Testing Becker's Theory of Positive Assortative Matching*

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Abstract

In a static frictionless transferable utilities bilateral matching market with systematic and idiosyncratic payoffs, supermodularity of the match output function implies a strong form of positive assortative matching: The equilibrium matching distribution has all positive local log odd ratios or totally positive of order 2 ($TP2$). A particular form of a preference for own type implies supermodularity of the match output function. Other forms imply non-$TP2$ behavior. Local odds ratios are not informative on whether a bilateral matching market equilibrates with or without transfers. Using white married couples in their thirties from the US 2000 census, spousal educational matching obeyed $TP2$ except for less than 0.2% of marriages with extreme spousal educational disparities. Using the $TP2$ order, there were more positive assortative matching by couples living in SMSA's than those who do not; but not more positive assortative matching in 2000 than in 1970. There were increases in specific local log odds over that period.

A landmark result in the theory of bilateral matching is Becker's theory

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of perfect positive assortative matching (PAM).[1] Under perfect PAM, when agents of ability i are matched with agents of ability j on the other side of the market, other type i and type j agents should not simultaneously be matched with lower ability agents, nor simultaneously be matched with higher ability agents. In a static frictionless transferable utilities matching market where the match output function is supermodular in the agents' unidimensional abiliities, Becker showed that:

1. There is perfect PAM in equilibrium.
2. Perfect PAM is independent of the population distributions of agents on both sides of the market.
   (a) Perfect PAM continues to apply when a new type of agent is introduced into the market or when an existing type is removed.
3. Perfect PAM is independent of the subset of agent types observed by a researcher.
4. There is no restriction on the distributions of the unmatched.

Becker's insight has generated a substantial theoretical literature which seeks to both extend and qualify it.[2] Using data from marriage and labor markets, empirical testing of his model are more preliminary.[3]

There are at least two related difficulties associated with empirical testing of Becker's theory. The first is that there is an obvious alternative explanation for PAM which is a preference for own type in a match. Becker's theory has empirical content if we can distinguish supermodularity against this alternative hypothesis.

The second problem is that perfect PAM is not observed in either marriage or labor markets. High ability agents on both sides of a market will match with lower ability agents on the other side of the market in violation of perfect

[1] Becker (1973); summarized in Becker (1991). Weiss (1997) has an elementary exposition. Roth and Sotomayor (1990) surveys the theory of static matching markets.

[2] E.g. Atakan (2006); Burdett and Coles (1997); Chiappori, et. al. (2008, forthcoming); Damiano, et. al. (2005); Iyigun and Walsh (2007); Legros and Newman (2002, 2007); Lundberg and Pollak (2003); Peters and Siow (2002); Shimer and Smith (2000).

[3] E.g. Abowd, et. al. (1999); Anderson and Leo (2007); Bagger and Lentz (2008); Fernandez, et. al. (2005); Galichon and Selanie (2009); Lise, et. al. (2008); Liu and Lu (2006); Lopes de Melo (2008); Mendes, et. al. (2007); Suen and Lui (1999).

PAM. Thus an empirical test of Becker's model should avoid rejecting the model based on observing inadmissible matches.

Most empirical tests of Becker's theory do it in two steps. Given a sample of matches, they first construct an index of ability for every agent.[4] Then they compute the correlation, or related summary measure of positive association, between the abilities of match partners. A positive correlation is interpreted as favoring Becker's theory. These papers deal with important and difficult issues, such as how to generate a one dimension ability index for agents on each side of the market, adding search friction and other dynamic considerations, in order to construct a relevant correlation. A correlation test has two advantages. First, it is non-parametric. Second, it avoids rejecting Becker's model based on observing inadmissible matches. However, a positive correlation has no power against the alternative of a preference for own type.

Building on the above papers, the first part of this paper constructs a stochastic Becker model which (1) delivers stochastic analogs of all the predictions of the Becker model discussed above; (2) delineates the empirical differences between supermodularity and a preference for own type; (3) provide a test of Becker's static theory which has the same advantages as, but is statistically more powerful than a correlation test; and (4) provide a rationale for the standard practice of testing Becker's theory on a subset of agent types, including ignoring the distributions of unmatched agents.

To describe patterns in bilateral matching data, let there be I types of agents on one side of the market, $i = 1, .., I$, where type $i + 1$ agents have higher ability than type i agents. Let there be J types of agents on the other side, $j = 1, .., J$, where type $j + 1$ agents have higher ability than type j agents. Let $\mu(i, j)$ be the number of type i agents who are matched with type j agents. The $I \times J$ matrix μ with a typical element $\mu(i, j)$ is known as the equilibrium matching distribution.

The $\{i, j\}$ local log odds ratio, $\ln[\mu(i, j)\mu(i + 1, j + 1)][\mu(i + 1, j)\mu(i, j + 1)]^{-1}$, is a local measure of association in μ. There are $(I - 1)(J - 1)$ of these local log odds ratios. They and the $I + J$ numbers of agents of each type in matches, i.e. marginal distributions of μ, provide a reparametrization of μ. Thus there is no loss of information in considering local log odds ratios rather than μ.

In the statistics literature (E.g. Douglas, et. al. (1991); Shaked and Shan-

[4] Galichon and Selanie (2009) is an exception.

thikumar (2007)), a common strong measure of PAM (positive dependence) is totally positive of order 2, $TP2$. μ is $TP2$ when all local log odds ratios are positive. When the index of ability is one dimensional as in Becker's model, $TP2$ is equivalent to stochastic dominance where the distributions of i's partners' types are ordered by i and similarly for j (Shaked and Shanthikumar). $TP2$ implies other weaker forms of PAM such as positive correlation.

The stochastic Becker model retains the static frictionless transferable utilities setup of Becker. It is a special case of Choo Siow (2006; hereafter CS).[5] Like Becker, each agent of type i has a systematic payoff which depends on the type of the partner, j, in the match. Unlike Becker's deterministic model, I instead follow CS where each agent of type i also obtains an idiosyncratic payoff from an $\{i, j\}$ match which is particular to that agent. Due to the idiosyncratic payoffs, every $\{i, j\}$ match will occur with positive probability. In fact without further restrictions, the stochastic Becker model will fit any observed matching distribution.

The first result of this paper shows that each $\{i, j\}$ local log odds ratio is equal to the degree of complementarity between the two partners i and j in producing match output.[6] If the the degree of complementarity is equal to zero for all $\{i, j\}$ matches, the local log odds ratios are all equal to zero, which imply that the equilibrium distribution of i's partners' types is independent of i and similarly for j.

μ is $TP2$ if and only if the match output function is supermodular. Like the correlation test, the $TP2$ test is non-parametric and also avoids rejecting the model based on inadmissible matches. Unlike the correlation test, the $TP2$ test is tight. Supermodularity of the match output function implies that μ is $TP2$ and vice versa. Furthermore, similar to Becker,

1. $TP2$ is independent of the distributions of the population vectors.

    (a) $TP2$ continues to apply when a new type of individual is introduced into the market or when an existing type is removed.

[5] Other applications include Brandt, et. al. (2008); Botticini and Siow (2008); Chiappori, Selanie and Weiss (in process); Choo, Seitz and Siow (CSSa, CSSb). Siow (2008) is a survey.

[6] The stochastic Becker model provides the behavioral foundation for Fox 2009 assumption of this result in his one to one matching, unidimensional ability case. Fox shows that such assumptions are useful in cases outside the Becker environment for learning about properties of the matching output function.

2. $TP2$ is independent of the subset of the agent types observed by the researcher.

3. There is no restriction on the unmatched.

What is the relationship between supermodularity of the marital output function and preference for own type? To define an own type, let agents on each side of the market share the same ability index. So if $i = j$, the agent of type i has the same ability as the agent of type j on the other side of the market. Define a preference for own type as a match output function with a penalty $d(i, j)$ which is increasing in the difference between i and j. Supermodularity of the marital output function is a special case of $d(i, j)$. If $d(i, j)$ is supermodular, the match output function is supermodular and μ is also $TP2$. So $TP2$ cannot be used to distinguish between a preference for own type and supermodularity alone. On the other hand, the general penalty function approach also allows zero and or negative off diagonal local log odds which is not $TP2$.

Although this paper derives the behavior of local log odds ratios from a transferable utilities model, the same local log odds ratios behavior can be derived from Dagsvik (2000) non-transferable utilities model. So local log odds ratios of a bilateral matching distribution is not informative on whether such a market equilibrates with or without transfers. As will be shown below, other implications can be used to empirically distinguish between these two classes of models.

$TP2$ is also used to stochastically order matching distributions by their degree of PAM (E.g. Shaked and Shanthikumar (2007)). Consider two ordered bilateral matching distributions μ_1 and μ_2, each of which has the same types of agents. μ_1 exhibits more positive dependence than μ_2 if the difference between the two distributions is $TP2$.[7]

There is a large empirical literature which estimates log linear models of positive assortative marriage matching.[8] A saturated log linear model is equivalent to the unrestricted log odds model. $TP2$ are inequality restrictions on the log linear model. The empirical literature generally estimate unsaturated log linear models. This paper provides a formal behavioral interpretation for their findings.

