

Costly Self Control and Random Self Indulgence¹

Eddie Dekel² Barton L. Lipman³

Preliminary and Incomplete Draft
March 2010

¹We thank Larry Epstein, Faruk Gul, Jawwad Noor, Andy Postlewaite, Todd Sarver, and numerous seminar audiences for helpful comments. We also thank the National Science Foundation, grants SES-0820333 (Dekel) and SES-0851590 (Lipman), for support for this research.

²Economics Dept., Northwestern University, and School of Economics, Tel Aviv University
E-mail: dekel@nwu.edu.

³Boston University. E-mail: blipman@bu.edu.

1 Introduction

On the surface, the Strotz (1955) and Gul and Pesendorfer (2001) (henceforth GP) models of temptation, while clearly related, seem quite different. Both models consider an agent who is currently not influenced by temptation but who chooses commitments today in anticipation of how his future, tempted self will choose tomorrow. In Strotz, the untempted self anticipates that his future self will behave in a completely self-indulgent fashion, maximizing his temptation-influenced utility (paying attention to the untempted self's preferences only when indifferent). By contrast, in GP's model, the current self anticipates that his future self will, in general, exert costly self control. The current and future selves have the same objective function: the utility of the current self minus the self-control costs. These costs are generated by the utility the future self could enjoy by succumbing to temptation, so the choosing self ends up effectively trading off temptation utility and the untempted self's utility. Because of these costs, the current self values commitment even when his future self will not succumb to temptation.

In this paper, we show some unexpected connections between the GP model and a version of the Strotz model with uncertainty about the nature of the temptation that will strike. Even though the models appear very different and even though the GP model has no uncertainty, we show that the commitment choices of the agent in GP are identical to the commitment choices in a Strotz model with appropriately chosen uncertainty. Interestingly, the proof of equivalence uses standard results on incentive compatibility, a fact which highlights the potential application of such techniques to the study of random Strotz models.

In light of this result, we are led to a more detailed exploration of what we call the *random Strotz* model. We relate it to a version of the GP model which adds similar uncertainty about temptations, which we call *random GP*. Restricting attention to Lipschitz continuous versions of the two models, we find that the commitment choices they predict are identical. In this sense, there is no observable difference in commitment choices in these models of (random) costly self control and random self indulgence.¹

We prove this direction of the relationship between random GP and random Strotz models via an axiomatic characterization of Lipschitz continuous versions of the two models. More specifically, we extend a result of Stovall (2009) to give axioms on preferences over commitments — that is, over menus — which are necessary and sufficient for the existence of a Lipschitz continuous version of either model, showing that the commitment behavior of the two models is equivalent.

¹There are, of course, other examples of preferences which have multiple interpretations, such as the overlap between multiple priors preferences and Choquet expected utility preferences — see, for example, Gilboa and Schmeidler (1994).

We also show that the random Strotz model is uniquely identified by the preference over menus (whether Lipschitz continuity is assumed or not) and use this identification to give an interesting comparative notion.

Next, we compare the two models in terms of the choices by the tempted self. We show that observations of choice from menus *alone* cannot, in general, distinguish the two models. In particular, any choice function which can be generated by the random Strotz model can also be generated by the random GP model. If we focus on random choice correspondences instead, the models have a large overlap, but are not identical. We show that any choice correspondence generated by a continuous random Strotz model can also be generated by a random GP model, but the converse is not true. Hence there is a large class of choices by the tempted self that can be explained by either costly self control or random self indulgence.

On the other hand, except in the trivial case of no temptation, the two models can *always* be distinguished by observing both *ex ante* menu choice and *ex post* choice from menus. In particular, the random Strotz model, all else equal, shows more temptation in the choices from menus than does the random GP model in a sense we make precise below.

One reason we find the random GP and random Strotz models of interest is that they are able to rationalize some very natural preferences which cannot be rationalized within GP's model. For example, consider a dieting agent who ranks menus which may contain broccoli (b), chocolate (c), and/or potato chips (p). As Dekel, Lipman, and Rustichini (2009) note, GP's representation rules out the seemingly natural ranking that has $\{b\}$ as best, then $\{b, c\}$, and $\{b, p\}$ in some order, followed by $\{b, c, p\}$. The problem noted there is that GP's model treats temptation as one-dimensional so only the most tempting item from $\{b, c, p\}$ can matter, so that this menu must be indifferent to either $\{b, c\}$ or $\{b, p\}$. It is not hard to see that the random Strotz representation is consistent with this ranking, though. If there is some probability of a temptation toward chocolate and some probability of a temptation toward potato chips, then the agent breaks her diet more often with $\{b, c, p\}$ than with $\{b, c\}$ or $\{b, p\}$ and thus this menu can be strictly worse than the other two.

The basic point that the GP representation can be rewritten in terms of a random determination of which self has control has been made before, though in very different ways. In particular, Benabou and Pycia (2002) note that the GP representation can be written as the equilibrium payoff of a game between the current and future self engaging in a costly battle for control. Also, Chatterjee and Krishna (2007) show that a preference with a GP representation also has a representation where there is a probability which depends on the menu that the choice is made by the tempted self, with the choice made by the untempted self otherwise. Unfortunately, the properties of the function

relating menus to probabilities over control make it difficult to interpret in general.² One appealing aspect of our result is that the random Strotz representation is a natural alternative formulation.

The next section defines the model and the representations considered. In Section 3, we relate random Strotz representations to GP and random GP representations. Section 4 turns to a characterization of random Strotz representations, showing the uniqueness and comparative results described above. In Section 5, we discuss choice from menus and the extent to which this information, possibly together with preferences over menus, enables us to distinguish the random GP and random Strotz models. Section 6 concludes. Proofs not contained in the text are in the Appendix.

2 Definitions

Fix a finite set Z of “prizes” or outcomes, let $\Delta(Z)$ denote the set of lotteries over Z and let X denote the set of *menus*, the set of compact, nonempty subsets of $\Delta(Z)$.³ The current self is modeled as having a preference over X , denoted \succeq , where this is interpreted as a preference regarding how much commitment to impose on later choices. Later, we discuss how we represent choices from menus.

Throughout, we assume that \succeq is *nontrivial* in the sense that there exist $x, y \in X$ such that $x \succ y$.

