# Inflation and Unemployment in the Long Run*

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**Abstract**

We study the long-run relation between money, measured by inflation or interest rates, and unemployment. We first discuss data, documenting a strong positive relation between the variables at low frequencies. We then develop a framework where both money and unemployment are modeled using explicit microfoundations, integrating and extending recent work in macro and monetary economics, and providing a unified theory to analyze labor and goods markets. We calibrate the model, to ask how monetary factors account quantitatively for low-frequency labor market behavior. The answer depends on two key parameters: the elasticity of money demand, which translates monetary policy to real balances and profits; and the value of leisure, which affects the transmission from profits to entry and employment. For conservative parameterizations, money accounts for some but not that much of trend unemployment – e.g. we can explain around 20% of the increase in unemployment during the 70s stagflation by monetary policy alone. For less conservative parameters, money accounts for much of the low-frequency movement in unemployment over the last half century, and explains between 68 and 86% of the increase in unemployment during stagflation.

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# 1 Introduction

Since it is very relevant for what we do in this project, we begin by reviewing an exercise in Lucas (1980). He was interested in two fundamental propositions from monetary economics: the quantity equation, which can be interpreted as saying that (other things equal) inflation moves one-for-one with the growth rate in the money supply; and the Fisher equation, which can be interpreted as saying that (other things equal) the nominal interest rate moves one-for-one with inflation.[1] These relations are derived from elementary economic principles, and are almost 'model free' in the sense e.g. that the quantity equation emerges from a variety of formalizations, and the Fisher equation is basically a no-arbitrage condition. This does not mean they are consistent with data. Indeed, as Lucas emphasized, one ought not expect them to hold at each point in time since there may be a lot going on to complicate matters in the short run; yet they may still be useful ideas if they are consistent with longer-run observations.

To investigate this, Lucas plotted inflation vs. the growth rate of $M1$, using annual data, from 1955-1975, which we reproduce in the upper left panel of Figure 1.1, except using quarterly data, and extended to 2005.[2] Although the simple regression line slopes upwards, it is not that easy to see the quantity equation in the picture – but, again, there may be a lot going on at high frequencies to obscure the relation. So Lucas filtered the data, using progressively stronger filters to remove more and more of the short-run 'noise.' We do the same in the other panels of Figure 1.1, using HP filters with a parameter varying

[1] Lucas actually looked not at the Fisher equation per se, but the relation between money growth and nominal rates. If the quantity equation is correct, this amounts to the same thing, but in any case we look at both.

[2] All figures are at the end of the paper. Also, we actually put together data for all variables discussed below going back to 1948, but focus on the sample starting in 1955 for three reasons: this is where Lucas started; it gives us exactly a half century of data, which is a nice round number; and certain series like inflation seem especially erratic in the late 40s and early 50s. But we are not trying to hide anything – results for the full data set are at http://www.wwz.unibas.ch/witheo/aleks/BMWII/BMWII.html.

from 0 to 160,000 as indicated on each panel (Lucas used moving average filters, but nothing hinges on this detail). As one can plainly see, with progressively stronger filtering, a distinct pattern emerges, and eventually it appears that the quantity equation looks really quite good.

This finding is robust on several dimensions. One can look at five-year averages, a different way to filter the data, shown in the final panel of Figure 1.1, and the message is the same. Or one can measure variables in different ways, as we do in Figures 1.2 and 1.3, where we replace $M1$ by $M2$ and by $M0$, and the picture is similar. One can also redo the exercise in levels (looking at $p$ vs. $M$ rather than growth in $p$ and $M$) and the results are similar.[3] In terms of the Fisher equation, Figure 1.4 plots inflation vs. the nominal interest rate using Aaa corporate bonds to define the nominal rate (the conclusions are similar using e.g. the T-Bill rate). After we filter out the 'noise' the Fisher equation also looks very good. Figures 1.5 makes a similar point when we replace inflation by $M1$ growth (results for $M2$ and $M0$ are similar). Just as Lucas concluded from his exercise, we conclude from this that the ideas represented by the quantity and Fisher equation hold up quite well in long-run data.

Lucas warns us, however, that the method is risky. Take any two series, he says, plot progressively stronger filtered versions, and one will see patterns emerge. To illustrate his point Lucas does the exercise for inflation and unemployment, two variables that he 'knew' were unrelated at low frequency, in the sense that he was persuaded by the arguments of Friedman (1967) and Phelps (1969) that the long-run Phillips curve must be vertical (although he does say this explicitly, it seems from related work such as Lucas 1973 he bought into the idea of a 'natural rate' independent of inflation). Lo and behold, with progressive filtering, a pattern between inflation and unemployment emerged when

[3] In the interest of space we do not show all the figures here; go to the address in footnote 2 for additional figures and much more information, including details of the data sources, calibration and simulation programs, etc.

Lucas did it, and emerges even more obviously when we redo it with updated data. As Figure 1.6 shows, contrary to what Lucas thought he 'knew' from theory, inflation and unemployment are related in the long run, and positively.

We like his method for extracting information about long-run relations, but do not agree with all of Lucas' conclusions. In terms of method, we are persuaded that this filtering technique, while perhaps not perfect, is a useful tool – in particular, while there is no guarantee that forces driving the short-run deviations are irrelevant for understanding the true long-run relation, the approach does have the virtue of allowing one to avoid taking a stand on exactly what the forces are behind the high frequencies. Where we think Lucas went wrong is his devotion to a vertical long-run Phillips curve. We find the evidence of a positive relation between inflation and unemployment about as clear as the evidence for the quantity or Fisher equation, and based on this data there seems little reason to deem one observation compelling and another statistical artifact; moreover, we will argue, a positive long-run relation between these variables is as much "an implication of a coherent economic theory" as Lucas said the other relations are.

We are not the first to suggest this, and Friedman (1977) himself was trenchant when he said the following: "There is a natural rate of unemployment at any time determined by real factors. This natural rate will tend to be attained when expectations are on average realized. The same real situation is consistent with any absolute level of prices or of price change, *provided allowance is made for the effect of price change on the real cost of holding money balances*" (emphasis added). He also noted that in the data he was examining at the time one could see emerging evidence of an upward slope to the long-run Phillips curve (others have discussed similar points; see Beyer and Farmer 2007 and the references therein). Again, we will show here that basic economic theory predicts such a pattern just as clearly as the data depicts such a pattern.

Before proceeding we mention some more evidence. In principle, if the Fisher and quantity equations are valid, it does not matter if we examine the relationship between unemployment and either inflation, interest, or money growth rates. But although the Fisher and quantity equations hold up well in the longer run, they do not hold exactly. In Figures 1.7 and 1.8 we redo the exercise replacing inflation with interest and $M1$ growth rates ($M2$ and $M0$ give similar results). Also, in Figures 1.9 to 1.11 we redo the exercises using employment rather than unemployment.[4] Based on all of this, we think there is a clear negative relation between monetary variables and labor market performance in the longer run, even if the relation may sometimes go the other way in the shorter run, including the 60s where a downward sloping Phillips curve is evident. While we welcome more, and more sophisticated, econometric analyses, for the purpose of this paper we take this fact as given.

As a final application of the method, and because we will need it later, Figures 1.12 to 1.14 show the relation between the nominal rate and the inverse of velocity, $M/pY$, commonly interpreted as money demand. The different plots use $M1$, $M2$ and $M0$. As has been documented many times, the relationship is negative, although it is confounded by what looks like a structural shift that occurs some time in the late 80s or early 90s, depending on which panel one looks at. Similar results obtain when we replace the nominal interest rate by the inflation or money growth rates.[5] In any event, we will use some version of this money demand relation in the calibration below, as is done in most quantitative monetary economics (see Cooley-Hansen 1989, Lucas 2000, Lagos-Wright 2005, and the references therein).

[4] This is complicated by a long-term trend in employment over the sample, presumably due to demographic and other factors. To control for this we filter the data twice: once to eliminate the very long-run trend, and again to eliminate very high-frequency fluctuations.

[5] That is, the results are similar except for one detail: while $M0/pY$ and $M1/pY$ behave as expected, a simple regression indicates $M2/pY$ rises with inflation or money growth; this may however be an artifact of the strucutral shift mentioned above (see the website mentioned in footnote 2).

We now proceed to theory. Since we are primarily for this paper interested in the longer-run relation between monetary variables and unemployment, we abstract from factors commonly believed to matter in the short run, including information problems or other forms of real-nominal confusion, as well as stickiness in wages or prices. Instead, we focus on Friedman's suggestion that to understand the effect of monetary variables on the "natural rate" allowance really must be made for "the effect of price change on the real cost of holding money balances." To this end, it seems obvious that it would be good to have a theory where the cost of holding money balances can be made precise, which suggests to us a theory where the benefits of holding money balances are made explicit. Additionally, it would seem good to have a theory of unemployment that has proven successful in other contexts.

In recent years much progress has been made studying monetary economics and unemployment using theories that incorporate frictions – in the case of unemployment, search and matching frictions; and in the case of money, some sort of double coincidence problem due to specialization and spatial separation, combined with information problems like imperfect record keeping.[6] It is not surprising that models with frictions are useful for understanding dynamic labor markets and hence unemployment, as well as for understanding the role of money and hence inflation. However, existing models along these lines analyze either unemployment *or* inflation in isolation. We integrate these models into a unified framework that allows one to analyze unemployment *and* money together using logically consistent microfoundations. This theory predicts that inflation and unemployment should move together.

[6] In terms of unemployment, we have in mind Mortensen and Pissarides (1994), as well as earlier work by Diamond (1981,1982), Mortensen (1982), Pissarides (1985,1990), Merz (1995), Andolfatto (1996), and recent contributions by Shimer (2005), Hall (2005), Hagedorn and Manovskii (2007) and others. In terms of money, we have in mind the model in Lagos and Wright (2005), but going back to Kiyoatki and Wright (1989,1993), Aiyagari and Wallace (1991), Matsuyama et al. (1993), Shi (1995,1997), Trejos and Wright (1995), Kochelakota (1998), Wallace (2001), Williamson (2006), Molico (2006) and others.

We then consider the issue quantitatively. To this end we calibrate the model to standard observations, including money demand, and ask how it accounts for long-run labor market observations over the last half century when we counterfactually assume monetary policy is the only impulse. This is a common method in modern macro, as epitomized when one asks of the Kydland and Prescott (1982) model how well it accounts for output fluctuations when the only impulse is a shock to productivity. As in that application, the target here is not 100%; we just want to know how much. Although there are details to discuss, one way to summarize the findings is to ask the following question: For reasonable parameter values, how much of an increase in unemployment does the model predict from a run up in inflation or nominal interest rates like we saw during the stagflation of the 70s? The answer, we show, depends on two key parameters: the value of leisure and money demand elasticity.