[7] Anderson and Leo (2007) provide an alternative stochastic ordering by measuring how far each matching distribution is from perfect assortative matching.

[8] E.g. Mare (1991); Qian (1998); Schwartz and Mare (1995). See Agresti (2002); Goodman (1972) for log linear models.

A caution is necessary. This paper shows that one can learn some properties of the match output function from studying the local log odds of μ. What can be learned is limited. As is known from CS, the entire match output function cannot be estimated from equilibrium matching data alone.

Following CS, I have modelled the idiosyncratic payoffs to spousal choice as identically and independently distributed (IID) Type I extreme value distributions. Assuming transferable utilities and competitive matching, Becker's theory is about how the structure of match output completely determines the equilibrium matching pattern. So when I add idiosyncratic payoffs, the distributions of these idiosyncratic payoffs should not systematically affect the matching pattern. The IID assumption forces any systematic matching pattern to come only from the structure of match output function. As shown by McFadden and CS, the Type I extreme value distribution generates very convenient functional forms which I exploit here.

The Type I extreme value distribution is the basic discrete choice model used by the economic profession. The main criticism against it is its independence of irrelevant alternative (IIA) property exemplified by McFadden's blue bus red bus discussion. This criticism does not apply if agents have unidimensional abilities. In his discussion, if researchers treat blue buses as a different category from red buses, they may make the wrong prediction about transportation choices if blue buses were removed from the choice set. If types are unidimensional as is the case here, this problem cannot arise. All buses are the same, independent of their color.

In fact, as will be discussed in the text, the IIA property is an important property of the stochastic Becker model. Unlike the multidimensional case, it imposes sensible behavioral and convenient empirical restrictions on μ.

The second part of this paper uses data from the United States 2000 census to test Becker's theory of positive assortative marriage matching by spousal educational attainment. The samples are restricted to white couples, females between 31-35 and males between 32-36.

The largest sample is the national sample with over 120,000 marriages. $TP2$ is rejected at the 0.001 significance level. The rejection of $TP2$ is localized. Ignoring the two local log odds ratios with the most dissimilar educational matches (husband with less than high school and wife with more than bachelor's degree, and vice versa), the hypothesis that all other local log odds are all positive cannot be rejected. Since there were few marriages at these extreme dissimilar educational matches, $TP2$ is not rejected except for 0.2 percent of marriages. This finding shows the value of looking at unrestricted local log odds ratios to describe association patterns in μ. One can pin down where the departures from $TP2$, or other models of association lie.

The rejection of $TP2$ for the national sample may due to inappropriate aggregation. I divide the national sample into two subsamples, a SMSA sample (where the couple resides in an SMSA) versus a non-SMSA sample. $TP2$ is rejected for both subsamples.

Recent researchers have argued that cities facilitate PAM in marriage relative to non-cities.[9] To investigate this hypothesis, I ask whether the SMSA subsample exhibit more positive dependence than the non-SMSA subsample in the $TP2$ order. Although each subsample do not satisfy $TP2$, the difference between them is $TP2$. Thus this paper provides evidence that there is more marital sorting by spousal educational attainment in cities.

Many observers have argued that PAM by educational attainment in the US has grown in the second half of the twentieth century.[10] Using the $TP2$ order, I find no evidence for a general increase in PAM by educational attainment between 1970 and 2000.[11] There were substantial increases in local log odds between 1970 and 2000 along the diagonal of μ.

Summarizing, this paper makes the following contributions. First, it develops an elementary empirical static stochastic Becker model which delivers stochastic analogs of all of Becker's predictions. It rationalizes standard practice in the empirical literature of testing Becker's model with a subset of agent types and also ignoring the distributions of the umatched. Second, the empirical framework delineates what can be distinguish between Becker's theory of PAM and a preference for own type marital output function. Third, the model provides an exact behavioral interpretation of local odds ratio, log linear models, and the $TP2$ order, a common statistical measure for positive dependence in bivariate matching distributions. Using geometric programming, these models are easy to estimate. Fourth, the local log odds and the $TP2$ order provide new insights on some well known findings on marriage matching in spousal educational attainment in the US. Finally, the estimates of the stochastic Becker model shows that his insight rationalizes spousal

[9] Costa and Kahn (2000); Compton and Pollak (2007); Edlund (2005); Gautier, Svarer, and Teulings (2005).

[10] E.g. Fernández et. al. (2005); Liu and Lu (2006); Schwartz and Mare (2005); Mare (1991); Qian and Preston (1993); Qian (1998).

[11] A more comprehensive study which reaches the same conclusion is Chiappori, Selanie and Weiss (in process).

educational matching patterns in the US.

1 The Stochastic Becker Model

Men and women are differentiated by ability. There are I types of men, $i = 1, .., I$. The ability of type $i+1$ men are higher than the ability of type i men. There are J types of women, $j = 1, .., J$. The ability of type $j+1$ women are higher than the ability of type j women. Unless stated otherwise, the ability rankings are ordinal. The ordering of individuals by ability types is what differentiates this model from CS.

M is a population vector where element m_i is the number of eligible (single) men of ability i. F is a population vector where element f_j is the number of eligible (single) women of ability j.

Each marital match between two different ability types of individuals constitute a distinct sub-marriage market. With I ability types of men and J ability types of women, there are $I \times J$ sub-marriage markets.

In an $\{i, j\}$ marriage, $\Pi(i, j)$ marital output is generated. Following Becker, let the marital output function satisfy:

Assumption 1 $\Pi(i, j)$ *is supermodular.*

where

Definition 1 *For $i < I$ and $j < J$, a function $\Phi(i, j)$ is supermodular if*[12]*:*

$$\Phi(i+1, j+1) + \Phi(i, j) \geq \Phi(i+1, j) + \Phi(i, j+1)$$

Assumption 1 says that the sum of the marital outputs from closest ability matching is higher than the sum of marital outputs from mixed ability matching for all $\{i, j\}$.

Definition 2 *For $i < I$ and $j < J$, a function $\Phi(i, j)$ is submodular if:*

$$\Phi(i+1, j+1) + \Phi(i, j) < \Phi(i+1, j) + \Phi(i, j+1)$$

[12] Apply induction to get the standard condition for supermodularity, $\Phi(i+k, j+h) + \Phi(i, j) \geq \Phi(i+k, j) + \Phi(i, j+h)$, $k, h > 0$.

The marital output, $\Pi(i,j)$, is divided between the two spouses. Let $\widetilde{\tau}(i,j)$ be the share of the marital output that is obtained by a type j wife. Each wife also gets an idiosyncratic payoff from marriage which depends on her specific identity, the type of spouse that she marries and not his specific identity. Her idiosyncratic payoff also does not depend on $\widetilde{\tau}(i,j)$.

In an $\{i,j\}$ marriage, $\Pi(i,j)-\widetilde{\tau}(i,j)$ is the share of marital output that is obtained by a type i husband. Each husband also gets an idiosyncratic payoff that is specific to him, the type of spouse that he marries and not her specific identity. His idiosyncratic payoff also does not depend on $\Pi(i,j)-\widetilde{\tau}(i,j)$.

The above assumptions imply that every type i male regards every type j female as perfect spousal substitutes and vice versa.

Each individual also gets a systematic payoff from remaining unmarried which depends on their type as well as an idiosyncratic payoff which depends on their specific identity.

Given their payoffs, both systematic and idiosyncratic, from every potential spousal choice including remaining unmarried, each individual will choose the spousal choice which maximizes their utility.

Given $\widetilde{\tau}(i,j)$, we can solve each individual's spousal choice problem. We can aggregate these individual decisions into demand and supply functions for spouses in every $\{i,j\}$ submarriage market.

Finally, we solve for the matrix of $\widetilde{\tau}(i,j)$ which will equilibrate demand with supply in every submarriage market simultaneously.

The equilibrium distribution of marriages is a function of population vectors and exogenous parameters which determine the systematic and idiosyncratic payoffs. The objective of this paper is to study the conditions for equilibrium PAM in this society.

Following the additive random utility model, let the utility of male g of ability i who marries a female of ability j be:

$$v_{ijg}=\Pi(i,j)-\widetilde{\tau}(i,j)+\varepsilon_{ijg} \tag{1}$$

As discussed above, $\Pi(i,j)-\widetilde{\tau}(i,j)$ is the systematic marital share of the husband. ε_{ijg} is his idiosyncratic payoff. The addition of an idiosyncratic payoff will make different men of type i make different choices. So we will not get the perfect assortative matching result of Becker. At the same time, we do not want the distributions of the idiosyncratic payoffs to systematically affect marriage matching patterns. To respect Becker's theory, the distribution of ε_{ijg} should be the same for all i, j and g. I will assume that ε_{ijg} is an IID type I extreme value random variable. As shown by McFadden, the type I extreme value distribution has analytically convenient properties which I will exploit.

