

Growth in the Shadow of Expropriation*

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Abstract

In this paper, we address two questions: (i) Why do developing countries with the highest growth rates export capital; and (ii) Why are some countries unable or unwilling to pursue the high growth/low debt strategies that has proven successful for many “miracle” economies. The model we study is a small open economy subject to political economy and contracting frictions. The political economy frictions involve polarization and political turnover, while the contracting friction is a lack of commitment regarding foreign debt and expropriation. We show that the political economy frictions induce growth dynamics in a limited-commitment environment that would otherwise move immediately to the steady state. In particular, greater polarization corresponds to a high tax rate on investment, which declines slowly over time, generating slow convergence to the steady state. Moreover, while political frictions shorten the horizon of the government, the government may still pursue a path of tax rates in which the first best investment is achieved in the long run, although the transition may be slow. The model rationalizes why openness has different implications for growth depending on the political environment, why institutions such as respect for property rights evolve over time, and why open countries that grow rapidly tend to accumulate net foreign assets rather than liabilities.

*Preliminary. Comments welcome.

1 Introduction

This paper studies why the most successful developing economies export capital along the transition path, and addresses the related question of why doesn't every country pursue a high growth-low debt strategy. The standard neoclassical growth model predicts that opening an economy to capital inflows will speed convergence, as the constraint that investment equals domestic savings is relaxed, where convergence is conditional on the economy's particular technology and associated steady state.¹ Moreover, there is a reasonable presumption (justified by a variety of models) that openness to capital flows facilitates technology transfers/spillovers. However, the evidence on openness and growth is at best mixed (see [Obstfeld, 2009](#) for a recent overview). The East Asian miracle economies perhaps provide the strongest evidence in favor of *trade* openness and growth. However, these economies did not rely on large net capital inflows during their growth takeoffs (see the evidence presented below). This is the case despite the fact that factor accumulation played a large role in generating the high growth rates ([Young, 1995](#)).

A predominant explanation of the poor growth performance of developing countries is that weak institutions in general and poor government policies in particular tend to deter investment in capital and/or productivity enhancing technology.² A literature has developed that suggests that weak institutions generate capital outflows rather than inflows (see, for example, [Tornell and Velasco, 1992](#) and [Alfaro et al., 2008](#)). While it is no doubt true that world capital avoids countries with weak property rights, there is more to the story.

The East Asian economies again provide a useful counter-example. These economies had high growth rates, high investment rates, large gross capital inflows, but even larger gross capital outflows. More generally, consider Figure 1 Panel A, which plots cumu-

¹Two important papers that study the neoclassical growth model in an open economy setting are [Barro et al. \(1995\)](#) and [Ventura \(1997\)](#).

²An important contribution in this regard is [Parente and Prescott \(2000\)](#). Similarly, a large literature links differences in the quality of institutions to differences in income per capita, with a particular emphasis on protections from governmental expropriation (for an influential series of papers along this line, see [Acemoglu et al., 2001, 2002](#) and [Acemoglu and Johnson, 2005](#)).

lative growth in real GDP against changes in net foreign asset positions for a number of developing countries.³ A clear (and statistically significant) positive relationship between growth and changes in net foreign assets emerges. In Panel B, we see that no such strong correlation exists for developed economies. [Gourinchas and Jeanne \(2007\)](#) have documented in a striking fashion that net capital flows in the last decades have been directed to economies with low growth rates, while capital has flown out of economies with high growth rates (see also [Aizenman et al., 2004](#) and [Prasad et al., 2006](#)). This allocation puzzle represents an important challenge to the standard open economy model.

To understand these various facts, we present a tractable growth model that highlights the interaction of political economy frictions and capital flows in a small open economy. The analysis focuses, given an underlying political economy environment, on the endogenous evolution of “institutions,” which corresponds to the level of expropriation in our analysis. Our model builds on a neoclassical foundation where growth occurs through capital accumulation. The government taxes capital income, where capital taxation proxies for the various means through which governments expropriate capital income in practice. Capital is fully mobile, and economic growth is governed by the endogenous evolution of capital taxes.

We break from the neoclassical framework by assuming that the economy is subject to two frictions. First, there is limited commitment on the part of the domestic government: investment could be expropriated ex-post and the government could default on the external stock of debt. Second, there is a political economy friction as the country is composed of parties with distinct objectives that compete for political power.

We model the evolution of political power in a simple way that summarizes two well studied forces: incumbent parties are afraid to lose office, and prefer consumption to occur when they are in power. We model the political parties as caring about the allocation

³Figure 1’s vertical axis is the log change in constant local currency GDP from the World Development Indicators between 1970 and 2004. The horizontal axis is the change over the same period in gross foreign assets minus gross foreign liabilities (both in current US dollars) normalized by the country’s average current dollar GDP between 1970 and 2004, using data from [Lane and Milesi-Ferretti \(2007\)](#) External Wealth of Nations (EWN) Mark II data set. “Developing” is defined as 1970 GDP per capita of less than USD 9,000, in 2000 dollars.

of consumption that the population receives (either because they themselves share out of that allocation, or because of electoral concerns), however they disagree about the timing of spending. In particular, incumbents prefer spending to occur while in office. This preference can be motivated by the polarization of the political process: If the next government has different priorities than the current incumbent, spending undertaken by the future government will yield less utility to the current incumbent than if the incumbent had held onto office and spent the same amount. This “love of incumbency” could also be interpreted as government office holders consuming a disproportionate share of aggregate consumption while in office.⁴ Our benchmark model assumes that each party has a constant (and exogenous) probability of being elected to rule in any period. We show how the results generalize to a Markov political process in which incumbents oscillate between incumbency and non-incumbency. In general, the love of incumbency together with a non-zero probability of losing office generates short-term discount rates for the incumbent parties that are higher than their long-term ones (as in [Amador, 2004](#)).

We consider the path of taxes, consumption, investment, and debt, that maximizes the population’s welfare subject to the constraint that each incumbent has the power to repudiate debt and expropriate capital. Deviation, however, leads to financial autarky and reversion to a high tax-low investment equilibrium. In this sense, we study self-enforcing equilibria in which allocations are constrained by the government’s lack of commitment, as in [Thomas and Worrall \(1994\)](#), [Albuquerque and Hopenhayn \(2004\)](#), and [Aguiar et al. \(2009\)](#). These papers discuss how limited commitment can slow capital accumulation. However, in this paper we show that the political economy frictions generate additional dynamics. To drive this point home, we first study the case where the utility function is linear in consumption and, absent political frictions, convergence to the steady state is immediate. We show that political economy frictions introduce non-degenerate transitional dynamics.

Specifically, we first show that if domestic agents discount at the world interest rate, the economy eventually reaches the first best level of capital, as long as parties place

⁴This interpretation depends on whether the preferences are such that the substitution effect dominates the income effect of a change in the price of current consumption.

a non-zero value on consumption while out of office. This reflects the fact that while incumbents discounts the near future more than do private citizens, this relative impatience disappears as the horizon is extended far into the future. Importantly, the speed of convergence is now bounded and determined by the level of political polarization.

One main feature of the model is debt overhang: A country with a large external debt position has a greater temptation to default, and therefore cannot credibly promise to leave large investment positions un-expropriated. Growth therefore requires the country to pay down its debt, generating a trade off between the incumbent's desire to consume while in office against reducing foreign liabilities and increasing investment. The mechanism is consistent with the empirical fact that fast growth is accompanied by reductions in net foreign liabilities. However, we are still left with the question of why all countries do not pursue the high growth, low debt strategy. Our model answers this by pointing out that countries with a greater polarization parameter are unwilling to reduce debt quickly, as the desire for immediate consumption outweighs the future benefits of less overhanging debt. In this manner, the model is able to reconcile the mixed results that countries have had with financial globalization. Countries with different underlying political environments will have different growth experiences after opening. Some economies will borrow and stagnate, while others will experience net outflows and grow quickly.

Moreover, as long as domestic agents discount at the world interest rate, the steady state level of debt *declines* as we increase the political economy frictions. That is, a greater love of incumbency leads to less debt in the long run. This is the counterpart to the fact that tax rates converge to zero and first best capital is sustained. To support the first best level of investment, an economy with greater political frictions must have disproportionately less debt.

When private agents are impatient relative to the world interest rate, the first best level of investment would not be achieved in the long run even in the absence of political economy frictions. In this environment, the steady state level of capital decreases as we increase polarization. Nevertheless, conditional on the steady state, the speed of convergence is still decreasing in polarization. Moreover, the results are robust to more general

utility functions. While the linear case is a useful expositional environment as it leads to linear dynamics that can be easily characterized, we show that the results carry over to a more realistic environment with concave utility.

Related Literature

Our paper addresses the issue of “global imbalances” as it relates to the interaction of developing economies with world financial markets. An alternative explanation is that developing economies have incomplete domestic financial markets and therefore higher precautionary savings, which leads to capital outflows (see [Willen, 2004](#) and [Mendoza et al., 2008](#)). However, this literature is silent on the heterogeneity across developing economies in terms of capital flows. For example, several Latin American economies have similar or even more volatile business cycle than South Korea ([Aguiar and Gopinath, 2007](#)) and less developed financial markets ([Rajan and Zingales, 1998](#)), yet Latin America is not a strong exporter of capital (Figure 1). [Caballero et al. \(2008\)](#) also emphasize financial market weakness as generating capital outflows. In their model, exogenous growth in developing economies generates wealth but not assets, requiring external savings. Our model shares their focus on contracting frictions in developing economies, but seeks to understand the underlying growth process. As noted above, our paper shares the feature of [Thomas and Worrall \(1994\)](#) that reductions in debt support larger capital stocks. [Dooley et al. \(2004\)](#) view this mechanism through the lens of a financial swap arrangement, and perform a quantitative exercise that rationalizes China’s large foreign reserve position. These papers are silent on why some developing countries accumulate collateral and some do not, a primary question of this paper. Our paper also explores how the underlying political environment affects the speed with which countries accumulate collateral or reduce debt.

