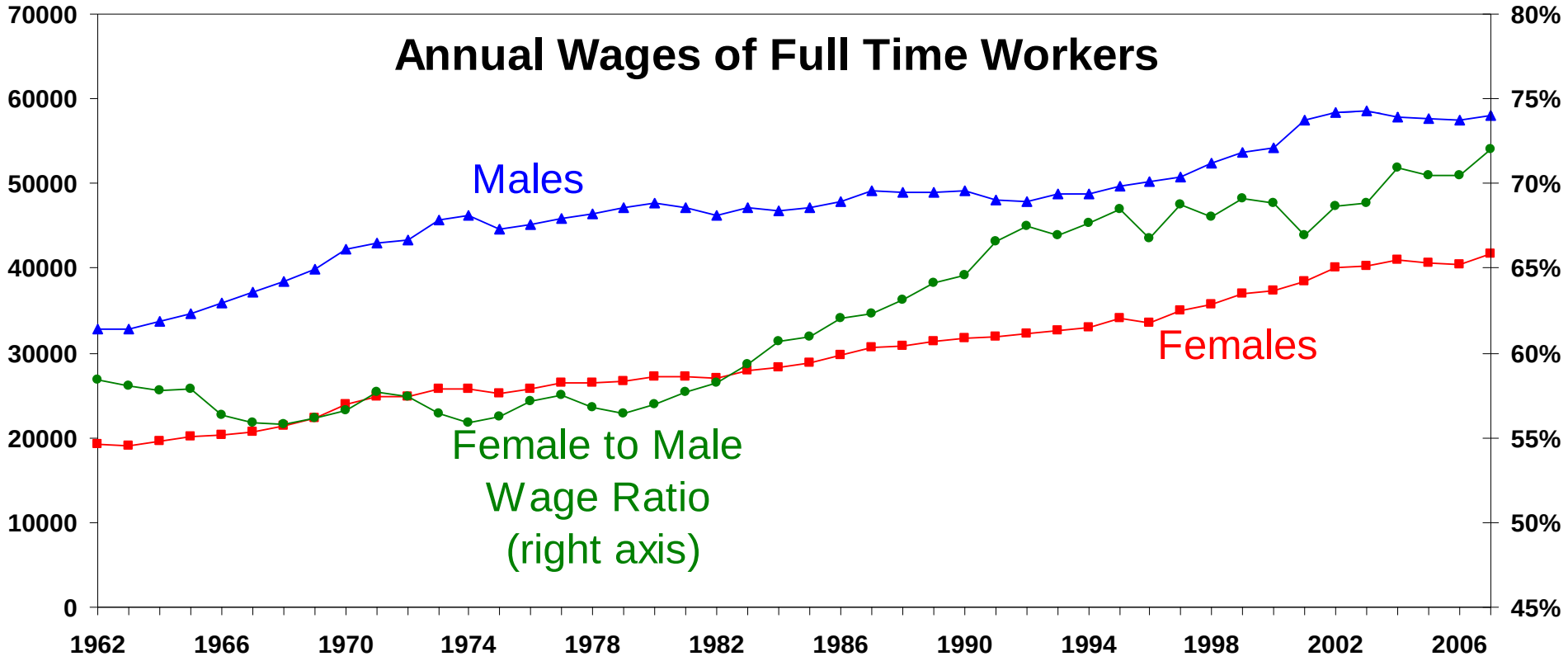


Women by cohort

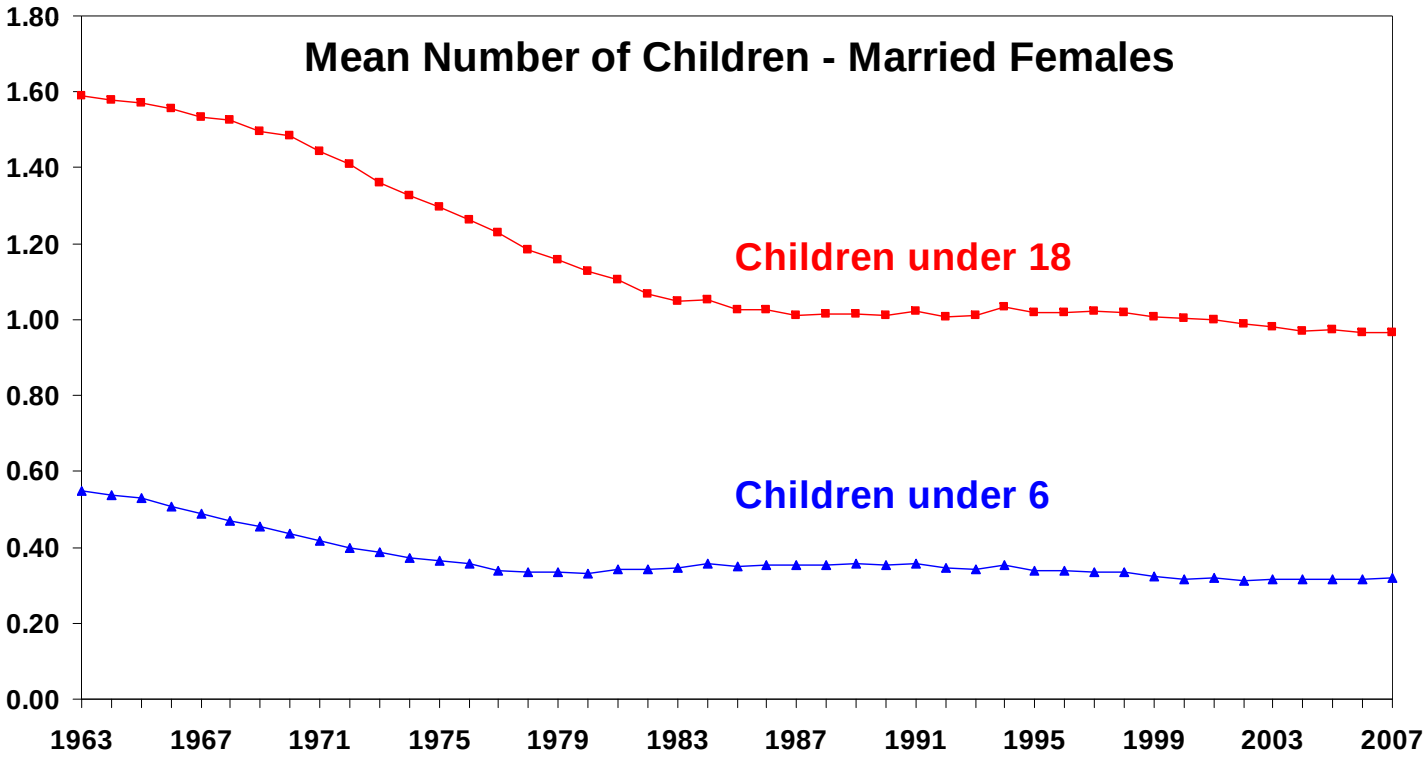
Men composition s.s.

Men by cohort

Wage increase – Gender Gap decline



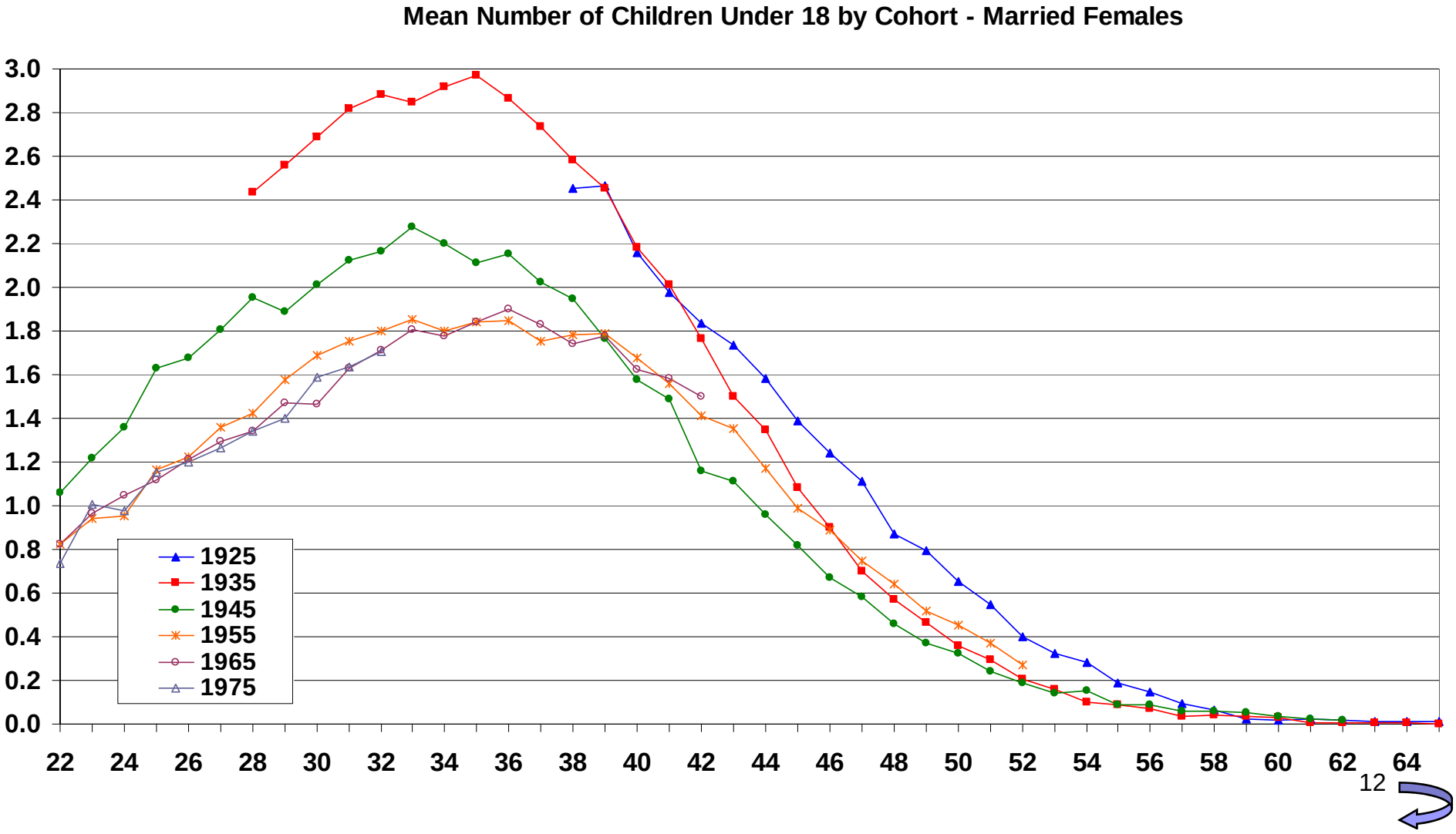
Fertility Decline



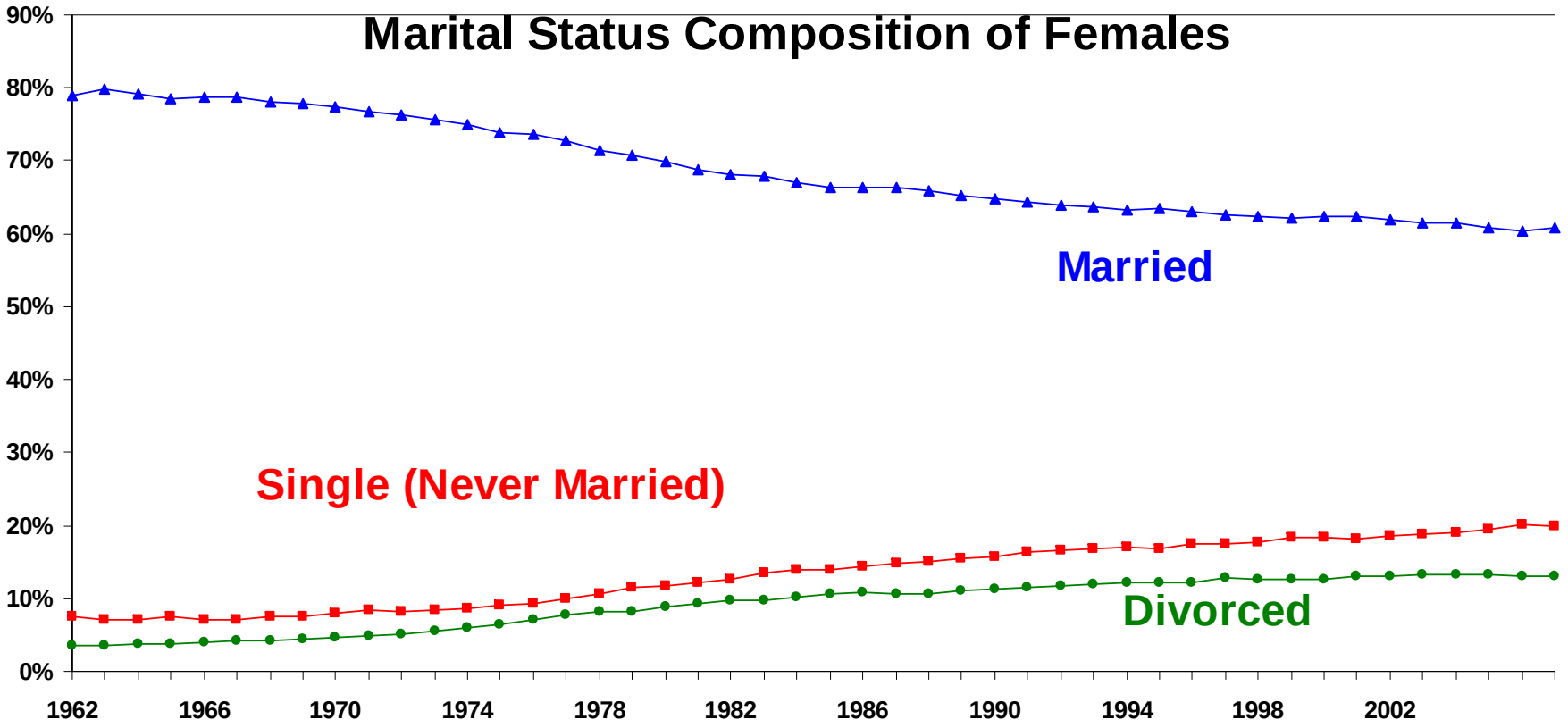
Ref.

by cohort

Decrease in the Fertility of Married Women



Marriage Declines – Divorce Increases



What are the ***Other*** Empirical Hypotheses?

