A model of political campaigns, lobbying and marketing over social networks: Whom to target?

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November 16, 2009

[*Jobmarket paper.*]

Abstract I study competitions between two agents (political parties, lobbies or firms) who wish to persuade public opinion when the individuals they compete to persuade are also influenced by the opinion of their neighbors on a social network. The persuaders must decide whom to target to maximize the impact of their resources. I propose a strategic model of competition and test it using data on lobbying expenditures in the US House of Representatives. In equilibrium, persuaders target their resources toward individuals with higher network influence, adjusting to spend less on individuals who are harder to persuade. This finding contrasts with the conventional wisdom from models on strategic spending (without network influence) which had found that parties (or lobbies) would spend more on individuals who have a higher probability of being pivotal voters. To test my model I combined data on cosponsorship networks in Congress with data on campaign contributions by lobbies over several electoral cycles. Both network influence and pivot probabilities are statistically significant predictors of time variations of lobby spending, but the estimate of network influence is larger.

Keywords: Network games, strategic spending, Colonel Blotto games, counteractive lobbying, viral marketing, Bonacich centrality.

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1 Introduction

This paper studies competitions to persuade public opinion when members of the public influence each other's opinion. When people are deciding how to vote or which product to buy, they discuss their decision with people in their social environment. Studying the pattern of social relationships is important in understanding how individuals are influenced directly and indirectly by the opinion of others. Currently we do not have a model of competition that takes these effects into account.

Competitions to persuade public opinion are the essence of political campaigns, but they also occur in marketing between rival firms or in lobbying with interests groups on opposite sides of a legislation. The model below can be used for all of these applications.

Using techniques from social network analysis, I propose a model where two *persuaders* target resources over a group of *choosers* that influence each other through a social network. My model allows a rich structure of influence between individuals. I allow for influence to be asymmetric between individuals and put no restriction on the number of people they talk to.

My main finding is that in equilibrium persuaders spend on each chooser in proportion to his network influence but adjust to spend less on choosers who are harder to persuade in the margin. In the unique pure-strategy nash equilibrium for political campaigns and lobbying I find that persuaders spend in proportion to the *DeGroot measure of network influence* of each chooser (voter/legislator). For marketing campaigns I find that persuaders spend in proportion to the *Bonacich measure of network influence* of each chooser (consumer). Both of these measures had been found in the sociology literature.

Previous papers on strategic spending in political campaigns and lobbying have found that resources should be targeted toward voters who have a higher probability of casting a pivotal vote. (See Shubik & Weber (1981)). In my model, influence is solely determined by the network structure, not by the ideology of each voter. This implies that influential voters could be completely different from pivotal voters. This yields new predictions on campaign spending. For example, my model predicts that resources will be spent on voters who have an influential position on the social network even if they are very unlikely to swing their vote.

Spending switches from pivotal voters to influential voters because network spillovers undermine targeting. Pivotal voters are important for elections because they have the maximum impact in the outcome of the election. This is still true in my model, but persuaders can no longer effectively target their resources in the presence of network effects. Voters cannot be persuaded individually because they mix their opinion with those of their neighbors.

To test the model I match data on campaign contributions by lobby groups with data on cosponsorships networks in the US House of Representatives. Using the variation in time in the contributions for each Representative, I identify the effect of network influence on the pattern of spending. I find that changes in both network influence and pivot probabilities are significant predictors of changes in campaign contributions. After controlling for several confounds I find that increasing network influence by one standard deviation increases the campaign contributions by 34,521 US dollars (9.4% of the average contributions received by a Representative) while increasing the probability of being pivotal by one standard deviation increases them by 21,962 USD (6% of the average contributions).

My paper brings together two literatures. On the social networks side there has been much work on identifying the influential members in networks but almost no work has been done on how this information would be used in competitions. My model predicts that two of these measures are related to strategic spending. These measures have been derived from graph characteristics of the network[1] and from processes of opinion formation[2], but they have not been tied to strategic spending.[3]

On the strategic persuasion side, there is a literature on counter-active lobbying[4] and

[1]Katz (1953); Bonacich & Lloyd (2001); Bonacich (1987).

[2]French (1956); Harary (1959); DeGroot (1974).

[3]See also Jackson (2008) and Wasserman & Faust (1994) for references on the broad number of measures of network influence that have been proposed.

[4]Austen-Smith & Wright (1994, 1996).

strategic spending in presidential elections,[5] but these papers do not allow for voters to influence one other.

My paper also contributes to the growing literature on advertising through social networks, also called "viral marketing".[6] Except for the paper by Galeotti & Goyal (2009), which has a section on price competitions and word-of-mouth advertising, this literature has focused on the problem of a single decision maker without analyzing the effects of competition.

Social networks will be increasingly more important for future political campaigns. In a survey by the Pew Research Center on the 2008 presidential election, 27% of people under 30 reported getting information on the campaign through social networking sites. The number rose to 37% if you only consider those between 18 and 24 years. This drastically differed from the 4% of people in their 30s and the less than 1% of people above 40 who reported using these sites.[7] Concurrently, the growth in social networking sites is generating more data than ever before on the structure of social networks. This allows levels of targeting that would have been inconceivable a decade ago.

The paper is structured as follows: Section 2 presents the model; Section 3 solves the model for majoritarian competitions, these include elections in presidential systems and lobbying; Section 4 solves the model for proportional competitions, these include marketing campaigns and elections in proportional representation systems; Section 5 tests the model with data on legislative cosponsorship networks and data on campaign contributions in the US House of Representatives; for most of the paper I assume persuaders have a fixed amount of resources, I provide two extensions to this: Section 6 analyzes the effect of changing the persuaders' relative amount of resources and Section 7 solves the model when persuaders have to raise their resources at a cost; Section 8 concludes.

[5] Merolla *et al.* (2005).

[6] Campbell (2008); Richardson & Domingos (2002); Leskovec *et al.* (2007).

[7] The next time somebody asks you where you get your political information, you might want to remember that those who do so through social networking sites are on average 21.25 years younger than those who don't.

2 The Model

2.1 The persuaders

Two persuaders, A and B, have to decide how to spend advertising resources over a group of individuals who will choose between them. The persuaders A, B can be thought of as political parties, competing lobbies or competing firms. The choosers can be thought of as voters, consumers, demographic groups, or members of congress.

In my model, the main difference between political and marketing campaigns is the objective function of the persuaders. In majoritarian competitions, persuaders need to convince a threshold number of choosers to win the competition. For example, they would need to convince half plus one of the electorate college to win a presidential race or convince a qualified majority to pass a bill in a commission. In proportional competition persuaders want to maximize the share of choosers who select them. For example, a firm would like to increase its customers even if its market share is more than 50%.

2.2 The timing of the game

The game is divided in several stages which are qualitatively different. Inside these stages are rounds which repeat similar actions.

- **The initial stage:** (Round 0) Choosers begin with a given probability of selecting A over B. The probability a chooser will select A over B depends on the chooser's opinion of A and B as well as his preferences. I explain this in detail in Section 2.3.

- **The persuasion stage:** (Round 1) Persuaders simultaneously spend resources to influence the decision of the choosers. I explain how spending changes the opinion choosers in Section 2.4.

- **The deliberation stage:** (Rounds 2 through τ) After persuaders spend all their budget, choosers start a series of deliberation rounds. Every round choosers update their opinion parameter by taking a weighted average of the opinion of their neighbors on a social network. I explain this in Section 2.5.

- **Final stage.** After updating their opinion $\tau - 1$ times through the network, choosers stochastically pick either A or B. The realization of these choices have different consequences for persuaders A, B under majoritarian or proportional competitions. (Respectively, sections 3 and 4).

2.3 The choosers: those to be persuaded

There is a finite number N of choosers that select between A and B. A subscript i denotes chooser i. Choosers decide between A and B to maximize a stochastic utility function. I assume that the utility function of a chooser is

$$U_i(\text{Choosing } A) = \frac{u_i + v_i}{2} - \epsilon_i$$

$$U_i(\text{Choosing } B) = 0$$

This utility function has three components:

- **An private taste parameter** $u_i \in (0, 1)$. This can be thought as an ideal point in a one dimensional spatial model. A parameter $u_i = 1$ represents the maximal preference for A and $u_i = 0$ represents the maximal preference for B.

