Competition relative to Incentive Contracts in Common Agency

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Abstract

In most common agency problems, competing principals non-cooperatively incentivize a privately-informed agent's action choice with monetary transfer by offering incentive contracts (e.g., nonlinear prices) that specify the amounts of monetary transfer as a *function* of the part of the agent's action that is contractible. This paper shows that whenever contracting in common agency involves monetary transfer, the set of *all* equilibrium allocations relative to incentive contracts is identical to the set of *all* equilibrium allocations relative to any complex mechanisms that assign incentive contracts contingent on the agent's messages.

1 Introduction

In common agency problems, competing principals try to control a privately-informed agent's action choice because their preferences on the agent's actions are not aligned with one another. Common agency problems are prevalent in practice, ranging from lobbying, public good provision, selling private goods, vertical contracting to financial contracting. In most common agency problems, each principal non-cooperatively incentivizes the agent's action

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choice with monetary transfer by offering to the agent an *incentive contract* that specifies the amount of monetary transfer as a *function* of the part of the agent's action that is contractible. Each principal's strategy space is therefore set up to the set of incentive contracts in the literature on most common agency problems.[1]

Equilibrium allocations however depend on the nature of competition embedded in principals' strategy spaces defined in common agency games. For example, when multiple sellers (principals) engage in negotiating with a potential buyer (agent), the buyer typically has private information not only on her payoff type but also on what's happening in the market. The latter is called the agent's market information, which may include the negotiation schemes that the sellers offer to the buyer, the information that the sellers receive from the buyer during negotiation, the incentive contracts that are determined through negotiation, and so on. A seller would want to make his negotiation scheme responsive to what the other sellers are doing with the buyer if he could gain by doing so instead of simply offering an incentive contract. Naturally, sellers should be able to use complex mechanisms as their negotiation schemes that assign their contracts contingent on the agent's true reports on her payoff type and her market information by providing the agent with incentives to reveal her true private information. Those mechanisms are however very complex.[2]

Peters (2001) and Martimort and Stole (2002) made breakthroughs in this regard by establishing menu theorems in common agency. They show that the competition relative to menus of alternatives does not restrict principals' ability to offer any complex mechanisms. The idea is that if the agent's preference ordering over alternatives in menus depends on what other principals have done with the agent, then it is enough for principals to create the punishments, in their menus, that support any equilibrium relative to any complex mechanisms, without explicit communication.

Menu theorems are sufficiently general to cover various contracting en-

[1]See section 2 for the literature review on common agency problems.

[2]The revelation principle for a single principal associated with incentive-compatible mechanisms defined over agents' payoff types does not hold when multiple principals are non-cooperatively competing in the market whether they deal with a single agent or multiple agents (For details and related examples, see Peck 1997, Epstein and Peters 1999, and Marimort and Stole 2003 among others). The reason is that a standard direct mechanism fixes the message space for an agent as her payoff type space and it is not big enough for the agent to report her private information because she also has the market information.

vironments where a principal incentivizes the agent's action choice with an arbitrary contracting variable so that an incentive contract specifies a principal's arbitrary contracting variable, say his action, as a function of the part of the agent's action that is contractible: Peters's menu theorem shows that the set of equilibrium allocations relative to menus of incentive contracts is the set of equilibrium allocations relative to any complex mechanisms.[3]

Subsequently, Pavan and Calzolari (2009) show that it is useful in identifying equilibrium menus to utilize a class of incentive-compatible extended direct mechanisms that ask the agent about her payoff type and also about her choice of payoff-relevant alternatives from the other principals. The competition relative to the class of incentive-compatible extended direct mechanisms does not generate all equilibrium allocations relative to any complex mechanisms, but it does equilibrium allocations associated with Markov pure-strategy equilibria relative to any complex mechanisms in which the agent's report depends only on the payoff-relevant information. Once a principal figures out an incentive-compatible extended direct mechanism that he wants to offer in a Markov pure-strategy equilibrium, he can equivalently offer a menu that is the image set of the extended direct mechanism.

Another research direction examines if or when even simpler competition models (e.g., the competition relative to incentive contracts or relative to payoff-type direct mechanisms) can be rationalized although it may not support all equilibrium allocations relative to any complex mechanisms. Peters (2003) shows that any pure-strategy equilibrium relative to payoff-type direct mechanisms that assign incentive contracts as a function of the agent's payoff type report continues to be an equilibrium relative to menus of incentive contracts (or equivalently any complex mechanisms). He also defines a "no externalities" condition for principals' contracting variables with which they incentivize the agent's action choice.[4] He shows that when the "no externalities" condition holds, payoffs associated with any pure-strategy

[3]In the competition relative to menus, principals offer menus of incentive contracts to the agent and then the agent chooses an incentive contract from each menu she accepts and makes her action choice. Finally, a principal takes his action that is specified in his incentive contract contingent on the part of the agent's action that is contractible.

[4]Section 4 discusses more on this. No externalities condition is satisfied if (i) each principal's payoff depends only on the agent's action and his contracting variable (i.e., his action) with which he incentivizes the agent's action choice and (ii) the agent's optimal choices of each principal j's action from each closed subset of the set of principal j's feasible actions only depend on the agent's action.

equilibrium allocation relative to menus of incentive contracts are preserved by a pure-strategy equilibrium relative to random incentive contracts that specify a probability distribution over a principal's actions as a function of the part of the agent's action that is contractible.[5] Attar, Majumdar, Piaser, and Porteiro (2008) show that the agent's separable preferences ensure that payoffs associated with any pure-strategy equilibrium allocation relative to complex mechanisms are preserved by a pure-strategy equilibrium relative to payoff-type direct mechanisms.[6]

This paper provides a *general* foundation of modeling common agency problems through the competition relative to incentive contracts when contracting involves monetary transfer. As explained later in Section 4, contracting involving monetary transfer leads to a strict form of the "no externalities" condition, but it is observed in most common agency problems in the literature. In most common agency problems, principals incentivize the agent's action choice with monetary transfer by offering incentive contracts that specify the amounts of monetary transfer as a function of the part of the agent's action that is contractible. It shows that whenever contracting involves monetary transfer, payoffs associated with *any* equilibrium relative to any complex mechanisms that assign incentive contracts as a function of the agent's message are preserved by an equilibrium relative to incentive contracts. Furthermore, whenever contracting involves monetary transfer, *any* equilibrium relative to incentive contracts continues to be an equilibrium relative to any complex mechanisms. Therefore, it establishes the complete equivalence between the set of *all* equilibrium allocations relative to incentive contracts and the set of *all* equilibrium allocations relative to any complex mechanisms that assign incentive contracts as a function of the agent's message.