For a conservatively low estimate of the money demand elasticity, if we set the value of leisure so that a real version of the model generates realistic unemployment fluctuations in response to productivity shocks, we account for only 20% of the increase in the raw unemployment series, and around 13% of filtered unemployment, during stagflation. This is nothing to scoff at, but obviously does leave plenty of room for other factors, including productivity, demographics, fiscal policy etc. However, if we set the value of leisure slightly higher, the model can account for virtually all of trend unemployment during the period, although of course it then generates excessive unemployment fluctuations in response to real shocks (about double the data). For a bigger money demand elasticity, the basic message is similar, although the model accounts for more of the data with a low value of leisure, and does not generate as excessive unemployment fluctuations in response to real shocks.

We conclude that while conservative parameter estimates imply monetary factors account for some but not the majority of trend unemployment, one does not have to stretch parameters too far to account for much more. It should be no surprise that some parameters matter a lot for the issues at hand. That the value of leisure can make a big difference in search-based models of the labor market is very well known; see e.g. the discussion of Shimer (2005) in Hagedorn and Manovskii (2007).[7] That the elasticity of money demand matters a lot for the effects of inflation is equally well known; see e.g. Lucas (2000). It is to be expected therefore that both matter in the integrated model. While our results do depend on parameters, and hence we cannot provide one definitive number, they indicate that monetary factors can be important for labor market outcomes not only theoretically but also quantitatively.

The rest of the paper is organized as follows. In Section 2 we describe the basic model. In Section 3 we show how to solve for equilibrium in the labor market taking the goods market as given, and vice-versa, and then put things together to get general equilibrium. In the presentation in Section 3 we focus on steady states, and relegate the dynamic-stochastic case to the Appendix. Also, in Section 3 we use Nash bargaining in both goods and labor markets, but in Section 4 we consider different pricing mechanisms, including price taking and posting. In Section 5 we present the quantitative analysis. Section 6 concludes wit a brief summary.[8]

[7] The macro-labor literature has not yet converged on the best way to extend the baseline search model to generate realistic unemployment responses to real shocks, but everyone agrees that a high value of leisure gets the job done. It is likely that some other 'tricks' to generate realistic unemployment responses to real shocks would also work for us. One way to summarize this is our robust finding that an increase in inflation from 0 to 10% will have the same impact as a drop in labor productivity of between 2/3 and 3/2 of 1%, independent of how we specify the labor market parameters, whose only role is to determine how the effect is propagated to unemployment.

[8] Some recent attempts to bring monetary issues to bear on search-based labor models include Farmer (2005), Blanchard and Gali (2005), and Gertler and Trigari (2006), but they take a different tact by assuming nominal rigidities. We generate interesting effects without nominal wage or price stickiness, as which seems distinctly preferable given we are interested in intermediate- to long-run phenomena. Lehmann (2006) is more in line with our approach, although details are different. Shi (1998,1999) and Shi and Wang (2006) are also worth mentioning. Rocheteau et al. (2006) and Dong (2007) integrate modern monetary economics into an alternative theory of unemployment – Rogerson's (1988) indivisible labor model – and while that approach leads to some interesting results, there are reasons to prefer Mortensen-Pissarides. Earlier, Cooley and Hansen (1989) stuck a cash-in-advance constraint

## 2 The Basic Model

Time is discrete and continues forever. Each period, there are three distinct locations where economic activity takes place: a labor market, in the spirit of Mortensen-Pissarides; a goods market, in the spirit of Kiyotaki-Wright; and a general market, in the spirit of Arrow-Debreu. For brevity we call these the MP, KW and AD markets. While it does not matter for the results, for concreteness we assume these markets convene sequentially, as shown in Figure 2. Also, without loss of generality we assume that agents discount at rate $\beta$ between one AD market and the next MP market, but not between the other markets. There are two types of private agents, firms and households, indexed by $f$ and $h$. The set of households is $[0, 1]$; the set of firms has arbitrarily large measure, although not all will be active at any point in time. Households work, consume, and enjoy utility; firms simply maximize profits and pay out dividends to households.

As is standard in modern theories of unemployment, a household and a firm can combine to create a job that produces output $y$. Let $e$ index employment status: $e = 1$ indicates that a household (firm) is matched with a firm (household); $e = 0$ indicates otherwise. For now, it is easiest to think of agents matching bilaterally in the MP and KW markets and multilaterally in AD, although we also discuss other interpretations below. As indicated in Figure 2, there are three value functions for the three markets, $U_e^i$, $V_e^i$ and $W_e^i$, which generally depend on type $i \in \{h, f\}$, employment status $e \in \{0, 1\}$, and possibly other state variables. Also, $\hat{U}_f^i$ in the Figure is the MP value function next period, since a "hat" indicates the value of variables next period (in stationary equilibrium these are the same).

into Rogerson, as Cooley and Quadrini (2004) and Andofatto et al. (2003) did to Mortensen-Pissarides. Our framework actually nests as special case something that looks like a standard cash-in-advance model, as well as a money-in-the-utility-function model. We prefer to lay out the role of money explicitly, however, because the additional generality is useful, and also because we find it easier than having to decide based on implicit theorizing when cash-in-advance applies, or how money enters utility.

In the benchmark model discussed in the text, we assume policy and productivity are constant, and focus on steady states; in this case, the only state variable for agents that we need to track, other than $i$ and $e$, is real balances $z$.[9] We adopt the following convention for measuring real balances, which facilitates presentation of the dynamic-stochastic model discussed in the Appendix. When an agent brings in $m$ dollars to the AD market, we let $z = m/p$, where $p$ is the current price level, denote his real balances. He then takes $\hat{z} = \hat{m}/p$ out of this market and into next period's MP market, still deflated by $p$. If he were to bring $\hat{z}$ through the next KW market and into the next AD market, its real value is then given by $\hat{z}\hat{\rho}$, where $\hat{\rho} = p/\hat{p}$ converts $\hat{z}$ into the units of the numeraire in that market. Notice $\hat{\rho} = 1/(1+\pi)$, where $\pi$ is the inflation rate between this and the next AD market.

## 2.1 Households

We now consider the different markets in turn, starting with AD. Household $h$ with employment status $e$ and real balances $z$ solves

$$W_e^h(z) = \max_{x,\hat{z}} \left\{ x + \beta \hat{U}_e^h(\hat{z}) \right\} \tag{1}$$
$$\text{st } x + \hat{z} = ew + (1-e)(b+\ell) + \Delta - T + z$$

where $x$ is consumption, $w$ the wage, $b$ UI benefits, $\ell$ production of $x$ by the unemployed, $\Delta$ dividend income, and $T$ a lump-sum tax. Employment status $e$ is carried out of AD into MP next period. Notice $w$ is paid in AD even though matching and bargaining occur in MP (this is not important, but it makes some things more transparent, as discussed below). Also, as in most of the literature using MP models, utility is linear in $x$, although we have other goods traded in

[9] For matched agents, in principle, the wage $w$ is a state, since it is set in MP and carried forward to KW and AD, although it can be renegotiated next MP. To reduce clutter in the text, $w$ is subsumed in the notation; in the Appendix we present the general case where policy and productivity follow stochastic processes and unemployment varies endogenously over time, and these variables as well as wages are explicit state variables.

the KW market where agents have general utility.[10]

It is useful to provide a few results concerning AD before discussing the rest of the model. Substituting $x$ from the budget equation into the objective function in (1), we get

$$W_e^h(z) = I_e + z + \max_{\hat{z}} \left\{ -\hat{z} + \beta \hat{U}_e^h(\hat{z}) \right\} \tag{2}$$

where $I_e = ew + (1-e)(b+\ell) + \Delta - T$ is income. Notice $W_e^h$ is linear in $z$ and $I_e$. Moreover, the choice of $\hat{z}$ is independent of $z$ and $I_e$, although it could depend on $e$ through $\hat{U}_e^h$. However, the KW utility function introduced below will be independent of $e$, which makes $\partial \hat{U}_e^h / \partial \hat{z}$ and hence $\hat{z}$ independent of $e$. This gives the convenient result that every $h$ exits the AD market with the same $\hat{z}$, as long as we have an interior solution for $x$, which we can guarantee by assuming that $b + \ell$ is not too small.

In the KW market, a different good $q$ is traded, which gives $h$ utility $\upsilon(\cdot)$, with $\upsilon(0) = 0$, $\upsilon' > 0$ and $\upsilon'' < 0$.[11] In this market, households are anonymous, which generates an essential role for a medium of exchange. To convey the idea, suppose $h$ asks $f$ for $q$ now and promises to pay later – say, in the next AD market. If $f$ does now know who $h$ is, the latter can renege on such promises without fear of repercussion, so the former insists on quid pro quo. If $x$ is not storable by $h$, money steps into the role of medium of exchange (see Kocherlakota 1997, Wallace 2001, Corbae et al. 2003, Araujo 2004, and Aliprantis et al. 2007 for formal discussions). Of course, to make money essential we need only *some* anonymous trade – we need not rule out *all* barter, credit, etc. A nonmonetary version of the model with perfect credit is of interest in its own right, embedding as it does a retail sector into MP, and can actually be rendered

[10] All we really need for tractability is quasi-linearity: everything goes through if we assume AD utility is $x + \Upsilon_e(\mathbf{x})$, where $\mathbf{x}$ is a vector of other AD goods. To reduce notation, we assume a single AD good in the text, and discuss the general case in footnotes.

[11] We use Greek 'upsilon' $\upsilon$ for utility since $U$ denotes the MP value function and $u$ unemployment. We apologize to those who have trouble distinguishing 'upsilon' $\upsilon$ from 'vee' $v$, which denotes vacancies, but it should always be clear what is meant by the context.

as a special case when we run the Friedman rule $i = 0$, since this makes cash equivalent to perfect credit. But to study the impact of nominal variables like inflation we obviously want to consider a monetary version of the model.

For $h$ with real balances $z$ and employment status $e$ in KW,

$$V_e^h(z) = \alpha_h \upsilon(q) + \alpha_h W_e^h \left[\rho \left(z - d\right)\right] + (1 - \alpha_h) W_e^h(\rho z), \tag{3}$$

where $\alpha_h$ is the probability of trade and $(q, d)$ the terms of trade. We multiply any real balances taken out of KW by $\rho$ to get their value in AD. Using the linearity of $W_e^h$, following from (2), we have

$$V_e^h(z) = \alpha_h \left[\upsilon(q) - \rho d\right] + W_e^h \left(\rho z\right). \tag{4}$$

The probability $\alpha_h$ is given by a CRS matching function: $\alpha_h = \mathcal{M}(B, S)/B$, where $B$ and $S$ are the measures of buyers and sellers in the market. Letting $Q = B/S$ be the queue length, or market tightness, we have $\alpha_h = \mathcal{M}(Q, 1)/Q$. Assume $\mathcal{M}(Q, 1)$ is strictly increasing in $Q$, with $\mathcal{M}(0, 1) = 0$ and $\mathcal{M}(\infty, 1) = 1$, and $\mathcal{M}(Q, 1)/Q$ is strictly decreasing with $\mathcal{M}(0, 1) = 1$ and $\mathcal{M}(\infty, 1) = 0$ (conditions satisfied by most standard matching functions; see Menzio 2007).