- **A common-utility parameter** $v \in (0, 1)$. This parameter is the same for all choosers. I call it the valence dimension. A value of v closer to 1 means choosers think A is more attractive versus than B and vice-versa. Each individual has an opinion v_i on the value of v. The opinion will evolve through the game. I use v_i^t to denote the opinion of chooser i at round t. Persuaders can only change valence dimension by spending resources. See Section 2.4.

There are two ways to interpret the valence dimension: It can be thought as a social taste that choosers update to match with their neighbors. They might want to match this dimension because of altruism or because they want to match individuals with high social status.

The parameter can also be interpreted as information on dimensions of the decision that all choosers agree they prefer more: everybody wants a better quality product and everybody wants a candidate who is more competent to deal with a financial crisis. In this interpretation there is a true value of v but choosers do not know it. Rather they have an opinion v_i^t about it's value which they update through the opinion of their neighbors. See Section 2.5.

- **A stochastic preference shock,** $\epsilon_i \in [0,1]$ **which is distributed** $U[0,1]$**.** This parameter represents unmodelled uncertainty about the elements that determine the final choice of a chooser. This shock need not be random from the point of view of the chooser, it only matters that it's unknown by the persuaders at the time they decide their spending. There can be all sorts of elements that make choosers have a change of heart when they make their final decision. For example, personal experiences can vary the attitude toward a candidate; and Gomez *et al.* (2008) reported that bad weather affects voter turnout differently for Democrats than Republicans.[8] These elements are hard for persuaders to forecast or control.

This formulation is convenient[9] to calculate the probability a chooser will pick persuader A or persuader B. A chooser picks A if and only if $\frac{u_i+v_i^t}{2} - \epsilon_i > 0$ which happens with probability $\frac{u_i+v_i^t}{2}$.

The utility of B is zero because of a normalization. The value $U_i(\text{Choosing } A)$ should be interpreted as the difference in utility between choosing A over B.

[8]Even though I do not explore here the relationship between voter turnout and social networks, I do believe this an important and promising issue for future research, but one that can be studied separately.

[9]I have implicitly assumed each voter gives equal weight to his idiosyncratic preferences and his valence opinion. The results are identical for to the case where choosers give weight $\alpha \in (0,1)$ the u_i dimension and weight $(1-\alpha)$ to the v_i^t dimension.

I assume the realization of ϵ_i is independent for every chooser. Conditional on $u_i + v_i^{\tau}$, choices are independent across choosers.

2.4 The Persuasion Stage

During the persuasion stage, persuaders simultaneously spend money on choosers to change their opinion. Every persuader has a fixed amount R_A, R_B of resources to spend. In Section 7 I solve the model when persuaders have to raise resources at a cost.

Let (a_i, b_i) be, respectively, the percentage of resources persuader A and persuader B spends on chooser i, so $(a_i R_A, b_i R_B)$ are the amounts in money.

Persuaders can only affect the opinion of choosers through the valence dimension. I assume persuaders do so through a contest success function: $v_i^1 : \mathbb{R}_+^2 \to [0,1]$. A contest success function takes as inputs the amount of resources each persuader spends on i and maps it into a new opinion. For tractability I assume the contest success function is:

$$v_i^1(a_i R_A, b_i R_B; v_i^0, \gamma_i) = \frac{v_i^0 (a_i R_A)^{\gamma_i}}{v_i^0 (a_i R_A)^{\gamma_i} + (1 - v_i^0)(b_i R_B)^{\gamma_i}}$$

This contest success function has four important properties:

- **The contest success function is scale-free:** it only depends on the ratio of resources spent on each chooser, $(a_i R_A)/(b_i R_B)$. If both persuaders scale the amount they are spending on chooser i by any positive factor, the opinion v_i^1 is left unaffected.
- It takes values in $[0, 1]$ and varies smoothly with the amount of resources each persuader spends.
- If both persuaders spend the same amount of resources, $a_i R_A = b_i R_B$, then the opinion of chooser i doesn't change: $v_i^1 = v_i^0$.
- If persuader A spends an infinite amount of resources she completely convinces chooser i on the valence dimension: $v_i^1 \to 1$ as $a_i R_A \to \infty$. Symmetrically, $v_i^1 \to 0$ as $b_i R_B \to \infty$.

Contest-success functions have been used in the economics literature to study strategic spending in tournaments, arms races and competitions.[10] Skaperdas provides axiomatizations for this and other contest-success functions.[11]

A particularly relevant type of models that uses contest-success functions are the "Colonel Blotto games". In a Colonel Blotto game, two opposing armies simultaneously allocate forces among different battlefields. Any given battlefield is won by the army that committed a larger force to that battlefield, and the overall winner is the army that wins a majority of the battlefields. This model has been also interpreted as a model of electoral competition.[12] The Shubik & Weber model is a Colonel Blotto game where spending resources changes the probability of winning a battlefield using the above scale-free contest success function. My model is different in that it allows resources spent on a given battlefield (chooser) to influence the outcome of other battlefields.[13]

The parameter $\gamma_i > 0$ is a sensitivity parameter that captures the responsiveness of choosers are to advertising. As all γ_i go to infinity, choosers become infinitely responsive and my game becomes a standard Colonel Blotto game.

2.5 The Deliberation Stage

After persuaders have spent all their budget, choosers update their v_i^t opinion by taking a weighted average of the opinion their neighbors on a social network. The network is exogenous and common-knowledge by the persuaders.

The network can be summarized by a matrix T with non-negative entries where entry T_{ij} represents the weight chooser i gives to the opinion of chooser j. An element in the main diagonal, T_{ii}, represents the weight chooser i assigns to his previous opinion. It parametrizes the persistency of opinions for each individual. I assume the rows of T add up to 1, which

[10] See Hirshleifer (1991); Skaperdas (1992); Siegel (2009, Forthcoming).

[11] Skaperdas (1996)

[12] See Merolla *et al.* (2005).

[13] See Roberson (2006) for a good review on on the Colonel Blotto games and characterizations of equilibria.

normalizes the total weight each chooser gives to the opinion of his social neighbors.

Every round of deliberation, the opinion v_i^t evolves according to

$$v_i^{t+1} = \sum_{j=1}^{N} T_{ij} v_j^t$$

Choosers can have asymmetric weights on each other's opinion; T_{ij} can be different than T_{ji}. This would be rational if the quality of information each chooser receives is different. It can even be that chooser i listens to chooser j but chooser j does not listen to chooser i. Opinion-followers are influenced by the editorials and blogs of opinion-leaders, but opinion-leaders do not have to know the opinion of all their readers.

It's convenient to describe the evolution of beliefs in matrix notation to apply tools from linear-algebra and markov-chain theory. Let $\mathbf{v^t}$ be the vector of opinions at time t. This vector evolves according to:

$$\mathbf{v^{t+1}} = T\mathbf{v^t} = T^t\mathbf{v^1}$$

This myopic linear-updating process was proposed by DeGroot (1974) as simple model of experts deliberating in a committee. The DeGroot model provides a tractable, yet intuitive, framework to study the diffusion of opinions through a network. More recently, DeMarzo *et al.* (2003) used the same model to study bounded-rational updating. With multidimensional opinions they find that disagreement collapses to a one dimensional vector of disagreement before eventually converging to a consensus.

There are two important ways in which this model diverges from an optimal bayesian update.[14] First, linear-updating implicitly assumes the weight given to the opinion of each individual is independent of the realized opinion. This is the optimal bayesian update for a model where choosers receive normal signals from a state-of-the-world parameter that has a normal prior. The optimal weights would simply correspond to the relative precision of

[14]See DeMarzo *et al.* (2003) and Golub & Jackson (2008) for more discussions on this issues.

the signal of each chooser receives. In other contexts one can easily come up with examples were this need not hold. For example, observing a well-known Republican figure endorse a Democratic candidate might be more informative than seeing him endorse a Republican candidate, independently of his ex-ante credibility.