This result may explain the prevalent practice of monetary transfer observed in practice and the literature on common agency. The results imply

[5]Note that the two competition models have different contracting primitives in that random incentive contracts are not included in the set of feasible incentive contracts in the competition relative to menus but they are in the competition relative to random incentive contracts.

[6]The separability is however restrictive in most common agency problems involving monetary transfer as Martimort (2006) and Pavan and Calzolari (2009) point out. For example, it rules out the possibility that a buyer's preferences for the quality/quantity of a seller's product may depend on the quality/quantity of the product purchased from another seller.

that whenever contracting involves monetary transfer, it is irrelevant whether principals are restricted to offer only incentive contracts or any complex mechanisms. Incentivizing the agent's action choice with monetary transfer, a principal can significantly simplify his mechanism design problem by focusing on incentive contracts that specify amounts of monetary transfer contingent on the part of the agent's action that is contractible.

Section 2 reviews common agency problems in the literature, sets up the payoff environments that encompass those common agency problems, and formulates the competition relative to any arbitrary complex mechanisms. Section 3 presents Theorems 1 and 2 that show the equivalence between the set of equilibrium allocations relative to incentive contracts and the set of equilibrium allocations relative to any complex mechanisms. Section 4 contrasts the results in this paper with the results from the literature on how to model common agency problems and concludes the paper. The proofs of Theorems 1 and 2 appear in Appendix.

2 Preliminaries

When a measurable structure is necessary, the corresponding Borel σ - algebra is used. For a set S, $\Delta(S)$ denotes the set of probability distributions on S. For any $s \in \Delta(S)$, supp s denotes the support of the probability distribution s. For any mapping L from G into Q, $L(G)$ denotes the image set of L.

There are a set of principals, $\mathcal{J} \equiv \{1, \cdots, J\}$, and a single agent. The agent has private information about her preferences. This information is parameterized by an element, called a (payoff) type, in a set Ω. Principals share a common prior belief that the agent's type follows a probability distribution F on Ω. The agent can take an action x from a set X. Each principal j incentivizes the agent's action choice with monetary transfer. If the agent of type ω takes an action x and the total monetary payment from principals is y, her payoff is $u(y, x, \omega)$. If principal j makes a monetary payment of y_j to the agent, the agent takes an action x, and her type is ω, principal j's payoff is $v_j(y_j, x, \omega)$. The agent's payoff is increasing in the total monetary payment y from principals at each (x, ω) and principal j's payoff is decreasing in his monetary payment y_j to the agent at each (x, ω). Note that payoff functions for the agent and principals allow for general preference orderings over the possible bundles of the agent's action and the amount of monetary transfer without quasilinearity or separability.

In most common agency problems in the literature, principals incentivize the agent's action choice with monetary transfer so that they compete in incentive contracts. An incentive contract $a_j : X \to Y_j$ that principal j offers to the agent specifies the amount of monetary transfer from the principal to the agent as a function of the part of the agent's action that is contractible between them. Let $Y_j \subseteq \mathbb{R}$ is the set of all possible amounts of monetary transfer, which may differ in the applications we consider. Let A_j be the set of all possible mappings from X into Y_j. Following Peters (2003), let $\mathcal{X}_j$ be a collection of measurable equivalence classes, whose union is X such that principal j is constrained to respond to each action in the same equivalence class the same way. The set of feasible incentive contracts for principal j is therefore defined as $\mathcal{A}_j \equiv \{a_j \in A_j : a_j \text{ is } \mathcal{X}_j\text{-measurable}\}$. $\mathcal{A}_j$ differs in what the part of the agent's action that is contractible between principal j and the agent. Let $\mathcal{A} \equiv \times_{k=1}^{J} \mathcal{A}_k$.

The set of feasible incentive contracts depends on the nature of common agency (i.e., private common agency vs. public common agency).[7] In *public common agency*, each principal can make monetary transfer fully contingent on the agent's action. Public common agency with asymmetric information includes lobbying games (Lebreton and Salanie 2003, Martimort and Semenov 2008, Martimort and Stole 2009c) and public good provision games (Martimort and Stole 2009b, Martimort and Stole 2009c) among others.[8] In lobbying games, interest groups (principals) offer monetary contribution schedules to a politician (agent) whose ideal policy or policy preferences are her own private information. In public good provision games, consumers (principals) offer nonlinear prices to a public good provider (agent) whose cost of providing a public good is her own private information. In those cases, an incentive contract (i.e., a monetary contribution schedule or nonlinear price) makes monetary transfer fully contingent on the agent's action (the whole characteristics of a policy or a public good). In public common agency, the set of feasible incentive contract, $\mathcal{A}_j$, is therefore defined as A_j with $Y_j = \mathbb{R}_+$ for each $j \in \mathcal{J}$.

[7] This terminology is coined by Marimort (2007).

[8] Public common agency problems with complete information can be founded in Bernheim and Whinston (1986), Dixit, Grossman, and Helpman (1997), and Grossman and Helpman (1994). Laussel and Lebreton (2001) characterize the set of equilibrium payoffs and evaluate the effects of competition among principals on the magnitude of the rent obtained by the agent in public common agency with complete information.

In *private common agency*, different principals can make their monetary transfer contingent on only different parts of the agent's action under their control. For example, when sellers (principals) contract with a buyer (agent), each seller can specify the amount of monetary transfer contingent on the quantity/quality of the good that the buyer purchases from him, but not the quantities/qualities of the goods that the agent purchases from the competing sellers.[9] Private common agency problems with asymmetric information are adopted in Calzolari and Scarpa (2008), Diaw and Pouyet (2005), Ivaldi and Martimort (1994), Martimort and Stole (2009a), Mezzetti (1997). They analyze sellers' competition in nonlinear prices in the environments without externalities in the sense that each seller's cost of producing a good depends only on the quantity/quality of the good that he sells. Martimort and Stole (2003) study private common agency with asymmetric information in vertical contracting problems, with externalities, where retailers (principals) offer nonlinear prices to a manufacturer (agent) and the market (inverse) demand depends on the total quantity that retailers sell in the market. Biais, Martimort, and Rochet (2000) and Khalil, Martimort, and Parigi (2007) study financial contracting problems through the competition relative to nonlinear prices as private common agency with asymmetric information.[10]

In private common agency, the agent's action x is decomposed into $[x_1, \dots, x_J] \in X \equiv \times_{k=1}^{J} X_k$ with $x_j \in X_j$. The jth component x_j of x is the part of the agent's action that is contractible between principal j and the agent. For example, it is a quantity/quality of the good that the buyer buys from seller j. Private common agency features both asymmetric information and moral hazard because x_{-j} is not contractible between principal j and the agent and it is not observable to principal j at the time he contracts with the agent. In private common agency, the set of feasible incentive contracts, $\mathcal{A}_j$, for principal j is therefore defined as A_j°:

$$A_j^{\circ} \equiv \{a_j \in A_j : a_j(x) = a_j(x') \text{ if } x_j = x'_j \ \& \ a_j(x) = 0 \text{ if } x_j = \underline{x}_j\},$$

where $\underline{x}_j$ means no trading with principal j. The first requirement implies

[9] Private common agency problems with complete information can be founded in Chinesa and Denicolo (2009) and D'Aspremont and Ferreira (2009).

