Sunk Costs, Depreciation, and Industry Dynamics

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Abstract: Dynamic competitive models of industry evolution predict higher variability of firm value and lower variability of the number of firms over time in industries exhibiting higher sunk entry costs. These predictions have done well empirically. Here we extend the theory to allow for depreciation and argue that this generates countervailing effects. We test this assertion empirically and find the results are broadly consistent with the theory.

Keywords: Sunk costs, depreciation, entry and exit.

JEL Classification: L00

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1. Introduction

There is a substantial literature on competitive industry dynamics. This literature includes both theoretical and empirical contributions, and these complement each other nicely.[1] Two predictions of the theory are (1) a positive relationship between sunk entry costs and the intertemporal range of firm value, and (2) a negative relationship between sunk entry costs and the intertemporal range of the number of firms. The first prediction arises from natural equilibrium conditions requiring that firm values be at most equal to entry costs (because higher values provoke entry) and at least equal to scrap value (because lower values provoke exit). This suggests that the range of firm value over time should be approximately equal to the difference between the entry cost and the scrap value, which in turn is a natural definition of sunk entry costs. It follows almost immediately that the range of firm value over time is equal to, and hence increasing in, the sunk entry cost. The second prediction arises because higher sunk entry costs make entry and exit more expensive and so tend to reduce the incidence of these activities. Although detailed data on sunk entry costs are not available, these predictions have been tested using various proxies and found to be consistent with the data.

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[1] Examples of empirical work include Deutsch (1984), Dunne, Roberts and Samuelson (1989), Geroski, Gilbert, and Jacquemin (1990), Geroski and Schwalbach (1991), Siegfried and Evans (1992,1994), Audretsch (1995), Lambson and Jensen (1995,1998), Gschwandtner and Lambson (2002,2006), Disney, Haskel, and Heden (2003), and many others. Examples of theoretical work include Jovanovic (1982), Ericson and Pakes (1989), Dixit (1989), Sutton (1991), Lambson (1991,1992), Hopenhayn (1992), Cabral (1995), Caballero and Pindyck (1996), and many others.

This paper extends the analysis to allow for depreciation, a factor that has not received much attention in this corner of the literature.[2] Depreciation dampens the effects of sunk entry costs. Intuitively, depreciation destroys the value of past investment and tends to return the firms to their initial conditions. To illustrate, consider the extreme case in which firms' assets completely depreciate each time period. Then when entry and exit decisions are made at the beginning of the subsequent period, the incumbent firms have no advantage over potential entrants. In equilibrium, entry and exit maintain firm value equal to the entry cost. Thus, in contrast to sunk entry costs, which are associated with high intertemporal variability of firm value and low intertemporal variability of the number of firms, high depreciation generates low intertemporal variability of firm value and high intertemporal variability of the number of active firms.

Section 2, along with Appendix D, spells out the theoretical arguments upon which the empirical implications rest. Section 3 describes the measurement of depreciation. It gathers market-based (not accounting-based) estimates of depreciation rates for various capital inputs from the existing literature and explains how we incorporate those into the subsequent empirical analysis. Section 4 presents the empirical results, which are broadly consistent with the theory. Section 5 concludes.

[2] There are, of course, exceptions, such as Kessides (1990), Farinas and Ruano (2005), and Vivek (2007).

2. Theory

Appendix D contains a formal discussion of the effects of sunk entry costs and depreciation on the dynamic behavior of firm value and the number of firms. This section contains a heuristic presentation of these issues with the goal of providing the reader with intuition for the results.

2a. Firm value

In the absence of depreciation, the theoretical result that the range of firm value over time is positively correlated with sunk entry cost is robust in that very little structure is required to demonstrate it. Specifically, let ξ_i be the expenditure on input i required to enter the industry and let χ_i be its scrap value. Natural equilibrium conditions require that firm value, V, be bounded above by $\sum_i \xi_i$ (because higher values provoke entry and thus cannot persist) and bounded below by $\sum_i \chi_i$ (because lower values provoke exit and thus cannot persist). If market conditions are variable enough and the observation time long enough for V to visit each end of its support, then the range of V over time is $\sum_i \xi_i - \sum_i \chi_i$. Since this is a plausible definition of sunk entry cost, the range of firm value is identical to, and hence positively correlated with, the sunk entry cost.[3]

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[3]The empirical importance of sunk entry costs in various contexts has been studied by Asplund (2000), Ramey and Shapiro (2001) and others.