- Social Norms

  Fernandez, Fogli and Olivetti(2004), Mulligan and Rubinstein(2004), Fernandez (2007)

- Cost of Children

  Attanasio, Low and Sanchez-Marcos(2008) Albanesi and Olivetti(2007)

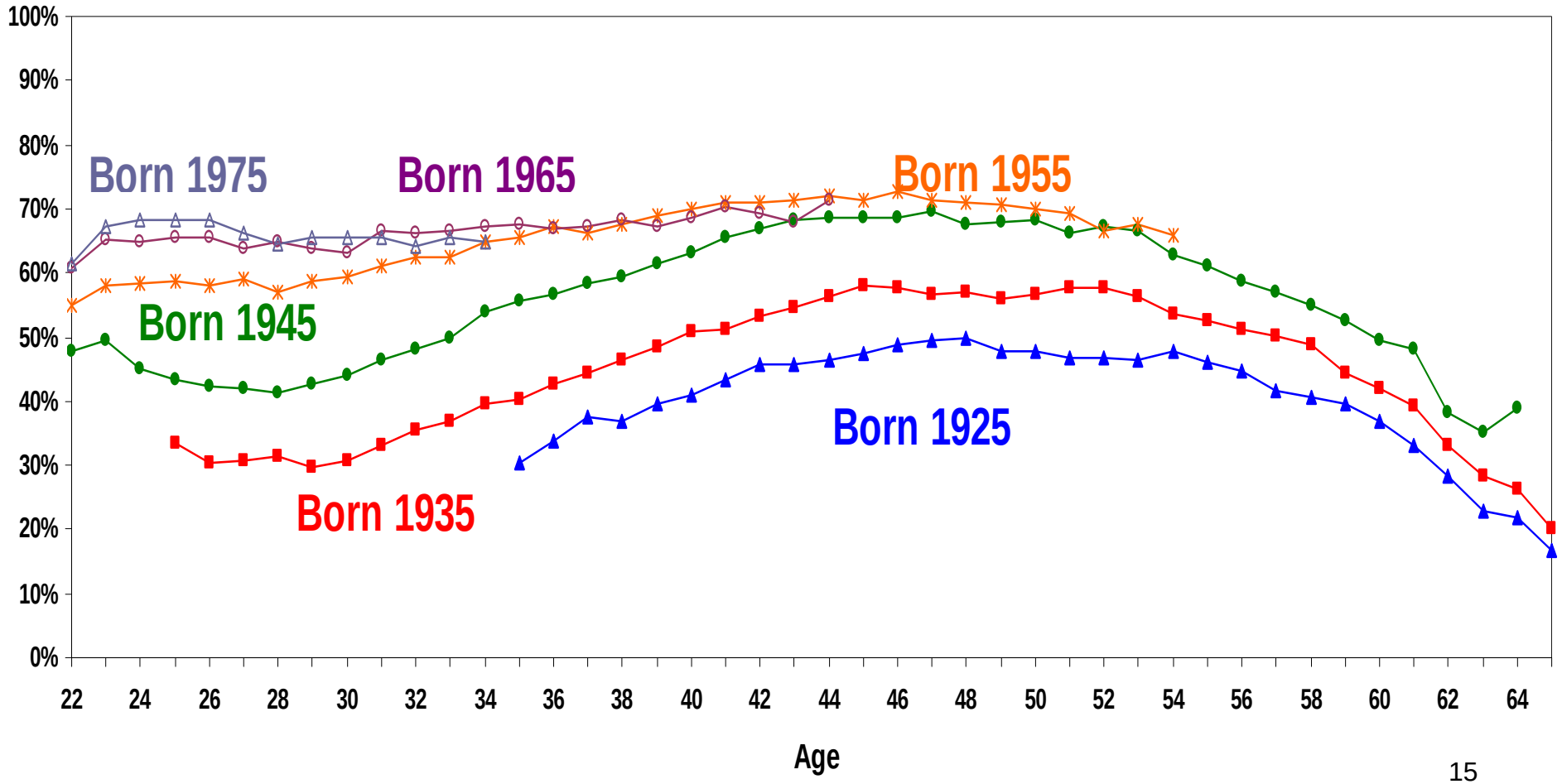
- Technical Progress

  Goldin(1991), Greenwood et. al.(2002),

Will show up as a c***ohort*** effects..

Employment rates by Age

Post baby-boomers Cohort's FE stabilized



An Accounting Exercise

- Measure female's employment due to:
  - **Education** increase
  - **Wage** increase/Gender Gap decrease
  - **Fertility** decline
  - **Marriage** decline/**Divorce** growth

- The "*unexplained*" is Others

Lee and Wolpin, 2008

An Accounting Exercise

- Need an empirical model
- Use Standard *Dynamic* Female Labor Supply Model – Eckstein and Wolpin 1989 (EW): "old" model

Later extensions (among others..): van der Klauw, 1996, Altug and Miller, 1998, Keane and Wolpin, 2006 and Ge, 2007.

Sketch of the Model

- Extension of Heckman (1974)
- Female maximizes PV utility
  - Chooses employment (p_t = *1 or 0*)
  - Takes as given:
    - Education at age 22
    - Husband characteristics
    - Processes for wages, fertility, marital status
- Estimation using SMM and 1955 cohorts from CPS

Model

The woman chooses employment in order to maximize:

$$E_t\left[\sum_{k=0}^{T-t}\delta^j U\left(p_{t+k}, x_{t+k}, K_{t+k-1}, N_{t+k,j}\,(j=1..J), S, M_{t+k}\right)\right]$$

$p_t = 1$ Employed

x_t Consumption

K_{t-1} Experience

N_{tj} Children age group j

S Schooling;

$M_t = 1$ Married

$$U_t = \alpha_1 p_t + x_t + \alpha_2 p_t x_t + \alpha_3 p_t K_{t-1} + \sum_{j=1}^{J}\alpha_{4j} N_{tj} p_t + \alpha_5 p_t S + f\left(N_{tj}\right)$$

The household's budget constraint:

$$((1-\alpha)(1-M_t)+\alpha)(y_t^w p_t + y_t^h M_t) = x_t + \sum_{j=1}^{J} c_j N_{tj} + bp_t$$

y_t^h Husband's earnings

y_t^w Wife's earnings

c_j Goods cost per child of age *j*;

b Fixed cost if working;

α Fraction of income consumed by the wife.

The Mincerian (Ben-Porat; Griliches) female's earning function

$$\ln y_t^w = \beta_1 + \beta_2 K_{t-1} + \beta_3 K_{t-1}^2 + \beta_4 S + \beta_5 t + \varepsilon_t .$$

Budget constraint and wage into utility imply:

Employment:
$$U^1{}_t\left(K_{t-1}, \varepsilon_t, \Omega_t\right) = \alpha_1 + \left(1+\alpha_2\right)\left(\exp\left\{\beta_1 + \beta_2 K_{t-1} + \beta_3 K^2{}_{t-1} + \beta_4 S + \varepsilon_t\right\} + \bar{y}_t^h - \sum_{j=1}^{J} c_j N_{tj} - b \right)$$
$$+ \alpha_3 K_{t-1} + \sum_{j=1}^{J} \alpha_{4j} N_{tj} + \alpha_5 S + f\left(N_{tj}\right)$$

Unemployment:
$$U^0{}_t\left(K_{t-1}, \varepsilon_t, \Omega_t\right) = \bar{y}_t^h - \sum_{j=1}^{J} c_j N_{tj} + f\left(N_{tj}\right).$$

Probabilities

Logistic form for: job offer probability, marriage and divorce probability and probability of having a new child

$V_t^1(\cdot)$ and $V_t^0(\cdot)$ are the maximum expected discounted utility if the woman at time *t* works $(p_t = 1)$ or does not work $(p_t = 0)$, respectively

$$v_t^1(\Omega_t, t) = U_t^1(K_{t-1}, \varepsilon_t, \Omega_t) + \beta \cdot E\left(V_{t+1}(K_t, \varepsilon_{t+1}, \Omega_{t+1}) \middle| \Omega_t, p_t = 1\right)$$

$$v_t^0(\Omega_t, t) = U_t^0(K_{t-1}, \varepsilon_t, \Omega_t) + \beta \cdot E\left(V_{t+1}(K_t, \varepsilon_{t+1}, \Omega_{t+1}) \middle| \Omega_t, p_t = 0\right)$$

$$V_t = \max(v_t^0, v_t^1)$$

Solution:

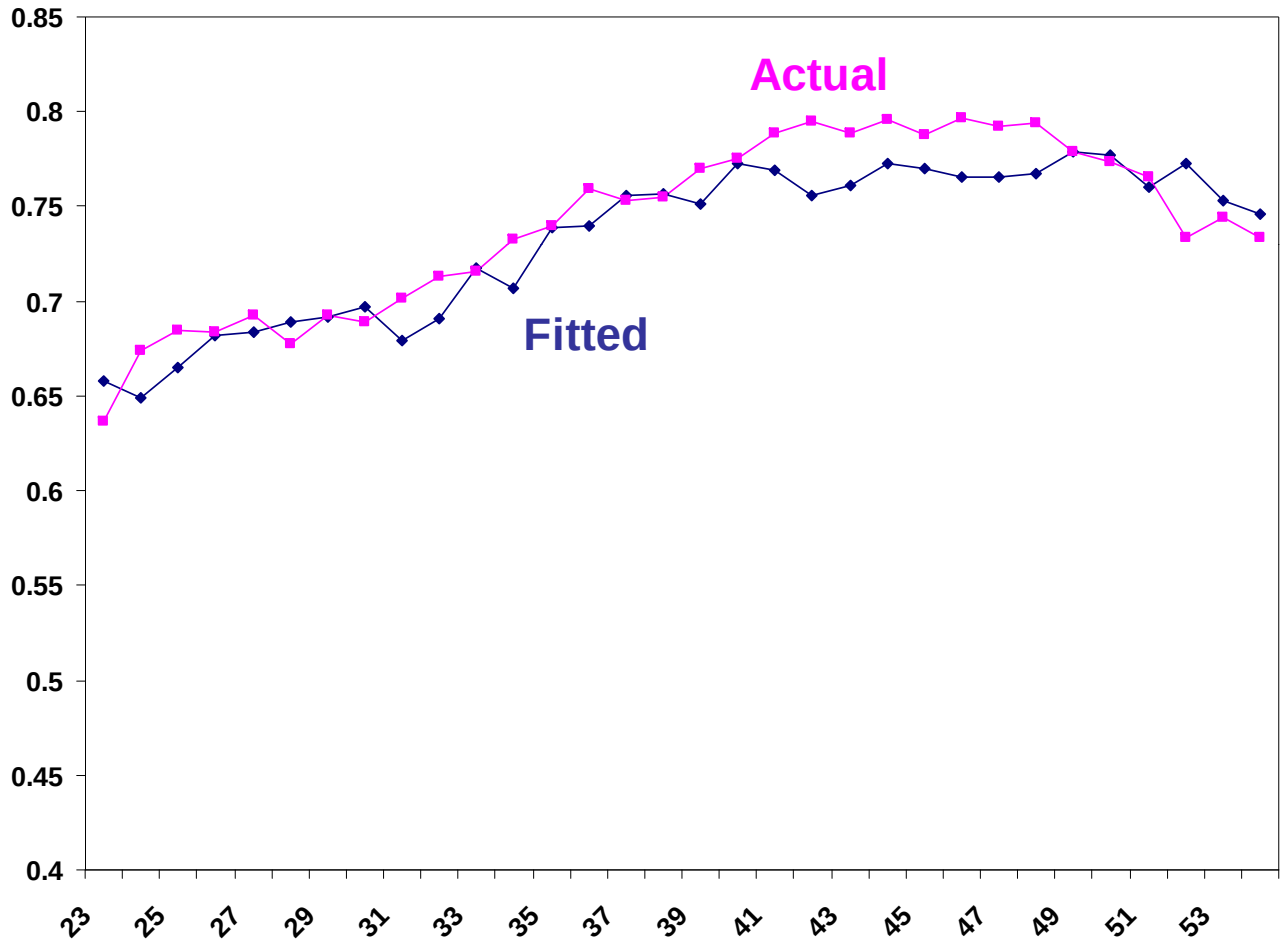
Backward Solution following Eckstein and Wolpin (1989) and Keane and Wolpin (1997)

Estimation: Structural DP model using CPS

- **Estimation EW**: SMM using 1955 cohort CPS data and choice of relevant cross-section moments. Joint estimation of the following equations :
  - **Female Employment dynamic discrete choice model with cross equation restrictions and rational expectations internal consistency** (Lucas, 1976, Sargent, 1983: Mix probit with logit FE offer rate)
  - Log wage with endogenous experience (not age).
  - MNL of Children, Marriage, Divorce
  - Random choice of husband conditional of characteristics;

  Female
- **Alternative:** MNL and Log Wage Alternative - Full Reduced form approximation. (KW, 2006, Del-Boca and Sauer 2008)

Estimation Fit – 1955 cohort FE



HSD
HSG
SC
CG
PC

Parameters

Back to Accounting Exercise

- For the 1955 cohort we estimated:

$$p^{55} = P^{55}(S, y^{w}, y^{h}, N, M) \quad \textit{for each age}$$

- Contribution of ***Education*** of 1945 cohort (S^{45}) for predicted FE of 1945 cohort is:

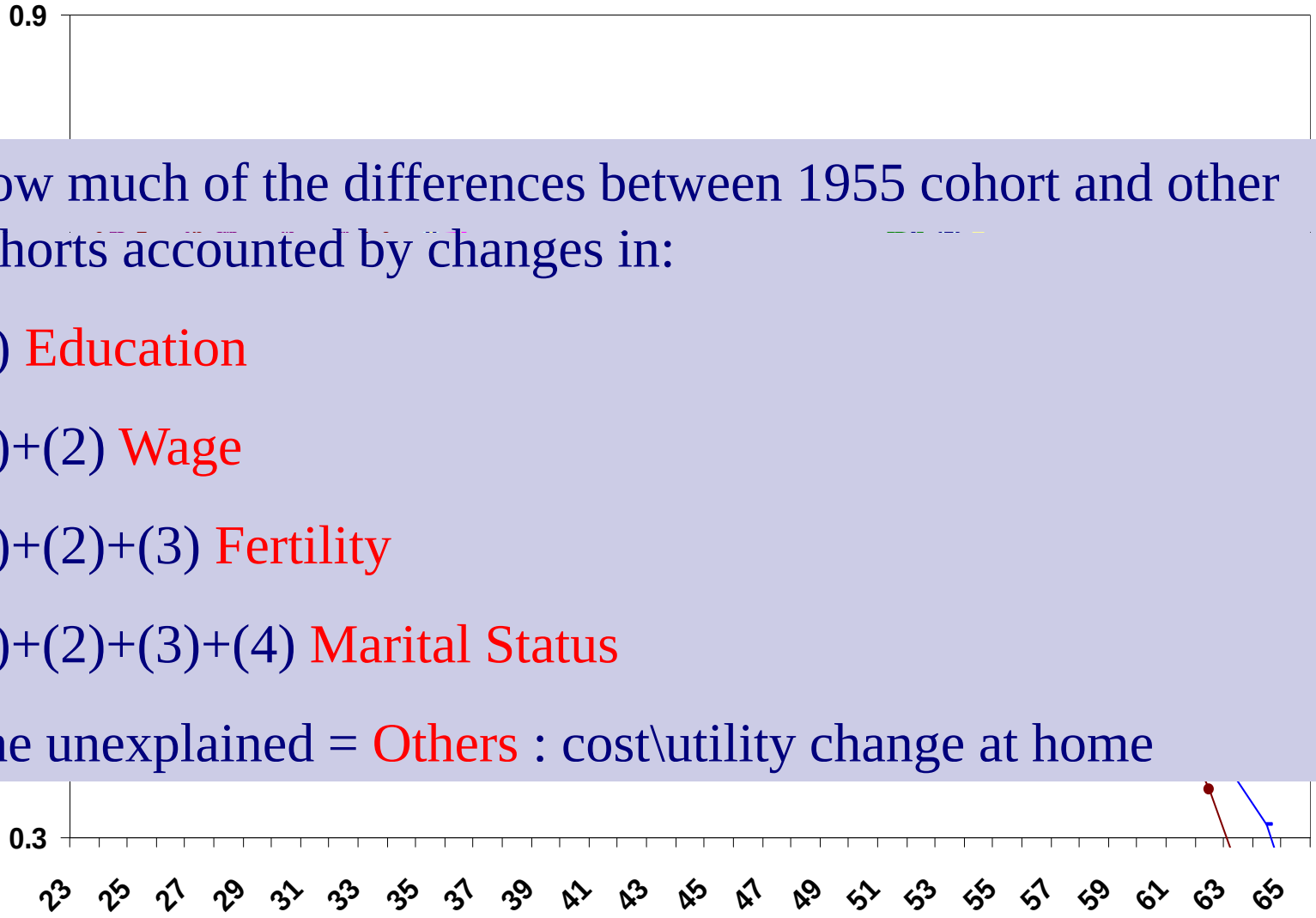
$$\textit{predicted } p^{45} = P^{55}(S^{45}, y^{w55}, y^{h55}, N^{55}, M^{55})$$

- ***….Education and Wage***

$$\textit{predicted } p^{45} = P^{55}(S^{45}, y^{w45}, y^{h45}, N^{55}, M^{55})$$

- *….Etc*

FE by Age per Cohort



How much of the differences between 1955 cohort and other cohorts accounted by changes in:

(1) Education

(1)+(2) Wage

(1)+(2)+(3) Fertility

(1)+(2)+(3)+(4) Marital Status

The unexplained = Others : cost\utility change at home

Accounting for changes in FE: **1945 cohort**

Age Group: 28-32 1955: Actual: 69% Fitted: 69%	
Actual 1945	**53%**
1 - Education	66%
1+ 2 Wage	65%
+ 3 Children	63%
+ 4 Martial Status	63%
Other	**10%**
Age Group: 38-42 1955:Actual: 78% Fitted: 76%	
Actual 1945	**73%**
1 - Education	73%
1+ 2 Wage	74%
+ 3 Children	73%
+ 4 Martial Status	73%
Other	**0%**

Early age total difference 16% - 10% is Other

Decomposition of the change in FE

	cohort 35	cohort 45	cohort 65	cohort 75
Age Group: 23-27	55 Actual: 67%, fitted: 67%			
Actual		**50%**	**74%**	**74%**
1 - Education		61%	69%	72%
1+ 2 Wage		59%	69%	73%
+ 3 Children		57%	69%	73%
+ 4 Martial Status		57%	69%	74%
unexplained Diff		**7%**	**5%**	**0%**
Age Group: 28-32	55 Actual: 69%, fitted: 69%			
Actual	**38%**	**53%**	**74%**	**72%**
1 - Education	57%	66%	70%	73%
1+ 2 Wage	55%	65%	70%	73%
+ 3 Children	55%	63%	72%	73%
+ 4 Martial Status	55%	63%	72%	74%
unexplained Diff	**17%**	**10%**	**2%**	**-2%**
Age Group: 33-37	55 Actual: 74%, fitted: 73%			
Actual	**47%**	**63%**	**76%**	
1 - Education	65%	70%	73%	
1+ 2 Wage	64%	69%	73%	
+ 3 Children	64%	68%	75%	
+ 4 Martial Status	64%	68%	75%	
unexplained Diff	**17%**	**5%**	**1%**	

	cohort 25	cohort 35	cohort 45	cohort 65
Age Group: 38-42	55 Actual: 78%, fitted: 76%			
Actual	**53%**	**58%**	**73%**	**76%**
1 - Education	63%	69%	73%	74%
1+ 2 Wage	63%	68%	74%	76%
+ 3 Children	63%	69%	73%	76%
+ 4 Martial Status	63%	68%	73%	77%
unexplained Diff	**10%**	**10%**	**0%**	**-1%**
Age Group: 43-47	55 Actual: 79%, fitted: 77%			
Actual	**54%**	**64%**	**76%**	
1 - Education	67%	69%	75%	
1+ 2 Wage	65%	69%	76%	
+ 3 Children	65%	69%	75%	
+ 4 Martial Status	65%	69%	75%	
unexplained Diff	**11%**	**5%**	**1%**	

Schooling composition graph 1945 Schooling composition graph 1965 Schooling composition graph 1975

Accounting for the change in FE: Cohorts of 1945, 65, 75 based on 1955

- Education: **~ 60%** of the change in FE
- Wages: **~ 10%**
- Fertility: **~ 10%**
- Marriage: **~ 0%**
- Other: **~ 20%**
  - **50%** at the early ages
  - **0%** for older ages

Accounting for the change in FE: Cohorts of 1925, 35: based on 1955 Cohort

- Education: **~ 47%** of the change in FE
- Wages: **~ 5%**
- Fertility: **~ 3%**
- Marriage: **~ 0%**
- Other: **~ 45%**
  - **55%** at the early ages
  - **35%** for older ages

What are the missing factors for "other"?

What is missing factor for **early ages**?

- **Childcare cost** if working

- Change 1 parameter (α_4) – get perfect fit
  - 1945 cohort childcare cost: **$5/hour higher**
  - 1965 cohort childcare cost: **$0.23/hour lower**
  - 1975 cohort childcare cost: **$0.34/hour lower**

What is missing factor for **all ages**?

- **Childcare cost** if working
- Value of staying at home
- Change 2 parameters (α_1, α_4) – get perfect fit
  - 1935,1925 cohorts childcare cost: **$5/hour higher**
  - 1935 cohort leisure value: **$3.9/hour higher**
  - 1925 cohort leisure value: **$3.5/hour higher**

How can we explain results?

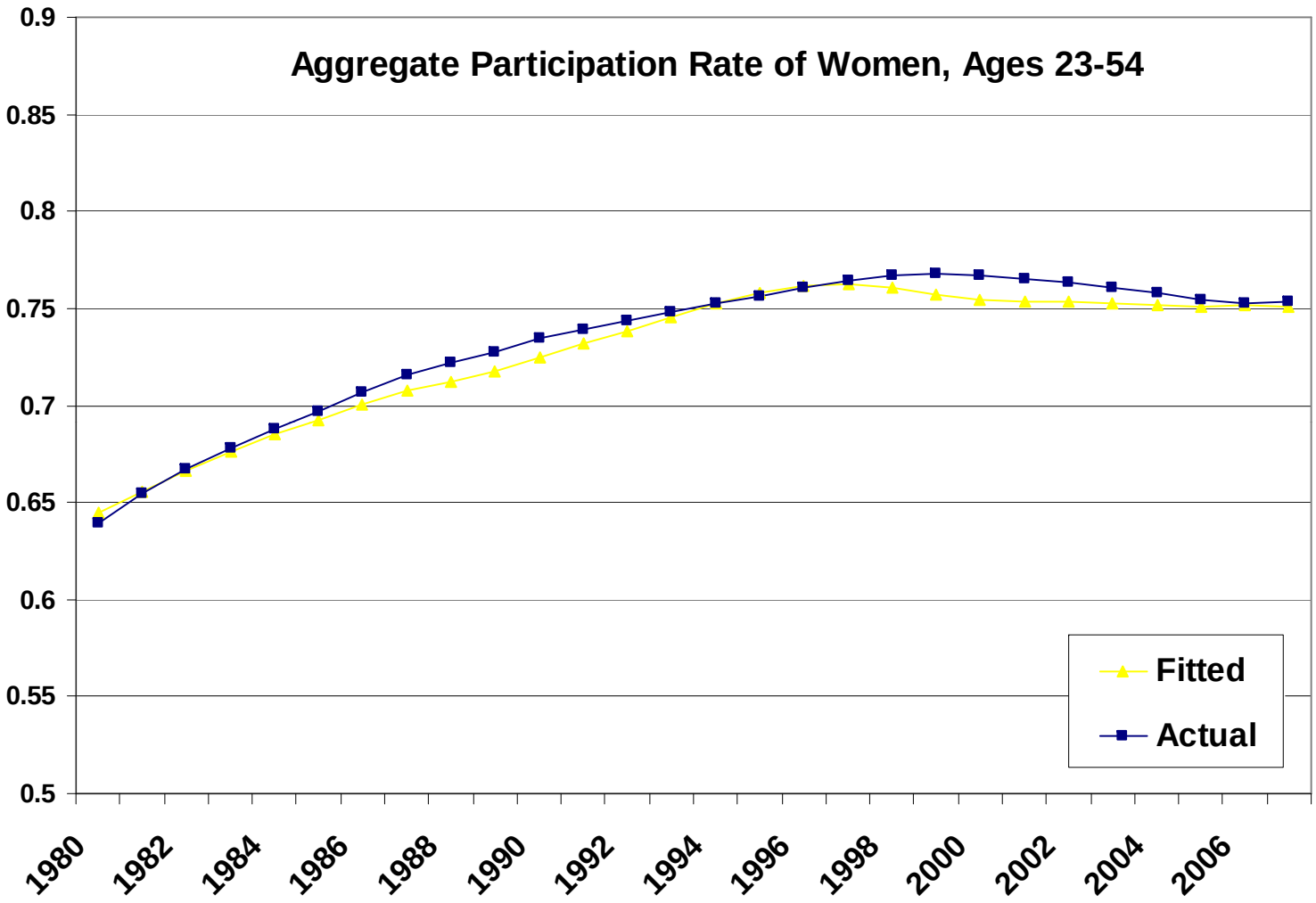
How can we explain results?

- Change in cost/utility interpreted as:
  - Technical progress in home production
  - Change in preferences or social norms

How do we fit the aggregate employment/participation?

Aggregate fit Simulation

- Simulate the participation rate for all the cohorts: 1923-1978.
- Calculate the aggregate participation for each cohort at each year by the weight of the cohort in the population.
- Compare actual to simulated participation 1980-2007.


Aggregate Participation Rate of Women, Ages 23-54
0.9
0.85
0.8
0.75
0.7
0.65
0.6
0.55
0.5
1980
1982
1984
1986
1988
1990
1992
1994
1996
1998
2000
2002
2004
2006
Fitted
Actual

Modeling change in cost/utility of leisure

- **Unobserved heterogeneity** regarding leisure/children
- **Bargaining** power of women changes
- Household game: a “new” empirical framework

Labor Supply of Couples: Traditional and Modern Households –"new" Model

- Internal family game (McElroy,1984, Chiappori, 1998)
- New empirical dynamic models of household labor supply: Lifshitz (2004), Flinn (2007), Tartari (2007)

The Model: Household Dynamic Game

- Two types of household
  - **Traditional (T):** Husband is **Stackelberg leader**.
    Every period after state is realized the husband makes the decision before the wife, and then she responds.
  - **Modern (M):** Husband & Wife play **Nash**.
    Husband & wife are symmetric, act simultaneously after state is realized, taking the other person actions as given**.**
- Both games are solved as **sub-game perfect**.

Sketch of Model: Choices

- Employment; Unemployment; Out of LF
- Initially UE or OLF - two sub-periods
  - Period 1: Search or OLF
  - Period 2: Accept a potential offer E or UE
- Initially E – one period
  - Quit to OLF
  - Fired to UE
  - Employment in a “new” wage.