[10] Attar, Mariotti, and Salanié (2009) apply private common agency to non-exclusive contracting in the market for lemons. They consider the competition among buyers (principals) in which buyers offer menus of quantity and price pairs. The equilibrium menu is a linear price schedule with the same unit price for any quantity. This continues to be an equilibrium relative to incentive contracts.

that principal j's monetary transfer only depends on the jth component of the agent action. The second requirement implies "no payment upon no trading." $\mathcal{A}_j = A^\circ_j$ is effectively the set of all incentive contracts that are mappings from X_j into Y_j with the "no payment upon no trading" clause. The set of feasible monetary payments Y_j is either $\mathbb{R}_+$ or $\mathbb{R}_-$ depending on whether the agent is a seller or a buyer. If the agent can be both (e.g., a trader in the financial asset market as in Biais, Martimort, and Rochet 2000), then $Y_j = \mathbb{R}$.

2.1 Competition relative to Complex Mechanisms

The competition relative to $\mathcal{A}$ well describes most common agency problems as discussed earlier. However, a principal may want to offer to the agent a mechanism that assigns an incentive contract as a function of the agent's message. By doing so, he can make his contract itself responsive to the agent's report on both her payoff type and what the other principals are doing in the market.

Formally, a mechanism that principal i offers is a measurable mapping $\gamma_j : M_j \to \mathcal{A}_j$, where M_j is a closed set of messages available for the agent. The set of messages can be quite general in the degree and nature of the communication that it permits regarding what the other principals are doing: It could allow the agent to report not only about her type but also about the whole mechanisms offered by the other principals, the incentive contracts that the agent chooses from the other principals, and so on. When the agent sends a message $m_j \in M_j$ to principal j, the incentive contract $\gamma_j(m_j) \in \mathcal{A}_j$ is assigned. Let Γ_j be the set of mechanisms available for principal j and $\Gamma \equiv \times_{k=1}^J \Gamma_k$.

The common agency game relative to Γ starts when each principal j simultaneously offers a mechanism from Γ_j. After seeing a profile of mechanisms $\gamma = [\gamma_1, \cdots, \gamma_I] \in \Gamma$, the agent decides which mechanisms to accept. Then, she sends messages to those principals whose mechanisms she has accepted and takes an action from X. Finally, payoffs are realized. In public common agency, it is weakly dominant for the agent to accept all principals' mechanisms because an amount of monetary transfer is non-negative at any action as pointed out in Martimort and Semenov (2008) and Martimort and Stole (2009c). In private common agency, choosing $x_j = \underline{x}_j$ given any incentive contract a_j is equivalent to not accepting principal j's mechanism. Therefore, the agent's participation decision is incorporated into her action choice so that we do not separately formulate the agent's participation decision for notational simplicity.[11]

To incorporate both public common agency and private common agency in the notation, define $\tau_j(x)$ as, for all $x \in X$,

$$\tau_j(x) \equiv \begin{cases} x & \text{if } \mathcal{A}_j = A_j \\ x_j & \text{if } \mathcal{A}_j = A_j^\circ \end{cases}, \tag{1}$$

where x_j is the jth component of $x = [x_1, \ldots, x_J]$.

The agent's continuation strategy is characterized by a measurable mapping $c : \Gamma \times \Omega \to \Delta(M \times X)$, where $M \equiv \times_{k=1}^J M_k$. Given a continuation strategy c, let $c(\gamma, \omega)$ denote a probability distribution on the agent's action and messages. Any continuation strategy can be decomposed into $c_x : \Gamma \times \Omega \to \Delta(X)$ and $c_m : X \times \Gamma \times \Omega \to \Delta(M)$. Let $c_x(\gamma, \omega)$ denote the probability distribution on the agent's action conditional on (γ, ω) and $c_m(x, \gamma, \omega)$ the probability distribution on the agent's messages across principals conditional on (x, γ, ω). When the agent's continuation strategy is c, her continuation payoff is

$$U(\gamma, c, \omega) \equiv \int_X \left[\int_M u \left(\sum\nolimits_{k=1}^J \gamma_k(m_k)(\tau_k(x)), x, \omega \right) dc_m(x, \gamma, \omega) \right] dc_x(\gamma, \omega)$$

at each $(\gamma, \omega) \in \Gamma \times \Omega$. Let C be the set of all continuation strategies for the agent. A continuation strategy c is a continuation equilibrium relative to Γ if $U(\gamma, c, \omega) \geq U(\gamma, c', \omega)$ for all $(\gamma, c', \omega) \in \Gamma \times C \times \Omega$.

Let $\mathcal{C}$ be the set of all continuation equilibria relative to Γ. A continuation equilibrium $c \in \mathcal{C}$ defines a normal-form game for principals where they simultaneously offer mechanisms from Γ. We assume that $\mathcal{C}$ is non-empty. Otherwise, a normal-form game for principals is not properly defined. Let $c_{m_j}(x, \gamma, \omega)$ be the marginal probability distribution on the agent's message for principal j conditional on (x, γ, ω) that is induced by the agent's continuation strategy c. Given $c \in \mathcal{C}$, principal j's continuation payoff is

$$V_j(\gamma, c) \equiv \int_\Omega \left[\int_X \left(\int_{M_j} v_j \left(\gamma_j(m_j)(\tau_j(x)), x, \omega \right) dc_{m_j}(x, \gamma, \omega) \right) dc_x(\gamma, \omega) \right] dF$$

[11]The literature on common agency takes this approach. Alternatively, one can incorporate the agent's participation decision into the agent's communication as follows: Include the null message $\emptyset$ in M_j and the null contract $\underline{a}_j$ (i.e., no trading) in $\mathcal{A}_j$. A mechanism γ_j satisfies $\gamma_j(\emptyset) = \underline{a}_j$. All the results in this paper hold even when we explicitly formulate the agent's participation decision.

at each $\gamma \in \Gamma$. Let $\delta_j \in \Delta(\Gamma_j)$ be principal j's strategy and $\delta = [\delta_1, \ldots, \delta_J]$ a profile of principals' strategies. Given a continuation equilibrium c and the other principals' strategies δ_{-j}, principal j's payoff associated with δ_j is

$$V_j(\delta_j, \delta_{-j}, c) \equiv \int_{\Gamma_j} \left(\int_{\Gamma_{-j}} V_j(\gamma_j, \gamma_{-j}, c) d\delta_{-j} \right) d\delta_j.$$

As in the literature on common agency, we adopt the solution concept of perfect Bayesian equilibrium.