Our paper also relates to the literature on optimal government taxation with limited commitment. Important papers in this literature are [Benhabib and Rustichini \(1997\)](#) and [Phelan and Stacchetti \(2001\)](#), who share our focus on self-enforcing equilibria supported

by trigger strategies (a parallel literature has developed that focuses on Markov perfect equilibria, such as [Klein and Rios-Rull, 2003](#), [Klein et al., 2005](#), and [Klein et al., 2008](#)). In this literature, our paper is particularly related to [Dominguez \(2007\)](#), which shows in the environment of [Benhabib and Rustichini \(1997\)](#) that a government will reduce its debt in order to support the first best capital in the long run (see also [Reis, 2008](#)).

The key parameter of our model captures the extent of polarization. [Esteban and Ray \(1994\)](#) discuss the theory behind the measurement of polarization. One interpretation of this parameter is the degree of ethnic or linguistic fragmentation, which [Easterly and Levine \(1997\)](#) document is inversely related to growth. We show in the concluding section that ethnic fragmentation is positively correlated with net financial inflows, which is perhaps puzzling from the viewpoint that capital should avoid political risk but is completely consistent with our model.

As noted above, a large literature has documented that respect for property rights is positively correlated with growth and investment. A classic paper is [North and Weingast \(1989\)](#), who argue that the Glorious Revolution in seventeenth century England constrained the government, allowing a deepening of financial markets. Our paper can be seen as a modern analogue of this phenomenon, but in the modern version developing economies exploit global financial markets to endogenously improve their domestic institutions.

The remainder of the paper is organized as follows. Section 2 presents the environment. Section 3 characterizes the path of equilibrium taxes, investment, and output. Section 4 revisits the empirical facts through the lens of the model and concludes.

2 Environment

In this section we describe the model environment (which is based on [Aguiar et al., 2009](#)). Time is discrete and runs from 0 to infinity. There is a small open economy which produces a single good, whose world price is normalized to one. There is also an inter-

national financial market that buys and sells risk-free bonds with a return denoted by $R = 1 + r$.

The economy is populated by capitalists, who own and operate capital, workers who provide labor, and a government. We draw an important distinction between workers and capitalists. Specifically, we assume that capitalists do not enter the government's objective function, defined below. This may be because capitalists are foreign and face a nationalistic government, or that capitalists are domestic but face a populist government. More generally, capitalists are "outsiders" and workers are "insiders" in terms of government priorities. This assumption is convenient in that the government has no hard-wired qualms about expropriating capital income and transferring it to its preferred constituency. The important distinction is that capitalists operate a technology otherwise unavailable to the government and are under the threat of expropriation. For convenience, we use the terminology of foreign capitalists and equate workers with domestic residents.

2.1 Firms

Domestic firms use capital together with labor to produce according to a strictly concave, constant returns to scale production function $f(k, l)$. We assume that $f(k, l)$ satisfies the usual Inada conditions. Capital is fully mobile internationally at the beginning of every period, but after invested is sunk for one period. Capital depreciates at a rate d .

Labor is hired by the firms in a competitive domestic labor market which clears at an equilibrium wage w_t . The government taxes the firm profits at a rate τ_t . Let $\pi = f(k, l) - wl$ denote per capita profits before taxes and depreciation, and so $(1 - \tau)\pi$ is after-tax profits. The firm rents capital at the rate $r + d$, where d is the rate of depreciation. Given an equilibrium path of wages and capital taxes, profit maximizing behavior of the

representative firm implies:

$$(1 - \tau_t)f_k(k_t, l_t) = r + d \quad (1)$$

$$f_l(k_t, l_t) = w_t. \quad (2)$$

For future reference, we denote k^* as the first best capital given a mass one of labor: $f_k(k^*, 1) = r + d$. When convenient in what follows, we will drop the second argument and simply denote production $f(k)$.

It is convenient to limit the government's maximal tax rate to $\bar{\tau}$. We assume that this constraint does not bind along the equilibrium path. Nevertheless, as discussed in Section 2.5, this assumption allows us to characterize possible allocations off the equilibrium path.

2.2 Domestic agents and the government

Labor is supplied inelastically each period by a measure-one continuum of domestic agents (there is no international mobility of labor). The representative domestic agent enjoys utility flows over per capita consumption, $u(c)$. Domestic agents discount the future with a discount factor $\beta \in (0, 1/R]$. The representative agent's utility is

$$\sum_{t=0}^{\infty} \beta^t u(c_t). \quad (3)$$

with $u' > 0$, $u'' \leq 0$, and where we normalize $u(0) \geq 0$.

We assume that domestic agents have no access to capital markets and consume their endowment: $c_t = w_t + T_t$, where T_t is the transfer from the government. Equivalently, one could assume that the government has enough policy instruments to control the consumption/savings decisions of its constituents (see for example the discussion in [Kehoe and Perri, 2004](#)).

The government every period receives the income from the tax on profits and transfers

resources to the workers subject to its budget constraint:

$$\tau_t \pi_t + b_{t+1} = Rb_t + T_t \quad (4)$$

where b_t is debt due in period t . The country's resource constraint is therefore:

$$c_t + (1 + r)b_t = b_{t+1} + \tau \pi_t + w_t. \quad (5)$$

Note that output is deterministic, and so a single, risk-free bond traded with the rest of the world is sufficient to insure the economy. However, as described next in the next subsection, political incumbents face a risk of losing office, and this risk is not insurable. The fact that sovereign debt is not contingent on individual leader's political fortunes is a realistic assumption. We leave the question of how debt contingent on political outcomes affects dynamics for future research.

2.3 Political Environment

There is a set $I \equiv \{1, 2, \dots, N\}$ of political parties, where N is the number of parties. At any time, the government is under the control of an incumbent party that is chosen at the beginning of every period from set I . As described below, an incumbent party may lose (and regain) power over time. A fundamental assumption is that the incumbent strictly prefers consumption to occur while in power:

Assumption 1 (Political Economy Friction). *A party enjoys a utility flow $\tilde{\theta}u(c)$ when in power and a utility flow $u(c)$ when not in power, where c is per capita consumption by the domestic agents and where $\tilde{\theta} > 1$.*

The parameter $\tilde{\theta}$ parametrizes the extra marginal utility benefits that a party extracts from expenditures when in power. One motivation for this parameter is political polarization. Specifically, suppose that the incumbent party selects the attributes of a public good that forms the basis of private consumption. If parties disagree about the desirable attributes of the consumption good, the utility stemming from a given level of spending will be

greater for the party in power. We model such political polarization in a simple, reduced form way with the parameter $\tilde{\theta}$. Alternatively, we can think of the incumbent capturing a disproportionate share of per capita consumption.⁵

The transfer of power is modeled as an exogenous Markov process. The fact that the transfer of power is exogenous can be considered a constraint on political contracts between the population (or other parties) and the incumbent. As will be clear, each incumbent will abide by the constrained efficient tax plan along the equilibrium path. However, doing so does not guarantee continued incumbency (although our results easily extend to the case where the incumbent loses office for sure if it deviates). That is, following the prescribed tax and debt plan does not rule out that other factors may lead to a change of government. We capture this with a simple parametrization that nests both perpetual office holding and hard term limits.

Specifically, let $\gamma_{i,o}$ be the probability that today's incumbent loses power next period, where the subscript "i,o" indicates the transition between "in power" to "out of power". If the incumbent loses office, it may regain power at some point in the future. Let $\gamma_{o,i}$ be the probability of moving from out of power back into office, and this probability is the same for all parties out of power. Our benchmark case is $1 - \gamma_{i,o} = \gamma_{o,i} = p$, so each party has the same probability of taking office next period, regardless of incumbency. The more general case in which an incumbent has a higher probability of retaining office is treated in Appendix A. The main results presented below extend to the more general political process. The appendix also discusses new results that arise when incumbency is persistent.

Given a deterministic path of consumption, the utility of the incumbent in period t can now be expressed as:

$$\tilde{W}_t = \tilde{\theta}u(c_t) + \sum_{s=t+1}^{\infty} \beta^{s-t}(p\tilde{\theta} + 1 - p)u(c_s). \quad (6)$$

⁵For example, suppose that when in power, a party receives a higher share ϕ of c . Then, the marginal utility when in power is $u'(\phi c)\phi$, and our assumption is that this marginal utility is increasing in ϕ .

Note that the incumbent discounts the future at the private agents' discount factor β , but incumbency implies that it discounts between the current and next period at the rate $\beta/(p\tilde{\theta} + 1 - p) < \beta$. That is, for a deterministic path of consumption, the incumbent's objective is equivalent to a quasi-hyperbolic agent as in [Laibson \(1997\)](#) (the fact that political turnover can induce hyperbolic preferences for political incumbents was explored by [Amador, 2004](#)). This framework is rich enough to capture several important cases. A situation where the country is ruled by a "dictator for life" who has no altruism for successive generations, can be analyzed by letting $\theta \rightarrow \infty$, reflecting the zero weight the dictator puts on aggregate consumption once it is out of power. Letting $\theta \rightarrow 1$, the government is benevolent and the political friction disappears.

We simplify the expression for the government's value function by introducing a sequence of fictitious "stand-in" governments, each of which is in power one period and has a value function W_t proportional to \tilde{W}_t . Specifically, we can normalize the government's value function (6) by the constant $p\tilde{\theta} + 1 - p$. As will be clear below, the scaling of the incumbent's utility has no effect on the equilibrium allocations, and so we work with W_t . In particular,

$$\begin{aligned}
 W_t &= \frac{\tilde{W}_t}{(p\tilde{\theta} + 1 - p)} \\
 &= \theta u(c_t) + \sum_{s=t+1}^{\infty} \beta^{s-t} u(c_s) \\
 &= \theta u(c_t) + \beta V_{t+1},
 \end{aligned} \tag{7}$$

where V_t is the value function of private agents and $\theta \equiv \frac{\tilde{\theta}}{p\tilde{\theta} + 1 - p}$. Note that θ is increasing in $\tilde{\theta}$, but is decreasing in p . That is, a lower probability of retaining office is another form of polarization from the perspective of our stand-in government.