Depreciation, the cost of wear and tear on or consumption of capital inputs, has countervailing effects relative to sunk entry costs. Assume each firm has for its objective to maximize its expected present value. Then when deciding whether to enter, potential entrants compare the cost of entry, $\sum_i \xi_i$, with their post-entry value, V. Similarly, active firms compare their current value with their post-exit value. Now an active firm can avoid the current depreciation cost and recoup the scrap value of each capital input by exiting. So if λ_i is the fraction of the expenditure ξ_i lost to depreciation each period then exit has a tendency to bound firm value below by $\sum_i \lambda_i \xi_i + \sum_i \chi_i$. This suggests that if the market conditions are variable enough and the observation time is long enough for firm value to visit the extreme points of its support, then the range of firm value over time is

(2.1) $$R(V) = (\sum_i \xi_i - \sum_i \chi_i) - (\sum_i \lambda_i \xi_i)$$

Inspection of (2.1) reveals that the range of firm value depends positively on sunk entry costs as before, but depends negatively on depreciation. Put another way, the theory predicts that the regression

(2.1') $$R(V) = \beta_0 + \beta_1(\sum_i \xi_i - \sum_i \chi_i) + \beta_2 (\sum_i \lambda_i \xi_i)$$

will generate a positive β_1 and a negative β_2. Indeed, with data on the actual variables (rather than proxies), $\beta_1 = -\beta_2 = 1$

2b. Number of firms

Lambson (1992) showed that if the sunk entry cost, $\sum_i\xi_i - \sum_i\chi_i$, is increased either by increasing the entry cost $\sum_i\xi_i$ while holding $\sum_i\chi_i$ fixed, or by decreasing the scrap value $\sum_i\chi_i$ while holding $\sum_i\xi_i$ fixed, then the range of the number (mass) of firms, R(y), is negatively related to sunk entry costs. As argued above, when depreciation is included in the model, the value of a firm is bounded between ($\sum_i\lambda_i\xi_i + \sum\chi_i$) and $\sum_i\xi_{i,}$ instead of between $\sum_i\chi_i$ and $\sum\xi_i$. So analogous reasoning establishes that the range of the number of active firms is negatively related to $\sum_i\xi_i - (\sum_i\lambda_i\xi_i + \sum_i\chi_i)$ when it is increased by increasing $\sum_i\xi_i$ with $\sum_i\lambda_i\xi_i + \sum_i\chi_i$ fixed, or by decreasing $\sum_i\lambda_i\xi_i + \sum_i\chi_i$ with $\sum_i\xi_i$ fixed. Put another way, the theory predicts that the regression

(2.2) $$R(y) = \beta_0 + \beta_1\sum_i\xi_i + \beta_2 (\sum_i \lambda_i \xi_i + \sum_i \chi_i) + \varepsilon$$

will generate a negative β_1 and a positive β_2. Putting (2.2) in a form more readily comparable to (2.1') results in:

(2.2') $$R(y) = \beta_0 + (\beta_1\sum_i\xi_i + \beta_2 \sum_i \chi) + \beta_2 (\sum_i \lambda_i \xi_i) + \varepsilon$$

Note that (2.2') is conceptually quite different from (2.1'). Given the equilibrium conditions, (2.1') is directly implied, including the magnitudes of the coefficients. Hence it is valid to rearrange the terms in (2.1) to highlight the separate effects of the sunk entry costs and depreciation.

By contrast, when the range of the number of firms is the dependent variable, it can be difficult to separate the effects of sunk entry costs from depreciation. No particular functional form is implied, so additional structure must be added to separate the effects. One approach is to suppose (as seems plausible) that there is not much variation in scrap values (perhaps because they are all close to zero). Then (2.2) implies that (2.3) holds approximately:

(2.3) $$R(y) = \beta_0 + \beta_1(\sum_i \xi - \sum_i \chi_i)_i + \beta_2 (\sum_i \lambda_i \xi_i) + \varepsilon$$

Theory then predicts $\beta_1 < 0$ and $\beta_2 > 0$.

3. Measuring depreciation

Jorgensen (1996) and Fraumeni (1997) discuss the empirical literature on depreciation. We agree with their view that economically relevant measures of depreciation are determined by the workings of resale markets for capital assets. Such measures are more likely to be economically relevant than, for example, accounting measures.