A function $w : \Delta(Z) \rightarrow \mathbf{R}$ is *linear* if $w(\lambda\alpha + (1 - \lambda)\beta) = \lambda w(\alpha) + (1 - \lambda)w(\beta)$ for all $\lambda \in [0, 1]$ and $\alpha, \beta \in \Delta(Z)$. We say that $w : \Delta(Z) \rightarrow \mathbf{R}$ is an *expected utility function* if it is linear.⁴

Both the Strotz and GP representations use two expected utility functions, $u, v : \Delta(Z) \rightarrow \mathbf{R}$. The Strotz representation uses the pair (u, v) to evaluate a menu x by

$$V_S(x) = \max_{\beta \in B_v(x)} u(\beta)$$

²The published version of Chatterjee and Krishna’s paper, Chatterjee and Krishna (2009), considers only the case where this probability is independent of the menu. While this provides more structure, the constant probability model no longer nests GP.

³Most of our results extend easily to the case where Z is compact. Rather than complicate the exposition by considering both the finite and compact cases, we focus on the finite case, noting where the proof also covers the compact case. When we say that results extend to compact Z , we consider a topology on Z and take $\Delta(Z)$ to be the set of distributions on the Borel field of Z .

⁴For the case where Z is compact instead of finite, we define w to be an expected utility function if it is linear and continuous.

where $B_v(x)$ is the set of best elements of x according to v . That is,

$$B_v(x) = \{\beta \in x \mid v(\beta) \geq v(\alpha), \forall \alpha \in x\}.$$

Intuitively, v represents the preference of the future self who will be completely self indulgent, choosing from the menu as he wishes, breaking ties in favor of the current self who has utility function u .

One unfortunate feature of the Strotz model is that the agent's utility depends discontinuously on the commitments he makes. This occurs because when the choosing self is almost indifferent, the current self may still have strong preferences regarding the choices. A small change in commitments can then create indifference for the chooser. Hence we can find such small changes in commitments that have big effects on the current self's payoff.⁵ This discontinuity is both intuitively implausible and analytically inconvenient. For example, because of the discontinuity, optimal policies for the current self may not exist.

The representation introduced by GP is continuous and hence avoids this problem. In their representation, the way that u and v are used to evaluate a menu x is by the function

$$V_{GP}(x) = \max_{\beta \in x} [u(\beta) + v(\beta)] - \max_{\beta \in x} v(\beta).$$

GP emphasize the idea that in their representation, the agent chooses from the menu the item which maximizes $u + v$, not v . In this sense, he shows partial self control by compromising between u and v instead of simply maximizing v . One intriguing interpretation offered by GP which highlights this idea can be seen by writing the representation as

$$V_{GP}(x) = \max_{\beta \in x} [u(\beta) - c(\beta, x)]$$

where $c(\beta, x) = [\max_{\alpha \in x} v(\alpha)] - v(\beta)$. This representation is written as if the agent chooses the β which maximizes $u(\beta) - c(\beta, x)$ which is the β which maximizes $u + v$. Under this interpretation, $c(\beta, x)$ is the cost of resisting temptation by choosing β instead of maximizing v .

As noted, we consider random versions of the GP and Strotz models. Hence we require a field for the set of EU functions. Letting K denote the number of elements of Z , we identify the set of such functions with \mathbf{R}^K since for any EU function, we only need to specify the payoffs to the pure outcomes. We use the Borel field over \mathbf{R}^K .

Definition 1. A random Strotz representation of \succeq is a pair (u, μ) such that u is an expected utility function and μ is a measure over expected utility functions such that the

⁵Note that this difficulty is not eliminated by changing the tie-breaking rule.

function

$$V_{RS}(x) = \int_{\mathbf{R}^K} \max_{\beta \in B_w(x)} u(\beta) \mu(dw)$$

represents the preference.

This is the Strotz representation but where the agent is not sure what his future self’s preference will be. It seems quite natural to suppose that an agent may not know exactly what temptations will strike him in the future or exactly how strong they will be. Adding uncertainty to the Strotz model also has the potential to resolve the continuity problems noted above. Intuitively, if the distribution over the chooser’s utility function is suitably atomless, then the probability the chooser is indifferent will be zero. Since it is this indifference which creates the discontinuities, making such events irrelevant to the current self resolves the discontinuity problem. As Caplin and Leahy (2006) show, such atomlessness can ensure existence of an optimal policy in Strotz’s sense.

A random GP representation generalizes the notion of a GP representation in a fashion exactly parallel to the way that random Strotz generalizes Strotz: specifically, the u is fixed but there is a probability measure over the “temptations.”⁶

Definition 2. A random GP representation is a pair (u, ν) such that u is an expected utility function and ν is a measure over expected utility functions such that the function

$$V_{RGP}(x) = \int_{\mathbf{R}^K} \left\{ \max_{\alpha \in x} u(\alpha) + v(\alpha) \right\} - \max_{\alpha \in x} v(\alpha) \nu(dv)$$

represents the preference.

3 Costly Self Control = Random Self Indulgence: Menu Choice

3.1 Costly Self Control \subseteq Random Self Indulgence

We begin by relating the GP and random Strotz representations.⁷

⁶There is one difference between the way randomization enters these two representations which will become important later. Specifically, in the random Strotz model, we could (and later will) normalize the space of EU functions, while in the random GP model, we cannot. For random GP, the scale of each v relative to u matters as it measures the “strength” of the temptation v , while for random Strotz, the choice made under a temptation is all that matters, not the scale of the temptation.

⁷The result of this subsection also holds for compact Z as is evident from the proof which makes no use of finiteness.

Theorem 1. Fix any GP representation (u, v) and the corresponding V_{GP} . Then there exists a measure μ over expected utility functions such that the function V_{RS} corresponding to the random Strotz representation (u, μ) satisfies $V_{GP}(x) = V_{RS}(x)$ for every menu x .

Proof. Let W denote the set of expected utility preferences such that $w \in W$ iff there exists $A \in [0, 1]$ with $w = v + Au$. Define a measure μ over W by taking the uniform distribution over A . That is, for a set $E \subseteq W$, we have

$$\mu(E) = \Pr[\{A \in [0, 1] \mid v + Au \in E\}],$$

where $\Pr(\cdot)$ is the uniform distribution. Finally, let V_{RS} denote the random Strotz representation generated by this measure.