2.4 Equilibrium Concept

The final key feature of the environment concerns the government's lack of commitment. Specifically, tax policies and debt payments for any period represent promises that can be broken by the government. Given the one-period irreversibility of capital, there exists the possibility that the government can seize capital or capital income. Moreover, the government can decide not to make promised debt payments in any period.

We consider self-enforcing equilibria that are supported by a "punishment" equilibrium. Specifically, let $\underline{W}(k)$ denote the payoff to the incumbent government after a deviation when capital is k , which we characterize in the next subsection. Self-enforcing implies that:

$$W_t \geq \underline{W}(k_t), \forall t, \quad (8)$$

where W_t is given by (7).

Our equilibrium concept is the following:

Definition 1. A *self-enforcing deterministic equilibrium* is a deterministic sequence of consumption, capital, debt, tax rates and wages $\{c_t, k_t, b_t, \tau_t, w_t\}$, with $\tau_t \leq \bar{\tau}$ for all t and such that (i) firms maximize profits given taxes and wages; (ii) the labor market clears; (iii) the resource constraint (5) and the associated no Ponzi condition hold given some initial debt b_0 ; and (iv) the participation constraint (8) holds given deviation payoffs $\underline{W}(k_t)$.

2.5 The punishment and the deviation payoff

For simplicity we defined the equilibrium conditional on deviation payoffs $\underline{W}(k)$. Towards obtaining this punishment payoffs, we assume that after any deviation from the equilibrium allocation, the international financial markets shut down access to credit and assets forever.

Given that the country has no access to borrowing nor savings after a deviation, we construct a punishment equilibrium of the game between investors and the government

that has the following strategies. For any history following a deviation, the party in power sets the tax rate at its maximum $\bar{\tau}$, and investors invest \underline{k} , where \underline{k} solves:

$$(1 - \bar{\tau})f'(\underline{k}) = r + d.$$

These strategies form an equilibrium. A party in power today cannot gain by deviating to a different tax rate, given that it is already taxing at the maximum rate and reducing taxes does not increase future investment. Investors understand that they will be taxed at the maximum rate, and thus invest up to the point of indifference.

The following proposition establishes that the above equilibrium is the harshest punishment:

Proposition 1. *The continuation equilibrium where $\tau_t = \bar{\tau}$ after any history and the country is in financial autarky generates the lowest utility to the incumbent party of any self-enforcing equilibrium.*

In any self-enforcing equilibrium, once the investors have invested k in the country, the party in power could deviate from the equilibrium path by choosing a different tax rate or by changing the equilibrium path of debt issuance. Such a deviation triggers financial autarky and the lowest possible investment. If the party in power where to deviate, it will find optimal to tax current k at the maximum possible rate, and its deviating payoff will be given by $\underline{W}(k)$:

$$\begin{aligned} \underline{W}(k) &= \theta u(\bar{c}(k)) + \sum_{t=1}^{\infty} \beta^t u(\bar{c}(\underline{k})) \\ &= \theta u(\bar{c}(k)) + \frac{\beta}{1 - \beta} u(\bar{c}(\underline{k})), \end{aligned} \tag{9}$$

where $\bar{c}(k) = f(k) - (1 - \bar{\tau})f'(k)k$.

3 Efficient Allocations

In this section we solve for the self-enforcing deterministic equilibrium that maximizes the utility of the population as of time 0 given an initial level of debt.

We first simplify the problem. Multiply the firms' first order conditions for capital by k to obtain: $(1 - \tau)f_k k = (r + d)k$. Similarly, multiply the first order condition for labor by $(1 - \tau)l$ to obtain: $(1 - \tau)f_l l = (1 - \tau)wl$. Adding together and using $f_k k + f_l l = f$, we have $(1 - \tau)f = (r + d)k + (1 - \tau)wl$. Tax revenue for the government is given by $\tau\pi = \tau(f - wl)$. Substituting into the previous expression, we have

$$\begin{aligned} wl + \tau\pi &= \tau f + (1 - \tau)wl \\ &= f - (r + d)k. \end{aligned} \tag{10}$$

We can therefore substitute τ out of the resource constraint for the economy: $c_t + (1 + r)b_t = b_{t+1} + f(k_t) - (r + d)k_t$. Given choices of c , k , and b , we can solve for the associated tax rate using the above expressions. However, given the revised resource constraint, we no longer need to consider τ as a choice variable and the firm's first order conditions as constraints.

We can then obtain the equilibrium allocation that maximizes the utility of the population at time zero, given an initial stock of debt b_0 by solving:

$$P : \quad V(b_0) = \max_{\{u_t, h_t\}} \sum_{t=0}^{\infty} \beta^t u_t$$

subject to:

$$b_0 \leq \sum_{t=0}^{\infty} R^{-t} (f(K(h_t)) - (r + d)K(h_t) - c(u_t)), \tag{11}$$

$$h_t \leq \sum_{s=t}^{\infty} \beta^{s-t} [(1 - \gamma)^{s-t} (\theta - 1) + 1] u_s, \quad \forall t. \tag{12}$$

The first constraint is the budget constraint, where we have imposed a No Ponzi con-

dition. The second constraint is the participation constraint, where we let the deviation utility $h_t = \underline{W}(k_t)$ be our choice variable instead of capital, and define $K(\underline{W}(k)) = k$ to be the inverse of $\underline{W}(k)$. Similarly, we make utility itself the choice variable and let $c(u)$ denote the inverse utility function, that is, the consumption required to deliver the specified utility. We make these substitutions to address the fact that the participation constraint may not be convex in the original choice variables. In our new notation, the following guarantees convexity of the problem:

Assumption 2. $f(K(h)) - (r + d)K(h)$ is strictly concave in h .

This assumption allows us to unambiguously sign the relationship between the Lagrange multipliers introduced below and the capital stock. In essence, it requires the production function to be concave relative to $\underline{W}(k)$. This condition is satisfied for linear utility as well as for concave utility in the neighborhood of k^* , which are the two cases explored below.

We also assume that the constraint $k_t \geq \underline{k}$ (or, equivalently, the constraint $\tau_t \leq \bar{\tau}$) is not binding along the equilibrium path. Moreover, we consider b_0 such that the constraint set is not empty (that is, initial debt can be paid back while satisfying the participation constraints). Inspection of problem (P) reveals that the value function $V(b_0)$ is strictly decreasing in debt.

Let μ_0 be the multiplier on the budget constraint (11) and $R^{-t}\eta_t/\mu_0$ be the multiplier on the sequence of constraints on participation (12). The necessary conditions for optimality, in an interior, are:

$$\begin{aligned} \eta_t &= \left(f'(K(h_t)) - (r + d) \right) K'(h_t) \\ &= \frac{f'(k_t) - (r + d)}{\underline{W}'(k_t)} \end{aligned} \tag{13}$$

and

$$c'(u_t) = (\beta R)^t \left(\frac{1}{\mu_0} \right) + (\theta - 1)\eta_t + \sum_{s=0}^t (\beta R)^{t-s} \eta_s. \tag{14}$$

The multiplier η_t represents the distortion in investment that arises from a binding participation constraint. As can be seen from (13) and given our concavity assumption, there is a decreasing and monotone relationship between η_t and k_t . Capital will be first best if the incumbent's participation is slack ($\eta = 0$), and is sub-optimal otherwise. Referring back to the firm's first order condition for capital, this implies that absent commitment problems, the incumbent would set $\tau = 0$. This follows from the fact that $\tau = 0$ maximizes domestic resources. However, under lack of commitment, the government may tax capital as $\tau = 0$ is not credible. Nevertheless, the first order conditions indicate that $\tau = 0$ will be sustained in the long run if private agents discount at the world interest rate:

Proposition 2. *If $\beta R = 1$ and $\theta < \infty$, then $k_t \rightarrow k^*$.*

The proof of this proposition (see Appendix B) relies on the fact that each time the participation constraint binds, $\eta > 0$ and we add to the sum on the right hand side of (14). There is a potentially countervailing force in that the current η_t is weighted by more than the past, as $\theta > 1$. However, eventually the (infinite) sum dominates and consumption levels off at a point such that participation no longer binds at k^* .

Note that the steady state level of capital when $\beta R = 1$ is invariant to θ (as long as $\theta < \infty$). However, as we will see, while the end point may be the same, the speed at which it gets there depends on and θ .

A common feature of models with one-sided limited commitment is that the optimal policy "back loads" incentives if agents are patient (see, for example, Ray, 2002). This is because, at the margin, agents are indifferent to consumption today and tomorrow. However, consumption tomorrow relaxes participation constraints going forward, meaning agents are inclined to save. Eventually, enough assets are built up to relax the constraints altogether. However, if agents are impatient, they prefer consumption today and may be unwilling to accumulate enough assets to completely relax the participation constraint. However, if the government (agent) is impatient, this is not necessarily the case. For example, in the environments of Aguiar et al. (2009) and Acemoglu et al. (2008), govern-

mental impatience prevents the first best level of investment from being achieved in the long run. In our environment, we approach this first best level despite the fact that the incumbent that chooses the tax rate at every period is discounting between today and tomorrow at a higher rate than private agents. However, each incumbent discounts between future periods at the rate β . For this reason, each government is willing to support a path of investment that approaches the first best limit because it values the limiting periods at the world interest rate. This case clarifies that short term impatience of the incumbents is not sufficient to generate distortions in the long-run.

If the government does not value private agent consumption (that is, $\theta \rightarrow \infty$), then as long as there is political turnover the economy will not reach first best capital in the steady state. In fact, as we let θ go to infinity, the problem collapses to a myopic representative government and the economy jumps immediately to a steady state with tax rate $\bar{\tau}$ and capital \underline{k} .

