Occupational Mobility and Consumption Insurance*

Job Market Paper

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Abstract

Evidence from the Consumer Expenditure Survey shows that over the 1980-1992 period, between-group consumption inequality for occupation groups has remained stable, while within-group consumption inequality has increased. Meanwhile, evidence from the Panel Study of Income Dynamics shows that involuntary occupational mobility has increased. Within a model with limited commitment, I show that under certain conditions a rise in occupational mobility increases the desire for insurance for low income individuals more than that of high income individuals, thereby increasing consumption inequality within that group. A calibrated version of the model shows that the rise in occupational mobility can account for up to 90% of the rise in within-group inequality observed in the data.

Journal of Economic Literature Classification Numbers: E21, D31, G22

Keywords: Consumption Inequality; Occupational Mobility; Limited Enforcement

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1 Introduction

In this paper, I explore the impact of the increase in occupational mobility on between- and within-group consumption inequality over the 1980-1992 period in the United States. Between- and within-group consumption inequality can be interpreted as a measure of consumption insurance against involuntary occupation switch risk and within-group (residual) income risk, respectively. Evidence from the Consumer Expenditure Survey (CEX) over the 1980-1992 period shows that between-group consumption inequality has remained stable, while within-group consumption inequality has increased. Meanwhile, evidence from the Panel Study of Income Dynamics (PSID) shows that involuntary occupational mobility has increased over the same period. I study the relationship between occupational mobility and between- and within-group consumption inequality in a model with limited enforcement of contracts where agents face involuntary occupation switches as well as within-group income shocks. I show that the model with a rise in occupational mobility can account for up to 90% of the rise in within-group inequality observed in the data.

In the literature on consumption insurance, a lot of effort has been devoted to measuring and accounting for the extent to which households insure their idiosyncratic income risk. The empirical measure of idiosyncratic income shocks commonly used in this literature consists of deviations from the mean income of the set of households that share the same characteristics such as head’s age and education. However, the analysis using this empirical measure often fails to take into account the possibility that households’ ability to insure against the measured idiosyncratic income shocks might depend on their ability to insure against other income risk such as group-specific income shocks or between-group mobility. This paper, by contrast, explicitly takes account of two types of idiosyncratic income shocks, namely involuntary occupational mobility and within-group income shocks, and shows that a rise in involuntary occupational mobility affects a measure of consumption insurance against within-group income shocks.

I use data from the PSID to document occupational mobility of household heads

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1 A notable exception is Attanasio and Davis (1996) who examine consumption insurance against relative wage movements among birth cohorts and education groups. The authors find a sharp rejection of the full insurance hypothesis for this type of between-group consumption insurance.
over the 1980-1992 period. Since it is extremely costly to incorporate many occupation groups in a quantitative model, I use only two occupations in defining groups: professional/managerial specialty and others. Following Kambourov and Manovskii (2006), I define occupational mobility as the fraction of currently employed individuals who report a current occupation different from their previously reported occupation. With this definition, average occupational mobility between professional/managerial specialty and other occupation groups for the 1980-1985 period is 8.0%, which increases to 10.2% for the 1986-1992 period. In this paper, however, I focus my attention on the impact of occupation switches that are exogenous to individuals and thus can be considered as shocks. In reality, individuals can choose to change their occupation or anticipate changes in their occupation. For example, individuals at firms may know the timing of their promotion. Thus, as a proxy for exogenous occupation switch risk, I examine occupation switches due to involuntary job losses defined as a plant closing, an employer going out of business, and a layoff. Occupational mobility due to involuntary job losses increases from 0.6% in 1980-1985 to 0.8% in 1986-1992.

In order to measure the effects of involuntary occupational mobility on consumption insurance, I decompose income and consumption inequality into observable and unobservable components. For the observable components, I focus my attention on inequality accounted for by occupation groups and refer to this component as between-group inequality. I refer to the component of overall inequality not accounted for by observable attributes as within-group inequality.

Using income data from the PSID and expenditure data from the CEX, I document changes in between- and within-group income and consumption inequality between the periods 1980-1985 and 1986-1992. Between these two periods, between-group income inequality mildly increases, whereas within-group income inequality exhibits a much larger increase. Meanwhile, between-group consumption inequality remains stable, while within-group consumption inequality increases.

To study the conceptual link between occupational mobility and between- and within-group consumption inequality, I construct a stylized pure exchange economy with limited enforcement of contracts along the lines of Kehoe and Levine (1993). I assume that there are two occupation groups, one with high mean income and one with low mean income. In addition, each occupation group has two idiosyncratic
income states so that there are a total of four income states. There are four agents, each of whom faces two types of idiosyncratic income shocks, namely involuntary occupation switches and within-group income shocks. This simple structure allows me to analytically characterize constrained efficient symmetric stationary Markov allocations, as well as to isolate a simple mechanism through which a rise in occupational mobility leads to an increase in within-group consumption inequality.

When occupational mobility increases, the value of autarky (agent’s outside option) for the low-income agent decreases more than that for the high-income agent in the high income occupation, in the presence of persistent within-group income shocks. According to the characterization result, both (all) of the agents in the high income occupation are constrained, which means that the agents contribute some of their resources to risk sharing. Given the changes in the value of autarky, the low-income agent in the high income occupation increases his/her contributions to risk sharing more than the high-income agent. As a result, consumption inequality within the high income occupation increases.

The next step is to study whether this mechanism is sufficiently important to explain the empirical facts described above. To do so, I use a model of a production economy with limited enforcement of contracts and a continuum of agents. In this environment, I calibrate the model and compute a stationary competitive equilibrium with solvency constraints that are not too tight along the lines of Alvarez and Jermann (2000) for the periods 1980-1985 and 1986-1992.

Under the benchmark parameter values, the model accounts for 33.4% of the observed increase in within-group consumption inequality. Under an alternative calibration, in which occupational mobility is calibrated to match the level of between-group consumption inequality in the periods 1980-1985 and 1986-1992, the model accounts for 91% of the increase in within-group consumption inequality observed in the data. In a version of this model which abstracts from occupational mobility, Krueger and Perri (2006) find that their model largely understates the increase in within-group consumption inequality. Consistent with the finding, the model without occupational mobility only accounts for 14.2% of the increase in within-group consumption inequality.

The rest of the paper is organized as follows. Section 2 presents empirical evi-
idence of occupational mobility and between- and within-group income and consumption inequality. Section 3 examines constrained efficient allocations in a stylized pure exchange economy with limited enforcement of contracts. Section 4 presents a quantitative model and describes my calibration strategy. Section 5 reports the main results. Section 7 concludes.

2 Empirical Evidence

2.1 Data

I use data from the PSID since the survey provides detailed panel data on respondent’s income and characteristics such as age, race, sex, education, and occupation. It is imperative to have the panel dimension in order to estimate occupational mobility and income process. However, the PSID does not collect detailed information on household expenditure, collecting only food and some housing-related (rent and property tax) expenditure on a regular basis. In order to address this problem, I report changes in consumption inequality using data from the CEX where more detailed expenditure information is available.

2.1.1 PSID

My benchmark PSID sample runs from 1980 to 1992. It starts in 1980 since consumption data from the CEX are not available prior to 1980. It ends in 1992 as many other studies on consumption insurance using the PSID data. The unit of analysis is a household. I define after-tax income as after-tax labor earnings plus transfers. The PSID stopped imputing income taxes in 1992, which affects the 1991 and 1992 income measures. To construct income measures for those years, I use the Cross-National Equivalent File that imputes federal and state income taxes for the PSID data using

\(^2\)It will be possible to extend the sample period to 1997 since the PSID have completed data quality checks on the public release for 1994-1997. However, because the PSID changed frequency of survey interviews from yearly to once every two years in 1997, it is not possible to extend the analysis to the post 1997 period in a consistent manner.

\(^3\)Since the PSID retrospectively asks the previous year’s income in each interview, income data in the 1993 file, for example, refer to income for 1992.
the National Bureau of Economic Research *TAXSIM* package.4

I restrict the original sample to households with reliable data for labor earnings and main characteristics including education and occupation. Table 12 in Appendix A reports the step-by-step selection of my PSID sample. First, I exclude households experiencing major family composition change, in particular changes in the head or the spouse, over the sample period. It is because income reported in year $t$, for example, may not correspond to household characteristics reported in $t-1$ if the family composition changes dramatically from $t-1$ to $t$. I exclude households with female heads. I exclude households with missing report on region of residence, education, and occupation (conditional on being employed) as they are necessary to construct between- and within-group inequality in the next section. I exclude households with topcoded income and food expenditures assigned by the PSID. I also eliminate households with income and consumption outliers.5 The PSID consists of two different subsamples: the first is representative of the US population; the second is a supplementary low income subsample (SEO sample). For the analysis below, I exclude the SEO sample. To focus on households whose head is of working age, I exclude households whose head is less than 30 or more than 65 years of age.

In what follows, I examine how households insure against income changes due to occupational mobility and other changes not explained by observable characteristics. Due to computational burden in quantitative exercises, I aggregate occupation groups into two broad groups: 1. professional, technical workers, and managers; 2. the rest. For the benchmark analysis, I exclude households whose head is classified as ‘armed services and protective service workers’ in the one-digit occupation code. This occupation is likely to feature distinct occupational mobility and income shocks, and accounts for less than 3% of the sample. I also exclude households whose head is unemployed for 5 consecutive years. This criterion drops only 2% of the remaining sample. As a result, my PSID sample contains 2,881 households and 22,299 observations.

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4The *Cross-National Equivalent File* is created and maintained by the Department of Policy Analysis and Management at Cornell University.

5Following Blundell et al. (2006), I consider income and consumption as an outlier if an annual income is below 100 dollars or below total food expenditure, if income growth is above 500 percent or below -80 percent, or if total food expenditure is zero or missing.
2.1.2 CEX

My CEX sample runs from 1980 to 1992. I impose basically the same sample selection as the one described above. Table 13 describes the step-by-step sample selection. Table 14 shows that the mean characteristics of the CEX sample closely match those of the PSID sample. Data in CEX are monthly in that expenditure, for example, refers to the last three months from the point of each interview. Following Blundell et al. (2006), I assign an observation to a given year if the last interview of that household is conducted between July of that year and June of the following year.

2.2 Occupational mobility

In this section, I document household head’s occupational mobility for the 1980-1985 period and the 1986-1992 period, using PSID data. As mentioned before, computational considerations lead me to use only two occupations in defining groups: professional/managerial specialty and others. For those who are unemployed at the point of interview, I refer to their most recent occupation by exploiting PSID’s long panel dimension. A timing issue arises for changes in occupation. Although people may change their job at any time of the year, for the benchmark analysis below, I refer to an occupation at the time of the interview as the status for the whole interview year. Furthermore, I only consider respondent’s main occupation even though people may have multiple jobs at a time.

Tables 1 and 2 report the transition matrix of household head’s mobility over the occupation groups for two subperiods: 1980-1985 and 1986-1992. I compute transition probabilities over the sample period as follows. Take the transition from Group 1 to Group 2, for example. First, compute the fraction of household heads making the transition in each year. Then calculate the average fraction over the sample period. I take the average fraction as the transition probability for the transition from Group 1 to Group 2. I repeat the same computation for all transitions.

I consider two concepts of occupational mobility, labelled Mobility 1 and Mobility 2. Mobility 1 counts all the household heads who change their occupation. However, involuntary switches are of more interest when considering occupational mobility as

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6 More than 80 percent of the interviews were conducted from March to May for 1980-1992.
income risk. The PSID provides information on the reason why a household head left his previous job, which is categorized as follows: 1. company folded/changed hands/moved out of town, employer died/went out of business; 2. strike, lockout; 3. laid off, fired; 4. quit, resigned, retired, pregnant, needed more money, just wanted a change in jobs, was self-employed; 5. no previous job; 6. promotion; 7. other, transfer, any mention of armed services; 8. job was completed, seasonal work, was a temporary job. Following Cochrane (1991), I consider job loss due to 1 - 3 to be involuntary. Then Mobility 2 only counts occupation switches due to involuntary job loss. Hence, note that ‘staying in the same group’ in Mobility 2 contains those who change their occupation voluntarily as well as those who stay in the same group.

Table 1 reports a transition matrix for Mobility 1 for two subperiods, 1980-1985 and 1986-1992, while Table 2 reports a transition matrix for Mobility 2. These two tables show that household heads do change their occupation over time. Although Mobility 2 features much less occupation switches, the numbers are still substantial (0.9% from Group 1 to Group 2, 0.6% from Group 2 to Group 1 for 1980-1992). In both tables, transition probabilities increase between the periods 1980-1985 and 1986-1992. Following Kambourov and Manovskii (2006), define occupational mobility as the fraction of currently employed individuals who report a current occupation different from their most previous report of an occupation. For Mobility 1, the value increases from 8.0% to 10.2%. For Mobility 2, the value increases from 0.6% to 0.8%.

Table 1: Mobility 1 (1980-1985, 1986-1992)

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<tr>
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<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>0.9121</td>
<td>0.0879</td>
</tr>
<tr>
<td>2</td>
<td>0.0766</td>
<td>0.9234</td>
</tr>
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</table>
Table 2: Mobility 2 (1980-1985, 1986-1992)

<table>
<thead>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>0.9920</td>
<td>0.9900</td>
</tr>
<tr>
<td>2</td>
<td>0.0057</td>
<td>0.9935</td>
</tr>
</tbody>
</table>

2.3 Between- and within-group inequality

To measure effects of occupational mobility on consumption insurance in detail, I decompose income and consumption inequality into observable and unobservable components. Furthermore, for the observable components, I focus my attention on inequality accounted for by occupation and refer to this component as between-group inequality. I refer to the component of overall inequality not accounted for by observable attributes as within-group inequality. In this section, I document the evolution of between- and within-group income and consumption inequality over the 1980-1992 period, using income data from the PSID and expenditure data from the CEX.\(^7\)

My income measure is after-tax labor income plus transfers. As for consumption, I use the following two measures: 1. expenditures on nondurables, services, and small durables (such as household equipment) plus imputed services from housing and vehicles (ND+); 2. nondurable plus semidurable (clothing and footwear) expenditure (ND). The ND+ measure includes imputed services from housing and vehicles, which is important as a measure of consumption. On the other hand, the ND measure does not involve imputation of consumption services from durables and is widely used in the literature.\(^9\) In this section, I also report consumption inequality using food expenditure (Food), another widely used measure of consumption. I deflate income and consumption measures by the relevant CPI and by an adult equivalence scale.

