LABOR MATCHING MODEL: PUTTING THE PIECES TOGETHER.

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Abstract. The original Mortensen-Pissarides model possesses two elements that are absent from the commonly used simplified version: the job destruction margin and training costs. I find that these two elements enable a model driven only by productivity shocks to simultaneously explain most of the movements in unemployment, vacancies, job destruction, job creation, the job finding rate and wages. The role of the job destruction margin in propagating productivity shocks is to create an additional pool of unemployed at the very beginning of a recession. The role of training costs is to explain the simultaneous decline in vacancies.

I. Introduction

The labor search model pioneered by Mortensen and Pissarides (1994 [23], 1998 [24]), the MP model, has attracted considerable attention recently because of its intuitive explanation of equilibrium unemployment. However, as described by Shimer (2005, [29]), a calibrated stochastic steady state version of the model with only productivity shocks is incapable of quantitatively explaining the behavior of unemployment and vacancies over the business cycle. Even under an alternative calibration proposed by Hagedorn and Manovskii (2008 [17]) additional exogenous shocks[1] that are correlated with productivity shocks are required to fit the data (see Lubik (2009 [20])). This suggests that the simple model studied by Shimer lacks some propagation mechanism.

There are two assumptions in the original MP framework that are absent in the simplified version and haven't been explored as extensively: endogenous job destruction and training costs. In this paper, I embed both of these features of the MP model into a real business cycle model with matching and explore its fit. I find that these two key elements enable a model driven by productivity shocks of reasonable magnitude to simultaneously explain most of movements in unemployment, vacancies, job

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[1]Exogenous shocks to matching efficiency are an example of such shocks.

destruction, job creation, the job finding rate and real wages. Thus, the original MP model possesses a propagation mechanism that the simplified version does not.

The first assumption is that a firm can choose whether to preserve or terminate a match with a worker based on match profits. This makes firms more eager to destroy jobs when aggregate conditions are worse and the value of a match is lower. The role of the job destruction margin in propagating productivity shocks is to immediately create an additional pool of unemployed at the very beginning of a recession.

The second assumption is that job creation costs are a mix of recruiting and training costs. Recruiting costs are specific to posting a vacancy, while training costs are specific to creating a match conditional on finding a worker to fill the vacancy[2]. When more jobs are destroyed and the labor market becomes tighter, it is much easier for firms to find workers. In absence of training costs this would lead to an increase in the number of vacancies. Introduction of training costs attenuates the effect of market tightness on the total cost of creating a new job. Firms which face a lower value of a prospective match and little change in the cost of job creation will post fewer vacancies and create fewer new jobs. The role of the dual structure of creation costs is to explain the decrease in vacancies and the slow response of job creation once jobs have been destroyed.

I embed these two assumptions into a real business cycle model augmented by a matching friction as in Merz (1995 [21]) and Andolfatto (1996 [2]). In order to abstract from interactions between capital adjustment and job destruction as in den Haan, Ramey and Watson (2000 [11]) I assume that labor is the only factor used in production. I allow for variations in the labor wedge - the difference between workers disutility of employment and average match productivity - by assuming that each firm-worker match produces a different variety of the final good and the degree of product differentiation varies over time.

I link the degree of product differentiation to the number of productive firm-worker matches. This assumption helps capture the idea that as relatively less productive matches are destroyed the surviving matches are better on average. As a result job destruction is short-lived: once relatively unproductive matches are destroyed the remaining jobs are safe.

The question of whether the labor search model of Mortensen and Pissarides (1994, [23]) can explain the cyclical behavior of unemployment and vacancies and enhance the explanatory power of the standard real business cycle model has been in the focus of macroeconomic research ever since it was published[3]. One of the most puzzling

[2] The idea that creation costs can be a mix of vacancy-specific and match-specific costs was recently revived and discussed by Pissarides (2009 [26]). Non-linear creation costs were also used by Yashiv (2006 [33]) and Rotemberg (2006 [28]).

[3] The extensive literature on the empirical performance of labor search models includes among others Merz (1995 [21]), Andolfatto (1996 [2]), Caballero and Hammour (1996, [3]), Cole and Rogerson (1999, [7]), den Haan, Ramey and Watson (2000 [11]), Costain and Reiter (2003, [9]), Cheron and Langot (2004, [6]), Hornstein et. al. (2005, [19]), Farmer and Hollenhorst (2006, [12]), Gertler

findings has been the observation of Shimer (2005, [29]) that in a calibrated MP model the impact of productivity shocks of reasonable magnitude on unemployment and vacancies is an order of magnitude smaller than observed fluctuations in these variables. At the same time the impact of separation shocks implies a counterfactually positive correlation between unemployment and vacancies. One potential solution is provided by Hall (2005, [18]), who argues that introducing wage stickiness into the model simultaneously increases the volatility of unemployment and vacancies and matches volatility of wages in the data. Another potential solution to the so called "Shimer puzzle" is an alternative calibration proposed by Hagedorn and Manovskii (2008, [17]). Motivated by micro estimates of job creation costs they assume a much smaller value of joint rents and attribute a smaller fraction of these rents to the workers compared to the standard calibration.

The first explanation has been disputed by Pissarides (2009, [26]). He points to microeconomic evidence that contrary to Hall's prediction wages in newly created jobs are more volatile than average wages in the economy. Lubik (2009, [20]) shows that the alternative calibration of Hagedorn and Manovskii, while matching the volatilities of unemployment and vacancies in simulations, does not pass a test of fit, which uses the likelihood function as a tighter measure of success.

The first contribution of this paper compared to the literature is to expand the set of variables the model is aimed at explaining. In addition to productivity, unemployment, vacancies and wages used in most of previous studies, I include job creation, job destruction and job finding rates into the analysis. I follow Lubik in applying a tighter measure of fit by using a likelihood-based Bayesian estimation strategy.

The second contribution of this paper is to construct a model which not only explains the magnitudes of observed fluctuations but also generates the impulse responses to productivity shocks reminiscent of the data. This result relies on two features of the model. I follow the ideas of Ramey and Watson (1997, [27]) and den Haan, Ramey and Watson (2000 [11]) in modeling the job destruction margin. I build upon the suggestion of Pissarides (2009, [26]) that introduction of training costs can improve the empirical fit of the model by affecting the response of job creation to productivity shocks. I show how the elasticity of the matching function, the elasticity of the job destruction margin and the ratio of training to recruiting costs jointly determine the slope of the Beveridge curve.

The third contribution of the paper is to provide a simple mechanism which endogenously generates fluctuations in the labor wedge. The strategy adopted in this paper is motivated by the decomposition of the labor wedge into frictions in job creation and job destruction provided by Cheremukhin and Restrepo-Echavarria (2009, [5]). In that paper authors find that the initial sharp increases in unemployment in recessions are driven mostly by increases in job destruction while impediments to job

and Trigari (2006, [15]), Yashiv (2006, [33]), Rotemberg (2006, [28]), Fujita and Ramey (2007, [14]), Mortensen and Nagypal (2007, [22]) and Cheremukhin and Restrepo-Echavarria (2009 [5]).

creation are responsible for the subsequent slow recoveries. In this paper I take a stand on what these frictions in job destruction and job creation are.

To explore the fit of the model I use Bayesian techniques recently developed for analyzing DSGE models. I form relatively wide priors for parameters of interest and let the data choose parameter combinations which are most likely to explain the data. I then measure the fraction of variations in the data that the model can explain under the best parameter combination. Posterior densities of parameters of interest not only tell me which values of parameters are preferred by the data, but also shed light on how well they are identified and hence how important they are for the propagation mechanism.

I use this estimation strategy instead of the commonly used calibration strategies for three reasons. First, there is no consensus in the literature over many of the parameters of interest. Recent debates between Shimer (2005 [29]) and Hagedorn and Manovskii (2008 [17]) demonstrate how different calibrations of the model can lead to different results. Second, a likelihood function, by giving natural weights to all the moments of the data, provides a tighter measure of success compared to most previous studies. Finally, the Bayesian approach provides a simple tool for understanding which assumptions of the model are important to fit the data and which are not.

My main finding is that a reasonably calibrated labor search model augmented by a job destruction margin and training costs can simultaneously explain two thirds of fluctuations in unemployment and more than 80 percent of fluctuations in vacancies, job creation, job destruction, the job finding rate, all as a result of a single shock to labor productivity. The model is also consistent with empirical volatility and cyclicality of real wages.

In addition to the main result the model has several implications. First, most of business cycle fluctuations in labor market variables can be accounted for by focusing on flows from terminated jobs to unemployment and from unemployment to newly created jobs. Similarly to Cole and Rogerson (1999 [7]), the model implies a relatively low steady-state job finding rate. However, I do not interpret this result as implying counterfactually long duration of unemployment. The model considers only permanent job destruction of old jobs and creation of new jobs, which are a fraction of total turnover. As a result the job finding rate in the model represents the probability that an unemployed worker matches with one of the new jobs created to replace one of the destroyed jobs. This probability varies as much as the total job finding rate because matches are random; but it has a lower steady-state value.

The second result is that the split of total job creation costs between recruiting and training costs implied by the model matches very closely the microeconomic evidence presented by Silva and Toledo (2009 [31]). The model also implies a relatively low value of surplus available to the worker and the firm. This result is in the ballpark of empirical evidence on the size of total job creation costs provided by Hagedorn and Manovskii (2008 [17]). However, the reason the data chooses a low value of joint rents is different. When rents are low, they are also volatile; and Hagedorn and Manovskii need high volatility of rents to increase volatility of job creation. In my model high volatility of rents is required to match the large empirical variations in job destruction.

The third result is that the calibration is not very sensitive to the wage-setting rule. The bargaining weight parameter can be set to almost any value without a significant effect on the propagation mechanism. This is because the volatility of real wages is governed mostly by changes in the price index and not in wages themselves. This is also a feature of US data.