If he chooses to remain unmarried, denoted by $j = 0$, his utility will be:

$$v_{i0g} = \Pi(i, 0) + \varepsilon_{i0g} \tag{2}$$

where ε_{i0g} is also an idiosyncratic payoff which is another IID extreme value random variable.

This man g can choose to marry one of J ability types of spouses or not to marry. The utility from his optimal choice will satisfy:

$$v_{ig} = \max_j[v_{i0g}, .., v_{ijg}, .., v_{iJg}] \tag{3}$$

Let $\underline{\mu}(i, j)$ be the number of men of ability i who want to marry women of ability j. $\underline{\mu}(i, 0)$ is the number of type i men who want to remain unmarried. When there are many type i males, McFadden (1974) showed that ability type i's quasi-demand for ability type j spouses satisfy:

$$\ln \frac{\underline{\mu}(i, j)}{\underline{\mu}(i, 0)} = \Pi(i, j) - \widetilde{\tau}(i, j) - \Pi(i, 0) \tag{4}$$

Turning to the marital choices of women, let the utility of female k of ability j who marries a male of ability i be:

$$V_{ijk} = \widetilde{\tau}(i, j) + \epsilon_{ijk} \tag{5}$$

As discussed above, $\widetilde{\tau}(i, j)$ is the systematic marital share of the wife. ϵ_{ijk} is her idiosyncratic payoff. Assume that ϵ_{ijk} is an IID extreme value random variable.

If she chooses to remain unmarried, denoted by $i = 0$, her utility will be:

$$V_{0jk} = \Pi(0, j) + \epsilon_{0jk} \tag{6}$$

where ϵ_{0jk} is also an idiosyncratic payoff which is another IID extreme value random variable.

Note that I assume that both men and women draw idiosyncratic payoffs for every choice from the same IID distribution. Becker's theory is not gender specific and so there is no reason to introduce any gender specific differences here.

This woman k can choose to marry one of I types of spouses or not to marry. The utility from her optimal choice will satisfy:

$$V_{jk} = \max_j [V_{0jk}, .., V_{ijk}, .., V_{Ijk}] \tag{7}$$

Let $\overline{\mu}(i,j)$ be the number of women of ability j who want to marry men of ability i. $\overline{\mu}(0,j)$ is the number of women of ability j who wants to remain unmarried. When there are many ability type j females, type j's quasi-supply for i spouses satisfy:

$$\ln \frac{\overline{\mu}(i,j)}{\overline{\mu}(0,j)} = \widetilde{\tau}(i,j) - \Pi(0,j) \tag{8}$$

For every $I \times J$ sub-marriage market, let $\widetilde{\tau}(i,j) = \tau(i,j)$ be the female equilibrium share of marital output in the $\{i,j\}$ sub-marriage market which equilibrates the demand and supply of spouses in all sub-markets simultaneously. In this case, the equilibrium number of $\{i,j\}$ marriages, $\mu(i,j)$, will satisfy:

$$\mu(i,j) = \underline{\mu}(i,j) = \overline{\mu}(i,j) \; \forall \; i,j \tag{9}$$

Imposing marriage market clearing, (9), to the quasi-demand equation, (4), and to the quasi-supply equation, (8), we get the male and female net gains equations respectively:

$$\ln \frac{\mu(i,j)}{\mu(i,0)} = \Pi(i,j) - \tau(i,j) - \Pi(i,0) \tag{10}$$

$$\ln \frac{\mu(i,j)}{\mu(0,j)} = \tau(i,j) - \Pi(0,j) \tag{11}$$

Add the two net gains equations to get the CS marriage matching function (MMF):

$$\ln \frac{\mu(i,j)}{\sqrt{\mu(i,0)\mu(0,j)}} = \frac{\Pi(i,j) - \Pi(i,0) - \Pi(0,j)}{2} \; \forall \; i,j \tag{12}$$

CS calls the left hand side of (12) the total gains to marriage. It is equal to the log ratio of the number of marriages to the geometric average of the unmarrieds. The right hand side is equal to the systematic marital output of an $\{i,j\}$ marriage minus their systematic surpluses from not marrying.

In a more general model, CSSa shows existence of marriage market equilibrium. So there exists an $I \times J$ marriage matching distribution, μ, with typical element $\mu(i,j)$, which satisfies (12).

2 Positive assortative matching

Without imposing structure on marital output, the CS model, and by implication also the stochastic Becker model, fits any equilibrium marriage matching distribution which does not have thin cells. This section provides definitions of PAM and relate them to restrictions on marital output.

Given two men of adjacent abilities, i and $i+1$, and two women of adjacent abilities, j and $j+1$, define marital matching by closest abilities as the marital matches $\{i, j\}$ and $\{i+1, j+1\}$. Define marital matching by mixed abilities as the marital matches $\{i, j+1\}$ and $\{i+1, j\}$.

Given a marriage distribution μ, a measure of association in matching is based on local log odds ratios:

Definition 3 *The local log odds ratio for an* $\{i, j\}$ *match is:*

$$l(i,j) = \ln\left[\frac{\mu(i+1,j+1)\mu(i,j)}{\mu(i,j+1)\mu(i+1,j)}\right];\ i < I, j < J$$

Altogether there are $(I-1)\times(J-1)$ local log odds ratios (log odds from hereon). The $(I-1)\times(J-1)$ log odds and the $I+J$ number of married individuals of each type are a reparmetrization of μ.

Consider all the men of adjacent abilities and women of adjacent abilities who form the log odds. There are $\widehat{m}_{i+1} = \mu(i+1, j+1) + \mu(i+1, j)$ high ability men, $\widehat{f}_{j+1} = \mu(i, j+1) + \mu(i, j)$ low ability men, $\widehat{f}_{j+1} = \mu(i+1, j+1) + \mu(i, j+1)$ high ability women, and $\widehat{f}_j = \mu(i+1, j) + \mu(i, j)$ low ability women. If there is random matching between all these men and women, using Definition 3, the log odds is:

$$\ln\left[\widehat{m}_{i+1}\frac{\widehat{f}_{j+1}}{\widehat{f}_{j+1}+\widehat{f}_j}\right]\left[\widehat{m}_i\frac{\widehat{f}_j}{\widehat{f}_{j+1}+\widehat{f}_j}\right] - \ln\left[\widehat{m}_i\frac{\widehat{f}_{j+1}}{\widehat{f}_{j+1}+\widehat{f}_j}\right]\left[\widehat{m}_{i+1}\frac{\widehat{f}_j}{\widehat{f}_{j+1}+\widehat{f}_j}\right] = 0$$

When a log odds is equal to zero, there is no local association in marital matching. Equivalently, there is local independence in marital matching. When all log odds are equal to zero, there is random marriage matching (Agresti (2002)).

If a log odds is larger than zero, there is local PAM. There are more closest abilities marital matching relative to mixed abilities matching than can be predicted by random matching.

Given a marriage distribution μ, a strong definition of positive assortative spousal matching requires that all the log odds of μ are larger than zero. In this case:

Definition 4 *μ is totally positive of order 2 (TP2) if*

$$l(i,j) \geq 0 \; \forall \; i < I, j < J \tag{13}$$

Equivalently,

Definition 5 *μ is $TP2$ if $\ln \mu$ is supermodular.*

When ability is unidimensional as is assumed here, $TP2$ is equivalent to stochastic dominance where the distributions of i's partners' types are ordered by i and similarly for j.

TP2 is a uniform notion of PAM (Douglas, et. al. (1991); Karlin and Rinott (1980)). Consider a less uniform notion of PAM. If i and j are comparable, and $I = J$,

Definition 6 *The $I \times I$ matrix μ has diagonal positive of order 2 (DP2) if*

$$l(i,i) \geq 0 \; \forall \; i < I$$

DP2 is weaker than *TP2.* $TP2$ also implies a positive Spearman correlation in spousal abilities (Nelson (1992); Yanagimoto and Okamoto (1969)).

I will now relate the definitions of PAM to the stochastic Becker model. Using the definition of total gains in (12),

Proposition 1 *The log odds of μ measures the degree of complementarity of the marital output function at $\{i,j\}$:*

$$\ln \frac{\mu(i+1,j+1)\mu(i,j)}{\mu(i,j+1)\mu(i+1,j)} = \Pi(i+1,j+1) + \Pi(i,j) - [\Pi(i,j+1) + \Pi(i+1,j)] \tag{14}$$

The log odds are observable. It is equal to the sum of the marital outputs from closest abilities matching minus the sum of the marital outputs from mixed abilities matching. Proposition 1 says that the log odds, i.e. local marriage matching behavior, measures the degree of complementarity of the marital output function at $\{i, j\}$.

The degree of complementarity of the marital output function is "technologically" determined. It is independent of the population vectors M and F.

Proposition 1 provides the microfoundation of equation (14) which Fox 2009 assumes in his paper on identifying properties of the marriage matching function for unidimensional ability.

Since the log odds and the marginal distributions of married men and women by types are a reparametrization of μ without any loss of information,

Proposition 2 *The degree of complementarity is all that can be learned from μ about the marital output function.*

Researchers can learn about other attributes of the marital output function from μ only by adding further identifying assumptions.

