Understanding Wage Growth:

Estimating and Testing Learning-by-Doing

Philippe Belley

November 12, 2009

Abstract

This paper focuses on the skill accumulation of workers who have completed their formal education. Skills are acquired through work experience in the learning-by-doing (LBD) model, whereas skills are acquired through costly investments that reduce current production in the on-the-job training (OJT) model. The latter creates a trade-off between current and future earnings. Recent papers assume either LBD or OJT, but the two models have very different implications for the impact of wage subsidies, wage measurement, and the effect of business cycles on hours worked. I use the National Longitudinal Survey of Youth 1979 cohort and GMM to obtain structural estimates of a pure LBD human capital production function. No assumptions about preferences or credit markets are required, and I account for permanent unobserved heterogeneity and the endogeneity of hours worked. Estimates of the elasticity of wages with respect to hours worked range from 0.03 to 0.065. I then investigate whether wage growth is fully explained by the pure LBD model. This model implies that once skills, permanent ability, and hours of work are held constant, wage growth should not be related to variables affecting the incentive to accumulate human capital (e.g. future hours of work). I test this prediction of the LBD model using different variables related to incentives to invest in human capital. Future hours worked affect men's wage growth but not that of women. This suggests that wage growth is consistent with a pure LBD model for women but not for men.

1 Introduction

This paper studies the accumulation of human capital through work experience. Researchers interested in human capital accumulation typically use one of two different models of skill acquisition: the learning-by-doing (LBD) model (Weiss [1971,1972]) or the on-the-job training (OJT) model (Becker [1964], Ben-Porath [1967], Mincer [1974]). Although both models are grounded in human capital theory, they differ on an important assumption about the cost of acquiring human capital. The OJT model assumes that human capital investment comes at the expense of productive work. Time spent investing is rivalrous with current production. This implies a trade-off between human capital investment (increasing future earnings) and current productive work (increasing current earnings). The LBD model assumes that workers acquire skills through work experience: human capital accumulates as a by-product of productive work. In its common form, there is no trade-off between increasing current earnings and future earnings.

Many recent papers assume one model or the other without discussing their different implications for policy. Shaw (1989) and Imai and Keane (2004) use the LBD framework to study the intertemporal elasticity of substitution of labour supply in models with endogenous human capital accumulation. Shaw (1989) finds evidence that the intertemporal elasticity increases with age since the value of accumulating skills decreases as workers get older. Imai and Keane (2004) estimate an intertemporal elasticity of substitution of labour supply more in line with those obtained in the macroeconomics literature, and larger than those previously obtained from micro data in models with exogenous wage growth.

Heckman, Lochner and Taber (1998), and Huggett, Ventura and Yaron (2006) use the OJT model to replicate observed patterns in life-cycle wages. The first paper reconciles an increase in the demand for skilled labour in the US with a decreasing wage gap between college educated workers and their less educated peers. Huggett, Ventura and Yaron (2006) study how a calibrated dynamic life-cycle model with endogenous human capital accumulation can replicate some properties of the earnings age profiles for a cohort of US workers. The model successfully generates increasing mean earnings, and increasing variance in earnings, as the cohort ages.

But the LBD and OJT models have different implications. Cossa, Heckman and Lochner (2003) show that the effect of a wage subsidy program on human capital accumulation differs depending on whether a LBD or OJT framework is assumed. Hansen and Imrohoroglu (2008) find that both mechanisms lead to different steady state and business cycle properties in a calibrated general equilibrium model. Moreover these two models have different implications about the measurement of workers' productivity. Because investment and productive work cannot be separately identified in data without assumptions about preferences and credit markets, observed wages (total income divided by total hours of work) provide a downward biased measure of potential wages when individuals invest in human capital. This means that studying observed wages across workers with different levels of investment leads to invalid conclusions about the distribution of productivity or the price of skills. Finally, the costly nature of investment in the OJT model implies that workers who face borrowing constraints might have to make sub-optimal investments in human capital. There is less scope for this in the LBD model where human capital accumulation is costless.

In this paper, I use data on hourly wages and annual hours of work from the 1979 cohort of the National Longitudinal Survey of Youth (NLSY79) to estimate a pure LBD human capital production function. The assumed parametric form of this production function shares features with that of Imai and Keane (2004): human capital is lost through depreciation and gained through learning, learning is more productive for workers with more skills, and the production function includes age effects. To address unobserved heterogeneity, I estimate a production function with multiplicative separability between its deterministic and unobserved heterogeneity components. This leads to a log wage growth equation that excludes permanent unobserved heterogeneity and that has an additive stochastic component. This skill production function has a direct structural interpretation and can be estimated using generalized method of moments (GMM) without any assumptions about preferences or the structure of credit markets. I use multi-period lagged values of hours worked and skills as instruments to address the endogeneity of the production function's inputs: current hours of work and skills. The estimates are broadly similar for men and women. They imply that additional hours of work lead to more skills accumulation, with the elasticity of skills with respect to hours worked ranging from 0.03 to 0.065 percentage points. Allowing the LBD production function to change with age dramatically increases the estimates' goodness-of-fit, suggesting that age effects are important.

Since human capital accumulation is a by-product of work in the LBD model, differences in hours worked account for all the variation in skill accumulation through learning. In the OJT model, unobserved costly investments determine the accumulation of skills, implying that workers with identical hours of work may experience different skill growth. Note, however, that workers accumulate more skills the stronger their incentives to do so are. Therefore, variation in incentives will generate variation in investment and skill growth in the OJT model even after holding hours worked constant. In the LBD model, any variation in learning is observed through hours worked so that variation in incentives to acquire skills should not correlate with skill growth after accounting for hours worked. Based on these predictions, I estimate the sensitivity of wage growth to variables affecting the incentives to accumulate human capital controlling for hours worked. Once the LBD human capital production function is accounted for, "excess sensitivity" of wage growth to variables affecting the incentive to accumulate skills amounts to a rejection of the pure LBD model, since incentives to invest correlate with wage growth only in the OJT model. This empirical test is borrowed from the permanent income hypothesis literature that studies how sensitive changes in consumption are to changes in income. See, for example, Flavin (1981).

The excess sensitivity tests are based on variables that can be argued to affect the returns to and costs of investment in human capital. The first variable I use is future hours of work since workers who plan to work more in the future reap greater benefit from an increase in skills. The second variable is per-capita family income, which may affect the cost of investment in the OJT model. If workers face constraints on borrowing, workers in lower per-capita income families may have less scope to trade-off current earnings for the accumulation of skills. After instrumenting these two variables with higher order lagged hours and wages, I find that male wage growth displays "excessive sensitivity" only to future hours of work. Female wage growth is not affected by either future hours of work or per-capita family income.

These results suggest that human capital is accumulated through both LBD and OJT. I, therefore, try to quantify the relative importance of each mechanism. First, I compute the elasticity of wages with respect to current hours of work, which captures the effects of learning on skill accumulation. Second, to capture the effects of costly investments on skill growth, I consider the elasticity of wages with respect to future hours worked and per-capita family income. In general, wages are more sensitive to current hours worked than to future hours and per-capita family income, consistent with a strong role for the LBD mechanism.

This paper is structured as follows. The next section formalizes both the LBD and OJT models and briefly reviews the relevant literature. The third section discusses the methodology I use to test the LBD model. The fourth section describes the data, while results are presented and discussed in the fifth section. Section six concludes.

2 Models of Human Capital Accumulation

In this section I first present a general life-cycle model of labour supply with human capital accumulation. I discuss the factors that affect skill accumulation in both the OJT and the LBD models. The aim is to provide some intuition about the variables that should be included in the human production function, as well as the variables that affect the incentives to invest in human capital, which I use to implement the "excess sensitivity" tests of the pure LBD model. This is followed by a formal description of the OJT and LBD models, their distinctive features, and the research concerned with their estimation. The last subsection reviews the differences between each model's predictions.

2.1 Model

Consider a life-cycle model of labor supply where individuals choose consumption and leisure in order to maximize lifetime utility. At the beginning of period t, state variables (W_{it}, X_{it}) are realized and observed by agent i. The scalar W_{it} represents the agent's human capital. The vector X_{it} contains other state variables, such as age or financial assets, that affect consumption and leisure decisions. Each period, an agent has one unit of time that must be allocated either to leisure or work. Let H_{it} denote time allocated to work.

In a model where the stock of human capital W_{it} grows exogenously, the opportunity cost of leisure is simply the worker's wage rate. When human capital accumulation is endogenous, the opportunity cost of leisure depends on the current wage rate as well as the marginal benefit of allocating time to skill augmentation, regardless of whether skills are acquired through OJT or LBD. The test of the pure LBD model that I implement depends on variables that reflect the marginal benefit of acquiring skills, or more generally, variables that measure a worker's incentive to invest in human capital.

The benefit of investing in human capital mostly depends on the worker's expected working life, as demonstrated in Ben-Porath (1967). Each hour of productive work increases the returns to previously accumulated human capital, so that workers with longer expected working life have a stronger incentive to acquire skills. This implies that skill accumulation at any point in the life-cycle is affected by the number of hours the individual expects to work from then on. Among workers of the same age, investment will be more substantial for those who plan to work more in the future. In the LBD model, individuals that expect to work more in the future will also choose to work more today in order to raise their future wages. Differences in the incentives to accumulate skills translate entirely into different hours worked. This is not true in the OJT model. In this model, workers may alter current investment without changing current work hours. We can, therefore, test the LBD model by testing whether variation in incentives to invest (e.g. expected future work) affect wage growth conditional on current hours worked.

If the incentive to accumulate skills is affected by the expected future work, then women's fertility decisions affect their incentives to acquire human capital, since child-rearing requires some time off from work. Differences in human capital accumulation between men and women have indeed been shown to explain part of the male-female wage gap (Blau and Kahn [2006]). These negative effects of fertility on female skill accumulation could be weaker if a mother's human capital increases the productivity of her maternal investments. Variables related to fertility choices might well provide a measure of women's incentives to invest in human capital.

Heckman (1976) shows in the OJT model that individuals will accumulate more skills if they are intrinsically more productive at it. Huggett, Ventura and Yaron (2006) establish the importance of heterogeneity in the learning ability of workers to generate increasing dispersion in life-cycle wages. This is also true for the LBD model and suggests that it is important to account for a permanent unobserved ability that affects the accumulation rate of skills. The pure LBD production function that I estimate, as described section 3, allows abler individuals to be more productive at accumulating human capital.

Ben-Porath (1967) shows that the current level of skills may affect the level of investment in skills in the OJT model. This certainly applies for the LBD model as well and I, therefore, let the current level of skill be an input in the LBD production function.

Thus far, I have discussed factors that affect incentives to accumulate skills in both the OJT and the LBD model. I next discuss in more details the production technology for both models.

2.1.1 On-The-Job Training

In the OJT model, the worker allocates hours at work H_{it} between investment I_{it} and productive work $H_{it} - I_{it}$. Earnings, E_{it}, and human capital accumulation are given by:

$$E_{it} = W_{it}\left(H_{it} - I_{it}\right) \quad (1)$$

$$W_{it+1} = g\left(I_{it}, W_{it}, X_{it}, \theta_i, \varepsilon_{it}\right), \quad (2)$$

where $g(\cdot)$ reflects the human capital production function. Inputs include investment I_{it}, as well as previous human capital W_{it} and other state variables X_{it}. It also includes unobserved inputs: θ_i represents permanent unobserved heterogeneity while ε_{it} represents transitory random disturbances to the human capital production process. For example, these transitory disturbances could reflect shocks to the individual's health that change the returns to human capital investment.

Equations (1) and (2) reveal the trade-off between current earnings and future earnings. The costly nature of investment in the OJT model implies that older workers eventually stop accumulating skills. Workers who are close to retirement have little time left to work, so the benefit of investing is much lower than the returns to productive work, which are especially high due to past skill investments. Although age is closely related to a worker's incentives to accumulate human capital, I do not use age to test the LBD model since it could be argued to directly affect to production of human capital.

Another impact of costly investments in the OJT model is that workers will take advantage of periods during which their wage is relatively low to invest in human capital. This provides an additional incentive to invest for young workers, who tend have lower wages. It also suggest that workers will allocate their investment to periods hit by aggregate negative shocks to productivity. Variables describing the state of the economy might be useful in testing the LBD model.

Equation (1) also shows that for a worker who invests in human capital ($I_{it} > 0$), the observed wage W_{it}^o (earnings divided by hours of work) is a downward biased measure of the potential wage rate given by human capital W_{it}:

$$W_{it}^o = \frac{E_{it}}{H_{it}} < W_{it}, \tag{3}$$

with the bias increasing in I_{it}. This wedge between actual wage and observed wage is a feature of the OJT model (and absent from the LBD model) that is critical to Heckman, Lochner and Taber (1998) in order to reconcile an increase in the demand for skilled labour with a decreasing wage gap between college educated workers and their less educated peers.

Because costly investments are not observed, estimation of the OJT model requires assumptions about agents' preferences and expectations, as well as the structure of credit markets. Heckman (1976) and Brown (1976) illustrate earlier attempts at estimating the OJT model. From their assumed preferences and technology, they derive a structural earnings equation that they estimate using life-cycle wage patterns. Heckman, Lochner and Taber (1998) solve their structural model to generate the distribution of life-cycle wages of workers and match it to that observed in the NLSY79 data to estimate their model's parameters.