We require two different measures of depreciation: a firm-level measure to test the implications of Section 2a and an industry-level measure to test the implications of Section 2b. To construct either measure requires an estimate of depreciation for each capital input. We have taken these from Hulten and Wykoff (1981) as summarized by Jorgenson (1996) Table II. Hulten and Wykoff apply the Box–Cox power transformation to prices of used assets in order to estimate the rate and form of economic depreciation. This allows them to statistically discriminate between various patterns of depreciation (such as geometric, linear and 'one-hoss-shay' depreciation patterns).

They find that the observed depreciation patterns are approximately geometric. In a later paper Hulten and Wykoff (1996) revised and extended these measures to include the effect of obsolescence, defined as the decline in price resulting from the introduction of new vintages of capital. As a result the revised rates are generally somewhat higher than the initial Hulten-Wykoff depreciation rates. These two sets of depreciation estimates will be referred to as HW1 and HW2, respectively. Finally, the Bureau of Economic Analysis (BEA) published its own estimates of depreciation rates for use in the National Income and Product Accounts. The estimates employed in the national accounts differ from the other two depreciation measures in that they incorporate information about lifetimes and salvage values of assets and accounting formulas permitted for tax purposes. The economic depreciation rates for nonresidential structures estimated by Hulten and Wykoff are much lower than those employed in the U.S. national accounts. The BEA depreciation rates can be found, for example, in Jorgenson and Sullivan (1981). More recent but not very different depreciation rates can be found in Fraumeni (1997). Despite the differences in construction, our results using the BEA depreciation rates do not differ significantly from the results using the two Hulten-Wykoff estimates.

We constructed the firm-level estimate of depreciation as a weighted sum of the depreciation rates of the capital inputs used by the firm. The weights are estimates of the firm's expenditures for the respective capital inputs. Specifically, the depreciation index for firm f in industry F is

(3.1) $$\Lambda_{Ff} = \sum_i \lambda_i P_{iF} (S_f/S_F),$$

where S_f is firm *f's* average sales over time, λ_i is the depreciation rate of input i, P_{iF} is the aggregate expenditure on input i in industry F, and S_F is industry F's average sales over time. Of course, it would be preferable to have a direct firm-level measure of the usage of the various inputs. However, if usage is roughly proportional to sales, then the index will generate fairly good estimates of firm-level use. For the industry's expenditures on input i, P_{iF}, we used the capital flows table constructed by the Industry Economics Division (IED) of the Bureau of Economic Analysis (BEA) of the United States Department of Commerce. The capital flows table is a supplement to the benchmark input-output accounts and it shows purchases of new structures, equipment and software by industry. Specifically, the capital flows table lists the capital inputs used in each industry; we multiplied these by the depreciation rates for the respective industries in which the inputs were produced and then summed over inputs.

We used similar methods to construct our depreciation measures for the analysis of the intertemporal number of firms in an industry. In contrast to the analysis of intemporal firm value, where each observation corresponds to a firm, here an observation corresponds to an industry. Of course, defining industries is seldom without difficulties. We assigned firms to industries according to SIC (NAICS) codes at the 2-4 digit level. It is well known that this approach is not perfect—for example, since many of the firms are diversified across product lines they might not be well described by a single SIC code—but we are unaware of any compelling reason to believe that these shortcomings introduce any biases. If expenditure per unit of output (sales) is interpreted as a cost of capacity, then

(3.2) $$\boldsymbol{\Lambda_F = \sum_i} \lambda_i P_{iF} / S_F$$

is roughly interpretable as the depreciation rate of capacity in industry F.

Finally, we emphasize again that these depreciation measures are based on market prices. These are more likely to reflect the economically relevant depreciation rates than are estimates that follow accounting rules that are not necessarily correlated with economic activity. Furthermore, the functional forms used to determine the depreciation rates are very flexible and therefore much more probable to approximate the real pattern of depreciation than typical accounting rules.

4. Evidence

The empirical results are broadly supportive of the theory. As anticipated by the discussion in section 2, the results for the intertemporal range of firm value are the strongest, always exhibiting the predicted signs with high statistical significance. The results for the intertemporal range of the number of active firms are weaker. The coefficients of the depreciation indexes are statistically significant and of the correct sign, but the coefficients of the sunk entry costs are not statistically distinguishable from zero. Each set of empirical results will now be discussed in turn.