Corollary 1 *If the degree of complementarity is zero for all $\{i, j\}$ then the log odds of μ are all equal to zero.*

If the degree of complementarity of the marital output function is zero, then the equilibrium distribution of marital partners for men is independent of their type, and similarly for women. This result shows that adding idiosyncratic payoffs to spousal choices as is done here do not by itself create any systematic marriage matching pattern.

Corollary 2 (IIA[1]) *If a new type of individuals is introduced into the marriage market, or an existing type is removed from the marriage market, the log odds of the other types are unaffected.*

The above corollary is the IIA property of the stochastic Becker model. It is also easy to understand. Because the log odds measure a property of the marital output function, as long as the introduction of a new type or removal of an existing type from the marriage market does not change the marital output function, the log odds of the other types in the market are unaffected.

As discussed in the introduction, the usual criticism about the IIA property of logit related models do not apply here where ability is unidimensional. The blue bus red bus critique is based on the observation that researchers may inappropriately treat the color of a bus as a salient characteristic with which consumers use to differentiate between buses. However because the ability of an individual is unidimensional in this paper, this problem does not arise. Every individual with the same ability is the same type.

Corollary 3 (IIA^2) *Censoring $\{i', j'\}$ log odds do not affect $\{i, j\}$ log odds for $i' \neq i$ and $j' \neq j$.*

From an empirical point of view, the above corollary is convenient. All empirical research, including this paper, test Becker's model using a subset of agent types. This corollary shows that such tests are valid.

It is now easy to connect the stochastic Becker model and $TP2$. Using Assumption 1 and proposition 1:

Proposition 3 *The marriage distribution μ is TP2 if and only if the marital output function is supermodular.*

The above proposition says that the $TP2$ test is the strongest test there is for the marital output function to be supermodular.

The $TP2$ test is non-parametric. It does not impose any parametric restriction on μ.

Like Becker's perfect PAM being independent of the population vectors M and F, μ being $TP2$ is also independent of the population vectors M and F.

Corollary 4 (IIA^3) *When the marital output function is supermodular, μ will continue to be $TP2$ after introducing a new type and/or removing an existing type from the marriage market.*

Petrin 2002 have shown that with multidimensional attributes, the logit IID assumption may lead to implausibly large estimated welfare gains for consumers from the introduction of a new type of good. The above corollary shows that this is not the case here. Because Π is supermodular, the degree of complementary involving the new type also has to respect supermodularity. It is be bounded above and below if the new type is ranked between existing types. In particular, μ remains $TP2$ after the introduction of a new type or removal of an existing type. Thus the IIA property in the stochastic Becker model imposes sensible behavioral restrictions on μ.

Becker's perfect PAM also survives the introduction and/or removal of an existing type from the marriage market.

The total gains to remaining unmarried, $\Pi(i,0)$ and $\Pi(0,j)$, do not affect the $TP2$ outcome. So similar to Becker, supermodularity of $\Pi(i,j)$ does not pin down the distributions of the unmarrieds.

A comparison of Becker and the stochastic Becker model is displayed in Table 1.

If a marriage distribution is not $TP2$, Proposition 1 shows that the local log odds provide information on where the specific departures from $TP2$ are located. As will be seen in the next section, there is a simple behavioral interpretation for these departures.

A caution to the casual reader: This and the next section shows that one can learn some properties of the marital output function, $\Pi(i,j)$, from studying the local log odds of μ. What can be learned is limited. As is known from CS, the entire marital output function, $\Pi(i,j)$, is not identified from marriage matching data alone.

3 Preference for Own Type

The most common explanation for PAM by spousal characteristics is that marital output is higher if spouses are more similar.

This section investigates what a preference for own type in producing marital output means for log odds and whether supermodularity of the marital output function can be distinguished from a preference for own type.

To define own types, let

Assumption 2 *Men and women share the same ability index.*

So if $i=j$, type i men has the same ability as type j women.

Let marital output be:

$$\Pi(i,j) = h(i) + k(j) - d(i,j) \tag{15}$$

$h(.)$ are $k(.)$ are bounded functions. $d(i,j)$ is a penalty function between i and j.

Assumption 3 $d(i,i)=0$, $d(i+1,i)-d(i,i)>0$ *and* $d(i,i+1)-d(i,i)>0$, $d(i+1,j)-d(i,j)>0$ *and* $d(i,j+1)-d(i,j)<0$ *if* $i>j$, $d(i,j+1)-d(i,j)>0$ *and* $d(i+1,j)-d(i,j)<0$ *if* $j>i$.

Table 1: Two marriage matching models

	Becker	Stochastic Becker
Assumptions		
Individuals ordered by single index	Yes	Yes
Static, frictionless marriage market	Yes	Yes
Π	Supermodular	Supermodular
Payoff of male g of type i in $\{i, j\}$	$\Pi(i, j) - \tau(i, j)$	$\Pi(i, j) - \tau(i, j) + \varepsilon_{ijg}$
Payoff of female k of type j in $\{i, j\}$	$\tau(i, j)$	$\tau(i, j) + \xi_{ijk}$
Given τ, choose spousal type	Yes	Yes
τ clears market	Yes	Yes
Results		
μ matching pattern	Perfect PAM	$TP2$
Restrict unmarrieds	None	None
Observe subset of agent types	Perfect PAM	$TP2$
Restrict M and F	None	None
Add or subtract a type	Perfect PAM	$TP2$

The penalty is zero if $i = j$. It increases as i differs more from j. The penalty need not be symmetric in i and j.

For $i < I$ and $j < J$, the local log odds is:

$$
\begin{aligned}
l(i,j) &= \Pi(i+1,j+1) + \Pi(i,j) - (\Pi(i+1,j) + \Pi(i,j+1)) && (16)\\
&= -[d(i+1,j+1) + d(i,j) - d(i+1,j) - d(i,j+1)] && (17)
\end{aligned}
$$

Assumption 3 and equation (17) imply that:

Proposition 4 *A preference for own type marital output function, modelled as a penalty function, implies that* $l(i,i) > 0$.

The above proposition says that, without further restrictions, the preference for own type marital output function only implies that the log odds along the main diagonal are all positive. There is no restriction on the off diagonal terms.

Corollary 5 *For* $i \neq j$*, if* $d(i,j)$ *is supermodular, all log odds are positive.*

If $d(i,j)$ is supermodular, the marital output function is supermodular and it will generate μ which is $TP2$. So $TP2$ cannot be used to differentiate between a supermodular marital output function versus a preference for own type with a supermodular penalty function for marital output.

On the other hand, proposition 4, equations (16) and (17) lead to

Corollary 6 *For* $i \neq j$*, if* $d(i,j)$ *is submodular, the log odds are strictly positive along the main diagonal and less than zero elsewhere.*

All off diagonal log odds must be negative. I.e. off the main diagonal, there is local negative assortative matching even though there is a preference for own type! Call this the $DPNE$ model, positive main diagonal and negative elsewhere log odds.

Corollary 7 *If* $d(i.j)$ *is separable in* i *and* j*, the log odds are strictly positive along the main diagonal and zero elsewhere.*

Call this the $DP0E$ model. $DP0E$ implies that there is random matching off the main diagonal.

One can generate both positive and negative off main diagonal log odds by using combinations of supermodular and submodular penalty functions for marital output.

As a special case, let i and j be cardinal and consider the distance penalty function $d(i,j) = D(i-j)$. If $D(.)$ is convex, $d(i,j)$ is supermodular and μ is $TP2$. If $D(.)$ is concave, $d(i,j)$ is submodular for $i \neq j$, and μ is $DPNE$. If $D(.)$ is linear, μ is $DP0E$. Finally, $D(.)$ and equations (16) and (17) also imply that all log odds only depend on $i-j$. So along any diagonal, all log odds are the same. There is nothing unusual about concave distance penalty functions. In other words, negative log odds should not be unexpected when there is a preference for own type.

4 Non-transferable utilities

Thus far, the restrictions on the log odds have been developed under the assumption of transferable utilities. The main alternative static model of the marriage market is the non-transferable utilities model (See Roth and Sotomayor (1990)).

Dagsvik (2000) proposed a static frictionless non-transferable utilities model of the marriage market where individuals also have additive random utility preferences over spouses. Each idiosyncratic payoff is also drawn from a Type I extreme value distribution. In addition to the non-transferable utilities assumption, the main departure from CS is that Dagsvik assumes that, conditioning on the type of the potential spouse, an individual's idiosyncratic payoff from a particular potential spouse depends on his and her specific identities. Instead, CS assumes that an individual is indifferent between all potential spouses of the same type.

Using the notation here, Dagsvik derives his MMF[13]:

$$\ln \frac{\mu(i,j)}{\mu(i,0)\mu(0,j)} = \frac{\Pi(i,j) - \Pi(i,0) - \Pi(0,j)}{2} \; \forall \, i,j \tag{18}$$

The only difference between Dagsvik's MMF in (18) and CS in (12) is the absence of the square root in the left hand side of (18). Specializing to the case of ability types considered in this paper, this difference does

[13] He uses Gale and Shapley's 1962 classic deferred acceptance algorithm to construct an equilibrium.

not affect the log odds computed from Dagsvik model versus the stochastic Becker model. Straightforward substitution shows that the Dagsvik MMF (18) generates the same log odds ratios as Proposition 1.