As long as the surplus for $h$ in KW is positive, all households participate and $B = 1$; since they are the only ones with output for sale, only firms with $e = 1$ can participate, and $S = 1 - u$ where $u$ is the unemployment rate.[12] Thus, $\alpha_h = \mathcal{M}(1, 1 - u)$. This gives us our first spillover across markets: buyers in the goods market are better off when there are more sellers, which means less unemployment in the labor market. While the exact relation depends on details, the robust idea is that it is better to be a buyer when unemployment is low, because the probability of trade can be better, and also because in equilibrium the terms of trade can be better.

[12] To be clear, let $u$ be the unemployment rate starting the period. After the current MP market, it changes to $\hat{u}$, the rate starting next period, and it is $\hat{u}$ rather than $u$ that determines $\alpha_h$ in the KW market. In steady state $u = \hat{u}$, and so we can ignore this for now, but we are more careful in the dynamic model presented in the Appendix.

For $h$ in the MP market,

$$U_1^h(z) = V_1^h(z) + \delta \left[V_0^h(z) - V_1^h(z)\right] \tag{5}$$

$$U_0^h(z) = V_0^h(z) + \lambda_h \left[V_1^h(z) - V_0^h(z)\right], \tag{6}$$

where $\delta$ is the exogenous rate at which matches are destroyed and $\lambda_h$ the endogenous rate at which they are created. The latter is determined by another CRS matching function, $\lambda_h = \mathcal{N}(u,v)/u = \mathcal{N}(\tau,1)/\tau$, where $u$ is unemployment, $v$ is the number of vacancies, and $\tau = v/u$ is labor market tightness. Assume $\mathcal{N}(1,\tau)$ is strictly increasing in $\tau$, with $\mathcal{N}(1,0) = 0$ and $\mathcal{N}(1,\infty) = 1$, and $\mathcal{N}(1,\tau)/\tau$ strictly decreasing with $\mathcal{N}(1,0)/0 = 1$ and $\mathcal{N}(1,\infty) = 0$ (again see Menzio 2007). Wages are determined when firms and households meet in MP, although they are paid in the next AD market, which means that we do not have to worry about whether $w$ is paid in dollars or goods. There is commitment to $w$ within a period, but in ongoing matches it can be renegotiated next period when MP reconvenes.

This completes the household problem. Before moving on, we show how to collapse the three markets into one handy equation. Substituting $V_e^h(z)$ from (4) into (5) and using the linearity of $W_e^h$, we have

$$U_1^h(z) = \alpha_h \left[\upsilon(q) - \rho d\right] + \rho z + \delta W_0^h(0) + (1-\delta) W_1^h(0)$$

Something similar can be done for $U_0^h$. Updating these to next period and inserting into (2), the AD problem becomes

$$W_e^h(z) = I_e + z + \max_{\hat{z}} \left\{ -\hat{z} + \beta\hat{\alpha}_h \left[\upsilon(\hat{q}) - \hat{\rho}\hat{d}\right] + \beta\hat{\rho}\hat{z} \right\} + \beta \mathbb{E}_{\hat{e}} \hat{W}_{\hat{e}}^h(0) \tag{7}$$

where the expectation is wrt next period's employment status $\hat{e}$ conditional on $e$. We will see that the terms of trade $(\hat{q}, \hat{d})$ in the next KW market do not depend on $\hat{e}$ – see (14) below – so therefore (7) implies $\hat{z}$ is independent of $e$, $I_e$ and $z$.

## 2.2 Firms

Firms obviously carry no money out of AD. In MP,

$$U_1^f = \delta V_0^f + (1-\delta) V_1^f \tag{8}$$

$$U_0^f = \lambda_f V_1^f + (1-\lambda_f) V_0^f, \tag{9}$$

where $\lambda_f = \mathcal{N}(u,v)/v = \mathcal{N}(1,\tau)/\tau$. This is completely standard. Where we deviate from textbook MP theory is that, rather than having $f$ and $h$ each consume a share of their output, in our model, $f$ takes $y$ to the goods market, where he looks trade with other agents. The uncontroversial idea is that people do not necessarily want to consume what they make each day at work. This generates a role for a separate goods, or retail, sector. Although it might be interesting to proceed differently in future work, for now we consolidate production and retail activity within the firm.

As we said above, $f$ participates in KW iff $e = 1$. When $f$ makes a sale of $q$ in this market, the rest of the output $y - q$ is transformed into $x = \zeta(y-q)$ units of the AD good later that period, with $\zeta' \geq 0$ and $\zeta'' \leq 0$ (there is a constraint $q \leq y$, but it is easy to give conditions making this slack). We could also simply assume unsold output vanishes between the KW and AD markets, but we like the idea of giving $f$ an opportunity cost of KW trade.[13] It is useful to write the opportunity cost as $c(q) = \zeta(y) - \zeta(y-q)$. Unless otherwise stated, we take $\zeta$ to be linear, so $x = y - q$ and $c(q) = q$, although in Section 4.2 we use the general case. With $\zeta$ linear, we can interpret $x$ and $q$ as one good that $f$ can store across markets, but since $h$ generally values it differently in KW and AD, $f$ wants to sell at least some of it in the first market.

[13] One could alternatively assume $y - q$ is carried forward to the next KW market, but then we would need to track the inventory distribution across firms. Having them liquidate inventories in the AD market allows us to have an opportunity cost of trade while avoiding this technical problem, just like the AD market allows us to avoid tracking a distribution of money holdings across households in the KW market.

For a firm in the goods market,

$$V_1^f = \alpha_f W_1^f(y-q,\rho d) + (1-\alpha_f) W_1^f(y,0) \tag{10}$$

where $\alpha_f = \mathcal{M}(B,S)/S$. The AD value of $f$ with $x$ in inventory and $z$ in cash receipts is

$$W_1^f(x,z) = x + z - w + \beta \hat{U}_1^f, \tag{11}$$

given wage commitment $w$. Simplifying, we get

$$V_1^f = R - w + \beta \left[ \delta \hat{V}_0^f + (1-\delta)\hat{V}_1^f \right], \tag{12}$$

where $R = y + \alpha_f(\rho d - q)$ is expected revenue in units of the AD good. This is our second spillover effect: the terms of trade in the goods market $(q,d)$ affects $R$, and in equilibrium this affects entry and ultimately employment. Again the exact relation depends on details, but the robust idea is that as long as firms are deriving at least some of their profits from cash transactions, monetary factors affect their decisions.

To model entry, as is standard, we assume any $f$ with $e=0$ has no current revenue or wage obligations, but can pay $k$ in units of $x$ in any AD market to enter the next MP market with a vacancy, which allows a probability of matching. Thus

$$W_0^f = \max\left\{0, -k + \beta\lambda_f \hat{V}_1^f + \beta(1-\lambda_f)\hat{V}_0^f \right\},$$

where $\hat{V}_0^f = \hat{W}_0^f = 0$ by free entry. In steady state $k = \beta\lambda_f V_1^f$, which by (12) can be written

$$k = \frac{\beta\lambda_f (R-w)}{1-\beta(1-\delta)}. \tag{13}$$

Average profit across all firms in a period is $(1-u)(R-w) - vk$. As we said, firms pay out profit as dividends. If we assume the representative $h$ holds the representative portfolio – say, shares in a mutual fund – this gives the equilibrium dividend $\Delta$.

## 2.3 Government

The government consumes $G$, pays UI benefit $b$, levies tax $T$, and prints money at rate $\pi$, so that $\hat{M} = (1+\pi)M$, where in steady state $\pi$ is inflation. Hence, their period budget constraint is $G + bu = T + \pi M/p$, which we assume holds at every date (without loss of generality, since Ricardian equivalence holds). For steady state analysis, we can equivalently describe monetary policy in terms of setting the nominal interest rate $i$ or the growth rate of money $\pi$, by virtue of the Fisher equation $1 + i = (1+\pi)/\beta$. In the stochastic model in the Appendix we specify policy in terms of interest rate rules. We always assume $i > 0$, although we can take the limit as $i \to 0$, which is the Friedman rule.

# 3 Equilibrium

Various assumptions can be made concerning price determination in our different markets, including bargaining, price taking, and price posting, with either directed or undirected search. We think the most reasonable scenario is the following: price taking in the AD market; wage bargaining with undirected search in MP; and price posting with directed search in KW. We like price taking in the AD market because it is simple, and in any case the AD market is not our prime focus. In the MP market, bargaining seems realistic and is standard in the literature, although it is actually a simple reinterpretation here to alternatively say that our labor market has wage posting with directed search. The issues are less clear for the KW market, so we explicitly analyze several options: bargaining, price taking, and price posting with directed search.[14]

Posting with directed search – also known as competitive search equilibrium

[14] We emphasize that in the labor market posting with directed search is equivalent to generalized Nash bargaining with a particular bargaining power (the Hosios 1990 condition), but this is *not* true for the goods market. This is because there is a double holdup problem in the goods market, with ex ante investment in money by $h$ and entry by $f$, that cannot be resolved for any bargaining power parameter.

– is attractive in the goods market for a variety of reasons.[15] However, we present bargaining first, mainly because it is easy and standard in the literature. In any case, we break the analysis into three parts. First, taking unemployment $u$ as given, we determine the value of money in the goods market $q$ as in Lagos-Wright (2005). Then, taking $q$ as given, we determine $u$ in the labor market as in Mortensen-Pissarides (1996). It is convenient to depict these results graphically in $(u, q)$ space as the *LW curve* and *MP curve*. Their intersection determines the unemployment rate and the value of money, from which all other variables follow, in steady state.

## 3.1 The Goods Market

Imagine for now that in the KW market $f$ and $h$ meet and bargain bilaterally over $(q, d)$, subject to $d \leq z$ and $q \leq y$, obviously, since neither party can trade more than they have. We use generalized Nash bargaining (Aruoba et al. 2007 study several other bargaining solutions in this kind of model). Let the threat points be given by continuation values, and let $\theta \in (0, 1]$ be the bargaining power of $h$. The surplus for $h$ is $\upsilon(q) + W_e^h[\rho(z - d)] - W_e^h(\rho z) = \upsilon(q) - \rho d$. Similarly, the surplus for $f$ is $\rho d - q$. It is easy to show $d = z$ (intuitively, because it is costly to carry cash when we are not at the Friedman rule; see Lagos-Wright for details). Given this, the first order condition from maximizing the Nash product wrt $q$ can be written $\rho z = g(q, \theta)$ where

$$g(q, \theta) \equiv \frac{\theta q \upsilon'(q) + (1 - \theta)\upsilon(q)}{\theta \upsilon'(q) + 1 - \theta}. \tag{14}$$

[15] First, it is fairly tractable after one incurs an initial set-up cost. Second, it has some desirable efficiency properties (see e.g. Kircher 2007). Third, directed search should seem like a big step forward to those who criticize monetary theory with random matching for the assumption of randomness per se (Howitt 2005). It should also appease those who dismiss modern monetary economics because they "don't like bargaining" (Phelan 2005). More seriously, posting models avoid the assumption that agents see each others' money balances, usually made in bargaining models to avoid technical difficulties with private information. Finally, competitive search eliminates bargaining power as a free parameter, which is useful in calibration.