4a. Firm value

The regressions in this subsection test the proposition from Section 2a that the range of firm value is positively correlated with sunk entry costs and negatively correlated with the rate of depreciation of its capital inputs. The database used for these purposes contains information on 162 companies. All of them are publicly traded manufacturing companies in the United States observed between 1950 and 2001. The sample is comprised of those firms among the largest 500 companies (in terms of sales) as of 1950 for which a complete time series on profits for the analyzed period existed. There is obvious selection bias in the firms that survive, but it is irrelevant because the theory makes predictions about surviving firms. Most of the database was compiled from Standard and Poor's Compustat. Gaps—mostly a problem in the early years—were filled from Moody's Industrial Manual.

As is commonly done, we measure firm value as the sum of stock market capitalization and total liabilities. Stock market capitalization is calculated as the year-end closing price of common shares times the number of common shares outstanding. The closing price is the closing trade price for shares traded on a national stock exchange and the closing bid price for shares trading over-the-counter. We measured intertemporal variability in two ways: range and variance. Although theory favors range as the appropriate measure, variance is less sensitive to data problems that result in outliers. It turns out that one may remain agnostic as to which is the better measure:

the regressions with range and the regressions with variance yield similar results. We also tried various dependent variables: range, variance, log of range, and log of variance.

The regressions include both a sunk entry cost proxy as in prior studies (namely, investment in property, plants and equipment) and a depreciation cost (the construction of which is explained in section 3). The coefficient of the sunk entry cost proxy is positive, as predicted. The coefficient of the depreciation cost is negative, as predicted. Concerned that fifty years might be too long to expect a firm to remain similar, we also divided the 50 years of the sample into ten year subsamples. Our conclusions were unaffected. In addition to what we have reported, we tried several other specifications of independent variables as robustness checks, including the number of employees, capital expenditures, capital intensiveness (as measured by the capital-labor ratio), and new capital expenditures. None of these variants had any significant effect on our conclusions.

4b. Number of firms

To explore the effects of sunk entry costs and depreciation costs on the range of the number of firms over time, we used annual data from the US Census Bureau. With sponsorship from the US Small Business Administration (SBA), the Census Bureau collects data on entry and exit by industry for the United States as a whole and for each state.[4] This database contains information about entry, exit, and employment from 1990-2000 for each included industry.[5]

Recall from Section 2b that the theory asserts that the range of the number of active firms over time should be negatively correlated with entry costs and positively correlated with depreciation costs. Our data do not include the number of firms. They do include annual data on entry and exit. Fortunately, this is adequate to calculate the range of the number of firms because, assuming there is enough variability over time and a long enough observation period to observe the extremes, the range of the number of firms is

$$R(y) = \max_{\tau}\{y_o + \sum_{t=1}^{\tau} [n_t - x_t]\} - \min_{\tau} \{y_o + \sum_{t=1}^{\tau} [n_t - x_t]\}$$

$$= \max_{\tau} \{ \sum_{t=1}^{\tau} [n_t - x_t]\} - \min_{\tau} \{ \sum_{t=1}^{\tau} [n_t - x_t]\}$$

where n_t and x_t are respectively entry and exit in period t. Since the initial number of firms, y_o, cancels, not observing it poses no problem. The results are reported in Appendix B. The various specifications all exhibit significant coefficients of the predicted sign for depreciation. The coefficients of the sunk cost proxies are not distinguishable from zero.

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[4] These data are available at: http://www.sba.gov/advo/stats/data.html.

[5] More specific information is at： http://www.census.gov/csd/susb/susb2.htm#godyn1 .

5. Concluding Remarks

The field of industrial organization began as the study of imperfect competition. Differences in profit rates across industries, a very well documented phenomenon, were taken to be evidence that competition was imperfect. However, differing profit rates across industries are consistent with perfect competition, even in the long run. (See Lambson (1992).) Since maximizing average profits is not the firms' objective, the market provides no mechanism to equalize them. Rather, if firms attempt to maximize the expected present value of investments, then it is the value of a marginal dollar of investment that will tend to equalize across investments. Under these circumstances, it seems likely that any robust empirical implications will be inherently dynamic. This paper has focused on some of these dynamic implications, showing them to be broadly consistent with the data.

Appendix A: Firm value over time

Numbers in parentheses are robust standard errors cluster corrected by industry. One, two and three stars denote significance at the 10%, 5% and 1% levels, respectively. HW1, HW2 and BEA denote the depreciation index calculated from (3.1) for the various data sources. Entry costs are proxied by a firm's average value over time of property, plant and equipment.