As discussed in CS, Dagsvik MMF obeys increasing returns to scale whereas CS has constant returns to scale. Using data from three different marriage markets, Botticini and Siow (2008) shows that constant returns to scale is a much better description of the data than increasing returns.

There are three lessons from this discussion. First, a transferable utilities model of the marriage market is not distinguishable from a non-transferable utilities model based on log odds of the equilibrium marriage matching distribution, μ. Second, independent of whether the marriage market clears with transfers or without, the log odds are informative about supermodularity or preference for own type penalty function of the marital output function. Third, implications other than log odds of μ can empirically distinguish between transferable utilities versus nontransferable utilities models of the marriage market.

5 Empirical Methodology

This section is known in the statistics literature and is included here for convenience. The empirical methodology is based on estimating log odds of the marriage matching distribution μ. Different models of marriage matching imply different inequality restrictions on these odds ratios. I will use maximum likelihood to estimate these models.

Consider the maximum likelihood estimation model k where μ is assumed to be $TP2$. Let there be a random sample of marriages of sample size N. Each marriage (observation) is assumed to follow the multinomial distribution. Let the expected number of observations in the $\{i, j\}$ cell be $\mu_{ij} > 0$. $\sum_{ij} \mu_{ij} = N$. The probability that a randomly chosen observation falls in the $\{i, j\}$ cell is $p_{ij} = \frac{\mu_{ij}}{N}$.

Let the observed number of marriages (observations) in the $\{i, j\}$ cell be n_{ij}.

Let L be the kernel of the log likelihood function. To find the maximum likelihood estimates of μ subject to $TP2$, I want to solve:

$$L_k \propto \max_{\mu_{ij}} L = \sum_{ij} n_{ij} \ln \mu_{ij} \tag{19}$$

subject to the $(I-1)(J-1)$ log odds constraints:

$$\ln \mu_{ij} + \ln \mu_{i+1,j+1} - \ln \mu_{i,j+1} - \ln \mu_{i+1,j} \geq 0 \tag{20}$$

and

$$N - \sum_{ij} \mu_{ij} = 0 \tag{21}$$

I solve the above problem by rewriting it as a geometric programming problem which is computationally easy to solve (Boyd, et. al. (2007); Lim, et. al. (2008)):[14]

$$\mu_{ij} = -\arg\min \sum_{ij} n_{ij} \ln \mu_{ij} \tag{22}$$

subject to the $(I-1)(J-1)$ log odds constraints:

$$-\ln \mu_{ij} - \ln \mu_{i+1,j+1} + \ln \mu_{i,j+1} + \ln \mu_{i+1,j} \leq 0 \tag{23}$$

and

$$\ln \left[\sum_{ij} \frac{\mu_{ij}}{N} \right] \leq 0 \tag{24}$$

The solution to (22), (23) and (24) will impose (24) as an equality constraint, otherwise the objective function in (22) would not have been minimized.

To test between the unrestricted model a and restricted model b where b is nested in a, I will compute the log likelihood ratio test (LR) statistic:

$$LR = 2(L_a - L_b)$$

where L_a and L_b are the values of the maximized kernels of the log likelihoods for models a and b respectively. Under the null, the distribution of LR which is a Chi-bar squared distribution, does not have a closed form solution. The p-value for the test statistic will be obtained by parametric bootstrap (1000 replications).[15] I will also provide bootstrap standard errors for the estimated log odds.

[14] Open source MATLAB code to solve geometric programming problems, CVX, is available at `http://www.stanford.edu/~boyd/cvx/`.

[15] Wang (1996) shows consistency of these parametric bootstrapping tests of stochastic ordering. An alternative is to use the chi-bar squared statistic (E.g. Anderson and Leo (2007), Wolak (1991)). See Garre, et. al. (2002) for an exposition of these two alternatives forms of likelihood ratio tests for the class of models considered here.

In large samples, the power of LR, any restrictive model b against the unrestricted alternative a, approaches one. I consider another test statistic, MRE, mean relative error which is not sensitive to sample size:

$$MRE = \frac{1}{IJ} \sum_{ij} \frac{|\mu_b(i,j) - \mu(i,j)|}{\mu(i,j)}$$

MRE has a value of zero if model b fits the data perfectly. With MRE, each cell gets equal weight, independent of the number of observations in a cell. Due to sampling error, thin cells will have more weight and thus MRE will be more sensitive to departures of the model from the data in thin cells.

5.1 2 samples test of stochastic ordering

Let μ^1 and μ^2 be two equilibrium matching distributions with the same types of participants, and with matrices of log odds l^1 and l^2 respectively. $l^1 - l^2 \geq 0$, i.e. the difference in local log odds is $TP2$, is a measure of whether μ^1 exhibits stronger PAM than μ^2.

Let the null hypothesis be the restricted model: $l^1 - l^2 \geq 0$. The alternative hypothesis is the unrestricted model: $l^1 \lesseqgtr -l^2$. Dykstra, et. al. (1995) shows how a likelihood ratio test can be used to test between these two hypotheses.

Let the sample size from the first and second distribution be N^1 and N^2 respectively. Let the observed numbers of marriages in the $\{i,j\}$ cell be n_{ij}^1 and n_{ij}^2 for the first and second sample respectively.

The restricted model can be estimated by solving:

$$L_z \propto \max_{\mu_{ij}^1, \mu_{ij}^2} \sum_{ij} n_{ij}^1 \ln \mu_{ij}^1 + \sum_{ij} n_{ij}^2 \ln \mu_{ij}^2 \tag{25}$$

subject to the $(I-1)(J-1)$ differences in log odds constraints:

$$l^1 - l^2 \geq 0 \tag{26}$$

and

$$N^k - \sum_{ij} \mu_{ij}^k = 0; \; k = 1,2 \tag{27}$$

The unrestricted model is estimated by estimating the restricted model without imposing the differences in local log odds constraints (26). I will do a likelihood ratio test between the two models and provide parametric bootstrap p-values for the test statistic (1000 replications).

6 Data and Empirical Results

The main data set is the 5% sample of the 2000 US census from IPUMS. Details on variable definitions and extraction are in the appendix. I consider white married couples, husbands and wives between ages 32-36 and 31-35 respectively. Educational attainment is divided into five categories: Less than high school (LHS), high school (HS), less than a bachelor's degree (LBA), bachelor's degree (BA) and more than a bachelor's degree (GBA). These five categories were chosen to provide roughly uniform marginal distributions and to be consistent with existing empirical work.

6.1 National sample

Table 1.a presents the national marriage matching distribution by educational attainment. There are 121418 marriages. About 10% of men and women have less than high school or more than a bachelor's degree. High school graduates, less than a bachelor's degree and a bachelor's degree occupy approximately 20% each. There are marriages for every feasible marital match. Perfect assortative matching is rejected.

Table 2.a shows the estimates of the unrestricted cell probabilities. In general, the estimated unrestricted cell probabilities are largest along the diagonal and the probabilities decline as the cells move away from the diagonal.

Table 2.b shows the estimated log odds of the unrestricted model. The estimated diagonal log odds are all larger than one with small standard errors. There are four off diagonal negative log odds, with two being larger in absolute value than twice their standard errors. These negative log odds suggest that $TP2$ may not describe the data.

While I have not done so, it is easy to reject the hypothesis of uniform log odds along every diagonal. Since the educational rankings are ordinal, such a test has no behavioral significance.

Each row of Table 3.a presents different statistics assessing the fit of a particular model. The name of the model is in Column (Model). Column (LL) presents the value of the estimated kernel of the log likelihood of that model. Column (LR) presents the likelihood ratio statistic for the model versus the unrestricted model. Column (p-value) presents bootstrap p-values for the LR test. MRE presents the mean relative error and Spearman presents the Spearman correlation for spousal educational attainment of the estimated model.

Row 1 of Table 3.a shows that the Spearman correlation for the unrestricted model is 0.6165. As discussed, it is not a strong test of Becker's theory.

Since all the estimated log odds along the diagonal is above one, row 2 of Table 3.a shows that the LR statistic of $DP2$ is zero. The p-value is larger than 0.999 and the MRE is 0.0. There is no statistical evidence against $DP2$.

Table 2.c presents the estimated cell probabilities of the $TP2$ model. Note that the marginal distributions in Table 2.c are unchanged from the unrestricted model Table 2.a. That is, restrictions on log odds do not change the marginal distributions of μ.

Table 2.d presents the estimated log odds for the $TP2$ model. There were four binding log odds constraints.

Row 3 of Table 3.a shows that the value of the LR for the $TP2$ is 28.2979 with a p-value less than 0.001. So $TP2$ does not hold at the national level.