Definition 1 *A strategy profile* $\{\delta, c\}$ *is a* perfect Bayesian equilibrium *(henceforth simply an* equilibrium*) relative to* Γ *if, for all* $j \in \mathcal{J}$*, all* $\delta'_j \in \Delta(\Gamma_j)$*, and some* $c \in \mathcal{C}$*,* $V_j(\delta_j, \delta_{-j}, c) \geq V_j(\delta'_j, \delta_{-j}, c)$.

3 Competition relative to Incentive Contracts

As shown in the previous section, most common agency problems in the literature are modeled through the competition relative to $\mathcal{A}$ where each principal j offers an incentive contract from $\mathcal{A}_j$. In the common agency game relative to $\mathcal{A}$, let $z : \mathcal{A} \times \Omega \to \Delta(X)$ denote the agent's action strategy. Let $\mathcal{Z}$ be the set of all continuation equilibria relative to $\mathcal{A}$. Given a continuation equilibrium $z \in \mathcal{Z}$, we can define the agent's continuation payoff $U(a, z, \omega)$ for all $(a, \omega) \in \mathcal{A} \times \Omega$. Let $\sigma_j \in \Delta(\mathcal{A}_j)$ denote principal j's strategy. Given an equilibrium $\{\sigma, z\}$ relative to $\mathcal{A}$, we can define principal j's equilibrium payoff $V_j(\sigma_j, \sigma_{-j}, z)$.

Theorem 1 shows that payoffs for the agent and principals associated with any equilibrium relative to any complex mechanisms are preserved by an equilibrium relative to $\mathcal{A}$. Formally, a set of mechanisms Γ_j is bigger than Γ'_j ($\Gamma_j \succcurlyeq \Gamma'_j$) if there exists an embedding $\phi_j : \Gamma'_j \to \Gamma_j$. When Γ_j is bigger than Γ'_j, there are additional mechanisms in Γ_j, compared to the mechanisms in Γ'_j. Let $\Gamma \succcurlyeq \Gamma'$ if $\Gamma_j \succcurlyeq \Gamma'_j$ for all $j \in \mathcal{J}$. If Γ is smaller than $\mathcal{A}$ (i.e., $\mathcal{A} \succcurlyeq \Gamma$), some equilibria relative to Γ may not be reproduced by equilibria relative to $\mathcal{A}$. However, it implies that those equilibria are no longer equilibria once principals are allowed to offer incentive contracts in $\mathcal{A}$. Those equilibria are not interesting because at least conceptually principals should be able to offer any mechanisms they like. Therefore, our interest is the set of equilibria relative to $\Gamma \succcurlyeq \mathcal{A}$.

Theorem 1 *For any equilibrium* $\{\delta, c\}$ *relative to any* $\Gamma \succcurlyeq \mathcal{A}$*, there exists an equilibrium* $\{\sigma, z\}$ *relative to* $\mathcal{A}$ *and a mapping* $\psi_j : \Gamma_j \to \mathcal{A}_j$ *for all* $j \in \mathcal{J}$ *such that (1) for all* $\omega \in \Omega$ *and all* $[\gamma_1, \ldots, \gamma_J] \in \Gamma$, $U(\gamma_1, \ldots, \gamma_J, c, \omega) = U(\psi_1(\gamma_1), \ldots, \psi_J(\gamma_J), z, \omega)$ *and (2) for all* $j \in \mathcal{J}$, $V_j(\delta_j, \delta_{-j}, c) = V_j(\sigma_j, \sigma_{-j}, z)$.

First consider public common agency. Because $\mathcal{A}_j = A_j$, the agent's action is fully contractible. If the agent takes an action x, it is always optimal for her to send a message m_j to principal j such that $\gamma_j(m_j)(x) \geq \gamma_j(m'_j)(x)$ for all $m'_j \in M_j$ because a larger amount of monetary transfer from principal j is preferred by the agent at any given x regardless of her payoff type and the amounts of monetary transfer from the other principals.

In private common agency, $\mathcal{A}_j = A^\circ_j$. If the agent takes x_j for principal j, in the continuation game relative to Γ, it is always optimal for her to send a message m_j to principal j such that $\gamma_j(m_j)(x_j) \geq \gamma_j(m'_j)(x_j)$ for all $m'_j \in M_j$ regardless of her payoff type, her choice of x_{-j} and the amounts of monetary transfer from the other principals. Therefore, the amount of monetary transfer from principal j in any continuation equilibrium is uniquely determined by principal j's mechanism and the agent's choice of the part of her action that is contractible with principal j.

The argument above implies that for any x in the support of $c_x(\gamma_j, \gamma_{-j}, \omega)$ given any $c \in \mathcal{C}$, the agent will send a message that induces principal j's monetary payment equal to $\max_{m'_j} \gamma_j(m'_j)(\tau_j(x))$. Of course, $\max_{m'_j} \gamma_j(m'_j)(\tau_j(x))$ may not exist for all x, but such x is not in the support of $c_x(\gamma_j, \gamma_{-j}, \omega)$. This leads us to construct the mapping $\psi_j : \Gamma_j \to \mathcal{A}_j$ such that, for all $\gamma_j \in \Gamma_j$ and all $x \in X$,

$$\psi_j(\gamma_j)(\tau_j(x)) \equiv \begin{cases} \max\limits_{m'_j} \gamma_j(m'_j)(\tau_j(x)) & \text{if } \exists \max\limits_{m'_j} \gamma_j(m'_j)(\tau_j(x)) \\ \sup\limits_{m'_j} \gamma_j(m'_j)(\tau_j(x)) - \varepsilon_j(\gamma_j, \tau_j(x)) & \text{otherwise} \end{cases}, \quad (2)$$

where $\varepsilon_j(\gamma_j, \tau_j(x)) > 0$ is properly chosen depending on $Y_j = \mathbb{R}_-, \mathbb{R}_+$, or $\mathbb{R}$.[12] Note that $\psi_j(\gamma_j)$ is in A_j in public common agency and it is in A°_j in private

[12]When $Y_j = \mathbb{R}_-$ or $\mathbb{R}$, $\varepsilon_j(\gamma_j, \tau_j(x))$ can be any arbitrary positive real number. Suppose that $\max_{m'_j} \gamma_j(m'_j)(\tau_j(x))$ does not exist given $Y_j = \mathbb{R}_+$. Then $\sup_{m'_j} \gamma_j(m'_j)(\tau_j(x))$ must be strictly postive because an amount of monetary transfer cannot be negative. Because $\sup_{m'_j} \gamma_j(m'_j)(\tau_j(x)) > 0$, we can find $\varepsilon_j(\gamma_j, \tau_j(x)) > 0$ such that $\sup_{m'_j} \gamma_j(m'_j)(\tau_j(x)) - \varepsilon_j(\gamma_j, \tau_j(x)) \geq 0$.

common agency so that $\psi_j(\gamma_j)$ itself is a feasible incentive contract in each class of common agency problems.