Finally, the exogenous shock to labor productivity, recovered from the estimation of the model and designed to simultaneously explain the behavior of other labor market variables, matches reasonably well the cyclical properties of labor productivity in the data. However, this exogenous shock should be viewed more broadly as a latent factor which captures a combination of supply and demand disturbances. In fact the model produces responses to demand and supply shocks that are almost indistinguishable from each other.

The paper is organized as follows. Section 2 lays out the model and explains its key assumptions. Section 3 describes the empirical methodology. Section 4 provides a discussion of the results and section 5 concludes.

II. Model

Before describing the primitives of the model I provide a motivation for some of the modeling choices I make. In the Mortensen-Pissarides framework every period each job is characterized by an individual productivity level. Differences in productivity lead to differences in profits and wages across jobs. A large enough decrease in the productivity of a job leads to an efficient termination of the job. In this model aggregate shocks have a non-trivial effect on the productivity distribution which becomes a state variable. Variations in the number of jobs destroyed are a result of shifts in the productivity distribution over time.

Instead of carrying the productivity distribution I choose to model the job destruction margin in a somewhat reduced form way. I assume that every period the idiosyncratic component of productivity, represented in my model by a taste shock, is drawn independently from the same distribution with varying support. The size of the support is equal to the number of existing jobs. This makes aggregate employment a state variable which characterizes the productivity distribution. I use variations in the support of the distribution to capture the idea that once a relatively unproductive job is destroyed the distribution of remaining jobs has a higher mean. This assumption ensures that a persistent aggregate productivity shock does not lead to a persistent increase in the rate of job destruction.

In order to further simplify the analysis I collapse the distribution of wages across jobs by assuming that wages cannot be set or renegotiated conditional on the idiosyncratic component of productivity. This leads to inefficient destruction of jobs. Assuming inefficient job destruction is not completely implausible given the large empirical literature on downward wage rigidity. The inability to renegotiate the wage contract is not a restriction, however. Ramey and Watson (1997, [27]) describe an environment where the inability to condition the labor contract on unobserved efforts leads to a fragile contract between workers and firms. In this environment negative productivity shocks lead to inefficient job destruction even though the possibilities to renegotiate wage contracts are unlimited.

The model I construct is a real business cycle model with a matching friction. I deliberately simplify the model to concentrate the discussion around the two key elements: endogenous job destruction and training costs. First, I describe the physical environment. Then I explain how employment relationships between workers and firms are formed, operated and terminated. I close the model with a description of the household's problem and equilibrium conditions. I then explain how the incorporation of endogenous job destruction and training costs affects propagation of shocks.

II.1. **Physical Environment.** Time is discrete and continues forever. The economy is populated by a unit measure of workers and a large number of firms. Workers can be unemployed searching for a job, head-hunting or engaged in a productive employment relationship. I denote the measure of head-hunters, X_t, the measure of unemployed,

U_t, and N_t represents the measure of workers engaged in productive activities. Their sum is equal to the total number of workers:

$$N_t + X_t + U_t = 1. \tag{1}$$

Each firm has a blueprint for producing a different variety of the consumption good, and needs a worker to be productive. A firm can be in one of three states: matched with a worker and producing, or searching for a worker, or idle. A firm can hire at most one worker which provides at most one unit of time. As operating firms will always demand the maximum amount of time, N_t represents both the measure of workers in productive activities and the measure of operating firms. I denote the measure of firms searching for a worker V_t, which also represents the number of vacancies. The measure of idle firms is large enough so that there are always enough potential entrants.

The production technology of a firm is linear in labor so that each worker produces A_t units of the final good. A_t represents aggregate labor productivity and follows an autoregressive process of order one governed by exogenous productivity shocks ε_t drawn from a standard normal distribution:

$$A_t = A_{ss}^{1-\rho} A_{t-1}^{\rho} e^{\sigma \varepsilon_t}, \quad \varepsilon_t \in \mathcal{N}(0,1) \tag{2}$$

where A_{ss} is the steady-state value of productivity, ρ is persistence, and σ is the standard deviation of shocks to labor productivity.

New employment relationships are formed through a matching process between vacant firms and unemployed workers. The mass V_t of firms that decide to post vacancies are matched with the mass of unemployed workers U_t according to a constant returns to scale matching function[4]:

$$M_t = B U_t^{\alpha} V_t^{1-\alpha}, \tag{3}$$

where M_t is the mass of new employment relationships starting to operate next period.

Job creation is costly with two components: a vacancy posting component includes costs of advertising and interviewing, and a match formation component includes costs of developing a working environment and training the worker to meet specific needs. Thus, firms post vacancies at a cost c and then firms which match workers incur an additional training cost K per match.

To cover these costs firms hire head-hunters in a specialized competitive labor market. Head-hunters have the same productivity, A_t, as employed workers. The total mass of head-hunters, X_t, required to cover the creation costs in period t, satisfies:

$$A_t X_t = cV_t + KM_t \tag{4}$$

[4]Given the estimated parameters values the condition $M_t \leq min(U_t, V_t)$ holds in all the simulations with a very high probability.

I assume that workers are members of a large family which pools income and then redistributes it equally to all members. The representative household then maximizes the expected discounted utility of a representative worker, who values consumption and leisure:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[u\left(C_t\right) - \left(N_t + X_t\right) \right]. \tag{$\bigstar$}$$

In equation ($\bigstar$) $u\left(C_t\right)$ is a concave utility function defined with respect to a consumption aggregator defined over different varieties of final goods produced by firms:

$$u\left(C_t\right) = C_t - \frac{\gamma}{2} C_t^2.$$

Here $C_t \in [0, 1/\gamma]$ is aggregate consumption of final goods - a weighted sum of demands for each variety produced by individual firm i:

$$C_t = \int_0^{N_t} z_{it} q_{it} di. \tag{5}$$

Variable z_{it} is the idiosyncratic taste shock for variety of firm i, and N_t is the measure of employment relationships operating in period t. I assume that the taste shock is governed by the following distribution with variable support:

$$z_{it} = e^{-gi}, \qquad i \in \mathcal{U}\left[0, N_t\right], \tag{6}$$

where i is an index of a firm uniformly distributed on a closed interval $[0, N_t]$.

Firms and workers discount future in the same way. Having described the primitives, technologies and preferences, I now describe how employment relationships are formed and operated. I divide each period t into three stages. During the first stage, uncertainty about aggregate productivity is realized. The state of the economy can then be summarized by two variables $\{A_t, N_t\}$. There is no aggregate uncertainty left after this stage.

During the second stage, production, recruiting, matching, bargaining and training take place. Each existing employment relationship indexed by i produces a different variety of the final good. Vacant firms hire head-hunters to advertise vacancies and train new employees. Unemployed workers search for and match with vacant firms.

I follow the literature in assuming that when vacant firms and unemployed workers meet, they split the future surplus using a Nash bargaining solution. I denote ψ the bargaining power of a worker. The threat point of the worker is to remain unemployed and the threat point of the firm is to remain vacant. Because taste shocks are i.i.d., at this stage all existing employment relationships are ex ante identical. A complete wage contract assigns a wage W_{t+s} to each aggregate state $\{A_{t+s}, N_{t+s}\}$ in all future periods $s \in [1, \infty]$. The assumption that a wage contract cannot be written conditional on the idiosyncratic taste shock z_{it} for each variety i collapses the wage distribution across workers and leads to inefficient separations in the third stage. After a worker and a firm sign a contract the new workers are trained by head-hunters to satisfy the specific needs of firms.

In the third stage an idiosyncratic taste shock z_{it} for variety of each firm i is realized. At this stage markets for goods clear. All goods produced by firms are consumed. Equilibrium prices determine firm profits. Firms which face low enough prices go bankrupt and terminate their contracts with workers. I denote the fraction of jobs that are destroyed ζ_t.

As a result, every period a fraction of existing jobs is destroyed and a number of new jobs are formed. The measure of employment relationships satisfies the following evolution equation:

$$N_{t+1} = N_t (1 - \zeta_t) + M_t, \tag{7}$$

where labor adjustment comes from creation of M_t new jobs and destruction of a fraction ζ_t of existing jobs to be endogenously determined later.

II.2. **Characterization of Equilibrium.** In this subsection I close the explanation of the model by describing a competitive equilibrium. I first describe the household's problem. Then I explain how prices and values of firms and workers in different states are determined. I then discuss the problem of the firm and the job destruction decision. I conclude by a definition of a competitive equilibrium.

The representative household chooses consumption q_{it} of each variety i to maximize utility (★) subject to (II.1), (5) and a budget constraint:

$$\int_0^{N_t} p_{it} q_{it} di = w_t X_t + W_t N_t + \Pi_t,$$

where all of the wage and profit income is spent on final goods produced by employment relationships in the same period. Aggregate profits Π_t are the sum of individual profits of firms net of head-hunting costs:

$$\Pi_t = \int_0^{N_t} (p_{it} A_t - W_t) \, di - w_t X_t.$$

Household optimization dictates that output of individual firms is priced using marginal utility of consumption with the price of each variety proportional to household taste for that variety:

$$p_{it} = \frac{1}{\lambda_t} \frac{\partial u (C_t)}{\partial C_t} z_{it}, \tag{8}$$

where λ_t is the lagrange multiplier on the budget constraint.