Fix any menu x . Let $\beta^*(A)$ denote any element of x which maximizes u over the set $B_{v+Au}(x)$. Let $\hat{u}(A) = u(\beta^*(A))$ and let $\hat{v}(A) = v(\beta^*(A))$. Note that if multiple elements of x maximize u over $B_{v+Au}(x)$, the values of $\hat{u}(A)$ and $\hat{v}(A)$ do not depend on the particular choice of $\beta^*(A)$. Also, it is easy to show that \hat{u} is nondecreasing in A and hence measurable. Since \hat{u} is also bounded, it is integrable. We have

$$V_{RS}(x) = \int_0^1 u(\beta^*(A)) dA = \int_0^1 \hat{u}(A) dA.$$

Define

$$\mathcal{U}(A) = \hat{v}(A) + A\hat{u}(A) = \max_{\bar{A} \in [0, 1]} \hat{v}(\bar{A}) + A\hat{u}(\bar{A}).$$

From the usual argument characterizing incentive compatibility with transferrable utility (see, e.g., Mas–Colell, Whinston, and Green (1995), Proposition 23.D.2, page 888, or Milgrom and Segal (2002), Theorem 2),⁸ we have

$$\mathcal{U}(s) = \mathcal{U}(0) + \int_0^s \mathcal{U}'(A) dA = \mathcal{U}(0) + \int_0^s \hat{u}(A) dA.$$

Hence

$$\mathcal{U}(1) - \mathcal{U}(0) = \int_0^1 \hat{u}(A) dA = V_{RS}(x).$$

But $\mathcal{U}(1) = \max_{\beta \in x} [v(\beta) + u(\beta)]$, while $\mathcal{U}(0) = \max_{\beta \in x} v(\beta)$. Hence the left-hand side is $V_{GP}(x)$. ■

Corollary 1. Every preference with a random GP representation also has a random Strotz representation.

⁸For intuition, consider a standard auction problem or other characterization of incentive compatibility with quasi-linear utility. View A as the type of the agent where this is his valuation for some good. Then \bar{A} plays the role of the agent's report of his type, $\hat{u}(\bar{A})$ is the probability the agent obtains the good if his report is \bar{A} , and $\hat{v}(\bar{A})$ is the transfer to him when his report is \bar{A} .

To see explicitly why the corollary follows, note that

$$V_{RGP}(x) = \int \left\{ \max_{\alpha \in x} u(\alpha) + v(\alpha) \right\} - \max_{\alpha \in x} v(\alpha) \nu(dv)$$

so

$$V_{RGP}(x) = \int \left\{ \int_0^1 \max_{\beta \in B_{v+Au}(x)} u(\beta) dA \right\} \nu(dv)$$

which is a random Strotz representation. Thus the random Strotz model is more general than the random GP model.

3.2 Random Self Indulgence \subseteq Costly Self Control?

This observation naturally leads one to ask what behavior random Strotz can accommodate which random GP precludes. A partial answer comes from the fact that random Strotz includes nonrandom Strotz as a special case and hence allows the possibility of discontinuous preferences. More formally, as mentioned in Section 2, a (nonrandom) Strotz representation need not be continuous. Since such representations are a special case of random Strotz, the same holds true for random Strotz representations. On the other hand, a random GP representation must be continuous. (Proofs of these claims are in Section of the Appendix.)

Is discontinuity the only property which random Strotz allows but random GP does not? We conjecture that the answer is yes — more specifically, that the set of preferences with a *continuous* random Strotz representation equals the set with a random GP representation. Our result, however, is more limited. Instead, we show that the set of preferences with a Lipschitz continuous random Strotz representation equals the set with a Lipschitz continuous random GP representation. We show by an example in the appendix that not every random GP representation is Lipschitz continuous (even though every (nonrandom) GP representation *is* Lipschitz continuous⁹).

We prove this result via an axiomatic characterization of the class of preferences with a Lipschitz continuous random GP representation and show that these axioms also characterize the set of preferences with a Lipschitz continuous random Strotz representation.

A function $V : X \rightarrow \mathbf{R}$ is *Lipschitz continuous* if there is a \bar{N} such that

$$V(y) - V(x) \leq \bar{N} d_h(x, y), \quad \forall x, y$$

⁹Dekel, Lipman, Rustichini, and Sarver (2007a) show that every additive EU representation is Lipschitz continuous. Since, as Dekel, Lipman, and Rustichini (2009) explain, every GP representation is an additive EU representation, the claim follows.

where d_h denotes Hausdorff distance.

Our axiomatic characterization begins with the additive EU representation of Dekel, Lipman, and Rustichini (2001) (henceforth DLR). As shown in Dekel, Lipman, Rustichini, and Sarver (2007a) (henceforth DLRS), this representation is Lipschitz continuous. DLRS show that \succeq has an additive EU representation iff it is continuous and satisfies the following three axioms.

Axiom 1 (Weak Order). \succeq is complete and transitive.

Axiom 2 (Continuity). For every $x \in X$, the sets $\{y \in X \mid x \succ y\}$ and $\{y \in X \mid y \succ x\}$ are open in the Hausdorff topology.

Let $\lambda x + (1 - \lambda)y = \{\gamma \in \Delta(Z) \mid \gamma = \lambda\alpha + (1 - \lambda)\beta \text{ for some } \alpha \in x, \beta \in y\}$.

Axiom 3 (Independence). $x \succeq y$ implies $\lambda x + (1 - \lambda)\bar{x} \succeq \lambda y + (1 - \lambda)\bar{x}$ for every $\lambda \in [0, 1]$ and $\bar{x} \in X$.

See GP or DLR for discussion of this axiom.

Axiom 4 (L-Continuity). There exist nonempty sets $x^*, x_* \subseteq \Delta(Z)$ and $N > 0$ such that for every $\varepsilon \in (0, 1/N)$, for every x and y with $d_h(x, y) \leq \varepsilon$,

$$(1 - N\varepsilon)x + N\varepsilon x^* \succeq (1 - N\varepsilon)y + N\varepsilon x_*.$$

See DLRS for a discussion of this axiom.

Definition 3. An additive EU representation of \succeq is a countably additive, signed measure η over expected utility functions such that the function

$$V_{AEU}(x) = \int_{\mathbf{R}^K} \max_{\beta \in x} w(\beta) \eta(dw)$$

represents \succeq .

The additive EU representation is rather general, saying nothing about temptation or other motivations of the agent. We now specialize by adding a ‘‘temptation’’ axiom. This axiom was first proposed by Dekel, Lipman, and Rustichini (2009). Stovall (2009) gives a different version which is equivalent given the other axioms.

Axiom 5 (Weak Set Betweenness (WSB)). If $\{\alpha\} \succeq \{\beta\}$ for all $\alpha \in x$ and $\beta \in y$, then $x \succeq x \cup y \succeq y$.