\(^{7}\) Using data only from the PSID, Section 7 documents the evolution of between- and within-group income inequality over the 1972-1992 period.

\(^{8}\) I follow Krueger and Perri (2006) to construct this ND+ measure.

\(^{9}\) Previous works using the ND measure include Attanasio and Weber (1995) and Blundell et al. (2006).
taken to be the square root of the family size.\textsuperscript{10} Income and consumption inequality are weighted by sample weights throughout this section.

I decompose income and consumption inequality into observable and unobservable components by a standard variance decomposition.\textsuperscript{11} First, I regress the logarithm of income and consumption on a constant, region of residence, marital status, the number of family members, the number of earners other than household head and spouse, household head’s age, household head’s and spouse’s (if present) race, education, and occupation. I allow coefficients to vary with time. Regression equations are as follows,

\begin{equation}
\ln y_{it} = \gamma_{0t}^{y} + z_{it}'\gamma_{1t}^{y} + d_{it}^{y}\alpha_{t}^{y} + u_{it}^{y}
\end{equation}

\begin{equation}
\ln c_{it} = \gamma_{0t}^{c} + z_{it}'\gamma_{1t}^{c} + d_{it}^{c}\alpha_{t}^{c} + u_{it}^{c}
\end{equation}

where $y_{it}$ and $c_{it}$ are, respectively, household $i$'s income and consumption at $t$, $d_{it}^{n}$ is the professional/manager dummy, and $z_{it}$ is a vector of the regressors except for a constant and the professional/manager dummy. The orthogonality of the OLS estimator implies

\begin{equation}
\text{Var}(\ln y_{it}) = \text{Var}(\gamma_{0t}^{y} + z_{it}'\gamma_{1t}^{y} + d_{it}^{y}\alpha_{t}^{y} + u_{it}^{y})
\end{equation}

\begin{equation}
\text{Var}(\ln c_{it}) = \text{Var}(\gamma_{0t}^{c} + z_{it}'\gamma_{1t}^{c} + d_{it}^{c}\alpha_{t}^{c} + u_{it}^{c})
\end{equation}

In this paper, instead of the quantitative contributions of all observable attributes to the overall inequality, I focus my attention on the contributions of occupation. Thus I measure between-group income and consumption inequality by $\text{Var}(d_{it}^{n}\alpha_{t}^{y})$ and $\text{Var}(d_{it}^{n}\alpha_{t}^{c})$. I measure within-group income and consumption inequality by $\text{Var}(u_{it}^{y})$ and $\text{Var}(u_{it}^{c})$.

### 2.3.1 Results

Figure 1 shows the evolution of between-group income and consumption inequality from its 1980 value, while Figure 2 shows the evolution of within-group income and consumption inequality. In both figures, the line with asterisk on data points represents consumption inequality (measured by ND+), and the line without asterisk

\textsuperscript{10}The square root of the family size is a widely used adult equivalence scale in the literature. See, for example, Blundell et al. (2006).

\textsuperscript{11}For example, see Katz and Autor (1999).
represents income inequality. As high-frequency variation in time series of income and consumption inequality is likely due to measurement error in the PSID and the CEX, I present average inequality for two sub-periods, 1980-1985 and 1986-1992 in Table 3.

Between the two periods, between-group income inequality mildly increases, while within-group income inequality exhibits a much larger increase. Meanwhile, in all measures of consumption (ND+, ND, Food), between-group consumption inequality remains stable, while within-group consumption inequality increases. Choice of a consumption measure matters for the magnitude of the increase in within-group consumption inequality. Within-group consumption inequality with CEX food expenditure increases the least of the four measures, while that with ND+ exhibits the largest increase. For all consumption measures, however, the increase in within-group consumption inequality is smaller than within-group income inequality.

![Figure 1: Between-Group](image1.png)

![Figure 2: Within-Group](image2.png)

3 Pure Exchange Economies

In this section, I examine a qualitative mechanism that links occupational mobility and between- and within-group consumption inequality in a stylized model of pure exchange economy. Key features of the model are two occupation groups with different mean income levels, two idiosyncratic income states within each occupation, and
Table 3: Between- and within-group inequality (1980-1992)

<table>
<thead>
<tr>
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<tbody>
<tr>
<td><strong>Between-group inequality</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Income (PSID)</td>
<td>0.0072</td>
<td>0.0116</td>
<td>0.0045</td>
</tr>
<tr>
<td></td>
<td>[0.0057, 0.0096]</td>
<td>[0.0096, 0.0140]</td>
<td></td>
</tr>
<tr>
<td>Consumption (ND+, CEX)</td>
<td>0.0040</td>
<td>0.0043</td>
<td>0.0003</td>
</tr>
<tr>
<td></td>
<td>[0.0033, 0.0061]</td>
<td>[0.0036, 0.0059]</td>
<td></td>
</tr>
<tr>
<td>Consumption (ND, CEX)</td>
<td>0.0034</td>
<td>0.0031</td>
<td>-0.0003</td>
</tr>
<tr>
<td>Consumption (Food, CEX)</td>
<td>0.0022</td>
<td>0.0020</td>
<td>-0.0002</td>
</tr>
<tr>
<td>Consumption (Food, PSID)</td>
<td>0.0035</td>
<td>0.0043</td>
<td>0.0008</td>
</tr>
<tr>
<td><strong>Within-group inequality</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Income (PSID)</td>
<td>0.2052</td>
<td>0.2320</td>
<td>0.0268</td>
</tr>
<tr>
<td></td>
<td>[0.1988, 0.2134]</td>
<td>[0.2205, 0.2358]</td>
<td></td>
</tr>
<tr>
<td>Consumption (ND+, CEX)</td>
<td>0.1286</td>
<td>0.1457</td>
<td>0.0171</td>
</tr>
<tr>
<td></td>
<td>[0.1232, 0.1338]</td>
<td>[0.1411, 0.1495]</td>
<td></td>
</tr>
<tr>
<td>Consumption (ND, CEX)</td>
<td>0.1310</td>
<td>0.1419</td>
<td>0.0109</td>
</tr>
<tr>
<td>Consumption (Food, CEX)</td>
<td>0.1467</td>
<td>0.1557</td>
<td>0.0090</td>
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<tr>
<td>Consumption (Food, PSID)</td>
<td>0.1815</td>
<td>0.1974</td>
<td>0.0159</td>
</tr>
</tbody>
</table>

Numbers in square brackets represent the 90 percent confidence interval of the corresponding number computed using a bootstrap procedure with 399 repetitions.
limited enforcement of contracts.

In Section 3.1, I set up a model with four agents and analytically characterize constrained efficient symmetric stationary Markov allocations (Propositions 1 and 2). Numerically computing constrained efficient symmetric stationary Markov allocations, I find that a rise in occupational mobility leads to an increase in within-group consumption inequality for the high-mean income group. In Section 3.2, I compute stationary constrained efficient allocations with a continuum of agents under various income distributions, generalizing the analysis in the four-agent model.

3.1 A model with four agents

I consider a pure exchange economy. Time is discrete. There are four agents, $i \in \{1, 2, 3, 4\}$. Every agent lives for infinitely many periods. There are two occupation groups, each of which consists of two agents. Every period, agents are endowed with perishable consumption goods. The endowments follow a stochastic process that I describe below.

There are two sources of income shocks in the model: occupational mobility and idiosyncratic income shocks. The process $y_t$ represents occupational mobility and $z_t$ represents idiosyncratic income shocks. Stochastic processes $y_t$ and $z_t$ are independent and both are symmetric stationary Markov chains with symmetric transition matrices. For all $t$, $y_t$ and $z_t$ take on only two values, $\{1, 2\}$ and $\{h, l\}$, respectively. Let $\rho$ and $\gamma$ denote the probability of staying in the same state for $y_t$ and $z_t$, respectively. Agents face idiosyncratic income shocks, that is, $0 < \gamma < 1$. For occupational mobility, I also consider the case with no such shocks, so $0 \leq \rho \leq 1$. Let $\theta_t = (y_t, z_t)$ and $\Theta = \{(y, z)|y \in \{1, 2\}, z \in \{h, l\}\}$: the set $\Theta$ is the range of $\theta_t$ for every $t$. Since $y_t$ and $z_t$ are independent, $\theta_t$ also follows a symmetric stationary Markov chain with a symmetric transition matrix denoted by $\Pi = (\pi(\theta'|\theta))$. Let $\theta^t = (y^t, z^t) = (y_0, \ldots, y_t, z_0, \ldots, z_t)$ denote a history up to period $t$ and $\pi(\theta^t)$ the probability of $\theta^t$. The distribution of $\theta_0$ is discrete uniform: $\pi(\theta_0) = 1/4$ for all $\theta_0 \in \Theta$.\(^\text{12}\) Let $\omega^t_i$ denote an endowment process of Agent $i$. The endowment process is adapted to the stochastic process $\theta_t$.

\(^\text{12}\)With the uniform distribution, agents are ex ante identical.
Furthermore, \( \omega_i^t(\theta^t) = \omega^i(\theta_t) \) and

\[
\begin{align*}
\omega^1 &= \omega_{1h}, \quad \omega^2 = \omega_{1l}, \quad \omega^3 = \omega_{2h}, \quad \omega^4 = \omega_{2l} \quad \text{if } \theta_t = (1, h) \\
\omega^1 &= \omega_{1l}, \quad \omega^2 = \omega_{1h}, \quad \omega^3 = \omega_{2l}, \quad \omega^4 = \omega_{2h} \quad \text{if } \theta_t = (1, l) \\
\omega^1 &= \omega_{2h}, \quad \omega^2 = \omega_{2l}, \quad \omega^3 = \omega_{1h}, \quad \omega^4 = \omega_{1l} \quad \text{if } \theta_t = (2, h) \\
\omega^1 &= \omega_{2l}, \quad \omega^2 = \omega_{2h}, \quad \omega^3 = \omega_{1l}, \quad \omega^4 = \omega_{1h} \quad \text{if } \theta_t = (2, l)
\end{align*}
\]

where \( \omega_{1h} > \omega_{2l} > 0 \) and \( \omega_{1l}, \omega_{2h} \in (\omega_{2l}, \omega_{1h}) \). Let us interpret the situation as follows. If \( \theta_t = (1, h) \), then Agent 1’s state is \((1, h)\), Agent 2’s state is \((1, l)\), and so on. The same interpretation applies to the cases of \( \theta_t = (1, l) \), \( \theta_t = (2, h) \), and \( \theta_t = (2, l) \). Agent 1 and Agent 2 \((3 \text{ and } 4)\) are in the same group and Agent 1 and Agent 3 \((2 \text{ and } 4)\) always have the same idiosyncratic income state. Assumption (1) guarantees that agents face occupational mobility when \( \rho \) is strictly less than one. That is, \((\omega_{1h} + \omega_{1l})/2 > (\omega_{2h} + \omega_{2l})/2\). Throughout this section, I do not impose any order between \( \omega_{1l} \) and \( \omega_{2h} \). Assumption (1) also guarantees that the amount of aggregate resources available in the economy remains constant over time.

Agents have identical preferences represented by the following standard expected utility function,

\[
\sum_{t=0}^{\infty} \sum_{\theta^t} \beta^t \pi(\theta^t) u(c^i_t(\theta^t)),
\]

where \( \beta \in (0, 1) \) is a discount factor. The period utility function \( u \) is given by

\[
u(c) = \begin{cases} 
\frac{c^{1-\sigma} - 1}{1-\sigma} & \text{if } \sigma \neq 1 \\
\ln c & \text{if } \sigma = 1.
\end{cases}
\]

Let \( V^i(\theta_t) \) denote the value of autarky of Agent \( i \) at \( \theta_t \), which is defined as follows:

\[
V^i(\theta_t) = \sum_{\tau=t}^{\infty} \sum_{\theta^\tau|\theta^t} \beta^{\tau-t} \pi(\theta^\tau|\theta^t) u(\omega^i_\tau) = \sum_{\tau=t}^{\infty} \sum_{\theta^\tau|\theta_t} \beta^{\tau-t} \pi(\theta^\tau|\theta_t) u(\omega^i_\tau).
\]

Note that \( V^i \) is well-defined since \( \theta_t \) and \( \omega^i_t \) are stationary Markov processes.