Sketch of Model: Dynamic program

- Max Expected PV as in EW
  - Utility functions are **identical** for both T and M
  - Characteristics of husband and wife **different**

- Game solved recursively backwards to wedding

Utility functions:

$$U_{jt} = u(x_t) + \alpha_j \cdot l_{jt} + f(N_t) \qquad l_{jt} \text{ = leisure}$$

$$u(x_t) = \frac{(x_t)^{\gamma j}}{\gamma_j} \qquad f(N_t) = \gamma_0 \cdot N_t + \gamma_2 c_t + \frac{\gamma_1}{\overline{age_t}} \left[\frac{l_{Wt} + l_{Ht}}{N_t} \right]$$

$$U^1_{Wt} = u\left((1-\alpha)\left(y^W_t + y^H_t \cdot d^1_{Ht}\right)\right) + f(N_t)$$

$$U^2_{Wt} = u\left((1-\alpha)\left(y^H_t \cdot d^1_{Ht}\right)\right) + f(N_t) + \alpha_W \cdot (l_{Wt} - SC) + \varepsilon^2_{Wt}$$

$$U^3_{Wt} = u\left((1-\alpha)\left(y^H_t \cdot d^1_{Ht}\right)\right) + f(N_t) + \alpha_W \cdot l_{Wt} + \varepsilon^3_{Wt}$$

$$U^1_{Ht} = u\left((1-\alpha)\left(y^H_t + y^W_t \cdot d^1_{Wt}\right)\right) + f(N_t)$$

$$U^2_{Ht} = u\left((1-\alpha)\left(y^W_t \cdot d^1_{Wt}\right)\right) + f(N_t) + \alpha_H \cdot (l_{Ht} - SC) + \varepsilon^2_{Ht}$$

$$U^3_{Ht} = u\left((1-\alpha)\left(y^W_t \cdot d^1_{Wt}\right)\right) + f(N_t) + \alpha_H \cdot l_{Ht} + \varepsilon^3_{Ht}$$

Sketch of model: Budget constraint

The household budget constraint

$$y_t^W \cdot d_{Wt}^1 + y_t^H \cdot d_{Ht}^1 = x_t + c_t \cdot N_t$$

y_t^W and y_t^H are the wife's and husband's earnings;

d_{jt}^a equals one if individual $j = H, W$ chooses alternative a at time t, and zero otherwise;

x_t is the joint couple consumption during period *t*;

c_t is the goods cost per child, $c_t = \alpha \cdot \left(\frac{y_t^W \cdot d_{Wt}^1 + y_t^H \cdot d_{Ht}^1}{N_t} \right)$

N_t is the number of children in the household.

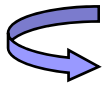
Sketch of model: Wage and probabilities (EW)

- **Mincerian wage** functions for each $j = H, W$

$$ln\, y_t^j = \beta_1^j + \beta_2^j K_{jt-1} + \beta_3^j K_{jt-1}^2 + \beta_4^j S_j + \varepsilon_{jt}^1 .$$

- **Endogenous experience** $k_{jt} = k_{jt-1} + d_{jt}^1$

- Logistic form for job offer probability, divorce probability and probability of having a new child (like EW model).



Logistics form for probability of employment, children and divorce:

$$\Pr\left(\phi\left(\Omega_t, P_t\right)\right) = \frac{\exp\left(\phi\left(\Omega_t, P_t\right)\right)}{1+\exp\left(\phi\left(\Omega_t, P_t\right)\right)}$$

Job Offer Probability

(function of: constant, schooling, experience and time trend):

$$\left(\phi\left(\Omega_t, P_t\right)\right) = \rho_0 + \rho_1 \cdot S + \rho_2 \cdot K_{t-1} + \rho_3 \cdot P_{t-1} + \rho_4 \cdot t$$

Probability of Having a New Child

(function of: constant, age of couple, schooling of couple, number of children and age of youngest child):

$$\left(\phi\left(\Omega_t, P_t\right)\right) = c_0 + c_1 \cdot age_W + c_2 \cdot age_W{}^2 + c_3 \cdot age_H + c_4 \cdot S_W + c_5 \cdot S_H + c_6 \cdot N_{t-1} + c_7 \cdot Age\_of\_Younges$$

Divorce Probability

(function of: constant, years of marriage, number of children, husband and wife previous state):

$$\left(\phi\left(\Omega_t, P_t\right)\right) = d_0 + d_1 \cdot y\_marriage + d_2 \cdot N_{t-1} + d_3 \cdot P^H{}_{t-1} + d_4 \cdot P^W{}_{t-1}$$

Sketch of Model: Main Result

- **Wives work more in M than T family** because:
  - Husband earnings and offer rates are **larger**
  - In **M family** she faces more uncertainty

    (Husband employment and earnings are uncertain when she makes the decision independently)

Estimation: SMM

Data

- **PSID – Panel -** 863 couples who got married between 83-84 **- Cohort of 1960**
- 10 years (40 quarters) sample (at most)

2 sets of moments:

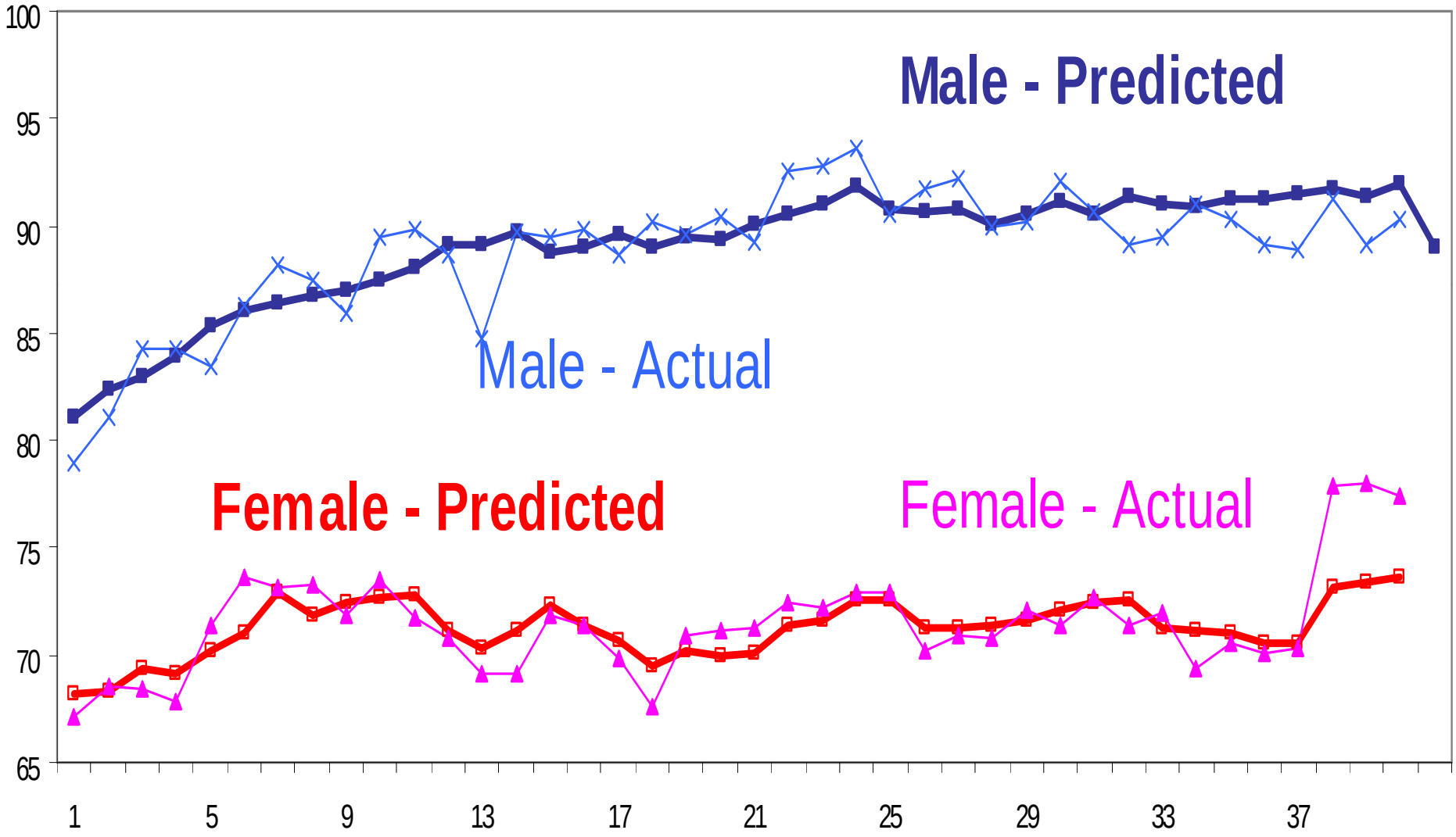
- Mean individual choice of (E; UE; OLF) by duration since marriage.
- Average predicted and actual wage for men and women by duration since marriage.

Estimation Results

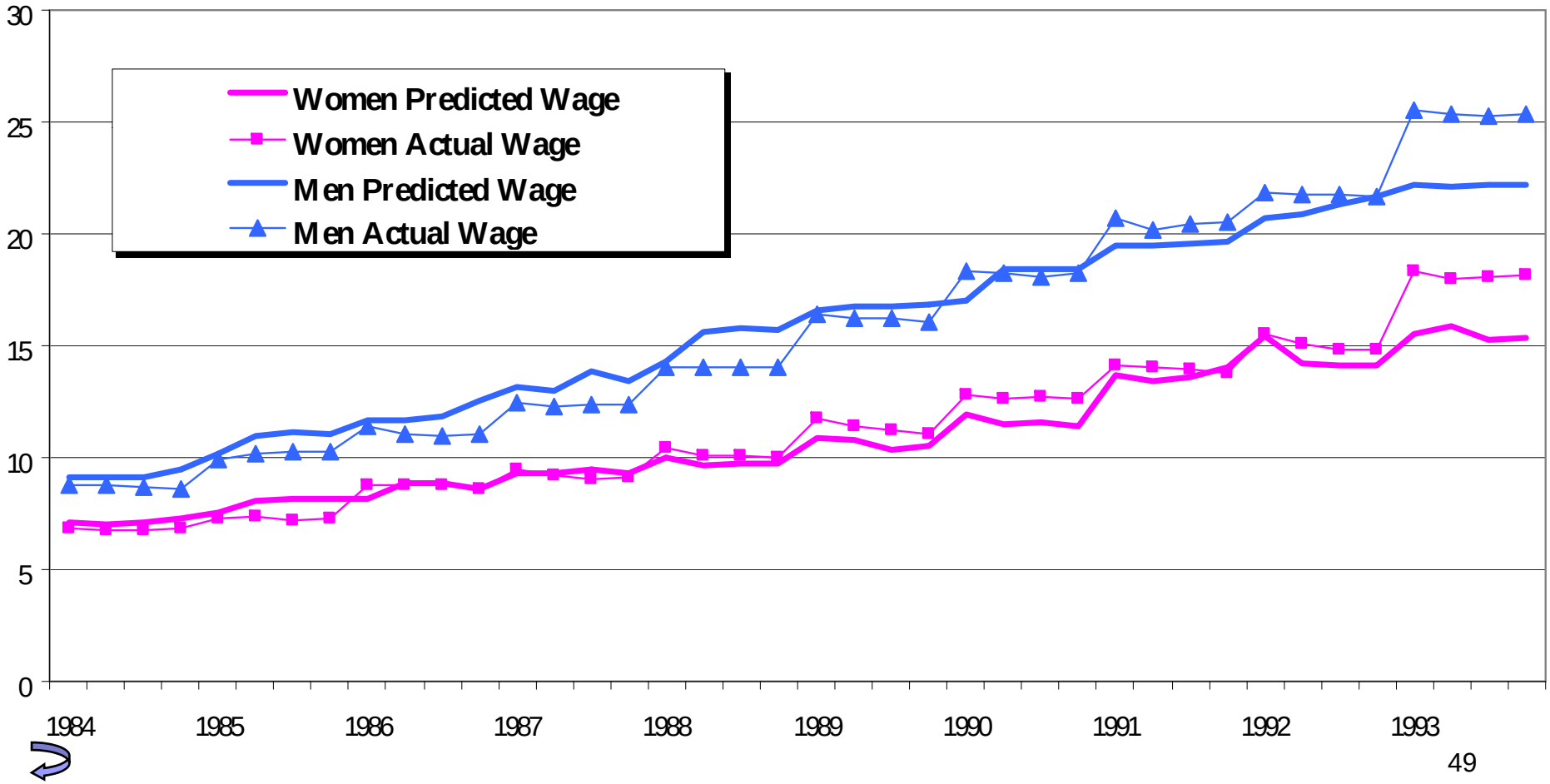
- **90%** of choices are correctly predicted
- **61%** is estimated proportion of T families

- Husbands in T & M have similar labor supply
- Wives participate **9%** more in M families