Any x at which $\max_{m'_j} \gamma_j(m'_j)(\tau_j(x))$ does not exist is not chosen in any continuation equilibrium relative to Γ and the amount of monetary transfer conditional on $\tau_j(x)$ is slightly reduced from $\sup_{m'_j} \gamma_j(m'_j)(\tau_j(x))$ through the mapping $\psi_j(\gamma_j)$. Therefore any x at which $\max_{m'_j} \gamma_j(m'_j)(\tau_j(x))$ does not exist will be never chosen under $\psi_j(\gamma_j)$ in any continuation equilibrium. Subsequently, the agent's optimal choice set of actions and monetary payments at any $[\gamma_1, \dots, \gamma_J] \in \Gamma$ is preserved at $[\psi_1(\gamma_1), \dots, \psi_J(\gamma_J)] \in \mathcal{A}$. As shown in the proof, this is the key to the construction of an equilibrium $\{\sigma, z\}$ relative to $\mathcal{A}$ that preserves payoffs for the agent and principals associated with any equilibrium $\{\delta, c\}$ relative to any $\Gamma \succcurlyeq \mathcal{A}$.

One of the concerns on competition models relative to simple mechanisms is that some equilibrium may disappear if principals deviate to more complex mechanisms. Therefore, it is important to check whether an equilibrium relative to $\mathcal{A}$ continues to be an equilibrium relative to $\Gamma \succcurlyeq \mathcal{A}$. If it is, it is said to be weakly robust to any $\Gamma \succcurlyeq \mathcal{A}$ according to Peters (2001). Theorem 2 shows that any equilibrium relative to $\mathcal{A}$ continues to be an equilibrium relative to any complex mechanisms.

Theorem 2 *Any equilibrium $\{\sigma, z\}$ relative to $\mathcal{A}$ is weakly robust relative to any $\Gamma \succcurlyeq \mathcal{A}$.*

Given any equilibrium $\{\sigma, z\}$ relative to $\mathcal{A}$, suppose that principal j deviates to an arbitrary complex mechanism γ_j while the other principals offer incentive contracts based on their equilibrium strategies over the set of incentive contracts. Because $\psi_j(\gamma_j)(\tau_j(x))$ is the maximum amount of monetary transfer that the agent can receive from the deviating principal given her optimal choice of action x in any continuation equilibrium, the agent's optimal communication with the deviator induces her to select her action as if she faces an incentive contract $\psi_j(\gamma_j)$. The proof of Theorem 2 shows that one can always assign a continuation equilibrium, upon any principal's deviation to any complex mechanism, that is payoff-equivalent to the continuation equilibrium z so that it punishes a principal's deviation to any complex mechanism. Theorems 1 and 2 show that payoffs associated with any equilibrium relative to any complex mechanisms can be preserved by an equilibrium relative to $\mathcal{A}$ and that vice versa, payoffs associated with any equilibrium relative to $\mathcal{A}$ can be preserved by an equilibrium relative to any complex mechanisms.

Theorems 1 and 2 are established with arbitrary preference orderings of the agent and principals over the possible bundles of the agent's action and the amount of monetary transfer.

The results shows how the agent's private information is revealed when common agency involves monetary transfer. Once the agent chooses the part of her action that is contractible with principal j, she always prefers a larger amount of monetary transfer from principal j regardless of her payoff type, her choice of the parts of her action that are contractible with the other principals, and the amounts of monetary transfer from the other principals and . This implies that even when the other principals *randomize* their mechanism offers, the agent's message to principal j, given his mechanism γ_j, only depends on her choice of the part of her action that is contractible with principal j regardless of the realization of the other principals' mechanisms. In other words, the agent's payoff type and her market information (e.g., the amounts of monetary transfer, the other principals' mechanisms, and her choice of the other parts of her action, and etc.) are revealed through her choice of the part of her action that is contractible with principal j and then her choice of the contractible part of her action uniquely determines the amount of principal j's monetary transfer, given γ_j, in any continuation equilibrium. If principal j wants to a mechanism γ_j, he can directly offer $\psi_j(\gamma_j)$ whether or not the other principals randomize their mechanism offers.[13]

If principal j incentivizes the agent's action choice with an arbitrary contracting variable, say his action, then the agent's preference ordering on principal j's action may depend not only on her action choice but also on the other principals' actions that their incentive contracts induce given her action choice. It implies that the agent's optimal communication with principal j will generally differ in what incentive contracts that she chooses with the other principals. This is why principals may want to keep alternative choices of incentive contracts for the agent in their mechanisms or menus when they incentivize the agent's action with arbitrary contracting variables.

[13]As long as principal j incentivizes the agent's action choice with monetary transfer, this is true even when the other principals incentivize the agent's action choice with arbitrary contracting variables.

4 Discussion

Menu theorems are established in rich environments where a principal incentivizes the agent's action choice with an arbitrary contracting variable, say a principal's action, so that an incentive contract specifies a principal's action as a function of the part of the agent's action that is contractible. Subsequent research on modeling common agency is devoted to how to derive equilibrium menus or if and when one can provide a rationale for simple mechanism games such as the competition relative to incentive contracts or payoff-type direct mechanisms while those simple mechanism games may not generate all interesting equilibrium allocations relative to complex mechanisms.

Pavan and Calzolari (2009) show that it is useful in identifying equilibrium menus to utilize a class of incentive-compatible extended direct mechanisms that ask the agent about her type and also about her choice of payoff-relevant alternatives from the other principals. The competition relative to the class of incentive-compatible extended direct mechanisms does not generate all equilibria relative to any complex mechanisms, but it generate equilibrium allocations associated with *Markov* pure-strategy equilibria relative to any complex mechanisms in which the agent's communication with each principal only depends on her payoff-relevant information. Once a principal figures out an incentive-compatible extended direct mechanism that he wants to use, he can equivalently offer a menu that is the image set of the extended direct mechanism.