There is no storage technology or capital in this environment. Thus, the resource constraint reads:

\[
\sum_{i=1}^{4} c^i_t(\theta^t) \leq \sum_{\theta \in \Theta} \omega_\theta \quad (\forall t)(\forall \theta^t).
\]
Agents can walk away from contract. If agents do so, then they are excluded from any risk sharing arrangement. With no storage technology or capital, agents must revert to autarky upon default. Hence participation constraints are formulated as

\[
\sum_{\tau=t}^{\infty} \sum_{\theta^t|\theta^t} \beta^{\tau-t} \pi(\theta^\tau|\theta^t) u(c^i_{\tau}(\theta^\tau)) \geq V^i(\theta^t) \quad (\forall i)(\forall t)(\forall \theta^t).
\] (2)

An allocation is feasible if it satisfies the resource and participation constraints as well as the nonnegativity constraints on consumption. An allocation is constrained efficient if it is feasible and there is no other feasible allocation that Pareto dominates it. Any constrained efficient allocation is a solution to the following planner’s problem with appropriate choice of weights \(\langle \lambda^i \rangle_{i=1}^4\):

\[
\max_{\langle c^i(\theta^t) \rangle_{i,t,\theta^t}} \sum_{i=1}^{4} \lambda^i \sum_{t=0}^{\infty} \sum_{\theta^t} \beta^{t} \pi(\theta^t) u(c^i_{t}(\theta^t))
\]

subject to

\[
\sum_{\tau=t}^{\infty} \sum_{\theta^t|\theta^t} \beta^{\tau-t} \pi(\theta^\tau|\theta^t) u(c^i_{\tau}(\theta^\tau)) \geq V^i(\theta^t) \quad (\forall i)(\forall t)(\forall \theta^t),
\]

\[
\sum_{i=1}^{4} c^i_{t}(\theta^t) \leq \sum_{\theta^t} \omega_{\theta^t} \quad (\forall t)(\forall \theta^t),
\]

\[
c^i_{t}(\theta^t) \geq 0 \quad (\forall i)(\forall t)(\forall \theta^t).
\]

Note that the solution to the planner’s problem exists uniquely, and it is characterized by first order conditions and complementary slackness conditions since the planner’s problem is a convex problem.\(^\text{13}\)

---

\(^\text{13}\)Suppose that \(\pi(\theta^0) > 0\) for all \(\theta^0 \in \Theta\). Define \(\mathcal{S}\) and \(\mathcal{C}^i\) by \(\mathcal{S} = \prod_{t=0}^{\infty} \mathbb{R}^{4^{t+1}}\) and \(\mathcal{C}^i = \{(c^i_0, c^i_1, \ldots) \in \mathcal{S} : (\forall t)(\forall \theta^t \in \Theta^t) \quad c^i_{t}(\theta^t) \geq 0\}\), respectively. The set \(\mathcal{C}^i\) is agent \(i\)'s consumption set. Define \(\mathcal{C} = \prod_{i=1}^{4} \mathcal{C}^i\). Then the objective function in the above planner’s problem is defined over \(\mathcal{C}\). Let \(\Phi \subset \mathcal{C}\) denote the constraint set of the problem. For each \(t \geq 0\), endow \(\mathbb{R}^{4^{t+1}}\) with the Euclidean metric. Endow \(\mathcal{S}\) with the product topology. With this topology, one can show that the objective function is (upper-semi) continuous on \(\mathcal{C}\) and the constraint set \(\Phi\) is compact. Furthermore, \(\Phi\) is convex and the objective function is strictly concave on \(\Phi\). Therefore, the maximum exists uniquely.
3.1.1 Constrained efficient symmetric stationary Markov allocations

This section examines properties of constrained efficient allocations under the endowment process specified above. As it is well known, constrained efficient allocations can feature either full, partial, or no consumption insurance. Since neither full nor no consumption insurance is consistent with empirical evidence reported in the literature, I focus my attention on the case of partial insurance and study how consumption inequality changes with the persistence of occupational mobility, $\rho$.

I focus on constrained efficient allocations of the following simple structure and call it a symmetric stationary Markov allocation:

$$c_i^t(\theta_t) = \begin{cases} 
  c_{1h} & \text{if } \omega_i^t(\theta_t) = \omega_{1h} \\
  c_{1l} & \text{if } \omega_i^t(\theta_t) = \omega_{1l} \\
  c_{2h} & \text{if } \omega_i^t(\theta_t) = \omega_{2h} \\
  c_{2l} & \text{if } \omega_i^t(\theta_t) = \omega_{2l}.
\end{cases}$$

Let $(c_{1h}, c_{1l}, c_{2h}, c_{2l})$ denote a symmetric stationary Markov allocation. In general, constrained efficient allocations are history dependent and, therefore, do not take this simple form.\textsuperscript{14} However, I present sufficient conditions for symmetric stationary Markov allocations to be constrained efficient (Proposition 1 in Section 3.1.1) and show that there exists a range of parameter values under which the sufficient conditions are satisfied (Proposition 3 in Appendix). For any symmetric stationary Markov allocation $(c_{1h}, c_{1l}, c_{2h}, c_{2l})$, define $V_m^\theta$ by

$$V_m^\theta = u(c_\theta) + \beta \sum_{\theta' \in \Theta} \pi(\theta'|\theta)V_m^{\theta'}.$$ 

With a little abuse of notation, $\theta$ here represents agent’s endowment $\omega_\theta$. Thus, $V_m^\theta$ is the present discounted value that agents with $\omega_\theta$ obtain in a risk sharing mechanism characterized by $(c_{1h}, c_{1l}, c_{2h}, c_{2l})$. Note that, under assumption (1), autarky is also a symmetric stationary Markov allocation. Thus, let $V_a^\theta$ denote the value of autarky, which is defined by,

$$V_a^\theta = u(\omega_\theta) + \beta \sum_{\theta' \in \Theta} \pi(\theta'|\theta)V_a^{\theta'}.$$ 

\textsuperscript{14}Several researchers have found that the solution to the planner’s problem stated above has a recursive structure if one expands the state space. See, for example, Marcet and Marimon (1998) or Rustichini (1998) for a recursive method to solve the planner’s problem.
With this notation, the participation constraint (2) for symmetric stationary Markov allocations can be written as follows: \( V_m^\theta \geq V_a^\theta \) for all \( \theta \in \Theta \).

I examine properties of constrained efficient allocations with some but not perfect risk sharing (partial risk sharing). An allocation is said to feature perfect risk sharing if it is the first best allocation, which is the unique solution to the planner’s problem without imposing the participation constraints (2). Since agents are ex ante identical, I consider the case of equal initial weights over agents \( \lambda^i = 1/4 \) for all \( i \). Under equal weights, the first best allocation is \( c_{\text{FB}} \).

Proposition 1 presents a set of sufficient conditions for symmetric stationary Markov allocations to be constrained efficient when agents face occupational mobility and some but not perfect risk sharing is attainable in the economy.

**Proposition 1** Let \( \rho \in [0,1) \). Suppose that perfect risk sharing is not attainable. Suppose that a feasible symmetric stationary Markov allocation \((c_{1h}, c_{1l}, c_{2h}, c_{2l})\) satisfies the following:

- **some risk sharing:**
  \[
  V_{2l}^m > V_{2l}^a. \tag{3}
  \]

- **resource constraint:**
  \[
  c_{1h} + c_{1l} + c_{2h} + c_{2l} = \omega_{1h} + \omega_{1l} + \omega_{2h} + \omega_{2l}. \tag{4}
  \]

- **participation constraint:**
  \[
  V_{1h}^m = V_{1h}^a, \quad V_{1l}^m = V_{1l}^a, \quad \text{and} \quad V_{2h}^m = V_{2h}^a. \tag{5}
  \]

\footnote{All propositions in this section also hold with asymmetric initial weights. (See Remark 2 in Appendix A1 for details.) Given \( \lambda^i \), the first best allocation is, for all \( i, t, \theta^t \), \( c_i^t(\theta^t) = c_{FB}^t \) such that \( u'(c_{FB}^t)/u'(c_i^t) = \lambda^i/\lambda^j \) for all \( i,j \). Therefore, the first best allocation is feasible if and only if \( \min\{u(c_{FB}^t)/(1 - \beta)\} \geq V_{1h}^a \). Note that if \( u(\sum_\theta \omega^\theta/4)/(1 - \beta) < V_{1h}^a \), then the first best allocation under any initial weight is not feasible.}
• non-degenerate distribution

\[ c_{1h} > \{c_{1l}, c_{2h}\} > c_{2l} \]  \hspace{1cm} (6)

• within-group inequality

\[ \frac{u'(c_{1h})}{u'(c_{1l})} \geq \frac{u'(c_{2h})}{u'(c_{2l})}. \]  \hspace{1cm} (7)

Then the symmetric stationary Markov allocation is constrained efficient.

**Proof.** See Appendix A1. □

All the conditions in Proposition 1, except for (6), are also necessary for optimality. As for (6), the second inequality is also a part of the necessary conditions.

**Proposition 2** Let \( \rho \in [0, 1) \). Suppose that a symmetric stationary Markov allocation \((c_{1h}, c_{1l}, c_{2h}, c_{2l})\) is constrained efficient and features some but not perfect risk sharing. Then the allocation satisfies the conditions (3), (4), (5) and (7) in Proposition 1, and \( c_{2l} < \min\{c_{1h}, c_{1l}, c_{2h}\} \).

**Proof.** See Appendix A2. □

Note that conditions (3), (4), and (5) pin down a feasible symmetric stationary Markov allocation and, thus, determine the structure of risk sharing arrangements. More precisely, under conditions (3) - (5), agents only insure against the worst income state \( \omega_{2l} \). In particular, note that the participation constraint binds for the agent at \((1, 1)\), the lowest income state in the high-mean income group. In Proposition 1, conditions (6) and (7) are imposed to guarantee constrained efficiency of such risk sharing arrangements. Condition (7) requires within-group consumption variability in the high-mean income group to be sufficiently low, allowing agents to focus on the insurance against the worst income state \( \omega_{2l} \). In Appendix A, I show that, for sufficiently high \( \rho \), the condition is satisfied when, given the persistence of within-group income shocks, within-group income inequality for the high-mean income group is small relative to within-group consumption inequality for the low-mean income group (Proposition 3).

---

16To see this, note that, for any feasible allocation, the following statement is true: if the participation constraint holds with equality for \( i \) at \( t \) and \( \theta^t \), then \( c_i^t(\theta^t) \leq \omega_i^t(\theta^t) \). The inequality becomes strict when there is some risk sharing. Therefore, conditions (3) and (5) imply that \( c_{1h} < \omega_{1h}, c_{1l} < \omega_{1l}, \) and \( c_{2h} < \omega_{2h} \).
3.1.2 Numerical example

In this section, I numerically examine how between- and within-group consumption inequality vary with the probability of switching occupation, denoted by $q = 1 - \rho$. For this exercise, I use the Theil index as the index is additively decomposable into between- and within-group components.\(^{17,18}\) Let $n$ denote the population size and $y_i$ the income (or consumption) of agent $i$. The Theil index, $T$, is defined by

$$T = \sum_{i=1}^{n} s_i \log(n s_i),$$

where $s_i = y_i / (n \overline{y})$. Here $\overline{y}$ is the arithmetic mean of $\{y_i\}_{i=1}^{n}$. Suppose that the population is partitioned into $k$ subgroups. Then it holds that

$$T_{total} = T_{between} + \sum_{j=1}^{k} g_j T_j,$$

where $g_j$ is the share of group $j$’s income in total income (or consumption), $T_j$ is the Theil index for group $j$, and $T_{between}$ is the Theil index for the allocation in which each group member is assigned the corresponding group-mean income (or consumption).

I set parameter values as follows: $\beta = 0.65$, $\sigma = 1.0$, $\gamma = 0.9$ and $(\omega_{1h}, \omega_{1l}, \omega_{2h}, \omega_{2l}) = (1.0, 0.5, 0.9, 0.1)$. The subjective discount factor $\beta$ is set to 0.65 so that perfect risk sharing is not attainable. In light of Proposition 1, one can find a constrained efficient allocation by solving Equations (4) and (5) for a symmetric stationary Markov allocation and checking its optimality by the conditions stated in the proposition. In Figures 3 - 5, the solid line represents the Theil index of the constrained efficient allocation (consumption inequality) for $q \in [0, 0.1]$, while the dashed line represents that of autarky (income inequality). (Recall that $q = 1 - \rho$, the occupation switch probability.) Figure 3 shows between-group inequality. Figures 4 and 5 show within-group inequality for the high and low mean income groups, respectively.

\(^{17}\)The Theil index belongs to the general entropy (GE) family of measures. All measures in the GE family exhibit additive separability. Through simulations, Kuga (1980) shows that the experimental rankings of the Theil index are similar to those of the Gini index.

\(^{18}\)The variance of logarithms can also be written as the sum of between- and within-group components. However, as Cowell (1995) has noted, it is not a valid decomposition since the between-group component is not independent of the within-group component. For example, take the allocations $(\omega_{1h}, \omega_{1l}, \omega_{2h}, \omega_{2l})$ and $(\omega_{1h}, \omega_{1l}, \omega_{2h} - x, \omega_{2l} + x)$ for some $x \in (0, \omega_{2h})$. These allocations share the same mean in each group. With variance of logarithms, however, the between-group inequality of $(\omega_{1h}, \omega_{1l}, \omega_{2h}, \omega_{2l})$ takes a different value from that of $(\omega_{1h}, \omega_{1l}, \omega_{2h} - x, \omega_{2l} + x)$.
Changes in occupational mobility affect both between- and within-group components of consumption inequality. Figure 3 shows that between-group inequality decreases as occupational mobility increases. Without occupational mobility \((q = 0)\), between-group consumption inequality takes the same value as between-group income inequality. This observation comes with no surprise given the fact that the Theil index is additively decomposable and the fact that obtaining the allocation \((\omega_{1h}, \omega_{1l}, \bar{c}_{2h}, \bar{c}_{2l})\) from \((\omega_{1h}, \omega_{1l}, \omega_{2h}, \omega_{2l})\) involves no between-group transfers. Figures 4 and 5 show that within-group consumption inequality for the high mean income group increases in \(q\), while that for the low mean income group decreases in \(q\).

For the increase in within-group consumption inequality for the high mean income group, the persistence of within-group income shocks and within-group consumption inequality for the low-mean income group play a crucial role. Since within-group income shocks are persistent, the agent with the low income state in the high-mean income group \((1, l)\) is more likely to transit to \((2, l)\) than the agent at \((1, h)\) in the incident of occupation switch. When occupational mobility increases, therefore, the agent at \((1, l)\) becomes worse off than the agent at \((1, h)\), given that within-group consumption inequality in the low-mean income group is strictly positive. Now recall that agents in the high-mean income group are constrained, thus giving up some of their resources. If the value of autarky for the agent at \((1, l)\) decreases sufficiently more than that for the agent at \((1, h)\), the agent at \((1, l)\) agrees to give up more resources than the agent at \((1, h)\).\(^{19}\) As a result, within-group consumption inequality for the high-mean income group increases as occupational mobility rises.