2.1.2 Learning-By-Doing

In the LBD model, human capital is accumulated as a by-product of productive work:

$$E_{it} = W_{it}H_{it} \tag{4}$$

$$W_{it+1} = f\left(H_{it}, W_{it}, X_{it}, \theta_i, \varepsilon_{it}\right). \tag{5}$$

The LBD production function (5) makes it clear that once skill level W_{it}, other state variables X_{it}, and unobserved ability θ_i are held constant, only observed hours of work H_{it} generate systematic variation in human capital W_{it+1}. This means that although agents' incentives to accumulate skills in the LBD model vary with the same variables as in the OJT model, these variables do not provide any additional information about skill level W_{it+1} once hours worked and observed factors in $f(\cdot)$ are taken into account.

I test the pure LBD model presented in equations (4) and (5) by studying how skill level W_{it+1} correlates with variables that affect the incentive to accumulate human capital, once the production technology is taken into account. According to this technology, any variation in skill accumulation should come through variation in hours of work H_{it} once the other state variables are taken into account. In the OJT model, individuals with identical work hours may achieve different skill levels through variation in unobserved investments I_{it}. Differences in investment levels should be the result of different incentives to invest in human capital. Variables that affect these incentives should therefore correlate with skills level W_{it+1} in the OJT model but not in the LBD model. A statistically significant relationship between variables affecting the incentives to invest and W_{it+1} is, therefore, interpreted as a rejection of the pure LBD model.

Although the incentives to accumulate human capital in the LBD model respond to most of the variables that affect them in the OJT model, equations (4) and (5) make it clear that investment is not costly in the LBD model. Individuals who work more earn more and accumulate more human capital; there is no trade-off between current and future earnings. The absence of this trade-off implies that skill accumulation in the LBD model is not subject to the same variation in the opportunity cost of investment that arises in the OJT model.

Costless accumulation also implies that wages are accurately measured by the ratio of earnings to hours worked. This allows the researcher to avoid having to identify unobserved investments in human capital, so it is possible to make valid inferences about workers' productivity using observed wages.

Imai and Keane (2004) use the NLSY79 data to solve and estimate a life-cycle model of consumption and labor supply that allows for endogenous human capital accumulation through the LBD mechanism. They show that allowing for the endogeneity of human capital leads to estimates of the intertemporal elasticity of substitution of labour supply that are more in line with those reported in the macroeconomics literature, and larger than estimates obtained with micro data and exogenous wage growth.

Shaw (1989) also estimates a life-cycle model of labour supply with LBD human capital accumulation with an eye on estimating the intertemporal elasticity of substitution of labour supply. In a two step estimation procedure, she first estimates the LBD production technology, a function of quadratic terms in current wages and hours worked, as well as a vector of demographic variables. She finds that hours of work are an important input to the accumulation of human capital. In a second step, she derives from her life-cycle model of labour supply an Euler equation that she estimates using GMM, her production technology estimates, and assumptions about preferences and expectations. Her model allows her to simulate age profiles for wages and hours of work, from which she infers that the intertemporal elasticity of labour supply increases with age. Young workers are less responsive to changes in their wage because the incentive to accumulate human capital is very strong for them.

Altuğ and Miller (1998) estimate a life-cycle model of labour supply for women using the assumption that observed allocations are Pareto optimal. Their main conclusion is that female labour supply is affected by work experience through its effect on LBD human capital accumulation but also through non-separable preferences over time. Their LBD production function is somewhat different than those discussed so far, because it does not depend on the current stock of skills. Instead it depends on multi-period lags of annual hours of work.

The methodology I use to estimate a pure LBD production function allows me to do so without having to specify workers' preferences and expectations, or the structure of credit markets. The GMM estimation framework is implemented using standard statistical software. The estimation, however, requires assumptions about the form of the production technology (to deal with unobserved ability) and the structure of the stochastic component of the production function (to deal with the endogeneity of its inputs).

2.2 Differences between LBD and OJT

The LBD and OJT models differ concerning the cost of human capital accumulation. This is important since the two models can lead to different policy predictions. Cossa, Heckman and Lochner (2003) show that the effect of a wage subsidy program on human capital accumulation differs depending on whether a LBD or OJT framework is assumed. For individuals who would work in the absence of wage subsidies, the LBD implies an increase in skill accumulation as long as the subsidy increases labour supply. In the OJT model where investment is rivalrous with production, these same workers might decrease investment since wage subsidies increase its opportunity cost.

Although the OJT and LBD models have different policy implications, only Heckman and Lochner (2003) investigate the relative importance of the two accumulation mechanisms. They note that short-term taxes and subsidies on wages are predicted to have different impacts on skill formation depending on whether the LBD or the OJT model prevails. They estimate the effects of a short-term increase in marginal tax rates on wage growth and find a negative effect consistent with the LBD framework.

Hansen and Imrohoroglu (2008) study the effect of allowing for endogenous human capital accumulation in a calibrated general equilibrium life cycle model. Their benchmark model is one where the age-specific stock of human capital is exogenously determined. This benchmark model's steady state and business cycle properties are found to be very close to that of a model where human capital accumulation is achieved through OJT. However, an economy where the LBD mechanism prevails leads to different steady state hours of work for the youngest and oldest workers. It also leads to higher variation in older workers' hours of work. This is due to the fact that investment ceases for older workers in the OJT framework, while human capital accumulation still takes place for older workers in the LBD model.

The OJT and LBD mechanisms also have different implications for human capital investment in an economy where workers face borrowing constraints. In the OJT model, the trade-off between current and future earnings implies that investment is costly. If workers are constrained in their borrowing they might have to make sub-optimal investments in human capital. If there are borrowing constraints preventing young adults from obtaining a post-secondary education (Belley and

Lochner [2007]), the OJT model suggests that these same young adults might continue making suboptimal investments in the labour market.

3 Estimating and Testing the Learning-by-Doing Model

3.1 Testing the LBD Model

In this paper, I estimate and test the pure LBD technology laid out in equations (4) and (5). The key difficulty in estimating the LBD production function (5) is to take into account unobserved ability θ_i to obtain consistent estimates despite the endogeneity of hours worked H_{it}. It is also important to choose a functional form that is flexible and interpretable. I address these concerns within a GMM framework.

In the pure LBD model, human capital accumulation comes only through productive work. Once hours of work are taken into account, along with previous skills, other state variables and ability, $(W_{it}, X_{it}, \theta_i)$ in (5), there should not be any systematic variation in human capital W_{it+1}. This is not true in the OJT model since hours of work do not measure all the variation in investments. The key insight in testing the LBD model is to find variables that measure variation in a worker's incentives to invest and test for any systematic association between these variables and human capital accumulation once hours of work are held constant.

Consider a modified version of a model proposed in Killingsworth (1982) where both OJT and LBD accumulation mechanisms play a role:

$$E_{it} = W_{it}\left(H_{it} - I_{it}\right) \tag{6}$$

$$W_{it+1} = f\left(H_{it}, W_{it}, X_{it}, \theta_i\right) + \beta \cdot g\left(I(H_{it}, W_{it}, X_{it}, S_{it}, \theta_i), W_{it}, X_{it}, \theta_i\right) + \varepsilon_{it}, \tag{7}$$

where β denotes the importance of the OJT mechanism relative to LBD, and where costly investments I_{it} are written as a function of hours worked H_{it}, human capital W_{it}, other state variables X_{it}, unobserved permanent ability θ_i, and some variables S_{it} (e.g. expected hours of work) that affect the incentive to accumulate skills: $I_{it} = I(H_{it}, W_{it}, X_{it}, S_{it}, \theta_i)$. These S_{it} factors cause workers with identical $(H_{it}, W_{it}, X_{it}, \theta_i)$ to choose different levels of investment. Consider the following estimating equation

$$W_{it+1} = f\left(H_{it}, W_{it}, X_{it}, \theta_i\right) + \delta S_{it} + \tilde{e}_{it}. \tag{8}$$

If skills are accumulated through the OJT mechanism (i.e. $\beta \neq 0$), then δ will not generally equal zero since the test variable S_{it} accounts for variation in $g(\cdot)$ that is not accounted for by the pure LBD production function. However, if $\beta = 0$ and all skills are acquired through LBD, the estimates for (eq:OJTLBDprodfunc) should yield $\widehat{\delta} = 0$.[1]

Simply adding a linear term in S_{it} as in (8) tends to detect first order effects of investment on skill accumulation. Higher order effects could also be studied by looking at the correlation between skills and interaction terms of S_{it} with elements of $(H_{it}, W_{it}, X_{it}, \theta_i)$.

Intuitively, the LBD framework predicts that $(H_{it}, W_{it}, X_{it}, \theta_i)$ should account for all systematic variation in human capital W_{it+1}, so δ should equal 0. In the OJT model, it is expected that $\delta \neq 0$ since the term δS_{it} picks up variation in $g\left(I_{it}, W_{it}, X_{it}, \theta_i\right)$ due to variation in I_it conditional on $(W_{it}, X_{it}, \theta_i)$. Assuming that one can deal with the unobserved components θ_i, then equation (8) can be estimated. Finding estimates of δ that are statistically different from zero is interpreted as a rejection of the LBD model.[2]

Based on human capital theory I consider two S_{it} variables that may affect a worker's investments. The first is expected hours of work which capture variation in a worker's returns to human capital investment. Incentives to invest are greater for those who expect to work more in the future. I use observed future work hours (H_{it+1}) as a proxy for expected hours of work.

The second variable relies on the fact that some workers might have less scope to substitute between current and future earnings. The fact that workers have to forego earnings to accumulate human capital in the OJT model implies that borrowing constrained workers may have to make

[1]Note also that if there is skill accumulation through costly investment, then observed wages W^o_{it+1} are a biased measure of the actual stock of human capital W_{it+1}, as shown in equation (3). This bias is an increasing function of costly investment I_{it}. This imply that the term δS_{it} also picks up variation in measurement bias due to variation in investment.

[2]This test is weaker if the S_{it} variables are correlated with the other inputs $(H_{it}, W_{it}, X_{it}, \theta_i)$ since S_{it} is then less likely to capture variation in investment incentives that is not captured by the other inputs. Another possibility is that $\delta \neq 0$ because S_{it} is correlated with $(H_{it}, W_{it}, X_{it}, \theta_i)$ while the production function is misspecified. This underlines the importance of choosing a functional form for the LBD production function that is flexible enough for the term δS_{it} to account for a negligible amount of variation in the other inputs. Conceptually it is easy to nonparametrically estimate the production function $f(\cdot)$ in (8). But this is more difficult due to the presence of unobserved heterogeneity θ_i.

sub-optimal investments. To account for these possibilities, I test whether per-capita family income affects skill accumulation conditional on hours worked. Workers whose families have low per-capita income should have less scope to substitute between investment and current earnings. Therefore, I expect per-capita family income to be positively correlated with skill accumulation.

While age might affect the incentives to invest, it is also likely that it directly affects the marginal productivity of hours worked in the the LBD model. This implies that it should be directly included in the LBD production function rather than used as a test variable.

The test variables I propose do not belong in production function (5). Future hours of work are determined jointly with current inputs to the production function but they are not direct inputs to the production of W_{it+1}. Current per capita family income is also determined jointly with current inputs but is unlikely to be an input itself.

This test methodology is inspired by the permanent income literature (Flavin [1981], Attanasio and Weber [1995], Stephens [2008]). In this literature, consumption equations are derived from life-cycle models of consumption. Broadly speaking, the permanent income hypothesis implies that predictable changes in income should not affect consumption behavior. This restriction is typically tested by estimating a structural consumption function to which income is added. If (predicted) income is found to have a statistically significant correlation with consumption, *i.e.* if consumption displays “excess sensitivity” to income, the permanent income model is rejected.

3.2 Econometric Issues

3.2.1 Unobserved Heterogeneity

The production function I use includes unobservable ability θ_i and random shocks ε_{it}. To deal with these unobserved variables, I assume multiplicative separability between the deterministic and unobserved/stochastic components of the production function:

$$W_{it+1} = \widetilde{f}\,(H_{it}, W_{it}, X_{it})\, e^{\theta_i + \varepsilon_{it}}. \tag{9}$$

This still allows for the worker's ability to affect the productivity of inputs (H_{it}, W_{it}, X_{it}). For example,

$$\frac{\partial^2 W_{it+1}}{\partial H_{it} \partial \theta_i} = \frac{\partial \widetilde{f}}{\partial H_{it}} e^{\theta_i + \varepsilon_{it}},$$

so if skills are increasing in hours worked then an increase in ability θ_i increases the marginal productivity of hours worked in the production of skills. Imai and Keane (2004) also assume multiplicative separability between the deterministic and stochastic components of their pure LBD production function, while Shaw (1989) only allows for observed permanent heterogeneity in the production of skills.

After taking the log of equation (9) one can simply first-difference it in order to eliminate unobserved ability θ_i:

$$\triangle \ln W_{it+1} = \ln \widetilde{f}\,(H_{it}, W_{it}, X_{it}) - \ln \widetilde{f}\,(H_{it-1}, W_{it-1}, X_{it-1}) + \triangle \varepsilon_{it} \tag{10}$$

where $\triangle$ denotes the first difference operator (*e.g.* $\triangle \ln W_{it+1} = \ln W_{it+1} - \ln W_{it}$). A test variable S_{it} can be included in (10) to implement the LBD excess sensitivity test

$$\triangle \ln W_{it+1} = \triangle \ln \widetilde{f}\,(H_{it}, W_{it}, X_{it}) + \delta S_{it} + \triangle \varepsilon_{it}. \tag{11}$$

3.2.2 Endogeneity

Both W_{it} and H_{it} are likely to be correlated with $\triangle \varepsilon_{it}$ in (10) and (11). First, note that W_{it} is a function of ε_{it-1} which directly generates a correlation between W_{it} and $\triangle \varepsilon_{it}$. Second, workers may have information about ε_{it} that is not available to the econometrician. Since the worker takes this information into account when choosing hours of work, it creates a correlation between H_{it} and ε_{it}, as well as between H_{it-1} and ε_{it-1}.