Peters (2003) focuses on pure-strategy equilibria where principals employ pure strategies. Theorem 2 in Peters (2003) shows that any pure-strategy equilibrium relative to payoff-type direct mechanisms that assign incentive contracts contingent on the agent's payoff type reports continues to be an equilibrium relative to menus (or equivalently any complex mechanisms) while competition relative to payoff-type direct mechanisms may not generate all equilibrium allocations relative to any complex mechanisms. Peters defines a "*no externalities*" condition for principals' contracting variables with which they incentivize the agent's action choice. Theorem 4 in Peters (2003) showed that if the "no externalities" condition holds, payoffs associated with any pure-strategy equilibrium relative to any complex mechanisms that assign incentive contracts in $\mathcal{A}$ are preserved by a pure-strategy equilibrium relative to random incentive contracts. Principal j's random incentive contract specifies a probability distribution on y_j contingent on the part of agent's action that is contractible while an incentive contract in $\mathcal{A}$ specifies

y_j directly.[14] The "no externalities" condition in Peters (2003, 2007) is stated as follows:

D1. Each principal j's payoff function is given by $v_j(y_j, x, \omega)$.

D2. For each $j \in \mathcal{J}$, each $x \in \mathcal{X}_j$, each closed subset $B_j \subset Y_j$, the set

$$G_j(B_j) \equiv \left\{y_j \in B_j : h(y_j, y_{-j}, x, \omega) \geq h(y'_j, y_{-j}, x, \omega) \text{ for all } y'_j \in B_j\right\}$$

is the same for all $y_{-j} \in Y_{-j}$, all $\omega \in \Omega$.

Even when we focus on pure-strategy equilibria, Theorem 4 does not give us a direct answer to whether or not payoffs associated with any pure-strategy equilibrium relative to complex mechanisms that assign incentive contracts in $\mathcal{A}$ can be reproduced by a pure-strategy equilibrium relative to $\mathcal{A}$ because two competition models compared in Theorem 4 have different contracting primitives in that random incentive contracts are not included in the set of feasible incentive contracts in the competition relative to menus (or equivalently complex mechanisms) but they are in the competition relative to random incentive contracts.

Note that if we rewrite the agent's payoff function in this paper as $h(y_1, \ldots, y_J, x, \omega) = u(y, x, \omega)$ with $y = \sum_{k=1}^{J} y_k$, the nature of monetary transfer leads to the following condition:

E2. For each $j \in \mathcal{J}$, each closed subset $B_j \subset Y_j$, $G_j(B_j)$ is the same for all $x \in \mathcal{X}_j$, all $y_{-j} \in Y_{-j}$, all $\omega \in \Omega$.

Given conditions D1 and E2, it is straightforward to show that our main results, Theorems 1 and 2, hold. Clearly, E2 is a stronger requirement than D2. However, it is satisfied by any arbitrary preference ordering over the possible bundles of the agent's actions and the amounts of monetary transfer without quasilinearity or separability because the agent's payoff is increasing in the amount of principal j's monetary transfer given any amounts of the other principals' monetary transfer, any action choice of the agent, and any payoff type: $u(y_j + \sum_{k \neq j} y_k, x, \omega)$ is increasing in y_j at each (y_{-j}, x, ω). Under arbitrary preference ordering for the possible bundles of the agent's actions

[14] A random incentive contract offered by principal j is defined as $\alpha_j : X \to \Delta(Y_j)$. While an incentive contract is defined as $a_j : X \to Y_j$.

and the amounts of monetary transfer, Theorems 1 and 2 establish the complete equivalence between the set of *all* equilibrium allocations relative to $\mathcal{A}$ and the set of *all* equilibrium allocations relative to any complex mechanisms that assign incentive contracts in $\mathcal{A}$ contingent on the agent's messages.

Attar, Majumdar, Piaser, and Porteiro (2008) show that the agent's *separable preferences* ensure that payoffs associated with any pure-strategy equilibria relative to complex mechanisms can be preserved by a pure-strategy equilibrium relative to payoff type direct mechanisms. However, the separability is restrictive in the most of applications mentioned in Section 2 as Martimort (2006) and Pavan and Calzolari (2009) point out. For example, x_{-j} is not contractible for principal j (e.g., seller) in private common agency. In this case, the separability means that if the agent (e.g., buyer) strictly prefers (y_j, x_j) when the agent contracts (y_{-j}, x_{-j}) with the other principals, then the agent also strictly prefers (y_j, x_j) at all (y'_{-j}, x'_{-j}). Therefore, the separability rules out the possibility that a buyer's preferences for the quality/quantity of a seller's product may depend on the quality/quantity of the product purchased from another seller. Importantly, the payoff function $u(y, x, \omega)$ for the agent with monetary transfer (equivalently E2) does not imply separability in any way.

In most common agency problems in the literature, principals incentivize the agent's action choice with monetary transfer so that competition among principals is modeled through the game where each principal offers an incentive contract that specifies the amount of monetary transfer contingent on the part of the agent's action that is contractible. Not only does the competition relative to $\mathcal{A}$ that specify the amounts of monetary transfer as a function of the part of the agent's contractible action well describe most common agency problems observed in practice but the dominance of such competition models in the literature comes from their tractability for equilibrium analysis.

Theorems 1 and 2 show that whenever contracting involves monetary transfer, the set of all equilibrium allocations relative to incentive contracts is identical to the set of all equilibrium allocations relative to any complex mechanisms that assign incentive contracts contingent on the agent's messages. Therefore, whenever contracting involves monetary transfer, it is irrelevant whether principals are restricted to offer only incentive contracts or any complex mechanisms. This allows us to study any equilibrium allocations relative to any complex mechanisms by focusing on equilibria relative to incentive contracts in common agency problems.[15] It also provides theoretical insight into the prevalent practice of monetary transfer observed in practice and the literature on common agency problems: Incentivizing the agent's action choice with monetary transfer in common agency problems, a principal can significantly simplify his *general* mechanism design problem in the decentralized market in the sense that he only need to focus on incentive contracts.