To see the idea, fix menus x and y satisfying the hypothesis of the axiom which says that the agent would rather commit himself to *any* option in x than to *any* option in y . Intuitively, then, *everything* in x is better for the agent than *anything* in y . Thus it seems very natural to require $x \succeq y$. Furthermore, since $x \cup y$ simply adds the inferior y elements to x , it seems natural to have $x \succeq x \cup y$. Finally, since $x \cup y$ just adds the superior x elements to y , it seems natural to have $x \cup y \succeq y$.

As the name suggests, WSB is implied by GP’s set betweenness axiom which states that $x \succeq y$ implies $x \succeq x \cup y \succeq y$. To see why, suppose we have x and y related as in the hypothesis of WSB. Let α_* be a “worst” $\alpha \in x$ in the sense that $\{\alpha\} \succeq \{\alpha_*\}$ for all $\alpha \in x$. Similarly, fix $\beta^* \in y$ such that $\{\beta^*\} \succeq \{\beta\}$ for all $\beta \in y$. It is easy to use set betweenness to show that $x \succeq \{\alpha_*\}$ and $\{\beta^*\} \succeq y$. Hence we have $x \succeq \{\alpha_*\} \succeq \{\beta^*\} \succeq y$, so $x \succeq y$. Using set betweenness again, we have $x \succeq x \cup y \succeq y$, so WSB holds.

Theorem 2. *The following statements are equivalent:*

1. \succeq is a weak order which satisfies continuity,¹⁰ independence, L -continuity, and weak set betweenness.
2. \succeq has a Lipschitz continuous random GP representation.
3. \succeq has a Lipschitz continuous random Strotz representation.

The main part of the proof is the demonstration that (1) implies (2). This part extends a result due to Stovall (2009) who shows that a weak order satisfying continuity, independence, weak set betweenness, and the *finiteness axiom* of Dekel, Lipman, and Rustichini (2009) has a random GP representation where the measure ν has finite support. His proof works by starting from the finite additive EU representation discussed in Dekel, Lipman, and Rustichini (2009) and then showing the implications weak set betweenness has for this representation. We start from a general additive EU representation instead. Once (1) implies (2) is shown, the rest of the proof is straightforward: Corollary 1 shows that (2) implies (3) and it is easy to show that (3) implies (1).

¹⁰An implication of the additive EU existence theorem in Dekel, Lipman, Rustichini, and Sarver (2007b) is that this theorem remains true if we weaken continuity to mixture continuity. See the proof of Theorem 2 in the appendix for a definition of mixture continuity.

4 Properties of Random Strotz Representations

4.1 Uniqueness

In this subsection, we discuss the uniqueness properties of the random Strotz representation, while the next subsection uses this uniqueness to characterize a natural comparison notion. While our axiomatic characterization only covers Lipschitz continuous random Strotz representations, all results in this section apply to *any* random Strotz representation, regardless of its continuity properties.

So suppose we have a preference \succeq with random Strotz representations (u, μ) and $(\bar{u}, \bar{\mu})$. What is the relationship between the representations? First, it is easy to see that u and \bar{u} must be the same up to a positive affine transformation. This follows from the fact that both u and \bar{u} must represent the preference over singleton menus. That is, for any α , the random Strotz representation (u, μ) evaluates the menu $\{\alpha\}$ by $u(\alpha)$. Hence $u(\alpha) \geq u(\beta)$ iff $\{\alpha\} \succeq \{\beta\}$, so $u(\alpha) \geq u(\beta)$ if and only if $\bar{u}(\alpha) \geq \bar{u}(\beta)$. Thus the usual uniqueness properties for expected utility representations imply that \bar{u} is a positive affine transformation of u .

To identify the measure, we must first normalize the space of expected utility functions. Since only the choices by each given w matter for the representation, we obviously cannot distinguish a representation that puts probability p on w from a representation that puts probability p on $2w$. Recall that we identified the space of EU functions with \mathbf{R}^K where K is the number of pure outcomes. We take the normalized space to be

$$\mathcal{W} = \{w \in \mathbf{R}^K \mid w \cdot \mathbf{1} = 0 \text{ and } w \cdot w = 1\}$$

where $\mathbf{1} = (1, \dots, 1) \in \mathbf{R}^K$. For the σ -algebra on \mathcal{W} , we use the Borel σ -algebra using as our topology on \mathcal{W} the (relativized) usual Euclidean topology on \mathbf{R}^K .

We refer to a random Strotz representation (u, μ) with $u \in \mathcal{W}$ and $\text{supp}(\mu) \subseteq \mathcal{W}$ as a *canonical random Strotz representation*. It is easy to show that our assumption that \succeq is nontrivial implies that if it has a random Strotz representation, then it has a canonical random Strotz representation.

It is also easy to show that if $u, \bar{u} \in \mathcal{W}$ are the same up to a positive affine transformation, then $u = \bar{u}$. Thus once we restrict attention to the canonical random Strotz representation, the u is unique. As the following theorem shows, the measure is uniquely identified given our normalization of the space of expected utility functions.¹¹

¹¹Since the measure is unique, obviously, the support is unique as well. We show in supplementary

Theorem 3. *If (u, μ) and $(\bar{u}, \bar{\mu})$ are canonical random Strotz representations of \succeq , then $(u, \mu) = (\bar{u}, \bar{\mu})$.*

4.2 Comparative

Given that the measure is uniquely identified, we turn to the behavioral implications of properties of the measure. The following definition gives a natural version of a comparative notion of temptation which relates in an interesting way to the measure. We say that \succeq_2 is *more temptation averse* than \succeq_1 if whenever $\{\alpha\} \succeq_1 x$, we have $\{\alpha\} \succeq_2 x$. In other words, whenever \succeq_1 prefers a commitment to leaving open the choice from x , \succeq_2 does as well.¹²

It is easy to see that since \succeq_1 and \succeq_2 are nontrivial, this notion requires \succeq_1 and \succeq_2 to rank singletons identically. To see this, note that $\{\alpha\} \sim_1 \{\beta\}$ implies $\{\alpha\} \sim_2 \{\beta\}$. Hence any \succeq_1 indifference curve over singletons must be contained in a \succeq_2 indifference curve over singletons. Since the \succeq_1 indifference curve is a hyperplane and the \succeq_2 indifference curve cannot be larger than this, the indifference curves must coincide. Also, the direction of increase for \succeq_1 must be the same as that of \succeq_2 since $\{\alpha\} \succ_1 \{\beta\}$ implies $\{\alpha\} \succeq_2 \{\beta\}$, a preference which must be strict since \succeq_2 is nontrivial. Hence they have the same preference over singletons. Therefore, if both have canonical random Strotz representations, the u 's must be the same.