The MRE for the $TP2$ model is 0.0343. Imposing $TP2$ results in a mean difference of three percent between estimated and the observed number of marital matches. The Spearman correlation for $TP2$ is 0.6169 which is less than 0.1 percent difference from the correlation of the unrestricted model. The Spearman correlation has little power against $TP2$.

I will investigate why $TP2$ fails at the national level. The unrestricted log odds in Table 2.b shows that only the two extreme log odds, bottom left and top right, have negative log odds which are more than twice the size of their standard errors. These two log odds involve marital matches in which spousal educational differences are largest, one spouse is a high school dropout and the other has more than a bachelor degree.

Table 2.e and 2.f present the estimates of the $TP2'$ model in which all log odds other than the top right and bottom left are restricted to be positive. The $TP2'$ cell probabilities and log odds estimates essentially match the unrestricted estimates.

Row 4 of Table 3.a shows the LR statistic for $TP2'$ against the unrestricted model is 0.4455 with a p-value of 0.483. So there is no statistical evidence against $TP2'$. MRE is 0.0010. One can provide a behavioral interpretation for $TP2'$. The penalty function for marital output at the national level is supermodular except for the extremes in spousal educational disparities where it is submodular.

$TP2$ is rejected because there are too many marital matches with the most extreme spousal educational differences. These marital matches account for less than 0.2 percent of marriages in the sample. From a behavioral significance point of view, the rejection of $TP2$ at the national level is modest.

For comparison, Tables 2.g and 2.h present estimates of the $DPNE$ model. The non-positive constraint binds for every off diagonal log odds. In other words, even for the top right and bottom left cells, the $DPNE$ model would like to make them positive if we require their adjacent cells to be non-positive Row 5 of Table 3.a presents the statistics for this estimated model. The LR statistic is 982.24. Not only is the $DPNE$ model rejected against the unrestricted model, it is also rejected against the $TP2$ model. The MRE is 0.1616 which is appreciably worse than the $TP2$ fit. Since all the off diagonal non-positive constraints bind, the $DP0E$ model has the same fit as the $DPNE$ model.

6.2 SMSA versus non-SMSA

This subsection studies two mutually exclusive subsamples of the national sample. I divide the national sample by couples who live in a standard metropolitan statistical area (SMSA) and those who do not. There are two reasons for studying these subsamples. First, even if $TP2$ applies to every sub-national marriage market, μ measured using national (aggregated) data need not be $TP2$. So my first objective is to construct two mutually exclusive marriage markets which are more homogenous than the national market.[16] Second, recent researchers have argued that cities facilitate PAM in marriage relative to non-cities. I will use the $TP2$ order to ask whether the SMSA marriage distribution has more positive dependence than the non-SMSA distribution.

Tables 1.b and 1.c present the 2000 marriage distributions for the SMSA and non-SMSA subsamples. The SMSA sample has more than twice the number of marriages compared with the non-SMSA sample. In terms of the marginal distributions, there were disproportionately more high school graduates and less than bachelor's in the non-SMSA sample, whereas the SMSA sample had disproportionately more bachelors and above.

Rows 3 and 4 in Table 3.b show that $TP2$ is rejected for both samples at $p-$values below 0.001. A comparison of the LR statistic and the MRE for both samples shows that $TP2$ is a worse description of the SMSA sample

[16] I have also worked with New York City which has a sample size of 1396 marriages. While $TP2$ was not rejected, this sample was too small to have much statistical power.

than the non-SMSA sample.

Table 4.a and 4.b show the unrestricted log odds for both samples. The rejection of $TP2$ for the SMSA sample is again due to the two extreme log odds, bottom left and top right, having negative log odds which are more than twice the size of their standard errors. Similar to the national sample, $TP2'$ cannot be rejected. The rejection of $TP2$ for the non-SMSA is less easy to characterize.

The results for the SMSA and non-SMSA samples show that the rejection of $TP2$ for the national sample is not just an inappropriate aggregation problem.

I now turn to the question as to whether there is more positive dependence in the SMSA sample. Rows 1 and 2 in Table 3.b show that the Spearman correlation is 0.6270 and 0.5399 for the SMSA and non-SMSA sample respectively. It suggests that there is more positive dependence in the SMSA sample.

Table 4.c provides the unrestricted difference in log odds between the SMSA and non-SMSA samples. There are six negative log odds, with two of them being larger than twice their estimated standard error ($\{1,4\}$ and $\{2,1\}$ cells).

Table 4.d provides the estimated differences in log odds between the SMSA and non-SMSA samples after imposing $TP2$ on the difference. There are six binding zero constraints corresponding to the six negative log odds in Table 4.c. Row 8 of Table 3.b shows that the $p-$value of the LR test that the difference is $TP2$ is 0.384. So in spite of the six binding zero constraints, and consistent with the Spearman coefficient ranking, the SMSA sample does show more positive dependence than the non-SMSA sample by the $TP2$ order. This finding supports the recent research which argued for more PAM in marriage in cities.

Rows 5 and 6 of Table 3.b show the MRE for the SMSA and non-SMSA samples after imposing $TP2$ on the difference. The MRE for the SMSA sample is 0.0094 and 0.0240 for the non-SMSA sample. These $MRE's$ are significantly smaller than those in rows 3 and 4 respectively where I impose $TP2$ on each sample separately.

Botticini and Siow 2008 show that marriage rates and total gains to marriage in cities are independent of the size of the city. Here I show that there is more marital sorting by spousal educational attainment in cities than non-cities. Taken together, these two studies suggest that conditional on marrying, most individuals care about the type of spouse that they marry. But their gains to marriage from different spousal choices, to a first order, do not affect their decision of whether to marry or not.

6.3 $2000 - 1970$ is $DP2$

Many researchers have argued that PAM by spousal educational attainment in the US has grown in recent decades. Most of these studies use correlation tests which have low power and/or tightly parametrized models to make their case. This subsection will use the $TP2$ order to investigate this claim.

Table 1.d presents the marriage counts for a 6% sample of the 1970 US census. Compared with the 2000 national sample, they are comparable in sample size. The 1970 individuals have lower educational attainment. There were marriages for all potential marital matches in 1970.

Table 5.a presents the unrestricted log odds for the 1970 sample. There are five negative log odds, with two of them exceeding twice their standard errors in magnitude. In row 1 of Table 3.c, the Spearman correlation of the unrestricted model is 0.4496 which is lower than for any other sample studied here. Row 2 of Table 3.c shows that the $p-$value for $TP2$ is less than 0.001. The MRE is 0.0586 which suggests that $TP2$ is a significantly worse fit of the 1970 data than any other sample studied here.

Table 5.b presents the unrestricted differences in log odds between 2000 and 1970. Surprisingly, there are seven negative log odds, with five of them larger than twice their standard errors. Table 5.c shows the estimated differences in log odds by imposing $TP2$ on them. There are eight binding zero constraints. Row 8 of Table 3.c shows that the difference in log odds being $TP2$ has a p-value less than 0.001. The MRE is 0.0230 which also suggests that the difference in log odds is not $TP2$. Thus although there is more positive dependence in 2000 than in 1970, the increase in dependence is not well captured by the $TP2$ order. This finding is anticipated by the comprehensive study by Chiappori, Selanie and Weiss (in process).

$TP2$ is a worse fit in 1970 than 2000 and the Spearman coefficient of the unrestricted model is lower in 1970 than in 2000. Both of these facts suggest that positive dependence has increased in 2000. Yet the difference in log odds being $TP2$ is strongly rejected. One potential reconciliation of the two findings is that the increase in positive dependence in 2000 is more localized than what a $TP2$ order would require. Returning to Table 5.b, the unrestricted differences in log odds along the diagonal are primarily positive. The one negative estimate is less than twice its standard error.

Table 5.d provides the estimated difference in log odds after imposing $DP2$ on the difference in log odds. Row 9 of Table 3.c shows that the p-value for the difference in log odds being $DP2$ is 0.369. The MRE for the difference in log odds is significantly less than one percent. I.e. there is almost no difference between imposing $DP2$ on difference in log odds and leaving them unrestricted.

Thus there was a localized increase in positive dependence between 1970 and 2000 along the diagonal log odds. The changes in off diagonal log odds were idiosyncratic, some being positive and others being negative. A behavioral interpretation of the finding is that a preference for own type has increased but the increase is non-monotone. Tightly parametrized empirical models of the increase in PAM in spousal educational attainment between 1970 and 2000 are misleading.

7 Conclusion

This paper makes the following contributions. First, it develops an empirical static stochastic Becker model which provides stochastic analogs of all the predictions of the original Becker model. Second, the empirical framework shows what can be differentiated between supermodularity and a preference for own type marital output function. Third, the paper provides an exact behavioral interpretation of local odds ratios, log linear models and the $TP2$ order, a common statistical measure for positive dependence in bivariate matching distributions. Fourth, the local log odds and the $TP2$ order provide new insights on some common findings on marriage matching by spousal educational attainment in the US. For the 2000 national sample, supermodularity of the marital output function cannot be rejected except for less than 0.2 percent of the sample. Thus Becker's insight on the importance of supermodularity of the marital output function in explaining PAM by spousal education is validated. Using the $TP2$ order, there was more PAM in SMSA than non-SMSA marriage markets. Finally, there were increases in specific local log odds at the national level between 1970 and 2000.