I assume that the following conditional moment condition holds

$$\mathbb{E}\left[\triangle \varepsilon_{it} | H_{it-2}, W_{it-1}, X_{it-1}\right] = 0. \tag{12}$$

Given this conditional expectation, I can use functions of $(H_{it-2}, W_{it-1}, X_{it-1})$ as instruments for the estimation of equations (10) and (11). Formally, let $F(H_{it-2}, W_{it-1}, X_{it-1})$ be a vector of functions of $(H_{it-2}, W_{it-1}, X_{it-1})$. GMM estimation below is based on the following vector of orthogonality conditions

$$\mathbb{E}\left[F(\cdot)\triangle\varepsilon_{it}\right] = 0. \tag{13}$$

This is in essence a non-linear instrumental variable estimator where functions of $(H_{it-2}, W_{it-1}, X_{it-1})$ are used as instruments for the endogenous variables $(H_{it}, H_{it-1}, W_{it}, X_{it})$.[3]

Lagged values of human capital, hours worked and other state variables are valid instrument since they do not correlate with $\triangle\varepsilon_{it}$. They also correlate with the endogenous variables since they are all jointly determined due to $(\varepsilon_{i1}, \varepsilon_{i2}, ..., \varepsilon_{it-2}, \theta_i)$. Note that I use the same set of moment conditions (13) to estimate the "excess sensitivity test" equation (11). For $S_{it} = H_{it+1}$, lagged hours of work provides a valid instrument for future hours worked. I also instrument per-capita family income with the same functions of $(H_{it-2}, W_{it-1}, X_{it-1})$.

If human capital is accumulated through regular involvement in productive work, then skill accumulation depends on the allocation of hours across current and past periods $(H_{it}, H_{it-1}, H_{it-2}, ...)$. This suggests that the skill production function should also include the vector of past hours of work, as in Altuğ and Miller (1998). If this is the case, then using lagged hours worked as an instrument for the estimation of (11) might lead to finding $\delta \neq 0$ because the LBD production function is misspecified. To facilitate the detection of deviations from the pure LBD model rather than misspecifications of the LBD production function, I use an alternative set of instruments that excludes lagged hours of work. The instruments are functions of lagged state variables Z_{it-1} that reflect household composition and family income: number of children in the household, ratio of number of earners to household members, and per capita family income. These variables and hours of work are jointly determined but the household related variables should not appear directly in the production function.

Conditional expectation (12) will generally hold if ε_{it} is serially independent. In this case, current shocks are unrelated to past choices. Some persistence can be introduced without the need

[3] Intuitively $\triangle\varepsilon_{it}$ is also uncorrelated with $(H_{it-3}, H_{it-4}, ...)$, $(W_{it-2}, W_{it-3}, ...)$, and $(X_{it-2}, X_{it-3}, ...)$. In practice, these additional instruments are weak.

to use further lags as instruments. Conditional expectation (12) still holds if

$$\triangle\varepsilon_{it} = \nu_{it} + \rho \cdot \nu_{it-1}, \tag{14}$$

where ν_{it} is an independently and identically distributed mean zero process. The second term on the right-hand side allows for shocks to be persistent if $\rho > 0$. These stochastic components reflect the fact that circumstances out of the worker's control affect his ability to learn (*e.g.* sudden illness, changes to work conditions, or family related concerns).[4]

3.2.3 Functional Form

With the aim of detecting deviation from the pure LBD model rather than misspecifications of LBD production function, I would ideally estimate equations (10) and (11) nonparametrically. Unfortunately, this is difficult and not very practical. I instead use a fairly general parametric model for the production function $\widetilde{f}(H_{it}, W_{it}, X_{it})$:

$$\widetilde{f}(H_{it}, W_{it}, X_{it}) = B(X_{it}) \cdot W_{it} + A(X_{it}) \cdot H_{it}^{\alpha_1} W_{it}^{\alpha_2} \tag{15}$$

A similar specification has been used by Imai and Keane (2004). It allows for depreciation in skills ($B(X_{it}) \cdot W_{it}$) and for accumulation through learning ($A(X_{it}) \cdot H_{it}^{\alpha_1} W_{it}^{\alpha_2}$). Parameters α_1 and α_2 determine the productivity of hours of work and lagged human capital. Parameters A and B are functions of state variables X_{it}, allowing the human capital production function to differ based on things like age or education.

Inserting equation (15) into (10) and (11) yields

$$\triangle \ln W_{it+1} = \triangle \ln \left[B(X_{it}) \cdot W_{it} + A(X_{it}) \cdot H_{it}^{\alpha_1} W_{it}^{\alpha_2}\right] + \triangle\varepsilon_{it} \tag{16}$$

$$\triangle \ln W_{it+1} = \triangle \ln \left[B(X_{it}) \cdot W_{it} + A(X_{it}) \cdot H_{it}^{\alpha_1} W_{it}^{\alpha_2}\right] + \delta S_{it} + \triangle\varepsilon_{it}. \tag{17}$$

These equations can be estimated by GMM through orthogonality conditions (13).

[4] Allowing instead ε_{it} to be a moving average of ν_{it} processes would require using further lags of hours worked and skill levels as instruments.

4 Data

I use the 1979 cohort of the National Longitudinal Survey of Youth (NLSY79). This is a random survey of American youth aged 14 to 21 in 1979 which provides extensive panel data on wages and work experience from 1979 to 2006. In this study, I use the random sample of whites excluding the poor white over-sample, and implement the analysis separately for men and women. The time unit I consider is a year, so I do not use wage and hours data collected after 1994, after which the NLSY79 surveys were conducted every two years.

Human capital is not observable, so I use hourly wages which, under the assumption of a pure LBD model, are a valid measure of skills. The NLSY79 provides hourly wages for each job held. It is, therefore, not unusual for an individual to report more than one wage in a particular survey. I first restrict my sample to wages related to jobs held at the time of the survey. If at that time the respondent holds more than one job, I select the job with the highest weekly hours of work. All wages are then adjusted to 2004 dollars using the Bureau of Labor Statistics Consumer Price Index for all urban consumers. To exclude outlier wage values, I restrict the sample to wages greater $1.90 and less than $100. I also exclude wage observations that display wage growth below -50% and above 100%.

Hours of work are measured as the sum of all hours worked at all jobs since the date of the last interview. Given the yearly time unit considered in this study, the last interview must have taken place no more than 14 months and no less than 10 months prior to the current interview.[5] To exclude outlier values and to focus the analysis on workers who demonstrate a significant level of participation in the labor market, I further restrict the sample to respondents who worked more than 780 hours (15 hours per week, for 52 weeks) and less than 4368 hours (84 hours per week, for 52 weeks). Moreover, I use the number of weeks worked since the date of the last interview to compute the average number of hours worked per week. This average must be above 15 and below 84 for a wage observation to remain in the sample.

Human capital investment behavior for workers who have initiated their career is most likely

[5] Although the NLSY79 survey are conducted every year between 1979 and 1994, interviews do not always take place in a 12 months interval. This interval also exceeds 12 months for respondents who have just been brought back in the survey after missing one interview or more.

different from that of students holding summer jobs. I focus on the former. To determine when a respondent has completed schooling, I use years of schooling from 1979 to 2004. Based on years of schooling, the respondent is categorized as high school dropout (less than 12), high school graduate (12 years), at least some college (between 13 to 15 years of schooling), and completed college (at least 16 years of schooling) in each survey year. I then identify the highest education category for each respondent as well as the first survey year in which it is observed. Wage observations preceding this first year are excluded from the sample.

I also apply age restrictions to my sample. My sample includes individuals aged 18 or older if their highest level of schooling is high school graduate or less. Respondents with at least some college must be at least 21 years old. I compute age in months for each survey year based on the interview date and the respondent's date of birth. After applying the relevant transformation, age in years, net of 18, are used in regressions for more straightforward interpretations.

The NLSY79 data also provides a wealth of information about respondents' household composition. Among these, I use per-capita family income, the number of children in the household, and the ratio of earners to members in the household as state variables Z_{it-1} to build the alternative set of instruments discussed in section 3.2.2.

After applying the above restrictions to the sample, I obtain a sample of 9,566 wage observations and 1,772 individuals for men and 7,560 wage observations for 1,580 women.[6] Table 1 contains sample descriptive statistics. Average hourly wage is \$17 for men and lower for women at \$13. Average wage growth is quite similar across gender at around 6% but its variance is higher for men. Men also work more hours, on average, than women. Given that the sample is relatively young (average age is 28), the average number of children in households is less than one and the ratio of earners to household members is between 0.5 and 1. For both men and women, more than 80% of the sample is made up of individuals who have at least graduated from high school.

Figures 1 and 2 present age profiles of wages and hours of work across education levels for men and women. As expected, wage profiles become steeper as education increases. Hours worked

[6] The initial random sample of white respondents includes 2,236 men and 2,279 women. My sample therefore represents 80% of the full sample for men and 70% for women. For men (women), 15% (25%) of the sample is lost due to missing wages, outlier wages, exclusion of years preceding the end of formal schooling, and the fact that I use only surveys from 1979 to 1994.

profiles are quite similar across education levels. Women's wage and hours profiles are always below that of men. Although hours of work are flat, wage profiles indicate that wage growth slows down as workers get older. There are two potential reasons for this: first, the wage production function may change with age. Second, investment may be an important component of human capital accumulation which decreases with age, leading to weaker wage growth.

The main focus of this paper is on the relationship between wages, wage growth, hours of work and past wages. Figure 3 reports wage and wage growth as a function of hours worked for both men and women. Figure 4 reports these profiles as a function of lagged wages. Both male and female samples are separated into deciles according to hours of work (Figure 3) and lagged wage (Figure 4). Average wage and average wage growth within each decile are graphed. In general, these profiles are consistent with the form of the human capital production function estimated below.

As can be seen in Figure 3, wages are roughly an increasing and concave function of hours of work for men and women. Given the functional form adopted in equation (15), this is consistent with $\alpha_1 \in (0,1)$.[7] Moreover, α_1 between 0 and 1 is also consistent with the roughly increasing and concave shape of the wage growth profile in Figure 3.

The relationship between current wages and wage growth with past wages is quite similar across gender as can be seen in Figure 4. As illustrated by the blue lines, wages are an increasing function of lagged wages, which is broadly consistent with $\alpha_2 > 0$. The negative slope of the red lines are also consistent with the chosen form since

$$\frac{\partial \triangle \ln W_{it+1}}{\partial W_{it}} = \frac{\partial \left(\ln \left[B(X_{it}) \cdot W_{it} + A(X_{it}) \cdot H_{it}^{\alpha_1} W_{it}^{\alpha_2}\right]\right)}{\partial W_{it}} - \frac{\partial \ln W_{it}}{\partial W_{it}} < 0$$

for $\alpha_2 \in (0,1)$.

The chosen human capital production function with $\alpha_1 \in (0,1)$ and $\alpha_2 \in (0,1)$ is generally consistent with the wage and wage growth profiles presented in Figures 3 and 4. The next section shows that most of the estimates are in line with these values.

[7] See the appendix for more details.

5 Results

Estimation focuses on two main models which incorporate age effects on skill production in different ways. In model A, I allow the productivity multiplier to be a quadratic function of age a_{it}, while model B allows the depreciation rate to be a quadratic function of age[8]:

$$
\begin{aligned}
\triangle \ln W_{it+1} &= \triangle \ln \left[B(a_{it}) \cdot W_{it} + A(a_{it}) \cdot H_{it}^{\alpha_1} \cdot W_{it}^{\alpha_2}\right] + \triangle \varepsilon_{it} \\
\text{Model A} &: \; A(a_{it}) = A_0 + A_1 \cdot a_{it} + A_2 \cdot a_{it}^2,\; B(a_{it}) = B_0 \\
\text{Model B} &: \; B(a_{it}) = B_0 + B_1 \cdot a_{it} + B_2 \cdot a_{it}^2,\; A(a_{it}) = A_0.
\end{aligned}
$$

At this point it should be noted that parameters A_0 and B_0 cannot be separately identified. To illustrate this, consider model A:

$$
\begin{aligned}
\triangle \ln W_{it+1} &= \triangle \ln \left[B_0 \cdot W_{it} + \left(A_0 + A_1 a_{it} + A_2 a_{it}^2\right) \cdot H_{it}^{\alpha_1} \cdot W_{it}^{\alpha_2}\right] + \triangle \varepsilon_{it} && (18) \\
&= \triangle \ln \left[\left(\frac{B_0}{A_0}\right) \cdot W_{it} + \left(1 + \left(\frac{A_1}{A_0}\right) a_{it} + \left(\frac{A_2}{A_0}\right) a_{it}^2\right) \cdot H_{it}^{\alpha_1} \cdot W_{it}^{\alpha_2}\right] + \triangle \varepsilon_{it}. && (19)
\end{aligned}
$$

where equation (19) is obtained by adding and subtracting $\ln A_0$ from equation (18). Equation (19) shows that we can only identify the ratios B_0/A_0, A_1/A_0 and A_2/A_0 and not A_0, A_1, A_2 and B_0 separately.

Note that in equation (19), I normalize using $\ln A_0$. In the case where $A_0 < 0$, this normalization is not possible because it is not possible to take the logarithm of a negative number. I instead have to normalize with $\ln B_0$ which leads to the following model:

$$
\triangle \ln W_{it+1} = \triangle \ln \left[W_{it} + \left(\frac{A_0}{B_0} + \left(\frac{A_1}{B_0}\right) a_{it} + \left(\frac{A_2}{B_0}\right) a_{it}^2\right) \cdot H_{it}^{\alpha_1} \cdot W_{it}^{\alpha_2}\right] + \triangle \varepsilon_{it}. \quad (20)
$$

When I implement estimation of model A, I estimate both equations (19) and (20) and present results for the model that yields the lowest GMM objective function. When presenting model estimates, I assume that $A_0 = 1$ if parameters have been normalized by A_0, and I assume that

[8] I also estimated other specifications where different combinations of the parameters A, B, α_1 and α_2 were quadratic functions of age. Models A and B provided the best fit for the data based on the model selection criteria proposed by Andrews and Lu (2001).