The second departure is that the weights stay constant each round. Even though linear weights are appropriate in a model with normal signals, these weights have to change at each round of communication. Individuals also need to keep track of their previous beliefs, because v_i^t is not a sufficient statistic of what they have observed.

Even with these restrictions, myopic linear-updating can still provide a reasonable estimate. The work in Golub & Jackson (2008) shows that in large societies myopic linear-updating provides a consistent estimate of the true state-of-the-world as long as the influence of any individual and of any finite group of individuals is not bounded away from zero.

Calculating the optimal bayesian estimates can be quite cumbersome even for simple networks, making the model intractable. In return for these shortcomings, the DeGroot model is tractable and yields sharp predictions on the influence of each individual over opinions in the long-run. There is also extensive work on the speed of convergence.[15]

The main result of the DeGroot model is that with enough rounds of network updating, everybody's opinion converges to a common estimate and social consensus is a weighted-sum of the initial opinion of the choosers, where the weights are given by an eigenvector of the network. I call these weights are the *DeGroot weights* of network influence.

This convergence result is so important for solving my model that I state it formally in Theorem (2). The result depends on the network being path-connected and aperiodic. A directed network is path connected if for every pair of nodes i, j there exists a directed path from i to j and directed path back. Aperiodicity is a technical condition that is verified if at least one chooser places a positive weight on his previous opinion. I will assume this throughout. See Jackson (2008) for more details on the definitions.

[15] See Golub & Jackson (2008, 2009).

Definition 1 (**The DeGroot Weights**)**.** *Let T be the matrix representation of a weighted directed network whose entries are non-negative and rows sum-up to one. Suppose the network is path-connected and aperiodic. Define the* ***DeGroot weights of network influence****, or simply the* ***DeGroot weights****, as the unique left-eigenvector of matrix T that corresponds to the eigenvalue 1 and whose entries have been normalized to one. I denote it by* $\mathbf{s}$. *In other words,* $\mathbf{s}$ *is the unique vector such that*

$$\mathbf{s}T = \mathbf{s}$$
$$\sum s_i = 1$$

Theorem 2 (DeGroot 1974)**.** *Take a weighted directed network that is path-connected and aperiodic. Let T be the matrix representation of a the network. Assume the entries of T are non-negative and the rows sum-up to one. Then for any initial vector of opinions $\mathbf{v^1} \in \mathbb{R}^N$ we have:*

$$\lim_{t\to\infty} T^t \mathbf{v} = v^* \begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix}$$

Where v^* is

$$v^* = \sum s_i v_i^1$$

It's important to emphasize that the DeGroot weights are only determined by the network structure and do not depend on the initial opinion of the choosers. This allows to identify the influential members before calculating how much persuaders spend to change opinions.

I will now proceed to solve for the equilibria of my model.

3 Majoritarian competition: political campaigns and lobbying

To interpret my model as a political campaign I take persuaders A and B to be two political parties trying to convince voters to choose them. A chooser i casts a vote for A with probability $\frac{u_i+v_i^\tau}{2}$ and votes for B with probability $1-\left(\frac{u_i+v_i^\tau}{2}\right)$. Whichever persuader gets a majority of the realized votes gets elected.

To interpret my model as a lobbying competition I take persuaders A and B to be two opposing lobbies who spend resources to persuade congressmen to vote for or against a bill. Lobby A wants the bill to pass while Lobby B wants to keep the status quo. I solve my model for voting rules that require any supermajority because different congressional committees have different voting rules.

I assume both persuaders receive a payoff of 1 if they win, 0 if they don't; and wish to maximize their expected payoff. This is equivalent to saying they want to maximize their probability of winning. Let $\bar{N}$ be the number of votes persuader A needs to win. Conditional on persuader B's strategy, persuader A solves:

$$\max_{(a_1,\ldots,a_N)} \sum_{\substack{S\subset N\\ |S|\geqslant\bar{N}}} \prod_{i\in S} \frac{u_i+v_i^\tau}{2} \prod_{j\notin S}\left(1-\frac{u_j+v_j^\tau}{2}\right)$$

I now present the main result of the paper. If the number of rounds of deliberation is large enough, in the unique pure-strategy equilibrium (if it exists) both persuaders spend the same percentage on every chooser i and this percentage is proportional to the DeGroot weights of each chooser. This is stated formally in Proposition (3). Proposition (4) shows this equilibrium exists and is the unique equilibrium of the game as long as the opinion of choosers is not too responsive to campaign spending. In Section 3.2 I explain the result through an example and in Section 3.3 I solve the model for networks that are composed of several disconnected groups.

3.1 Solving for equilibria

Remark 1. ***If society reaches a consensus on the valence dimension, then each persuader would strictly prefer to have the consensus closer to his side.*** *This informal statement is obvious, but stating it formally requires some attention. Let* $v' > v$*, then the distribution of votes if all choosers have a common valence assessment of* v' *first-order stochastically dominates the distribution under* v*. Assuming all probabilities are between zero and one, the probability of winning the election is strictly increasing in the probability of any given chooser.*

Remark 2. ***As the rounds of deliberation tend to infinity, a pure-strategy best-response must maximize the DeGroot consensus.*** *To be precise, for a large enough* τ*, the maximizer of the DeGroot consensus is an epsilon-optimum of the problem with finite* τ*. This follows from Remark (1), from the uniform convergence of opinions to the DeGroot consensus and from the fact that the objective function is uniformly continuous. I will not dwell in this point, as the details for proving this are well-established but cumbersome. Rather I directly assume that persuaders maximize a monotone transformation of the DeGroot weights.*

Therefore, a strategy profile $(\mathbf{a}, \mathbf{b})$ *constitutes pure-strategy nash equilibrium if and only if* $(\mathbf{a}, \mathbf{b})$ *solve*

$$\max_{(a_1,\ldots,a_n)} \sum s_i v^1(a_i R_A, b_i R_B; v_i^0)$$

$$s.t. \sum a_i = 1$$

and

$$\min_{(b_1,\ldots,b_n)} \sum s_i v^1(a_i R_A, b_i R_B; v_i^0)$$

$$s.t. \sum b_i = 1$$

Note that the taste parameter u_i for each individual does not figure in the persuaders maximization problem. This happens for three reasons. First, persuaders cannot influence the choosers' preferences. Second, preferences and opinions are additively separable. Third, the DeGroot consensus is a linear combination, so the u_i's change the value of the objective function without changing the maximizer.

Define $\tilde{v}_i$ as $v^1(R_A/N, R_B/N; v_i^0, \gamma_i)$. This variable will play an important role in the equilibrium of the game. In words it is the valence opinion for i if both persuaders spend their resources equally over all choosers. It only matters if persuaders have a different amount of resources. If $R_A = R_B$ then $\tilde{v}_i = v_i^0$. Intuitively, if $R_A > R_B$, party A will be able to change the opinion all choosers closer to her side. The variable $\tilde{v}_i$ is a measure of how much these opinions will change.

Proposition 3 (On the structure of equilibria)**.** *Let T be a path-connected, aperiodic network. Suppose $\tau = \infty$. Let $\tilde{v}_i$ be $v^1(R_A/N, R_B/N, v_i^0)$. Then unique pure-strategy nash equilibrium in spending, if there exists such an equilibrium, is:*

$$(a_i^*, b_i^*) = \frac{s_i \tilde{v}_i (1 - \tilde{v}_i)}{\sum s_j \tilde{v}_j (1 - \tilde{v}_j)}$$

Proof. This proof is an adaptation of the Shubik & Weber proof to my environment. I first prove that a pure-strategy equilibrium must be in the interior by the contrapositive. Suppose that $a_i = 0$, then B can spend an arbitrarily small quantity on i to obtain $v_i^1 = 1$. Since persuader B has no best-response the strategies cannot constitute an equilibrium.