The fact that capital is first best in the long run depends on openness as well as patience. In particular, it requires access to foreign financial markets. The ability to support first best capital depends on the ability to accumulate net foreign assets. If debt were constant, then capital would also be constant, and there would be no mechanism through which the economy could converge to the first best. This is true even with foreign direct investment, highlighting a distinction between net financial assets as discussed in Section 2 and inward FDI. Our environment provides a stark counterpoint to the analysis of [Gourinchas and Jeanne \(2006\)](#). That paper noted that a closed economy operating according to the neoclassical growth model converges to the same steady state as an open economy. Openness in that environment shortens the transition, but does not change the ultimate steady state. This limits the welfare gain of openness, the primary message of [Gourinchas and Jeanne \(2006\)](#). Conversely, in our model, the steady state level of capital depends crucially on openness, and this may increase the welfare gains from openness.

3.1 Linear Utility

In this subsection, we study the case of linear utility, $u'(c) = 1$, for which we can solve for the equilibrium dynamics in closed form. Although an extreme case, the intuition of the linear case carries over to the concave case studied next. For what follows, we will ignore the non-negativity constraint on consumption (or else, the reader can assume that the analysis is in the neighborhood of the steady state of the economy, which will turn out to feature positive consumption levels).

In the case of linear utility, the first order condition for consumption becomes:

$$1 = (\beta R)^t \left(\frac{1}{\mu_0} \right) + (\theta - 1)\eta_t + \sum_{s=0}^t (\beta R)^{t-s} \eta_s, \forall t \geq 0 \quad (15)$$

The initial period η_0 is therefore $\theta\eta_0 = 1 - 1/\mu_0$. As μ_0 is the multiplier on b_0 , more debt in period 0 is associated (weakly) with a larger μ_0 and a larger η_0 . After the initial period, the dynamics of η_t are linear, as the next proposition states:

Proposition 3. *The multiplier η_t that solves (15) satisfies the following difference equation:*

$$\eta_{t+1} = \frac{1 - \beta R}{\theta} + \beta R \left(1 - \frac{1}{\theta} \right) \eta_t, \forall t \geq 0 \quad (16)$$

with $\eta_0 = \frac{1 - \mu_0^{-1}}{\theta}$. The sequence of η_t converges monotonically towards its steady state value η_∞ :

$$\eta_\infty = \frac{1 - \beta R}{\theta(1 - \beta R) + \beta R}.$$

Letting $\hat{\eta}_t \equiv \eta_t - \eta_\infty$, convergence to the steady state can be characterized by:

$$\hat{\eta}_{t+1} = \beta R \left(1 - \frac{1}{\theta} \right) \hat{\eta}_t. \quad (17)$$

From (16), if $\beta R < 1$, then $\eta_t > 0$ for $t \geq 1$, regardless of initial debt. If $\beta R = 1$, then $\eta_t > 0$ for all t if and only if $\eta_0 > 0$, which is the case if and only if $\mu_0 > 0$. That is, if $\beta R < 1$, or if initial debt is large enough that we cannot support the first best at $t = 0$ when $\beta R = 1$, the

economy will have non-trivial dynamics. For the remainder of the analysis, we assume this is the case.

Before discussing this proposition, we note that the path of capital inherits the dynamics of η . Specifically, recall that there is a one to one decreasing mapping from η_t to k_t . A particularly convenient benchmark is $\bar{\tau} = 1$. In this case, $\tau_t = \theta\eta_t = 1 - (r + d)/f'(k_t)$, so the tax rate is proportional to the multiplier on participation. More generally, monotone convergence of η_t implies monotone convergence of k_t (and of the tax rates that decentralize it). The sequence of k_t then, also converges to a steady state:

Corollary 1. *The sequence of capital, k_t , converges monotonically to its steady state level of capital, k_∞ :*

$$\frac{f'(k_\infty) - (r + d)}{c'(k_\infty)} = \frac{\theta(1 - \beta R)}{\theta(1 - \beta R) + \beta R} \quad (18)$$

and k_∞ is decreasing in θ when $\beta R < 1$. If country A starts with a higher debt level than country B, then all else equal, the path of capital for country A will be (weakly) lower than that for country B.

We start by discussing the case in which there are no political economy considerations, which can be obtained when $\theta = 1$. From the propositions above, note that in this case $\eta_t = (1 - \beta R)$ for all $t > 0$. That is the economy converges in one period to its steady state level. Namely, because we do not impose the non-negative consumption constraint, in period 0 the country could post a large bond abroad or reduce the debt sufficiently, such that the steady state level of capital can be sustained starting from $t = 1$.

The linear case in standard models of expropriation has been studied in detail by [Thomas and Worrall \(1994\)](#) and [Albuquerque and Hopenhayn \(2004\)](#) for $\beta R = 1$. In those papers, non-trivial transition dynamics are generated because of the binding requirement that consumption must be positive. The results here make clear that the speed of convergence around the steady state in these models is infinity (independently of whether βR is equal to or less than one), and also that these linear economies will immediately converge if they start with sufficiently low debt. Moreover, transition dynam-

ics that are driven solely by the non-negativity constraint on consumption leaves open the question of why some developing countries converge faster than others. For that, we re-introduce the political economy frictions.

The political economy frictions are at play when $\theta > 1$. As long as private agents are impatient, (18) implies that increased polarization (θ) leads to a decline in steady state capital. In this situation, political economy considerations exacerbate the relative impatience of private agents. Moreover, from (17) we see that $\theta > 1$ introduces transitional dynamics, and the greater is θ the slower the rate of convergence. To the extent that political polarization varies across countries, we will have heterogeneity in growth rates, and, if $\beta R < 1$, in steady states as well. This addresses the question posed in the introduction regarding cross-country differences in growth rates observed in the data. In the concluding section, we provide some suggestive evidence that this mechanism is at work empirically using a popular measure of polarization.

Before characterizing the path of consumption and debt, we provide some intuition for why θ governs the rate of convergence. The intuition for why this is the case is related to why we observe dynamics at all in an environment where absent political economy frictions we jump immediately to the steady state. To understand the comparative statics for growth rates, consider the following perturbation exercise. For this exercise, suppose $\beta R = 1$ and $\bar{\tau} = 1$. The latter assumption implies that $\bar{c}(k) = f(k)$ and so $\underline{W}'(k) = \theta f'(k)$.

Suppose we (or the period-0 private agents) increase output by one unit in period t , by investing an additional $1/f'(k_t)$ units of capital. As participation binds, we need to increase consumption by one unit as well, given that $\Delta W_t = \theta \Delta c_t = \theta \Delta f(k_t) = \Delta \underline{W}(k_t)$. The net cost of this perturbation in t is therefore the opportunity cost of capital, $(r + d)/f'(k_t)$.

The fact that consumption increases in t relaxes the participation constraint in period $t - 1$. In particular, we reduce c_{t-1} so that $\Delta W_{t-1} = 0$. This ensures that k_{t-1} is sustainable. In particular, $\Delta c_{t-1} = -\beta/\theta$, so the planner gets some of the additional cost back. However, the political economy frictions mean it is less than one for one. That is, c_{t-1}

cannot be decreased by the full present value of the increase of c_t as $\beta/\theta < \beta$. Collecting terms, the net benefit from period $t - 1$ and t , in period 0 units and using $\beta = R^{-1}$, is:

$$\beta^{t+1} \left(\frac{1}{\theta} - \frac{r+d}{f'(k_t)} \right). \quad (19)$$

Considering the exercise so far, we have increased c_t and reduced c_{t-1} in such a way that W_{t-1} remains unchanged. If government preferences were exponential, we would stop here – the fact that W_{t-1} is unchanged implies that W_s , $s < t - 1$, also remains unchanged. Before proceeding, it is useful to pause and consider the exercise as if incumbent preferences were exponential. Optimality would imply (19) is set to zero. Using the fact that $\tau_t = (r+d)/f'(k_t)$, we would have $\tau_t = (1 - 1/\theta)$. A few things are worthy of note. First, the right hand side of this equality is a constant, so taxes would be constant and we would have no dynamics. Second, the tax rate is increasing in θ , and if $\theta = 1$, which implies no political frictions, we obtain the first best capital stock. This exercise makes clear that the non-recursive structure is at the heart of dynamics.

Returning to the actual problem, we need to incorporate the fact that $\Delta W_{t-1} = 0$ does not imply that $\Delta W_s = 0$ for $s < t - 1$. In fact, we can reduce consumption in each period $s < t - 1$. To keep W_{t-2} unchanged, period $t - 2$ consumption can be reduced $\frac{\beta^2}{\theta} \left(1 - \frac{1}{\theta}\right)$, and so on moving backward through time to period 0. Summing these terms, the we can reduce the present value of consumption for $s < t$ by a factor:

$$- \sum_{s=0}^{t-1} \beta^s \Delta c_s = \frac{\beta^t}{\theta} \sum_{s=0}^{t-1} \beta^s \left(1 - \frac{1}{\theta}\right)^s. \quad (20)$$

Equating the net cost $\beta^t(r+d)/f'(k_t)$ to the net benefit in saved consumption (20), we have

$$\frac{r+d}{f'(k_t)} = \frac{1}{\theta} \sum_{s=0}^{t-1} \left(1 - \frac{1}{\theta}\right)^s = 1 - \left(1 - \frac{1}{\theta}\right)^t \text{ for } t \geq 1. \quad (21)$$

Equation (21) highlights why θ governs the speed of capital accumulation. Increasing k in period t requires increasing c in period t to satisfy that period's participation constraint. This relaxes the participation constraint of previous incumbents, but not one for

one. Time inconsistent preferences imply that each incumbent has a different view on future incumbent's consumption, so as we move forward in time each incumbent enters separately in the summation. The further forward in time we move, the more incumbents we add and the lower the cost of increasing k . However, the higher the θ , the slower the decline in costs and the longer it takes to converge to the steady state.