$B_0 = 1$ if I normalized with B_0. A similar normalization with respect to A_0 or B_0 is applied for the estimation of model B where the depreciation rate depends on a quadratic function of age.

These different models are estimated in a GMM framework using instrumental variables for endogenous variables $(H_{it}, H_{it-1}, W_{it})$. These instruments are essentially non-linear functions of lagged wage W_{it-1}, second order lag of hours worked H_{it-2}, as well as age and lagged age. These instruments are meant to capture the non-linear effects of $(H_{it}, H_{it-1}, W_{it})$ on wage growth implied by the form of the production function.[9]

Table 2 presents estimates of the benchmark production function with no test variables. The first column of Tables 2a and 2b shows the estimation results for model A where the productivity multiplier A is a quadratic function of age. In the second column, the human capital retention rate B changes with age (model B). In the third column, I present results for a model where age does not affect the production function. For both men and women, this last specification is rejected by the over-identifying restrictions test proposed by Hansen (1982). The last row in Tables 2a and 2b shows that the *p-value* for this test is above the standard level of rejection when age is accounted for in the production function.[10] Thus, allowing for age effects on skill production appears to be important.

It is interesting to note that for both men and women, parameters A and B are concave functions of age, increasing at early ages and then decreasing by the time workers are 30 years old. In model A, this implies that individuals become more proficient at accumulating human capital in their early career. The interpretation is slightly different in model B. Its estimates imply that workers retain more of their previously accumulated human capital as they age. The estimates are consistent with the increasing and concave profiles of wages with respect to age observed in figures 1a and 2a.

Estimates for α_1 are roughly 0.08 for men and 0.03 for women, suggesting that hours of work are less productive for women. Estimates of α_2 are larger than for α_1, ranging from 0.1 to 0.5.

[9] More specifically, the instrument set includes an intercept, linear and quadratic first order lags of wage, linear and quadratic logarithm of first order lags of wage, linear and quadratic second order lags of hours of work, linear and quadratic logarithm of second order lags of hours of work, lagged wage times second order lagged hours of work, log of lagged wage times log of second order lagged hours, linear and quadratic age, linear and quadratic lag of age.

[10] If the instruments are valid, this serves as a test of the functional form chosen for the production technology. It can be implemented here since the GMM estimation is based on more orthogonality conditions ($r = 15$) than parameters ($q = 5$ for models A and B, $q = 3$ for the no age effect model). The GMM objective function and overidentification test statistic is asymptotically distributed as a $\chi^2_{(r-q)}$.

These estimates are broadly consistent with the general pattern of wage and wage growth observed in figures 3 and 4 as discussed in the previous section. These exponents imply that the elasticity of wages W_{it+1} with respect to hours of work H_{it} ranges from 0.03 to 0.065, while the elasticity with respect to lagged wage W_{it} ranges between 0.2 to 0.4.

Table 3 presents results for a test of the LBD model. Tables 3a and 3b report tests for males with model A and B, respectively. For women, Table 3c contains the results for model A and Table 3d for model B. In each table, the first column reproduces the relevant estimates from Table 2 for comparison. Other columns introduce a single test variable S_{it} to the specification. The test variable in the second column is future hours worked, while the last column uses per capita family income (in \$10,000) as a test variable. These test variables are assumed to be endogenous and instrumented with the same set of instruments used for the endogenous inputs $(H_{it}, H_{it-1}, W_{it})$. Lagged hours of work provide a valid instrument for future hours of work, while per-capita family income is codetermined with lagged wages and hours worked.

Finally, Table 3 includes a row for the elasticity of wage W_{it+1} with respect to hours of work H_{it}, lagged wage W_{it}, and the test variable S_{it}. These elasticities are meant to assess the relative importance to wage growth of both human capital accumulation mechanisms (LBD versus OJT). If wage growth is found to be more sensitive to test variables than to hours worked, this indicates that the investment mechanism of the OJT model is more important than that of the LBD model.

In Table 3, future hours of work H_{it+1} is found to have a positive correlation with wage growth; individuals who work more in the future experience more wage growth. An obvious interpretation is that individuals who are going to work more in the future have more incentive to accumulate human capital. This positive correlation is found to be statistically significant only for estimates based on the male sample. This may be because hours of work exhibit less variation in the male sample, so that H_{it+1} gives a more reliable picture of the complete future stream of annual hours of work than it does for women. Elasticities with respect to future hours worked are smaller than elasticities for current hours of work. One might expect that a more precise measure of the stream of all future hours may affect wages more than the single measure of hours worked next year.

Per capita family income does not display any statistically significant correlation with wage growth. This may reflect that it is a poor measure of the trade-off between investment and productive work in the OJT model. It might also be the case that few workers are constrained by the trade-off's cost.

In summary, the evidence suggests that future work correlates with wage growth even after holding hours of work and lagged wage constant. This suggest that the assumed LBD human capital model where accumulation is a by-product of work does not fully account for the variation in human capital accumulation across workers. Moreover, the sensitivity of wages with respect to these test variables is roughly within the same range as the sensitivity to hours of work. If both test variables provide a good measure of the incentive to invest in skills, then this suggests that both LBD and OJT play more or less comparable roles in human capital accumulation.

One caveat has to be mentioned at this point. All models estimated in Tables 2 and 3 use the same set of instruments. These instruments include functions of lagged hours worked H_{it-2}. As explained in section 3.2.2, the production function may be misspecified in that it should account for longer lags of hours of work, including H_{it-2}. In this case, finding that variables S_{it} are correlated with wage growth may be an indication that the LBD production function is misspecified, rather than an indication that skills are accumulated through OJT.

Table 4, therefore, presents estimates of the models in Table 3 using a set of instruments that exclude all lags of hours worked. With this alternative set of instruments, I find few statistically significant correlations between test variables and wage growth. However, it should also be noted that estimates for the exponent α_1 are also noisier than with the original set of instruments. These tests are inconclusive since these alternative variables are weak instruments for hours of work, are likely poor instruments for the test variables who identify variation in the incentives to accumulate human capital.

In appendix Tables A1 to A4, I assess the robustness of the production function estimates presented in Table 2. To account for the possibility that human capital accumulation is different for workers who have different levels of education, I estimate the production function separately for high school graduates and workers who complete some college.[11] Estimates for workers who

[11] As was shown in Table 1, high school dropouts represent less than 15% of the male sample and less than 10% of the female sample, so I do not present estimates for them here.

completed some college are quite similar to those presented in Table 2, while estimates for high school graduates are somewhat different. The profiles of parameters A and B with respect to age are less steep and less concave than those of college-goers. This is consistent with figures 1a and 2a where age profiles of wages for high school graduates are flatter than those of college educated workers. For high school graduates, estimates of α_1 (effect of hours worked) are larger while α_2 (effects of lagged wages) is smaller. This might be the result of the higher variation in hours worked and the weaker variation in lagged wages observed for high school graduates in figure 1 and 2.

Tables A1 and A2 also report estimates of models A and B for broader samples of males and females where there is no restriction on wages, wage growth, hours worked or the time span since the last interview. Only the restrictions on age and having completed schooling are applied. The estimates of model A for males are roughly similar to those obtained from the main sample presented in the first column of Table A1. Model B's estimates for most parameters are however quite different from those presented earlier, and the over-identifying restrictions test indicates that the model provides a poor fit of the data. For females, the estimates are roughly in line with those obtained with the main sample. However, the assumptions of the identifying restrictions test are rejected for both models, suggesting that those models are inappropriate for this broader sample.

6 Conclusion

Research using human capital theory to explain wage determination has assumed either the on-the-job training (OJT) model or the learning-by-doing (LBD) model. In the OJT model, human capital investments are costly. Skills are accumulated as a by-product of production in the LBD model. Studies have shown that these two models have different implications in terms of the impact of wage subsidy policies, wage measurement, and the effect of business cycles on hours worked (Cossa, Heckman and Lochner [2003], Hansen and Imrohoroglu [2008]).

In this paper, I use the NLSY79 data to estimate and test a pure LBD model of skill accumulation. Estimation is implemented via GMM and does not require any assumptions regarding preferences or the structure of credit markets. I deal with permanent unobserved heterogeneity and the endogeneity of inputs in the production of skills.

Estimates of the pure LBD production function indicate that workers who work more accumulate more human capital. Estimates are broadly similar across genders and education levels. The elasticity of wages with respect to hours worked range from 0.03 to 0.065. Allowing for age effects in the the production function is important since it dramatically improves the estimation goodness-of-fit. The productivity of learning is a concave function of age, initially increasing at younger ages.

The pure LBD model is then tested based on its strong prediction that once hours of work and other inputs are held constant, there should be no systematic variation in wage growth. If skills are acquired through investment as in the OJT model, variables that reflect variation in the incentives to invest should affect wage growth since hours worked do not capture all the variation in unobserved investments in human capital. Finding that wage growth correlates with the incentives to accumulate skills, even after accounting for hours worked, ability and current skill levels, is interpreted as a rejection of the pure LBD framework. This test is inspired by the "excess sensitivity" test used in the literature that tests the permanent-income hypothesis.

My test variables include future hours of work and per-capita family income. Individuals who will work more hours in the future have a greater incentive to invest in human capital. Individuals in families with low per-capita family income may have less scope to engage in costly human capital investments. I find that future hours of work have a statistically significant and positive effect on wage growth for men, suggesting that men acquire human capital through costly investments. For women, future hours of work are not correlated with wage growth once the pure LBD production function is taken into account. Per-capita family income is not found to affect wage growth conditional on hours worked.

Future work will focus on testing the LBD model using a wider variety of variables that affect the incentives to accumulate human capital. Economic theory suggests that investment in health and in human capital should go hand in hand as healthier individuals live longer and, therefore, extract more benefits from the accumulation of skills. One would therefore expect health to affect the incentives to acquire human capital.

I also plan to study the effect of variables measuring fertility expectations and decisions for women. Having children almost invariably requires women to take some time off from work, reducing their hours of work and, therefore, their incentives to accumulate skills. This implies that fertility variables might provide information about women's unobserved investment in human capital in the OJT model.

I plan to look at how individuals' expectations about their career affects wage growth. Different careers, *e.g.* professional versus non-professional, require the acquisition of different levels of skills. These expectations should provide some information about workers' incentives to accumulate skills, and correlate with costly investment in the OJT model.

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Appendix

Consider the human capital production function

$$W_{it+1} = [B(X_{it}) \cdot W_{it} + A(X_{it}) \cdot H_{it}^{\alpha_1} \cdot W_{it}^{\alpha_2}] \cdot e^{\theta_i + \varepsilon_{it+1}}, \tag{21}$$

and its first difference

$$\triangle \ln W_{it+1} = \triangle \ln [B(X_{it}) \cdot W_{it} + A(X_{it}) \cdot H_{it}^{\alpha_1} \cdot W_{it}^{\alpha_2}] + \triangle \varepsilon_{it+1}. \tag{22}$$

Taking the derivative of the production function (21) with respect to H_{it} yields

$$\frac{\partial W_{it+1}}{\partial H_{it}} = \left[\alpha_1 \cdot A(X_{it}) \cdot H_{it}^{\alpha_1 - 1} \cdot W_{it}^{\alpha_2}\right] \cdot e^{\theta_i + \varepsilon_{it+1}}. \tag{23}$$

This derivative is positive if $\alpha_1 > 0$ and yields a slightly concave profile of W_{it+1} with respect to H_{it} if $\alpha_1 < 1$. Looking at wage growth from equation (22), its derivative with respect to hours of work yields

$$\frac{\partial \triangle \ln W_{it+1}}{\partial H_{it}} = \frac{\alpha_1 \cdot A(X_{it}) \cdot H_{it}^{\alpha_1 - 1} \cdot W_{it}^{\alpha_2}}{B(X_{it}) \cdot W_{it} + A(X_{it}) \cdot H_{it}^{\alpha_1} \cdot W_{it}^{\alpha_2}}. \tag{24}$$

This derivative is positive if $\alpha_1 > 0$ and concave if $\alpha_1 < 1$.

Taking the derivative of the production function (21) with respect to past wage W_{it} yields

$$\frac{\partial W_{it+1}}{\partial W_{it}} = \left[B(X_{it}) + \alpha_2 \cdot A(X_{it}) \cdot H_{it}^{\alpha_1} \cdot W_{it}^{\alpha_2 - 1}\right] \cdot e^{\theta_i + \varepsilon_{it+1}}. \tag{25}$$

This derivative is positive if $\alpha_2 > 0$ and yields a slightly concave profile of W_{it+1} with respect to H_{it} if $\alpha_2 < 1$. Looking at wage growth from equation (22), its derivative with respect lagged wage yields

$$\frac{\partial \triangle \ln W_{it+1}}{\partial W_{it}} = \frac{B(X_{it}) + \alpha_2 \cdot A(X_{it}) \cdot H_{it}^{\alpha_1} \cdot W_{it}^{\alpha_2 - 1}}{B(X_{it}) \cdot W_{it} + A(X_{it}) \cdot H_{it}^{\alpha_1} \cdot W_{it}^{\alpha_2}} - \frac{1}{W_{it}}. \tag{26}$$

This derivative is positive if $\alpha_2 > 0$ and concave if $\alpha_2 < 1$.