\(^{19}\)In all numerical experiments I have conducted, within-group consumption inequality for the high-mean income group increases with occupational mobility. Note that Condition 7 in Proposition 1 requires within-group consumption inequality for the low-mean income group to be relatively large.
3.2 A model with a continuum of agents

The above model with four agents is of interest as I can analytically characterize constrained efficient symmetric stationary Markov allocations. However, the analysis only deals with a special case in which agents in the high-mean income group only insure against future occupation switches. It is because in general, constrained efficient allocations do not feature the Markov property in models with limited enforcement of contracts. Hence, in this section, I consider a model with a continuum of agents
and generalize the analysis in the last section by numerically computing stationary constrained efficient allocations. Krueger and Perri (2005) examine stationary constrained efficient allocations in this type of models in detail and provide a numerical algorithm to compute such allocations.

The environment is the same as the four-agent model, except that there is now a continuum of agents and every agent faces an independent endowment process. The latter means that every agent faces a stochastic process $\theta_i^t = (y_i^t, z_i^t)$ that is described above, and $\omega_i^t = \omega(\theta_i^t)$ with $\omega(1,h) = \omega_{1h}$, $\omega(1,l) = \omega_{1l}$, $\omega(2,h) = \omega_{2h}$, and $\omega(2,l) = \omega_{2l}$. In this environment, I compute stationary constrained efficient allocations. As in the previous section, I set $\beta = 0.65$, $\sigma = 1.0$, and $\gamma = 0.9$.

Table 4 reports inequality of stationary constrained efficient consumption allocations for various endowment processes. Columns designated by Between, Within (Low), Within (High), and Total present between-group inequality, within-group inequality for the low-mean income group, within-group inequality for the high-mean income group, and total inequality, respectively. The table consists of four panels with different income distributions. The group-specific mean for each group is kept constant across these four panels. Each panel shows income inequality on the top row and consumption inequality corresponding to the given income inequality. For each panel, I report three cases: 1. no mobility; 2. mobility with the transition probability equal to 0.05; 3. mobility with the transition probability 0.10.

In Panels A - D, between-group consumption inequality decreases with occupational mobility. However, the effect of occupational mobility on within-group consumption inequality depends on income risk that agents face. Note that risk sharing structure for constrained efficient allocations considered in this section is richer than the one considered in Section 3.1. For example, constrained efficient allocations can have both constrained and unconstrained agents at $(1,l)$ and $(2,h)$ in the continuum economy in contrast to constrained efficient symmetric stationary Markov allocations in the four-agent economy. Agents are not constrained at $(1,l)$ and $(2,h)$ when they transit from the highest income state $(1,h)$.

However, the analysis with the four-agent model helps us make sense of the results in Table 3.2. The analysis points out that a rise in occupational mobility leads to an increase in within-group consumption inequality for the given group when within-
group consumption inequality in the other group is relatively large and within-group income shocks are persistent. Let us start with Panel C that corresponds to the case studied in the four-agent model, featuring relatively large within-group income (and thus consumption) inequality for the low-mean income group. As in the four-agent model, within-group consumption inequality for the high-mean income group increases as occupational mobility rises. In Panel B, the situation is opposite with within-group income inequality for the high-mean income group larger than that for the low-mean income group. Then, as expected, within-group consumption inequality for the low-mean income group increases as occupational mobility rises. In Panel D, both within-group income inequality for the high-mean income group and that for the low-mean income group are relatively large. Thus, consistent with the above observations, a rise in occupational mobility leads to an increase in within-group consumption inequality for both groups.
### Table 4: Consumption and Income Inequality

<table>
<thead>
<tr>
<th>Panel</th>
<th>Type</th>
<th>Between (Low)</th>
<th>Within (Low)</th>
<th>Within (High)</th>
<th>Total</th>
</tr>
</thead>
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<tr>
<td>A</td>
<td>Income</td>
<td>0.0566</td>
<td>0.0201</td>
<td>0.0201</td>
<td>0.0768</td>
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<tr>
<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>No Mobility</td>
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<td>0.0191</td>
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<td>0.0757</td>
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<td>0.0182</td>
<td>0.0695</td>
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<tr>
<td></td>
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<td>0.0263</td>
<td>0.0179</td>
<td>0.0641</td>
</tr>
<tr>
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<td>0.0201</td>
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<td>0.1506</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>No Mobility</td>
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<td>0.0179</td>
<td>0.0885</td>
<td>0.1215</td>
</tr>
<tr>
<td></td>
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<td>0.0335</td>
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<tr>
<td>C</td>
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<td>0.0201</td>
<td>0.1137</td>
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<tr>
<td></td>
<td>No Mobility</td>
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</tr>
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<td>0.0750</td>
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</tr>
<tr>
<td>D</td>
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<tr>
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<tr>
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</tr>
</tbody>
</table>

Inequality is measured by the Theil index. The number in parentheses refers to the probability of transiting to a new occupation.

Panel A: \((\omega_1, \omega_{11}, \omega_{2h}, \omega_{2l}) = (1.2, 0.8, 0.6, 0.4)\)

Panel B: \((\omega_1, \omega_{1l}, \omega_{2h}, \omega_{2l}) = (1.5, 0.5, 0.6, 0.4)\)

Panel C: \((\omega_1, \omega_{11}, \omega_{2h}, \omega_{2l}) = (1.2, 0.8, 0.75, 0.25)\)

Panel D: \((\omega_1, \omega_{1l}, \omega_{2h}, \omega_{2l}) = (1.5, 0.5, 0.75, 0.25)\).
4 Quantitative Exercise

In Section 2.3, I divide the sample period into two subperiods, 1980-1985 and 1986-1992. For each subperiod, I calibrate the model, compute a stationary competitive equilibrium, and simulate it to compute between- and within-group consumption inequality. In the rest of this section, I describe the model and discuss its calibration.

4.1 The Model

I introduce occupational mobility in a model economy developed by Krueger and Perri (2006). The economy is a production economy. The representative firm produces a single good that can be used for consumption or investment in physical capital $K$. The aggregate resource constraint is as follows,

$$C_t + K_{t+1} = AK_t^\alpha L_t^{1-\alpha} + (1 - \delta)K_t,$$

where $L_t$ is aggregate labor, $K_t$ the aggregate capital stock, $C_t$ aggregate consumption, $A$ is a technology parameter, $\alpha$ is the capital income share, and $\delta$ is the depreciation rate.

There is a continuum of households of measure 1. I consider two occupation groups: 1. professional/managerial specialty; 2. the other. Every household belongs to either one of these two groups by their head’s occupation. Households face a stochastic labor endowment process $\{\alpha_t, u_t\}$, where $\alpha_t$ and $u_t$ represent the mean income of the occupation group to which household head belongs and the within-group idiosyncratic component, respectively. Household heads can change their occupation over time. Both $\alpha_t$ and $u_t$ follow stationary Markov chains, with the support $\{\alpha_1, \alpha_2\}$ for $\alpha_t$ and the finite support $U$ for $u_t$. These two stochastic processes are independent. Let $\theta_t = (\alpha_t, u_t)$ and $\Theta = \{(\alpha_t, u_t) | \alpha_t \in \{\alpha_1, \alpha_2\}, u_t \in U\}$. The stochastic process $\theta_t$ also follows a stationary Markov chain with $\Theta$ as its support. Let $\pi(\theta^t | \theta)$ denote the transition probabilities of the Markov chain. Let $\theta^t = (\alpha^t, u^t) = (\alpha_0, \ldots, \alpha_t; u_0, \ldots, u_t)$ denote a history up to period $t$. Also, $\pi(\theta^t | \theta_0) = \prod_{s=1}^t \pi(\theta_s | \theta_{s-1})$. Let $\Phi_0$ denote the initial distribution over types $(\theta_0, a_0)$.
where $a_0$ is household’s initial asset holdings. Total labor supply is given by

$$L_t = \int \sum_{\theta^t} \alpha_t u_t \pi(\theta^t|\theta_0) d\Phi_0. \tag{9}$$

Household preferences are represented by the same utility function described in Section 3.

Households trade Arrow securities subject to borrowing constraints denoted by $B_t(\theta^t, \theta_{t+1})$. Let $q_t(\theta^t, \theta_{t+1})$ denote the prices for Arrow securities. Households face the following problem:

$$\max \left\{ u(c_t(a_0, \theta_0)) + \sum_{t=1}^{\infty} \sum_{\theta^t|\theta_0} \beta^t \pi(\theta^t|\theta_0) u(c_t(a_0, \theta^t)) \right\} \tag{10}$$

subject to

$$c_t(a_0, \theta^t) + \sum_{y_{t+1}} q_t(\theta^t, \theta_{t+1}) a_{t+1}(a_0, \theta^t, \theta_{t+1}) = w_t \alpha_t y_t + a_t(a_0, \theta^t) \quad (\forall t)(\forall \theta^t) \tag{11}$$

$$a_{t+1}(a_0, \theta^t, \theta_{t+1}) \geq B_{t+1}(\theta^t, \theta_{t+1}) \quad (\forall t)(\forall \theta^t)(\forall \theta_{t+1}). \tag{12}$$

where $\theta^t|\theta_0$ means that $\theta^t$ is a possible continuation from $\theta_0$, and $w_t$ is the economy-wide wage per efficiency unit of labor.

The constraints $B_t(\theta^t, \theta_{t+1})$ are specified as solvency constraints that are not too tight. First, if households default, they start the next period with neither assets or liabilities. In addition, after default, households do not have access to the markets for Arrow securities, but they are allowed to save (but not borrow) at a state-uncontingent interest rate $r_d$. Let $U^d_t(\theta_t)$ denote the continuation value of default from $\theta^t$. Then,

$$U^d_t(\theta_t) = \max_{\{c_t, b_{t+1}\}} \left\{ u(c_0) + \sum_{s=t+1}^{\infty} \sum_{\theta^s|\theta^t} \beta^{s-t} \pi(\theta^s|\theta^t) u(c_s) \right\}$$

subject to

$$c_s + \frac{b_{s+1}}{1 + r_d} = w_s \alpha_s u_s + b_s \quad (\forall s \geq t)(\forall y^s|y^t)$$

$$b_{s+1} \geq 0 \quad (\forall s \geq t)$$

subject to $b_t = 0$. Next, define the continuation utility $V_t(\theta^t, a)$ of a household with history $\theta^t$ and current asset holdings $a_t$ at time $t$ as follows.

$$V_t(\theta^t, a_t) = \max_{\{c_t, a_{t+1}\}} \left\{ u(c_t) + \sum_{s=t+1}^{\infty} \sum_{\theta^s|\theta^t} \beta^{s-t} \pi(\theta^s|\theta^t) u(c_s) \right\}$$
subject to (11) and (12). Then, the solvency constraints \( \{B_{t+1}(\theta^t, \theta_{t+1})\} \) that are not too tight solve the following equation:

\[
V_{t+1}(\theta^{t+1}, B_{t+1}(\theta^t, \theta_{t+1})) = U_{t+1}^d(\theta_{t+1}) \quad (\forall(\theta^t, \theta_{t+1})).
\] (13)

If the economy is in the steady state with aggregate quantities \((K_t, L_t)\) and prices \((r_t, w_t)\) constant, the Markov property of \(\theta_t\) and the assumption that defaulting households start with neither assets nor liabilities imply that \(U_{t+1}^d(\theta_t) = U_{t}^d(\theta_{t-1}), V_{t+1}(\theta_t, a) = V(\theta_t, a),\) and \(B_{t+1}(\theta_t, \theta_{t+1}) = B(\theta_{t+1})\). Thus, one can reformulate the household problem recursively with \((\theta, a)\) as state variables.

I am now in a position to define stationary competitive equilibrium with solvency constraints that are not too tight.

**Definition 1** A stationary competitive equilibrium with solvency constraints that are not too tight is a list of a value function \(V\), policy functions for the household problem \(\{c, a'\}\), solvency constraint \(B\), aggregate quantities \(\{K, L\}\), prices \(\{w, r, q\}\) and a measure \(\Phi\) defined over \((\Theta \times \mathbb{R}, \mathcal{B})\) where \(\mathcal{B}\) is the Borel \(\sigma\)-algebra on \(\Theta \times \mathbb{R}\) such that

1. Given prices and solvency constraints, \(V\) solves the following functional equation, and \(c\) and \(a'\) are the associated policy functions.

\[
V(\theta, a) = \max\{u(c) + \beta \sum_{\theta' \in \Theta} \pi(\theta'|\theta) V(\theta', a'(\theta, a, \theta'))\}
\]

subject to

\[
a + \sum_{\theta' \in \Theta} q(y'|y)a'(\theta, a, \theta') \leq w\alpha u + a
\]

\[
a'(\theta, a, \theta') \geq B(\theta') \quad (\forall \theta' \in \Theta).
\]

2. Solvency constraints are not too tight.

\[
V(\theta, B(\theta)) = U^d(\theta) \quad (\forall \theta \in \Theta).
\]

3. Given prices, the representative firm maximizes profits.

\[
w = (1 - \alpha)A\left(\frac{K}{L}\right)^{\alpha}
\]

\[
r = \alpha A\left(\frac{K}{L}\right)^{\alpha-1} - \delta
\]

27
4. All markets clear.

\[
\int c(\theta, a)d\Phi + \delta K = A K^{\alpha} L^{1-\alpha}
\]

\[
L = \int \alpha ud\Phi
\]

\[
K = \frac{1}{1 + r} \sum \alpha' (\theta, a, \theta')d\Phi.
\]

5. \(\Phi\) is stationary, that is, \(\Phi(Z) = \int_{\Theta \times R} Q((\theta, a), Z)d\Phi((\theta, a))\) for all \(Z \in \mathcal{B}\) where \(Q\) is the transition kernel generated by transition probabilities \(\pi(\theta' | \theta)\) and policy functions \((c, a')\).