5 Appendix

Suppose that principals offer mechanisms $\gamma = [\gamma_1, \ldots, \gamma_J] \in \Gamma$. Let $\gamma(m)(\tau(x)) = [\gamma_1(m_1)(\tau_1(x)), \ldots, \gamma_J(m_J)(\tau_J(x))] \in Y \equiv \times_{k=1}^{J} Y_k$ be the profile of principals' monetary payment when the agent chooses x and sends messages $m = [m_1, \ldots, m_J]$, one for each principal. Given γ, the optimal set of actions and monetary payments for the agent of type ω is defined as

$$H(\gamma,\omega) \equiv \Big\{(x, \gamma(m)(\tau(x))) \in X \times Y : u\left(\sum\nolimits_{k=1}^{J} \gamma_k(m_k)(\tau_k(x)), x, \omega\right) \\ \geq u\left(\sum\nolimits_{k=1}^{J} \gamma_k(m'_k)(\tau_k(x')), x', \omega\right),\ \forall x \in X, \forall m' \in M\Big\}$$

$H(\gamma,\omega)$ is non-empty for all $(\gamma,\omega) \in \Gamma \times \Omega$ because $\mathcal{C}$ is assumed to be non-empty.

Because the agent's payoff is monotonic in the total amount of monetary transfer from principals, the following relations hold respectively for public common agency and for private common agency:

(a) Given any $x \in X$, $y_j > y'_j$ if and only if $u(y_j + y_{-j}, x, \omega) > u(y'_j + y_{-j}, x, \omega)$ for all (y_{-j}, ω).

[15] When there are multiple agents in the market, Folk theorems (Yamashita 2010; Peters and Troncoso Valverde 2010) provide the characterization of feasible equilibrium payoffs in that any payoffs above min-max payoffs can be supported in equilibrium relative to complex mechanisms. This characterization is not easily applied to common agency problems. Yamashita's results requires three or more agents and Peters and Troncoso Valverde's result does not cover the common agent problems with two principals, which are often used in the literature. The environments in Peters and Troncoso Valverde (2010) differ from ones considered in common agency. They consider the environments where there is no distinction between principals and agent in the sense that every player offers mechanisms to everyone else. They establish Folk theorem with four or more players.

(b) Given any $x_j \in X_j$, $y_j > y'_j$ if and only if $u(y_j + y_{-j}, x_j, x_{-j}, \omega) > u(y'_j + y_{-j}, x_j, x_{-j}, \omega)$ for all (y_{-j}, x_{-j}, ω).

(a) implies that if the agent chooses x in public common agency, she will send a message that maximizes the monatary payment from principal j given x regardless of her type and the monetary payments from the other principals. (b) implies that the agent chooses x_j in private common agency, she will send a message that maximizes the monatary payment from principal j given x_j regardless of her type, her choice of x_{-j}, and the monetary payments from the other principals. From (a) and (b), any $(x, \gamma(m)(\tau(x))) \in H(\gamma, \omega)$ for any ω satisfies, for all $j \in \mathcal{J}$

$$\gamma_j(m_j)(\tau_j(x)) = \max_{m'_j} \gamma_j(m'_j)(\tau_j(x)) \tag{3}$$

Furthermore, any x without $\max_{m'_j} \gamma_j(m'_j)(\tau_j(x))$ for some j is never an optimal action for the agent of any type at γ.

Suppose that principals offer incentive contracts $a = [a_1, \ldots, a_J] \in \mathcal{A}$. Let $a(\tau(x)) = [a_1(\tau_1(x)), \ldots, a_J(\tau_J(x))] \in Y$. be the profile of principals' monetary payments given a and x. Given a, the optimal set of actions and monetary payments for the agent of type ω is defined as

$$H(a, \omega) \equiv \left\{ (x, a(\tau(x))) \in X \times Y : u\left(\sum_{k=1}^{J} a(\tau_j(x)), x, \omega\right) \right.$$
$$\left. \geq u\left(\sum_{k=1}^{J} a(\tau_j(x')), x', \omega\right) \ \forall x \in X \right\}$$

The mapping $\psi_j : \Gamma_j \to \mathcal{A}_j$ defined in (2) is an injective mapping because any incentive contract can be viewed as a mechanism that assigns the same incentive contract regardless of the agent's message. Let ψ_j^{-1} be the inverse correspondence of ψ_j: For all $a_j \in \mathcal{A}_j$, $\psi_j^{-1}(a_j) \equiv \{\gamma_j \in \Gamma_j : \psi_j(\gamma_j) = a_j\}$. Let $\psi^{-1}(a) \equiv \times_{k=1}^{J} \psi_k^{-1}(a_k)$. Compare $H(\gamma, \omega)$ and $H(a, \omega)$ for any $\gamma \in \psi^{-1}(a)$ given each $a \in \mathcal{A}$. Any action x without $\max_{m'_j} \gamma_j(m'_j)(\tau_j(x))$ for some j is never an optimal action for the agent at γ. It implies that it is also never an optimal action at a becuase a_j reduces a monetary payment slightly from $\sup_{m'_j} \gamma_j(m'_j)(\tau_j(x))$. Any action x with $\max_{m'_j} \gamma_j(m'_j)(\tau_j(x))$ for all j directly induces $\max_{m'_j} \gamma_j(m'_j)(\tau_j(x))$ under a_j. Because any optimal action and monetary payments for the agent at γ satisfies (3), we subsequently have, for any $a \in \mathcal{A}$, any $\gamma \in \psi^{-1}(a)$ and any $\omega \in \Omega$,

$$H(a, \omega) = H(\gamma, \omega). \tag{4}$$

Now we present proofs of Theorems 1 and 2.

Proof of Theorem 1. Fix an equilibrium $\{\delta, c\}$ relative to Γ for any $\Gamma \succcurlyeq \mathcal{A}$. Let σ_j be the measure induced by δ_j through the map ψ_j for each $j \in \mathcal{J}$. For any $a_j \in A_j$, define the set $D_j(a_j) \subset \Gamma_j$ as

$$D_j(a_j) \equiv \begin{cases} \psi_j^{-1}(a_j) \cap \text{ supp } \delta_j & \text{if } \psi_j^{-1}(a_j) \cap \text{ supp } \delta_j \neq \varnothing \\ \bar{\psi}_j^{-1}(a_j) & \text{otherwise,} \end{cases}$$

where $\bar{\psi}_j^{-1}(a_j)$ is an arbitrary mechanism in $\psi_j^{-1}(a_j)$. For any $a = [a_1, \ldots, a_J] \in \mathcal{A}$, let $D(a) = \times_{k=1}^{J} D_k(a_k) \in \Gamma$.