We should note that convergence depends on βR as well as θ , as seen from (17). However, this is simply a reflection of the fact that the steady state capital falls, and η_∞ increases, as agents become more impatient. In the limit, as $\beta \rightarrow 0$, the only sustainable tax is $\bar{\tau}$ and we jump immediately to a steady state in which $k = \underline{k}$. Conversely, as $\beta \rightarrow R^{-1}$, steady state capital becomes first best and the economy has a longer transition.

Now that we have solved for the dynamics of η and k , we turn to the dynamics of consumption and debt. The sequence of binding participation constraints, $W_t = \underline{W}(k_t)$ map the dynamics of capital into that of incumbent utility, given that $\underline{W}(k)$ is strictly increasing in k . Therefore, W_t also monotonically approaches its steady state value starting from $t = 1$. To recover the path of private agent utility, we need to recover private utility from incumbent utility. Recall that these differ due to the presence of θ in the incumbent's utility function. However, calculations similar to the one behind equation (20) implies the following relationship between V_t and the sequence of W_t :

Lemma 1. *The utility to the population as of time t , $V_t = \sum_{s=0}^{\infty} \beta^s c_{t+s}$, is given by:*

$$V_t = \frac{1}{\theta} \sum_{s=0}^{\infty} \beta^s \left(1 - \frac{1}{\theta}\right)^s W_t.$$

Note the fact that the adjustment factor $\left(1 - \frac{1}{\theta}\right)^t$ is the same as in (20), and for the same reason – this is the wedge between the private agent's and the incumbent's valuations of future consumption. Given that the values k_t are monotonic from $t = 1$ onwards and that $W_t = \underline{W}(k)$ is an increasing function of k , it follows that:

Proposition 4. *The utility to of the population, V_t , converges monotonically to its steady state value from $t = 1$ onwards. The value of V_t increases (decreases) if and only if k_t increases (de-*

creases) for $t \geq 1$.

We have now shown that the discounted utility of the population and the sequence of incumbent utility move monotonically in the same direction towards their respective steady states. Given that W_t and V_t increase monotonically, it follows that outstanding debt decreases monotonically:

Corollary 2. *The stock of the economy's outstanding debt decreases (increases) monotonically to its steady state value from $t = 1$ onwards if k_t is increasing (decreasing) from $t = 1$ onwards.*

The above is a main result that closes the loop between growth and debt and brings us back to our original motivation. It states, quite generally, that capital accumulation will be accompanied with a reduction in the external debt of the country. Similarly, a country that shrinks, does so while accumulating foreign debt liabilities. Therefore, the model delivers the relationship between growth and accumulation of external debt that was depicted in Figure 1.

To characterize the steady state consumption and debt, we use the fact that $W_\infty = \underline{W}(k_\infty)$, and so

$$c_\infty = \frac{\underline{W}(k_\infty)}{\theta + \frac{\beta}{1-\beta}} = \frac{\theta \bar{c}(k_\infty) + \frac{\beta}{1-\beta} \bar{c}(k)}{\theta + \frac{\beta}{1-\beta}}. \quad (22)$$

The steady state level of debt then follows from the fact that debt equals the present discounted value of net payments to the foreign financial markets:

$$B_\infty = \left(\frac{1+r}{r} \right) (f(k_\infty) - (r+d)k_\infty - c_\infty). \quad (23)$$

The case of $\beta R = 1$ provides a clear example of the impact of political economy on steady state debt. For this discount factor, the steady state capital is k^* regardless of $\theta \in [1, \infty)$; that is, $\partial k_\infty / \partial \theta = 0$. Therefore, $\partial B_\infty / \partial \theta = - \left(\frac{1+r}{r} \right) \partial c_\infty / \partial \theta$, and this expression is negative from (22). We thus have the following proposition:

Proposition 5. *Suppose $\beta R = 1$. Then steady state consumption is increasing and steady state debt is decreasing in $\theta \in [1, \infty)$.*

Proposition 5 states that when private agents discount at the world interest rate an increase in political polarization *reduces* debt levels in the steady state. That is, a country with larger political frictions has a smaller amount of debt in the long run. The intuition for this result follows from the fact that when $\beta R = 1$ the optimal allocation follows a path that leads to first best capital in the long run. However, to sustain this first best capital, a government with a higher θ must have a lower debt, or else it will deviate and expropriate the capital.

While Proposition 5 states that debt will be lower in the steady state for countries with greater polarization when $\beta R = 1$, this does not imply that debt will be negatively correlated with polarization in a cross-section of countries. This does not follow for two reasons. The first is that countries with a large θ converge very slowly to the steady state, with convergence being arbitrarily slow as $\theta \rightarrow \infty$. That is, while polarized countries may be heading to a steady state in which debt is low, it will take a very long time to arrive there. Therefore, at any moment in time, countries will be at different points on their transition paths, and the steady state comparative statics are not sufficient to sign the cross-sectional correlation.

Moreover, the result depends on $\beta R = 1$. It may be the case that agents in developing countries are impatient relative to the world interest rate. In this case, k_∞ declines with θ , as well as c_∞ , making the net effect of θ on B_∞ ambiguous. In fact, if $\beta R < 1$, then $\partial B_\infty / \partial \theta$ can have either sign, depending on β .

Similarly, if $\beta R < 1$, whether the economy converges to the steady state from above or below is ambiguous. That is, if $\beta R = 1$, then the fact that $k_1 < k^* = k_\infty$ implies that the country unambiguously converges to the steady state from below for all $\theta \in (1, \infty)$. However, if $\beta R < 1$, convergence may be from above, with capital falling over time and debt increasing. This is consistent with the unfortunate development experiences of several poor countries.

3.2 Concave Utility

The linear utility case provides simple analytical expressions for the dynamics of the economy. However, linear utility is an extreme assumption and begs the question of whether the insights gleaned from the linear case are robust to more realistic utility functions. To answer this question, we explore the dynamics around the steady state for concave utility.

The dynamics of the economy are characterized by the following four equations:

$$c'(u_t) - \beta R c'(u_{t-1}) = \theta \eta_t - \beta R(\theta - 1) \eta_{t-1} \quad (24)$$

$$\eta_t \underline{W}(k_t) = f'(k_t) - (r + d) \quad (25)$$

$$\theta u_t = \beta V_{t+1} = \underline{W}(k_t) \quad (26)$$

$$V_t = u_t + \beta V_{t+1}, \quad (27)$$

where the first equation is obtained by differencing (14), the second equation is (13), the third equation is the participation constraint with $W_t = \theta u_t + \beta V_{t+1}$, and the last equation is the recursion of V_t .

We linearize the system around the steady, letting a hat denote deviations from the steady state values:

$$c''(u_\infty) \hat{u}_t - \beta R c''(u_\infty) \hat{u}_{t-1} = \theta \hat{\eta}_t - \beta R(\theta - 1) \hat{\eta}_{t-1} \quad (24a)$$

$$\eta_\infty \underline{W}''(k_\infty) \hat{k}_t + \underline{W}'(k_\infty) \hat{\eta}_t = f''(k_\infty) \hat{k}_t \quad (25a)$$

$$\theta \hat{u}_t + \beta \hat{V}_{t+1} = \underline{W}'(k_\infty) \hat{k}_t \quad (26a)$$

$$\hat{V}_t = \hat{u}_t + \beta \hat{V}_{t+1}. \quad (27a)$$

The system can be simplified down to one pre-determined variable and one “jump” variable, and expressed as $\mathbf{x}_t = \mathbf{B}\mathbf{x}_{t-1}$, where \mathbf{B} is a 2×2 matrix of parameters and associated steady state values and \mathbf{x} are our two endogenous variables. The matrix \mathbf{B} has two eigenvalues, one larger than one in absolute value and the other less than one in absolute value. The convergence of the system is governed by the eigenvalue less than one

in magnitude. Setting $c''(u) = 0$ (the linear case), we recover an eigenvalue $\beta R \left(1 - \frac{1}{\theta}\right)$. For $c''(u) > 0$, convergence is a more complicated expression of underlying parameters. To assess the response of the system to θ , we present some numerical examples in Figure 2 [analytical results to be added]. Specifically, we consider power utility: $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$, for $\sigma = 0, 1.01, 1.5$, and 2 .⁶ The figure plots the convergence rate, defined as 1 minus the relevant eigenvalue, for each σ . Looking across the plotted lines, we see that all else equal, a higher level of risk aversion implies slower convergence. This is intuitive, especially when θ is close to one, for which the linear case jumped immediately to the steady state. More importantly, a larger value of θ is associated with slower convergence, confirming the comparative statics of the linear case.

4 Conclusion

In this paper, we presented a tractable variation on the neoclassical growth model that helps explain why small open economies have dramatically different growth outcomes, and the ones that grow fast do so while increasing their net foreign asset position. Now that the model has been presented, we can revisit the motivational facts presented in Figure 1 and discussed in the introduction. Figure 1 presented the relationship between growth and changes in net foreign assets. However, the state variable of the model is financial liabilities. Net foreign assets, as measured in Figure 1, include equity and FDI liabilities, which in the model are choice variables given the stock of outstanding debt. Figure 2 presents an adjusted measure of net foreign assets, which is gross foreign assets minus debt liabilities. The relationship predicted by the model is evident in the figure. Moreover, growth is associated with an increase in FDI liabilities; that is, fast growing economies accumulate FDI liabilities while reducing net debt liabilities, which is consistent with the model (Prasad et al., 2006 also document that the allocation puzzle does not hold for FDI).