Now recall (7), which in terms of the choice of $\hat{z}$ is summarized by

$$\max_{\hat{z}} \left\{-\hat{z} + \beta\hat{\alpha}_h \upsilon(\hat{q}) + \beta(1-\hat{\alpha}_h)\hat{\rho}\hat{z}\right\},$$

where we inserted $\hat{d} = \hat{z}$, and it is understood that $\hat{q}$ is a function of $\hat{z}$ as given in (14). Taking the FOC for an interior solution, then inserting $\partial\hat{q}/\partial\hat{z} = \hat{\rho}/g_1(\hat{q},\theta)$, by virtue of (14), we arrive at

$$\frac{1}{\beta\hat{\rho}} = \hat{\alpha}_h \frac{\upsilon'(\hat{q})}{g_1(\hat{q},\theta)} + 1 - \hat{\alpha}_h.$$

To reduce this to one equation in $(u,q)$ we do three things: (i) use the Fisher equation for the nominal interest rate to eliminate $1/\beta\hat{\rho} = 1+i$; (ii) insert the arrival rate $\hat{\alpha}_h = \mathcal{M}(1, 1-\hat{u})$; and (iii) impose steady state. The result is

$$\frac{i}{\mathcal{M}(1,1-u)} = \frac{\upsilon'(q)}{g_1(q,\theta)} - 1. \tag{15}$$

This is the LW curve, determining $q$ exactly as in Lagos and Wright (2005), except there $\alpha_h$ was fixed and now $\alpha_h = \mathcal{M}(1, 1-u)$. An increase in $u$ makes it less attractive to be a buyer, as discussed above. This reduces the choice of $\hat{z}$, and hence reduces $q$ via the bargaining solution. The LW curve is convenient because properties follow from well-known results in the literature – e.g. simple conditions guarantee the RHS of (15) is monotone in $q$, and hence a unique $q > 0$ solves (15) for any $u \in (0,1)$, with $\partial q/\partial u < 0$.[16] Also, letting $q^*$ solve $\upsilon'(q^*) = 1$, we know $q < q^*$ for all $i > 0$. Summarizing these and some other properties, we have:

**Proposition 1** *For all $i > 0$ the LW curve slopes downward in $(u,q)$ space, with $u = 0$ implying $q \in (0, q^*)$ and $u = 1$ implying $q = 0$. It shifts down with $i$ and up with $\theta$. In the limit as $i \to 0$, $q \to q_0$ for all $u < 1$, where $q_0$ is independent of $u$, and $q_0 \leq q^*$ with $q_0 = q^*$ iff $\theta = 1$.*

[16] Conditions that make the RHS of (15) monotone are: (i) $u'$ log-concave; or (ii) $\theta \approx 1$. Wright (2008) dispenses with these kinds of conditions entirely and proves there is generically a unique steady state $q$ with $\partial q/\partial u < 0$ even if the RHS of (15) is not monotone.

## 3.2 The Labor Market

Suppose that when $f$ and $h$ meet in MP they bargain over $w$, with threat points equal to continuation values, and $\eta$ the bargaining power of $f$. It is routine to solve this for

$$w = \frac{\eta\left[1-\beta\left(1-\delta\right)\right]\left(b+\ell\right)+\left(1-\eta\right)\left[1-\beta\left(1-\delta-\lambda_h\right)\right]R}{1-\beta\left(1-\delta\right)+\left(1-\eta\right)\beta\lambda_h}. \tag{16}$$

If we substitute this and $R = y + \alpha_f(\rho d - q)$ into (13), the free entry condition becomes

$$k = \frac{\lambda_f \eta\left[y-b-\ell+\alpha_f\left(\rho d-q\right)\right]}{r+\delta+(1-\eta)\lambda_h}.$$

To reduce this to one equation in $(u,q)$ we do three things: (i) use the steady state condition $(1-u)\delta = \mathcal{N}(u,v)$ to write $v = v(u)$ and $\tau = \tau(u) = v(u)/u$; (ii) insert the arrival rates $\lambda_f(u) = \mathcal{N}\left[1,\tau(u)\right]/\tau(u)$, $\lambda_h(u) = \mathcal{N}\left[1,\tau(u)\right]$ and $\alpha_f(u) = \mathcal{M}(1,1-u)/\left(1-u\right)$; and (iii) use the bargaining solution to eliminate

$$\rho d - q = g(q,\theta) - q = \frac{\left(1-\theta\right)\left[\upsilon(q)-q\right]}{\theta\upsilon'(q)+1-\theta}.$$

The result is

$$k = \frac{\lambda_f(u)\eta\left\{y-b-\ell+\alpha_f(u)\frac{(1-\theta)[\upsilon(q)-q)]}{\theta\upsilon'(q)+1-\theta}\right\}}{r+\delta+(1-\eta)\lambda_h(u)}. \tag{17}$$

This is the MP curve, determining $u$ as in Mortensen-Pissarides (1996), except the total surplus here is not just $y-b-\ell$, but includes as an extra term the expected surplus from retail trade. Routine calculations show this curve is downward sloping. Intuitively, there are three effects from an increase in $u$, two from the textbook model plus a new one, all of which encourage entry: (i) $\lambda_f(u)$ goes up (it is easier for $f$ to hire); (ii) $\lambda_f(u)$ goes down (it is harder for $h$ to get hired, which lowers $w$); and (iii) $\alpha_f(u)$ goes up (it is easier for $f$ to compete in the goods market). Summarizing this and other properties:

**Proposition 2** *The MP curve slopes downward in $(u,q)$ space. It shifts in with $y$ or $\eta$, and out with $k$, $r$, $\delta$, $\theta$, $b$ or $\ell$.*

### 3.3 Steady State Equilibrium

Propositions 1 and 2 imply LW and MP both slope downward in a box $\mathcal{B} = [0,1] \times [0, q^*]$ in $(u, q)$ space, shown in Figure 3. Notice LW enters $\mathcal{B}$ from the left at $u = 0$ and $q_0 \leq q^*$ and exits at $(1, 0)$, while MP enters where $q = q^*$ at some $\underline{u} > 0$, with $\underline{u} < 1$ iff $k$ is not too big, and exits by either hitting the horizontal axis at $u_0 \in (0, 1)$ or hitting the vertical axis at $q_1 \in (0, q^*)$. It is easy to check the former case, shown by the curve labeled 1, occurs iff $\eta (y - b - \ell) > k(r + \delta)$, which is the usual condition required for $u < 1$ in the MP model. In this case, there exists a nonmonetary steady state at $(u_0, 0)$, which is the standard MP equilibrium, plus at least one monetary steady state with $q > 0$ and $u < u_0$. The Figure also shows cases labeled 2 and 3, where there either exist multiple or no monetary steady states, plus a steady state at $(u, q) = (1, 0)$ where the KW and MP markets shut down.

To understand which case is more likely, look at Propositions 1 and 2, since those results tell us how the curves shift with parameters, and hence how the configuration depends on parameters. In any case, the discussion in the previous paragraph establishes existence of steady state equilibrium. Clearly we do not have uniqueness, in general. Monetary and nonmonetary equilibria may coexist, but it is possible for monetary steady state to be unique, as turns out to be the case in the calibrations below. If there exists any steady state with $u < 1$, which again is true iff $\eta (y - b - \ell) > k(r + \delta)$, then there will exist a monetary steady state. Once we have $(u, q)$, we easily recover all other endogenous variables, including vacancies $v$, arrival rates $\alpha_j$ and $\lambda_j$, real balances $z = g(q, \theta)$, and so on.[17]

[17] In particular, the nominal price level is $p = M/g(q, \theta)$, and the AD budget equation yields $x$ for every $h$ as a function of $z$ and $I_e$. In the general case where AD utility is $x + \Upsilon_e(\mathbf{x})$, utility maximization determines individual demand as a function of $e$ and $\mathbf{p}$ (plus $p$ which we already know), say $\mathbf{x} = \mathbf{D}_e(\mathbf{p})$. Market demand is $\mathbf{D}(\mathbf{p}) = u\mathbf{D}_0(\mathbf{p}) + (1 - u)\mathbf{D}_1(\mathbf{p})$, and equating this to the endowment $\bar{\mathbf{x}}$ yields a standard system of GE equations that solve for $\mathbf{p}$. We get classical neutrality: if $M$ changes, we can change $p$ and $\mathbf{p}$ proportionally without affecting the AD equilibrium conditions or $(u, q)$. We do not generally get superneutrality:

A convenient result from Propositions 1 and 2 is that changes in $i$ shift only the LW curve, while changes in $y$, $\eta$, $r$, $k$, $\delta$, $b$ or $\ell$ shift only the MP curve, which makes it easy to analyze changes in parameters. An increase in $i$ shifts the LW curve in toward the origin, reducing $q$ and $u$ if equilibrium is unique (or in the 'natural' equilibria if we do not have uniqueness). The result $\partial q/\partial i < 0$ holds in the standard LW model, with fixed $\alpha_h$, but now there is a general equilibrium multiplier effect via $u$ that reduces $\alpha_h$ and further reduces $q$. An increase in $b$ shifts the MP curve out, increasing $u$ and reducing $q$ if equilibrium is unique (or in the 'natural' equilibria). The result $\partial u/\partial z > 0$ $u$ holds in the standard MP model, with fixed $R$, but now there is a multiplier effect via $q$ that reduces $R$ and further increases $u$. Other experiments can be analyzed similarly.

Summarizing, we have established the following results:

**Proposition 3** *Steady state equilibrium always exist. One steady state is the nonmonetary equilibrium, which entails* $u < 1$ *iff* $\eta(y - b - \ell) > k(r+\delta)$. *If this inequality holds, there exists at least one monetary steady state. Assuming the monetary steady state is unique, a rise in* $i$ *decreases* $q$ *and increases* $u$*, while a rise in* $y$ *or* $\eta$*, or a fall in* $k$, $r$, $\delta$, $b$ *or* $\ell$*, increases* $q$ *and decreases* $u$*.*

# 4 Alternative Pricing Mechanisms

As discussed, there are reasons to consider alternatives to bargaining in the goods market. Here we consider price posting and directed search. We also consider price-taking, which may be of interest because it can be reduced as a special case to something that looks like a common cash-in-advance or money-in-the-utility-function specification. We maintain bargaining in the labor market, although, as mentioned above, one can reinterpret the same equations as coming

any change in $i$ shifts the LW curve, which affects $(q,u)$ and the rest of the system. When $\Upsilon_e$ does not depend on $e$, however, neither does $\mathbf{D}_e(\mathbf{p})$, in which case $\mathbf{D}(\tilde{\mathbf{p}})$ is independent of $u$ and hence $\mathbf{x}$ is independent of monetary factors – a version of the neoclassical dichotomy.

from competitive search by setting bargaining power in MP according to the Hosios (1990) condition.