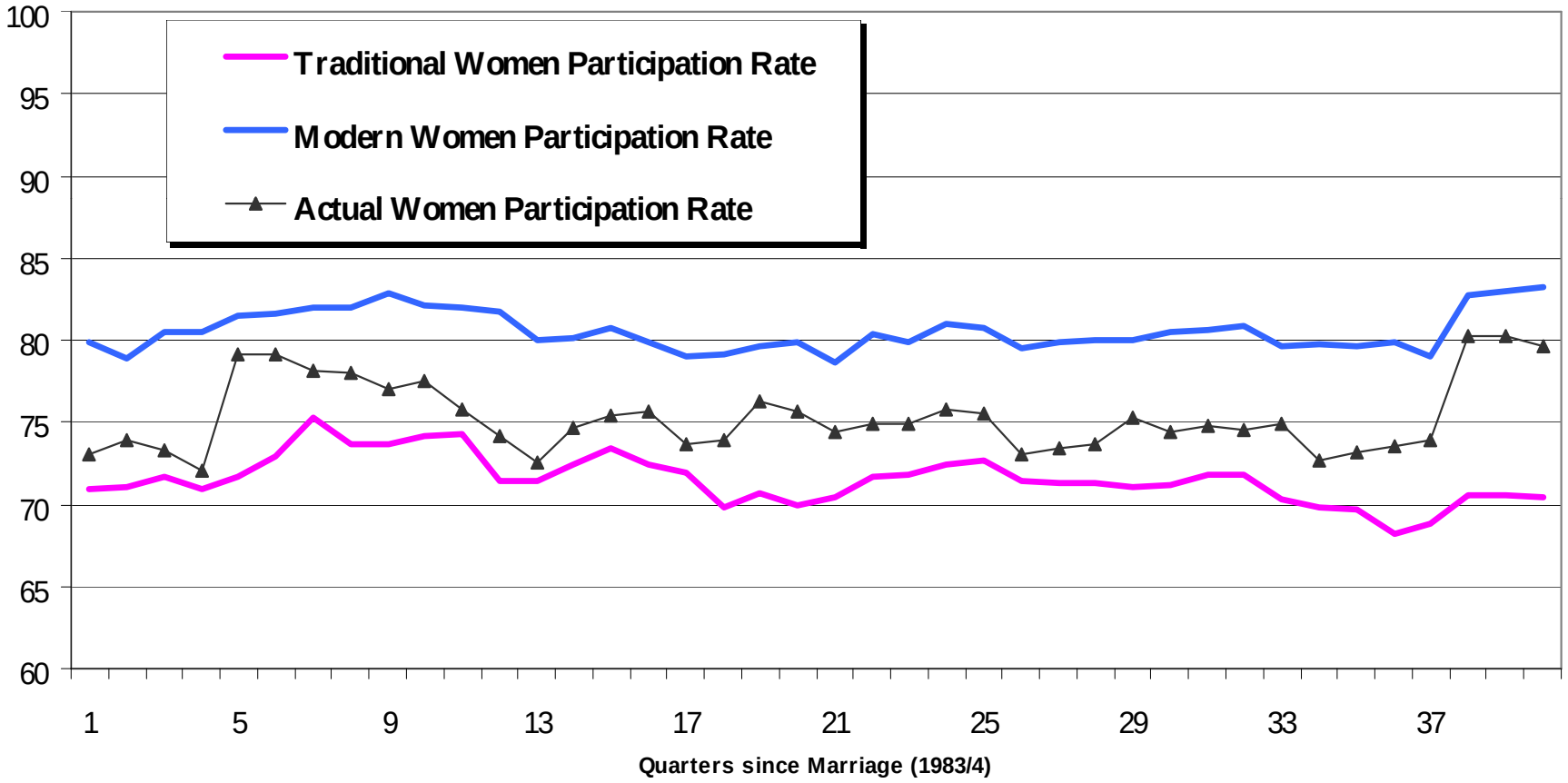
Fit: Employment rate



Actual vs. Predicted Average Wage



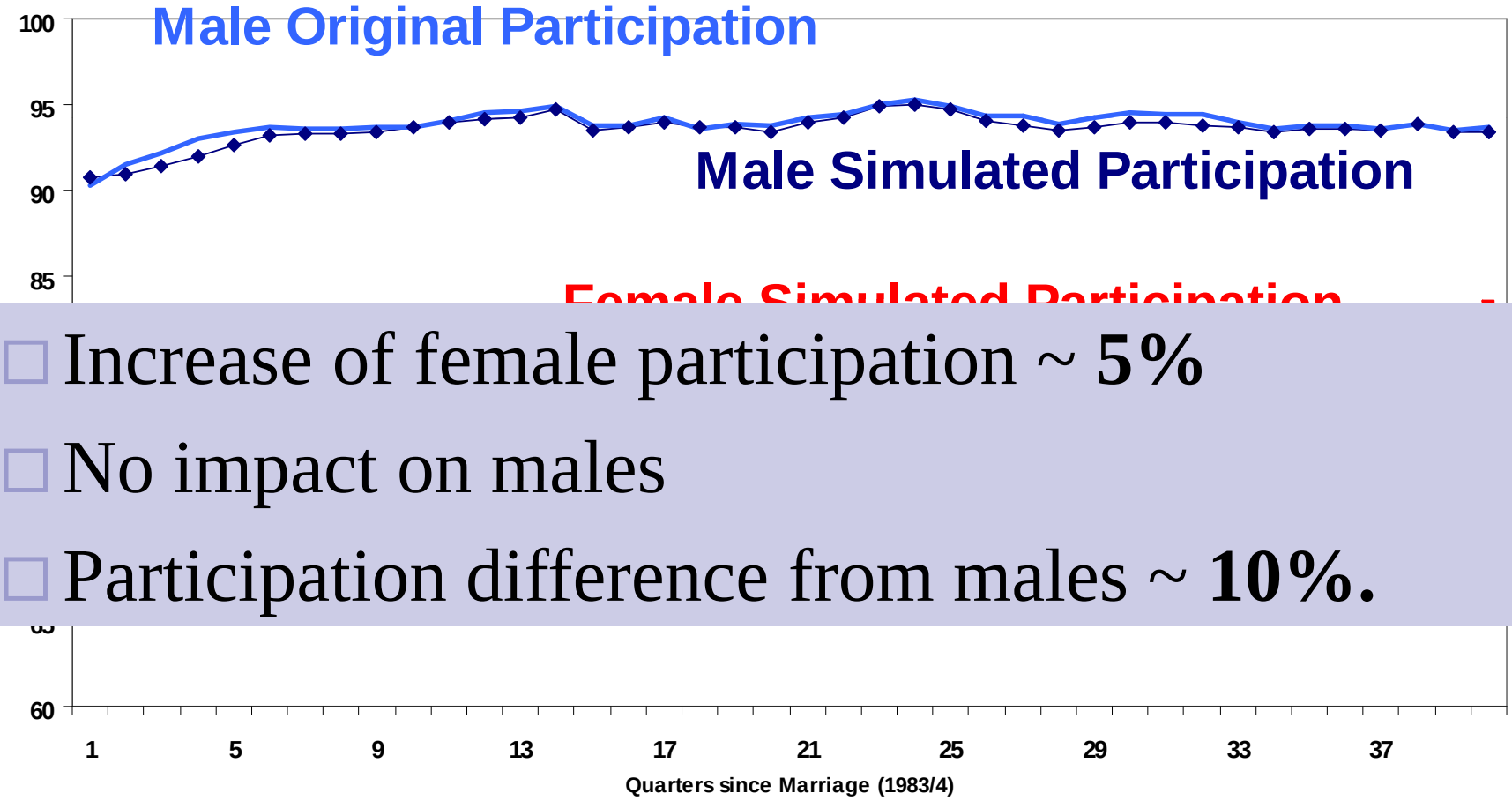
Predicted LFP: Traditional and Modern Women



Probability of Family type

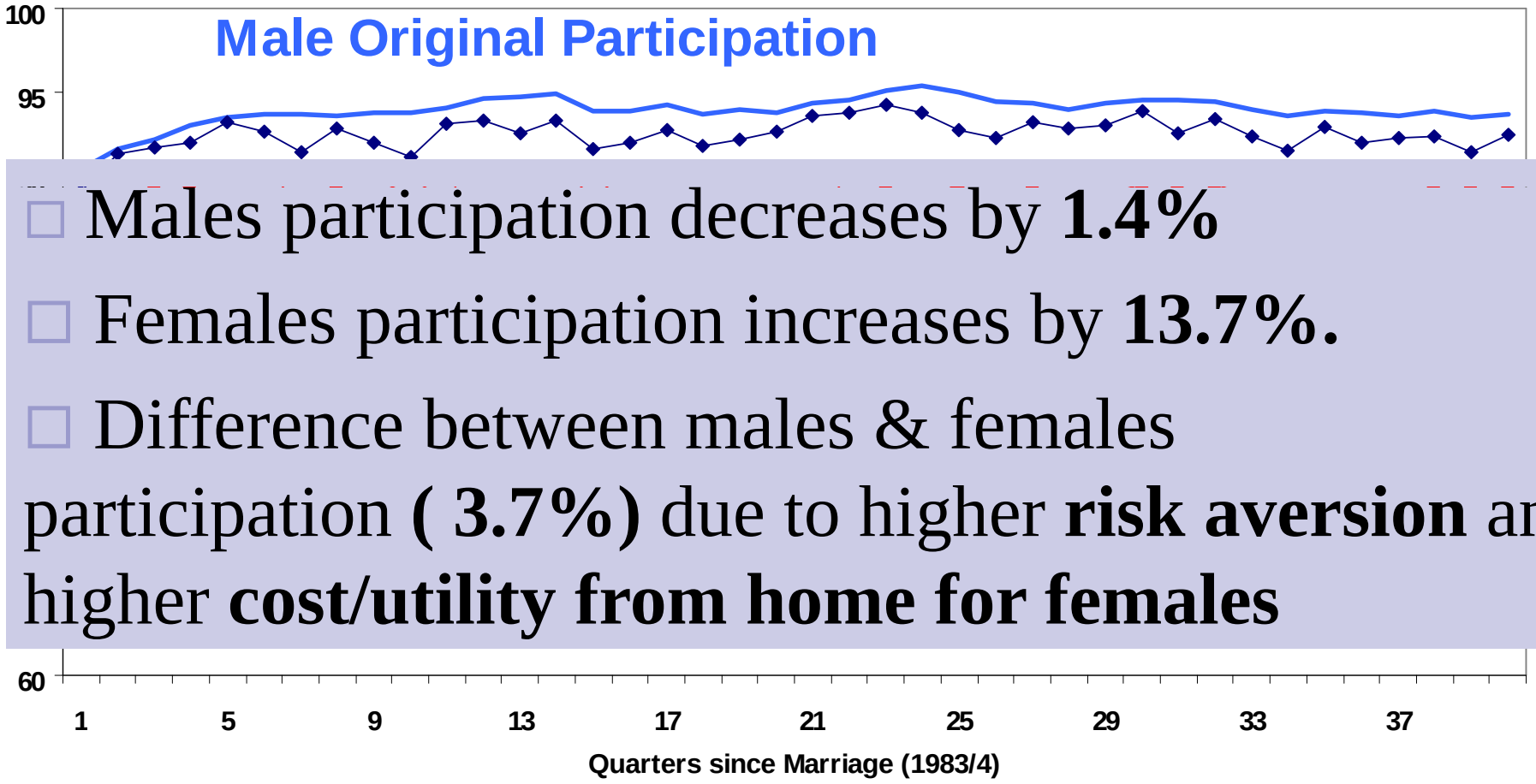
- **Posterior probability** of **M** family is:
  - **Negatively** correlated with: husband age at wedding, number of children, husband is black or Baptist.
  - **Positively** correlated with: couples education, wife age at wedding; husband is white, Catholic; potential divorce.

Counterfactual: 100% of Families are Modern



- Increase of female participation ~ **5%**
- No impact on males
- Participation difference from males ~ **10%.**

Counterfactual: Full Equality - 100% of Families are Modern; Equal Wages & Job Offers for Males and Females



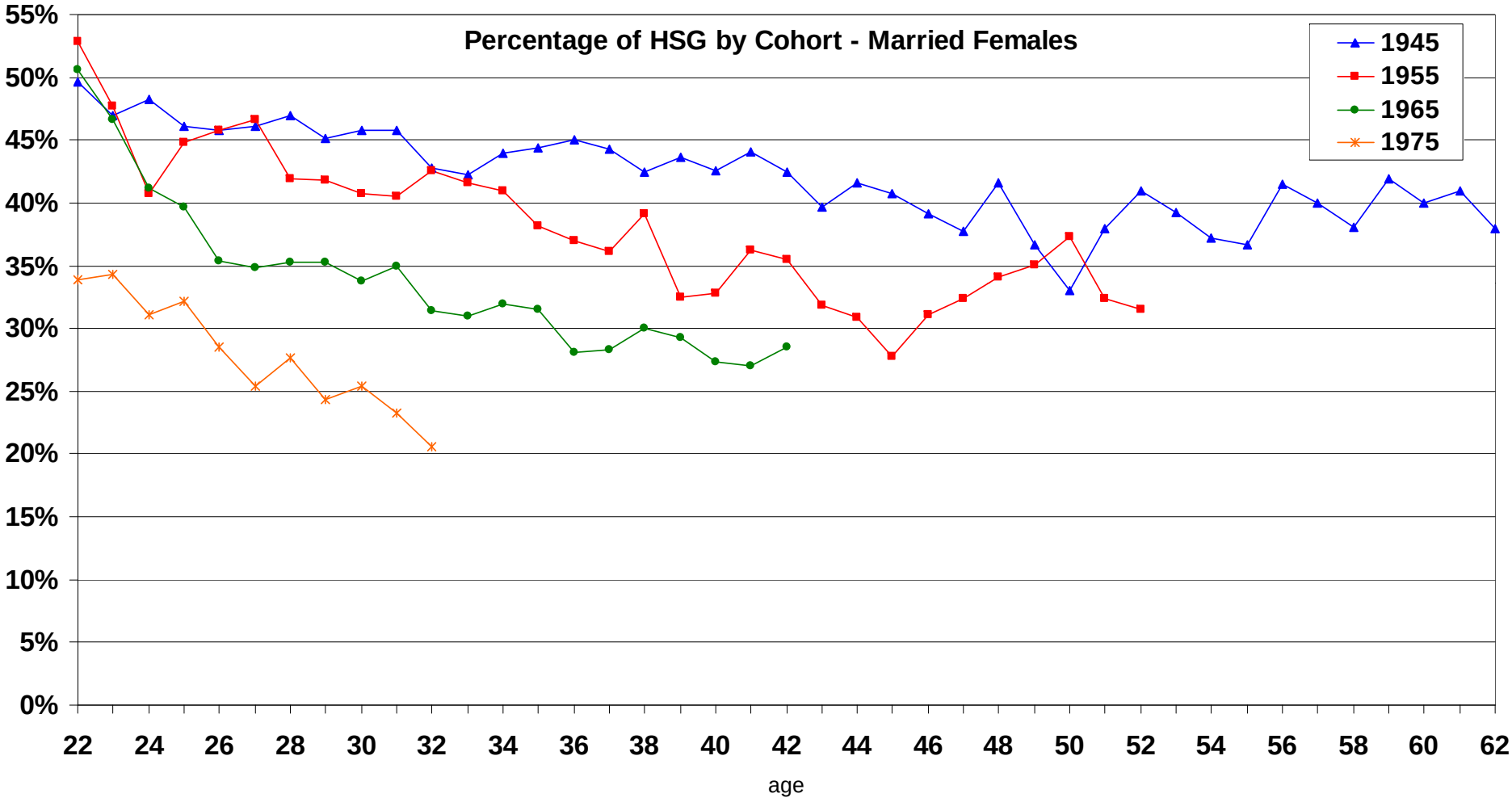
- Males participation decreases by **1.4%**
- Females participation increases by **13.7%.**
- Difference between males & females participation **(3.7%)** due to higher **risk aversion** and higher **cost/utility from home for females**