The wages of head-hunters w_t are set competitively such that individual workers are indifferent between head-hunting and being unemployed searching for a job. Therefore, the head-hunter wage compensates for the disutility of work and the option value of finding a job while being unemployed:

$$w_t = \frac{1}{\lambda_t} + \Gamma_t^W \frac{M_t}{U_t}, \tag{9}$$

The ratio of matches to unemployment $\frac{M_t}{U_t}$ is the probability of finding a job and Γ_t^W is the worker's expected discounted lifetime benefit from engaging in an employment relationship:

$$\Gamma_t^W = E_t \beta \frac{\lambda_{t+1}}{\lambda_t} \left[W_{t+1} - w_{t+1} + \Gamma_{t+1}^W (1 - \zeta_{t+1}) - \Delta_{t+1} \right]. \tag{10}$$

The instantaneous net benefit of having a job is equal to the difference between the wage W_{t+1} paid on the job and the head-hunter's wage w_{t+1}, which serves as an outside option. This benefit is summed over time discounted and adjusted for the probability of being terminated. The term Δ_{t+1} reflects the net average loss in case the employment relationship is terminated and the worker is not paid the full wage. Similarly, the lifetime value of an employment relationship to a firm, Γ_t^F, is the expected discounted sum of future profits:

$$\Gamma_t^F = E_t \beta \frac{\lambda_{t+1}}{\lambda_t} \max\{p_{it+1} A_{t+1} - W_{t+1} + \Gamma_{t+1}^F, 0\}. \tag{11}$$

When each employment relationship is formed firms bargain with workers on a wage contract $W_t (A_t, N_t)$ which is contingent on the aggregate state but not on idiosyncratic taste shocks. This contract sets wages in such a way as to give the worker a fraction ψ of total match surplus:

$$\Gamma_t^W = \psi \left(\Gamma_t^W + \Gamma_t^F - \frac{w_t}{A_t} K \right) \tag{12}$$

The total surplus is the sum of lifetime values to the worker and to the firm net of training costs. Training costs are taken into account and shared by the worker and the firm to avoid holdup. Advertising costs are not included because they are sunk at the moment a worker and a firm bargain on the wage contract. In Appendix A I compare the competitive equilibrium to a constrained planner's solution and show that they coincide if and only if the Hosios condition is satisfied:

$$\psi = \frac{\partial M_t}{\partial U_t} \frac{U_t}{M_t} = \alpha$$

Free entry of new firms into the labor market guarantees that vacancies are posted until their expected marginal costs are equal to their expected marginal benefits:

$$\frac{w_t}{A_t} \left(c + K \frac{M_t}{V_t} \right) = \Gamma_t^F \frac{M_t}{V_t}, \tag{13}$$

where w_t is the competitive wage paid to head-hunters and $\frac{M_t}{V_t}$ is the vacancy filling rate, which firms take as given. Thus, exactly enough firms advertise vacancies such that the sum of the marginal cost of posting an extra vacancy and the marginal cost of training the worker if the vacancy is filled, all measured in units of head-hunter wages, are equal to the expected discounted lifetime profits of a potential new match.

In the second stage productive firms choose labor input l_{it} in order to maximize the sum of expected profits:

$$E_i \max\{p_{it}A_t l_{it} - W_t l_{it} + \Gamma_t^F, 0\},$$

subject to a time constraint $l_{it} \leq 1$. The assumption that the worker and the firm cannot write a wage contract contingent on the realization of the price p_{it} and cannot renegotiate the contract in the third stage makes all firm-worker pairs agree on the same wage level W_t.

In equilibrium each firm uses the maximum of one unit of worker's time $l_{it} = 1$ and produces A_t units of the final good. In equilibrium markets for all varieties of the final good clear:

$$q_{it} = A_t. \tag{14}$$

In the third stage, conditional on the realization of the individual price, the employment relationship between a worker and a firm can either be preserved or terminated. A relationship is terminated if profits are low enough to outweigh the expected discounted value of future profits, Γ_t^F. Since profits are increasing in p_{it}, each period there is a unique cut-off price $\underline{p}_t$, such that a relationship is terminated if $p_{it} < \underline{p}_t$. The cut-off level $\underline{p}_t$ satisfies:

$$\underline{p}_t A_t - W_t + \Gamma_t^F = 0. \tag{15}$$

The firm compares its payoff inside the relationship with its value outside the relationship which is zero due to free entry. As a result, every period a fraction ζ_t of employment relationships is terminated:

$$\zeta_t = \int_0^{\underline{p}_t} d\mu\left(p_{it}\right), \tag{16}$$

where μ is the cumulative distribution function of individual prices defined by the combination of equations (8) and (6).

A competitive equilibrium of the model economy is a solution to equations (1)-(16), where $\{U_t; A_t; M_t; X_t; C_t; N_{t+1}; p_{it}; P_t; w_t; \Gamma_t^W; \Gamma_t^F; W_t; V_t; \underline{p}_t; q_{it}; \zeta_t\}$ are endogenous variables and $\{\varepsilon_t; z_{it}\}$ are exogenous shocks.

II.3. **Propagation Mechanism.** In this subsection I describe how the two key elements work. The endogenous job destruction margin is the first key element of the model. Figure 2 depicts the price distribution, $\mu\left(p_{it}\right)$, and the cutoff price level, $\underline{p}_t$. A negative productivity shock ε_t leads to a persistent decrease in productivity A_t and results in a decrease in expected future profits, Γ_t^F. This shifts the cutoff price upwards and leads to a spike in job destruction and a consequent increase in unemployment.

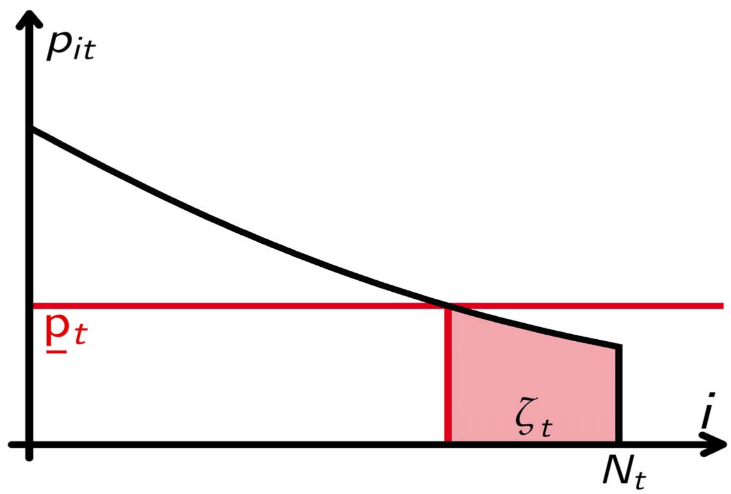

FIGURE 1. Price Distribution and the Cutoff Price.

Training costs are the second key element of the model. They help explain the response of vacancies and job creation to productivity shocks. To demonstrate the effect of training costs I rearrange equation (13) and substitute in the matching function (3):

$$\frac{c}{B} + K\left(\frac{U_t}{V_t}\right)^{\alpha} = \Gamma_t^F \frac{A_t}{w_t}\left(\frac{U_t}{V_t}\right)^{\alpha}, \tag{17}$$

Figure 2 depicts both sides of equation (17) as a function of $\left(\frac{U_t}{V_t}\right)^{\alpha}$. The right-hand side is an increasing function going through the origin. The left-hand side is a positively sloped line if training costs are present and a horizontal line if they are absent.

A persistent negative productivity shock lowers the expected discounted sum of future profits shifting the right-hand side of equation (17) downwards. This leads to an increase in market tightness, which is smaller in the absence of training costs. Intuitively, in the presence of training costs a decrease in the value of a new match not only leads to a decrease in potential future profits of a match, but also, through market tightness, to an increase in expected costs of a vacancy. The increase in expected costs further diminishes the incentives of firms to post vacancies.

To illustrate the combined effect of the job destruction margin and training costs I use a comparative statics exercise. I look at percentage changes in steady state unemployment, Δu, vacancies, Δv, job creation, Δm, job destruction, Δs, and the value of a match, $\Delta\Gamma$, resulting from a change in aggregate productivity ΔA.

In a steady-state the number of jobs destroyed is equal to the number of jobs created which in turn is determined by the numbers of unemployment and vacancies through

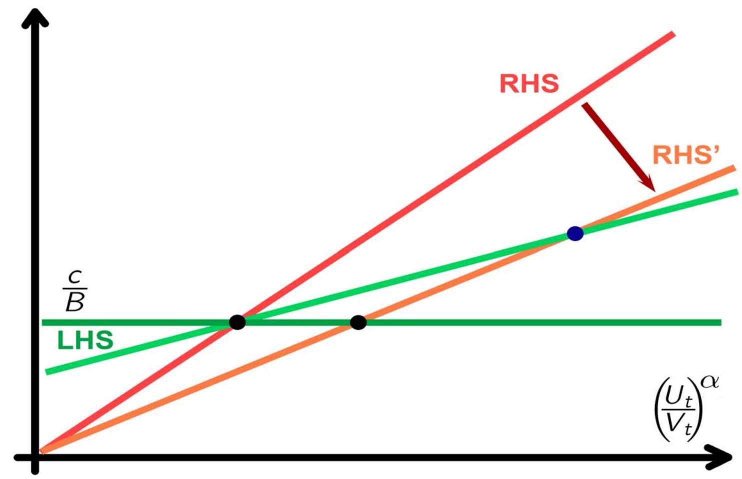

FIGURE 2. Effect of Training Cost on Market Tightness.

the matching function. Combining the steady-state version of equation (7) with the matching function (3) and taking the first difference it follows that the change in job destruction is equal the change in job creation which is a linear combination of changes in unemployment and vacancies between steady-states:

$$\Delta s = \Delta m = \alpha \Delta u + (1-\alpha)\Delta v. \tag{M}$$

Similarly, using equation (13) I derive the relationship between percentage changes in job creation, vacancies and the value of a match[5]:

$$\Delta v - \Delta m = \frac{\Delta \Gamma}{1-\varphi}, \tag{JC}$$

where $\varphi \in [0,1]$ is the fraction of training costs among total job creation costs. The job destruction margin represented in the model by equation (15) implies a negative relationship between job destruction and the value of a match:

$$\Delta s = -\tau \Delta \Gamma, \tag{JD}$$

where τ is the elasticity of the job destruction margin. I call equation (M) the matching curve, equation (JC) - the job creation curve, and equation (JD) - the job destruction curve. I combine these three equations to derive the slope of the Beveridge curve:

$$\frac{\Delta v}{\Delta u} = \frac{-\alpha\left(1-\tau\left(1-\varphi\right)\right)}{1-\alpha\left(1-\tau\left(1-\varphi\right)\right)}, \tag{BC}$$

[5]I ignore changes in wages and productivity in this equation because they are an order of magnitude smaller compared to changes in the rest of the variables and outweigh each other

The slope of the Beverdige curve is jointly determined by the elasticity of the matching function, α, the elasticity of the job destruction margin, τ, and the fraction of training costs, φ. Finally, as summarized by equation (11) the value of a match, the sum of expected discounted future profits, is driven by shocks to labor productivity:

$$\Delta\Gamma = \chi\Delta A, \tag{P}$$

where the elasticity χ is determined by the size of total job creation costs. As emphasized by Hagedorn and Manovskii (2008 [17]) we need the size of total job creation costs and the value of the match to be small in steady-state to match the empirically large response of labor market variables to productivity shocks. This holds true in my model. However, the slope of the Beveridge curve derived above does not depend on this elasticity.