There are some directions for further research. First, there is a need to extend Becker's model to multidimensional matching. Since $TP2$ has a multidimensional analog, such an extension using $TP2$ may be fruitful (Galichon and Selanie (2009)). Second, with small marriage markets, there are two issues, one theoretical and one empirical. The theoretical issue is to investigate how the market clears with finite number of agents. The empirical issue is one of thin cells in estimating multinomial models of bivariate matching. I encountered this problem when estimating twenty five marital matches models using a sample size of around 1400 for New York City. Finally, this paper has scratched the surface in terms of using local log odds to studying empirical marital matching. Extensions of the framework developed here to investigate other bivariate matching markets remain open.

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Appendix

2000 USA Data: 2000 5% national sample extracted from “usa.ipums.org”.

White males and females: RACED = 100

Married individuals: MARST = 1 or 2 for married individuals with spouse present and spouse absent respectively. Also SPRULE = 1 for husband has same serial number as wife and is listed directly above and SPRULE = 2 for wife has same serial number as husband and is listed directly above.

Females between the ages of 31 – 35

Males between the ages of 32 – 36

SMSA: MEATAREAD: Identifies whether not an individual lives in a Metropolitan Area (MA). In this sample, an MA refers to the same thing as a Standard Metropolitan Statistical Area (SMSA) in previous sample years. Refer to (usa.ipums.org) for the definition of an MA. METAREAD = 0 for individuals who do not live in a Metropolitan Area. METAREAD not = 0 for individuals who do live in a Metropolitan Area (specific value gives the MA in which the individual lives).

Education: LHS: Less than High School (EDUC99 = 1 - 9); HS: High School (EDUC99 = 10); LBA: Less than a Bachelor’s Degree (EDUC99 = 11 - 13); BA: Bachelor’s Degree (EDUC99 = 14); GBA: More than a Bachelor’s Degree (EDUC99 = 15 – 17).

1970 USA Data: 1970 national data is the sum of six 1% samples (for state, metro and neighbourhood samples, forms 1 and 2 are used). These were combined to create a 1970 6% national sample extracted from “usa.ipums.org”.

Age ranges, race, marital status as above.

Education: LHS: Less than High School (HIGRADED = 000 - 142); HS: High School (HIGRADED = 150); LBA: Less than a Bachelor’s Degree (HIGRADED = 151 - 182); BA- Bachelor’s Degree (HIGRADED = 190); GBA: More than a Bachelor’s Degree (HIGRADED >= 191)

Table 1: US Censuses

(a) 2000 national sample

FEMALE EDUC. \ MALE EDUC.	LHS	HS	LBA	BA	GBA	Tot.
LHS	5071	2746	1288	220	91	9416
HS	3980	16712	7650	1983	475	30800
LBA	2333	10918	17999	6714	1868	39832
BA	398	2933	7010	13906	5588	29835
GBA	150	776	1853	4123	4633	11535
Tot.	11932	34085	35800	26946	12655	121418

(b) 2000 SMSA subsample

FEMALE EDUC. \ MALE EDUC.	LHS	HS	LBA	BA	GBA	Tot.
LHS	3459	1656	870	163	75	6223
HS	2274	9404	4907	1476	358	18419
LBA	1424	6330	11996	5088	1471	26309
BA	267	1873	5108	11273	4698	23219
GBA	97	448	1399	3424	4036	9404
Tot.	7521	19711	24280	21424	10638	83574

(c) 2000 non-SMSA subsample

FEMALE EDUC. \ MALE EDUC.	LHS	HS	LBA	BA	GBA	Tot.
LHS	1612	1090	418	57	16	3193
HS	1706	7308	2743	507	117	12381
LBA	909	4588	6003	1626	397	13523
BA	131	1060	1902	2633	890	6616
GBA	53	328	454	699	597	2131
Tot.	4411	14374	11520	5522	2017	37844

(d) 1970 national sample

FEMALE EDUC. \ MALE EDUC.	LHS	HS	LBA	BA	GBA	Tot.
LHS	20409	11219	3244	1164	1209	37245
HS	15950	28339	9607	3942	3316	61154
LBA	2451	4330	4461	3002	3406	17650
BA	746	1164	1121	2329	2956	8316
GBA	343	418	453	495	1800	3509
Tot.	39899	45470	18886	10932	12687	127874

Table 2: 2000 national sample models

(a) Unrestricted probabilities

		MALE EDUC.					
		LHS	HS	LBA	BA	GBA	Tot.
FEMALE EDUC.	LHS	0.0418	0.0226	0.0106	0.0018	0.0007	0.0776
	HS	0.0328	0.1376	0.0630	0.0163	0.0039	0.2537
	LBA	0.0192	0.0899	0.1482	0.0553	0.0154	0.3281
	BA	0.0033	0.0242	0.0577	0.1145	0.0460	0.2457
	GBA	0.0012	0.0064	0.0153	0.0340	0.0382	0.0950
	Tot.	0.0983	0.2807	0.2948	0.2219	0.1042	1.0000

(b) Unrestricted log odds

LOCAL LOG ODDS				
	LHS,HS	HS,LBA	LBA,BA	BA,GBA
LHS,HS	2.0482 (0.0289)	-0.0244 (0.0370)	0.4171 (0.0762)	-0.5463 (0.1344)
HS,LBA	0.1084 (0.0293)	1.2813 (0.0185)	0.3640 (0.0292)	0.1497 (0.0556)
LBA,BA	0.4541 (0.0580)	0.3714 (0.0248)	1.6711 (0.0202)	0.3676 (0.0307)
BA,GBA	-0.3538 (0.1020)	-0.0009 (0.0481)	0.1148 (0.0318)	1.0283 (0.0271)

(c) TP2 probabilities

		MALE EDUC.					
		LHS	HS	LBA	BA	GBA	Tot.
FEMALE EDUC.	LHS	0.0418	0.0228	0.0105	0.0020	0.0005	0.0776
	HS	0.0328	0.1375	0.0632	0.0161	0.0041	0.2537
	LBA	0.0192	0.0899	0.1482	0.0553	0.0154	0.3281
	BA	0.0036	0.0241	0.0575	0.1145	0.0460	0.2457
	GBA	0.0010	0.0065	0.0155	0.0340	0.0382	0.0950
	Tot.	0.0983	0.2807	0.2948	0.2219	0.1042	1.0000

(d) TP2 log odds

LOCAL LOG ODDS				
	LHS,HS	HS,LBA	LBA,BA	BA,GBA
LHS,HS	2.0405 (0.0272)	0 (0.0210)	0.2690 (0.0655)	0 (0.0888)
HS,LBA	0.1095 (0.0292)	1.2778 (0.0180)	0.3803 (0.0289)	0.0797 (0.0503)
LBA,BA	0.3690 (0.0514)	0.3712 (0.0233)	1.6746 (0.0200)	0.3676 (0.0304)
BA,GBA	0 (0.0607)	0 (0.0255)	0.0983 (0.0290)	1.0283 (0.0271)

(e) TP2' probabilities

		MALE EDUC.					
		LHS	HS	LBA	BA	GBA	Tot.
FEMALE EDUC.	LHS	0.0418	0.0228	0.0105	0.0018	0.0007	0.0776
	HS	0.0328	0.1375	0.0632	0.0163	0.0039	0.2537
	LBA	0.0192	0.0899	0.1482	0.0553	0.0154	0.3281
	BA	0.0033	0.0242	0.0577	0.1145	0.0460	0.2457
	GBA	0.0012	0.0064	0.0153	0.0340	0.0382	0.0950
	Tot.	0.0983	0.2807	0.2948	0.2219	0.1042	1.0000

(f) TP2' log odds

LOCAL LOG ODDS				
	LHS,HS	HS,LBA	LBA,BA	BA,GBA
LHS,HS	2.0405 (0.0272)	0 (0.0210)	0.4005 (0.0726)	*-0.5463 (0.1344)*
HS,LBA	0.1095 (0.0292)	1.2778 (0.0180)	0.3664 (0.0291)	0.1497 (0.0566)
LBA,BA	0.4542 (0.0577)	0.3712 (0.0234)	1.6712 (0.0200)	0.3676 (0.0307)
BA,GBA	*-0.3544 (0.1020)*	0 (0.0481)	0.1145 (0.0318)	1.0283 (0.0271)

(g) DPNE probabilities

		MALE EDUC.					
		LHS	HS	LBA	BA	GBA	Tot.
FEMALE EDUC.	LHS	0.0418	0.0225	0.0095	0.0029	0.0010	0.0776
	HS	0.0306	0.1400	0.0590	0.0178	0.0063	0.2537
	LBA	0.0183	0.0838	0.1604	0.0485	0.0170	0.3281
	BA	0.0059	0.0270	0.0516	0.1194	0.0418	0.2457
	GBA	0.0016	0.0075	0.0144	0.0333	0.0382	0.0950
	Tot.	0.0983	0.2807	0.2948	0.2219	0.1042	1.0000