### 4.2 Calibration


#### 4.2.1 Technology and preference parameters

For technology and preference parameters, I choose the same values as Krueger and Perri (2006), since the authors also calibrate their model to the US economy in the 1980’s. The degree of relative risk aversion \(\sigma\) is set to 1, which implies that \(u(c) = \log(c)\), and a wage rate is normalized to 1. The authors calibrate \((A, \alpha, \delta)\) and \(\beta\) to the following empirical evidence: a capital income share of 30%, a return on physical capital of 4% per annum (McGrattan and Prescott (2003)), a capital-to-output ratio of 2.6 (The author’s calculation of the average wealth (including financial wealth and housing wealth) using CEX data for 1980-1981). The resulting parameter values are: \(\alpha = 0.3, A = 0.9637, \delta = 0.0754\) and \(\beta = 0.959\).
4.2.2 Within-group income process

The within-group income process is as follows.

\[
\begin{align*}
    u_{it} &= \eta_{it} + \epsilon_{it} \\
    \eta_{it} &= \rho \eta_{i,t-1} + \xi_{it}
\end{align*}
\]

where \( \xi_{it} \) and \( \epsilon_{it} \) represent persistent and transitory shocks, respectively. Here \( \epsilon_{it} \) and \( \xi_{it} \) are independent, serially uncorrelated, and normally distributed random variables. I omit measurement error in the benchmark analysis as the variance of measurement error is not separately identified in this model.\(^{20}\)

For the benchmark analysis, I fix \( \rho \) to 0.9989, the value found in Storesletten et al. (2004) for a similar income process.\(^{21}\) Krueger and Perri (2006) also take this estimate of the autocorrelation in their calibration. Then I estimate \( \sigma\xi \) and \( \sigma\epsilon \) by the following two (unconditional) moment conditions. I compute empirical moments using PSID data. Table 5 reports the estimation results.

\[
\begin{align*}
    \text{Cov}(u_{it}, u_{it-1}) &= \rho \frac{\sigma^2_{\xi}}{1 - \rho^2} \\
    \text{Var}(u_{it}) &= \frac{\sigma^2_{\xi}}{1 - \rho^2} + \sigma^2_{\epsilon}
\end{align*}
\]

4.2.3 Occupation-specific mean income and occupational mobility

I take occupation-specific mean income directly from the cross-sectional regression used to construct between- and within-group income inequality in Section 2.3. The cross-sectional regression equation is as follows:

\[
\ln y_{it} = \gamma_{0t} + z_{it}' \gamma_{1t} + d_{it}' \alpha_t + u_{it},
\]

\(^{20}\)One can use external evidence in order to take measurement error into account. See Meghir and Pistaferri (2004) and Heathcote et al. (2004) for estimation of similar income processes with measurement error.

\(^{21}\)Although their income process is similar, it is not exactly the same as the income process considered here. In particular, the persistent component in Storesletten et al. (2004) might contain income accounted for by occupation that I explicitly control for. Thus, for sensitivity analysis, I estimate the autocorrelation using the PSID data. However, due to the short panel dimension (6 periods for 1980-1985 and 7 periods for 1986-1992), the resulting estimates are likely to be downward biased, providing a lower bound for the autocorrelation. See Section 6.2.
Table 5: Estimation of the income process

<table>
<thead>
<tr>
<th></th>
<th>$\hat{\sigma}_\xi$</th>
<th>$\hat{\sigma}_\epsilon$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1980-1985</td>
<td>0.0183</td>
<td>0.2226</td>
</tr>
<tr>
<td>1986-1992</td>
<td>0.0193</td>
<td>0.2300</td>
</tr>
</tbody>
</table>

where $z_{it}$ is a vector of all the regressors except for a constant and the professional/manager dummy ($d_{it}^1$). I set the mean income for the professional/managerial occupation, $\alpha_1$, to $(\sum_{t=1980}^{1985} \exp(\hat{\alpha}_t))/6$ for the 1980-1985 period and $(\sum_{t=1986}^{1992} \exp(\hat{\alpha}_t))/7$ for the 1986-1992 period. Since I take the other occupation as the base group in cross-sectional regressions, I set $\alpha_2$ to 1.

For the benchmark calibration of occupational mobility, a relevant estimate is occupational mobility due to involuntary job loss presented in Section 2.2, since occupation switch is exogenous in the current model. However, Kambourov and Manovskii (2006) document that around 30% of the workers switching occupations return to their one-digit occupation within a three-year period. To take account of this type of occupation choice, I decrease the transition probability by 30% in Table 2. Table 6 presents the benchmark calibration of occupational mobility.

Table 6: Transition Matrix

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>0.9944</td>
<td>0.0056</td>
</tr>
<tr>
<td>2</td>
<td>0.0040</td>
<td>0.9960</td>
</tr>
</tbody>
</table>

Occupation groups are defined as follows: 1. professional/managerial occupation; 2. the rest.
5 Quantitative Results

[THIS VERSION OMITS TRANSITORY SHOCKS.]

In Table 7, I present changes in between- and within-group consumption inequality from the 1980-1985 period to the 1986-1992 period observed in the CEX data and in the model. The columns designated with ‘$\Delta$ between’, ‘$\Delta$ within’, and ‘$\Delta r\%$’ refer to the change in between-group consumption inequality, the change in within-group consumption inequality, and the change in the equilibrium interest rate in percentage point, respectively.

The rows with CEX (ND+) and CEX(ND) report statistics of the ND+ and ND consumption measures using the CEX data, respectively. I present the model’s implications in two different cases in order to examine the effects of occupational mobility on the evolution of consumption inequality. The row designated by ‘Model (Benchmark)’ reports the benchmark results. The row designated by ‘Model (No mobility)’ reports results of the model with no mobility.

Comparing to the model with no occupational mobility, the benchmark model better explains changes in both between- and within-group consumption inequality observed in CEX data (ND+ and ND). The change in between-group consumption inequality generated by the benchmark model quantitatively match up the empirical counterpart. Note that between-group consumption inequality in the model with no occupational mobility is equal to between-group income inequality since households do not transfer their resources between groups in the absence of occupation switch risk. The benchmark model generates a larger increase in within-group consumption inequality than the model with no occupational mobility. The former accounts for 33.4% of the observed increase, while the latter only accounts for 14.2%. However, both the benchmark model and the model with no occupational mobility understate the increase in within-group consumption inequality observed in data.
Table 7: Changes in between- and within-group consumption inequality

<table>
<thead>
<tr>
<th></th>
<th>△ between</th>
<th>△ within</th>
<th>△r%</th>
</tr>
</thead>
<tbody>
<tr>
<td>CEX (ND+)</td>
<td>0.0003</td>
<td>0.0171</td>
<td>n.a.</td>
</tr>
<tr>
<td>CEX (ND)</td>
<td>-0.0003</td>
<td>0.0109</td>
<td>n.a.</td>
</tr>
<tr>
<td>Model (Benchmark)</td>
<td>0.0005</td>
<td>0.0057</td>
<td>-0.06%</td>
</tr>
<tr>
<td>Model (No mobility)</td>
<td>0.0045</td>
<td>0.0024</td>
<td>-0.04%</td>
</tr>
</tbody>
</table>

△ between and △ within refer to the change in between- and within-group consumption inequality. △r% refers to the change in the equilibrium interest rate in percentage point. Each column reports changes from 1980-85 to 1986-92.

6 Sensitivity Analysis

6.1 Occupational mobility

In this paper, I focus on effects of an exogenous part of occupational mobility on consumption insurance. In the benchmark calibration, I take observed occupational mobility due to involuntary job loss as a proxy. In this section, I calibrate the exogenous occupation switch probability indirectly by using consumption data. Note that, everything else being constant, the higher the occupational mobility, the smaller between-group consumption inequality. Exploiting this relationship, I calibrate occupation switch probability to the level of between-group consumption inequality (measured in ND+), assuming symmetry of the transition matrix. Table 8 presents the resulting transition matrix for the 1980-1985 period and the 1986-1992 period. Between-group consumption inequality observed in CEX data and that generated by the model are presented in Table 9. Since the model generates too much consumption insurance, the calibrated occupational mobility is lower than occupational mobility presented in Section 2.2. However, note that the calibrated occupational mobility exhibits an increase from 0.01% in 1980-1985 to 0.04% in 1986-1992.

The row designated by Model (Calibrated mob) in Table 10 presents the results with the calibrated occupational mobility considered in this section. Other parameter
Table 8: Calibrated Transition Matrix

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>0.9999</td>
<td>0.0001</td>
</tr>
<tr>
<td>2</td>
<td>0.0001</td>
<td>0.9999</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.9996</td>
<td>0.0004</td>
</tr>
<tr>
<td>2</td>
<td>0.0004</td>
<td>0.9996</td>
</tr>
</tbody>
</table>

Table 9: Between-group consumption inequality

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>CEX (ND+)</td>
<td>0.0040</td>
<td>0.0043</td>
<td>0.0003</td>
</tr>
<tr>
<td>Model (Calibrated mob)</td>
<td>0.0037</td>
<td>0.0041</td>
<td>0.0004</td>
</tr>
</tbody>
</table>

values are unchanged. First of all, the model with the calibrated mobility now accounts for 91% of the increase in within-group consumption inequality measured with the ND+ consumption measure using CEX data. Note that the model matches the change in between-group consumption inequality with CEX(ND+) by construction.

6.2 Persistence of within-group income shocks

Recall within-group income process:

\[ u_{it} = \eta_{it} + \epsilon_{it} \]
\[ \eta_{it} = \rho \eta_{it-1} + \xi_{it}. \]

To identify the parameters, \( \rho, \sigma_\xi, \sigma_\epsilon \), I use the following (unconditional) moment conditions. Table 6.2 reports the estimation results. Since I can only use the short panel dimension (6 periods in 1980-1985 and 7 periods in 1986-1992), the estimates of the autocorrelation are likely to be downward biased. Hence, these values can be
Table 10: Changes in between- and within-group consumption inequality

<table>
<thead>
<tr>
<th></th>
<th>△ between</th>
<th>△ within</th>
<th>△r%</th>
</tr>
</thead>
<tbody>
<tr>
<td>CEX (ND+)</td>
<td>0.0003</td>
<td>0.0171</td>
<td>n.a.</td>
</tr>
<tr>
<td>CEX (ND)</td>
<td>-0.0003</td>
<td>0.0109</td>
<td>n.a.</td>
</tr>
<tr>
<td>Model (Calibrated mob)</td>
<td>0.0004</td>
<td>0.0156</td>
<td>-0.01%</td>
</tr>
<tr>
<td>Model (No mobility)</td>
<td>0.0045</td>
<td>0.0024</td>
<td>-0.04%</td>
</tr>
</tbody>
</table>

△ between and △ within refer to the change in between- and within-group consumption inequality. △r% refers to the change in the equilibrium interest rate in percentage point. Each column reports changes from 1980-85 to 1986-92.

\[
\text{Cov}(u_{it}, u_{it-1}) = \rho \frac{\sigma^2 \xi}{1 - \rho^2}
\]

\[
\text{Cov}(u_{it}, u_{it-2}) = \rho^2 \frac{\sigma^2 \xi}{1 - \rho^2}
\]

\[
\text{Var}(u_{it}) = \frac{\sigma^2 \xi}{1 - \rho^2} + \sigma^2 \epsilon
\]

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>̂ρ</td>
<td>0.8932</td>
<td>0.8853</td>
</tr>
<tr>
<td>̂σ_ξ</td>
<td>0.1857</td>
<td>0.2030</td>
</tr>
<tr>
<td>̂σ_ε</td>
<td>0.1775</td>
<td>0.1767</td>
</tr>
</tbody>
</table>

Using exactly identified estimates for within-group income shocks and benchmark values for other parameters, I recompute stationary equilibrium. In Table 11, the row designated by Model (Persistence) presents the results with the parameter estimates for within-group income process considered in this section. As one expects from the qualitative mechanism found in Section 3, the model generates a smaller increase in within-group consumption inequality than the benchmark case in which the autocorrelation is set to 0.9989. However, the main results are robust, that is, the increase in within-group consumption inequality is larger in the model with occupational mobility than the one without occupational mobility.
Table 11: Changes in between- and within-group consumption inequality

<table>
<thead>
<tr>
<th></th>
<th>△ between</th>
<th>△ within</th>
<th>△r%</th>
</tr>
</thead>
<tbody>
<tr>
<td>CEX (ND+)</td>
<td>0.0003</td>
<td>0.0171</td>
<td>n.a.</td>
</tr>
<tr>
<td>CEX (ND)</td>
<td>-0.0003</td>
<td>0.0109</td>
<td>n.a.</td>
</tr>
<tr>
<td>Model (Persistence)</td>
<td>0.0006</td>
<td>0.0044</td>
<td>-0.04%</td>
</tr>
<tr>
<td>Model (No mobility)</td>
<td>0.0045</td>
<td>0.0008</td>
<td>-0.03%</td>
</tr>
</tbody>
</table>

△ between and △ within refer to the change in between- and within-group consumption inequality. △r% refers to the change in the equilibrium interest rate in percentage point. Each column reports changes from 1980-85 to 1986-92. Benchmark occupational mobility is used.

7 Conclusion

In this paper, I examine the impact of the rise in involuntary occupational mobility on between- and within-group consumption inequality in a model with limited enforcement of contracts. First, to study a qualitative mechanism that links between occupational mobility and between- and within-group consumption inequality, I use a stylized pure exchange economy with two occupation groups with different mean incomes, two income states in each group, and limited enforcement of contracts. In the model, agents face two types of shocks, namely involuntary occupation switch and within-group income shocks. Through numerical experiments, I find that an increase in occupational mobility leads to an increase in within-group consumption inequality for the given group, when within-group income shocks are persistent and within-group consumption inequality in the other group is sufficiently large.