I illustrate how both the job destruction margin and training costs affect the slope of the Beveridge curve. I look at four cases: constant or variable exogenous job destruction, as well as endogenous job destruction with or without training costs.

The case when all job destruction is exogenous and constant is represented by $\tau = 0$. In this case the Beveridge curve coincides with the matching curve. Its slope is determined by the elasticity of the matching function, α. One can infer the elasticity of the matching function directly from comparing the volatilities of market tightness and the job finding rate following Shimer (2005, [29]). I use this method to set α to 0.72. In this case the slope of the Beveridge curve is negative:

$$\tau = 0: \quad \frac{\Delta v}{\Delta u} = \frac{-\alpha}{1-\alpha} = -2.57, \tag{I}$$

The case of exogenous shocks to the job destruction rate is summarized by the extreme value of $\tau = +\infty$. In this case the Beveridge curve is positively sloped and coincides with the job creation curve:

$$\tau = +\infty: \quad \frac{\Delta v}{\Delta u} = 1, \tag{II}$$

Now let me look at the case of endogenous job destruction. In the data the elasticity of job destruction to productivity shocks is around 30. Taking into account the elasticity of a match value to productivity χ of around 10 leads to a value of $\tau \approx 3$. Under this calibration a model with an operational job destruction margin but in absence of training costs also leads to a positively sloped Beveridge curve:

$$\tau = 3, \varphi = 0: \quad \frac{\Delta v}{\Delta u} = 0.59, \tag{III}$$

Increasing the fraction of training costs solves this problem. I need training costs to correspond to 90 percent of job creation costs leaving 10 percent to recruitment costs in order to generate the slope of -1 which is the property of US data.

$$\tau = 3, \varphi = 0.9: \quad \frac{\Delta v}{\Delta u} \approx -1, \tag{IV}$$

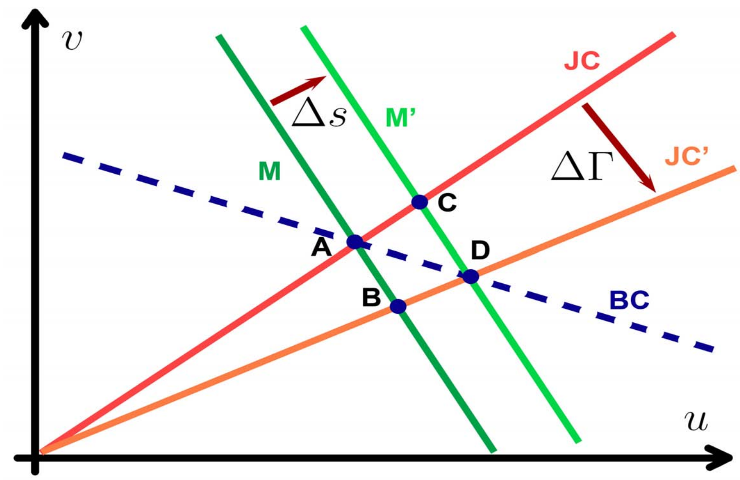

FIGURE 3. Beveridge Curve.

Figure 3 illustrates how the slope of the Beveridge curve arises as a combination of shifts in the matching curve and the job creation curve. Point A is the original steady-state. Point B represents the new steady-state when the job destruction margin is absent and corresponds to case I. Point C represents the new steady-state if job destruction is exogenous or if training costs are absent, corresponding to cases II and III. Point D represents the new steady-state when job destruction is endogenous and training costs are a large fraction of total costs. It corresponds to case IV with the slope of the Beveridge curve resembling the behavior of the data.

The dynamic response of the calibrated model is summarized by impulse response functions to a productivity shock depicted in Figure 4. It works as follows. A negative productivity shock lowers contemporaneous profits of firms leading to a sharp increase in job destruction. As more workers lose their jobs the number of unemployed workers increases making the market tighter. A decline in contemporaneous productivity also leads to a decline in expected future profits. This lowers the benefits of firms from creating new jobs. An increase in market tightness led by an increase in unemployment additionally increases the expected costs of posting a vacancy. This significantly undermines the incentives for firms to post vacancies. The number of vacancies falls. As the number of employment opportunities shrinks, the number of new jobs also falls.

Because of sharp employment adjustment in the first period the least productive jobs have already been destroyed and the job destruction rate quickly returns close to

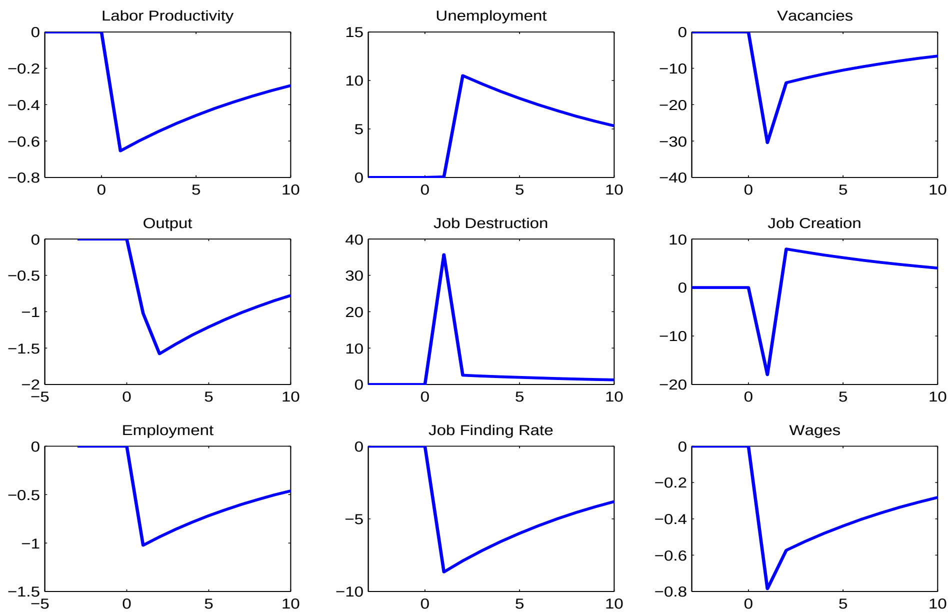

FIGURE 4. Impulse Responses to a 1 Standard Deviation Productivity Shock.

its original level. As productivity slowly recovers the cutoff price for job destruction slowly returns to its original level. As firms see an increase in future profits, they start posting more vacancies and creating more jobs.

III. Empirical Methodology

To explore the ability of the model to fit the data I use recently developed Bayesian methods for analyzing DSGE models.[6] This methodology has several advantages when compared to commonly used calibration strategies. In the context of vigorous debates over parameters of the standard matching model, the Bayesian framework allows me to remain more agnostic. I let the data choose a calibration which is most likely to explain its behavior.

The second advantage of this methodology is that a likelihood function gives natural weights to different moments of the data instead of focusing on just a few. In addition, setting relatively wide priors allows me to conduct a sensitivity analysis of model performance to the parameter combination. If I find that a posterior estimate is as wide as the prior, then the exact value of the corresponding parameter is not important for explaining the data. Conversely, if a posterior estimate is very narrow, this means that model dynamics are very sensitive to the exact calibration of a parameter.

In this section I describe the strategy that is used to evaluate the model. I also discuss data sources and priors distributions.

First, I solve for the steady-state of the model. I then log-linearize the equations of the model around the steady-state and solve the resulting system of linear forward-looking equations using a method developed by Sims (2002, [32]). This gives me the state-space representation of the model:

$$X_t = FX_{t-1} + G\varepsilon_t \tag{18}$$

$$Y_t = HX_t + \upsilon_t, \tag{19}$$

where X_t is the vector of state variables and Y_t is the vector of observables. I assume that the innovation to labor productivity, ε_t, is the only exogenous shock in the model. I attribute all the residual variation in observed fluctuations to a vector of measurement errors, υ_t. The fraction of variations in Y_t explained by the model is represented by HX_t and the unexplained component is captured by the error term. To allow for enough variation in the data and to avoid stochastic singularity I assume there are as many sources of measurement error as there are observables so that each measurement equation has its own error term[7].

I treat the model as the data-generating process and use the Kalman filter to construct the likelihood function of the data conditional on parameters. I combine the likelihood function with the prior distribution of parameters to obtain the posterior

[6] A survey of these methods is provided for instance by An and Schorfheide (2007 [1]).

[7] To avoid stochastic singularity I need at least as many shocks as observed variables. If I include productivity shocks then I can exclude one of the measurement errors. I choose not to do so because that would imply a prior choice of the variable I want the model to explain exactly. I choose to remain agnostic about the choice of variables the model can explain best.

distribution of parameters and use the random-walk Metropolis-Hastings algorithm to explore it numerically[8]. I then use the Kalman filter to obtain smoothed estimates of the shock process for labor productivity using parameter values at posterior mode.

III.1. **Data.** For estimation I use eight observables: output, unemployment, vacancies, job destruction, job creation, the job finding rate, real wages and labor productivity. All data is quarterly, seasonally adjusted for the period 1951:1 - 2004:4. The output series is the real GDP index provided by the BEA divided by the labor force. The unemployment series is the unemployment rate for the over-16-year-olds provided by the BLS. The vacancy series is the index of help-wanted advertisements provided by the Conference Board. The series for real wages is constructed by dividing average hourly earnings in private nonfarm payrolls by the consumption price index.