Given the equilibrium strategy profile $\{\delta, c\}$, we can derive a joint probability distribution $b(D, \omega)$ on $M \times X$ for all $D \subset \Gamma$ and all $\omega \in \Omega$. Let $b_m(x, D, \omega)$ be the probability distribution on M conditional on (x, D, ω) that $b(D, \omega)$ induces. Let $b_x(D, \omega)$ be the marginal probability distribution on X that $b(D, \omega)$ induces. Construct the agent's continuation strategy $z : \mathcal{A} \times \Omega \to \Delta(X)$ as

$$z(a, \omega) = b_x(D(a), \omega) \tag{5}$$

for all $(a, \omega) \in \mathcal{A} \times \Omega$. (4) ensures that for any $x \in \text{supp } b_x(D(a), \omega)$ and all $(a, \omega) \in \mathcal{A} \times \Omega$,

$$x \in \arg\max_{x' \in X} u\left(\sum\nolimits_{k=1}^{J} a_k(\tau_k(x')), x', \omega\right),$$

so that the continuation strategy z constructed by (5) is a continuation equilibrium relative to $\mathcal{A}$. Now we prove part 1 of Theorem 1. The agent's continuation equilibrium payoff satisfies, for all $\omega \in \Omega$ and all $\gamma = [\gamma_1, \ldots \gamma_J] \in \Gamma$ with $\psi(\gamma) = a$,

$$\begin{gathered} U(a, z, \omega) = \\ \int_X u\left(\sum\nolimits_{k=1}^{J} a_k(\tau_k(x)), x, \omega, \right) db_x(D(a), \omega) = \\ \mathbb{E}_{\gamma' \in D(a)}\left[\int_X u\left(\sum\nolimits_{k=1}^{J} a_k(\tau_k(x)), x, \omega\right) db_x(\gamma', \omega)\right] = \\ \int_X u\left(\sum\nolimits_{k=1}^{J} \psi_k(\gamma_k)(\tau_k(x)), x, \omega\right) dc_x(\gamma, \omega) = \\ \int_X \left[\int_M u\left(\sum\nolimits_{k=1}^{J} \gamma_k(m_k)(\tau_k(x)), x, \omega\right) dc_m(x, \gamma, \omega)\right] dc_x(\gamma, \omega) = \\ U(\gamma, c, \omega). \end{gathered} \tag{6}$$

The first equality in (6) follows the definition of the continuation equilibrium z defined in (5). The second, third, and fourth equalities follow the definition of $D(a)$ and (4). The last inequality is simply the definition of $U(\gamma, c, \omega)$.

Finally, consider each principal j's payoff. For any $(a_j, a_{-j}) \in \mathcal{A}$, let

$$\begin{aligned} v_j^*(a_j, a_{-j}) &= \int_\Omega \left(\int_X v_j\left(a_j(\tau_j(x)), x, \omega, \right) db_x(D(a), \omega) \right) dF \\ &= \mathbb{E}_{\gamma' \in D(a)} \left[\int_\Omega \left(\int_X v_j\left(a_j(\tau_j(x)), x, \omega\right) dc_x(\gamma', \omega) \right) dF \right]. \end{aligned}$$

Integrating $v_j^*(a_j, a_{-j})$ using σ_{-j} yields

$$\begin{aligned} & V_j(a_j, \sigma_{-j}, z) \qquad (7) \\ = \; & \mathbb{E}_{\gamma_j' \in D_j(a_j)} \left[\int_{\Gamma_{-j}} \left\{ \int_\Omega \left(\int_X v_j\left(a_j(\tau_j(x)), x, \omega\right) db_x(\gamma_j', \gamma_j, \omega) \right) dF \right\} d\delta_{-j} \right] \\ = \; & \mathbb{E}_{\gamma_j' \in D_j(a_j)}[V_j(\gamma_j', \delta_{-j}, c)]. \end{aligned}$$

If $a_j = \psi_j(\gamma_j)$ for some $\gamma_j \in \text{supp } \delta_j$, then $\gamma_j \in D_j(a_j)$ belongs to supp δ_j by the definition of $D_j(a_j)$. Because δ_j is principal j's equilibrium strategy in the common agency game relative to Γ, any mechanism in supp δ_j yields the same payoff for principal j. This implies that the integrand in the third line of (7) is constant and equal to $V_j(\gamma_j, \delta_{-j}, c)$ as required. Also, $\psi_j^{-1}(a_j) = \bar{\gamma}_j \notin$ supp δ_j,then

$$\mathbb{E}_{\gamma_j' \in D_j(a_j)} \left[V_j(\gamma_j', \delta_{-j}, c) \right] = V_j(\bar{\gamma}_j, \delta_{-j}, c) \leq V_j(\delta_j, \delta_{-j}, c).$$

Therefore, σ_j is a best response for principal j when the other principals use σ_{-j} given a continuation equilibrium z and condition 2 is satisfied. ■

Proof of Theorem 2. Fix an equilibrium $\{\sigma, z\}$ relative to $\mathcal{A}$. Note that any incentive contract a_k can be viewed as a mechanism γ_k that assigns a_k regardless of the agent's message. For principal j's deviation to mechanisms in Γ_j, one can associate z, due to (4), with a continuation equilibrium strategy $\tilde{c} : \Gamma_j \times \mathcal{A}_{-j} \times \Omega \to \Delta(M_j \times X)$ relative to $\Gamma_j \times \mathcal{A}_{-j}$ as follows. The probability distribution $\tilde{c}_{m_j}(x, \gamma_j, a_{-j}, \omega)$ on M_j satisfies, for all $m_j \in$ supp $\tilde{c}_{m_j}(x, \gamma_j, a_{-j}, \omega)$,

$$\gamma_j(m_j)(\tau_j(x)) = \max_{m_j'} \gamma_j(m_j')(\tau_j(x)) = \psi_j(\gamma_j)(\tau_j(x)) \qquad (8)$$

and the probability distribution $\tilde{c}_x(\gamma_j, a_{-j}, \omega)$ on X satisfies

$$\tilde{c}_x(\gamma_j, a_{-j}, \omega) = z(\psi_j(\gamma_j), a_{-j}, \omega). \tag{9}$$

If principal j deviates to a mechanism γ_j in Γ_j, his payoff becomes

$$V_j(\gamma_j, \sigma_{-j}, \tilde{c}) = V_j(\psi_j(\gamma_j), \sigma_{-j}, z) \tag{10}$$

because of (9). Because $\{\sigma, z\}$ is an equilibrium relative to $\mathcal{A}$ and $\psi_j(\gamma_j) \in \mathcal{A}_j$, we have

$$V_j(\sigma_j, \sigma_{-j}, z) \geq V_j(\psi_j(\gamma_j), \sigma_{-j}, z). \tag{11}$$

Combining (10) and (11) yields

$$V_j(\sigma_j, \sigma_{-j}, z) \geq V_j(\gamma_j, \sigma_{-j}, \tilde{c}),$$

which completes the proof. ■

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