Age Profiles

Now lets consider the effect of age a_{it} on those different derivatives. In most estimates I find that both $A(a_{it})$ and $B(a_{it})$ are concave and increasing up to their maximum. In most cases this leads the marginal productivity of lagged wage W_{it-1} and hours worked H_{it} with respect to wages W_{it+1} to have concave and increasing age profiles whether model A or B is considered. I show after that however that Model A and B have different implication regarding the marginal productivity profiles of lagged wage and hours worked with respect to wage growth $\triangle \ln W_{it+1}$.

Let $X_{it} = a_{it}$ and consider the derivative of (23) with respect to age:

$$\frac{\partial^2 W_{it+1}}{\partial H_{it} \partial a_{it}} = \left[\alpha_1 \cdot A'(a_{it}) \cdot H_{it}^{\alpha_1 - 1} \cdot W_{it}^{\alpha_2}\right] \cdot e^{\theta_i + \varepsilon_{it+1}}. \tag{27}$$

In model A, $A'(a_{it}) = A_1 + 2A_2 a_{it}$ so that $\partial^2 W_{it+1}/\partial H_{it} \partial a_{it} > 0$ if $A_1 + 2A_2 a_{it} > 0$. In most estimates I find that $A_1 + 2A_2 a_{it} > 0$ at early ages but negative afterward. Therefore the productivity of hours of work in human capital accumulation is concave and increasing until its maximum. In model B age has no impact on the productivity of hour worked since $A'(a_{it}) = 0$ leading to $\partial^2 W_{it+1}/\partial H_{it} \partial a_{it} = 0$.

Age implies that the productivity of lagged wage is concave and increasing (until its maximum) in both model A and B. Consider the derivative of (25) with respect to age:

$$\frac{\partial^2 W_{it+1}}{\partial W_{it} \partial a_{it}} = \left[B'(a_{it}) + \alpha_2 \cdot A'(a_{it}) \cdot H_{it}^{\alpha_1} \cdot W_{it}^{\alpha_2 - 1} \right] \cdot e^{\theta_i + \varepsilon_{it+1}}. \tag{28}$$

In model A, $A'(a_{it}) = A_1 + 2A_2 a_{it}$ while $B'(a_{it}) = 0$. Model B implies that $A'(a_{it}) = 0$ while $B'(a_{it}) = B_1 + 2B_2 a_{it}$. For most estimates I present both $A(a_{it})$ and $B(a_{it})$ are concave and increasing up to their maximum so that the marginal productivity of lagged wage is also increasing and concave whether it is the productivity multiplier A or the retention rate B that depends on age.

Now consider the marginal effect of hours of work on wage growth given in equation (24) and take its derivative with respect to age a_{it}:

$$\begin{aligned}\frac{\partial^2 \triangle \ln W_{it+1}}{\partial H_{it} \partial a_{it}} &= \frac{\alpha_1 \cdot A'(a_{it}) \cdot H_{it}^{\alpha_1 - 1} \cdot W_{it}^{\alpha_2}}{B(a_{it}) \cdot W_{it} + A(a_{it}) \cdot H_{it}^{\alpha_1} \cdot W_{it}^{\alpha_2}} \\ &- \frac{(B'(a_{it}) \cdot W_{it} + A'(a_{it}) \cdot H_{it}^{\alpha_1} \cdot W_{it}^{\alpha_2}) \cdot (\alpha_1 \cdot A(a_{it}) \cdot H_{it}^{\alpha_1 - 1} \cdot W_{it}^{\alpha_2})}{(B(a_{it}) \cdot W_{it} + A(a_{it}) \cdot H_{it}^{\alpha_1} \cdot W_{it}^{\alpha_2})^2}.\end{aligned} \tag{29}$$

In model A where $B'(a_{it}) = 0$ and $A'(a_{it}) = A_1 + 2A_2 a_{it}$ it can be shown that $\partial^2 \triangle \ln W_{it+1}/\partial H_{it} \partial a_{it} > 0$ if $B > 0$. In model B $\partial^2 \triangle \ln W_{it+1}/\partial H_{it} \partial a_{it}$ is always smaller than 0. Whereas model A implies that the marginal effect of hours worked on wage growth increases with age, model B implies they are decreasing with age.

Something similar holds for the effect of age on the marginal effect of lagged wage on wage growth. Take the derivative of (26) with respect to age:

$$\frac{\partial^2 \triangle \ln W_{it+1}}{\partial W_{it} \partial a_{it}} = \frac{B'(a_{it}) + \alpha_2 \cdot A'(a_{it}) \cdot H_{it}^{\alpha_1} \cdot W_{it}^{\alpha_2 - 1}}{B(a_{it}) \cdot W_{it} + A(a_{it}) \cdot H_{it}^{\alpha_1} \cdot W_{it}^{\alpha_2}} \tag{30}$$

$$- \frac{(B'(a_{it}) \cdot W_{it} + A'(a_{it}) \cdot H_{it}^{\alpha_1} \cdot W_{it}^{\alpha_2}) \cdot (B(a_{it}) + \alpha_2 \cdot A(a_{it}) \cdot H_{it}^{\alpha_1} \cdot W_{it}^{\alpha_2 - 1})}{(B(a_{it}) \cdot W_{it} + A(a_{it}) \cdot H_{it}^{\alpha_1} \cdot W_{it}^{\alpha_2})^2} \tag{31}$$

In model A where $B'(a_{it}) = 0$ it can be shown that $\partial^2 \triangle \ln W_{it+1}/\partial W_{it} \partial a_{it} < 0$ if the exponent on lagged wage α_2 is between 0 and 1. In model B the derivative is positive when α_2 is between 0 and 1.