A.1: Dependent variable is the logarithm of the range of firm values

Intercept	HW1	HW2	BEA	Entry Costs	Adj.R^2	Obs
8.74 (0.184)***	-3.34 (0.803) ***			0.21 (0.099)**	0.12	162
8.74 (0.185)***		-3.43 (0.812)***		0.21 (0.099)**	0.12	162
8.69 (0.184)***			-4.40 (0.950)***	0.21 (0.10)**	0.14	162

Note: An observation corresponds to a firm over the time period 1950-2001.

A.2: Dependent variable is the logarithm of the variance of firm values

Intercept	HW1	HW2	BEA	Entry Costs	Adj.R^2	Obs
14.98 (0.371)***	-6.53 (1.61) ***			0.42 (0.198)**	0.11	162
14.96 (0.37)***		-6.71 (1.63)***		0.42 (0.198)**	0.11	162
14.88 (0.370)***			-8.61 (1.91)***	0.42 (0.196)**	0.12	162

Note: An observation corresponds to a firm over the time period 1950-2001.

A.3: Dependent variable is the logarithm of the range of firm values within decades

Intercept	ln(HW1)	ln(HW2)	ln(BEA)	ln(Entry Costs)	Adj.R^2	Obs
6.221 (0.582)***	-0.294 (0.07) ***			0.803 (0.088)***	0.52	803
6.197 (0.586)***		-0.291 (0.071)***		0.806 (0.891)***	0.52	803
6.121 (0.572)***			-0.283 (0.692)***	0.813 (0.087)***	0.52	803

Note: An observation corresponds to a firm over a decade.

A4: Dependent variable is the logarithm of the variance of firm values within decades

Intercept	ln(HW1)	ln(HW2)	ln(BEA)	ln(Entry Costs)	Adj.R^2	Obs
10.558 (1.177)***	-0.610 (0.142) ***			1.559 (0.178)***	0.52	803
10.51 (1.186)***		-0.604 (0.143)***		1.564 (0.180)***	0.52	803
10.370 (1.156)***			-0.587 (0.140)***	1.579 (0.175)***	0.52	803

Note: An observation corresponds to a firm over a decade.

A.5: Dependent variable is the logarithm of the range of firm values within decades

Intercept	ln(HW1)	ln(HW2)	ln(BEA)	ln(Entry Costs)	ln(L)	Adj.R^2	Obs
6.025 (0.576)***	-0.224 (0.071) ***			0.699 (0.082)***	0.001 (0.000)***	0.57	803
6.000 (0.578)***		-0.221 (0.071)***		0.702 (0.082)***	0.000 (0.000)***	0.57	803
5.958 (0.569)***			-0.216 (0.069)***	0.706 (0.08)***	0.000 (0.000)***	0.57	803

Note: An observation corresponds to a firm over a decade. ln(L) is the logarithm of the average number of employees during the decade.

A6: Dependent variable is logarithm of the variance of firm values within decades

Intercept	ln(HW1)	ln(HW2)	ln(BEA)	ln(Entry Costs)	ln(L)	Adj.R^2	Obs
10.17 (1.163)***	-0.471 (0.142) ***			1.352 (0.165)***	0.001 (0.000)***	0.57	803
10.12 (1.167)***		-0.465 (0.143)***		1.357 (0.166)***	0.000 (0.000)***	0.57	803
10.026 (1.148)***			-0.454 (0.141)***	1.367 (0.162)***	0.000 (0.000)***	0.57	803

Note: An observation corresponds to a firm over a decade. ln(L) is the logarithm of the average number of employees during the decade.

Appendix B: Number of firms over time

Numbers in parentheses are robust standard errors. One, two and three stars denote significance at the 10%, 5% and 1% levels, respectively. HW1, HW2 and BEA denote the depreciation index calculated from (3.2) for the various data sources. Sunk entry costs are proxied by K/L=Capital Expenditures/Employees and by NewK/L= New Capital Expenditures/Employees. The number of industries is 61.

Dependent variables: Logarithm of the Range of the Number of Active Firms

Eq.	ln(HW1)	ln(IRB)	ln(BEA)	ln(K/L)	ln(NewK/L)	Adj.R^2
1	0.466 (0.223)**			0.144 (0.249)		0.12
2		0.479 (0.22)**		0.136 (0.248)		0.12
3			0.403 (0.23)**	0.161 (0.257)		0.10
4	0.468 (0.223)**				0.137 (0.245)	0.12
5		0.482 (0.22)**			0.130 (0.244)	0.13
6			0.405 (0.23)*		0.161 (0.257)	0.010

Note: An observation corresponds to an industry over time.