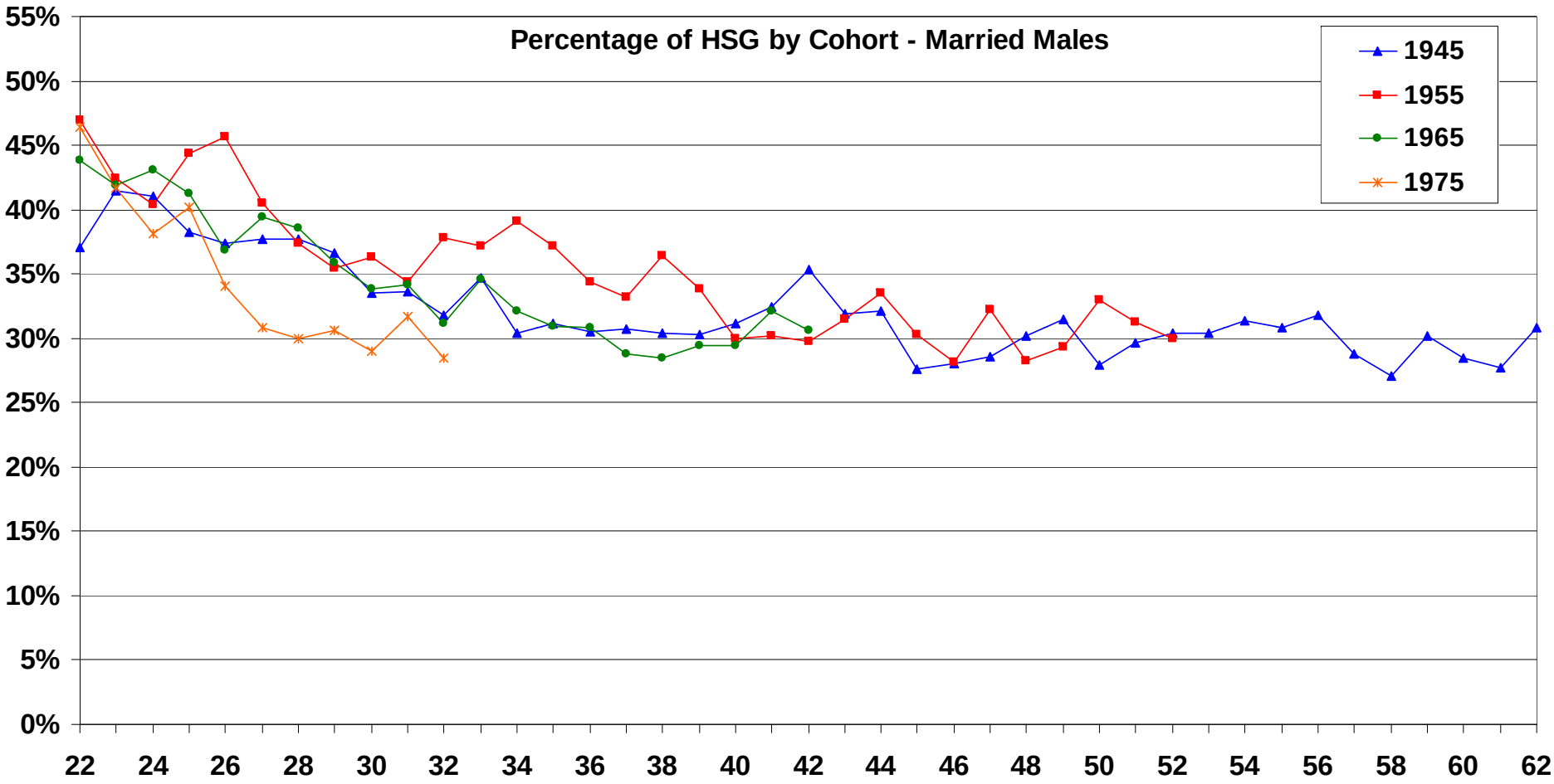
Summary of results

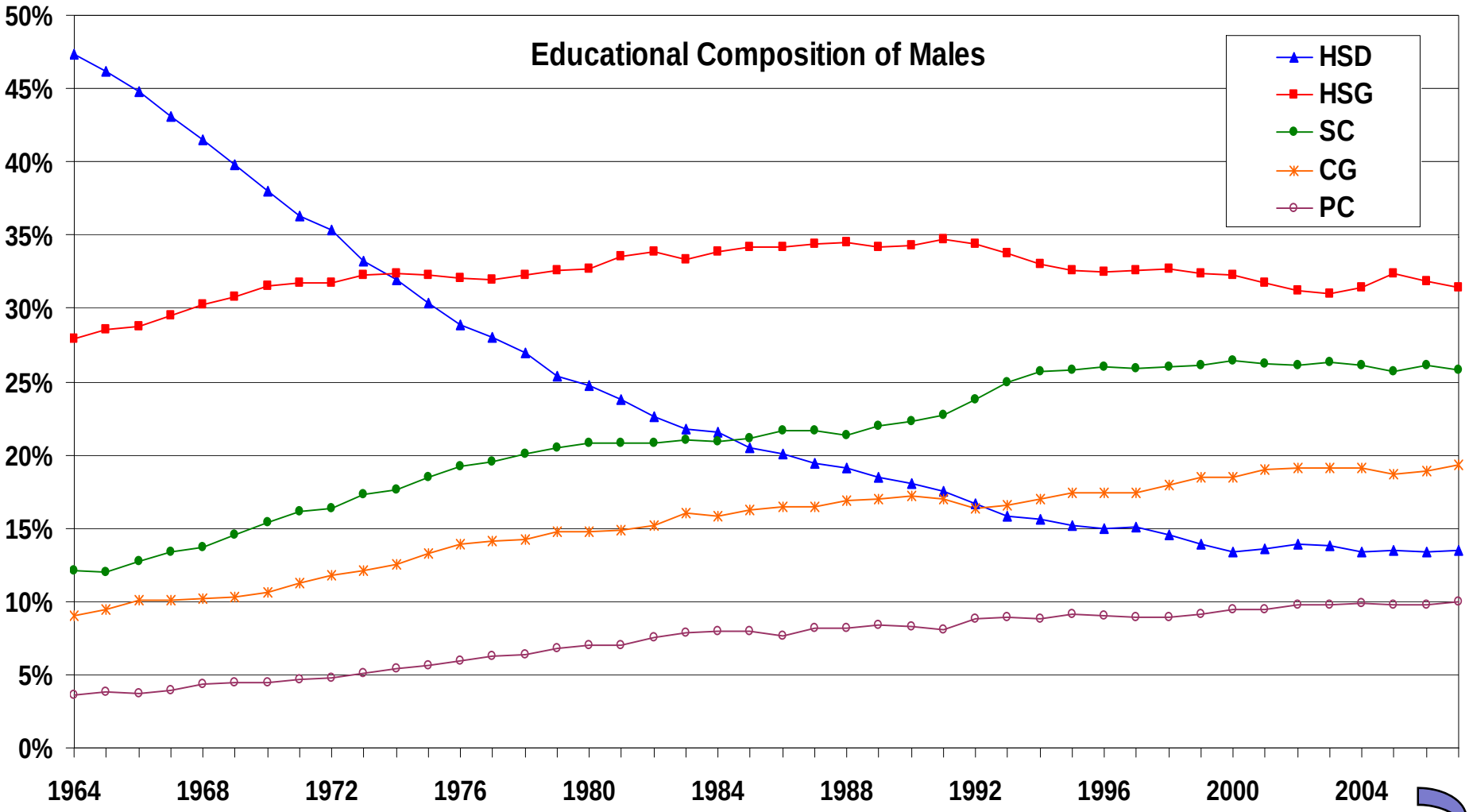
- Education – **50%** of increase in Married FE
- Other – **25-35%** of increase in Married FE
- Household *game model* for change in **Social Norms (T and M families)** can account to large change in Married FE – **5% to 10%**

Concluding remarks

- The two examples demonstrate the gains from using **Stochastic Dynamic Discrete models:**
  - *Dynamic selection method, rational expectations, and cross-equations restrictions are imposed*
  - *Accounting for alternative explanations for rise in US Female Employment*
- **Dynamic couples game models** are the framework for ***future*** empirical labor supply


Percentage of HSG by Cohort - Married Females
1945
1955
1965
1975
55%
50%
45%
40%
35%
30%
25%
20%
15%
10%
5%
0%
22
24
26
28
30
32
34
36
38
40
42
44
46
48
50
52
54
56
58
60
62
age


Percentage of HSG by Cohort - Married Males
1945
1955
1965
1975
55%
50%
45%
40%
35%
30%
25%
20%
15%
10%
5%
0%
22
24
26
28
30
32
34
36
38
40
42
44
46
48
50
52
54
56
58
60
62


Educational Composition of Males
HSD
HSG
SC
CG
PC
50%
45%
40%
35%
30%
25%
20%
15%
10%
5%
0%
1964
1968
1972
1976
1980
1984
1988
1992
1996
2000
2004

Appliances in U.S. Households, Selected Years, 1980-2001 (Percentage)

	Survey Year								
	1980	1981	1982	1984	1987	1990	1993	1997	2001
Clothes Dryer	47	45	45	46	51	53	57	55	57
Clothes Washer	74	73	71	73	75	76	77	77	79
Microwave	14	17	21	34	61	79	84	83	86
Dishwasher	37	37	36	38	43	45	45	50	53

Logistics form for probability of employment, children, marriage and divorce:

$$\Pr\left(\phi\left(\Omega_t, P_t\right)\right) = \frac{\exp\left(\phi\left(\Omega_t, P_t\right)\right)}{1+\exp\left(\phi\left(\Omega_t, P_t\right)\right)}$$

Job Offer Probability

(function of: constant, schooling, experience and previous state):

$$\left(\phi\left(\Omega_t, P_t\right)\right) = \rho_0 + \rho_{11} \cdot HSG + \rho_{12} \cdot SC + \rho_{13} \cdot CG + \rho_{14} \cdot PC + \rho_2 \cdot K_{t-1} + \rho_2 \cdot K^2{}_{t-1} + \rho_3 \cdot P_{t-1}$$

Marriage Probability

(function of: constant, age, schooling, previously divorced):

$$\left(\phi\left(\Omega_t, P_t\right)\right) = m_0 + m_1 \cdot age + m_2 \cdot age^2 + m_3 \cdot S + m_4 \cdot D$$

Probability of Having a New Child

(function of: constant, age, schooling, marital status, number of children and previous state):

$$\left(\phi\left(\Omega_t, P_t\right)\right) = c_0 + c_1 \cdot age + c_2 \cdot age^2 + c_3 \cdot S + c_4 \cdot M_{t-1} + c_5 \cdot N_{t-1} + c_6 \cdot N^2{}_{t-1} + c_7 \cdot P_{t-1}$$

Divorce Probability

(function of: constant, years of marriage, schooling, number of children, husband wage and previous state):

$$\left(\phi\left(\Omega_t, P_t\right)\right) = d_0 + d_1 \cdot y\_marriage + d_2 \cdot y\_marriage^2 + d_3 \cdot S + d_4 \cdot N_{t-1} + d_5 \cdot y^H{}_{t-1} + d_6 \cdot P_{t-1}$$

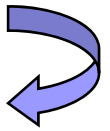
Estimated Parameters

Job offer probability Parameter	
Constant	2.412 (0.00)
Return to Experience	-0.001 (0.00)
Return to Experience^2	0.0007 (0.00)
Previous State	0.0065
Return to HSG	0.007 (0.00)
Return to SC	0.223 (0.00)
Return to CG	0.486 (0.00)
Return to PC	0.821 (0.00)

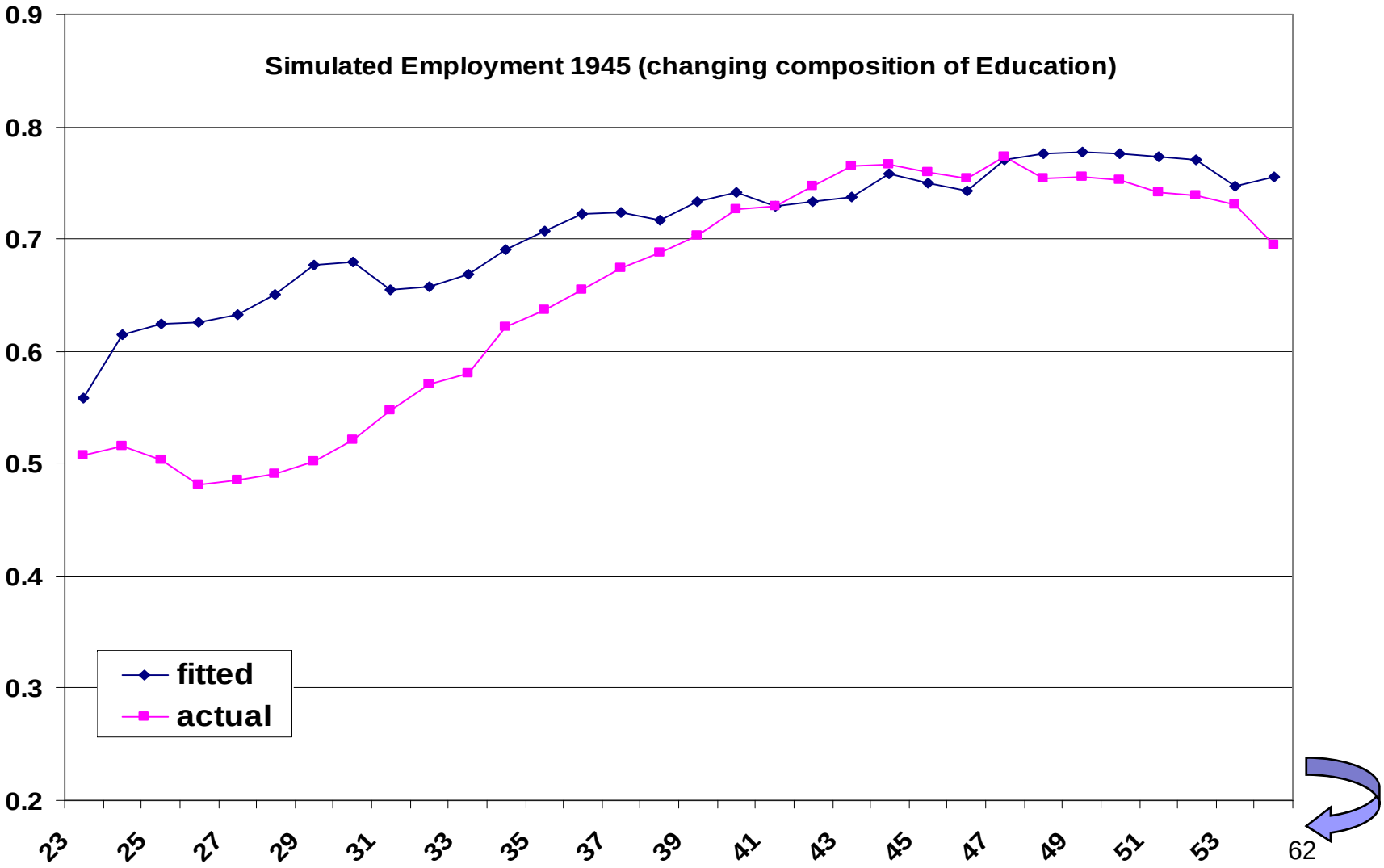
Marriage Parameter	
Constant	2.412 (0.00)
Return to Age	-0.001 (0.00)
Return to Age^2	0.0007 (0.00)
Divorce	0.0065
Return to Schooling	0.007 (0.00)