Perhaps the most important prediction of the model is that economies with a high de-

⁶The other parameters are $f(k) = k^\alpha$, $\alpha = 1/3$, $r = d = 0.05$, $\beta = 1/(1+r)$, and $\bar{\tau} = 0.99$.

gree of polarization have low growth rates due to the unwillingness to pay down debt. A literature has documented that polarization is negatively correlated with growth (for example, [Easterly and Levine, 1997](#)), but to our knowledge no one has focused on the empirical relationship between polarization and net foreign assets. Figure 3 provides evidence that there is a relationship. Specifically, we follow [Alessina et al. \(2003\)](#) and use their measure of ethnic fragmentation as our proxy for polarization. That paper documents that ethnic fragmentation is negatively correlated with growth. In Figure 3, we show that more ethnically diverse countries also accumulate assets or reduce debt liabilities at a slower rate than less polarized economies. This is consistent with the model's mechanism. While Figure 3 is suggestive, we leave to future research a more complete study of political polarization and net foreign liabilities. The mechanism identified in this paper suggests there may indeed be a tight link between political polarization and overhanging debt that underlies the dramatically different growth outcomes observed in the data.

Appendix A: Generalized Political Process

The model presented in the text assumed that each party had the same odds of being the next period's incumbent, regardless of which party is currently in power. In this appendix, we extend the model to the case where there may be an advantage to incumbency. Specifically, as before, let $\gamma_{i,o}$ be the probability that the current incumbent loses office, and $\gamma_{o,i}$ be the probability a party will regain office, where all party's out of office are treated symmetrically. We deviate from the text by dropping the assumption $1 - \gamma_{i,o} = \gamma_{o,i}$. If $1 - \gamma_{i,o} > \gamma_{o,i}$, then the incumbent has an advantage in retaining office, and vice versa if $1 - \gamma_{i,o} < \gamma_{o,i}$.

Define $p_{t,s}$ as the probability the incumbent in period t is in power at time $s \geq t$, so $p_{t,s}$ satisfies the difference equation $p_{t,s+1} = (1 - \gamma_{i,o})p_{t,s} + \gamma_{o,i}(1 - p_{t,s})$ with initial condition $p_{t,t} = 1$. Solving for $p_{t,s}$:

$$p_{t,s} = \frac{\gamma_{o,i}}{\gamma_{i,o} + \gamma_{o,i}} + \frac{\gamma_{i,o}(1 - \gamma_{i,o} - \gamma_{o,i})^{s-t}}{\gamma_{i,o} + \gamma_{o,i}}.$$

That is, the probability of being in power at some date s in the future is composed of two terms: a constant term, representing the unconditional probability of being in power, plus an incumbency advantage which vanishes as s goes to infinity.

The counterpart to (6) in the generalized case is:

$$\tilde{W}_t = \sum_{s=t}^{\infty} \beta^{s-t} p_{t,s} \tilde{\theta} u(c_s) + \sum_{s=t}^{\infty} \beta^{s-t} (1 - p_{t,s}) u(c_s). \quad (\text{A6})$$

We obtain the generalization of our stand-in government as follows. Define $\gamma = \gamma_{i,o} + \gamma_{o,i}$, and $\theta = 1 + \frac{(\tilde{\theta}-1)\gamma_{i,o}}{\gamma_{o,i}\tilde{\theta} + \gamma_{i,o}} \in (1, \tilde{\theta}]$. Note that this collapses to the case studied in the text by setting $\gamma_{o,i} = 1 - \gamma_{i,o} = p$, so $\gamma = 1$ and $\theta = \tilde{\theta}/(p\tilde{\theta} + 1 - p)$. More generally, $\gamma \in (0, 1]$.

Substituting in to (A6), we have the counterpart to (7):

$$\begin{aligned}
W_t &= \frac{\gamma_{i,o} + \gamma_{o,i}}{\gamma_{o,i}\bar{\theta} + \gamma_{i,o}} \tilde{W}_t \\
&= \sum_{s=t}^{\infty} \beta^{s-t} (1-\gamma)^{s-t} \theta u(c_s) + \sum_{s=t}^{\infty} \beta^{s-t} (1 - (1-\gamma)^{s-t}) u(c_s). \tag{A7}
\end{aligned}$$

Note that we recover the case studied in the text by setting $\gamma = 1$. As in the benchmark case, W_t is proportional to the incumbent's true utility \tilde{W}_t , and therefore an allocation satisfies the participation constraints for \tilde{W}_t if and only if it does so for W_t .

All propositions from the benchmark model extend to the general case, as proven in Appendix B, with the appropriate modification of the expressions for general $\gamma \leq 1$.

Appendix B: Proofs

We provide proofs using the the general political process described in Appendix A. The proofs for the benchmark case follow by setting $\gamma = 1$.

We begin with a generalized version of Lemma 1:

Lemma 1 (A). *The utility to the population as of time t , $V_t = \sum_{i=0}^{\infty} \beta^i u_{t+i}$, is given by:*

$$V_t = \left(\frac{1}{\theta}\right) W_t + \frac{\beta\gamma}{\theta} \left(1 - \frac{1}{\theta}\right) \sum_{i=0}^{\infty} \beta^i \left(1 - \frac{\gamma}{\theta}\right)^i W_{t+1+i}.$$

Proof. Using the definitions, we have

$$V_t = u_t + \beta V_{t+1}$$

$$W_t = \theta u_t + \beta(1-\gamma)W_{t+1} + \beta\gamma V_{t+1}.$$

Eliminating u_t from the above and re-arranging:

$$\begin{aligned}\theta \left(V_t - \beta \left(1 - \frac{\gamma}{\theta} \right) V_{t+1} \right) &= W_t - \beta(1 - \gamma)W_{t+1} \\ \theta \left(1 - \beta \left(1 - \frac{\gamma}{\theta} \right) F \right) V_t &= (1 - \beta(1 - \gamma)F) W_t,\end{aligned}$$

where F is the forward operator. Solving for V_t and eliminating explosive solutions:

$$\begin{aligned}\theta V_t &= \left(\frac{1 - \beta(1 - \gamma)F}{1 - \beta \left(1 - \frac{\gamma}{\theta} \right) F} \right) W_t \\ &= W_t + \beta\gamma \left(1 - \frac{1}{\theta} \right) \sum_{i=0}^{\infty} \beta^i \left(1 - \frac{\gamma}{\theta} \right)^i W_{t+1+i}.\end{aligned}$$

Dividing through by θ yields the expression in the lemma. □

Proof of Proposition 1:

Proof. Define $\underline{W}(k)$ to be the incumbent's value function if it deviates given capital k . We can write this as

$$\underline{W}(k) = \theta u(\bar{c}(k)) + \beta(1 - \gamma)\underline{W} + \beta\gamma\underline{V}, \quad (28)$$

where \underline{W} is the continuation value under the punishment if the incumbent retains power next period, and \underline{V} is the continuation value if it loses power. We normalize $t = 0$ to be the time of the deviation, so we have $\underline{W} = W_1$ and $\underline{V} = V_1$. From Lemma A1, we have:

$$\theta\underline{V} = \theta V_1 = W_1 + \beta\gamma \left(1 - \frac{1}{\theta} \right) \sum_{i=0}^{\infty} \beta^i \left(1 - \frac{\gamma}{\theta} \right)^i W_{1+i}.$$

As the punishment must be self-enforcing, we have $W_t \geq \theta u(\bar{c}(k_t)) + \beta(1 - \gamma)\underline{W} + \beta\gamma\underline{V}$ at each t . Note that a second deviation is punished in the same way as the first. The fact that $\underline{W}(k)$ is the worst possible punishment implies that this maximizes the set of possible self-enforcing allocations, from which we are selecting the one with minimum

utility. Substituting in the participation constraint in the above expression yields:

$$\begin{aligned} \theta \underline{V} &\geq \theta u(\bar{c}(k_1)) + \beta(1 - \gamma)\underline{W} + \beta\gamma\underline{V} \\ &+ \beta\gamma \left(1 - \frac{1}{\theta}\right) \sum_{i=0}^{\infty} \beta^i \left(1 - \frac{\gamma}{\theta}\right)^i (\theta u(\bar{c}(k_{1+i})) + \beta(1 - \gamma)\underline{W} + \beta\gamma\underline{V}) \\ &\geq \left(\frac{1 - \beta(1 - \gamma)}{1 - \beta(1 - \frac{\gamma}{\theta})}\right) (\theta u(\bar{c}(\underline{k})) + \beta(1 - \gamma)\underline{W} + \beta\gamma\underline{V}), \end{aligned}$$

where the last inequality uses the fact that $k_t \geq \underline{k}$, for all t . Rearranging, we have

$$\underline{V} \geq \frac{(1 - \beta(1 - \gamma)) (u(\bar{c}(\underline{k})) + \beta(1 - \gamma)\underline{W})}{\theta(1 - \beta) + \beta^2\gamma(1 - \gamma)}. \quad (29)$$

Recall that $W_1 = \underline{W}$. Participation at $t = 1$ requires $W_1 \geq \theta u(\bar{c}(k_1)) + \beta\underline{W} + \beta\gamma\underline{V}$, or using the fact that $k_1 \geq \underline{k}$:

$$\underline{W} \geq \theta u(\bar{c}(\underline{k})) + \beta(1 - \gamma)\underline{W} + \beta\gamma\underline{V}.$$

Substituting (29) in for \underline{V} and rearranging yields:

$$\underline{W} \geq \left(\frac{\theta - 1}{1 - \beta(1 - \gamma)} + \frac{1}{1 - \beta}\right) u(\bar{c}(\underline{k})).$$

Substituting back into (29), we have

$$\underline{V} \geq \frac{u(\bar{c}(\underline{k}))}{1 - \beta}.$$

The left hand sides of these last two inequalities are the government's and private agent's utility, respectively, from the Nash equilibrium repeated ad infinitum. As repeated Nash is a self enforcing equilibrium and bounds from below the punishment payoff, it is the self-enforcing equilibrium that yields the lowest utility for the deviating government.