Appendix C: Descriptive Statistics

	Mean	Median	Std. Dev.
Log T	0.73016	0.8407	1.96843
HW1	0.05055	0.0104	0.16631
HW2	0.05108	0.0109	0.16417
BEA	0.05030	0.01064	0.1389
$\lambda 1$	0.09084	0.06115	0.07537
λ 2	0.09452	0.06115	0.07917
λ BEA	0.12628	0.1053	0.13085
LogRange Firm Val	9.1805	9.2414	1.78776
LogVar Firm Val	15.836	15.8587	3.57534
Entry	1.2479	0.9655	1.34524
LogK	8.6709	8.8776	1.21190
LogNew K/L	4.7117	4.9782	1.14404
L	20.801	7.9000	51.4931

Appendix D: Mathematical appendix

Consider a discrete time model with a continuum of identical, infinitesimal (price-taking) firms. Index the time periods by the positive integers, $t \in \{1,2,3,\ldots\}$. A market condition, $m \in M$, describes the values of the relevant exogenous variables such as factor prices and demand conditions. Market conditions are generated by an exogenous stochastic process known to the firms. Let $H_t := \times_{\tau=1}^{t} M$ be the set of conceivable *t*-period histories of market conditions and define $H := \cup_{t=1}^{\infty} H_t$. For $h \in H_t$ and $g \in H_s$, where *s>t*, let $\rho(g \mid h)$ be the probability that the history g is realized given that the history h is realized.

If g is the history of market conditions through time t and y is the number (measure) of active firms at time t then the current profit of an active firm at time *t* is denoted $\pi(y,g)$. It can be interpreted as having been derived from the competitive equilibrium given *y* firms and the last market condition in g. Firms maximize current profit given the price and the price adjusts to clear the market.

An exit rule is a set of histories, $\gamma \subset H$, such that if $h, g \in \gamma$ then $\rho(g \mid h) = 0$. If $g \in \gamma$ then a firm implementing the exit rule γ exits if the history g is realized. Given a stochastic process, $Y = \{y_h\}_{h \in H}$, governing the number of active firms, the value of an active firm given the history $h \in H_t$ is

$V(Y,h) =$

$$\pi(y_h, h) + \sup_{\gamma \in \Gamma}\{\sum_{s=t+1}^{\infty} \delta^{s-t} \sum_{g \in \eta(\gamma,s)} \rho(g \mid h)[\pi(y_g, g) - \Sigma_i \lambda_i \xi_i] + \sum_{s=t+1}^{\infty} \delta^{s-t} \sum_{g \in \gamma \cap H_s} \rho(g \mid h)\Sigma_i \chi_i\}$$

where δ is the discount factor, Γ is the set of all exit rules and $\eta(\gamma, s) \subset H_s$ is the set of *s*-period histories such that a firm using the exit rule γ would not exit before or during the period *s*. Note that V(Y,h) is the expected present value of a firm gross of entry costs or current depreciation. For $h \in H_t$, let h^{-1} be the $t-1$ period history comprised of the first $t-1$ market conditions of *h*. An equilibrium is a stochastic process *Y* such that, for all *t* and all $h \in H_t$,

$V(Y,h) \leq \sum_i \xi_i$	
$V(Y,h) = \sum_i \xi_i$	if $y(h) > y(h^{-1})$
$V(Y,h) \geq \sum_i \chi_i + \sum_i \lambda_i \xi_i$	if $y(h) > 0$
$V(Y,h) = \sum_i \chi_i + \sum_i \lambda_i \xi_i$	if $y(h^{-1}) > y(h) > 0$
$V(Y,h) \leq \sum_i \chi_i + \sum_i \lambda_i \xi_i$	if $y(h^{-1}) > y(h) = 0$.

Informally, in equilibrium all active firms maximize current profits, markets clear in every period, firms enter only if the expected present value of an active firm is not less than the entry cost, and firms exit only if the expected present value of an active firm net of depreciation does not exceed its scrap value.