As a proxy for job destruction and job creation I use destruction and creation rates in manufacturing constructed by Davis et. al. (2006 [10]). Davis, Faberman and Haltiwanger also provide series for all sectors for a much shorter period of time. The series for manufacturing and for all sectors have notably different volatilities, but a correlation close to one (see appendix). I use this observation to scale the series for manufacturing to represent the whole economy. Finally, I use the job finding rate series computed from CPS data by Shimer (2005 [29]). I use the series for labor productivity, measured as real output per worker in the non-farm business sector. This series is constructed by the BLS from the National Income and Product Accounts and the Current Employment Statistics. I apply the Hodrick-Prescott filter with smoothing parameter 1600 to detrend all series.

III.2. **Priors.** There are eight structural parameters in the model, of which $\{g, B, c, K\}$ are hard to compare with micro studies. Instead of estimating them directly I construct an alternative set of steady-state values which I then treat as parameters. I define $u = U_{ss}$ - the steady-state unemployment rate, $s = \zeta_{ss}$ - the job destruction rate, $\varphi = \frac{KM_{ss}}{cV_{ss}+KM_{ss}}$ - the fraction of training costs in total head-hunting services and $\pi = \frac{X_{ss}}{X_{ss}+N_{ss}}$ - the fraction of head-hunters among total employment. I then use the fact that conditional on the rest of the parameters there is a one-to-one mapping between $\{g, B, c, K\}$ and $\{u, s, \varphi, \pi\}$.

Prior distributions are reported in Table 1. I choose prior means based on values used in previous studies. I make the priors uninformative by setting prior standard deviations to relatively large values whenever possible. This allows me to remain agnostic and let the data choose the parameter combination which is most likely to capture the dynamic properties of the data. For parameters with support on the unit interval I use the Beta distribution and for real-valued parameters I use the Gamma distribution.

[8]The algorithm is extensively discussed in Geweke (1999 [16]). I use the open source DYNARE software developed by Collard and Juillard (2003, [8]) and collaborators.

Parameter		Density	Mean	Std. Dev.
Discount factor	β	Fixed	0.99	-
Matching elasticity	α	Beta	0.5	0.2
Bargaining power of worker	ψ	Gamma	0.5	0.2
Curvature of demand	γ	Gamma	0.25	0.125
Unemployment rate	u	Fixed	0.056	-
Job destruction rate	s	Gamma	0.03	0.015
Fraction of match-specific costs	φ	Beta	0.5	0.2
Fraction of head-hunters	π	log Normal	0.0006	0.0004
Persistence of productivity	ρ	Beta	0.50	0.20

Table 1. Prior Distributions.

I set the discount factor β to 0.99. The unemployment rate is fixed at its historical mean of 5.6 percent. The fraction of head-hunters is set to 0.06 percent which in steady-state implies total job creation costs per match to be 4 percent of the quarterly wage of a new hire. This is close to the upper bound for these costs according to micro estimates emphasized by Hagedorn and Manovskii (2008 [17]).

I choose the prior for the fraction of head-hunters in total employment to be log-normal to allow for a wider prior distribution. I set the mean of the steady-state job destruction rate at 3 percent to match the average flow from employment to unemployment during a quarter.[9] I choose to be completely agnostic about the bargaining power, the matching elasticity, the fraction of match-specific costs, the curvature of demand and the autoregressive parameter of labor productivity. As priors for standard deviations of errors I choose inverse-gamma distributions with standard deviations of 0.5 percent for productivity, 1 percent for output and wages and 5 percent for all other variables. I run 10 blocks 5000 iterations each from different starting points and target an acceptance rate of 30 percent.

IV. Results

In this section I describe the posterior estimates and discuss their implications for calibration of labor matching models. I then evaluate the fit of the model along different dimensions and explore the importance of the two key elements.

IV.1. **Parameter Estimates.** I report means and 90 percent confidence intervals of posterior estimates in Table 2. Curvature of demand is tightly estimated at 22 percent and implies demand elasticity of 0.8. The matching elasticity is estimated to be 0.68, very close to Shimer's estimate of 0.72. This is not surprising given that

[9] This is close to the finding of Nagypal (2008 [25]) that only about 20 percent of all separations (which are approximately 10 percent per quarter) correspond to transitions from employment to unemployment. Also according to the distribution of unemployment duration provided by the BLS about 60 percent of all unemployed find jobs within a quarter which is about 3 percent of the labor force in steady-state.

the parameter is identified in the same way through the relationship between the job finding rate and market tightness.

The estimated fraction of head-hunters of 0.04 percent is slightly less than the prior and has a relatively tight confidence interval, corresponding to total job creation costs of approximately 2 percent of quarterly wages of a new hire. This number is smaller than the micro estimates of 4 percent mentioned above and is identified in a somewhat similar way. By varying this parameter I find that it is identified by the volatility of job destruction. In order to generate large enough variations in job destruction the value of a match needs to be sufficiently volatile. This requires a relatively low value of a match and, hence, relatively small costs. In previous calibrations small and volatile rents were used to increase the volatility of job creation. The low estimate of total costs required to match volatility of job destruction in my model can be a consequence of the simplifying assumptions I made to model the job destruction margin and should be used with caution.

Parameter		Prior	Posterior	
		Mean	Mean	90% conf. interval
Matching elasticity	α	0.5	0.66	[0.64 0.67]
Bargaining power of worker	ψ	0.5	0.28	[0.06 0.47]
Curvature of demand	γ	0.25	0.22	[0.21 0.23]
Job destruction rate	s	0.03	0.0211	[0.0195, 0.0223]
Fraction match-specific costs	φ	0.5	0.912	[0.907 0.917]
Fraction of head-hunters	π	.0006	.0004	[.0003 .0007]
AR parameter of technology	ρ	0.5	0.92	[0.88 0.95]

Table 2. Posterior Estimates

The fraction of training costs and other costs specific to a match in total creation costs is tightly estimated to be 91 percent. This matches surprisingly closely the evidence presented by Silva and Toledo (2009, [31]). They estimate average recruiting costs to be about 9% of average on-the-job training costs. As shown below this parameter value is key to explaining the behavior of vacancies, in particular the negative correlation between the number of vacancies and unemployment.

The posterior estimate of the bargaining power of workers in steady-state has a very wide confidence interval. This implies that the value of the bargaining power has little or no effect on the dynamic properties of the model. Despite the low value of rents the proportion in which they are split between the worker and the firm has almost no effect on the labor market allocation. This result is different from the one commonly found in the literature for the following reason. Variations in real wages in my model are driven by changes in the consumption price index while nominal wages remain almost unchanged. This is also a feature of US data.

The posterior mean of the job destruction rate is tightly estimated at 2 percent. This value is substantially lower than the prior and implies a job finding rate of 34 percent. This estimate is close to that of Cole and Rogerson (1999 [7]). However, it is misleading to interpret this finding as implying unreasonably high duration of unemployment.

Figure 5 illustrates my interpretation of this result. Every quarter about 10 percent of employed workers separate from their jobs. Most of these separations are job-to-job transitions, with only about 3.5 percent of workers becoming unemployed. Not all of these jobs are destroyed. Some workers leave jobs which are quickly filled by other workers. Job destruction is measured as the sum of job decreases in contracting establishments and in the model corresponds to about 2 percent of employed workers transiting from employment to unemployment. Similarly, job creation is measured as the sum of job increases in expanding establishments and corresponds to a fraction of jobs filled by unemployed workers. Thus, the job finding rate of 34 percent corresponds to the probability of an unemployed worker matching with one of the newly created jobs, not all jobs filled by unemployed workers. This job finding rate is a fraction of the total job finding rate.

Turnover between employment and unemployment unrelated to job creation and destruction can potentially explain the discrepancy in the cyclicality of job destruction rates measured from surveys of firms and separation rates to unemployment measured from surveys of workers. If the matching process is random, the matching rates of workers both with new jobs and with existing jobs should fall in a recession. That implies a decline in turnover between employment and unemployment. If this is the case, then separation rates measured using surveys of workers include two components. The job destruction component increases in recessions and the turnover component falls. The sum of these two rates will vary, if at all, much less than the job destruction rate. This is what both Shimer (2007 [30]) and Fujita and Ramey (2007 [13]) find.

The finding that a relatively low job finding rate is required to match data on job creation and job destruction leads Cole and Rogerson (1999 [7]) to conclude that in order for the MP model to match data one needs to assume counterfactually long duration of unemployment. Given the ability of the model to replicate the most salient features of empirical behavior of labor market variables this finding leads me to a somewhat opposite conclusion. I conclude that accounting for job-to-job flows and turnover between employment and unemployment might be of little value for understanding the business cycle behavior of the labor market.

IV.2. **Model Fit.** To evaluate the fit of the model I compare the second moments of the data with moments of artificial data generated by the model when hit by the estimated productivity shock. Table 3 compares standard deviations of eight observables of interest as well as their correlations with output. The results indicate that the model fits the data well, explaining virtually all of the fluctuations in job destruction and job creation rates, vacancies, the job finding rate, about two thirds of

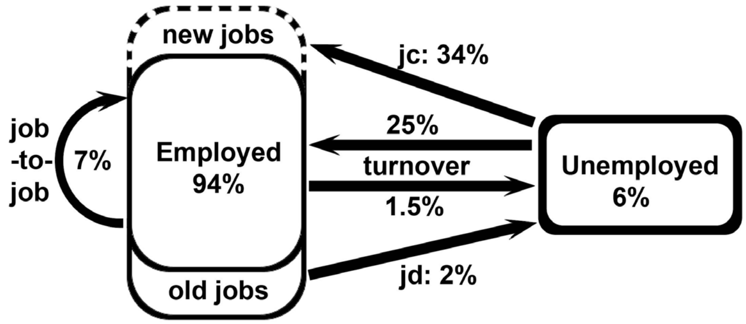

FIGURE 5. Quarterly Worker Flows.

fluctuations in unemployment and half of fluctuations in wages with a single shock to labor productivity. The required variations in labor productivity also have reasonable magnitude. Figures in appendix 3 illustrate the fit of the model. Given the simplicity of the model this is a remarkable result.