□

Proof of proposition 2:

Proof. We first note that in the generalized case discussed in Appendix A, the first order condition (14) for u_t takes the form:

$$c'(u_t) = (\beta R)^t \left(\frac{1}{\mu_0} + \sum_{s=0}^t (\beta R)^{-s} ((1-\gamma)^{t-s}(\theta-1) + 1) \eta_s \right) \quad (\text{A14})$$

The first order condition with respect to k_t remains the same. To prove the proposition, suppose that k_t did not converge to k^* . Define $T_\epsilon = \{t | k_t < k^* - \epsilon\}$. It follows that for some $\epsilon > 0$, T_ϵ has infinite members. Then

$$c'(u_t) = \frac{1}{\mu_0} + \sum_{s=0}^t ((1-\gamma)^{t-s}(\theta-1) + 1) \eta_s \geq \frac{1}{\mu_0} + \sum_{s \in T_\epsilon, s \leq t} \eta_s \geq \frac{1}{\mu_0} + \sum_{s \in T_\epsilon, s \leq t} C_\epsilon$$

where $C_\epsilon \equiv (f'(k^* - \epsilon) - (r+d))/\underline{W}'(k^* - \epsilon) > 0$, and the inequalities reflect $\eta_s \geq 0$ for all s and $\eta_s \geq C_\epsilon$ for $s \in T_\epsilon$. It follows then that $c'(u_t)$ diverges to infinity, and thus u_t converges to its maximum. But this implies that the participation constraints will stop binding at some finite t_0 , which leads to η_s that are zero for all $s > t_0$, a contradiction. \square

The generalized version of Proposition 3 is as follows:

Proposition 3 (A). *The multiplier η_t satisfies the following difference equation:*

$$\eta_{t+1} = \frac{(1 - (1-\gamma)\beta R)(1 - \beta R)}{\theta} + \beta R \left(1 - \frac{\gamma}{\theta}\right) \eta_t, \quad \forall t \geq 1 \quad (\text{A16})$$

with $\eta_0 = \frac{1-\mu_0^{-1}}{\theta}$ and $\eta_1 = \frac{1-\beta R}{\theta} + \frac{\beta R(\theta-1)\gamma}{\theta} \eta_0$. The sequence of η_t converges monotonically from $t \geq 1$ towards its steady state value η_∞ :

$$\eta_\infty = \frac{(1 - (1-\gamma)\beta R)(1 - \beta R)}{\theta(1 - \beta R) + \beta R\gamma}.$$

Letting $\hat{\eta}_t \equiv \eta_t - \eta_\infty$, convergence to the steady state can be characterized by:

$$\hat{\eta}_{t+1} = \beta R \left(1 - \frac{\gamma}{\theta}\right) \hat{\eta}_t. \quad (\text{A17})$$

Proof. The generalized counterpart of (15) is:

$$(\beta R)^{-t} - \sum_{s=0}^t (\beta R)^{-s} ((1 - \gamma)^{t-s}(\theta - 1) + 1) \eta_s = \frac{1}{\mu_0}, \forall t \geq 0 \quad (\text{A15})$$

We have that from (A15) at $t + 1$:

$$(\beta R)^{-(t+1)} - \sum_{s=0}^t (\beta R)^{-s} ((1 - \gamma)^{t-s}(\theta - 1) + 1) \eta_s - (\beta R)^{-(t+1)} \theta \eta_{t+1} = \mu_0$$

which can be written as:

$$\begin{aligned} (\beta R)^{-(t+1)} - (1 - \gamma) \sum_{s=0}^t (\beta R)^{-s} \left((1 - \gamma)^{t-s}(\theta - 1) + 1 + \frac{1}{1 - \gamma} - 1 \right) \eta_s \\ - (\beta R)^{-(t+1)} \theta \eta_{t+1} = \mu_0 \end{aligned}$$

where using (A15) at t we get:

$$(\beta R)^{-(t+1)} - (1 - \gamma)(\beta R)^t - \sum_{s=0}^t (\beta R)^{-s} \gamma \eta_s - (\beta R)^{-(t+1)} \theta \eta_{t+1} = \mu_0 - (1 - \gamma)\mu_0 \quad (30)$$

Subtracting (30) at t from (30) at $t + 1$:

$$\begin{aligned} (\beta R)^{-(t+1)} - (\beta R)^{-t} - (1 - \gamma)((\beta R)^t - (\beta R)^{t-1}) \\ - \gamma(\beta R)^{-t} \eta_t + (\beta R)^{-t} \theta \eta_t - (\beta R)^{-(t+1)} \theta \eta_{t+1} = 0 \end{aligned}$$

which delivers the result once simplified for $t \geq 1$. Using (A15) at $t = 0$ delivers $\eta_0 = (1 - \mu_0)/\theta$. And using (A15) at $t = 1$ delivers that

$$\eta_1 = \frac{1 - \beta R}{\theta} + \frac{\beta R(\theta - 1)\gamma}{\theta} \eta_0$$

The steady state value can be computed in the usual way. Given that the slope of (A16) is positive and less than one, convergence and monotonicity follow. \square

Proof of Corrollary 1:

Proof. The first part of the proposition follows directly from the value of η_∞ . For the second part, note that higher debt implies a (weakly) higher multiplier μ_0 , and a higher η_0 . Given that η_1 and η_t are monotonic in previous values, it follows that the entire path of η increases with μ_0 and debt. That is, a higher level of debt leads to a lower capital path. \square

Proof of Corollary 2:

Proof. Let $B_t = \sum_{s=t}^{\infty} R^{s-t}(f(k_s) - (r + d)k_s - c_s)$ denote the stock of debt outstanding in period t . Suppose, to generate a contradiction, that $B_{T+1} > B_T$ for some $T \geq 1$. Let $\{u_t, k_t\}$ denote the equilibrium allocation. Now consider the alternative allocation: $\tilde{u}_t = u_t$ and $\tilde{k}_t = k_t$ for $t < T$, and $\tilde{c}_t = c_{t+1}$ and $\tilde{k}_t = k_{t+1}$ for $t \geq T$. That is, starting with period T , we move up the allocation one period. As $\tilde{V}_0 - V_0 = \beta^T(\tilde{V}_T - V_T) = \beta^T(V_{T+1} - V_T) > 0$, the objective function has increased and where the last inequality follows from the monotonicity of V_t . Similarly, $\tilde{B}_0 - B_0 = R^{-T}(\tilde{B}_T - B_T) = R^{-T}(B_{T+1} - B_T) > 0$, the budget constraint is relaxed, where the last inequality follows from the premise $B_{T+1} > B_T$. For $t \geq T$, we have $\tilde{W}_t = W_{t+1} \geq \underline{W}(k_{t+1}) = \underline{W}(\tilde{k}_t)$, so participation holds for period T and after. For $t < T$, note that $W_t = \sum_{s=t}^{T-1} \beta^{s-t} [(1 - \gamma)^{s-t}(\theta - 1) + 1] u_s + \beta^T(1 - \gamma)^T W_T + \beta^T(1 - (1 - \gamma)^T)V_{T+1}$. As $\tilde{W}_T > W_T$ and $\tilde{V}_T > V_T$, we have $\tilde{W}_t > W_t$ for all $t < T$. As $\tilde{k}_t = k_t$ for $t < T$, our new allocation satisfies the participation constraints of the governments along the path. Therefore, we have found a feasible allocation that is a strict improvement, a contradiction of optimality. \square

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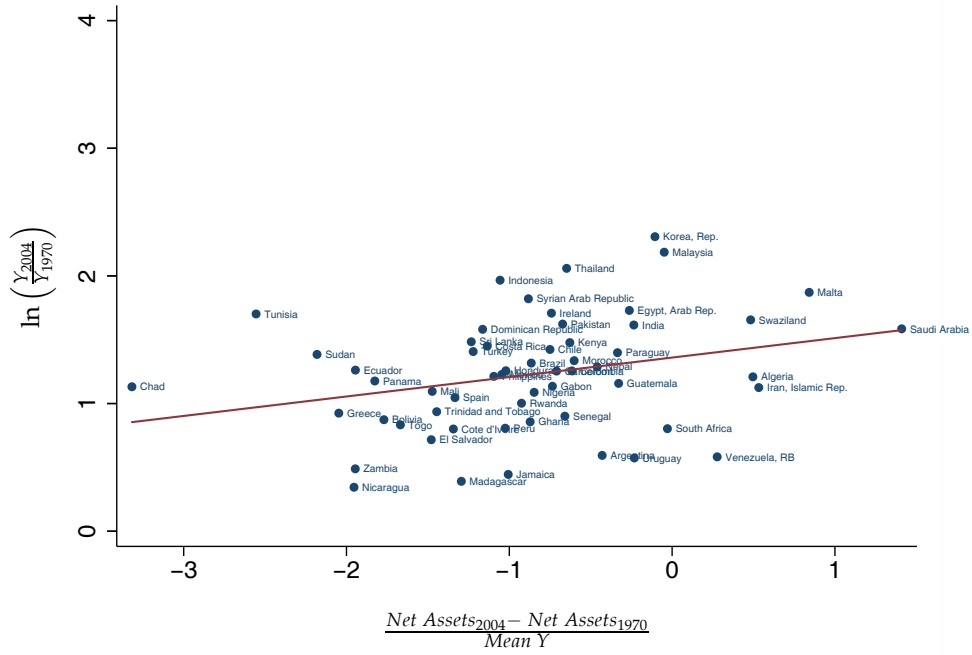
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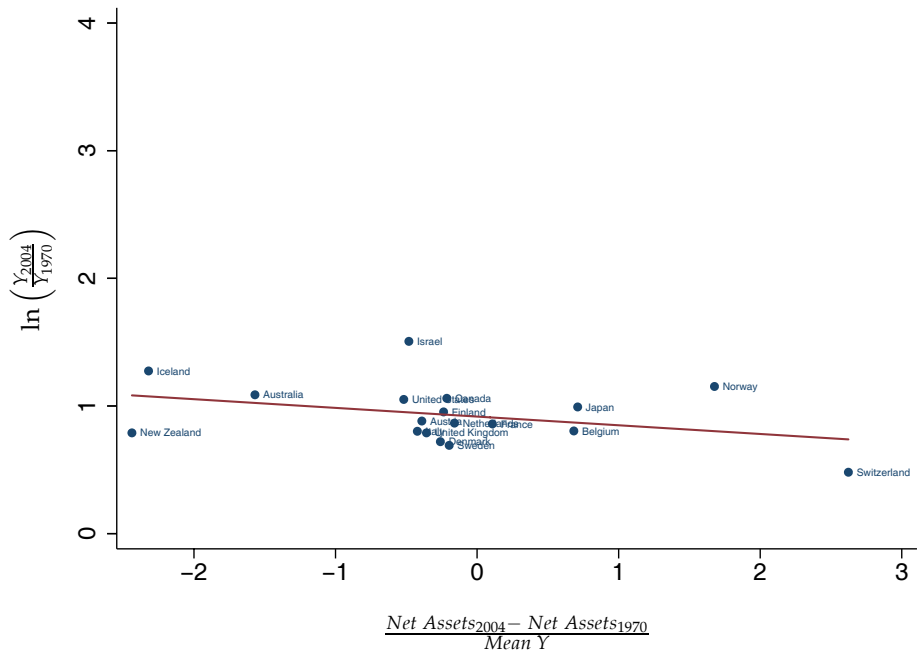
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A: Developing Economies