Second, I quantify the impact of involuntary occupational mobility on between- and within-group consumption inequality using a production economy model with limited enforcement of contracts. I find that the increase in involuntary occupational mobility helps account for changes in both between- and within-group consumption inequality. With the benchmark calibration of parameters, the model accounts for 33.4% of the observed increase in within-group consumption inequality measured in the ND+ consumption measure, while the model without occupational mobility
accounts for only 14.2%. I also calibrate involuntary occupational mobility to the level of between-group consumption inequality, a measure of consumption insurance against occupation switch risk. The resulting occupational mobility increases from 0.1% in 1980-1985 to 0.4% in 1986-1992. Under this calibration, the model accounts for 91% of the increase in within-group consumption inequality.
## Data Appendix

### Table 12: Sample Selection in the PSID

<table>
<thead>
<tr>
<th>Category</th>
<th>Observations deleted</th>
<th>Remaining observations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original data set</td>
<td>162297</td>
<td></td>
</tr>
<tr>
<td>Interviewed prior to 1979</td>
<td>58357</td>
<td>103940</td>
</tr>
<tr>
<td>Change in family composition</td>
<td>9909</td>
<td>94031</td>
</tr>
<tr>
<td>Change in marital status</td>
<td>2181</td>
<td>91850</td>
</tr>
<tr>
<td>Female head</td>
<td>26327</td>
<td>65523</td>
</tr>
<tr>
<td>Missing values and topcoding</td>
<td>3434</td>
<td>62089</td>
</tr>
<tr>
<td>Income and consumption outliers</td>
<td>4576</td>
<td>57513</td>
</tr>
<tr>
<td>Poverty subsample</td>
<td>23149</td>
<td>34364</td>
</tr>
<tr>
<td>Aged less than 30 or more than 65</td>
<td>11143</td>
<td>23221</td>
</tr>
<tr>
<td>Armed services, protective workers</td>
<td>756</td>
<td>22465</td>
</tr>
<tr>
<td>Unemployed for 5 years</td>
<td>507</td>
<td>21958</td>
</tr>
</tbody>
</table>

“Missing values and topcoding” excludes households if household’s region of residence is missing, household head’s or spouse’s (if present) education or occupation (conditional on working) is missing, household’s income is topcoded, or food expenditure is assigned by the PSID. “Income and consumption outliers” excludes households if an annual income is below $100 or below total food expenditure, an income growth is above 500 percent or below -80 percent, or total food expenditure is zero or missing.
Table 13: Sample Selection in CEX

<table>
<thead>
<tr>
<th></th>
<th>Observations deleted</th>
<th>Remaining observations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original data set</td>
<td></td>
<td>167919</td>
</tr>
<tr>
<td>Incomplete income respondents</td>
<td>32863</td>
<td>135056</td>
</tr>
<tr>
<td>Zero food consumption</td>
<td>674</td>
<td>134382</td>
</tr>
<tr>
<td>Only food consumption</td>
<td>94</td>
<td>134288</td>
</tr>
<tr>
<td>Missing interviews</td>
<td>72219</td>
<td>62069</td>
</tr>
<tr>
<td>Inconsistent characteristics</td>
<td>4141</td>
<td>57928</td>
</tr>
<tr>
<td>Missing main characteristics</td>
<td>5014</td>
<td>52914</td>
</tr>
<tr>
<td>Change in marital status</td>
<td>572</td>
<td>52342</td>
</tr>
<tr>
<td>Non-positive, missing annual income</td>
<td>1172</td>
<td>51170</td>
</tr>
<tr>
<td>Non-positive, missing labor income</td>
<td>12553</td>
<td>38617</td>
</tr>
<tr>
<td>Positive labor income with zero weeks worked</td>
<td>1</td>
<td>38616</td>
</tr>
<tr>
<td>Income less than food</td>
<td>516</td>
<td>38100</td>
</tr>
<tr>
<td>Aged less than 30 or more than 65</td>
<td>7427</td>
<td>30673</td>
</tr>
<tr>
<td>Armed services, protective workers</td>
<td>226</td>
<td>30477</td>
</tr>
<tr>
<td>Unemployed for 2 years</td>
<td>1426</td>
<td>29021</td>
</tr>
<tr>
<td>Observed after 1992</td>
<td>12836</td>
<td>16185</td>
</tr>
</tbody>
</table>

“Missing main characteristics” excludes CU’s if the reference person’s or spouse’s (if present) sex, race, education, or occupation (conditional on working) is missing.
<table>
<thead>
<tr>
<th></th>
<th></th>
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<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Age</td>
<td>45.10</td>
<td>45.10</td>
<td>45.29</td>
<td>43.89</td>
<td>44.82</td>
<td>44.68</td>
<td>44.77</td>
<td>44.01</td>
<td>44.88</td>
<td>44.19</td>
</tr>
<tr>
<td>Family size</td>
<td>3.34</td>
<td>3.44</td>
<td>3.25</td>
<td>3.15</td>
<td>3.12</td>
<td>3.09</td>
<td>3.03</td>
<td>3.07</td>
<td>3.00</td>
<td>2.99</td>
</tr>
<tr>
<td>White</td>
<td>0.92</td>
<td>0.89</td>
<td>0.94</td>
<td>0.89</td>
<td>0.94</td>
<td>0.86</td>
<td>0.93</td>
<td>0.86</td>
<td>0.94</td>
<td>0.86</td>
</tr>
<tr>
<td>HS graduate</td>
<td>0.33</td>
<td>0.34</td>
<td>0.33</td>
<td>0.33</td>
<td>0.29</td>
<td>0.30</td>
<td>0.30</td>
<td>0.30</td>
<td>0.29</td>
<td>0.30</td>
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Between- and within-group income inequality for 1972-1992

In this section, I present the trend of between- and within-group income inequality for the 1972-1992 period, using the PSID data. Figure 6 shows changes in between- and within-group income inequality from their 1980 value. The line with asterisk represents within-group income inequality, while the line without asterisk represents between-group income inequality. I fit both between- and within-group income inequality by linear lines. Unfortunately, spouse’s occupation is only available from 1979. Thus, I drop spouse’s occupation from the cross-sectional regressions in constructing between- and within-group income inequality. Effects of the modification has turned out to be negligible, since spouse’s education picks up most of the effects of occupation. For 1972-1992, both between- and within-group income inequality increase. The slope of between-group income inequality is much smaller than that of within-group income inequality. Both slopes are significantly greater than zero.
Figure 6: Between- and Within-Group Income Inequality

The figure shows the change in between- and within-group income inequality from the 1972 value over the years from 1972 to 1992. The data indicates an increase in income inequality over the years, with fluctuations in the trend line.
A. Appendix

A1. Existence of constrained efficient symmetric stationary Markov allocations

Proposition 3 below shows that the sufficient conditions stated in Proposition 1 are satisfied when occupational mobility are sufficiently persistent (\( \rho \) is sufficiently close to 1) and idiosyncratic income risk in the high mean income group is sufficiently low (\( \omega_{1h} \) and \( \omega_{1l} \) are sufficiently close). The following lemma plays a key role to prove Proposition 3.

**Lemma 1** Suppose that there exists a non-autarkic solution, denoted by \((\bar{c}_{2h}, \bar{c}_{2l})\), to the following system of equations.

\[
\begin{align*}
(1 - \beta \gamma)(u(\omega_{2h}) - u(\bar{c}_{2h})) - \beta(1 - \gamma)(u(\bar{c}_{2l}) - u(\omega_{2l})) &= 0 \\
\bar{c}_{2h} + \bar{c}_{2l} - (\omega_{2h} + \omega_{2l}) &= 0.
\end{align*}
\]  

(15)

Then there exists \( \bar{\rho} \in [0, 1) \) such that for \( \rho \in [\bar{\rho}, 1) \), the system of equations consisting of (4) and (5) has a non-autarkic solution \((c_{1h}, c_{1l}, c_{2h}, c_{2l})\) that tends to \((\omega_{1h}, \omega_{1l}, \bar{c}_{2h}, \bar{c}_{2l})\) as \( \rho \to 1 \).

**Proof.** See Appendix A3. \( \Box \)

As Proposition 2 states, any constrained efficient symmetric stationary Markov allocation with some but not perfect risk sharing must satisfy (4) and (5). Hence Lemma 1 implies that the type of allocations must converge to \((\omega_{1h}, \omega_{1l}, \bar{c}_{2h}, \bar{c}_{2l})\) as \( \rho \to 1 \). It is this strong convergence result that helps identify sufficient conditions for existence in Proposition 3.

Note that the first equation of (15) is equivalent to \( V_{2h}^m = V_{2h}^a \) with \( \rho = 1 \). Thus, there exists a non-autarkic solution to (15) if and only if there exists a solution to (15) that satisfies \( V_{2l}^m > V_{2l}^a \) with \( \rho = 1 \). The condition means that some risk sharing is attainable in the low mean income group when there is no occupational mobility (\( \rho = 1 \)). The necessary and sufficient condition for the existence of a non-autarkic solution to (15) can also be written as \( u'(\omega_{2h})/u'(\omega_{2l}) < \beta(1 - \gamma)/(1 - \beta) \).
Proposition 3 Suppose that \( u(c_{FB})/(1 - \beta) < V^a_{1h} \) at \( \rho = 1 \) and \( u'(\omega_{2h})/u'(\omega_{2l}) < \beta(1 - \gamma)/(1 - \beta) \). Let \((\bar{c}_{2h}, \bar{c}_{2l})\) be the non-autarkic solution to the system (15) as in Lemma 1. Furthermore, suppose that \( \omega_{1h} > \{\omega_{1l}, \bar{c}_{2h}\} > \bar{c}_{2l} \) and

\[
\frac{u'(\omega_{1h})}{u'(\omega_{1l})} > \frac{u'(\bar{c}_{2h})}{u'(\bar{c}_{2l})}.
\]

Then, there exists \( \bar{\rho} \in [0, 1) \) such that for \( \rho \in [\bar{\rho}, 1] \), the non-autarkic symmetric stationary Markov allocation \((c_{1h}, c_{1l}, c_{2h}, c_{2l})\) satisfying (4) and (5) is constrained efficient. (At \( \rho = 1 \), the allocation \((\omega_{1h}, \omega_{1l}, \bar{c}_{2h}, \bar{c}_{2l})\) satisfies (4) and (5) and is constrained efficient.)

Proof. Immediate from Proposition 1 and Lemma 1. Note that the non-autarkic solution \((c_{1h}, c_{1l}, c_{2h}, c_{2l})\) is continuous in \( \rho \) in the neighborhood of \( \rho = 1 \). Optimality of \((\omega_{1h}, \omega_{1l}, \bar{c}_{2h}, \bar{c}_{2l})\) at \( \rho = 1 \) can be checked in the same manner as in the proof of Proposition 1. □

Proposition 3 tells us that constrained efficient allocations take the simple form of symmetric stationary Markov allocations when occupational mobility are sufficiently persistent \((\rho \text{ is sufficiently close to 1})\) and idiosyncratic income risk in the high mean income group is sufficiently low so that condition (16) holds. The results, in particular the optimality of the allocation \((\omega_{1h}, \omega_{1l}, \bar{c}_{2h}, \bar{c}_{2l})\) at \( \rho = 1 \) hold since condition (16) rules out many allocations. First, under condition (16), autarky in the high mean income group, \((\omega_{1h}, \omega_{1l})\), is the unique solution to the system of equations consisting of \( V^m_{1h} = V^a_{1h} \) with \( \rho = 1 \) and \( c_{1h} + c_{1l} = \omega_{1h} + \omega_{1l} \).\(^{22}\) Note that if there were another solution with some risk sharing to the system of equations, denoted by \((\bar{c}_{1h}, \bar{c}_{1l})\), then the allocation \((\bar{c}_{1h}, \bar{c}_{1l}, \bar{c}_{2h}, \bar{c}_{2l})\) would Pareto dominate \((\omega_{1h}, \omega_{1l}, \bar{c}_{2h}, \bar{c}_{2l})\). Therefore,

\(^{22}\)The system of equations is equivalently written as follows.

\[
\begin{cases}
(1 - \beta \gamma)(u(\omega_{1h}) - u(\bar{c}_{1h})) - \beta (1 - \gamma)(u(\bar{c}_{1l}) - u(\omega_{1l})) = 0 \\
\bar{c}_{1h} + \bar{c}_{1l} - (\omega_{1h} + \omega_{1l}) = 0.
\end{cases}
\]

To prove the statement, first suppose that there exists a non-autarkic solution to the above system. Then, \( u'(\omega_{1h})/u'(\omega_{1l}) < \beta(1 - \gamma)/(1 - \beta) \). In the meantime, Fact 1 in Appendix 3 tells us that \( u'(\bar{c}_{2h})/u'(\bar{c}_{2l}) > \beta(1 - \gamma)/(1 - \beta \gamma) \). Combining these two inequalities yields

\[
\frac{u'(\bar{c}_{2h})}{u'(\bar{c}_{2l})} > \frac{\beta(1 - \gamma)}{1 - \beta \gamma} > \frac{u'(\omega_{1h})}{u'(\omega_{1l})},
\]

contradicting the assumption that \( u'(\omega_{1h})/u'(\omega_{1l}) > u'(\bar{c}_{2h})/u'(\bar{c}_{2l}) \). □.
the nonexistence of such allocations confirms constrained efficiency of \((\omega_{1h}, \omega_{1l}, \bar{c}_{2h}, \bar{c}_{2l})\) at \(\rho = 1\). Second, under the current specification of the period utility function, condition (16) and \(\omega_{1h} > \omega_{1l}\) imply that

\[
1 < \frac{\omega_{1h}}{\omega_{1l}} < \frac{\bar{c}_{2h}}{\bar{c}_{2l}}.
\]

Therefore, \(\bar{c}_{2h} > \bar{c}_{2l}\), which means that perfect risk sharing is not attainable in the low mean income group when \(\rho = 1\).

One may see Proposition 3 from the agent’s point of view since constrained efficient allocations considered can be decentralized (with transfers): \(^{23}\) with endowment processes satisfying (16), agents in the high mean income group cannot share their idiosyncratic income risk since the risk is too small. In other words, no borrowing-lending contract can provide some risk sharing and, at the same time, prevent defaults since the value of autarky is too high. Hence, when agents are in the high mean income group and face occupational mobility, they only insure against occupational mobility. Meanwhile, agents who belong to the low mean income group insure only against the idiosyncratic income risk. As a result, the constrained efficient (equilibrium) allocation exhibits the simple structure.