The model matches well most of the cross correlations between observables with one exception. In the data job creation responds to productivity shocks slower than the model predicts. When compared to the model the data on job creation has a lag of about one quarter. This is essentially the only dimension on which the model doesn't perform well. The gap between wages in the model and in the data is satisfactory given that the discrepancy between the two commonly used series for real wages is large[10].

Standard Deviations								
	Y	U	V	JD	JC	JF	W	Y/N
Data	1.51	12.3	14.4	12.6	7.5	7.7	0.88	1.05
Model	1.17	7.6	14.0	10.3	6.7	7.1	0.47	0.46
Procyclicality								
Data	1	-0.80	0.80	-0.64	0.21	0.76	0.28	0.60
Model	1	-0.96	0.91	-0.26	-0.31	0.98	0.96	0.99

Table 3. Comparison of Second Moments

To study the importance of the two key assumptions for model performance I compare the performance of the benchmark model with four alternative specifications. In the first alternative specification I set the fraction of training costs in job creation costs to zero and re-estimate the model. The second alternative specification is the model of Lubik (2009 [20]), where jobs are destroyed exogenously at a constant rate, but non-linear job creation costs are allowed. In this specification I measure the joint

[10]The two commonly used series for real wages are average hourly earnings in private nonfarm payrolls divided by the consumption price index and the labor share times labor productivity. The root mean square difference between the two detrended series is 0.94 log points which is comparable to average wage variability over the cycle of 0.97 log points.

explanatory power of shocks to productivity, preferences and markups[11]. Lubik applies similar methods to the same data on GDP, unemployment, vacancies and wages. He allows for exogenous shocks to preferences and market power, which are somewhat similar to the preference specification I use. Thus, this specification provides a comparable account for the explanatory power of a model with constant exogenous job destruction but variable creation costs.

In the third alternative specification I apply the same estimation strategy to Shimer's model allowing for variations in the value of the outside option and the bargaining weight. I denote this specification "H-M" because the resulting estimates replicate the calibration of Hagedorn and Manovskii (2008, [17]). The last specification is Shimer's original calibration.

Fraction of Variation Explained								
	Y	U	V	JD	JC	JF	W	Y/N
1. Benchmark	.77	.62	.96	.82	.90	.91	.45	.70
2. K=0	.78	.66	.25	.85	.89	.85	.45	.66
3. Lubik	.79	.08	.61	.00	–	–	.85	–
4. H-M	–	.42	.99	.00	.42	.68	.44	.82
5. Shimer	–	.05	.09	.00	.06	.06	.91	.93

Table 4. Explanatory Power of Alternative Specifications

Results of model comparison are summarized in Table 4. Numbers in the rows of Table 4 describe fractions of standard deviations of the data explained by the four alternative specifications and the benchmark specification. Comparison of lines 4 and 5 indicates that the calibration of Hagedorn and Manovskii indeed improves the performance of the labor search model, explaining almost half of variations in unemployment and most of variations in vacancies. Line 3 of the table demonstrates that even while allowing for vacancy creation costs the addition of other sources of fluctuations to the model significantly lowers the contribution of productivity shocks. Direct introduction of training costs into Shimer's model under both calibrations does not alter its empirical performance and, therefore, is not reported.

Comparison of lines 1 and 2 to line 4 demonstrates that both the job destruction margin and training costs are key to the empirical performance of the benchmark model. Explaining variations in job destruction enhances the ability of the model to explain the behavior of unemployment, accounting for its initial increases during recessions. Incorporating training costs is crucial for explaining the decrease in vacancies and the sluggish response of job creation.

Additional dimensions where the benchmark model outperforms its predecessors are impulse response functions to a recessionary shock obtained by Caballero and

[11]The only residual source of variations not included is represented by shocks to matching efficiency

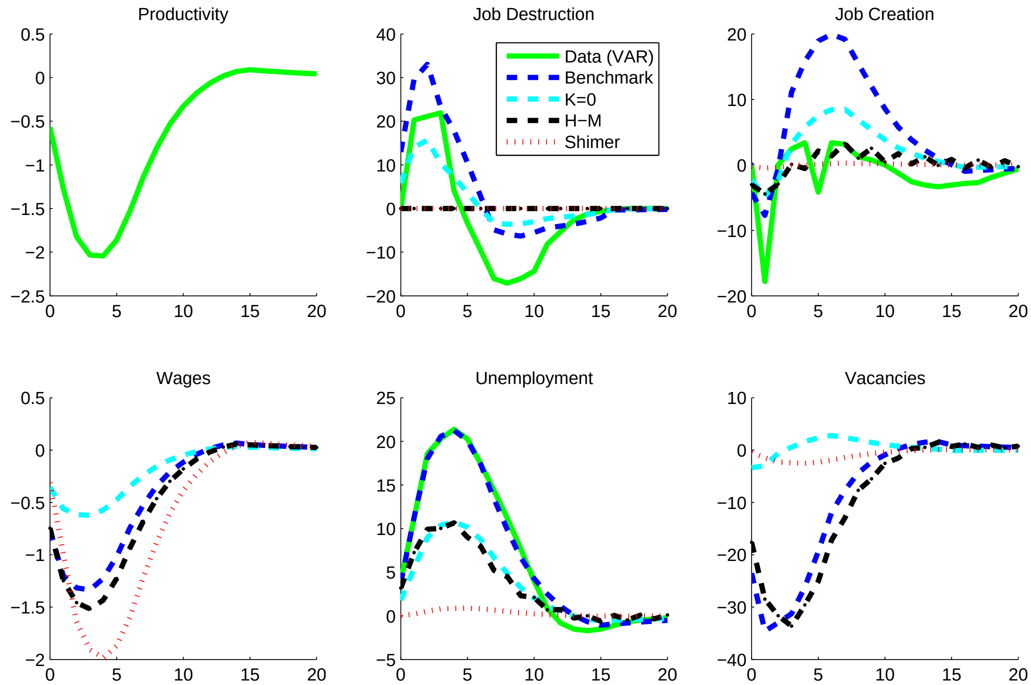

Figure 6. Impulse Responses of Alternative Specifications to a Recessionary Shock.

Hammour (1998, [4]) by estimating a semi-structural VAR. Figure 6 compares impulse responses of a VAR to impulse responses generated by four model specifications[12]. Shimer's calibration generates almost no response to a recessionary shock. The calibration of Hagedorn and Manovskii explains about half of the response of unemployment, all of it through the job creation margin. Introduction of job destruction alone also explains a large fraction of fluctuations in unemployment, but is much worse at explaining the behavior of vacancies. The benchmark model explains, both quantitatively and qualitatively, a large fraction of the observed fluctuations in all the variables of interest.

To my knowledge this model is the first incarnation of the MP model capable of generating realistic patterns of a large set of labor market variables as a result of a single shock to labor productivity of reasonable magnitude. It also generates the celebrated counter-clockwise loops around the Beveridge curve as depicted in Figure 7. The Figure compares the relatively flat movement along the matching curve (M) in the absence of the job destruction margin and the loop around the Beverdige curve (BC) when it is present.

[12] I construct a sequence of shocks to productivity such that the response generated by my model matches closely the path of unemployment in the VAR. I then use the same sequence of shocks for all four models.

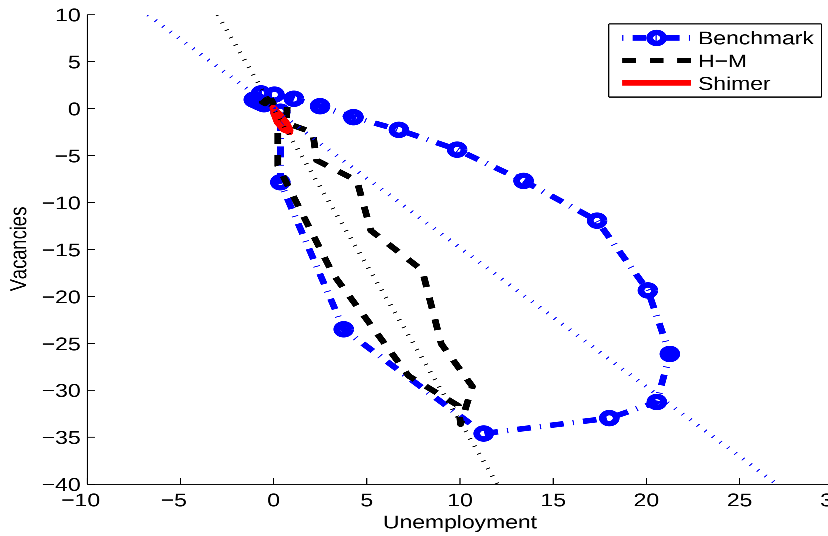

FIGURE 7. Counter-clockwise Loops Around the Beveridge Curve as a Response to a Recessionary Shock.

Finally, the exogenous shock to labor productivity, recovered from the estimation of the model and designed to simultaneously explain the behavior of other labor market variables, matches reasonably well the cyclical properties of labor productivity in the data. However, this exogenous shock should be viewed more broadly as a latent factor which captures a combination of supply and demand disturbances. In fact the model produces responses to demand and supply shocks that are almost indistinguishable from each other.

V. Conclusion

In this paper I emphasize two elements of the original Mortensen-Pissarides model: the job destruction margin and training costs. I show that these two elements are crucial for explaining the sharp increases in unemployment and the sluggish response of vacancies and job creation in recessions. I embed these two key elements into a general equilibrium model with matching. I show that such a model driven by a single shock to labor productivity can simultaneously explain most of variations in output, unemployment, vacancies, job creation, job destruction, the job finding rate and real wages. I estimate parameter values which provide best fit of the data and find that they are all of plausible magnitude.