The following technical assumptions are employed. Assumption 1 rules out the empirically irrelevant case of entry inducing exit by all incumbent firms. Incumbent firms, deciding between remaining active and exiting, compare V - $\Sigma_i\lambda_i\xi_i$ against $\Sigma_i\chi_i$. When there is entry, $V = \sum_i\xi_i$, so the relevant comparison is $\sum_i\xi_i - \Sigma_i\lambda_i\xi_i$ against $\Sigma_i\chi_i$. If $\sum_i\xi_i < \Sigma_i\lambda_i\xi_i+\Sigma_i\chi_i$ then entry provokes exit by all of the incumbent firms: incumbency is a disadvantage. This strange result is ruled out by the condition $\sum_i\xi_i > \Sigma_i\lambda_i\xi_i+\Sigma_i\chi_i$. Assumption 2 ensures existence of equilibrium values by simple intermediate value theorems. The variable ϕ_h can be interpreted as a fixed production cost. Assumption 3 implies that there is no incentive for entry if there is no expectation of profitable production.

Assumption 1: $\sum_i\xi_i > \sum_i\lambda_i\xi_i + \sum_i\chi_i$.

Assumption 2: For all *h*, $\pi(y,h)$ is continuous and decreasing in *y* with $\lim_{y\to 0}\pi(y,h)=\infty$ and $\lim_{y\to\infty}\pi(y,h) = -\phi_h < 0.$

Assumption 3: For all t and all $h\in H_\tau$, $\overline{V}(h) < \xi$ where

$$\overline{V}(h) = -\phi_h + \sup_{\gamma\in\Gamma}\left\{\sum_{s=t+1}^{\infty}\delta^{s-t}\sum_{g\in\eta(\gamma,s)}\rho(g\,|\,h)\left[-\phi_g - \Sigma_i\lambda_i\xi_i\right] + \sum_{s=t+1}^{\infty}\delta^{s-t}\sum_{g\in\gamma\cap H_s}\rho(g\,|\,h)\Sigma_i\chi_i\right\}$$

Lemma: There exists a stochastic sequence of ordered pairs, $\{N_h, X_h\}_{h\in H}$ *such that Y satisfying* $y_h = \min\left\{X_h, \max\left\{N_h, y_{h^{-1}}\right\}\right\}$ *for all* $h\in H$ *is an equilibrium.*

Proof: For all τ, all $t\le\tau$, and all $h\in H_t$, define

$$w_\tau(y,h) = \Sigma_i\lambda_i\xi_i + \Sigma_i\chi_i \qquad \text{if } h\in H_\tau$$

$$w_\tau(y,h) = \pi(y,h) + \delta \sum_{g \in H_{t+1}} \rho(g \mid h) w_\tau(y_{g\tau}, g) \text{ otherwise}$$

where $y_{g\tau} = \min\left\{x_{g\tau}, \max\left\{N_{g\tau}, y\right\}\right\}$. The pairs $\left\{N_{g\tau}, X_{g\tau}\right\}$ are defined by backward induction as follows.

For $g \in H_{\tau-1}$, $N_{g\tau}$ is the lowest value that satisfies $w_\tau\left(N_{g\tau}, g\right) = \Sigma_i \xi_i$. This is well defined because $\lim_{y \to 0} w_\tau\left(y, g\right) = \lim_{y \to 0} \pi\left(y, g\right) + \delta\left(\Sigma_i \lambda_i \xi_i + \chi_i\right) > \Sigma_i \xi_i$ by *A2* and $\lim_{y \to 0} w_\tau\left(y, g\right) = -\phi_g + \delta\left(\Sigma_i \lambda_i \xi_i + \Sigma_i \chi_i\right)_i < \Sigma_i \xi_i$ by *A1* and *A2*. Since $w_\tau\left(y, g\right)$ is continuous in *y*, $N_{g\tau}$ exists.

For $g \in H_{\tau-1}$, $X_{g\tau}$ is the smallest value that satisfies $w_\tau\left(x_{g\tau}, g\right) = \Sigma_i \lambda_i \xi_i + \Sigma_i \chi_i$. If $-\phi_g + \delta\left(\Sigma_i \lambda_i \xi_i + \Sigma_i \chi_i\right) < \Sigma_i \lambda_i \xi_i + \Sigma_i \chi_i$ then reasoning similar to the previous paragraph implies that $X_{g\tau}$ is finite. Otherwise $X_{g\tau}$ is infinite.