## 4.1 Price Posting

We assume sellers post, and buyers direct their search to preferred sellers.[18] Agents take into account that if a group of $B$ buyers direct their search towards a group of $S$ sellers, the number of meetings is $\mathcal{M}(B, S)$. Thus, $Q = B/S$ determines the trade probabilities $\alpha_f = \mathcal{M}(Q, 1)$ and $\alpha_h = \mathcal{M}(Q, 1)/Q$. To be precise, imagine $f$ posting the following message in the AD market: "Conditional on $e = 1$ in the next MP market, I commit to sell $q$ units for $d$ dollars in the KW market, but I can serve at most one customer, and you should expect queue length $Q$."

The equilibrium surplus $h$ gets from participating in the KW market, from the perspective of the AD market, where he has to acquire the cash, is given by

$$\tilde{\Sigma} = -\tilde{d} + \beta\alpha_h(\tilde{Q})\upsilon(\tilde{q}) + \beta\left[1 - \alpha_h(\tilde{Q})\right]\rho\tilde{d},$$

where $(\tilde{q}, \tilde{d})$ and $\tilde{Q}$ are the the equilibrium terms of trade and queue length; $h$ also has the option of not participating, which yields $\tilde{\Sigma} = 0$. Thus, $f$ posts $(q, d)$ to maximize $V_1^f$, which from (12) is simply $\alpha_f(Q)(\rho d - q)$ plus a constant, st the constraint that in order to get $Q > 0$ his buyers must receive a surplus $\Sigma$ equal to the market surplus $\tilde{\Sigma}$. Formally, assuming $f$ wants $Q > 0$, think of him choosing $Q$ as well as $(q, d)$ to solve

$$\begin{aligned} &\max_{q,d,Q} \mathcal{M}(Q, 1)(\rho d - q) \qquad (18) \\ \text{st } \tilde{\Sigma} &= -d(1 - \beta\rho) + \beta\frac{\mathcal{M}(Q, 1)}{Q}\left[\upsilon(q) - \rho d\right]. \end{aligned}$$

[18] One can also have buyers post to attract sellers, or have third parties (market makers) post to attract buyers and sellers, and get the same set of equilibrium conditions; see Moen (1997), Shimer (1996), Acemoglu and Shimer (1999), Julien et al. (2000), Burdett et al. (2001), Mortensen and Wright (2002), Rochteau and Wright (2005), Faig and Huangfu (2005) and Menzio (2007).

Using $\beta\rho = 1/(1+i)$ and the equilibrium condition $\Sigma = \tilde{\Sigma}$, we derive the following conditions characterizing the solution

$$\upsilon'(q) = 1 + \frac{i}{\alpha_h(Q)} \tag{19}$$

$$\rho d = g\left[q, \epsilon(Q)\right] \tag{20}$$

$$\Sigma = \beta\alpha_h(Q)\left\{\upsilon(q) - \upsilon'(q)g\left[q, \epsilon(Q)\right]\right\}, \tag{21}$$

where $g(\cdot)$ is defined in (14), and $\epsilon(Q)$ is the elasticity of $\mathcal{M}$ wrt $B$ evaluated at $Q$. Notice (19) looks like the equilibrium condition in a standard search-and-bargaining model of money when buyers make take-it-or-leave-it offers, while (20) looks like the usual bargaining solution with $\theta$ replaced by $\epsilon(Q)$. As in the related the literature, this means competitive search eliminates holdup problems on both the trade and entry (intensive an extensive) margins.

Let $q(i, Q)$ be the $q$ that solves (19), and notice it is strictly decreasing in $i$ and $Q$. Substituting $q(i, Q)$ into (21) give us an equation in $Q$ and $\Sigma$. Denote the LHS of (21) by $\Phi(q, Q)$, and for the sake of tractability assume $\Phi_1(q, Q) > 0$, $\Phi_2(q, Q) < 0$.[19] This implies there is a unique solution $Q = Q(\Sigma) \geq 0$ to (21). Moreover, it is strictly decreasing, equals $\overline{Q}(i) > 0$ when $\Sigma = 0$, and equals 0 when $\Sigma \geq \upsilon(q^*) - q^* - iq^*$. Often $Q(\Sigma)$ is interpreted as the 'demand' for $Q$, determining the queue length a seller wants as a function of the market 'price' $\Sigma$. The 'supply' of $Q$ is simple: if $\Sigma > 0$ then every $h$ participates in KW, so $Q = (1-u)^{-1}$; and if $\Sigma = 0$ then $h$ is indifferent to participating, so the number of participants can be any $B \in [0, 1]$. See Figure 4.1.

Equilibrium equates 'supply' and 'demand' for $Q$. Letting $\overline{u}(i) \equiv 1 - 1/\overline{Q}(i)$, we have that $q$ then depends on $u$ as follows:

$$u \leq \overline{u}(i) \Longrightarrow Q = (1-u)^{-1} \text{ and } \upsilon'(q) - 1 = i/\alpha_h(Q) \tag{22}$$

$$u > \overline{u}(i) \Longrightarrow Q = \overline{Q}(i) \text{ and } \upsilon'(q) - 1 = i/\alpha_h\left[\overline{Q}(i)\right]. \tag{23}$$

[19] Note that $\Phi_2 < 0$ holds for the usual matching functions, while a sufficient condition for $\Phi_1 > 0$ is that $\epsilon(Q)$ is not too small.

This is the LW curve with competitive search. It is downward sloping in $(u, q)$ space and shifts in with $i$, as under bargaining. The only complication is that once we increase $u$ beyond $\overline{u}(i)$, there is no $\Sigma > 0$ that clears the market for $Q$, so we get $\Sigma = 0$, $Q = \overline{Q}(i)$, and $q = \overline{q}(i)$ where $\overline{q}(i)$ solves (23). That is, the LW curve kinks and becomes horizontal at $\overline{u}(i)$. To find the point where it kinks, solve (21) with $\Sigma = 0$ for $\overline{q}(i) = \psi\left[\overline{u}(i)\right]$, which implies $\psi' \geq 0$. If $\mathcal{M}(B, S)$ is Cobb-Douglas, e.g., then $\overline{q}(i)$ is independent of $\overline{u}(i)$ and $\psi(u)$ is horizontal.

The MP curve also needs to be modified. First, for $u < \overline{u}(i)$, we have

$$k = \frac{\lambda_f(u)\eta\left\{y - b - \ell + \alpha_f(u)\frac{(1-\epsilon)[\upsilon(q)-q)]}{\epsilon\upsilon'(q)+1-\epsilon}\right\}}{r + \delta + (1-\eta)\lambda_h(u)}, \tag{24}$$

which is identical to (17) except we replace $\theta$ by the elasticity $\epsilon = \epsilon\left(Q\right)$, with $Q = 1/(1-u)$. Second, for $u > \overline{u}(i)$, the result is the same except $\alpha_f(u) = \mathcal{M}\left[\overline{Q}(i), 1\right]$ and $\epsilon = \epsilon\left[\overline{Q}(i)\right]$ no longer depend on $u$. As Figure 4.2 shows, the MP curve is downward sloping with a kink at $\overline{u}(i)$. Note that when $u > \overline{u}(i)$, the MP curve now depends on $i$ directly (this happens in one version of the calibrated model, as seen Figure 7 below). But apart from these minor technical modifications, the model with posting is similar to bargaining.

## 4.2 Price Taking

Search models with Walrasian price taking go back to the Lucas and Prescott (1974) model of unemployment, where it may take time to get from one local labor market to another, but each one contains large numbers of workers and firms who behave competitively. We can tell the same story about our goods market, and have agents take the price of $q$ in terms of AD goods parametrically (money remains essential, because of anonymity, even with Walrasian pricing). We also generalize Lucas-Prescott by allowing agents to get into the goods market only probabilistically. Additionally we now allow a nonlinear opportunity cost, so that revenue is $R = \zeta(y) + \alpha^f\left[q^f\rho - c(q^f)\right]$, because a linear cost implies profits are 0 in equilibrium, which would make $u$ independent of $q$, as we discuss below.

Every $f$ with $e = 1$ wants to get into the KW market. Those that do get in choose $q^f$ to maximize $q^f \rho - c(q^f)$, which implies $c'(q^f) = \rho$. Then in AD, with the usual manipulations, free entry implies

$$k = \frac{\lambda_f \eta \left\{ \zeta(y) - b - \ell + \alpha_f \left[ q^f c'(q^f) - c(q^f) \right] \right\}}{r + \delta + (1-\eta)\lambda_h}, \tag{25}$$

where $\alpha_f$ is the probability $f$ gets into KW.[20] Every $h$ wants to get into this market, and those that do choose $q^h$ to maximize $\upsilon(q^h) + W_e^h \left[ \rho \left( z - q^h \right) \right]$ st $q^h \leq z$. The constraint binds as usual in equilibrium. In AD, $h$ chooses $\hat{z}$ to $\max_{\hat{z}} \{-\hat{z} + \beta \hat{\alpha}_h \upsilon(\hat{q}) + \beta(1 - \hat{\alpha}_h)\hat{\rho}\hat{z}\}$, where $\hat{\alpha}_h$ is his probability of getting into KW. Taking the FOC, then using $\partial q^h / \partial z = 1$ and $\rho = c'(q^f)$, we get

$$\frac{i}{\alpha_h} = \frac{\upsilon'(q^h)}{c'(q^f)} - 1. \tag{26}$$

Search-type frictions are captured by letting the measures of agents that get in to a market depend on the measures that try to get in, which means $\alpha^h(u) = \mathcal{M}^h(1, 1-u)$ and $\alpha^f(u) = \mathcal{M}^f(1, 1-u)/(1-u)$. Goods market clearing implies $\mathcal{M}^h q^h = \mathcal{M}^f q^f$. Inserting $q^h = q$ and $q^f = q\mathcal{M}^h/\mathcal{M}^f = q/(1-u)$, as well as $\lambda_h$ and $\lambda_f$, into (26) and (25), we get the LW and MP curves with Walrasian pricing in the goods market. A special case is the frictionless version, where everyone who wants gets in, $\mathcal{M}^h = 1$ and $\mathcal{M}^f = 1 - u$. In this special case, the LW and MP curves are

$$i = \frac{\upsilon'(q)}{c'\left(\frac{q}{1-u}\right)} - 1$$

$$k = \frac{\lambda_f(u)\eta \left\{ \zeta(y) - b - \ell + \left[ \frac{q}{1-u} c'\left(\frac{q}{1-u}\right) - c\left(\frac{q}{1-u}\right) \right] \right\}}{r + \delta + (1-\eta)\lambda_h(u)}$$

[20] We emphasize that that there are two distinct notions of entry here: first $f$ pays $k$ to get into the MP market (post a vacancy); then, once $f$ produces, there is a probability $\alpha_f$ that he gets into the KW market (if he does not he transits directly to AD). Similarly, $h$ only gets into KW with probability $\alpha_h$. As a special case, of course, $\alpha_f$ or $\alpha_h$ or both can be 1.

If we additionally impose linear cost, $c(q) = q$, then $u$ vanishes from LW and $q$ vanishes from MP. In this special case, therefore, the LW curve is horizontal and the MP curve vertical.