The contribution of the paper is not only to explain a large set of a labor market variables with a single aggregate shock but also to provide a simple mechanism which endogenously generates fluctuations in the labor wedge. When analyzing the labor wedge through a prism of a labor matching model, Cheremukhin and Restrepo-Echavarria (2009, [5]) find that a friction in job destruction is responsible for the initial sharp increases in unemployment in recessions while frictions to job creation are responsible for the subsequent slow recoveries. In this paper I take a stand on what these frictions are.

Desirable extensions of this model include specifications of job creation costs that allow for additional delays in creation and detailed microeconomic studies of creation costs. A further direction of research is the interaction of matching frictions with market power as discussed by Rotemberg (2006, [28]). Among the main unresolved puzzles are the source of shocks driving the economy and the possibility of endogenous cycles in models with matching frictions.

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VI. Appendix

VI.1. **Competitive Equilibrium.** In this section I briefly describe the setup and solution of the competitive equilibrium. I then compare it with the planner's solution to draw conclusions about the optimal division of rents.

A representative agent acts both as a household and a firm to maximize discounted utility of the form:

$$\max_{\left\{q_{it}^d, X_t^s, N_{t+1}^s, U_t^s\right\}, \left\{q_{it}^s, X_t^d, N_{t+1}^d, V_t^d\right\}} E_0 \Sigma_t^{\infty} \beta^t U\left(C_t, N_t^s, X_t^s\right) \tag{20}$$

where the first set of variables corresponds to the choices of the household and the second set corresponds to the choices of the firm. Welfare is maximized subject to the joint budget constraint:

$$\int_0^{N_t^s} p_{it} q_{it}^d di = w_t X_t^s + W_t N_t^s + N_t^d \frac{1}{N_t} \int_0^{N_t} \left(p_{it} q_{it}^s - W_t\right) di - w_t X_t^d. \tag{21}$$

The household also takes into account the structure of the consumption aggregator:

$$C_t = \int_0^{N_t^s} z_{it} q_{it}^d di, \tag{22}$$

the labor accumulation constraint, where the job finding rate and the job destruction rate are taken as given:

$$N_{t+1}^s = N_t^s \left(1 - \zeta_t\right) + U_t^s \frac{M\left(U_t, V_t\right)}{U_t}, \tag{23}$$

and the constraint on aggregate time use:

$$N_t^s + X_t^s + U_t^s = 1. \tag{24}$$

The firm takes as given the vacancy filling rate and optimizes profits subject to the head-hunting technology:

$$A_t X_t^d = \left(K \frac{M\left(U_t, V_t\right)}{V_t} + c\right) V_t^d, \tag{25}$$

the labor accumulation equation:

$$N_{t+1}^d = N_t^d \left(1 - \zeta_t\right) + V_t^d \frac{M\left(U_t, V_t\right)}{V_t}, \tag{26}$$

and the production technology:

$$q_{it}^s = A_t. \tag{27}$$

The values of different states of the firms and the workers arise as Lagrange multipliers on the corresponding constraints. The solution of the joint problem satisfies the following first-order conditions. The values of the numerair and of individual prices satisfy:

$$
\begin{aligned}
U'_{C_t} z_{it} &= \lambda_t p_{it}, && (28)\\
U'_{C_t} &= P_t \lambda_t. && (29)
\end{aligned}
$$

The wage of the head-hunter:

$$w_t = -\frac{U'_{X_t}}{\lambda_t} + \mu_t. \tag{30}$$

Value of unemployed:

$$\mu_t = \Gamma_t^W \frac{M_t}{U_t}. \tag{31}$$

Value of a job to the household:

$$\Gamma_t^W = E_t \beta \frac{\lambda_{t+1}}{\lambda_t} \left[W_{t+1} + \frac{U'_{N_{t+1}}}{\lambda_{t+1}} - \mu_{t+1} + \Gamma_{t+1}^W \left(1 - \zeta_{t+1}\right) \right]. \tag{32}$$

Value of a head-hunter:

$$w_t = \eta_t. \tag{33}$$

Free entry of vacancies:

$$\eta_t \frac{c}{A_t} = \left(\Gamma_t^F - \eta_t \frac{K}{A_t} \right) \frac{M_t}{V_t}. \tag{34}$$

Value of a job to the firm:

$$\Gamma_t^F = E_t \beta \frac{\lambda_{t+1}}{\lambda_t} \left(\frac{P_{t+1} C_{t+1}}{N_{t+1}} - W_{t+1} + \Gamma_{t+1}^F \left(1 - \zeta_{t+1}\right) \right). \tag{35}$$

VI.2. **Planner's solution.** The above competitive equilibrium lacks an equilibrium condition which would split the rents between the worker and the firm and pin down wages. In this subsection I derive the first-order conditions for the planner and compare them with the competitive equilibrium described above. The planner maximizes utility:

$$\max_{\left\{q_{it}^d, X_t, N_{t+1}, U_t, V_t\right\}} E_0 \Sigma_t^\infty \beta^t U\left(C_t, N_t, X_t\right) \tag{36}$$

subject to the budget constraint:

$$\int_0^{N_t} p_{it} q_{it}^d di = N_t \frac{1}{N_t} \int_0^{N_t} \left(p_{it} q_{it}^s\right) di \tag{37}$$

the consumption aggregator:

$$Q_t = \int_0^{N_t} z_{it} q_{it}^d di \tag{38}$$

the production technology:

$$q_{it}^s = A_t \tag{39}$$

the head-hunting technology:

$$A_t X_t = KM(U_t, V_t) + cV_t \tag{40}$$

the labor accumulation equation:

$$N_{t+1} = N_t (1 - \zeta_t) + M(U_t, V_t) \tag{41}$$

the constraint on time use:

$$N_t + X_t + U_t = 1 \tag{42}$$

First order conditions determine individual prices and the value of the numeraire:

$$U'_{C_t} z_{it} = \lambda_t p_{it} \tag{43}$$

$$U'_{C_t} = P_t \lambda_t \tag{44}$$

the value of a head-hunter:

$$\eta_t + \frac{U'_{X_t}}{\lambda_t} = \mu_t \tag{45}$$

the value of an unemployed:

$$\mu_t = \left(\Gamma_t - \eta_t \frac{K}{A_t}\right) \frac{\partial M}{\partial U_t} \tag{46}$$

the value of a job:

$$\Gamma_t = E_t \beta \frac{\lambda_{t+1}}{\lambda_t} \left[\frac{P_{t+1} C_{t+1}}{N_{t+1}} + \frac{U'_{N_{t+1}}}{\lambda_{t+1}} - \mu_{t+1} + \Gamma_{t+1} \left(1 - \zeta_{t+1}^s\right) \right]. \tag{47}$$

Optimal number of vacancies satisfies:

$$\eta_t \frac{c}{A_t} = \left(\Gamma_t - \eta_t \frac{K}{A_t}\right) \frac{\partial M_t}{\partial V_t} \tag{48}$$

First, comparing equations (32) and (35) to equation (47) we can conclude that the social value of a match is equal to the sum of private values. Second, comparing the equations for the competitive equilibrium, (31) and (34), and for the planner's solution, (46) and (48), it follows that the competitive equilibrium is Pareto-optimal if and only if the following analog of the Hosios condition is satisfied:

$$\Gamma_t^W \frac{\partial M}{\partial V_t} \frac{V_t}{M_t} = \frac{\partial M}{\partial U_t} \frac{U_t}{M_t} \left(\Gamma_t^F - \eta_t \frac{K}{A_t} \right) \tag{49}$$

VI.3. **Tables and Graphs.**

std	1.51	12.4	14.4	12.6	7.5	7.8	0.88	1.05
corr	Y	U	V	JD	JC	JF	W	Y/N
GDP		-0.80	0.80	-0.64	0.21	0.76	0.28	0.60
U			-0.95	0.45	0.05	-0.93	-0.19	-0.13
V				-0.50	0.01	0.92	0.23	0.16
JD					-0.59	-0.45	-0.25	-0.55
JC						0.04	0.07	0.36
JF							0.17	0.11
W								0.42

Table 5. Moments of the Data

std	1.17	7.6	14.0	10.3	6.7	7.1	0.47	0.46
corr	Y	U	V	JD	JC	JF	W	Y/N
GDP		-0.96	0.92	-0.26	-0.31	0.98	0.97	0.99
U			-0.79	0.01	0.54	-0.91	-0.88	-0.91
V				-0.60	0.07	0.96	0.98	0.96
JD					-0.82	-0.40	-0.48	-0.40
JC						-0.17	-0.09	-0.17
JF							0.99	0.99
JF								0.99

Table 6. Model Generated Moments

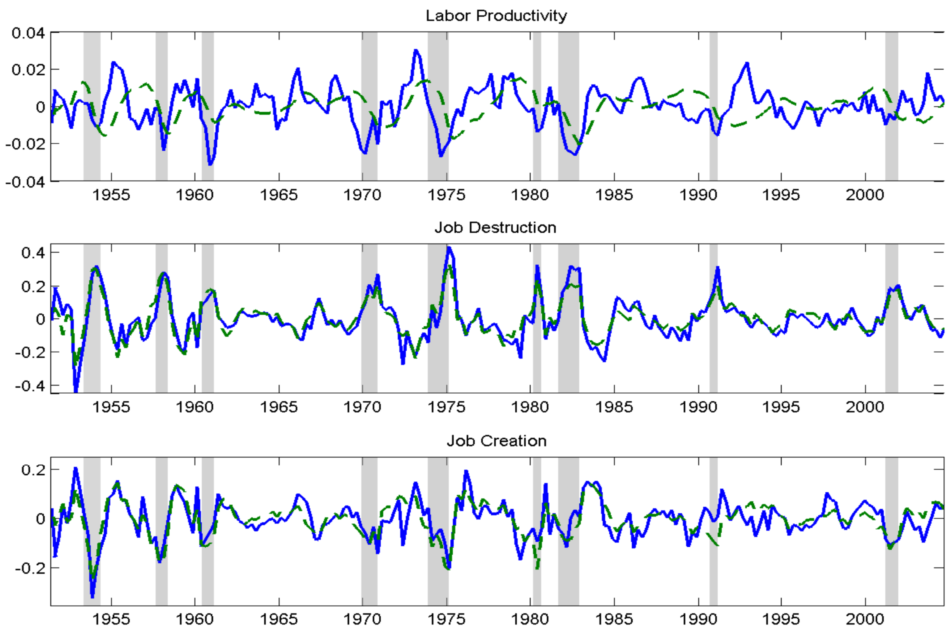

FIGURE 8. Fit of Productivity, Job Destruction and Job Creation.