Make the induction hypothesis that $w_\tau\left(y, g\right)$ is continuous and negatively dependent on *y* and that $N_{g\tau}$ and $X_{g\tau}$ are well defined for $g \in H_s$ and $t+1 \le s < \tau$. Then $\lim_{y \to 0} w_\tau\left(y, h\right) = \lim_{y \to 0} \pi\left(y, h\right) + \delta \Sigma_i \xi_i > \Sigma_i \xi_i$ by *A2* and $\lim_{y \to \infty} w_\tau\left(y, h\right) \le -\phi_h + \delta \Sigma_i \xi_i < \Sigma_i \xi_i$ by the definition of $y_{g\tau}$. So $N_{h\tau}$ is well defined for $h \in H_t$. Similar arguments establish $X_{h\tau}$ is well defined (although possibly infinite).

Next note that $N_{h\tau}$ and $X_{h\tau}$ are monotonically increasing in τ. This follows because $w_\tau\left(y, h\right) \le w_{\tau+1}\left(y, h\right)$ for all *y* and all $h \in H_t$ when $t \le \tau$. Then an application of

A3 establishes that $N_{h\tau}$ is bounded in τ. So $N_h = \lim_{\tau\to\infty} N_{h\tau}$ and $X_h = \lim_{\tau\to\infty} X_{h\tau}$ are well defined (although X_h may be infinite). These ordered pairs fulfill the requirements of the lemma. *QED*

Theorem: (a) The range of firm value over time, $\sum_i \xi_i$ - ($\sum_i \lambda_i \xi_i + \sum_i \chi_i$), is increasing in $\sum_i \xi_i - \sum_i \chi$, and decreasing in $\sum_i \lambda_i \xi_i$. (b) The range of the measure of active firms is decreasing in $\sum_i \xi_i$ and increasing in ($\sum_i \lambda_i \xi_i + \sum_i \chi_i$).

Proof: (a) Trivial. (b) For arbitrary τ, for all $t < \tau$, and for all $h \in H$, compare $(N_{h\tau}, X_{h\tau})$ to $(N'_{h\tau}, X'_{h\tau})$, when the former are equilibrium values given $\sum_i(\lambda_i\xi_i+\chi_i)$ and the latter are equilibrium values given $\sum_i(\lambda_i\xi_i+\chi_i)' > \sum_i(\lambda_i\xi_i+\chi_i)$. Specifically, if $h \in H_{\tau-1}$ then $N'_{h\tau} = N_{h\tau}$ and $X'_{h\tau} \le X_{h\tau}$. Make the induction hypothesis that for all $g \in H_s$, where $u < s < \tau$, $N'_{g\tau} \ge N_{g\tau}$ and $X'_{g\tau} \le X_{g\tau}$. For $h \in H_u$ define

$$G(\tau, h) = \left\{ g \in H \mid \rho(g \mid h) \ne 0, g \in H_t, u < t < \tau \right\}.$$

Define $\left\{y_{g\tau}\right\}_{g\in G(\tau,h)}$ and $\left\{y'_{g\tau}\right\}_{g\in G(\tau,h)}$ inductively by $y_{g\tau} = \min\left\{X_{g\tau}, \max\left\{N_{g\tau}, y_{g-1}\right\}\right\}$ and $y'_{g\tau} = \min\left\{X'_{g\tau}, \max\left\{N'_{g\tau}, y'_{g-1}\right\}\right\}$ respectively with $y_{h\tau} = y'_{h\tau} = N_{h\tau}$. Define $A = \left\{ g \in H \mid y'_{g\tau} > y_{g\tau} \textit{ and } y'_{g^{-k}\tau} \le y_{g^{-k}\tau} \textit{ for all } k \textit{ such that } g^{-k} \in G(\tau, h) \right\}$.

Note that for $g \in A$ either $y'_{g\tau} = N'_{g\tau}$ or $y_{g\tau} = X_{g\tau}$. In the former case, the continuation value of a firm is ξ for the path $\left\{y_{g\tau}\right\}$ and in the latter case the continuation value of a firm is $\lambda\xi + x$ for the path $\{y'_{g\tau}\}$ by virtue of their smaller mass. So $N'_{g\tau} \geq N_{g\tau}$. A similar argument shows $X'_{g\tau} \leq X_{g\tau}$. Taking limits shows $X'_{g} \leq X_{g}$ and $N'_{g} \geq N_{g}$ for all *g*. So the range of the mass of firms is increasing in $\sum_i(\lambda_i\xi_i+\chi_i)$ if $\sum_i\xi_i$ is held fixed, and decreasing in $\sum_i\xi_i$ if $\sum_i(\lambda_i\xi_i+\chi_i)$ is held fixed. QED

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