Figure 1a: Age Profiles of Hourly Wage by Education Level for Men

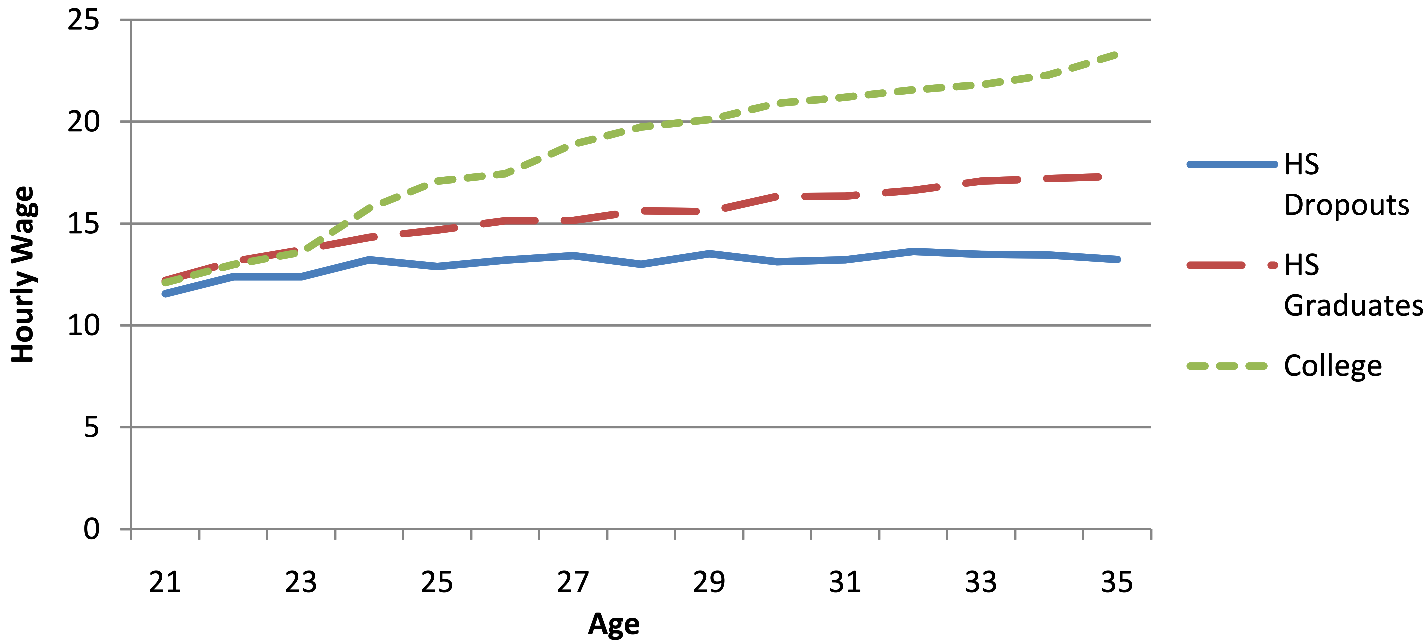

Figure 1b: Age Profiles of Hours Worked by Education Level for Men

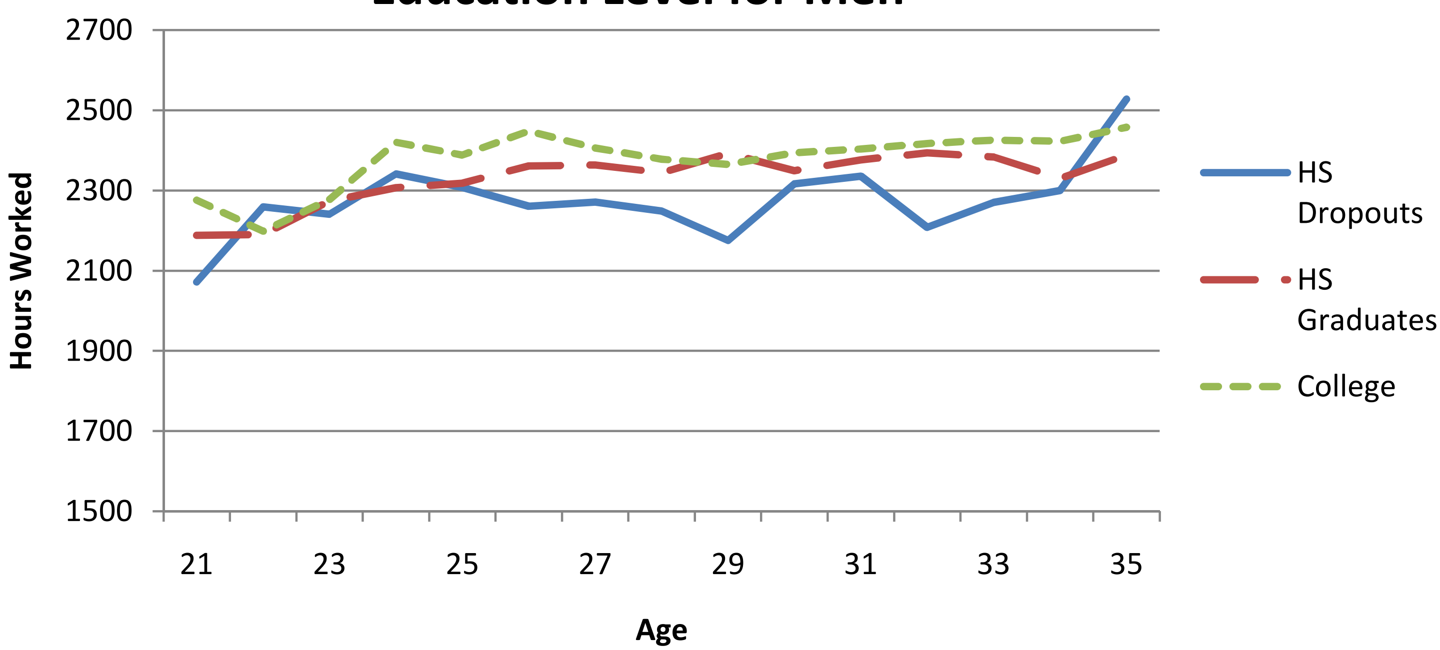

Figure 2a: Age Profiles of Hourly Wage by Education Level for Women

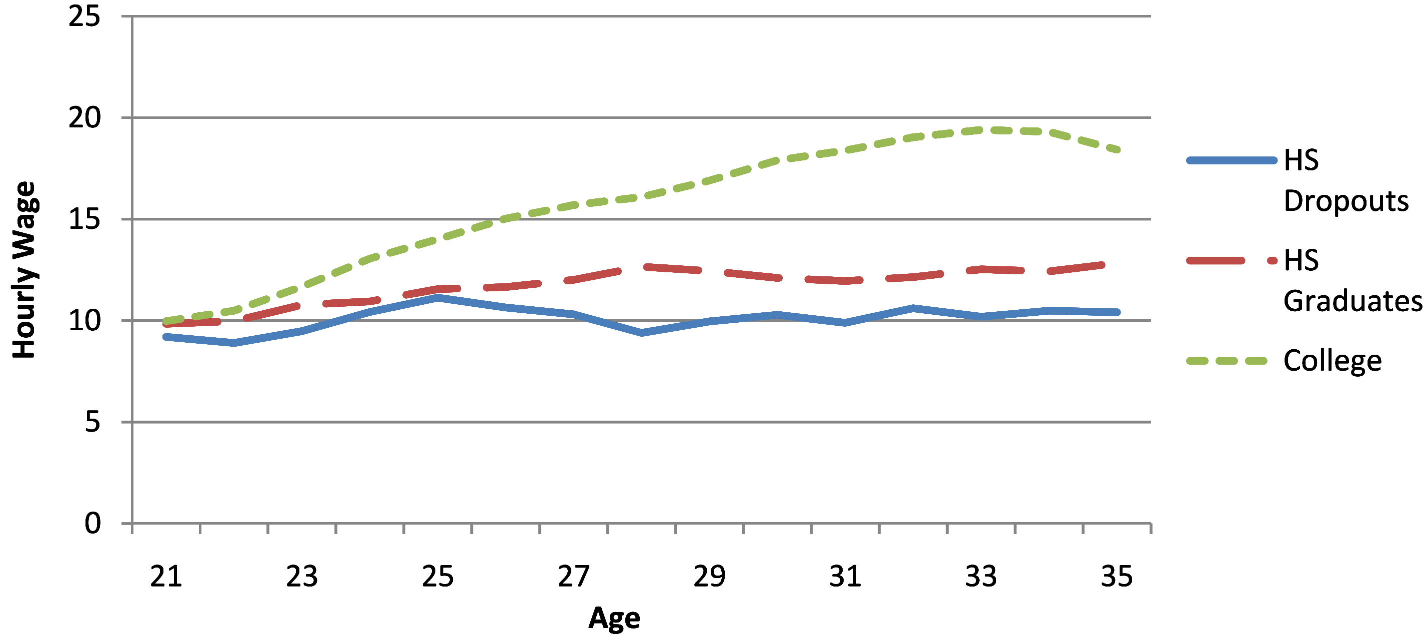

Figure 2b: Age Profiles of Hours Worked by Education Level for Women

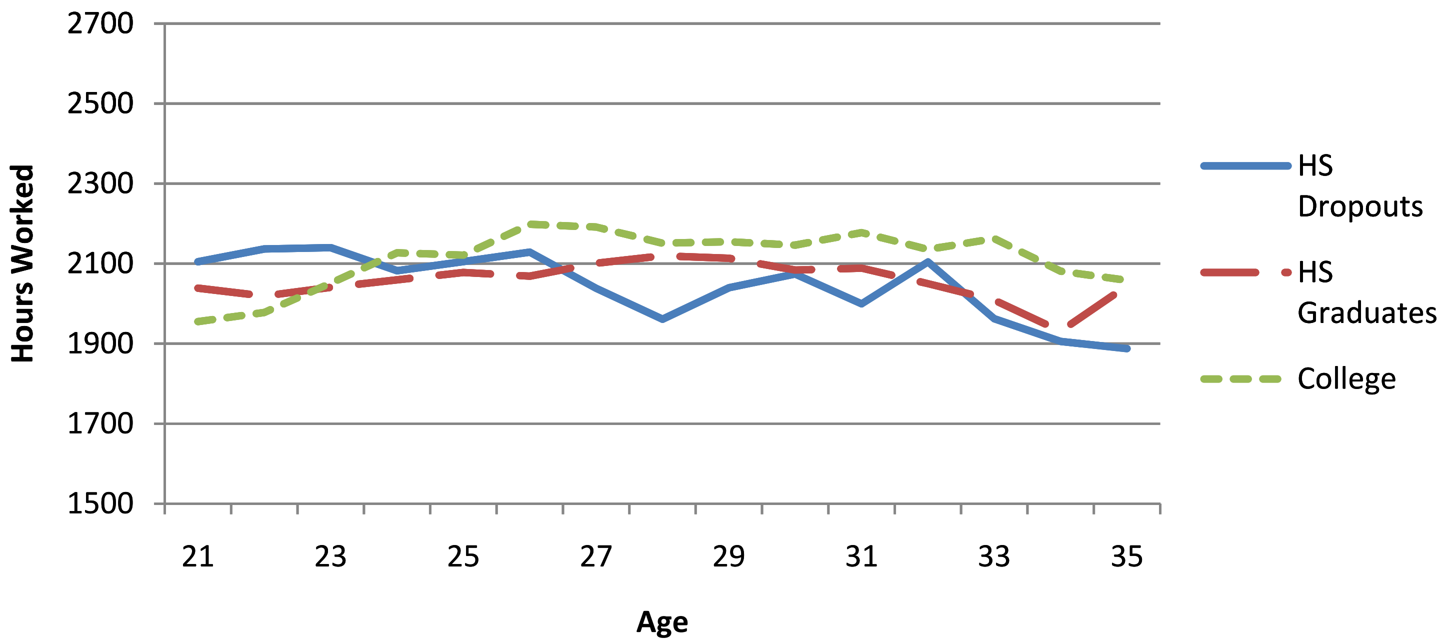

Figure 3a: Hours of Work Profiles of Wages and First Difference Log Wages for Men

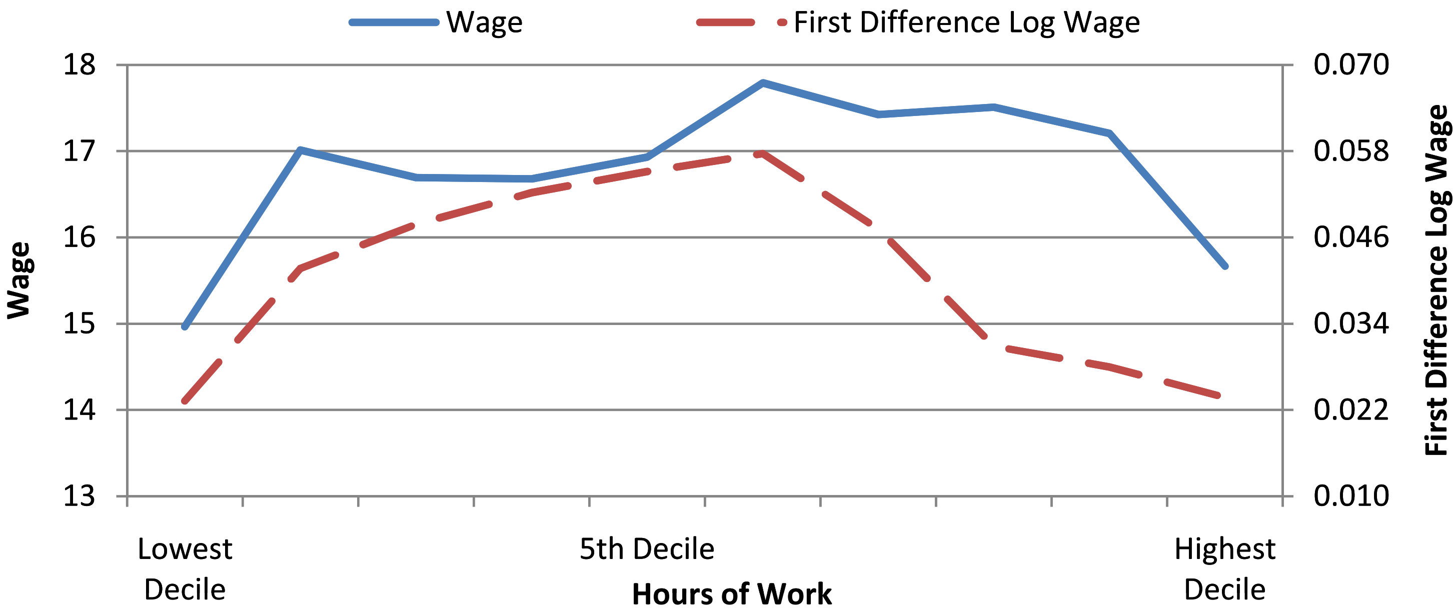

Figure 3b: Hours of Work Profiles of Wages and First Difference Log Wages for Women

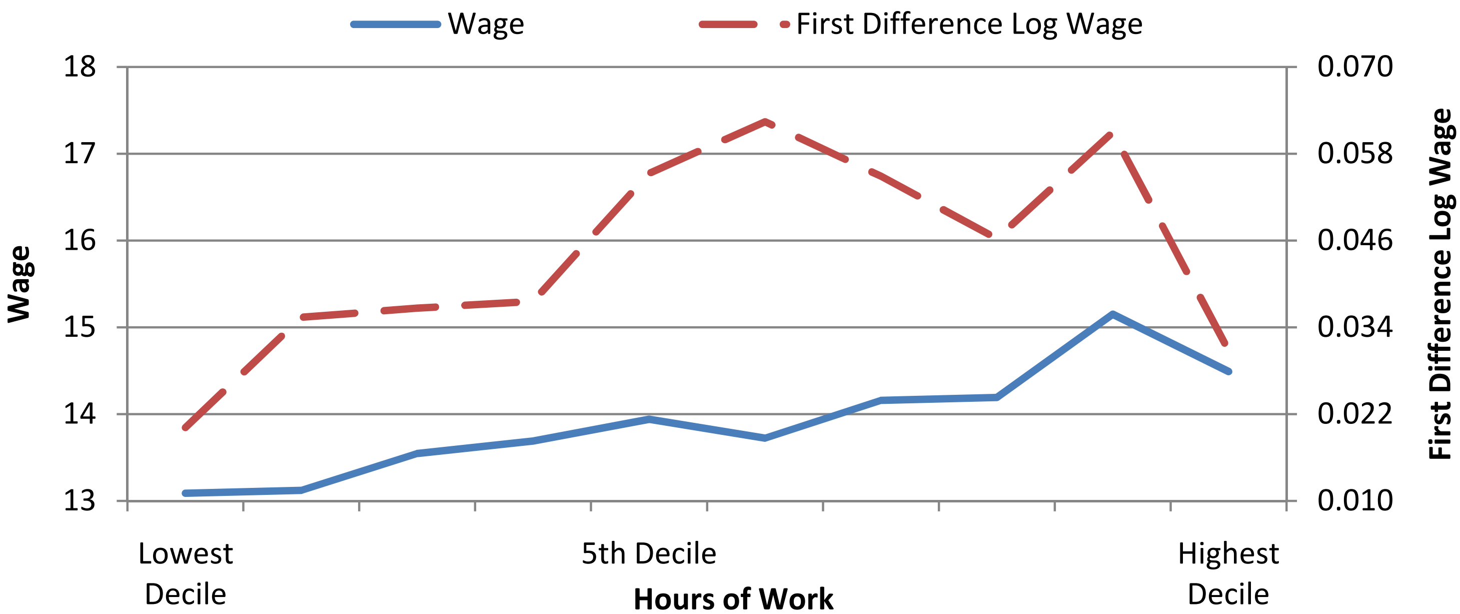

Figure 4a: Lagged Wage Profiles of Wages and First Difference Log Wages for Men

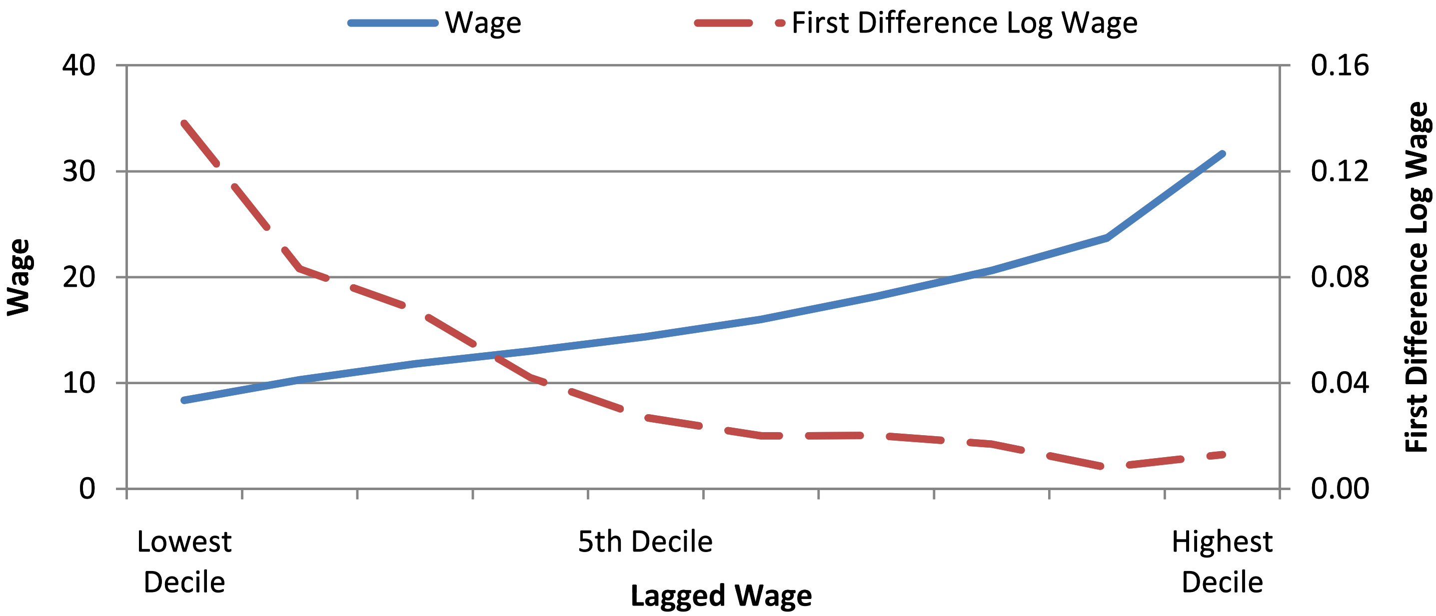

Figure 4b: Lagged Wage Profiles of Wages and First Difference Log Wages for Women