Knowing this I can use the first-order conditions (FOCs) to characterize the equilibrium strategies. For each persuader I equate the marginal benefit of the percentage spent on i with the marginal benefit on j to get the following equations.

$$s_i \frac{\partial v_i^1}{\partial a_i} = s_j \frac{\partial v_j^1}{\partial a_j}$$

$$s_i \frac{\partial v_i^1}{\partial b_i} = s_j \frac{\partial v_j^1}{\partial b_j}$$

Additionally by homogeneity of v^1 I can apply Euler's law to get

$$\begin{aligned} a_i \frac{\partial v_i^1}{\partial a_i} + b_i \frac{\partial v_i^1}{\partial b_i} &= 0 \\ -\frac{\partial v_i^1 / \partial b_i}{\partial v_i^1 / \partial a_i} &= \frac{a_i}{b_i} \end{aligned}$$

From the FOCs we know that the left-hand side must be the constant across i. Therefore a_i/b_i must be constant for all choosers. This means both A and B must be spending the same fraction of their resources on each chooser: $a_i^* = b_i^*$.

At this stage we know both persuaders spend the same percentage on a given chooser, but we don't know what this percentage is. To find out I use the FOCs.

$$\frac{\partial v^1}{\partial a}(a_i^* R_A, b_i^* R_B; v_i^0) = \frac{\partial v^1}{\partial a}(b_i^* R_A, b_i^* R_B; v_i^0) = \frac{1}{N b_i^*} \frac{\partial v^1}{\partial a}({}^{R_A}/_{N}, {}^{R_B}/_{N}; v_i^0) = \frac{\gamma_i}{N b_i^*} \tilde{v}_i (1 - \tilde{v}_i)$$

Where the second equality comes from the fact that the partial derivative of v^1 is homogenous of degree -1. I now substitute this in the first order condition for A.

$$s_i \frac{\gamma_i}{N b_i^*} \tilde{v}_i (1 - \tilde{v}_i) = s_j \frac{\gamma_j}{N b_j^*} \tilde{v}_j (1 - \tilde{v}_j)$$

$$\frac{b_j^*}{b_i^*} = \frac{s_j \gamma_j \tilde{v}_j (1 - \tilde{v}_j)}{s_i \gamma_i \tilde{v}_i (1 - \tilde{v}_i)}$$

Since this is true for any two choosers and the a_i, b_i must sum to one, I conclude that

$$a_i^* = b_i^* = \frac{s_i \gamma_i \tilde{v}_i (1 - \tilde{v}_i)}{\sum s_j \gamma_j \tilde{v}_j (1 - \tilde{v}_j)}$$

□

It's illuminating to contrast this result with the equilibrium-spending in a model without network influence. This model was solved by Shubik & Weber (1981). They find that the percentage spent on each chooser is proportional to the probability that chooser is pivotal. The table below summarizes this.

A chooser is pivotal for the decision if conditional on the votes of the others, his choice changes the outcome. Let q_i **be the probability chooser** i **is pivotal** under $\tilde{\mathbf{v}}$. (Remember $\bar{N}$ is th enumber of votes A needs to win.)

$$q_i = \sum_{\substack{S \subset N \setminus \{i\} \\ |S| = \bar{N}-1}} \prod_{j \in S} \frac{u_j + \tilde{v}_j}{2} \prod_{\substack{j' \notin S \\ j' \neq i}} \left(1 - \frac{u_{j'} + \tilde{v}_{j'}}{2}\right)$$

Equilibrium spending for Majoritarian Competition		
Resources for each persuader	With the network as $\tau \to \infty$	Without the network
If $R_A = R_B$	$a_i^* = b_i^* = \frac{s_i \gamma_i v_i^0 (1-v_i^0)}{\sum s_j \gamma_j v_j^0 (1-v_j^0)}$	$a_i^* = b_i^* = \frac{q_i \gamma_i v_i^0 (1-v_i^0)}{\sum q_j \gamma_j v_j^0 (1-v_j^0)}$
If $R_A \neq R_B$	$a_i^* = b_i^* = \frac{s_i \gamma_i \tilde{v}_i (1-\tilde{v}_i)}{\sum s_j \gamma_j \tilde{v}_j (1-\tilde{v}_j)}$	$a_i^* = b_i^* = \frac{q_i \gamma_i \tilde{v}_i (1-\tilde{v}_i)}{\sum q_j \gamma_j \tilde{v}_j (1-\tilde{v}_j)}$
a_i^*, b_i^* are the percentage persuaders A, B spend on chooser i in equilibrium.		

Network influence replaces pivot probabilities. Pivot probabilities are important because a chooser only has an impact in the outcome when he is pivotal. Changing choosers with a higher pivot probability has a higher expected benefit. This is still true with the network because objective of the persuaders does not change. What changes are the available persuasion tools. Because of the network consensus, persuaders can only change the valence opinion of all choosers simultaneously. They are unable to target the pivotal choosers. Influential choosers have a higher impact on the consensus, so choosers spend on them until the marginal persuasion becomes equal across all choosers.

Proposition (3) does not prove existence of a pure-strategy equilibrium. It only shows that if there exists one, it must have the stated strategies. To complement this Proposition (4) shows that when $\gamma_i < 1$ for all i the objective function is strictly quasi-concave. The FOCs are then sufficient to find an a best-response.

Proving the persuader's objective function is strictly quasi-concave implies persuaders have a unique best-response which implies this is the unique equilibrium because in zero-sum games equilibrium strategies are interchangeable.

If $\gamma_i > 1$ there might be situations where the previous strategies are an equilibrium but there exist other equilibria in non-degenerate mixed-strategies. In those situations **all equilibria would be payoff equivalent** because this is a zero-sum game.[16]

Proposition 4 (Existence and uniqueness of an equilibrium)**.** *Take the same assumptions as in Proposition (3). If for all i we have $\gamma_i < 1$, the stated strategies are the unique equilibrium of the game.*[17]

Proof. Because the probability of winning is a monotone transformation of v^*, they share the same maximizers. I will show that the DeGroot consensus is concave over a^*.

$$\frac{\partial^2 v^*}{\partial^2 a} = s_i \Big(\frac{\gamma_i}{a}\Big)^2 v_i^1(a, b^*)\big(1 - v_i^1(a, b^*)\big)\left(1 - 2v_i^1(a, b^*) - \frac{1}{\gamma_i}\right)$$

Which is strictly negative whenever $\gamma_i < 1$. Therefore a^* is the unique maximizer of $v^*(a, b^*)$. We conclude that a^* is the unique best-response to b^*. Mutato mutandis b^* is the unique best-response to a^*. This proves existence. Uniqueness follows because equilibria for zero-sum games are interchangeable. Since a^* is the unique best-response to b^*, there can be no mixed-strategy equilibrium.

□

[16]See the minimax theorem in Mas-Colell *et al.* (1995).

[17]In their model, Shubik & Weber were only able to show that the strategies constitute a local-best response.

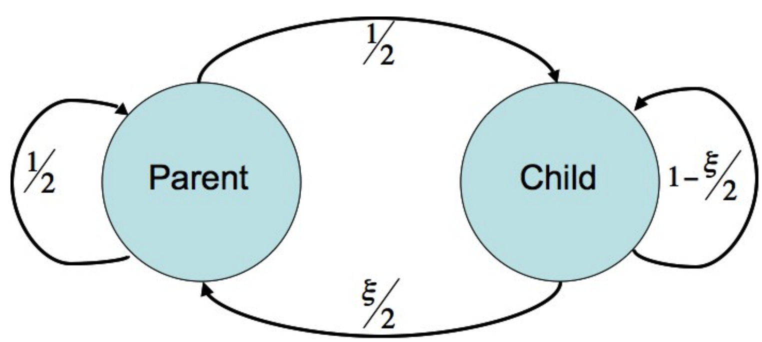
3.2 A Parent-Child example

Two choosers, a parent and a child, have to decide between two (almost) identical products: A and B. The main difference is that product A is sponsored by a popular cartoon character. The child is very much convinced that A is better than B, both on the valence and on the preference dimension. Assume $u_{child} = v^0_{child} = p \approx 1$. The parent is of the opposite state of mind. For symmetry, assume $u_{parent} = v^0_{parent} = 1 - p$.