In other words, the model dichotomizes in the case where: (i) there are no frictions; and (ii) $c(q) = q$. Actually, while (i) and (ii) are needed to solve LW for $q$ independently of $u$, only the latter is needed to solve MP for $u$ independently of $q$. Based on this, one can reinterpret the standard MP model as one where firms indeed sell their output in a market to households other than their own employees – for cash or credit, it is irrelevant in this case – since as long as the cost in this market is linear and pricing is Walrasian, firms get none of the gains from trade, and $u$ is determined as in the standard model. There may or may not be monetary exchange lurking behind the scenes, but this does not affect vacancy creation or unemployment.[21]

One could say that when $\alpha^h = \alpha^f = 1$ this model looks like a standard cash-in-advance economy, in the sense that there are no search or non-competitive pricing issues. One could also say it looks like a money-in-the-utility-function specification, since after all real balances do appear in the value (indirect utility) functions. This is all fine. It is because we like to go into more detail about the assumptions that make money useful, and to allow for search frictions and alternative pricing mechanisms, that we did not start with a cash-in-advance or money-in-the-utility-function specification. But for those wed to a reduced-form approach, we point out that a frictionless version of our model with Walrasian pricing leads to the same set of equations. To put it another way, one can derive the LW and MP curves *without* microfoundations for money. We prefer to be more explicit about the exchange process, not only for aesthetic reasons, but because this gives more general results, leads to additional insights, and is no more difficult.

[21] One can get something similar in the bargaining model by setting $\theta = 1$.

# 5 Quantitative Analysis

Theory predicts an increase in inflation or interest rates increases unemployment, because this raises the effective tax on cash-intensive goods markets, which reduces profit and employment (note that for the parameter values calibrated below equilibrium is unique, so these results are unambiguous). We now ask how big the effects might be. As we said, the model is best suited to lower-frequency observations, since we abstract from complications that may matter in the short run, like imperfect information, rigidities etc. Although the model could be used to address many quantitative issues, here we focus on examining how Friedman's "natural rate of unemployment" is affected by monetary factors. Thus, we ask, how well can the model account for low-frequency dynamics in unemployment when the driving force is counterfactually assumed to be nothing except changes in monetary policy?[22]

We use the version with competitive search in the goods market and bargaining in the labor market. However, as is common, we set bargaining power $\eta$ in the latter to the elasticity of the matching function a là Hosios (1990), which can be interpreted as imposing competitive search in MP. We also tried other versions, including bargaining and price taking in KW, and the results were similar in terms of the big picture if not all the details.[23] Although so far we focused on steady states, the dynamic-stochastic generalization is presented in the Appendix. There we allow randomness in monetary policy, as described by an interest rate rule $\widehat{i} = \bar{i} + \rho_i(i - \bar{i}) + \epsilon_i$, $\epsilon_i \sim N(0, \sigma_i)$, and in productivity, as described by a similar stochastic process for $y$. For most of our experiments we

[22] To be clear, for the purpose of this exercise, we take money to be exogenous and look at the response of $u$. A different approach might assume $u$ varies for some other reason and look at the endogenous response of policy – but that would be different approach.

[23] We think this is important, and helps to motivate studying the different price mechanisms in the first place – how else would one know if it matters? To be clear, we are *not* saying the price mechanism is unimportant: for given parameters, it makes a difference if we assume bargaining or posting, e.g., but if we change the mechanism and then recalibrate parameters we get similar results.

take $y$ to be constant in order to isolate the effects of money, and only use the model with real shocks as an aid in calibration.

## 5.1 Parameters and Targets

We choose a quarter as the period and look at 1955-2005, as in the Introduction. We need to calibrate: (i) preferences as described by $\beta$, $\ell$ and $v(q)$; (ii) technology as described by $\delta$, $k$, $\mathcal{N}(u,v)$ and $\mathcal{M}(B,S)$; (iii) policy as described by $b$ and the process for $i$. Utility is given by $v(q) = Aq^{1-a}/(1-a)$. Following much of the literature, we take the MP matching function to be $\mathcal{N}(u,v) = Zu^{1-\sigma}v^{\sigma}$ (truncated to keep probabilites below 1). We take the KW matching function to be $\mathcal{M}(B,S) = S\left[1-\exp\left(-B/S\right)\right]$, the so-called urn-ball technology, which is a parsimonious specification and one that can be derived endogenously using directed search theory (Burdett et al. 2001; Albrect et al. 2006). This leaves four parameters describing preferences $(\beta,\ell,A,a)$, four describing technology $(\delta,k,Z,\sigma)$, and four describing policy $\left(b,\bar{\imath},\rho_i,\sigma_i\right)$.

Calibration is fairly standard. First, set $\beta$ to match the average quarterly real interest rate, measured as the difference between the nominal rate and inflation. Then set the elasticity $1-\sigma$ of the MP matching function to the regression coefficient of the job-finding rate on labor market tightness, both expressed in logs. Then set UI so that in equilibrium the replacement rate is $b/w = 0.5$. Then set parameters of the $i$ process $(\bar{\imath},\rho_i,\sigma_i)$ to match the average quarterly nominal rate, its autocorrelation and variance. Then set $(\delta,k,Z)$ to match the average unemployment, vacancy and and job-finding rates – although we can normalize the vacancy rate to $v=1$ by choice of units, which affects the calibrated value of $Z$ but nothing else. This leaves only the preference parameters $(A,a,\ell)$, which we now discuss.

We set $(A,a)$ to match money demand in the data with that implied by theory. In the model, $M/pY$ is given by $M/p = g(q)$ over $Y = \mathcal{M}(1,1-$

$u) [g(q) - q] + (1 - u)y$, where both $q$ and $u$ depend on $i$. In terms of data, we target average $M/pY$ plus some measure of its responsiveness to $i$, using $M1$ as our notion of money.[24] One method is to target directly the elasticity of $M/pY$ wrt $i$, which we estimate to be around $-0.7$ using several specifications and periods of different lengths, summarized in Figure 5.1. The implied money demand curve is shown in Figure 5.2, and fits well, at least up to the 90s. A different tack is to simply match the slope of a regression line through the data in Figure 5.2, which also fits well, at least ignoring the 80s. Although this method does not target elasticity directly, the implied parameters do generate a money demand relation, with a bigger elasticity of around $-1.4$. We present results for both low and high elasticities, since both generate what look to us like reasonable money demand curves.[25]

Table 1: Calibration Targets

| Description | Value |
|---|---|
| average real rate $r$ (quarterly) | 0.00816 |
| average nominal rate $i$ (quarterly) | 0.01803 |
| autocorrelation of $i$ | 0.990 |
| standard deviation of $i$ | 0.006 |
| average money demand $M/pY$ (annual) | 0.169 |
| money demand elasticity (negative) | 0.7 or 1.4 |
| average unemployment $u$ | 0.058 |
| average vacancies $v$ (normalization) | 1 |
| average UI replacement rate $b/w$ | 0.500 |
| average job-finding rate $\lambda_h$ (monthly) | 0.450 |
| elasticity of $\lambda_h$ wrt $v/u$ | 0.280 |

The targets described above are summarized in Table 1. They are sufficient to pin down all but one of our 12 parameters, the value of leisure $\ell$. As is well

[24] We use $M1$ mainly to facilitate comparison with the literature. Although at first blush it may seem $M0$ is better suited to the theory, one can reformulate this kind of model so that demand deposits circulate in KW, either instead of or along with currency; see Berentsen et al. (2007), He et al. (2007), Chiu and Meh (2007), or Li (2007).

[25] These observations also pin down the share of the KW market: simply divide nominal spending in KW $\mathcal{M}(1, 1-u)M$ by total nominal spending $pY$ to get $\mathcal{M}(1, 1-u)$ times money demand $M/pY$. Adjusting from an annual to a quarterly frequency, $M1/pY$ is 0.676, and at the steady state $u = 0.058$ our matching function yields $\mathcal{M}(1, 1-u) = 0.616$, implying the KW market contributes around 42% and the AD market around 58% of total spending.

known, $\ell$ is difficult to calibrate and can matter a lot – this is at the heart of the difference between Shimer (2005) and Hagedorn-Manovskii (2007). Our approach is to be agnostic and consider various strategies for $\ell$. In our UI (for 'unemployment insurance') calibration we impose $\ell = 0$, as in Shimer. In our BC (for 'business cycle') calibration we set $\ell$ so that a real version of the model, with shocks to $y$ calibrated to the data and constant $i$, generates cyclical fluctuations in $u$ consistent with the evidence, which is close to Hagedorn-Manovskii.[26] Finally, in our BF (for 'best fit') calibration we choose $\ell$ to minimize the deviations between HP-filtered $u$ in the data and in the model, using either a low or a high HP parameter of 1600 or 160000.

Although we report the calibrated parameter values from the UI method, we will actually spend very little time in the rest of the paper on this version of the model (but see the webpage mentioned in footnote 2) because, as one should expect from the literature, it generates almost no response of $u$ to shocks, to either $y$ or to $i$. As mentioned above, our position is macro-labor economists have yet to settle on the definitive way to solve the 'puzzle' of getting $u$ to move more in response to shocks, but all seem to agree that a high value for $\ell$ (given $b$) gets the job done. So for this exercise we let $\ell$ do the work. But, in principle, some other ways of making $u$ more response to shocks, including e.g. those discussed in Nagapal and Mortensen (2007) or in Menzio and (Shi 2008), could work as well for our purposes.

Thus, for both the case of a low and a high money demand elasticity, we focus on three $\ell$ calibrations: the BC method, the BF method with a low HP filter, and the BF method with a high filter. Calibrated preference and technology parameters are reported in Tables 2.1 and 2.2 for the case of a low and high

[26] For the record, Hagedorn-Maovskii do *not* pick $\ell$ to match the volatility of $u$, but target other observables. When we say we are close to them, we mean that our ratio of $b + \ell$ to $y$, which is what matters, is close to theirs. We are aware of issues involved with high values of $\ell$ as in Hagedorn-Maovskii, including the critique by Costain and Ritter (2007), but we think the approach in Rogerson et al. (2008) can in principle address that problem.

money demand elasticity, respectively; to save space, the tables omit the policy parameters, which in all cases are $b = 0.5$, $\bar{i} = 0.018$, $\rho_i = 0.984$ and $\sigma_i = 8.9 \cdot 10^{-4}$. Notice $b + \ell$ is close to and sometimes above $y = 1$. This is not a problem, since the suplus from creating a job here is $y - b - \ell$ *plus* expected profit from retail trade; when $y - b - \ell < 0$, it merely means firms would not hire if the retail sector shut down, which seems reasonable.