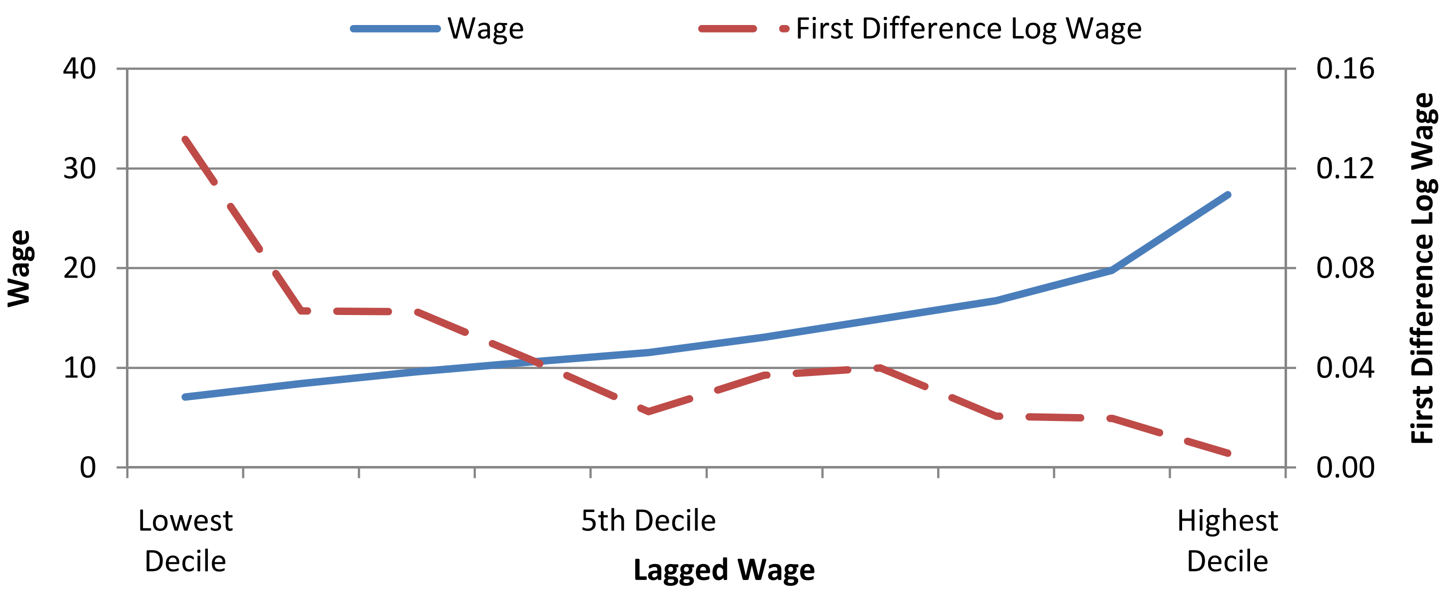

Table 1: Sample Descriptive Statistics

	Men	Women
Hourly Wage	16.79	13.91
	(7.69)	(6.70)
Wage Growth	6.26%	6.32%
	(21.69)	(19.96)
Annual Hours of Work	2348.03	2100.13
	(535.26)	(497.67)
Age	28	28
	(4)	(4)
Per Capita Family Income	18,585	19,167
	(24,132)	(31,142)
Number of Children	0.71	0.61
	(1.01)	(0.91)
Ratio Earners to Household Members	0.64	0.71
	(0.34)	(0.31)
High School Dropout	14%	7%
High School Graduates	44%	42%
Some College	42%	52%
Sample Size	9,566	7,560

Standard deviation in parenthesis. The men sample includes 1,772 individuals. The women sample includes 1,580 individuals. Both sample include individuals who have completed their formal schooling. Individuals who completed high school or less must be at least 18 years old. Individuals with some college or more must be at least 21 years old. Samples exclude individual with outlier values for hourly wage, hourly wage growth and hours of work.

Table 2a: Production Function Estimates White Males

		Model A	Model B	No Age Effect
Depreciation Rate	B_0	1	-0.031	1
			(0.050)	
	B_1		0.015	
			(0.003)	
	B_2		-0.0004	
			(0.0001)	
A	A_0	18.585	1	0.325
		(12.491)		(0.284)
	A_1	1.536		
		(0.901)		
	A_2	-0.027		
		(0.019)		
Exponents	α_1	0.078	0.087	0.509
		(0.028)	(0.034)	(0.131)
	α_2	0.093	0.212	-0.575
		(0.117)	(0.119)	(0.212)
Hansen Over-Identifying Restrictions Test	P-value	0.903	1.000	0.000

Standard errors in parenthesis, sample size = 9474

Sample includes white men who have completed formal schooling. Respondents who did not attend college must be at least 18 years old. Respondents who attended college must be at least 21 years old. Wages below $1.90 and above $100 are excluded. Wage observations with wage growth below -50% and above 100% are excluded. Annual hours of work must range between 780 and 4368. Average hours worked per week of work must be between 15 and 84. Wage observations for which the previous interview took place less than 10 months or more than 14 months are excluded. Instruments include: linear and quadratic second order lagged wage, linear and quadratic logarithm of second order lagged wage, linear and quadratic second order lagged hours of work, linear and quadratic logarithm of second order lagged hours of work, second order lag of wage times hours of work, second order lag of log wage times log hours, linear and quadratic age, linear and quadratic lagged age.

Table 2b: Production Function Estimates White Females

		Model A	Model B	No Age Effect
Depreciation Rate	B_0	1	-0.185	1
			(0.081)	
	B_1		0.013	
			(0.002)	
	B_2		-0.0003	
			(0.0001)	
A	A_0	414.090	1	0.502
		(6737.800)		(0.612)
	A_1	27.300		
		(439.200)		
	A_2	-0.304		
		(4.915)		
Exponents	α_1	0.030	0.034	0.410
		(0.019)	(0.018)	(0.177)
	α_2	0.198	0.514	-0.740
		(0.132)	(0.100)	(0.253)
Hansen Over-Identifying Restrictions Test	P-value	0.987	0.179	0.000

Standard errors in parenthesis, sample size = 7477

Sample includes white women who have completed formal schooling. Respondents who did not attend college must be at least 18 years old. Respondents who attended college must be at least 21 years old. Wages below $1.90 and above $100 are excluded. Wage observations with wage growth below -50% and above 100% are excluded. Annual hours of work must range between 780 and 4368. Average hours worked per week of work must be between 15 and 84. Wage observations for which the previous interview took place less than 10 months or more than 14 months are excluded. Instruments include: linear and quadratic second order lagged wage, linear and quadratic logarithm of second order lagged wage, linear and quadratic second order lagged hours of work, linear and quadratic logarithm of second order lagged hours of work, second order lag of wage times hours of work, second order lag of log wage times log hours, linear and quadratic age, linear and quadratic lagged age.

Table 3a: Production Function Estimates of Model A with Test Variables, White Males

		Model A	H_{it+1}	Per Capita Family Income ($10,000)
Depreciation Rate	B_0	1	1	0.143
				(0.175)
A	A_0	18.585	0.343	1
		(12.491)	(0.501)	
	A_1	1.536	-0.015	0.082
		(0.901)	(0.024)	(0.025)
	A_2	-0.027	0.000	-0.003
		(0.019)	(0.001)	(0.002)
Exponents	α_1	0.078	0.522	0.097
		(0.028)	(0.185)	(0.102)
	α_2	0.093	-0.113	-0.059
		(0.117)	(0.234)	(0.326)
LBD Test Variables	Linear		0.024	0.009
			(0.008)	(0.008)
Elasticities	H_{it}	0.065	0.146	0.054
	W_{it}	0.249	0.689	0.416
	Test Var.		0.058	0.015
Hansen Over-Identifying Restrictions Test	P-Value	0.903	0.897	1.000
Sample Size		9474	7968	8521

Standard errors in parenthesis

Sample includes white men who have completed formal schooling. Respondents who did not attend college must be at least 18 years old. Respondents who attended college must be at least 21 years old. Wages below $1.90 and above $100 are excluded. Wage observations with wage growth below -50% and above 100% are excluded. Annual hours of work must range between 780 and 4368. Average hours worked per week of work must be between 15 and 84. Wage observations for which the previous interview took place less than 10 months or more than 14 months are excluded. Instruments include: linear and quadratic second order lagged wage, linear and quadratic logarithm of second order lagged wage, linear and quadratic second order lagged hours of work, linear and quadratic logarithm of second order lagged hours of work, second order lag of wage times hours of work, second order lag of log wage times log hours, linear and quadratic age, linear and quadratic lagged age.

Table 3b: Production Function Estimates of Model B with Test Variables, White Males

		Model B	H_{it+1}	Per Capita Family Income ($10,000)
Depreciation Rate	B_0	-0.031	1	-0.098
		(0.050)		(0.146)
	B_1	0.015	4.69E+14	0.013
		(0.003)	(5.90E+14)	(0.003)
	B_2	-0.0004	-4.80E+13	-0.0002
		(0.0001)	(3.98E-15)	(0.0003)
A	A_0	1	1.81E+17	1
			(1.04E+17)	
Exponents	α_1	0.087	0.078	0.035
		(0.034)	(0.021)	(0.053)
	α_2	0.212	0.205	0.369
		(0.119)	(0.047)	(0.243)
LBD Test Variables	Linear		0.018	-0.006
			(0.007)	(0.018)
Elasticities	H_{it}	0.065	0.080	0.033
	W_{it}	0.409	0.177	0.394
	Test Var.		0.044	-0.010
Hansen Over-Identifying Restrictions Test	P-Value	1.000	1.000	1.000
Sample Size		9474	7968	8521

Standard errors in parenthesis

Sample includes white men who have completed formal schooling. Respondents who did not attend college must be at least 18 years old. Respondents who attended college must be at least 21 years old. Wages below $1.90 and above $100 are excluded. Wage observations with wage growth below -50% and above 100% are excluded. Annual hours of work must range between 780 and 4368. Average hours worked per week of work must be between 15 and 84. Wage observations for which the previous interview took place less than 10 months or more than 14 months are excluded. Instruments include: linear and quadratic second order lagged wage, linear and quadratic logarithm of second order lagged wage, linear and quadratic second order lagged hours of work, linear and quadratic logarithm of second order lagged hours of work, second order lag of wage times hours of work, second order lag of log wage times log hours, linear and quadratic age, linear and quadratic lagged age.

Table 3c: Production Function Estimates of Model A with Test Variables, White Females

		Model A	H_{it+1}	Per Capita Family Income ($10,000)
Depreciation Rate	B_0	1	-0.077	1
			(0.057)	
A	A_0	414.090	1	28.568
		(6737.800)		(43.754)
	A_1	27.300	0.043	2.070
		(439.200)	(0.030)	(2.819)
	A_2	-0.304	-0.001	-0.026
		(4.915)	(0.000)	(0.031)
Exponents	α_1	0.030	0.025	0.030
		(0.019)	(0.030)	(0.030)
	α_2	0.198	0.383	0.112
		(0.132)	(0.127)	(0.185)
LBD Test Variables	Linear		0.007	0.001
			(0.016)	(0.004)
Elasticities	H_{it}	0.029	0.033	0.026
	W_{it}	0.205	0.187	0.237
	Test Var.		0.014	0.002
Hansen Over-Identifying Restrictions Test	P-Value	0.987	0.932	0.718
Sample Size		7477	6258	6605

Standard errors in parenthesis

Sample includes white women who have completed formal schooling. Respondents who did not attend college must be at least 18 years old. Respondents who attended college must be at least 21 years old. Wages below $1.90 and above $100 are excluded. Wage observations with wage growth below -50% and above 100% are excluded. Annual hours of work must range between 780 and 4368. Average hours worked per week of work must be between 15 and 84. Wage observations for which the previous interview took place less than 10 months or more than 14 months are excluded. Instruments include: linear and quadratic second order lagged wage, linear and quadratic logarithm of second order lagged wage, linear and quadratic second order lagged hours of work, linear and quadratic logarithm of second order lagged hours of work, second order lag of wage times hours of work, second order lag of log wage times log hours, linear and quadratic age, linear and quadratic lagged age.

Table 3d: Production Function Estimates of Model B with Test Variables, White Females

		No Test Variable	H_{it+1}	Per Capita Family Income ($10,000)
Depreciation Rate	B_0	-0.185	-0.139	-0.243
		(0.081)	(0.093)	(0.091)
	B_1	0.013	0.007	0.012
		(0.002)	(0.004)	(0.002)
	B_2	-0.0003	-0.0003	-0.0002
		(0.0001)	(0.0001)	(0.0001)
A	A_0	1	1	1
Exponents	α_1	0.034	0.035	0.016
		(0.018)	(0.018)	(0.018)
	α_2	0.514	0.452	0.582
		(0.100)	(0.140)	(0.095)
LBD Test Variables	Linear		0.0134	-0.004
			(0.0070)	(0.008)
Elasticities	H_{it}	0.043	0.051	0.024
	W_{it}	0.375	0.198	0.355
	Test Var.		0.029	-0.008
Hansen Over-Identifying Restrictions Test	P-Value	0.179	1.000	0.325
Sample Size		7477	6258	6605

Standard errors in parenthesis

Sample includes white women who have completed formal schooling. Respondents who did not attend college must be at least 18 years old. Respondents who attended college must be at least 21 years old. Wages below $1.90 and above $100 are excluded. Wage observations with wage growth below -50% and above 100% are excluded. Annual hours of work must range between 780 and 4368. Average hours worked per week of work must be between 15 and 84. Wage observations for which the previous interview took place less than 10 months or more than 14 months are excluded. Instruments include: linear and quadratic second order lagged wage, linear and quadratic logarithm of second order lagged wage, linear and quadratic second order lagged hours of work, linear and quadratic logarithm of second order lagged hours of work, second order lag of wage times hours of work, second order lag of log wage times log hours, linear and quadratic age, linear and quadratic lagged age.