To decide which product they want, the parent and the child are going to take a vote. Product B is the status quo object, both the parent and the child have to vote for A to buy it. Suppose the persuaders, firms A and B, have the same amount of resources to spend on advertising.

In this simple example each chooser is pivotal only if the other is voting for A. The parent will be pivotal with probability p and the child with probability $1 - p$. Without a network, the parent will be heavily advertised by both companies because firms spend on the chooser that is more likely to be pivotal. Both firms would spend a fraction p of their budget on persuading the parent and a fraction $1 - p$ on persuading the child.

Suppose instead that before taking the decision the parent and the child will deliberate about the decision. Suppose the parent feels it's important to give an equal weight in the decision to his child's opinion. The child, being a childish, pays very little attention to the parent. She places $\xi/2 \approx 0$ weight on the parent's opinion and $1 - \xi/2$ on her own.



The matrix representation of the network is

$$T = \begin{pmatrix} T_{parent,parent} & T_{parent,child} \\ T_{child,parent} & T_{child,child} \end{pmatrix} = \begin{pmatrix} 1/2 & 1/2 \\ \xi/2 & 1-\xi/2 \end{pmatrix}$$

The corresponding DeGroot weights are

$$s = \begin{pmatrix} s_{parent} \\ s_{child} \end{pmatrix} = \begin{pmatrix} \frac{\xi}{1+\xi} \\ \frac{1}{1+\xi} \end{pmatrix}$$

Given this, if the parent and the child talk for long enough, the opinion of the child will mostly prevail. Knowing this, the firms would spend a large fraction of their resources on the child, $\xi/(1+\xi)$.

Which is the right model? Different products might have different levels of communication. The parent might not be willing to discuss with the child what is the right type clothes for playing in the snow. On the other hand, the car drive from San Francisco to LA will give the child ample time to convince the parent they should go to Disneyland instead of the LA Museum of Contemporary Art.

This simple example is useful to show that persuaders would still like to target pivotal voters, but the network effects prevent them to do so. To see this let's solve for the equilibrium spending assuming the parent and the child both started at the network consensus v^* but that there were no more rounds of network deliberation.

Even after the network consensus, the parent is more likely to be pivotal because of his idiosyncratic preference u_p. The final probability the parent will vote for A is $\frac{u_p+v^*}{2}$ while the child will vote with probability $\frac{u_c+v^*}{2}$. Therefore the relative probability the parent is pivotal is $\frac{u_c+v^*}{u_p+u_c+2v^*}$, which corresponds to the percentage of resources both firms spend on him in the Shubik & Weber model. Because $u_p < u_c$, the parent is still more likely to be a pivot voter and both firms spend more on him.

Therefore if the firms could spend their resources after the network consensus, they would spend more on the parent. Under majoritarian competition, persuaders always care more about pivotal voters. They spend on influential voters instead because the available strategies change, not because of a change in their objectives.

3.3 Targeting disconnected groups.

The previous analysis focused on networks that were path-connected and used the long-run DeGroot consensus to solve for equilibrium spending. The analysis can be easily extended to networks with disconnected groups.

Assume the choosers can be partitioned into M disjoint groups such that each group is path-connected and aperiodic. Label them $\{I_1, \ldots, I_m, \ldots, I_M\}$.

Theorem (2) implies each group will reach a "consensus" the long-run, but different groups might end up with different opinions. The DeGroot weights can be constructed for each group. Let $\mathbf{s}$ be the eigenvector of stacked DeGroot weights for each group. (The components of the vector belonging to the same group must sum-up to 1.)

Let N_m be the group size of I_m and let $\bar{q}_m$ be the average pivot probability in I_m.

$$\bar{q}_m = \frac{1}{N_m} \sum_{i \in I_m} q_i$$

Following the same line of proof as in Propositions (3) and (4) I can solve for unique pure-strategy equilibrium of the game. Let i be an element of I_m. Then

$$a_i^* = b_i^* = \frac{N_m \bar{q}_m s_i \gamma_i \tilde{v}_i (1 - \tilde{v}_i)}{\sum_{m'} N_{m'} \bar{q}_{m'} \sum_{j \in I_{m'}} s_j \gamma_j \tilde{v}_j (1 - \tilde{v}_j)}$$

From this I can rewrite the relative spending across choosers and across groups as

$$\frac{a_i^*}{a_j^*} = \frac{s_i \gamma_i \tilde{v}_i (1-\tilde{v}_i)}{s_j \gamma_j \tilde{v}_j (1-\tilde{v}_j)};\ \text{if } i, j \in I_m.$$

$$\frac{\sum_{i \in I_m} a_i^*}{\sum_{j \in I_{m'}} a_j^*} = \frac{\bar{q}_m N_m \sum_{i \in I_m} s_i \gamma_i \tilde{v}_i (1-\tilde{v}_i)}{\bar{q}_{m'} N_{m'} \sum_{j \in I_{m'}} s_j \gamma_j \tilde{v}_j (1-\tilde{v}_j)}$$

Spending across groups is proportional to the average pivot probability, the size of the group and an network average of the marginal persuadability. Spending across choosers inside each group is proportional to the DeGroot weights inside the group.

Even if the relevant is path-connected, reaching a consensus could take an arbitrary long-period of time. Several real-world networks exhibit homophily in that individuals tend to interact with individuals that have similar opinions to their own. Homophily decreases the speed of convergence of opinions across groups, but increases the convergence within groups, as shown in Golub & Jackson (2009). For these societies the disconnected network might be a better approximation to model campaign spending.

4 Proportional competition: advertising and elections in proportional representation systems

In this section I solve for equilibria when persuaders want to maximize the share of choosers who select them. This can be interpreted as a model of advertising, where persuaders are firms that spend resources to persuade consumers to choose them over their competitor. In my model consumers will always choose one of the two products, so maximizing the number of sales is equal to maximizing the share of sales.[18]

This model can also be applied to electoral systems with proportional representation,

[18]Social networks can also influence the number of people who are aware of a product, making market share less important for firms. The literature on viral or word-of-mouth marketing focuses on this question. See Richardson & Domingos (2002); Leskovec *et al.* (2007).

where parties get seats in parliament in proportion to the share of votes they get in the election.

The main result is qualitatively the same as before: persuaders spend over choosers in proportion to another eigenvector based measure of network influence: Bonacich influence.[19]

For majoritarian competition, I could only get results in the limit as the number of rounds of network deliberation tended to infinity; for proportional competition I can allow for a finite but random number of rounds of deliberation. I will assume that the number of rounds of deliberation follows a geometric distribution. That is, I assume that the probability the game moves to $t+1$ rounds of deliberation conditional on reaching t rounds is constant for all t. Let $\delta \in (0,1)$ be this probability. After persuaders spend to on advertising over choosers, they will deliberate for an uncertain number of periods. When they stop deliberating they choose stochastically with probability $\frac{u_i + v_i^\tau}{2}$.

Conditional on B's strategy, persuader A solves

$$\max_{(a_1,\ldots,a_N)} E_\tau \Big[\sum \frac{u_i + v_i^\tau}{2} \Big] = \max_{(a_1,\ldots,a_N)} (1-\delta) \sum_{\tau=0}^{\infty} \delta^\tau \sum_i \frac{u_i + v_i^\tau}{2}$$

I can get this stronger result because in proportional competition the objective function of the persuaders is linear, while in majoritarian competition it was highly non-linear near the threshold of votes required to win. The non-linearity doesn't matter in the limit, but it's hard to analyze for any finite time-horizon.

Definition 5. *Fix $\delta \in (0,1)$. The vector $\hat{s}$ of **Bonacich influence weights** for a matrix T is*

$$\hat{s} = (1-\delta)(1/N, \ldots, 1/N)(I - \delta T)^{-1}$$

[19] This measure is known as Bonacich centrality in the sociology literature, but to be consistent with my application I call it influence.