Hence, the two preferences have canonical representations which differ only in terms of the measure μ . Let the measure for \succeq_i be denoted μ_i . We give two statements of the relationship between μ_1 and μ_2 each of which is equivalent to \succeq_2 being more temptation averse than \succeq_1 . Both involve certain first-order stochastic dominance (FOSD) comparisons.

material available at <http://people.bu.edu/blipman> that the support consists of those $\bar{w} \in \mathcal{W}$ which are *relevant* in the sense that for every neighborhood N of \bar{w} , there exist menus x and x' with $x \not\sim x'$ and

$$\max_{\beta \in B_w(x)} u(\beta) = \max_{\beta \in B_w(x')} u(\beta), \quad \forall w \in \mathcal{W} \setminus N.$$

That is, \bar{w} is relevant if we need it (or small neighborhoods of it) to “see” why the agent is not indifferent between x and x' since all the other w 's behave the same way on x and x' as far as u is concerned. This is analogous to the characterization of the support of the measure in DLR. As observed by Chatterjee and Krishna (2009), while the characterization is analogous to that of DLR, it is not the same.

¹²This definition is also used by Ahn (2007) to compare ambiguity aversion, Sarver (2008) to compare regret attitudes, and Higashi, Hyogo, and Takeoka (2009) to compare aversion to commitment. It is also similar in spirit to the way Epstein (1999) and Ghiradato and Marinacci (2002) define comparisons of ambiguity aversion. Since the random Strotz representation is very different from the representations considered in these papers, their characterization results are quite different as well.

The first statement is based on generalizing the usual notion of FOSD. To generalize, we define an order over \mathcal{W} which will replace the usual \geq in our notion of FOSD. Define an order over \mathcal{W} by $w C_u \hat{w}$ (read “ w is closer to u than \hat{w} ”) if

$$u(\alpha) > u(\beta), \hat{w}(\alpha) \geq \hat{w}(\beta) \text{ implies } w(\alpha) \geq w(\beta).$$

In other words, w is willing to “go along with” u at least as often as \hat{w} .

It will prove useful to give a geometric restatement of this notion. Note that we defined \mathcal{W} to be the surface of a sphere. For intuition, think of \mathcal{W} as the points on a globe where u is the North Pole and $-u$ the South Pole. As we now show, $w C_u w'$ if and only if we can move north from w' to w along a longitude line on this globe.

The precise form of this idea works as follows. Let

$$\mathcal{V} = \{v \in \mathcal{W} \mid v \cdot u = 0\}.$$

Think of \mathcal{V} as the equator. The following lemma shows how any given point in \mathcal{W} can be rewritten in terms of a choice of an equator point and a movement along the longitude line through that point.

Lemma 1. *For every $w \in \mathcal{W}$, there exists $v \in \mathcal{V}$ and $A \in [-1, 1]$ such that $w = v\sqrt{1 - A^2} + Au$. If $w = u$, then this holds for every $(A, v) \in \{1\} \times \mathcal{V}$, while if $w = -u$, it holds for every $(A, v) \in \{-1\} \times \mathcal{V}$. For every other $w \in \mathcal{W}$, the (A, v) is unique.*

For each $v \in \mathcal{V}$, let $L(v)$ denote the “longitude” generated by v . That is, let $L(v)$ denote the set of $w \in \mathcal{W}$ such that $w = v\sqrt{1 - A^2} + Au$ for some $A \in [-1, 1]$. Intuitively, $A > 0$ corresponds to a movement north along the longitude $L(v)$, while $A < 0$ corresponds to a movement to the south. Clearly, the sets $\{L(v) \mid v \in \mathcal{V}\}$ form a partition of $\mathcal{W} \setminus \{u, -u\}$. Note that every $L(v)$ set includes both u and $-u$, just as every longitude line on a globe runs from the North Pole to the South Pole.

To complete the argument that C_u is related to movements along these longitude lines, we have the following lemma.

Lemma 2. *$w_1 C_u w_2$ if and only if there exists $v \in \mathcal{V}$ such that $w_i = v\sqrt{1 - A_i^2} + A_i u$, $i = 1, 2$, with $A_1 \geq A_2$.*

In words, w_1 and w_2 can be compared under C_u iff they are on the same longitude. In this case, the point further north is the one closer to u (the “North Pole”) in the sense of C_u .

Having provided this geometric intuition for C_u , we return to defining our first FOSD notion. Define a set $W \subseteq \mathcal{W}$ to be *closed under C_u* if $w' \in W$ and $w C_u w'$ implies

$w \in W$. Geometrically, a set is closed under C_u if it equals every point north of some curve which circumnavigates the globe (and may or may not include parts of the curve itself). Say that μ_1 C_u -FOSD μ_2 if for every closed and measurable W which is closed under C_u , we have $\mu_1(W) \geq \mu_2(W)$. This is a natural analog of the statement that a measure F on \mathbf{R} FOSD G if for every s , $F(\{r \in \mathbf{R} \mid r \geq s\}) \geq G(\{r \in \mathbf{R} \mid r \geq s\})$.

We will show that \succeq_2 is more temptation averse than \succeq_1 if and only if μ_1 C_u -FOSD μ_2 . Before discussing this further, though, we present the second version of the comparison.

In light of Lemma 1, we can obviously take a change of variables and write the distribution over \mathcal{W} as a distribution over (A, v) pairs in $[-1, 1] \times \mathcal{V}$. This is more complex when $\mu(\{u, -u\}) > 0$ since, as noted, each $L(v)$ set contains both of these points. Intuitively, we can divide these mass points and allocate part of each mass point to each v in order to eliminate this ambiguity. Thus there will be a family of measures over $[-1, 1] \times \mathcal{V}$ which correspond to μ .

Given any of these distributions, we can construct a version of the conditional probabilities to write it as a marginal over \mathcal{V} times a conditional on $L(v)$ given each $v \in \mathcal{V}$. Intuitively, this is just rewriting our initial measure on points on the globe as follows. First, we draw a point from the equator at random. Then, conditional on the point selected, we choose a distance to move up or down the longitude line through this point.

Given Lemma 2, a natural version of a comparison of measures on the globe which seems to match our C_u -FOSD notion is that μ_1 's conditionals on each $L(v)$ dominate μ_2 's conditionals in the usual first-order stochastic dominance sense.