Divorde Parameter	
Constant	2.412 (0.00)
Years of marriage	-0.001 (0.00)
Years of marriage ^2	0.0007 (0.00)
Number of children	0.0065
Previous state	0.007 (0.00)
Schooling	0.223 (0.00)
Husband Wage	0.486 (0.00)

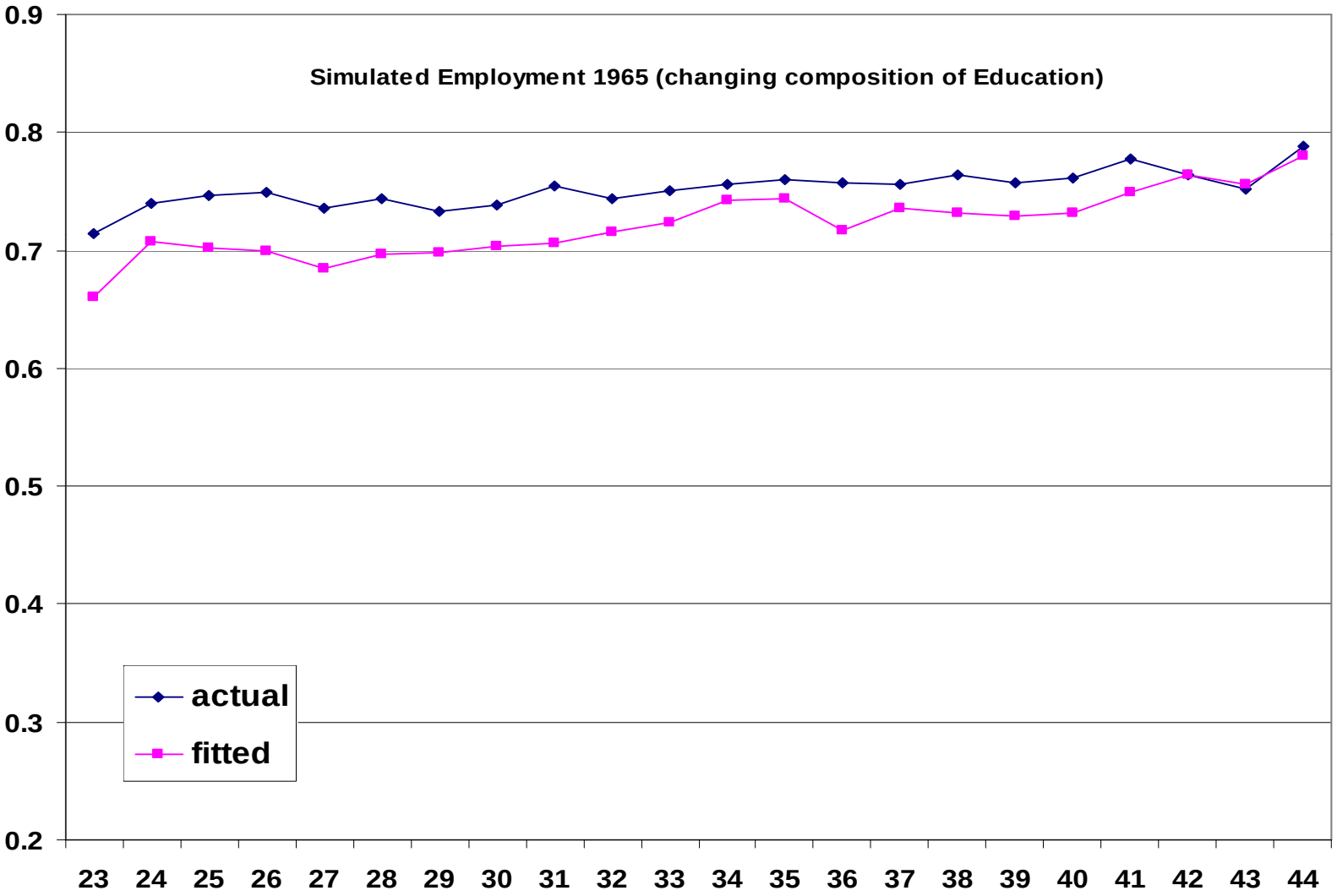
Children Parameter	
Constant	2.412 (0.00)
Return to Age	-0.001 (0.00)
Return to Age^2	0.0007 (0.00)
Number of children	0.0065
Number of children^2	0.007 (0.00)
Previous State	0.223 (0.00)
Marital Status	0.486 (0.00)
Schooling	0.821 (0.00)