Proposition 6. *Suppose each persuader wants to maximize the percentage of choosers that selects him. Then the unique pure-strategy nash equilibrium, if it exists, is:*

$$a_i^* = b_i^* = \frac{\hat{s}_i \gamma_i \tilde{v}_i (1 - \tilde{v}_i)}{\sum \hat{s}_j \gamma_j \tilde{v}_j (1 - \tilde{v}_j)}$$

If $\gamma_i < 1$ for all i, this is the unique equilibrium of the game.

Proof. Take $(\mathbf{a}, \mathbf{b}) \in (0, 1)^n$. I simply show that the objective function of each persuader is equal to $\sum \hat{s}_i v_i^1$. From the first order conditions I get the stated strategies just as in the proof for Proposition (3).

Setting-up the persuader A's maximization problem we have

$$\begin{aligned} \max_{a_1,\ldots,a_N} (1-\delta) \sum_{t=0}^{\infty} \delta^t \sum_i \frac{u_i + v_i^{t+1}}{2} &\sim \max_{a_1,\ldots,a_N} (1-\delta)(1,\ldots,1) \sum_{t=0}^{\infty} \delta^t T^t \mathbf{v}^1 \\ &= \max_{a_1,\ldots,a_N} (1-\delta)(1,\ldots,1)(I - \delta T)^{-1} \mathbf{v}^1 \\ &= \max_{a_1,\ldots,a_N} \hat{s} \cdot \mathbf{v}^1 \end{aligned}$$

Just as in Proposition (4), $\gamma_i < 1$ for all i implies the objective function is strictly concave, which guarantees existence. Uniqueness follows from because equilibria in zero-sum games are interchangeable. □

5 Using network influence to predict lobbying in Congress

To show how my model can be used I put together data on campaign contributions by interest groups with data on cosponsorship networks in Congress. My main aim is to test if lobbyists spend more on legislators with a larger network influence. The exercise will also be valuable to showcase the issues with taking the model to the data.

The estimation proceeds in three steps. First I need a way to measure to the bilateral influence across legislators: the weights of the links. Next I construct the global influence of each legislator by calculating the DeGroot weights. Finally I regress campaign contributions on network influence. Since I observe legislators several times, I will exploit the variation in time of network influence to explain the variation in time of campaign contributions.

To build the network I use data on the cosponsorship structure of Congress. Every time a bill is proposed in Congress, legislators can sign up as cosponsors of the bill. I will form a link from legislator j to legislator i if j cosponsored a bill sponsored by i. I will interpret this link as legislator i influence over his cosponsors.

This data is very convenient for my purposes because links have a direction (from cosponsor to sponsor) and because I can observe multiple interactions between legislators, which allows me to build a weight for each link.

The cosponsorship data I use ranges from 1972 to 2006, from the 93rd to the 109th Congress. The data was collected from the library of Congress by Fowler (2006a,b). To measure the influence of legislator i in electoral year t I used the links with the members of the legislature just before t. To measure the weight of the link I count the times j cosponsored i's bill in any previous Congress. For example, to construct the network for the 2006 election, I used the Representatives that served from 2004 to 2006. To construct the links between Nancy Pelosi and Tom Delay I measured the times they cosponsored each other's bills in any previous congress they served together. If Delay cosponsored Pelosi's bill (something unlikely) I interpret Pelosi has some influence of Delay.

Links in the network accumulate over time for legislators that remain in Congress. Since this might bias the influence measure in favor of more senior legislators I control for seniority when regressing campaign contributions with network influence.

A problem with the data is that some bills are cosponsored by a majority of the House. This probably has more to do with the content of the bill rather than the influence legislator involved. The distribution of cosponsors decreases exponentially but spikes up when the number of cosponsors approaches the half of the House (225 legislators). This peek hints that some cosponsored bills involve position signaling by the majority party instead of influence by the sponsoring legislator.

To deal with this I do two things: I drop the bills that have more than 215 cosponsors, the threshold where the distribution of cosponsors peaks up again. I then weigh down the links between cosponsors and sponsors by the number of cosponsors in a bill. So if j cosponsored i's bill along with 9 other legislators, I assign a weight of 1/10 from j to i. [20]

To measure lobbying expenditures I use the campaign contributions by Political Action Committees (PACs) using data from the Federal Elections Committee from 1990 to 2006. The data is made available by the Center for Responsive Politics.[21]

These expenditure do not correspond exactly to the lobbying expenditures in my model. PACs donate to get access to legislators and influence their vote, but they also donate to help elect legislators who are affine to their positions. As such, the estimates will suffer attenuation bias. For now my objective is to show that there is a positive relationship and leave for future work the exercise of coming with better estimates.

In each electoral year, many bills are presented and many different lobbies compete over separate issues. Here I am summing all the campaign contributions without separating them by issue. I also make no distinction if the lobbies are in favor or against the bill. In the context of my model this is appropriate. My model predicts that spending is linear in influence independent of the issue and that lobbies on both sides target the same legislators. The correlation between network influence and campaign contributions should still be there after summing all contributions.

If my model is wrong and lobbies target different legislators there will be an extraneous of variation that is uncorrelated with network influence. This would bias against my results.

In fact I know that PACs contributions are biased in a predictable way. As reported in Cox & Magar (1999), business PACs tend to favor Republican candidates and labor union

[20]Running the regression without these adjustments yields similar coefficients but higher standard errors.
[21]http://www.opensecrets.org

PACs tend to favor Democratic candidates. This does not immediately invalidate my model, which assumes that lobbies only spend money to persuade legislators. When lobbies spend money to get affine legislators elected they should spend asymmetrically. The data does not allow me to separate the two types of contributions. But this only adds noise to the data biasing against my results.

Whether lobbies spend resources at all on legislators with views opposite to their own is a subject of debate in the political science literature. Some authors claim lobbies only focus on legislators who are friendly to their position. But a strand of papers on counteractive lobbying have found that for the number of affine lobby groups engaged in persuading a particular legislator is positively correlated with the number of rival lobby groups who try to persuade him.[22]

Different categories of PACs distribute their expenditure very differently. Union PACs tend to have a strong ideological bias while corporation PACs tend to contribute more evenly.[23] In the 2006 electoral cycle, the top contributing PAC was the National Association of Realtors which gave to 49% to Democratic candidates and 51% to Republican. In the future, I plan to look at separate PAC contributions by the interests they represent and see if DeGroot influence can predict variations within groups.

After I added the total number of times each legislator cosponsored a bill of his colleagues (adjusted as above), I normalize each row to sum to 1 to be consistent with the interpretation that each rows represent the weights by which legislator j updates his opinion using the opinion of his neighbors.

5.1 Constructing the influence measures

I now use my model to build the influence measures. One way to interpret my model is that it assumes the researcher can measure "bilateral" or "local" influence directly, as I did when building the weighted links above, and in exchange provides a framework to translate

[22]Austen-Smith & Wright (1996, 1994)
[23]See Tripathi *et al.* (2002).

this to "global" influence.

Once I have the network matrix in the right form (with rows summing up to 1) I simply calculate the largest left-eigenvector and normalize it to sum to 1.

The model assumes legislators also place weight to their own opinion, but I do not observe self-links in the data. One approach is to ignore this and try to proceed to calculate the influence vector. Doing this corresponds to the identifying assumption that all individuals place the same weight on their previous opinion. To see this let α be the weight each legislator puts on himself and T be the network matrix whose main diagonal is zero and whose rows sum to one. The true network would be $\alpha I + (1-\alpha)T$. The largest eigenvector of $\alpha I + (1-\alpha)T$ is also the largest eigenvector of T.

There is one caveat. If $\alpha > 0$, the true matrix is always aperiodic but the "off-diagonal" matrix T might not. In that case the DeGroot weights of $\alpha I+(1-\alpha)T$ are still an eigenvector of T but there are other, non-trivial, largest eigenvectors, and the DeGroot consensus is not guaranteed. This did not occur in the empirical estimation of T.