To state this more precisely, we say that a pair $(\mu_{\mathcal{V}}, \mu_L(\cdot \mid v))$ is a *version* of μ if $\mu_{\mathcal{V}}$ is a probability measure over \mathcal{V} and, for each v , $\mu_L(\cdot \mid v)$ is a probability measure over $[-1, 1]$ such that for every measurable $E \subseteq \mathcal{W}$,¹³

$$\mu(E) = \int_{v \in \mathcal{V}} \mu_L \left(\left\{ A \in [-1, 1] \mid Au + v\sqrt{1 - A^2} \in E \right\} \mid v \right) \mu_{\mathcal{V}}(dv). \quad (1)$$

It is easy to see that any μ has at least one such version and has exactly one if $\mu(\{u, -u\}) = 0$. If $\mu(\{u, -u\}) > 0$, then μ has infinitely many versions since equation (1) only states that

$$\mu(\{u\}) = \int_{v \in \mathcal{V}} \mu_L(\{1\} \mid v) \mu_{\mathcal{V}}(dv)$$

and analogously for $\mu(\{-u\})$ without pinning down the versions any further on either subspace.

¹³ $\mu_{\mathcal{V}}$ and each $\mu_L(\cdot \mid v)$ are defined on the Borel σ -algebras.

In light of Lemma 2, the versions enable us to translate the somewhat abstract notion of C_u -FOSD into the usual first-order stochastic dominance notion on A 's. More specifically, we have the following result.

Theorem 4. *Fix \succeq_i with canonical random Strotz representation (u_i, μ_i) , $i = 1, 2$. Then the following statements are equivalent:*

1. \succeq_2 is more temptation averse than \succeq_1 .
2. $u_1 = u_2$ and μ_1 C_u -FOSD μ_2 .
3. $u_1 = u_2$ and there exists versions of μ_i , $(\mu_{\mathcal{V}}^i, \mu_L^i)$, $i = 1, 2$, such that $\mu_{\mathcal{V}}^1 = \mu_{\mathcal{V}}^2$ and for almost every $v \in \mathcal{V}$, the conditional distribution $\mu_L^1(\cdot | v)$ first order stochastically dominates $\mu_L^2(\cdot | v)$.

5 Costly Self Control and Random Self Indulgence: Choice from Menus

To this point, we have focused on the random Strotz and random GP models as representations of preferences over menus. In this sense, we have treated them as models of choice *of* a menu. As we have seen, at least if we restrict attention to Lipschitz continuous models, we cannot use choice of menus to distinguish the random GP and random Strotz models.

On the other hand, each model also makes predictions about choice *from* menus. In the case of random Strotz, it is natural to interpret the representation (u, μ) as saying that with probability $\mu(w)$, the choice is the one made by w with ties broken in favor of u (where this is stated for measures with finite support for simplicity). In the case of a GP representation (u, ν) , Gul and Pesendorfer argue that the natural interpretation of the choice from a menu x is that it is some maximizer of $u + v$ from that menu. It is natural to interpret a random GP representation (u, ν) analogously as saying that with probability $\nu(v)$, the choice is that which maximizes $u + v$. If we adopt these interpretations as parts of the models and observe choices from menus, can we distinguish random GP and random Strotz? If we observe *both* choices of menus and choices from menus, can we distinguish the two models?

One difficulty in answering these questions is that the randomness of the choices from menus makes it difficult to say what we should assume is observed. Obviously, if we only observe one instance of the agent choosing from a particular menu, we cannot possibly

identify the probability distribution over choices from this menu. Making matters still more complex, for some menus, some w 's or $u + v$'s will have multiple best choices. Thus there will typically be many probability distributions over choices from a given menu that can be generated by the model.

One approach is to hypothesize that we have repeated observations of the agent and that any given w or $u + v$ always breaks ties from any given menu in the same fashion. That is, if $\alpha, \beta \in B_u(B_w(x))$, then the random Strotz model says that when w is the chooser's utility function, either α or β could be chosen from x . One hypothesis is to assume we have repeated observations of the agent and that when w is the chooser from x , w always makes the same choice, either always α or always β . Under this hypothesis, the behavior of the agent is given by a unique probability distribution over choices from each menu.

Another approach is to assume that we have the maximum conceivable amount of data, namely, that we observe *all* possible probability distributions over choices for each menu. In terms of the example above, this means that we would observe the distribution of choices from x when w always chooses α *and* the distribution when w always chooses β . It is harder to interpret how we could obtain such data, but, clearly, this case will tell us what the limits of identification based on observing choices from menus will be.

More formally, we define a *random choice correspondence* to be a function $C : X \rightarrow 2^{\Delta(\Delta(Z))}$ where $C(x) \neq \emptyset$ for all x and $\rho \in C(x)$ implies $\text{supp}(\rho) \subseteq x$. That is, for each x , $C(x)$ gives a set of probability distributions over x interpreted as possible distributions over choices.

For a nontrivial u ,¹⁴ the random Strotz representation (u, μ) *RS rationalizes* a random choice correspondence C if for every menu x , the following two conditions hold. First, for every $\rho \in C(x)$, there exists a *selection function* $\beta^* : \text{supp}(\mu) \rightarrow x$ such that for every measurable $E \subseteq x$,

$$\rho(E) = \mu(\{w \in \mathcal{W} \mid \beta^*(w) \in E\}) \quad (2)$$

and $\beta^*(w)$ maximizes u over $B_w(x)$. In other words, for every probability distribution in C , there is an optimal choice for each w such that the probability distribution is generated by the random Strotz representation.¹⁵ Second, for every measurable selection function β^* , there is a $\rho \in C(x)$ such that equation (2) holds. That is, C is consistent with the random Strotz model and describes all behavior the random Strotz model could generate. Note that our definition rules out rationalization based on a trivial u .

¹⁴That is, u such that $u(\alpha) \neq u(\beta)$ for some $\alpha, \beta \in \Delta(Z)$.

¹⁵This definition is essentially the same as Gul and Pesendorfer's (2006b) definition of random expected utility maximization with a tie-breaker except that they focus on random choice functions rather than correspondences.

Similarly, for a nontrivial u , the random GP representation (u, ν) *RGP rationalizes* a random choice correspondence C if for every menu x , the following conditions hold. First, for every $\rho \in C(x)$, there exists a selection function $\beta^* : \text{supp}(\nu) \rightarrow x$ such that for every measurable $E \subseteq x$,

$$\rho(x) = \nu(\{v \in \mathbf{R}^K \mid \beta^*(v) \in E\})$$

and $\beta^*(v)$ maximizes $u + v$. In other words, we can rationalize every choice distribution using the random GP representation and the hypothesis that conditional on v , the choice made is one which maximizes $u + v$.¹⁶ Second, for every measurable selection function, the ρ generated as above is contained in $C(x)$.