B: Developed Economies

Figure 1: Growth in Output and Net Foreign Assets

Notes: This figure plots growth in real GDP against the change in net foreign assets between 1970–2004, where net foreign assets for each country are normalized by average US dollar GDP over the sample period. Net foreign assets are gross foreign assets minus gross liabilities in current US dollars. Real GDP is constant local currency GDP from WDI. Net assets and average GDP are current US dollar from EWN Mark II. Panel A omits Singapore, which is an outlier in terms of net foreign asset position, to maintain reasonable scaling for the remaining countries.

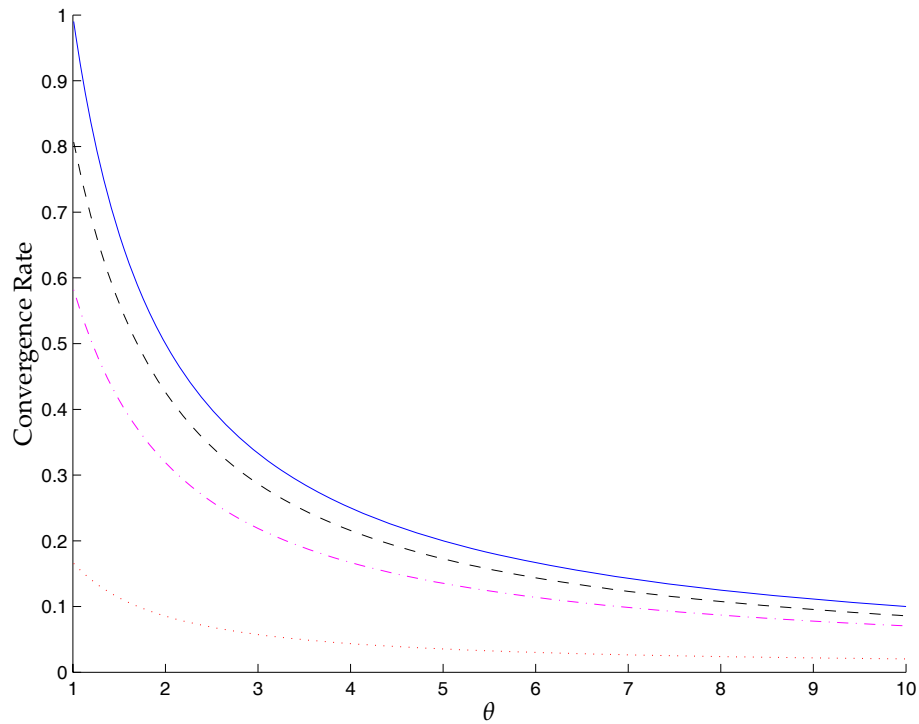


Figure 2: Growth Rates and θ for Nonlinear Utility

Notes: This figure depicts the sensitivity of convergence rates to θ . The vertical axis is the convergence rate, which is calculated as 1 minus the largest eigenvalue less than one of the matrix \mathbf{B} discussed in Section 3.2. Each line represents a different coefficient of relative risk aversion (σ). From top to bottom, the blue solid line is $\sigma = 0$, the black dashed line is $\sigma = 1.01$, the magenta dashed-dot line is $\sigma = 1.5$, and the red dotted line is $\sigma = 2.0$. Other parameters are: $f(k) = k^{1/3}$, $r = 0.05$, $d = 0.05$, and $\bar{\tau} = 0.99$.

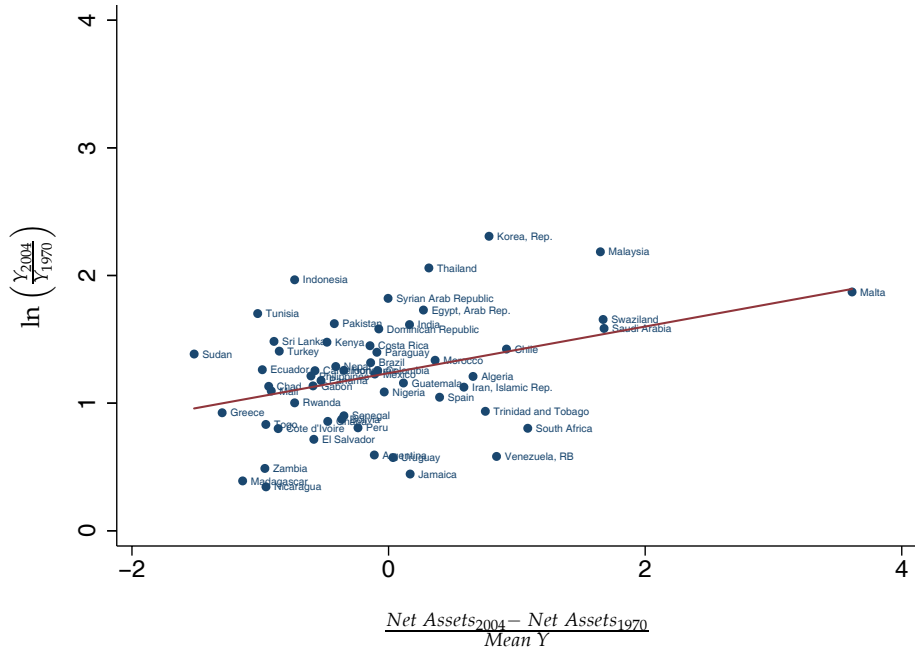


Figure 3: Growth in Output and Adjusted Net Foreign Assets

Notes: This figure plots growth in real GDP against the change in net foreign assets between 1970–2004, where net foreign assets for each country are normalized by average US dollar GDP over the sample period. Net foreign assets are gross foreign assets minus debt liabilities in current US dollars. Equity and FDI liabilities are excluded. Real GDP is constant local currency GDP from WDI. Net assets and average GDP are current US dollar from EWN Mark II. Figure omits Singapore and Ireland, which are outliers in terms of net foreign asset position, to maintain reasonable scaling for the remaining countries.

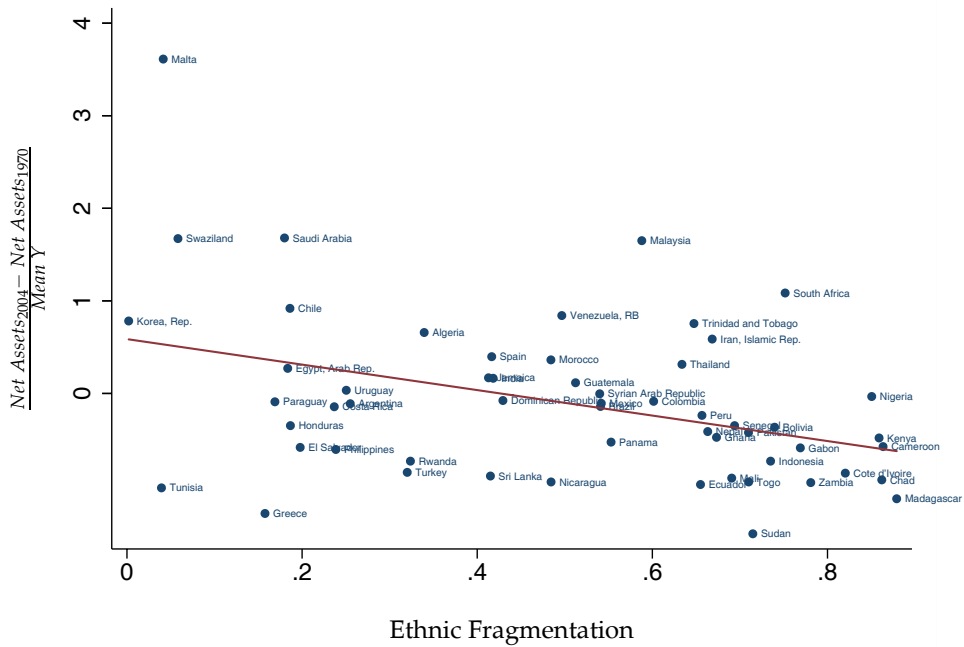


Figure 4: Growth in Output and Adjusted Net Foreign Assets

Notes: This figure plots the change in net foreign assets between 1970–2004 against a measure of ethnic fragmentation. Net foreign assets are as defined in Figure 4. Ethnic fragmentation data is from [Alessina et al. \(2003\)](#), and corresponds to $\sum_i(1 - s_i^2)$, where s_i is the share of the population of ethnic origin i . See [Alessina et al. \(2003\)](#) for details.