5.2 The specification

To test out hypothesis I run ordinary least-squares (OLS) with legislator fixed-effects and congress fix-effects.

$$Contributions_{i,t+1} = \alpha_i + \beta_1 DeGrootWeight_{i,t} + \beta_2 RelativePivot_{i,t} + \beta X_{i,t} + e_{i,t}$$

The $X_{i,t}$ is a matrix of controls that includes the following variables:

1. **Seniority** and seniority squared. Measured from the first time a legislator entered the House.[24] It's particularly important to control for seniority because the measure of network influence accumulates over time, albeit in a non-linear way. Even so, network influence is strongly correlated with seniority so omitting this would bias my results.

[24] This is almost identical as a number of years a legislator has served. In general, legislators leave Congress only once.

2. **Number of sponsored bills.** Also very important for my results because legislators who sponsor more bills will have more coauthors. If I did not control for this the network influence measure could act as a proxy for legislator productivity.

3. **Leadership dummies:** I include dummies for the House Speaker, the Democratic and the Republican leaders and whips, as well as for the chairmen of the influential Ways and Means Committee and the Appropriations Committee.[25]

4. **Congress year dummies.** My theory of lobbying spending is a theory on the relative contributions each legislator receives with respect to the other legislators. In my model network influence does not predict the total amount lobbies would spend. In the data I observe a lot of year to year variation in total contributions. The standard deviation of total contributions from year to year is 35 per cent of the mean. These could be driven by the economic activity or how much money the lobbies are willing to spend on the issues presented in a given year. Adding these dummies helps control the year to year variations.

The **legislator fixed-effects** help control for unobservable variables that do not change in time but might be correlated with the measures of network influence. The main concern is that some legislators have a better intrinsic ability to collect campaign funds and that this might correlate with their influence in the House. The effects will also eliminate the variance due to different wealth levels in each district. Since legislators almost never switch party, this also controls for any systematic difference in contributions received by Democrats and Republicans.

I include the **relative probability that a legislator is pivotal** to compare my theory with the predictions of the Shubik and Weber model, which stated that legislators who have a higher probability of casting the pivotal vote should receive more contributions.

[25] Actually, for my data the dummy for chairman of the House and Ways Committee is practically a dummy for Representative Bill Archer, so the fixed-effect forces me to drop it from the regression.

To calculate the probabilities I used Poole and Rosenthal's DW-Nominate scores[26] to predict the probabilities the legislators would vote in favor or against the bills presented in the last congress. I then simulated a vote on each bill many times using independent draws for each legislator. After running the votes for tens of thousands of times I can estimate how often a legislator would've been pivotal for a given bill. I then average across all bills to measure his average pivot probability.

I had to simulate the pivot probabilities because empirically bills are almost never passed by a single vote, so legislators are very infrequently pivotal. It turns out that this is also consistent with the theoretic prediction: even when simulating the votes tens of thousands of times, for 5 out of 9 of the congresses in the sample I did not observe a single simulated bill that was decided by single vote.[27]

Consistent with my model I normalize the pivot probabilities to sum to one, because only the relative probabilities matter when deciding where to spend money. For those legislatures where everybody had a zero probability of being pivotal, I assigned an equal value to each legislator. Using the pivot probability directly yields similar coefficients in the regression.

Theoretically, pivot probabilities and DeGroot weights are completely unrelated. Legislators can be influential while being firmly grounded on one side of an issue. Empirically the pivotality measure is not correlated with DeGroot weights. (See the scatter plot at the end).

Table 1 presents the main specification. After controlling for the other potential confounds, both the DeGroot weights and the pivot probability are statistical significant predictors of campaign contributions. The units are hard to interpret, so Table 2 reports the marginal effects at the mean of changing one standard deviation of each variable. This allows to get a sense of how much variation I can explain due to these variables.

[26] Data available at www.voteview.com.

[27] My best intuition for this is that if each legislator were deciding his or her vote independently with .5 probability, the chances of getting exactly 225 and 224 votes is minimal. Unfortunately, the intuition is not tight, as having heterogenous probabilities of voting for a bill might increase or decrease the probability in a hard to predict way.

An increase in the DeGroot weight by one standard deviation predicts an increase in the campaign contributions of the average legislator by 9.43% or 34,521 dollars. An increase of one standard deviation in the probability a legislator is pivotal would increase his campaign contributions by 6% or 21,962 dollars. The point estimate of the DeGroot weights is larger, but the difference is not statistically different. The largest variation in the data comes from differences in seniority, which in the margin are associated with an increase on the average campaign contributions of 38.62%. Network influence is the second largest source of variation.

6 Extension: Comparative statics on the relative amount of resources

If Persuader A increases it resources relative to Persuader B, the battle will shift to B's base. Both parties will spend a larger fraction of their resources on choosers who begin with a more favorable valence opinion on B. This is stated formally in Proposition 7.

Proposition 7. *Let choosers i, j be such that $v_i^0 < v_j^0$ and $\gamma_i = \gamma_j \equiv \gamma$. Define r as $\frac{R_A}{R_B}$. Then:*

$$\frac{\partial}{\partial r}\left(\frac{a_i^*}{a_j^*}\right) < 0$$

Proof. I will show this for the equilibrium for proportional competition, but a similar proof works for both kinds competitions and with or without the network.

It's easier to work with the derivative of the log.

$$
\begin{aligned}
\frac{\partial \log \left({a_i^*}/{a_j^*}\right)}{\partial r} &= \frac{\partial}{\partial r}\left(\log \left(\frac{\hat{s}_i v_i^0 (1 - v_i^0)}{\hat{s}_j v_j^0 (1 - v_j^0)} \right) - 2 \log \left(v_i^0 r^\gamma + (1 - v_i^0) \right) + 2 \log \left(v_j^0 r^\gamma + (1 - v_j^0) \right) \right) \\
&= \frac{2\gamma v_j^0 r^{\gamma-1}}{v_j^0 r^\gamma + (1 + v_j^0)} - \frac{2\gamma v_i^0 r^{\gamma-1}}{v_i^0 r^\gamma + (1 + v_i^0)} \\
&= \frac{2\gamma r^{\gamma-1}}{\left(v_j^0 r^\gamma + (1 + v_j^0) \right) \left(v_i^0 r^\gamma + (1 + v_i^0) \right)} \left(v_j^0 (1 - v_i^0) - v_i^0 (1 - v_j^0) \right) < 0
\end{aligned}
$$

□

7 Competition with fund-raising

Until now I have assumed the the amount of resources every persuader has is fixed. In this section I analyze the possibility that persuaders have to raise resources at a cost. I find that the relative amount of resources spent is independent of the network influence, the specific campaign rules and the initial distribution of opinions. The relative amount of resources only depends on the relative costs each persuader has for raising resources. On the other hand, the absolute level of resources spent does depend on the rules and the distribution of opinions, but in ways that are hard to characterize.

For example, firms advertising want to convince the largest number of consumers but are also want to keep their costs low. Spending too much to convince consumers would defeat the purpose of advertising. Similarly, interest groups lobbying for legislation in Congress would prefer to achieve their aims with as little resources as possible.

Assume each persuader has to pay an cost $c_j R_j^k$ to raise resources. ($k > 1$). I allow persuaders to have different marginal costs of raising resources through their parameter c_j.

I assume both players simultaneously choose their their level of resources and where spend it. In a pure-strategy nash equilibrium each persuader knows how much resources the other persuader will raise and where he will spend it. I can simplify the full maximization problem of each persuader to one where they first choose their level of resources and then get their equilibrium payoff.