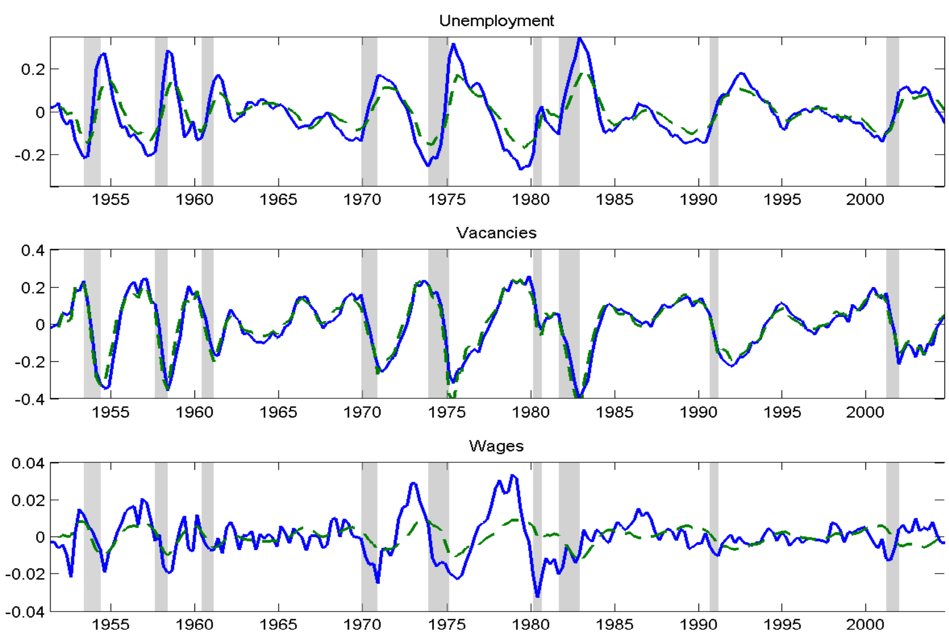

FIGURE 9. Fit of Unemployment, Vacancies and Wages.

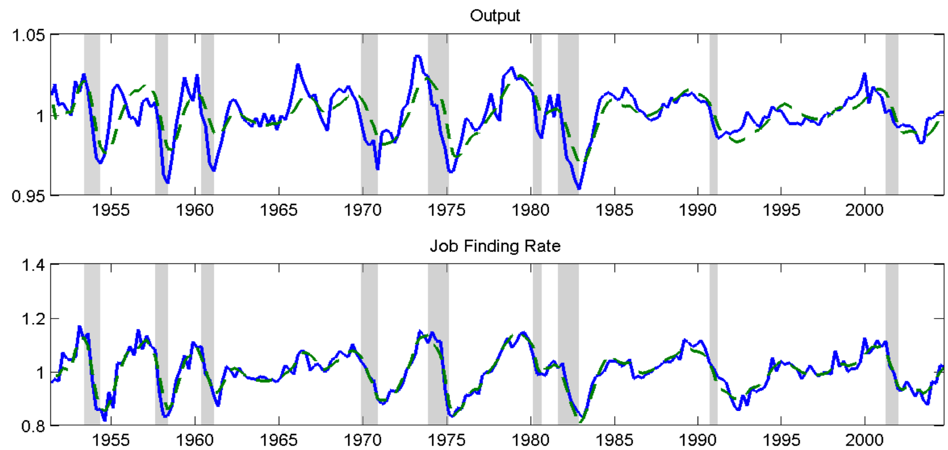

FIGURE 10. Fit of Output and the Job Finding Rate.

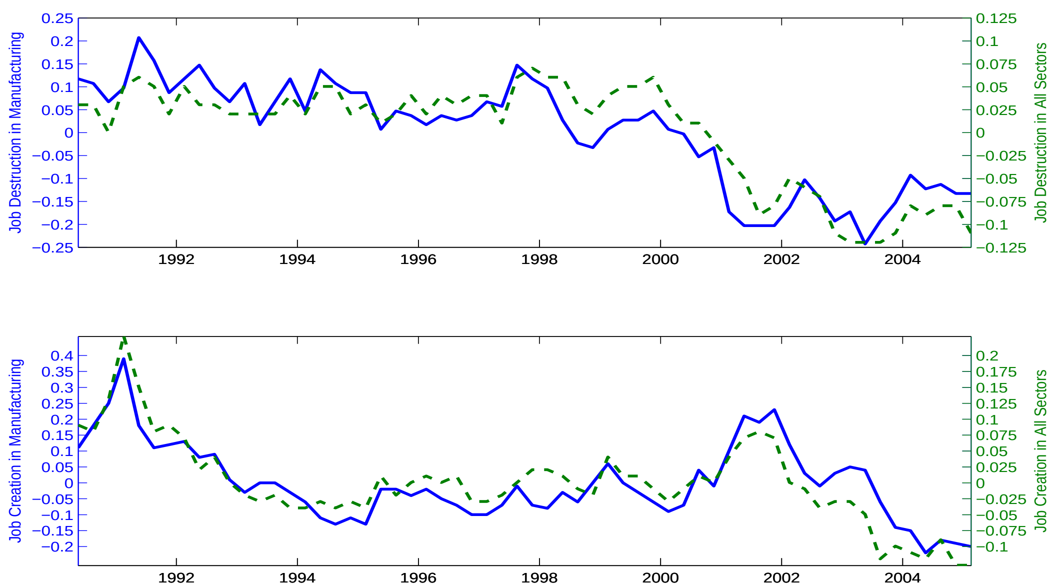

FIGURE 11. Comparison of Creation and Destruction for Manufacturing and All Sectors.

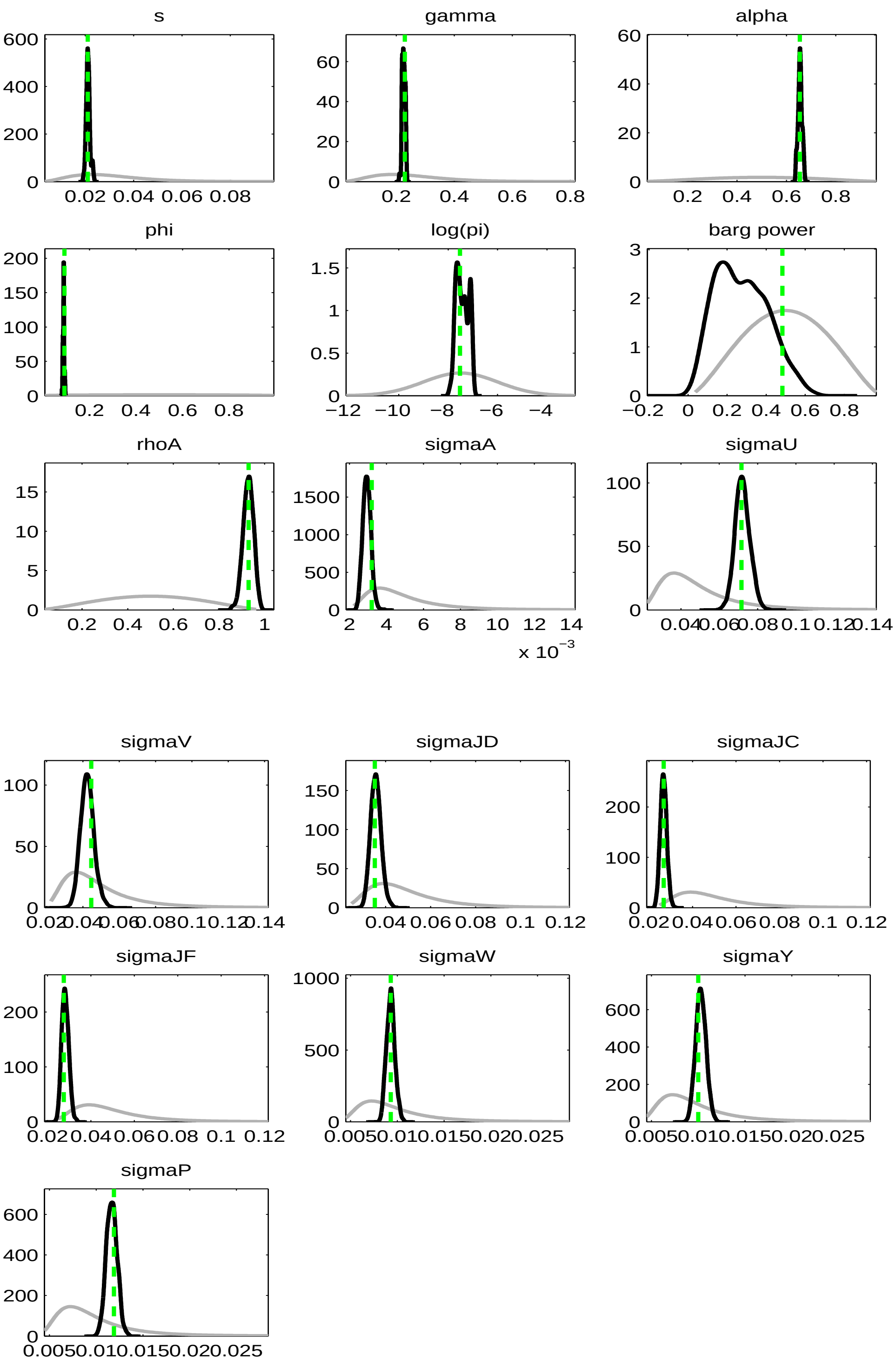

FIGURE 12. Comparison of Prior and Posterior Distributions.