In Section 5.1, we consider whether the models can be distinguished when our observations consist of a selection from the random choice correspondence. Implicitly, this case assumes that we have repeated observations of choices from each menu where the selection function β^* is constant over time. In Section 5.2, we analyze whether the models can be distinguished when we observe much more data on choices, namely, the entire random choice correspondence. Finally, in Section ??, we turn to the most heroic assumption, where we can observe the entire random choice correspondence as well as the preference over menus. As we will see, we cannot generally separate the two models in either of the first two cases, but can in the third.

5.1 Choice from Menus: Selections

In this subsection, we assume that the only data available to potentially distinguish the random GP and random Strotz models is a selection from the random choice correspondence. With this data alone, we cannot, in general, distinguish the two models. More specifically, we have the following result.

Theorem 5. *If C is a random choice correspondence which is RS rationalizable, then there exists an RGP rationalizable \hat{C} such that $C(x) \subseteq \hat{C}(x)$ for all menus x . Similarly, if \hat{C} is an RGP rationalizable random choice correspondence, then there exists an RS rationalizable C with $C(x) \subseteq \hat{C}(x)$ for all menus x .*

Proof. Suppose (u, μ) is a random Strotz representation that rationalizes C . Use a change of variables to construct a random GP representation where for each $w \in \text{supp}(\mu)$, we have $v = w - u \in \text{supp}(\nu)$ with $\nu(w - u) = \mu(w)$.¹⁷ It is then easy to see that for

¹⁶This definition is essentially a set-valued version of Gul and Pesendorfer's (2006a) definition of random expected utility maximization. Their definition focuses on the case where the probability of multiple optima is zero where the definitions are the same.

¹⁷For expositional simplicity, this is stated as if the supports are finite. The more general case is straightforward.

every $\rho \in C(x)$, ρ must be rationalized by this random GP representation since it simply embodies a particular form of tie-breaking for the $u + v$'s. That is, letting \hat{C} denote the random choice correspondence generated by the random GP representation, we have $C(x) \subseteq \hat{C}(x)$ for all x .

For the other direction, suppose (u, ν) is a random GP representation that rationalizes \hat{C} . Define a distribution μ over w 's by the same change of variables $w = u + v$. Choose any nontrivial \hat{u} and consider the random Strotz representation (\hat{u}, μ) . It is easy to see that this simply adds a tie-breaking rule to the random GP so that the choice correspondence C generated by (\hat{u}, μ) must satisfy $C(x) \subseteq \hat{C}(x)$ for all x . ■

Theorem 5 says that if we observe a selection from the choice correspondence which is consistent with the random Strotz model, then it is consistent with the random GP model. This result does not tell us whether there is a selection from an RGP rationalizable C which is not a selection from any RS rationalizable choice correspondence. However, it is not hard to see that such selections exist. The construction of such an example is tedious, though, and so is relegated to Section ?? of the Appendix.

5.2 Choice from Menus: Correspondences

We now turn to the case where the entire choice correspondence is observed. Our main result is that we achieve equality in Theorem 5 if and only if the random Strotz model generating the correspondence is continuous. To be more precise, we show that while there are random choice correspondences with an RGP rationalization and no RS rationalization and conversely, there is a very large set of random choice correspondences with both kinds of rationalization. In particular, the set of choice correspondences with both kinds of rationalization is precisely the set with an RS rationalization such that (u, μ) represents a continuous preference over menus. Put differently, if we restrict attention to continuous models, then strictly more choices can be interpreted using the random GP model than using the random Strotz model.

The key to our characterization of the overlap is the following result.

Lemma 3. *A random Strotz representation (u, μ) is continuous if and only if for every menu x , u is indifferent over all of $B_w(x)$ with probability 1. That is,*

$$\mu(\{w \in \mathcal{W} \mid u(\alpha) = u(\beta), \forall \alpha, \beta \in B_w(x)\}) = 1.$$

In other words, a random Strotz is continuous if and only if the tie-breaking assumption that w breaks ties in favor of u is irrelevant. If C has an RS rationalization

where (u, μ) satisfies the property identified in Lemma 3, we say it has a continuous RS rationalization.

The following lemma shows that the change of variables construction we used in the proof of Theorem 5 is precisely how models with the same choice correspondence are related.

Lemma 4. *Suppose random choice correspondence C has an RS rationalization (u, μ) and an RGP rationalization (\hat{u}, ν) . Then the measures μ and ν are related by the change of variables $w = \hat{u} + v$.*

Proof. Let x be a sphere in the interior of $\Delta(Z)$. Then every point on the sphere is the unique optimum for exactly one $w \in \mathcal{W}$. Hence the unique ρ in $C(x)$ identifies μ exactly up to sets of measure zero. Hence μ is identified through the change of variables defined by $\rho(\beta) = \mu(w)$ for the unique w which is maximized over x at β .

Similarly, ν is uniquely identified from ρ as well. Again, $\nu(v)$ is identified through the change of variables which replaces $\rho(\beta)$ with $\nu(v)$ for the unique v such that $\hat{u} + v$ is maximized over x at β . Hence ν and μ are related as described. ■

These lemmas make it easy to show the following.

Theorem 6. *A random choice correspondence C has both an RS and a RGP rationalization if and only if it has a continuous RS rationalization.*

Proof. Suppose C has a continuous RS rationalization. By definition, the selection function β^* must satisfy $\beta^*(w) \in B_u(B_w(x))$. But if the RS rationalization is continuous, then $B_u(B_w(x)) = B_w(x)$ with probability 1. Hence the choice correspondence also has an RGP rationalization where we construct ν from μ by taking the change of variables $v = w - u$. Obviously, for $v = w - u$, we have $B_{u+v}(x) = B_w(x)$. Hence C has both an RS and an RGP rationalization.

We complete the proof by showing that if C does not have a continuous RS rationalization, then it cannot have both an RS and an RGP rationalization. Suppose, to the contrary, that C has an RS rationalization (u, μ) and an RGP rationalization (\hat{u}, ν) . Since C has no continuous RS rationalization, there is some menu, denoted by y , such that for a positive measure set of w 's, we have $u(\alpha) \neq u(\beta)$ for some $\alpha, \beta \in B_w(y)$.