A2. Proofs

A2.1. Proof of Proposition 1

Solution to the planner problem is characterized by the first order conditions and the complementary slackness conditions associated with participation constraints and resource constraints. Let \(\pi(\theta^t)\) denote the probability of \(\theta^t\). Let \(\beta^t \pi(\theta^t) \varphi^t_i(\theta^t)\) denote a Lagrange multiplier associated with the participation constraint for Agent \(i\) at history \(\theta^t\). Following Marcet and Marimon (1998), I define a (modified) cumulative multiplier \(\psi^t_i(\theta^t)\) recursively by \(\psi^t_i(\theta^t) = \psi^t_{i-1}(\theta^{t-1}) + \varphi^t_i(\theta^t)\) for all \(t\) and \(\psi^t_{i-1} = \lambda^i\). Then the first order condition reads:

\[
\frac{u'(c^t_i(\theta^t))}{u'(c^t_j(\theta^t))} = \frac{\psi^t_i(\theta^t)}{\psi^t_j(\theta^t)} = \frac{\psi^t_{i-1}(\theta^{t-1}) + \varphi^t_i(\theta^t)}{\psi^t_{j-1}(\theta^{t-1}) + \varphi^t_j(\theta^t)}.
\]

\(^{23}\)Kehoe and Levine (1993, 2001), Kocherlakota (1996), and Alvarez and Jermann (2000) provide decentralization results for the type of constrained efficient allocations consider here.
Note that conditions (3), (4) and (5) pin down a feasible symmetric stationary Markov allocation \((c_{1h}, c_{1l}, c_{2h}, c_{2l})\). In this proof, I check if the first order condition (17) is satisfied with the symmetric stationary allocation \((c_{1h}, c_{1l}, c_{2h}, c_{2l})\) and nonnegative Lagrange multipliers \(\{\varphi^i_t\}_{i,t}\). As shown below, conditions (6) and (7) guarantee the nonnegativity of Lagrange multipliers.

Suppose that \(\theta_t = (1, h)\). Recall that when \(\theta_t = (1, h)\), Agents 1, 2, 3, and 4 receive \(\omega_{1h}, \omega_{1l}, \omega_{2h},\) and \(\omega_{2l}\), respectively. Since \(V^a_{2t} > V^a_{2l}\), the complementary slackness condition implies that \(\varphi^4_t(\theta_t) = 0\). There are three cases to consider. (In what follows, I suppress \(t\) and \(\theta_t\) in \(\varphi^i_t(\theta_t)\).)

1. \(\theta_{t-1} = (1, l)\): I check if the first order conditions hold with nonnegative Lagrange multipliers for the transition from \(\theta_{t-1} = (1, l)\) to \(\theta_t = (1, h)\). First, for Agents 1 and 4,

\[
\frac{\psi^4_{t-1}}{\psi^3_{t-1}} = \frac{u'(c^1_{t-1})}{u'(c^2_{t-1})} > \frac{u'(c^1_{1h})}{u'(c^1_{2l})} = \frac{\psi^4_t}{\psi^3_t} = \frac{\psi^4_{t-1}}{\psi^3_{t-1} + \varphi^3}.
\]

From the left, the first equality is the first order condition at \(t - 1\), the second equality holds since \(\theta_{t-1} = (1, l)\), the strict inequality holds since \(c_{1h} > \{c_{1l}, c_{2h}\} > c_{2l} > 0\), the third equality holds since \(\theta_t = (1, h)\), the fourth equality is the first order condition at \(t\), and the last equality holds since \(\varphi^4_t = 0\). Therefore, \(\varphi^3 > 0\). Similarly,

\[
\frac{\psi^4_{t-1}}{\psi^3_{t-1}} = \frac{u'(c^2_{2l})}{u'(c^2_{2l})} > \frac{u'(c^1_{1l})}{u'(c^1_{2l})} = \frac{\psi^4_{t-1}}{\psi^3_{t-1} + \varphi^3}.
\]

Therefore, \(\varphi^3 > 0\). Note that

\[
\frac{\psi^4_{t-1}}{\psi^3_{t-1}} = \frac{u'(c^1_{1h})}{u'(c^1_{2l})} > \frac{u'(c^1_{1l})}{u'(c^1_{2l})} = \frac{\psi^4_{t-1}}{\psi^3_{t-1} + \varphi^3},
\]

where the weak inequality follows from the assumption \(u'(c_{1h})/u'(c_{1l}) \geq u'(c_{2h})/u'(c_{2l})\). Therefore, \(\varphi^2 \geq 0\).

2. \(\theta_{t-1} = (2, h)\): As in Case 1, \(\varphi^1 > 0\) and \(\varphi^2 > 0\) due to the first order conditions and the fact that \(c_{1h} > \{c_{1l}, c_{2h}\} > c_{2l} > 0\). As for \(\varphi^3\), note that

\[
\frac{\psi^4_{t-1}}{\psi^3_{t-1}} = \frac{u'(c^1_{1h})}{u'(c^1_{2l})} > \frac{u'(c^1_{1l})}{u'(c^1_{2l})} = \frac{\psi^4_{t-1}}{\psi^3_{t-1} + \varphi^3}.
\]

Therefore, \(\varphi^3 \geq 0\).
3. $\theta_{t-1} = (2, l)$: $\varphi^1 \geq 0$ since $c_{1h} > c_{2l}$. As for $\varphi^2$, note that
\[
\frac{\psi^4_{t-1}}{\psi^2_{t-1}} = \frac{u'(c_{2h})}{u'(c_{1h})} > \frac{u'(c_{1h})}{u'(c_{2l})} = \frac{\psi^4_{t-1}}{\psi^2_{t-1}} + \varphi^2,
\]
where the strict inequality holds because $c_{1h} > c_{2h}$ and $c_{1l} > c_{2l}$. Therefore, $\varphi^2 > 0$. As for $\varphi^3$, note that
\[
\frac{\psi^4_{t-1}}{\psi^3_{t-1}} = \frac{u'(c_{1l})}{u'(c_{1h})} > \frac{u'(c_{2h})}{u'(c_{2l})} = \frac{\psi^4_{t-1}}{\psi^3_{t-1}} + \varphi^3,
\]
where the strict inequality holds because $c_{1h} > c_{1l}$ and $c_{2h} > c_{2l}$. Hence $\varphi^3 > 0$.

The Lagrange multiplier $\varphi^i_t(\theta^t)$ is also well defined and non-negative at $t = 0$ since initial weights $\langle \lambda^i \rangle_{t=1}^4$ are the same for all agents. The above and the complementary slackness conditions imply that $V_{1h}^m = V_{1h}^a$, $V_{1l}^m = V_{1l}^a$, and $V_{2h}^m = V_{2h}^a$. Apparently, the same argument applies to $\theta_{t} = (1, l)$, $(2, h)$, and $(2, l)$.

Therefore, one can conclude that the symmetric stationary Markov allocation $(c_{1h}, c_{1l}, c_{2h}, c_{2l})$ and the associated nonnegative Lagrange multipliers $\{\varphi^i_t\}_{i,t}$ satisfy the first order condition (17) and the complementary slackness condition for all $t$ and $\theta^t$. Hence the symmetric stationary Markov allocation satisfying all the conditions in Proposition 1 is constrained efficient. □

Remark 1. Marcet and Marimon (1998) show that the solution to the planner problem has a feedback representation with $\langle \psi^j_{t-1} \rangle_{j=1}^4, \theta_t$ as state variables. The feedback representation is,
\[
\begin{cases}
  c^i_t(\theta^t) = c^i(\langle \psi^j_{t-1} \rangle_{j=1}^4, \theta_t), \quad \varphi^i_t(\theta^t) = \varphi^i(\langle \psi^j_{t-1} \rangle_{j=1}^4, \theta_t), \\
  \psi^i_t = \psi^i_{t-1} + \varphi^i(\langle \psi^j_{t-1} \rangle_{j=1}^4, \theta_t), \quad \theta_t \text{ is a first-order Markov chain.}
\end{cases}
\]

For the current problem, $\varphi^i_t((\theta^{t-1}, \theta_t)) = \psi^i_{t-1} \left( \frac{u'(c^i_t(\theta^t))}{u'(c^i_{t-1}(\theta^{t-1}))} \right) - \psi^i_{t-1} = \varphi^i(\langle \psi^j_{t-1} \rangle_{j=1}^4, \theta_t)$ for $i \in \{1, 2, 3\}$, when $\theta_t = (1, h)$. The term $u'(c^i_t(\theta^t))/u'(c^i_{t-1}(\theta^{t-1}))$ depends only on $\theta_t$ because the consumption allocation is symmetric stationary Markov and endowments are deterministically linked across agents as specified above. By this argument, it is clear that $\varphi^i_t(\theta^t)$ can be written as a function of $\langle \psi^j_{t-1} \rangle_{j=1}^4$ and $\theta_t$ for all $\theta_t \in \Theta$. Therefore, the consumption-multiplier pair has the recursive structure proven by Marcet and Marimon (1998). □
Remark 2. Symmetric stationary Markov allocations can be optimal under asymmetric initial weights. However, initial weights must satisfy the following conditions. Let \((c_{1h}, c_{1l}, c_{2h}, c_{2l})\) be an allocation satisfying (4) and (5), and suppose that \(\theta_0 = (1, h)\). The first order conditions hold at \(t = 0\) with nonnegative Lagrange multipliers \((\varphi'^i)_{i=1}^4\) if and only if initial weights \((\lambda^i)_{i=1}^4\) satisfy the following:

\[
\frac{u'(c_{1h}^i)}{u'(c_{2l}^i)} = \frac{u'(c_{1l}^i)}{u'(c_{2l}^i)} \leq \frac{\lambda_4}{\lambda_1}, \quad \frac{u'(c_{2h}^i)}{u'(c_{2l}^i)} \leq \frac{\lambda_4}{\lambda_2} \quad \text{and} \quad \frac{u'(c_{2l}^i)}{u'(c_{2l}^i)} \leq \frac{\lambda_4}{\lambda_3}.
\]

Equivalently,

\[
\lambda^4 \geq \max\{\frac{u'(c_{1h}^i)}{u'(c_{2l}^i)} \lambda^1, \frac{u'(c_{1l}^i)}{u'(c_{2l}^i)} \lambda^2, \frac{u'(c_{2h}^i)}{u'(c_{2l}^i)} \lambda^3\}.
\]

Equal weights \((\lambda^i = 1/4 \text{ for all } i)\) satisfy this condition. However, it is not the only case in which the condition holds. For other \(\theta_0\)'s, similar conditions on initial weights must be satisfied. To summarize, the following weights work.

\[
(\lambda^i)_{i=1}^4 \text{ such that } \begin{cases}
\lambda^4 \geq \max\{\frac{u'(c_{1h}^i)}{u'(c_{2l}^i)} \lambda^1, \frac{u'(c_{1l}^i)}{u'(c_{2l}^i)} \lambda^2, \frac{u'(c_{2h}^i)}{u'(c_{2l}^i)} \lambda^3\} \quad \text{(for } \theta_0 = (1, h)\}) \\
\lambda^3 \geq \max\{\frac{u'(c_{1l}^i)}{u'(c_{2l}^i)} \lambda^1, \frac{u'(c_{1h}^i)}{u'(c_{2l}^i)} \lambda^2, \frac{u'(c_{2h}^i)}{u'(c_{2l}^i)} \lambda^4\} \quad \text{(for } \theta_0 = (1, l)\}) \\
\lambda^2 \geq \max\{\frac{u'(c_{2h}^i)}{u'(c_{2l}^i)} \lambda^1, \frac{u'(c_{1h}^i)}{u'(c_{2l}^i)} \lambda^3, \frac{u'(c_{1l}^i)}{u'(c_{2l}^i)} \lambda^4\} \quad \text{(for } \theta_0 = (2, h)\}) \\
\lambda^1 \geq \max\{\frac{u'(c_{2l}^i)}{u'(c_{2l}^i)} \lambda^1, \frac{u'(c_{1l}^i)}{u'(c_{2l}^i)} \lambda^2, \frac{u'(c_{1h}^i)}{u'(c_{2l}^i)} \lambda^4\} \quad \text{(for } \theta_0 = (2, l)\}).
\]

In words, the weight of the agent with \(\omega_{2l}\) at \(t = 0\) must be high enough. Otherwise, the planner would find it optimal to extract more resources from the agent with \(\omega_{2l}\) at \(t = 0\) since the agent’s participation constraint is yet to bind.

As long as the agent with the lowest income at \(t = 0\) is assigned a weight high enough, symmetric allocations can be optimal even with asymmetric initial weights. It is because the planner’s ability to reallocate resources can be highly restricted by the participation constraints and the endowment process is symmetric. In fact, Proposition 2 tells us that constrained efficient symmetric stationary Markov allocations with some but not perfect risk sharing must satisfy \(V_{\theta}^m = V_{\theta}^a\) for all \(\theta \in \Theta \setminus \{(2, l)\}\) and \(\sum_{\theta} c_{\theta} = \sum_{\theta} \omega_{\theta}\). These conditions are enough to pin down an allocation. This fact indicates that such allocations can be optimal only in special cases. It is indeed the case as shown in Proposition 3. □
A2.2. Proof of Proposition 2

Let us first prove that if a feasible symmetric stationary Markov allocation is constrained efficient and features some risk sharing, then it must be that $V_{2l}^m > V_{2l}^a$ (Condition (3)). I prove the following contrapositive of the statement.

Suppose that a feasible symmetric stationary Markov allocation features $V_{2l}^m = V_{2l}^a$. Then it is either autarky or not constrained efficient.