Table 2.1: Parameters with MD elasticity 0.7

| | Description | UI | BC | BF 1600 | BF 160000 |
|---|---|---|---|---|---|
| $\beta$ | discount factor | 0.992 | 0.992 | 0.992 | 0.992 |
| $\ell$ | value of nonmarket activity | 0 | 0.489 | 0.502 | 0.502 |
| $A$ | KW utility weight | 1.013 | 1.013 | 1.013 | 1.013 |
| $a$ | KW utility elasticity | 0.04 | 0.04 | 0.04 | 0.04 |
| $\delta$ | job destruction rate | 0.05 | 0.05 | 0.05 | 0.05 |
| $k$ | vacancy posting cost | $1.05 \cdot 10^{-2}$ | $3.72 \cdot 10^{-4}$ | $9.67 \cdot 10^{-5}$ | $9.44 \cdot 10^{-5}$ |
| $Z$ | MP matching efficiency | 0.364 | 0.364 | 0.364 | 0.364 |
| $\sigma$ | MP matching $u$ elasticity | 0.72 | 0.72 | 0.72 | 0.72 |

Table 2.2: Parameters with MD elasticity 1.4

| | Description | UI | BC | BF 1600 | BF 160000 |
|---|---|---|---|---|---|
| $\beta$ | discount factor | 0.992 | 0.992 | 0.992 | 0.992 |
| $\ell$ | value of nonmarket activity | 0 | 0.485 | 0.500 | 0.500 |
| $A$ | KW utility weight | 1.020 | 1.020 | 1.020 | 1.020 |
| $a$ | KW utility elasticity | 0.02 | 0.02 | 0.02 | 0.02 |
| $\delta$ | job destruction rate | 0.05 | 0.05 | 0.05 | 0.05 |
| $k$ | vacancy posting cost | $1.05 \cdot 10^{-2}$ | $3.93 \cdot 10^{-4}$ | $8.79 \cdot 10^{-5}$ | $8.72 \cdot 10^{-4}$ |
| $Z$ | MP matching efficiency | 0.364 | 0.364 | 0.364 | 0.364 |
| $\sigma$ | MP matching $u$ elasticity | 0.72 | 0.72 | 0.72 | 0.72 |

## 5.2 Results

We first solve for recursive equilibrium in the general dynamic model presented in the Appendix, where we allow stochastic processes for both the interest rate $i$ and productivity $y$. We then feed in the actual path of $i$, holding $y$ constant, and calculate the implied path for $u$. This is our prediction for unemployment in the counterfactual case where the only impulses over the period were changes in monetary policy. We compare the predictions of the model and the data in terms of $u$, where $u$ has been filtered to various degrees (in both the model and the data). We then look at statistics and plots of the variables in question.

Consider first the case of a low money demand elasticity. Figures 6.1 and 6.2 each summarize the results of the BC calibration in two ways: scatter plots of trend (filtered) $i$ vs. $u$ and $\pi$ vs. $u$; and the time series of trend (filtered) $u$ as well as the raw (unfiltered) series. In Figure 6.1 we use a high HP filter parameter of 160000, while in Figure 6.2 we use the lower filter of 1600. As one can see, this BC version of the model with relatively inelastic money demand implies that monetary policy alone can account for a little, but not that much, of the behavior of $u$ over the last fifty years. We get nothing like the big swings in $u$ observed in the data, even after filtering, although qualitatively the model clearly does correctly predict the broad pattern of $u$ rising in the first half and falling in the second half of the sample.

To use one summary statistic, consider the runup in $u$ over the worse part of the stagflation episode, between the first quarters of 1972 and 82.[27] As shown in Table 4.1, this version of the model accounts for only part of the increase in $u$ during this episode. Depending on how much of the high frequency we filter out, trend $u$ rose between 22% and 43% during this ten-year period, while the model predicts much less of an increase. Looking at unfiltered data, e.g., we only predict 8%, as compared to the actual 41% increase in $u$ – that is, we get only 20% of the observed increase. Similarly, the model can account for 12% and 14% of the observed increase in low and high filtered $u$ during the episode. Again, with the BC calibration and a low money demand elasticity, one might conclude the model accounts for something, but not all that much.

[27] We did not chose this subsample to represent stagflation in order to 'cook the results' in any sense, but for the following three reasons. First, both $i$ and $u$ are close to their steady state values in 1972Q1. Second, 1982Q1 has the highest value of $i = 15.1$ in the sample, as well as a a very high $u = 0.088$. Third, this gives us exactly a decade of data.

Table 4.1: Low MD elasticity, BC calibration

| Observation | $u$ 1972Q1 | $u$ 1982Q1 | % change |
|---|---|---|---|
| unfiltered data | 5.8 | 8.8 | 41 |
| unfiltered model | 5.8 | 6.3 | 08 |
| low filtered data | 5.3 | 8.2 | 43 |
| low filtered model | 5.8 | 6.1 | 05 |
| high filtered data | 5.7 | 7.1 | 22 |
| high filtered model | 5.8 | 6.0 | 03 |

Table 4.2: Low MD elasticity, BF calibration

| Observation | $u$ 1972Q1 | $u$ 1982Q1 | % change |
|---|---|---|---|
| unfiltered data | 5.8 | 8.8 | 41 |
| unfiltered model | 5.4 | 12.6 | 80 |
| low filtered data | 5.3 | 8.2 | 43 |
| low filtered model | 5.8 | 8.4 | 37 |
| high filtered data | 5.7 | 7.1 | 22 |
| high filtered model | 6.1 | 7.1 | 15 |

Table 4.3: High MD elasticity, BC calibration

| Observation | $u$ 1972Q1 | $u$ 1982Q1 | % change |
|---|---|---|---|
| unfiltered data | 5.8 | 8.8 | 41 |
| unfiltered model | 5.8 | 6.4 | 10 |
| low filtered data | 5.3 | 8.2 | 43 |
| low filtered model | 5.8 | 6.3 | 08 |
| High filtered data | 5.7 | 7.1 | 22 |
| High filtered model | 5.8 | 6.1 | 05 |

Table 4.4: High MD elasticity, BF calibration

| Observation | $u$ 1972Q1 | $u$ 1982Q1 | % change |
|---|---|---|---|
| unfiltered data | 5.8 | 8.8 | 41 |
| unfiltered model | 5.8 | 12.5 | 73 |
| low filtered data | 5.3 | 8.2 | 43 |
| low filtered model | 5.8 | 8.4 | 37 |
| high filtered data | 5.7 | 7.1 | 22 |
| high filtered model | 6.1 | 7.0 | 14 |

Figures 6.3 and 6.4 report results of the same exercises using the BF calibration. Now the model accounts for much of the movement in trend $u$ using a medium filter, and basically all of it using a high filter. And it is not as if we filtered out everything of interest: even with the high HP parameter $u$ goes

from below 5% to above 7% and back. Table 4.2 shows we can account for 68% and 86% of the runup in low and high filtered $u$ during stagflation, although at the cost of overpredicting somewhat the increase in unfiltered $u$. One may conclude from this that we can account for most of the low frequency behavior of $u$. But the BF calibration method is extreme, in that its implied value of $\ell$ generates excess volatility in $u$ wrt $y$ shocks (obviously, since the BC calibration generates just the right volatility wrt $y$ shocks). In fact, $u$ is about twice as volatile wrt $y$ shocks in the BF calibration. We nevertheless find it interesting that the theory can in principle account for as much as it does in this case.

Figures 6.5-6.8 report results with a more elastic money demand. The BC version now generates a little more movement in $u$: as Table 4.3 indicates, we can now account for 25, 19 or 23% of the runup in $u$ during stagflation, depending on which filter we use (compared to 20, 12 and 14% with a less elastic money demand). Also, the BF version again does quite well, but now with somewhat less excess volatility in $u$ wrt real shocks. To describe the results another way, with a low HP filter the scatter plot between $i$ and $u$ generated by the model looks pretty similar to the data, and with a high filter the scatter plots look indistinguishable (this was pretty much true with a lower money demand elasticity, too). We conclude that the general message is similar with a more elastic money demand, although with a bigger elasticity we can do a little more in terms of accounting for the data.

To understand these results, consider the following intuitive argument. The initial impact of a change in $i$ is to reduce $M/p$, which affects revenue $R$ and ultimately employment. The size of the effect of $i$ on $M/p$ and hence $R$ is determined by the money demand elasticity, as in any monetary model. The size of the effect of $R$ on entry and hence $u$ is then determined by the value of leisure, as in the usual macro-labor model. Either a bigger money demand elasticity or a bigger value of $\ell$ generate similar net effects. One way to see this is to consider the MP and LW curves drawn for the actual calibrated parameters in Figure 7. More elastic money demand implies the LW curve shifts more with $i$, while a higher value of $\ell$ makes the MP curve flatter, and both make $u$ respond more to monetary policy. Of course, shifting the curves only describes comparisons across steady states, but this conveys the main economic insight.

We conclude that combinations of parameters that are not unreasonable allow one to account for some and possibly a lot of the behavior in trend $u$ - just how much depending on the exact calibration. We have no problem with the idea that part of trend $u$ should be explained by productivity, demographics, taxes etc. We still think it is interesting that money in principle has a role to play. One thing to do to make the results less dependent on $\ell$ is to ask the following: how big would a shock to $i$ have to be to make it equivalent to a given shock to $y$? The answer is shown in Figure 8. For a low money demand elasticity, going from the Friedman rule $i = 0$ to $i = 0.13$ (i.e. 10% annual inflation) is equivalent to a reduction in $y$ of around 3/4 of 1%. For a higher money demand elasticity, the answer is closed to 1.5%.

This suggests that money may be important for labor market performance in the long run, independent of nominal rigidities, imperfect information, and other channels that may or may not be relevant in the short run. And these numbers are independent of the value of $\ell$ or other aspects of the labor market. Monetary policy, like productivity, has an impact on $R$, and Figure 8 simply gives the equivalent effect on $R$ from either $i$ or $y$. The degree that changes in $R$ translate into changes in $u$ depends on how one calibrates the labor market, but the comparison between changes in $i$ and changes in $y$ on $R$ does not. Also, to be clear, we are referring to changes in $y$ holding other things constant, including $b$ and $\ell$. It is well known in the standard macro-labor model that the interesting equilibrium variables are independent of changing productivity in market $y$ and nonmarket activities $b$ and $\ell$ at the same rate.

Finally, we can also ask about the welfare cost of inflation. Some recent models where money is modeled with relatively explicit microfoundations generate bigger costs than traditional models. The reduced-form literature typically finds that eliminating 10% inflation is worth less than 1% of consumption, and often much less, while models that explicitly incorporate frictions that make money useful find this same policy can be worth 3 to 4% or more.[28] However, these big effects usually occur only when there are holdup problems, as occur in bargaining models, and not in competitive search models like the one here. As Figure 9 shows, we can generate big welfare effects here even with competitive search. One reason is that here inflation affects unemployment $u$ (and hence trade on the extensive margin) as well as the quantity $q$ (trade on the intensive margin).

We do not dwell too much on welfare, however, since the results depend on the assumption that the Hosios condition is satisfied in the labor market. We can show the following. Given no constraints on policy, the optimum is to set $i = 0$ and set fiscal policy (any combination of UI and a wage tax that can easily be added to the model) to correct for discrepancies between bargaining power $\eta$ and the elasticity of the matching function $\sigma$ in the labor market. Given fiscal policy is set exogenously incorrectly, however, we would like to set $i \neq 0$. We of course are constrained to have $i \geq 0$, but if e.g. UI is exogenously set too low then the optimal monetary policy is $i > 0$. Intuitively, if we have excessive firm entry, we improve efficiency by the inflation tax. The main point however is simply that the cost of inflation is sensitive, and can even be negative, depending on bargaining power and fiscal policy. Additional exploration of welfare and optimal policy is therefore left for future work.