(h) DPNE log odds

LOCAL LOG ODDS				
	LHS,HS	HS,LBA	LBA,BA	BA,GBA
LHS,HS	2.1402 (0.0258)	$> -10^{-4}$ (0.0183)	$> -10^{-4}$ (0.0220)	$> -10^{-4}$ (0.0401)
HS,LBA	$> -10^{-4}$ (0.0145)	1.5130 (0.0157)	$> -10^{-4}$ (0.0135)	$> -10^{-4}$ (0.0173)
LBA,BA	$> -10^{-4}$ (0.0160)	$> -10^{-4}$ (0.0112)	2.0340 (0.0166)	$> -10^{-4}$ (0.0142)
BA,GBA	$> -10^{-4}$ (0.0330)	$> -10^{-4}$ (0.0156)	$> -10^{-4}$ (0.0146)	1.1857 (0.0244)

Table 3: Statistics

(a): 2000 national sample (121418 observations)

	Model	LL	LR (vs. unres)	p-value	MRE	Spearman
1	Unres.	1.0890*10e6	-	-	-	0.6165
2	DP2	1.0890*10e6	0	>0.999	0	0.6165
3	TP2	1.0889*10e6	28.2979	<0.001	0.0343	0.6169
4	TP2'	1.0890*10e6	0.4455	0.483	0.0010	0.6165
5	DPNE	1.0885*10e6	982.24	<0.001	0.1616	0.5939

(b): 2000 SMSA & non-SMSA subsamples

	Sample	Model	N	LL	LR (vs. unres)	p-value	MRE	Spearman
1	SMSA	unres.	83574	7.1707*10e5	-	-	-	0.6270
2	Non-SMSA	unres.	37844	2.9942*10e5	-	-	-	0.5399
3	SMSA	TP2	83574	7.1706*10e5	26.7121	<0.001	0.0387	0.6272
4	Non-SMSA	TP2	37844	2.9941*10e5	16.6702	<0.001	0.0337	0.5408
5	SMSA	Δ TP2	83574	-	-	-	0.0094	0.6274
6	Non-SMSA	Δ TP2	37844	-	-	-	0.0246	0.5383
7	Δ	Δ unres.	-	1.0165*10e6	-	-	-	-
8	Δ	Δ TP2	-	1.0165*10e6	6.2167	0.384	0.0170	-

(c) 2000-1970 national models

	Sample	Model	N	LL	LR (vs. unres)	p-value	MRE	Spearman
1	1970 US	unres.	127874	1.1739*10e6	-	-	-	0.4496
2	1970 US	TP2	127874	1.1738*10e6	177.8652	<0.001	0.0586	0.4524
3	1970 US	Δ TP2	127874	-	-	-	0.0128	0.4479
4	2000 US	Δ TP2	121418	-	-	-	0.0324	0.6184
5	1970 US	Δ DP2	127874	-	-	-	0.0008	0.4496
6	2000 US	Δ DP2	121418	-	-	-	0.0002	0.6165
7	Δ	Δ unres.	-	2.2628*10e6	-	-	-	-
8	Δ	Δ TP2	-	2.2628*10e6	53.3501	<0.001	0.0230	-
9	Δ	Δ DP2	-	2.2628*10e6	0.1455	0.369	0.0005	-

Table 4: 2000 SMSA & non-SMSA subsamples

(a) SMSA unrestricted log odds

LOCAL LOG ODDS				
	LHS,HS	HS,LBA	LBA,BA	BA,GBA
LHS,HS	2.1562 (0.0374)	-0.0068 (0.0443)	0.4734 (0.0901)	-0.6403 (0.1510)
HS,LBA	0.0722 (0.0366)	1.2897 (0.0235)	0.3436 (0.0350)	1.1756 (0.0677)
LBA,BA	0.4562 (0.0707)	0.3640 (0.0319)	1.6493 (0.0239)	0.3657 (0.0352)
BA,GBA	-0.4180 (0.1291)	0.1355 (0.0600)	0.1034 (0.0363)	1.0397 (0.0285)

(b) Non-SMSA unrestricted log odds

LOCAL LOG ODDS				
	LHS,HS	HS,LBA	LBA,BA	BA,GBA
LHS,HS	1.8461 (0.0473)	-0.0215 (0.0607)	0.3041 (0.1489)	-0.1959 (0.3043)
HS,LBA	0.1640 (0.0441)	1.2487 (0.0293)	0.3822 (0.0556)	0.0564 (0.1156)
LBA,BA	0.4720 (0.0980)	0.3158 (0.0425)	1.6314 (0.0411)	0.3253 (0.0656)
BA,GBA	-0.2681 (0.1700)	-0.2596 (0.0802)	0.1063 (0.0672)	0.9269 (0.0686)

(c) SMSA – Non-SMSA unrestricted log odds

LOCAL LOG ODDS				
	LHS,HS	HS,LBA	LBA,BA	BA,GBA
LHS,HS	0.3101 (0.0540)	0.0147 (0.0577)	0.1693 (0.1193)	-0.4444 (0.2119)
HS,LBA	-0.0918 (0.0428)	0.0410 (0.0416)	-0.0386 (0.0460)	1.1192 (0.0987)
LBA,BA	-0.0158 (0.0851)	0.0482 (0.0419)	0.0179 (0.0359)	0.0404 (0.0589)
BA,GBA	-0.1499 (0.1601)	0.3951 (0.0913)	-0.0029 (0.0537)	0.1128 (0.0663)

(d) SMSA – Non-SMSA TP2 log odds

LOCAL LOG ODDS				
	LHS,HS	HS,LBA	LBA,BA	BA,GBA
LHS,HS	0.2670 (0.0558)	0.0266 (0.0476)	0.0418 (0.0773)	0 (0.1317)
HS,LBA	0 (0.0277)	0.0189 (0.0227)	0 (0.0253)	0.0407 (0.0674)
LBA,BA	0 (0.0457)	0.0512 (0.0394)	0.0003 (0.0237)	0.0460 (0.0499)
BA,GBA	0 (0.0933)	0.3670 (0.0862)	0 (0.0369)	0.1117 (0.0620)

Table 5: 2000 - 1970 is DP2

(a) 1970 unrestricted log odds

LOCAL LOG ODDS				
	LHS,HS	HS,LBA	LBA,BA	BA,GBA
LHS,HS	1.1731 (0.0154)	0.1591 (0.0232)	0.1341 (0.0381)	-0.2109 (0.0467)
HS,LBA	-0.0057 (0.0264)	1.1116 (0.0255)	0.4947 (0.0312)	0.2992 (0.0337)
LBA,BA	-0.1242 (0.0540)	-0.0674 (0.0479)	1.1273 (0.0417)	0.1121 (0.0372)
BA,GBA	0.2471 (0.0847)	0.1181 (0.0810)	-0.6426 (0.0738)	1.0526 (0.0564)

(b) 2000 - 1970 unrestricted log odds

LOCAL LOG ODDS				
	LHS,HS	HS,LBA	LBA,BA	BA,GBA
LHS,HS	0.8751 (0.0339)	-0.1835 (0.0443)	0.283 (0.0859)	-0.3354 (0.1423)
HS,LBA	0.1141 (0.0397)	0.1697 (0.0308)	-0.1307 (0.0427)	-0.1495 (0.0681)
LBA,BA	0.5783 (0.0786)	0.4388 (0.0526)	0.5438 (0.0471)	0.2555 (0.0475)
BA,GBA	-0.6009 (0.1368)	-0.119 (0.0939)	0.7574 (0.0810)	-0.0243 (0.0645)

(c) 2000 - 1970 TP2 log odds

LOCAL LOG ODDS				
	LHS,HS	HS,LBA	LBA,BA	BA,GBA
LHS,HS	0.8283 (0.0312)	0 (0.0222)	0 (0.0310)	0 (0.0647)
HS,LBA	0.1223 (0.0397)	0.0857 (0.0276)	0 (0.0187)	0 (0.0294)
LBA,BA	0.5468 (0.0711)	0.4384 (0.0472)	0.5213 (0.0426)	0.1996 (0.0421)
BA,GBA	0 (0.0710)	0 (0.0472)	0.665 (0.0635)	0 (0.0378)

(d) 2000 - 1970 DP2 log odds

LOCAL LOG ODDS				
	LHS,HS	HS,LBA	LBA,BA	BA,GBA
LHS,HS	0.8751 (0.0335)	-0.1835 (0.0442)	0.283 (0.0859)	-0.3354 (0.1420)
HS,LBA	0.1141 (0.0396)	0.1697 (0.0309)	-0.1307 (0.0429)	-0.1495 (0.0682)
LBA,BA	0.5783 (0.0782)	0.4388 (0.0524)	0.5468 (0.0467)	0.2494 (0.0461)
BA,GBA	-0.1067 (0.1366)	-0.119 (0.0936)	0.7408 (0.0730)	0 (0.0377)