Table 4a: Production Function Estimates of Model A with Test Variables, White Males, Alternate Instrumental Variables

		Model A	H_{it+1}	Per Capita Family Income ($10,000)
Depreciation Rate	B_0	1	1	0.068
				(0.036)
A	A_0	11.571	8.451	1
		(6.044)	(5.776)	
	A_1	1.779	0.790	0.138
		(0.715)	(0.568)	(0.044)
	A_2	-0.048	-0.033	-0.004
		(0.021)	(0.017)	(0.001)
Exponents	α_1	0.035	0.083	0.022
		(0.037)	(0.053)	(0.037)
	α_2	-0.004	0.025	0.030
		(0.188)	(0.216)	(0.163)
LBD Test Variables	Linear		0.007	0.001
			(0.005)	(0.002)
Elasticities	H_{it}			
	W_{it}			
	Test Var.			
Hansen Over-Identifying Restrictions Test	P-Value	0.070	0.897	0.058
Sample Size		6878	6654	6348

Standard errors in parenthesis

Sample includes white men who have completed formal schooling. Respondents who did not attend college must be at least 18 years old. Respondents who attended college must be at least 21 years old. Wages below $1.90 and above $100 are excluded. Wage observations with wage growth below -50% and above 100% are excluded. Annual hours of work must range between 780 and 4368. Average hours worked per week of work must be between 15 and 84. Wage observations for which the previous interview took place less than 10 months or more than 14 months are excluded. Instruments include: second order lagged wage, logarithm of second order lagged wage, second order lagged number of children, second order lagged interaction of number of children and age, second order lagged per capita family income, second order lagged interaction of per capital family income and age, second order lagged ratio of earners to household members, second order lagged interaction of ratio of earners to household members with age, linear and quadratic term in age, linear and quadratic term in lagged age, linear and quadratic term in first order lead in age.

Table 4b: Production Function Estimates of Model B with Test Variables, White Males, Alternate Instrumental Variables

		Model B	H_{it+1}	Per Capita Family Income ($10,000)
Depreciation Rate	B_0	0.002	1	-0.027
		(0.053)		(0.047)
	B_1	0.016	8.35E+24	0.014
		(0.006)	(2.99E+24)	(0.004)
	B_2	-0.0005	-3.24E+23	-0.0004
		(0.0002)	(3.37E-23)	(0.0002)
A	A_0	1	6.73E+26	1
			(2.18E+26)	
Exponents	α_1	0.070	0.077	0.038
		(0.049)	(0.038)	(0.042)
	α_2	0.097	0.161	0.191
		(0.194)	(0.077)	(0.159)
LBD Test Variables	Linear		0.005	-0.002
			(0.009)	(0.003)
Elasticities	H_{it}			
	W_{it}			
	Test Var.			
Hansen Over-Identifying Restrictions Test	P-Value	0.122	0.702	0.057
Sample Size		6878	6654	6348

Standard errors in parenthesis

Sample includes white men who have completed formal schooling. Respondents who did not attend college must be at least 18 years old. Respondents who attended college must be at least 21 years old. Wages below $1.90 and above $100 are excluded. Wage observations with wage growth below -50% and above 100% are excluded. Annual hours of work must range between 780 and 4368. Average hours worked per week of work must be between 15 and 84. Wage observations for which the previous interview took place less than 10 months or more than 14 months are excluded. Instruments include: second order lagged wage, logarithm of second order lagged wage, second order lagged number of children, second order lagged interaction of number of children and age, second order lagged per capita family income, second order lagged interaction of per capital family income and age, second order lagged ratio of earners to household members, second order lagged interaction of ratio of earners to household members with age, linear and quadratic term in age, linear and quadratic term in lagged age, linear and quadratic term in first order lead in age.

Table 4c: Production Function Estimates of Model A with Test Variables, White Feales, Alternate Instrumental Variables

		Model A	H_{it+1}	Per Capita Family Income ($10,000)
Depreciation Rate	B_0	0.009	-0.057	-0.118
		(0.053)	(0.078)	(0.140)
A	A_0	1	1	1
	A_1	0.075	0.079	0.049
		(0.022)	(0.034)	(0.025)
	A_2	-0.001	-0.001	-0.001
		(0.001)	(0.001)	(0.001)
Exponents	α_1	0.021	0.011	0.009
		(0.022)	(0.027)	(0.019)
	α_2	0.240	0.368	0.454
		(0.155)	(0.160)	(0.212)
LBD Test Variables	Linear		-0.005	0.003
			(0.013)	(0.004)
Elasticities	H_{it}			
	W_{it}			
	Test Var.			
Hansen Over-Identifying Restrictions Test	P-Value	0.545	0.641	0.710
Sample Size		5503	5232	5003

Standard errors in parenthesis

Sample includes white women who have completed formal schooling. Respondents who did not attend college must be at least 18 years old. Respondents who attended college must be at least 21 years old. Wages below $1.90 and above $100 are excluded. Wage observations with wage growth below -50% and above 100% are excluded. Annual hours of work must range between 780 and 4368. Average hours worked per week of work must be between 15 and 84. Wage observations for which the previous interview took place less than 10 months or more than 14 months are excluded. Instruments include: second order lagged wage, logarithm of second order lagged wage, second order lagged number of children, second order lagged interaction of number of children and age, second order lagged per capita family income, second order lagged interaction of per capital family income and age, second order lagged ratio of earners to household members, second order lagged interaction of ratio of earners to household members with age, linear and quadratic term in age, linear and quadratic term in lagged age, linear and quadratic term in first order lead in age.

Table 4d: Production Function Estimates of Model B with Test Variables, White Females, Alternate Instrumental Variables

		Model B	H_{it+1}	Per Capita Family Income ($10,000)
Depreciation Rate	B_0	-0.149	-0.280	-0.403
		(0.099)	(0.170)	(0.155)
	B_1	0.014	0.017	0.011
		(0.003)	(0.005)	(0.003)
	B_2	-0.0004	-0.0004	-0.0003
		(0.0001)	(0.0001)	(0.0001)
A	A_0	1	1	1
Exponents	α_1	0.027	0.014	0.007
		(0.023)	(0.022)	(0.011)
	α_2	0.478	0.615	0.723
		(0.143)	(0.172)	(0.116)
LBD Test Variables	Linear		-0.004	0.001
			(0.012)	(0.005)
Elasticities	H_{it}			
	W_{it}			
	Test Var.			
Hansen Over-Identifying Restrictions Test	P-Value	0.121	0.313	0.379
Sample Size		5503	5232	5003

Standard errors in parenthesis

Sample includes white women who have completed formal schooling. Respondents who did not attend college must be at least 18 years old. Respondents who attended college must be at least 21 years old. Wages below $1.90 and above $100 are excluded. Wage observations with wage growth below -50% and above 100% are excluded. Annual hours of work must range between 780 and 4368. Average hours worked per week of work must be between 15 and 84. Wage observations for which the previous interview took place less than 10 months or more than 14 months are excluded. Instruments include: second order lagged wage, logarithm of second order lagged wage, second order lagged number of children, second order lagged interaction of number of children and age, second order lagged per capita family income, second order lagged interaction of per capital family income and age, second order lagged ratio of earners to household members, second order lagged interaction of ratio of earners to household members with age, linear and quadratic term in age, linear and quadratic term in lagged age, linear and quadratic term in first order lead in age.

Table A1: Assessing Robustness of Production Function Estimates of Model A, White Males

		Model A	High School Graduates	At Least Some College	No Sample Restriction
Depreciation Rate	B_0	1	0.095	1	-0.00003
			(0.049)		(0.00002)
A	A_0	18.585	1	18.622	1
		(12.491)		(28.772)	
	A_1	1.536	0.068	3.552	0.110
		(0.901)	(0.021)	(3.678)	(0.015)
	A_2	-0.027	-0.001	-0.070	-0.002
		(0.019)	(0.001)	(0.079)	(0.001)
Exponents	α_1	0.078	0.107	0.036	0.050
		(0.028)	(0.043)	(0.037)	(0.025)
	α_2	0.093	-0.035	0.158	0.036
		(0.117)	(0.172)	(0.243)	(0.016)
Hansen Over-Identifying Restrictions Test	P-Value	0.903	0.511	0.931	0.125
Sample Size		9474	4148	3963	13399

Standard errors in parenthesis

Sample includes white men who have completed formal schooling. Respondents who did not attend college must be at least 18 years old. Respondents who attended college must be at least 21 years old. Wages below $1.90 and above $100 are excluded. Wage observations with wage growth below -50% and above 100% are excluded. Annual hours of work must range between 780 and 4368. Average hours worked per week of work must be between 15 and 84. Wage observations for which the previous interview took place less than 10 months or more than 14 months are excluded. None of these restrictions were applied to the sample used to obtain the estimates presented in the fourth column. Instruments include: linear and quadratic second order lagged wage, linear and quadratic logarithm of second order lagged wage, linear and quadratic second order lagged hours of work, linear and quadratic logarithm of second order lagged hours of work, second order lag of wage times hours of work, second order lag of log wage times log hours, linear and quadratic age, linear and quadratic lagged age.

Table A2: Assessing Robustness of Production Function Estimates of Model B, White Males

		Model B	High School Graduates	At Least Some College	No Sample Restriction
Depreciation Rate	B_0	-0.031	0.051	-0.135	1
		(0.050)	(0.089)	(0.113)	
	B_1	0.015	0.014	0.023	-0.030
		(0.003)	(0.006)	(0.007)	(0.007)
	B_2	-0.0004	-0.0003	-0.0006	0.0012
		(0.0001)	(0.0002)	(0.0002)	(0.0003)
A	A_0	1	1	1	-293.477
					(0.000)
Exponents	α_1	0.087	0.149	0.072	-62.847
		(0.034)	(0.060)	(0.048)	(0.000)
	α_2	0.212	0.100	0.370	-44.508
		(0.119)	(0.184)	(0.172)	(0.000)
Hansen Over-Identifying Restrictions Test	P-Value	1.000	0.099	0.282	0.000
Sample Size		9474	4148	3963	13399

Standard errors in parenthesis

Sample includes white men who have completed formal schooling. Respondents who did not attend college must be at least 18 years old. Respondents who attended college must be at least 21 years old. Wages below $1.90 and above $100 are excluded. Wage observations with wage growth below -50% and above 100% are excluded. Annual hours of work must range between 780 and 4368. Average hours worked per week of work must be between 15 and 84. Wage observations for which the previous interview took place less than 10 months or more than 14 months are excluded. None of these restrictions were applied to the sample used to obtain the estimates presented in the fourth column. Instruments include: linear and quadratic second order lagged wage, linear and quadratic logarithm of second order lagged wage, linear and quadratic second order lagged hours of work, linear and quadratic logarithm of second order lagged hours of work, second order lag of wage times hours of work, second order lag of log wage times log hours, linear and quadratic age, linear and quadratic lagged age.

Table A3: Assessing Robustness of Production Function Estimates of Model A, White Females

		Model A	High School Graduates	At Least Some College	No Sample Restriction
Depreciation Rate	B_0	1	1	-0.098	1
				(0.106)	
A	A_0	414.090	14.315	1	1.1E+10
		(6737.800)	(6.014)		(2.5E+09)
	A_1	27.300	1.537	0.036	6.6E+08
		(439.200)	(0.720)	(0.015)	(1.7E-08)
	A_2	-0.304	-0.023	0.00007	3.0E+06
		(4.915)	(0.017)	(0.00040)	(7.1E+06)
Exponents	α_1	0.030	0.002	0.037	0.029
		(0.019)	(0.047)	(0.023)	(0.029)
	α_2	0.198	-0.293	0.447	0.069
		(0.132)	(0.257)	(0.158)	(0.018)
Hansen Over-Identifying Restrictions Test	P-Value	0.987	0.930	0.513	0.047
Sample Size		7477	3131	3853	10777

Standard errors in parenthesis

Sample includes white women who have completed formal schooling. Respondents who did not attend college must be at least 18 years old. Respondents who attended college must be at least 21 years old. Wages below $1.90 and above $100 are excluded. Wage observations with wage growth below -50% and above 100% are excluded. Annual hours of work must range between 780 and 4368. Average hours worked per week of work must be between 15 and 84. Wage observations for which the previous interview took place less than 10 months or more than 14 months are excluded. None of these restrictions were applied to the sample used to obtain the estimates presented in the fourth column. Instruments include: linear and quadratic second order lagged wage, linear and quadratic logarithm of second order lagged wage, linear and quadratic second order lagged hours of work, linear and quadratic logarithm of second order lagged hours of work, second order lag of wage times hours of work, second order lag of log wage times log hours, linear and quadratic age, linear and quadratic lagged age.

Table A4: Assessing Robustness of Production Function Estimates of Model B, White Females

		Model B	High School Graduates	At Least Some College	No Sample Restriction
Depreciation Rate	B_0	-0.185	1	-0.280	-0.004
		(0.081)		(0.165)	(0.002)
	B_1	0.013	6.9E+38	0.011	0.001
		(0.002)	(1.9E+37)	(0.004)	(0.000)
	B_2	-0.0003	-2.8E+37	-0.0001	0.0000
		(0.0001)	(1.3E-39)	(0.0001)	(0.0000)
A	A_0	1	1.8E+10	1	1
			(2.1E+12)		
Exponents	α_1	0.034	3.428	0.041	0.073
		(0.018)	(7.818)	(0.027)	(0.033)
	α_2	0.514	13.191	0.615	0.234
		(0.100)	(16.651)	(0.148)	(0.029)
Hansen Over-Identifying Restrictions Test	P-Value	0.179	1.000	1.000	0.000
Sample Size		7477	3131	3853	10777

Standard errors in parenthesis

Sample includes white women who have completed formal schooling. Respondents who did not attend college must be at least 18 years old. Respondents who attended college must be at least 21 years old. Wages below $1.90 and above $100 are excluded. Wage observations with wage growth below -50% and above 100% are excluded. Annual hours of work must range between 780 and 4368. Average hours worked per week of work must be between 15 and 84. Wage observations for which the previous interview took place less than 10 months or more than 14 months are excluded. None of these restrictions were applied to the sample used to obtain the estimates presented in the fourth column. Instruments include: linear and quadratic second order lagged wage, linear and quadratic logarithm of second order lagged wage, linear and quadratic second order lagged hours of work, linear and quadratic logarithm of second order lagged hours of work, second order lag of wage times hours of work, second order lag of log wage times log hours, linear and quadratic age, linear and quadratic lagged age.