[28] See e.g. Rocheteau and Wright (2007) or Craig and Rocheteau (2007) for summaries of recent findings, as well as a discussion of more traditional studies.

## 6 Conclusion

This paper has studied a venerable issue in macroeconomics: the relation between unemployment and monetary variables like inflation or nominal interest rates. We began by reviewing the data, and documenting a clear positive long-run relationship between these variables after filtering out higher-frequency movements. We then built a model, based on explicit microfoundations for both money and unemployment, consistent with these observations. The model takes seriously Friedman's (1977) suggestion that the natural rate of unemployment is determined by real factors, including the cost of holding real balances. We think the framework provides a natural integration and extension of existing models of unemployment and monetary economics. We then considered some quantitative issues, focusing on asking how the model accounts for the low-frequency patterns in unemployment when the sole driving process is monetary policy.

The answer depends mainly on two key parameters: the elasticity of money demand and value of leisure. The former influences the effect of monetary policy on real balances and hence on retail profits, while the latter determines how profits translate to entry and employment. For conservative values of the money demand elasticity and value of leisure, we can account for about 20% of the increase in unemployment during the 1970s stagflation episode, which is not insignificant but does leave room for other factors. For less conservative but not unreasonable parameters, the model can account for the lion's share of movements in trend unemployment over the last half century. These results suggest that monetary factors may be important for labor market outcomes, not only theoretically but also quantitatively. Future research could attempt to hone these numerical results and explore other quantitative and theoretical questions in the general framework.

# Appendix

We define equilibrium in the dynamic-stochastic model for case of wage bargaining in MP, price posting in KW, and price taking in AD (other combinations are similar). At the start of a period the state is $s = (u, i, y)$, where $u$ is unemployment, $y$ productivity, and $i$ nominal interest on bonds purchased in the previous and redeemed in the current AD market. The state $s$ was known in the previous AD market, including the return on the nominal bonds maturing this period. Although these bonds are not traded in equilibrium, $i$ matters for the following reason. When $s_+$ is revealed in the current AD, there is a response in the price $p = p(s_+)$ and hence in the return $\rho(s_+) = p(s)/p(s_+)$ on money brought in from the previous AD; this implies the no-arbitrage condition

$$1 = \beta(1 + i)\hat{\rho}(s),$$

where $\hat{\rho}(s) = \mathbb{E}_{s_+}[\rho(s_+)|s]$.

We assume $i$ and $y$ follow exogenous and independent processes,

$$\begin{aligned} y_+ &= \overline{y} + \rho_y(y - \overline{y}) + \epsilon_y,\ \epsilon_y \sim N(0, \sigma_y) \\ i_+ &= \overline{i} + \rho_i(i - \overline{i}) + \epsilon_i,\ \epsilon_i \sim N(0, \sigma_i). \end{aligned}$$

Unemployment changes endogenously as follows. The probability in MP an unemployed $h$ finds a job and $f$ fills a vacancy are $\lambda_h[\tau(s)]$ and $\lambda_f[\tau(s)]$, where $\tau(s)$ is the $v/u$ ratio and $v = v(s)$ was set in the previous AD market as a function of the current state, so that

$$u_+(s) = u - u\lambda_h[\tau(s)] + (1 - u)\delta.$$

Similarly, in KW the probability $h$ meets a seller and $f$ meets a buyer are $\alpha_h[Q(s)]$ and $\alpha_f[Q(s)]$, where the $B/S$ ratio $Q(s)$ and terms of trade $[d(s), q(s)]$ were posted in the previous AD market, and $d$ is measured in units of $x$ from that market.

After MP and KW, in the current AD market the realization of $s_+$ becomes known. Firms then liquidate inventories, pay wages and dividends, create vacancies for the next MP, and post terms for the next KW. Also, households choose real balances for the next KW, while government collects taxes, pays UI and announces $i_+$. Once $s_+$ is observed in AD, the real value of money brought in from KW is adjusted from $z(s)$ to $z(s)\rho(s_+)$; hence, in the KW market real balances are valued at $z(s)\hat{\rho}(s)$. Also, agents can commit within the period to any wage negotiated in MP, to be paid in units of $x$ in the current AD market, but $w(s)$ can be renegotiated when MP reconvenes next period.

We now present the value functions for $h$, keeping track of $s$ as well as individual state variables, as appropriate. In MP, taking as given the equilibrium wage function $w(s)$,

$$\begin{aligned} U_0^h(z;s) &= V_0^h(z;s) + \lambda_h\left[\tau(s)\right]\left\{V_1^h\left[z,w(s);s\right] - V_0^h(z;s)\right\} \\ U_1^h(z;s) &= V_1^h\left[z,w(s);s\right] - \delta\left\{V_1^h\left[z,w(s);s\right] - V_0^h(z;s)\right\}. \end{aligned}$$

Consider $h$ in KW with arbitrary real balances $z$ and, if $e=1$, arbitrary wage $w$, taking as given $[q(s),d(s),Q(s)]$. In equilibrium it should be clear that $h$ chooses either $z=0$ or $z=d(s)$. If $z=0$ then $V_e^h(\cdot;s) = \mathbb{E}_{s_+}W_e^h(\cdot;s_+)$; if $z=d(s)$ then

$$\begin{aligned} V_1^h\left(z,w;s\right) &= \alpha_h\left[Q(s)\right]\left\{\upsilon\left[q(s)\right] - d(s)\hat{\rho}(s)\right\} + d(s)\hat{\rho}(s) + \mathbb{E}_{s_+}W_1^h\left(0,w;s_+\right) \\ V_0^h(z;s) &= \alpha_h\left[Q(s)\right]\left\{\upsilon\left[q(s)\right] - d(s)\hat{\rho}(s)\right\} + d(s)\hat{\rho}(s) + \mathbb{E}_{s_+}W_0^h\left(0;s_+\right), \end{aligned}$$

using the linearity of $W_e^h(\cdot;s_+)$. And finally, in AD,

$$\begin{aligned} W_1^h\left(z,w;s_+\right) &= z + w + \Delta(s_+) - T(s_+) + \max_{z_+\geq 0}\left\{-z_+ + \beta U_1^h(z_+;s_+)\right\} \\ W_0^h\left(z;s_+\right) &= z + b + \ell + \Delta(s_+) - T(s_+) + \max_{z_+\geq 0}\left\{-z_+ + \beta U_0^h(z_+;s_+)\right\} \end{aligned}$$

We now present the value functions for $f$. In MP, given the equilibrium wage function $w(s)$,

$$\begin{aligned} U_0^f(s) &= \lambda_f\left[\tau(s)\right]V_1^f\left[w(s);s\right] \\ U_1^f(s) &= (1-\delta)V_1^f\left[w(s);s\right]. \end{aligned}$$

For $f$ in KW with $e=1$ and wage obligation $w$, given $[q(s),d(s)]$ and $Q(s)$,

$$V_1^f\left(w;s\right) = y - w + \alpha_f\left[Q(s)\right]\left[d(s)\mathbb{E}_{s_+}\rho(s_+) - q(s)\right] + \beta\mathbb{E}_{s_+}U_1^f\left(s_+\right).$$

We do not actually need $W_e^f$, although it should be clear how to define it.

In MP, wage bargaining implies

$$w(s) = \max_w [V_1^f(w;s)]^\eta\left[V_1^h(z,w;s) - V_0^h(z;s)\right]^{1-\eta}$$

where we note that $z$ vanishes on the RHS. In KW, let the surplus for $h$ with wage $w$ from either participating or not in the market be

$$\Sigma(s) = \max\left\{V_e^h\left[d(s),w;s\right] - V_e^h(0,w;s),0\right\},$$

where we note that $w$ vanishes from the RHS. Then $[d(s), q(s), Q(s), \Sigma(s)]$ solve the generalized conditions for competitive search given in the text:

$$\upsilon'[q(s)] - 1 = \frac{i}{\alpha_h [Q(s)]}$$
$$d(s)\mathbb{E}_{s_+}\rho(s_+) = g[q(s), \epsilon(s)]$$
$$\Sigma(s) = \beta\alpha_h [Q(s)] \{\upsilon [q(s)] - \upsilon' [q(s)] g [q(s), \epsilon(s)]\}$$
$$\Sigma(s) > 0 \Longrightarrow Q = [1 - u(s)]^{-1} \text{ and } \Sigma(s) = 0 \Longrightarrow Q = \overline{Q}(i).$$

Finally, we construct a probability transition function $\mathcal{P}(s_+, s)$ from the laws of motion for $u$, $i$ and $y$ given above in the obvious way.

We can now define equilibrium as a list of value functions $\left(U_e^j, V_e^j, W_e^j\right)$, prices $(w, d, q, \rho)$, market tightness measures $(\tau, Q)$, and distribution $\mathcal{P}$ satisfying the above conditions. More compactly, define the surplus from a match by

$$S(s) = V_1^h [z, w(s); s] + V_1^f [w(s); s] - V_0^h(z; s),$$

where $w$ and $z$ both vanish on the RHS. Then the list $(S, d, q, \tau, Q, \mathcal{P})$ constitutes an equilibrium as long as:
(i) the surplus solves

$$S(s) = y - b - \ell + \alpha_f [Q(s)] \left[\frac{d(s)}{\beta(1+i)} - q(s)\right] + \beta\mathbb{E}_{s_+} \{1 - \delta - (1 - \eta)\lambda_h [\tau(s_+)]\} S(s_+)$$

(ii) the KW terms of trade solve

$$\begin{aligned} \upsilon' [q(s)] &= 1 + i/\alpha_h [Q(s)] \\ d(s) &= \beta(1 + i)g [q(s), \epsilon(Q(s))] \end{aligned}$$

(iii) KW tightness $Q(s)$ solves

$$Q(s) = \begin{cases} [1 - u_+(s)]^{-1} & \text{if } u_+(s) \le \overline{u}(i) \\ [1 - \overline{u}(i)]^{-1} & \text{if } u_+(s) > \overline{u}(i) \end{cases}$$

where $\overline{u}(i)$ solves $\upsilon [\phi (u, i)] - \upsilon' [\phi (u, i)] g \left[\phi (u, i), \epsilon \left(\frac{1}{1-u}\right)\right] = 0$, and $\phi(u, i)$ is defined by $\upsilon' [\phi(u, i)] - 1 = i/\alpha_h \left(\frac{1}{1-u}\right)$, as in the text.
(iv) MP tightness $\tau(s)$ solves

$$k = \beta\lambda_f [\tau(s)] \eta S(s)$$

(v) $\mathcal{P}(\,\cdot\,)$ is derived from the laws of motion.

It is a standard exercise to solve numerically for functions $(S, d, q, \tau, Q, \mathcal{P})$. See http://www.wwz.unibas.ch/witheo/aleks/BMWII/BMWII.html for details, including programs for calibration and simulation of the model.

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# Figure Headings