Let $r = {}^{R_A}/_{R_B}$ be such ratio and let $\pi(r)$ be the equilibrium pay-off for persuader A. From that I can write each persuader's maximization problem as:

$$\max_{R_A} \pi({}^{R_A}/_{R_B}) - c_A R_A^k = \max_r \pi(r) - c_A (r R_B)^k$$

$$\max_{R_B} (1 - \pi({}^{R_A}/_{R_B})) - c_B R_B^k = \max_r (1 - \pi(r)) - c_B ({}^{R_A}/_r)^k$$

To solve for the ratio I again appeal to the 'hidden symmetry' of the game. The FOCs for the problem are

$$\frac{d\pi}{dr} - k c_A r^{k-1} R_B^k = 0$$

$$-\frac{d\pi}{dr} + k c_B r^{-k-1} R_A^k = 0$$

Solving this yields a solution that is independent of $\pi(r)$.

$$r^* = \left(\frac{c_B}{c_A}\right)^{1/k}$$

For example, if $c_A = c_B$ both persuaders will raise the same amount of resources and their probability of winning will not change from that determined by the initial opinion of choosers plus the network deliberation.

That determines the relative amount of resources. In the absolute level of resources each persuader equates marginal cost to the marginal benefit under the equilibrium ratio. From this I can derive two easy comparative statics.

- Constants everything else, if a chooser is less persuadable, lower γ_i, the total amount of resources raised by each persuader decreases.

- Suppose the marginal cost of raising funds changes in such a way to keep the ratio of marginal costs constant. That is, (c_A, c_B) changes to $(\lambda c_A, \lambda c_B)$ with $\lambda > 1$. Then the total amount of resources raised by each persuader decreases.

For majoritarian elections the marginal benefit of resources increases with the probability the election will be decided by a single vote. Therefore elections that are likely to be close make persuaders spend more money.

The network has an ambiguous effect on campaign spending. The network deliberation can make the election more or less close. For example, if everybody is very likely to choose for A except for one very influential chooser, then the competition with the network will be more close than without it. In the other direction, even if initially an large number of choosers are equally likely to vote for A or B, one very influential chooser can make everybody more likely to choose one alternative, making the competition less close.

8 Conclusion

I propose a model of strategic persuasion over social networks. This is one of the first models to address the role of network influence under competitions to persuade public opinion. The model is tractable and allows me to solve for the equilibrium spending over each individual in the network.

In equilibrium, spending is proportional to network influence. The result contrasts with previous findings on strategic spending for majoritarian competitions, which found that in equilibrium, spending targets voters who are more likely to be pivotal for the outcome of an election.

Network influence replaces pivot probabilities because the network spillovers preclude targeting. It is impossible to change the opinion of a single chooser because frequent network updating moves all opinions toward a consensus. Therefore persuaders can only persuade the group as a whole. They do so by convincing influential choosers.

When the network is not path-connected, disconnected groups the opinion only converge within each group. In this case persuaders spend more groups that are more likely to be pivotal for the election, but spend on the influential members inside of each group.

The model predicts that the relevant measure of network influence is an eigenvector measures of influence. Eigenvector measures of influence are self-referential: individuals are influential if influential individuals listen to them. Enough structure on the network is required to simultaneously solve for the influence of each individual. These measure highlight the quality rather than the quantity of connections.

To test my model I put together data on lobbying expenditures by Political Action Committees with data on cosponsorship networks in the US House of Representatives for the electoral cycles from 1990 to 2006. After controlling for several confounding variables, I found that both network influence and pivot probabilities are significant predictors for the variations across time of a legislator's campaign contributions.

An increase of network influence by one standard deviation from one electoral year to the next predicts an increase of \$34,521 ($p = 0.048$) in the campaign contributions of a Representative. This amount corresponds to 9.4% of the average campaign contributions. An equivalent increase in the relative pivot probability predicts an increase of \$21,962 ($p < 0.001$) or 6% of the average campaign contributions.

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Summary statistics				
Variable	Mean	Std. Dev.	Min	Max
Campaign contributions	365,954	316,887	23	4,200,000
DeGroot Weights	0.23	0.24	0.00	1.85
Relative Pivot Probability	0.23	0.11	0.01	0.43
Seniority	10.8	8	2	52
Bills Sponsored	17.6	14	0	158
Observations =3745				

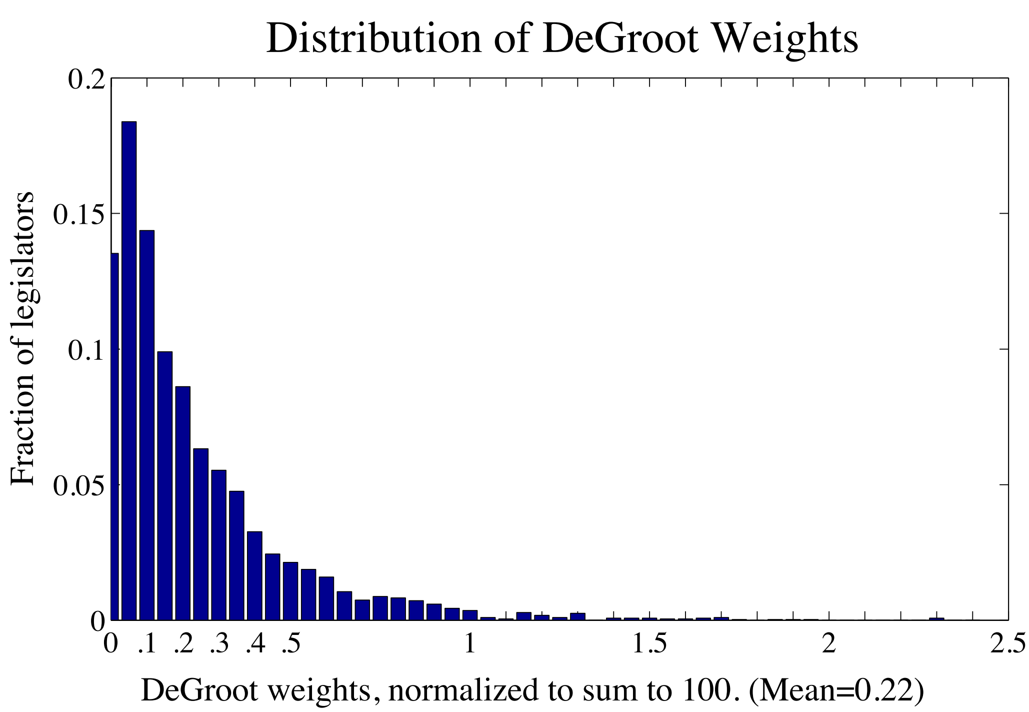

Table 1: OLS with individual and year fixed-effects.

VARIABLES	Contributions (in dollars)	Contributions (as percentage of mean)
DeGroot Weights	1419**	0.39**
(Normalized to 100)	(0.048)	(0.048)
Relative Pivot Probability	2074***	0.57***
(Normalized to 100)	(< 0.001)	(< 0.001)
Seniority	12999***	3.55***
	(< 0.001)	(< 0.001)
Seniority Squared	216.2**	0.06**
	(0.044)	(0.044)
Number of Bills Sponsored	1188**	0.33**
	(0.034)	(0.034)
Chairman Appropriations	168483***	46.04***
	(0.007)	(0.007)
House Speaker	1.100e+06***	300.7***
	(< 0.001)	(< 0.001)
Democratic Leader	733032***	200.3***
	(6.81e-07)	(6.81e-07)
Democratic Whip	452856***	123.7***
	(< 0.001)	(< 0.001)
Republican Leader	484348**	132.4**
	(0.0281)	(0.0281)
Republican Whip	553532***	151.3***
	(< 0.001)	(< 0.001)
Constant	122420	33.45
Observations	3735	
R^2 (within)	0.203	
Number of legislators	928	

Robust p-values in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Table 2: Marginal effect at mean of increasing one standard deviation.

VARIABLES	Dollar change per std dev.	Percent change per std dev.
DeGroot Weights	34521**	9.43**
	(17431)	(4.76)
Relative Pivot Probability	21962***	6.00***
	(6133)	(1.68)
Seniority	141313.7***	38.62***
	(15894)	(4.34)
Number of Bills Sponsored	20928**	5.72**
	(9845)	(2.69)
Observations	3735	
R^2	0.203	
Number of legislators	928	

Robust standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1