First of all, note that if $V_{2l}^m = V_{2l}^a$, then $c_{2l} \leq \omega_{2l}$. The inequality becomes strict if and only if $V_{\theta}^m > V_{\theta}^a$ for some $\theta \in \Theta \setminus \{(2, l)\}$. Therefore, if $c_{2l} = \omega_{2l}$, then $V_{\theta}^m = V_{\theta}^a$ for all $\theta$, which implies that the allocation is autarky. Next, let us consider the case of $c_{2l} < \omega_{2l}$. In this case, there exists $\theta \in \Theta \setminus \{(2, l)\}$ such that $V_{\theta}^m > V_{\theta}^a$ and $c_{2l} > \omega_{\theta}$. Then, one can construct a new feasible allocation $(c_{1h}^l, c_{1l}^l, c_{2h}^l, c_{2l}^l)$ from $(c_{1h}, c_{1l}, c_{2h}, c_{2l})$ by moving some resources from $c_{2l}$ to $c_{2l}$. Since the period utility function is strictly concave, the new allocation $(c_{1h}^l, c_{1l}^l, c_{2h}^l, c_{2l}^l)$ obtains a higher value of the planner’s objective function (with equal initial weights) than the original allocation $(c_{1h}, c_{1l}, c_{2h}, c_{2l})$. Therefore, if a feasible symmetric stationary Markov allocation features $V_{2l}^m = V_{2l}^a$ and $c_{2l} < \omega_{2l}$, then it is not constrained efficient.

From the proof of Proposition 1, it is clear that the conditions (4) and (7) are not only sufficient but also necessary for optimality. As for (5), the first order condition and the complementary slackness condition imply (5) provided $c_{2l} < \min\{c_{1h}, c_{1l}, c_{2h}\}$. Let us prove this statement here and I conclude this proof by confirming $c_{2l} < \min\{c_{1h}, c_{1l}, c_{2h}\}$. Recall the first order condition. (See the proof of Proposition 1 for notation.)

$$
\frac{u'(c_i^l(\theta^t))}{u'(c_i^l(\theta^t))} = \frac{\psi_i^l(\theta^t)}{\psi_i^l(\theta^t)} = \frac{\psi_i^{l-1}(\theta^{t-1}) + \phi_i^l(\theta^t)}{\psi_i^{l-1}(\theta^{t-1}) + \psi_i^l(\theta^t)}. \tag{18}
$$

Note that the transition probability from the state $(2, l)$ to any other state is strictly positive. Suppose that Agent 1 is in the state $(2, l)$ in period $t - 1$ and transits to the state $(1, l)$ in period $t$. Equation (18) implies that $\psi_i^{l-1}(\theta^{t-1}) < \psi_i^{l-1}(\theta^{t-1})$ and $\psi_i^l(\theta^t) > \psi_i^l(\theta^t)$. (Recall that whenever Agent 1 is in the state $(2, l)$, Agent 3 is in the state $(1, l)$, and vice versa.) In order for the above two inequalities to hold, it must be that $\phi_i^l(\theta^t) > 0$. Then the complementary slackness condition implies that the participation constraint for Agent 1 at $t$ holds with equality. Since symmetric stationary Markov allocations are Markovian, it means that participation constraints
for those who are currently in the state \((1,l)\) always hold with equality. The same argument applies to the participation constraints for \((1,h)\) and \((2,h)\).

In the rest of the proof, I show that any constrained efficient symmetric stationary Markov allocation with some but not perfect risk sharing features \(c_{2l} < \min\{c_{1h}, c_{1l}, c_{2h}\}\). First, \(c_{2l}\) cannot exceed \(c_{1h}\), \(c_{1l}\), or \(c_{2h}\). To see this, suppose that \(c_\theta < c_{2l}\) for some \(\theta \in \Theta \setminus \{(2,l)\}\). Then the first order condition and the complementary slackness condition imply that \(V^m_{2l} = V^\alpha_{2l}\). It contradicts the fact that \(V^m_{2l} > V^\alpha_{2l}\) shown above. Hence, \(c_{2l} \leq \min\{c_{1h}, c_{1l}, c_{2h}\}\).

There are essentially two cases other than \(c_{2l} < \min\{c_{1h}, c_{1l}, c_{2h}\}\), namely, \(c_{1h} > c_{1l} \geq c_{2h} = c_{2l}\) and \(c_{1h} = c_{1l} > c_{2h} = c_{2l}\). (The rest of the proof goes through with only notational modifications when \(c_{1h}, c_{1l}\), and \(c_{2h}\) are interchanged.) First, consider the former case. Suppose that \(\theta_{t-1} = (1,h)\) and \(\theta_t = (2,h)\). Then Agent 1 transits from \((1,h)\) to \((2,h)\), and Agent 2 transits from \((1,l)\) to \((2,l)\). The following holds:

\[
\frac{\psi^2_{t-1}}{\psi^1_{t-1}} = \frac{u'(c^1_{l-1})}{u'(c^1_{l-1})} = \frac{u'(c_{1h})}{u'(c_{2h})} < \frac{u'(c_{1l})}{u'(c_{2l})} = \frac{u'(c^2_{l-1})}{u'(c^2_{l-1})} = \frac{\psi^2_t}{\psi^1_t} = \frac{\psi^2_{t-1} + \varphi^2_t}{\psi^1_{t-1} + \varphi^1_t},
\]

where, from the left, the first equality is the first order condition at \(t - 1\), the second equality holds because \(\theta_{t-1} = (1,h)\), the inequality holds because \(c_{1h} > c_{1l}\) and \(c_{2h} = c_{2l}\), the third equality holds because \(\theta_t = (2,h)\), the fourth equality is the first order condition at \(t\), and the last equality holds by definition of cumulative multipliers \(\psi^i_t(\theta^t)\).\(^{24}\) Since the Lagrange multiplier must be nonnegative, \(\varphi^2_t > 0\), which implies that the participation constraint for \((2,l)\) binds. It contradicts the assumption that there is some risk sharing. Second, consider the case in which \(c_{1h} = c_{1l} > c_{2h} = c_{2l}\). In this case, we have \(V^m_{1h} = V^m_{1l}\) and \(V^m_{2h} = V^m_{2l}\). Again, the first order condition and the complementary slackness condition tell us that \(V^m_{1l} = V^\alpha_{1h}\) and \(V^m_{1l} = V^\alpha_{1l}\). But it is impossible since \(V^\alpha_{1h} > V^\alpha_{1l}\). Therefore, it must be that \(c_{2l} < \min\{c_{1h}, c_{1l}, c_{2h}\}\). \(\square\)

**A2.3. Proof of Lemma 1**

The following is the system of equations consisting of (4) and (5), together with the definition of \(V^\alpha_\theta\) and \(V^m_\theta\). (With group-specific income shocks, Proposition 2 shows that the constrained efficient symmetric stationary Markov allocation solves the system of equations.)

\(^{24}\)Refer to the proof of Proposition 1 for the first order conditions and the cumulative multipliers.
\[
\begin{align*}
V_\theta^a &= u(\omega_\theta) + \beta \sum_{\theta' \in \Theta} \pi(\theta' | \theta) V_{\theta'}^a \quad (\forall \theta \in \Theta), \\
V_\theta^m &= u(c_\theta) + \beta \sum_{\theta' \in \Theta} \pi(\theta' | \theta) V_{\theta'}^m \quad (\forall \theta \in \Theta), \\
\sum_{\theta \in \Theta} c_\theta &= \sum_{\theta \in \Theta} \omega_\theta.
\end{align*}
\]

The above system simplifies to the following four equations. Define \( F_1, F_2, F_3, F_4 \) by

\[
F_1(c_{1h}, c_{1l}, c_{2h}, c_{2l}; \rho) = a(u(\omega_{1h}) - u(c_{1h})) + b(u(\omega_{1l}) - u(c_{1l})) + d(u(\omega_{2h}) - u(c_{2h})) - e(u(c_{2l}) - u(\omega_{2l})) = 0
\]

\[
F_2(c_{1h}, c_{1l}, c_{2h}, c_{2l}; \rho) = b(u(\omega_{1h}) - u(c_{1h})) + a(u(\omega_{1l}) - u(c_{1l})) + e(u(\omega_{2h}) - u(c_{2h})) - d(u(c_{2l}) - u(\omega_{2l})) = 0
\]

\[
F_3(c_{1h}, c_{1l}, c_{2h}, c_{2l}; \rho) = d(u(\omega_{1h}) - u(c_{1h})) + e(u(\omega_{1l}) - u(c_{1l})) + a(u(\omega_{2h}) - u(c_{2h})) - b(u(c_{2l}) - u(\omega_{2l})) = 0
\]

\[
F_4(c_{1h}, c_{1l}, c_{2h}, c_{2l}; \rho) = c_{1h} + c_{1l} + c_{2h} + c_{2l} - (\omega_{1h} + \omega_{1l} + \omega_{2h} + \omega_{2l}) = 0
\]

where

\[
a = \frac{e}{\beta(1 - \gamma)(1 - \rho)} - 3\beta^3 \rho \gamma - 2\beta^2 \rho^2 - \beta^3 \gamma \rho - 2\beta^2 \rho^2 - 4\beta^3 \rho^2 \gamma + 2\beta^3 \rho^2 \gamma + 2\beta^3 \gamma \rho + 4\beta^2 \rho^2 \gamma
\]

\[
b = \beta(1 - \gamma)(2\rho + 2\beta \gamma - 4\beta \rho \gamma) - \frac{\rho e}{1 - \rho}
\]

\[
d = \beta(1 - \rho)(2\gamma + 2\beta \rho - 4\beta \rho \gamma) - \frac{\gamma e}{1 - \gamma}
\]

\[
e = \beta(1 - \gamma)(1 - \rho)(1 - \beta^2 + 2\beta^2 \rho + 2\beta^2 \gamma - 4\beta^2 \rho \gamma).
\]

I use the following fact to prove Lemma 1.

**Fact 1** Suppose that there exists a non-autarkic solution, denoted by \((\overline{c}_{2l}, \overline{c}_{2h})\), to the following system of equations.

\[
(1 - \beta \gamma)(u(\omega_{2h}) - u(\overline{c}_{2h})) - \beta(1 - \gamma)(u(\overline{c}_{2l}) - u(\omega_{2l})) = 0, \quad (20)
\]

\[
\overline{c}_{2h} + \overline{c}_{2l} - (\omega_{2h} + \omega_{2l}) = 0. \quad (21)
\]

Then, \(-(1 - \beta \gamma)u'(\overline{c}_{2h}) + \beta(1 - \gamma)u'(\overline{c}_{2l}) < 0\).
Proof. Note the following.

\[ \frac{u(\omega_{2h}) - u(c_{2h})}{u(c_{2l}) - u(\omega_{2l})} < \frac{u'(\tau_{2h})(\omega_{2h} - c_{2h})}{u'(\tau_{2l})(\tau_{2l} - \omega_{2l})} = \frac{u'(\tau_{2h})}{u'(\tau_{2l})}, \]

where the inequality holds by the strict concavity of \( u \) and the equality holds due to Equation (21).

Equation (20) and the above yield,

\[
\frac{\beta(1 - \gamma)}{1 - \beta \gamma} < \frac{u'(\tau_{2h})}{u'(\tau_{2l})} \quad \Leftrightarrow \quad -(1 - \beta \gamma)u'(\tau_{2h}) + \beta(1 - \gamma)u'(\tau_{2l}) < 0. \quad \square
\]

Proof of Lemma 1. Substitute \( \rho = 1 \) in \( F(c_{1h}, c_{1l}, c_{2h}, c_{2l}; \rho) \). Then, \( a = (1 - \beta \gamma)(1 - \beta)(1 + \beta - 2\beta \gamma), b = \beta(1 - \gamma)(1 - \beta)(1 + \beta - 2\beta \gamma), d = 0, \) and \( e = 0 \). Solutions to the system \( F(c_{1h}, c_{1l}, c_{2h}, c_{2l}; 1) = 0 \) satisfies \( c_{1h} = \omega_{1h}, c_{1l} = \omega_{1l}, \) and

\[
\begin{align*}
(1 - \beta \gamma)(u(\omega_{2h}) - u(c_{2h})) - \beta(1 - \gamma)(u(c_{2l}) - u(\omega_{2l})) &= 0, \quad (22) \\
c_{2h} + c_{2l} - (\omega_{2h} + \omega_{2l}) &= 0. \quad (23)
\end{align*}
\]

Equations (22) and (23) are identical to Equations (20) and (21), respectively. Therefore, the non-autarkic solution is \( (\omega_{1h}, \omega_{1l}, \bar{c}_{2h}, \bar{c}_{2l}) \).\textsuperscript{25} Jacobian \( |J| \) of \( F(c_{1h}, c_{1l}, c_{2h}, c_{2l}; \rho) \) with respect to \( (c_{1h}, c_{1l}, c_{2h}, c_{2l}) \) at \( (\omega_{1h}, \omega_{1l}, \bar{c}_{2h}, \bar{c}_{2l}; 1) \) is as follows,

\[
|J| = (-au'(\omega_{1h}))(\omega_{1h} - \sigma_{2h})(-au'(\tau_{2h})) - a\sigma_{1l}u'(\sigma_{1l})b\sigma_{2l}u'(\tau_{2l})
+ (1)(-bu'(\omega_{1l}))(\omega_{1l} - \sigma_{2l})(-au'(\sigma_{2l})) - b\omega_{1l}u'(\omega_{1l})b\tau_{2l}u'(\tau_{2l})
- \{-1 - \beta \gamma\}u'(\tau_{2h}) + \beta(1 - \gamma)u'(\tau_{2l})
\times \{(1 - \beta \gamma)^2 - \beta^2(1 - \gamma)^2\}(1 - \beta)^3(1 + \beta - 2\beta \gamma)^3u'(\omega_{1h})u'(\omega_{1l}).
\]

By Fact 1, \(|J| < 0\). Hence the Implicit Function Theorem tells us that there exits a neighborhood of \( \rho = 1 \) in which the solution to the system \( F = 0 \) is continuous in \( \rho \). \square

\textsuperscript{25}The other solution is autarky \( (\omega_{1h}, \omega_{1l}, \omega_{2h}, \omega_{2l}) \).
References