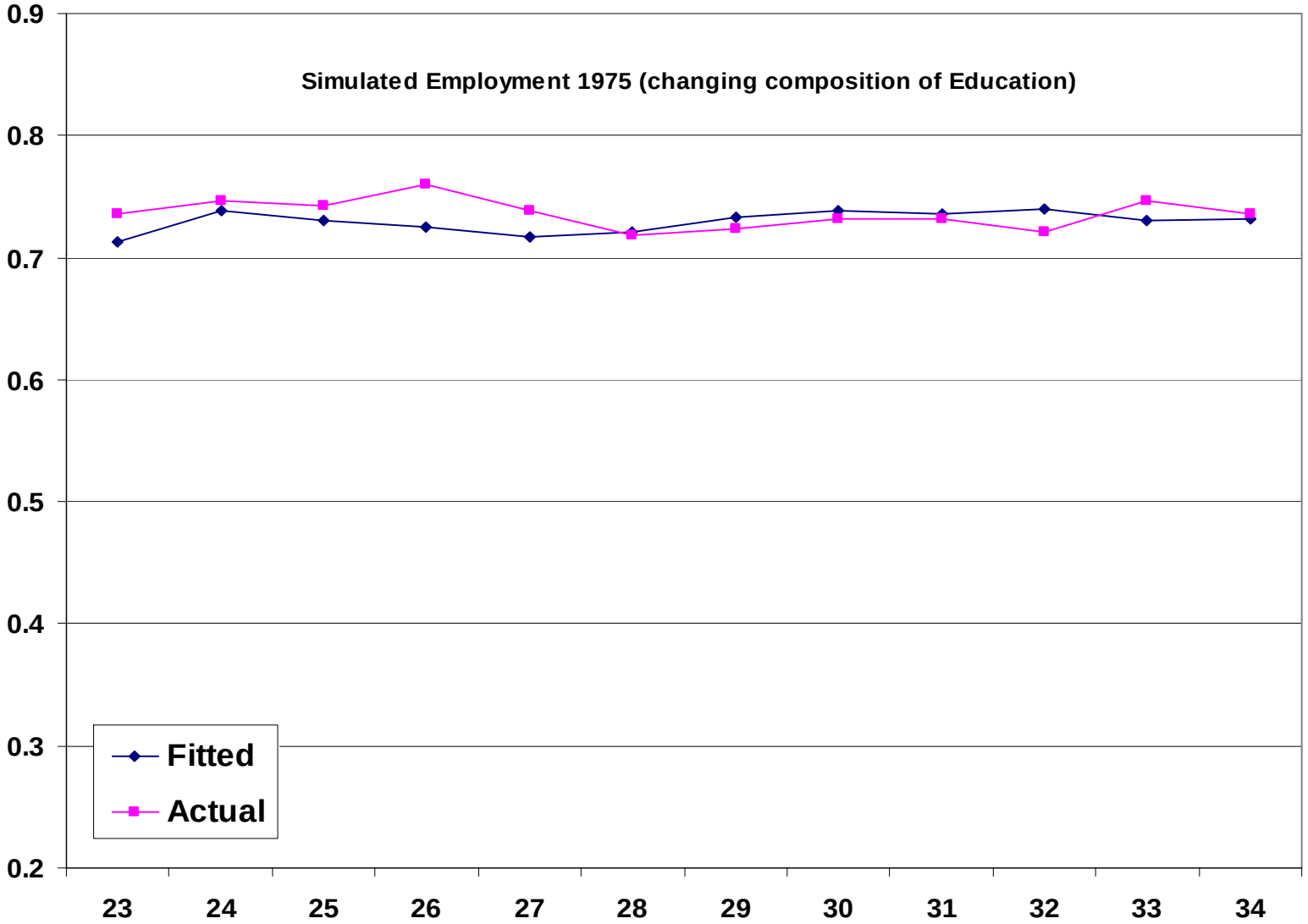
Simulation 1945

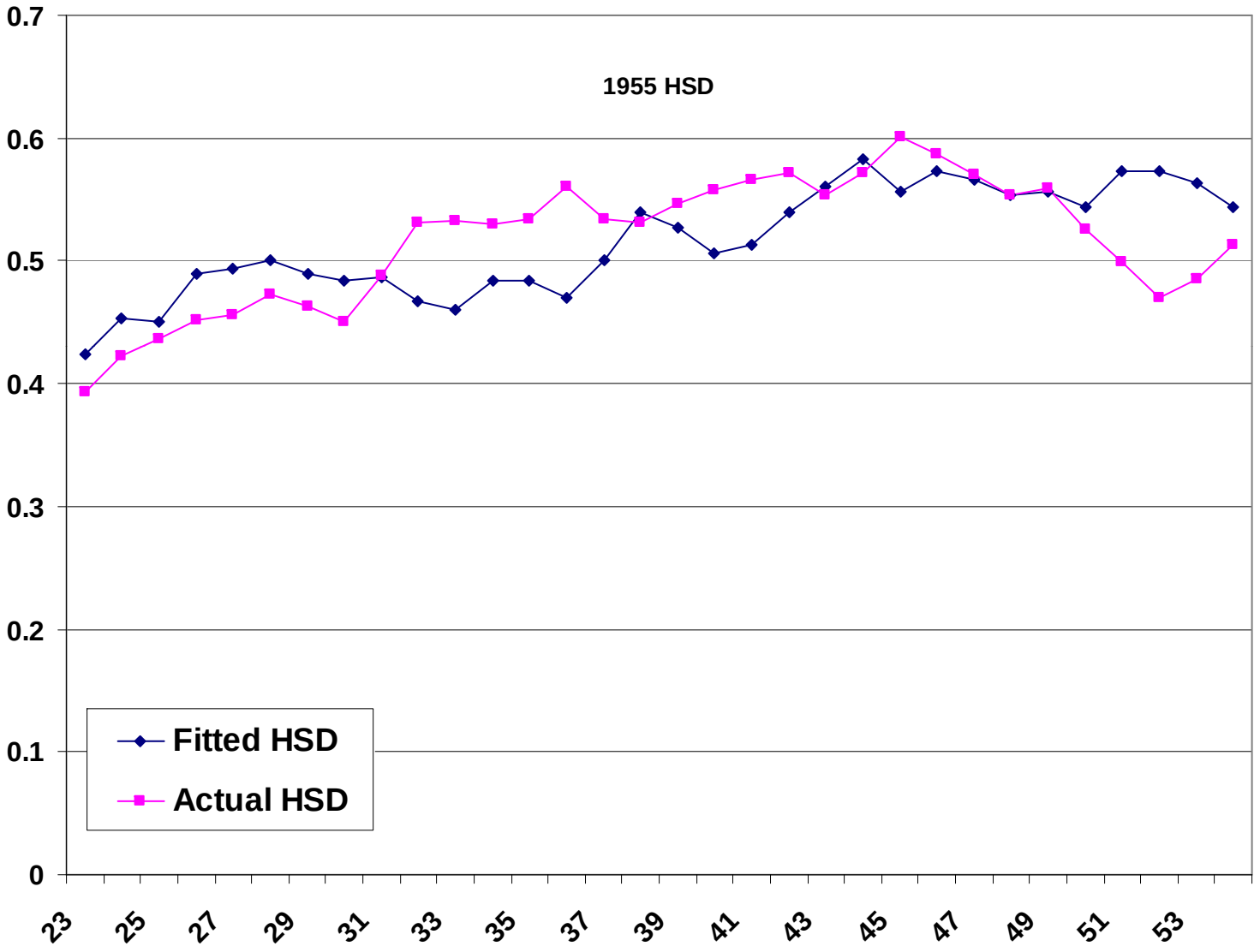


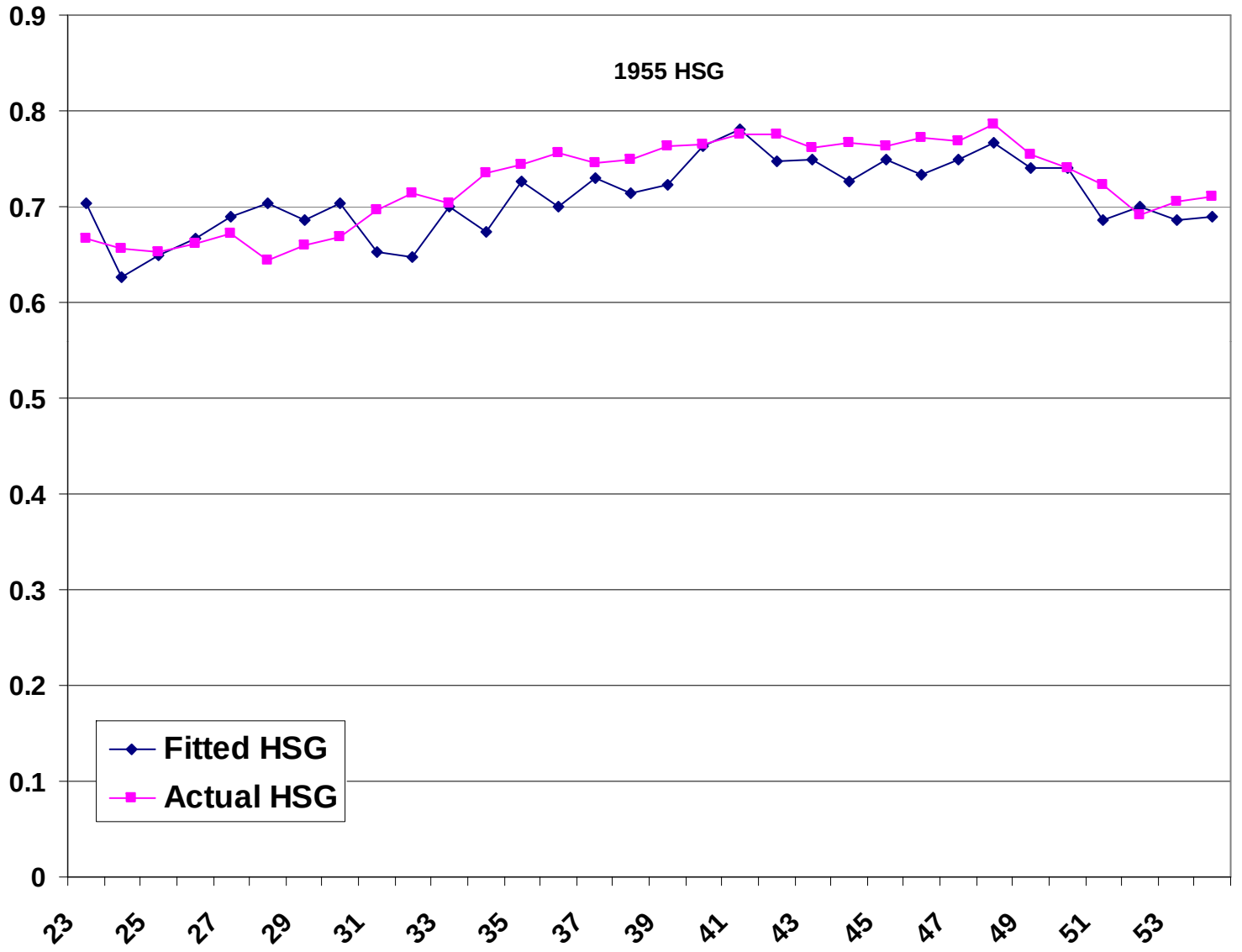
Simulation 1965

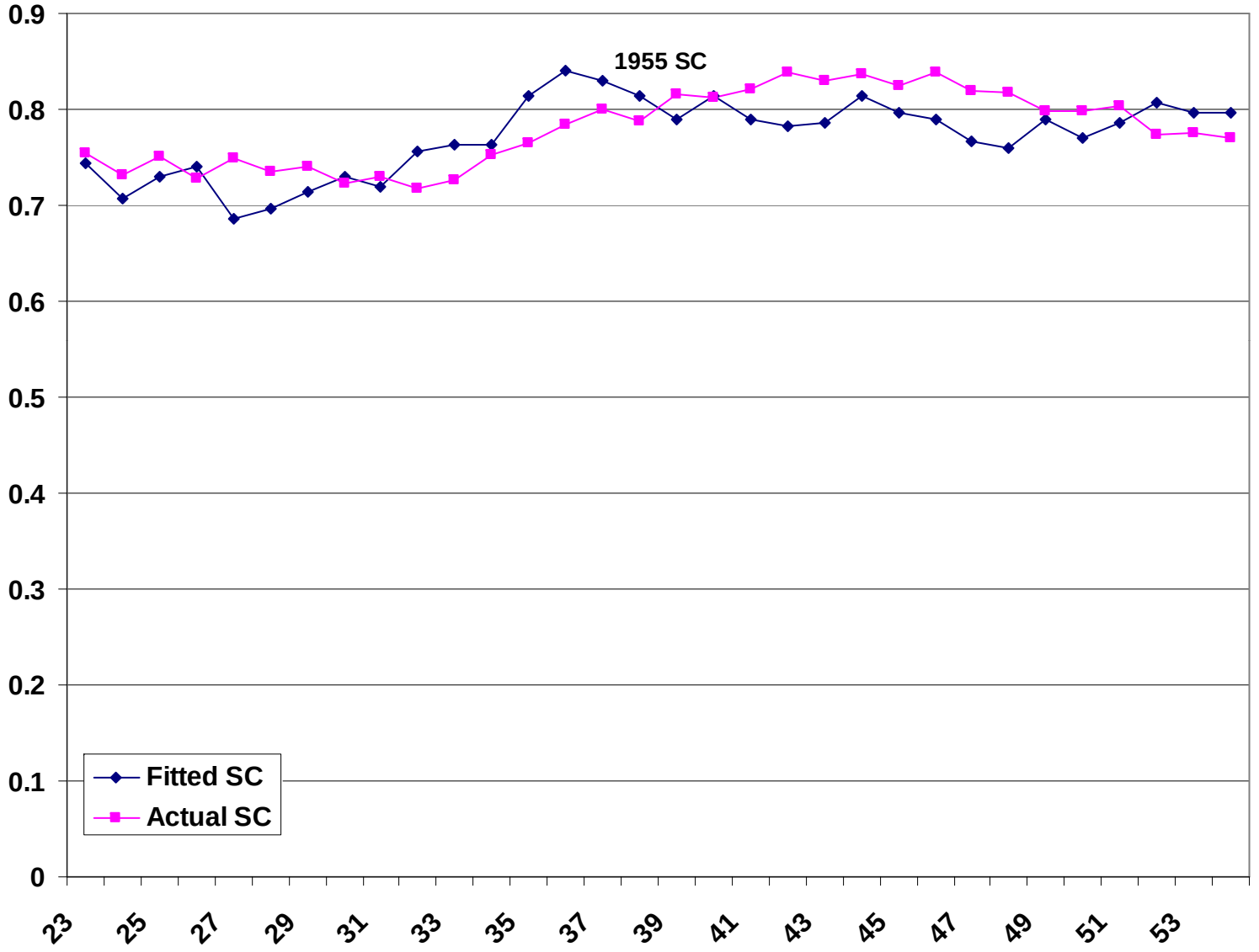


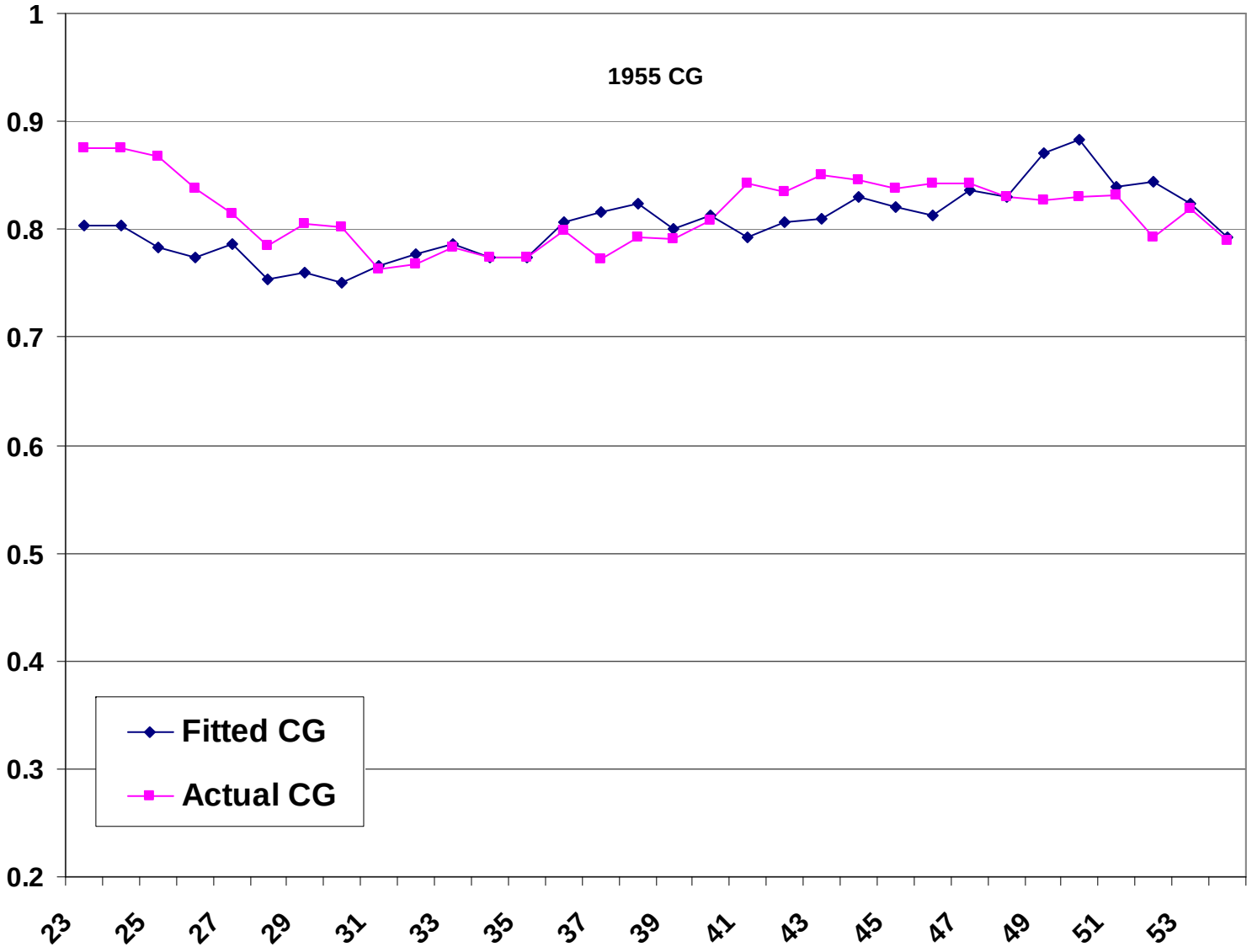
Simulation 1975

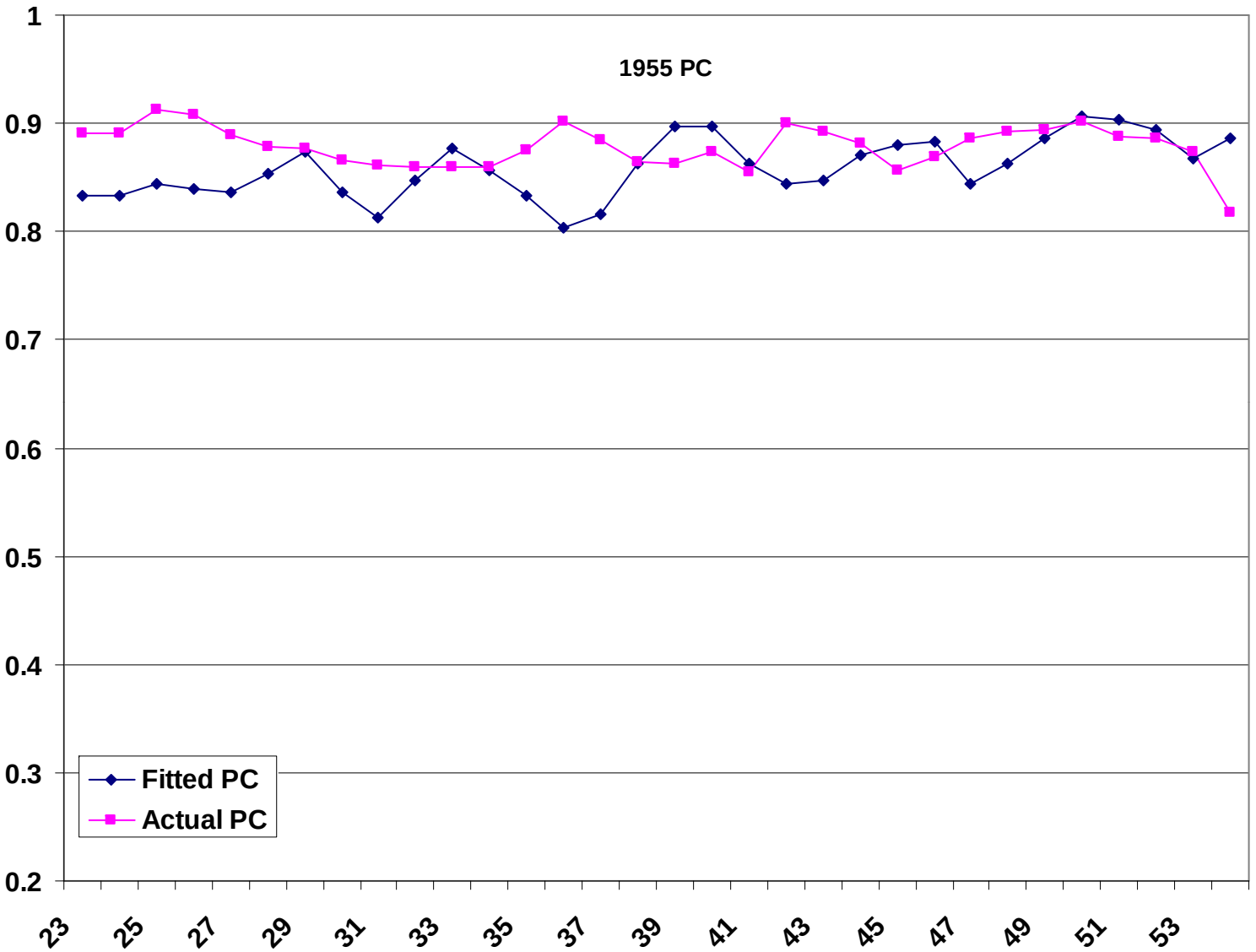



1955 HSD
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
23
25
27
29
31
33
35
37
39
41
43
45
47
49
51
53
Fitted HSD
Actual HSD


1955 HSG
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
23
25
27
29
31
33
35
37
39
41
43
45
47
49
51
53
Fitted HSG
Actual HSG


1955 SC
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
23
25
27
29
31
33
35
37
39
41
43
45
47
49
51
53
Fitted SC
Actual SC


1955 CG
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
23
25
27
29
31
33
35
37
39
41
43
45
47
49
51
53
Fitted CG
Actual CG


1955 PC
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
23
25
27
29
31
33
35
37
39
41
43
45
47
49
51
53
Fitted PC
Actual PC