Fix any selection rule β^* for the RGP rationalization such that $\beta^*(v) \in B_{-\hat{u}}(B_{\hat{u}+v}(y))$. That is, β^* breaks $\hat{u} + v$ ties *against* u . Let ρ be the random choice function generated by this selection. That is, for any measurable $E \subseteq y$, $\rho(E) = \nu(\{v \in \mathbf{R}^K \mid \beta^*(v) \in E\})$. Obviously, this ρ is contained in $C(y)$. Hence this ρ must be generated by some selection

function in the RS rationalization. That is, there must exist $\hat{\beta}^* : \mathcal{W} \rightarrow y$ such that $\hat{\beta}^*(w) \in B_u(B_w(y))$ and for any measurable $E \subseteq y$, $\rho(E) = \mu(\{w \in \mathcal{W} \mid \hat{\beta}^*(w) \in E\})$.

But this is impossible. Simply note that this implies

$$\int_{v \in \mathbf{R}^K} u(\beta^*(v)) \nu(dv) = \int_{w \in \mathcal{W}} u(\hat{\beta}^*(w)) \mu(dw),$$

which cannot hold since $u(\beta^*(w - u)) \leq u(\hat{\beta}^*(w))$ for all w , strictly so on a positive measure set. ■

It is worth noting that any choice that can be rationalized by the (nonrandom) GP model can be rationalized trivially by a (nonrandom) Strotz model. To be specific, take the (nonrandom) choice correspondence generated by the GP (u, ν) , namely, $C(x) = B_{u+\nu}(x)$. This is obviously rationalized by the ‘‘Strotz’’ model (\hat{u}, μ) where $\hat{u} = u + \nu$ and μ puts probability 1 on $u + \nu$. This is simply a reflection of the fact, noted by GP, that in their model, it is choice *of* a menu which reflects temptation, not choices from menus.

Theorem 6 obviously implies that there are random choice correspondences with an RS rationalization but no RGP rationalization since this is true of any correspondence rationalized by a discontinuous RS model. We complete the analysis by showing that there are random choice correspondences with an RGP rationalization but no RS rationalization. Fix a random GP model (u, ν) where ν has support $\{v_1, v_2\}$ where $u + v_1$ and $u + v_2$ represent different preferences over lotteries and also such that neither represents the same preference as the negation of the other. Let C denote the random choice correspondence this model generates and suppose that C has an RS-rationalization (\hat{u}, μ) . By Lemma 4, we know that $\mu(u + v_i) = \nu(v_i)$, $i = 1, 2$. For each i , let x_i denote a menu equal to a $u + v_i$ indifference curve which intersects the interior of $\Delta(Z)$, $i = 1, 2$. By Theorem 6, the RS rationalization must be continuous. Hence, by Lemma 3, \hat{u} must be indifferent over all of x_i . Therefore, each x_i must be contained in a \hat{u} indifference set. Since \hat{u} is nontrivial, this requires each x_i to be a \hat{u} indifference set. Since $u + v_1$ and $u + v_2$ cannot have indifference curves in common, there is no \hat{u} which satisfies this.

Summing up, we see that there are random choice correspondences with an RGP rationalization but no RS rationalization, correspondences with an RS rationalization and no RGP rationalization, and correspondences with each type of rationalization. The random choice correspondences in this last category are exactly the ones which have a continuous RS rationalization.

5.3 Choice of Menu Plus Choice from Menu

Thus as a general statement, neither preferences over menus alone nor choices from menus alone can distinguish these models. However, the two objects together *always* can, except in the degenerate case of no temptation. There are two (essentially equivalent) ways to see the point. First, we could consider a random Strotz and a random GP representation of the same preference over menus and compare the choices they make from menus. Second, we could consider a random Strotz representation and a random GP representation that produce indistinguishable choices from menus and compare them in terms of preferences over menus. In doing so, we must rule out the case of no temptation which is a special case of both models and obviously one where the two representations are equivalent. We say that a preference \succeq over menus *exhibits temptation* if there exist $\alpha, \beta \in \Delta(Z)$ with $\{\alpha\} \succ \{\beta\}$ and $\{\alpha\} \succ \{\alpha, \beta\}$. It's not hard to show that if (u, μ) is a random Strotz representation of \succeq , then \succeq exhibits temptation if and only if there is a $w \in \text{supp}(\mu)$ such that w does not represent the same preference over lotteries as u . Similarly, if (u, ν) is a random GP representation of \succeq , then \succeq exhibits temptation iff there exists $v \in \text{supp}(\nu)$ such that v does not represent the same preference over lotteries as u .

First, we show that if a random Strotz representation and a random GP representation generate the same preference over menus, then the random Strotz choices from menus exhibit strictly more temptation in a certain sense. Fix any preference over menus \succeq which has both a random Strotz representation, (u, μ) , and a random GP representation (u, ν) .¹⁸ Let C_{RS} be the random choice correspondence generated by the random Strotz representation and C_{RGP} the one generated by the random GP. Fix any menu x , any $\rho_{RS} \in C_{RS}(x)$, and $\rho_{RGP} \in C_{RGP}(x)$. Then the random Strotz choices from menus show more temptation in the sense that the agent prefers the expected behavior under the random GP. That is,

$$\left\{ \int \beta \rho_{RGP}(d\beta) \right\} \succeq \left\{ \int \beta \rho_{RS}(d\beta) \right\}.$$

Furthermore, this inequality must be strict for some x , ρ_{RGP} , and ρ_{RS} if \succeq exhibits temptation.

To see this, first consider the random GP representation. For each $v \in \text{supp}(\nu)$, let

¹⁸Note that since the two representations have the same preferences over menus, they must have the same preferences over singleton menus in particular. Hence, up to normalization, they must have the same u .

$\beta(v)$ denote a maximizer of $u + v$. Then for any x ,

$$\begin{aligned}
V_{RGP}(x) &= \int \left\{ \max_x [u(\beta) + v(\beta)] - \max_x v(\beta) \right\} \nu(dv) \\
&= \int \left\{ u(\beta(v)) + v(\beta(v)) - \max_x v(\beta) \right\} \nu(dv) \\
&\leq \int \left\{ u(\beta(v)) + v(\beta(v)) - v(\beta(v)) \right\} \nu(dv) \\
&= \int u(\beta(v)) \nu(dv) = u \left(\int \beta \rho_{RGP}(d\beta) \right) \\
&= V_{RGP} \left(\left\{ \int \beta \rho_{RGP}(d\beta) \right\} \right)
\end{aligned}$$

Hence $\left\{ \int \beta \rho_{RGP}(d\beta) \right\} \succeq x$. But $\rho_{RS} \in C_{RS}(x)$ implies that $x \sim \left\{ \int \beta \rho_{RS}(d\beta) \right\}$. (To see this, recall that in the random Strotz model, the agent evaluates the menu x as if he receives β with probability $\rho_{RS}(\beta)$.) Therefore, $\left\{ \int \beta \rho_{RGP}(d\beta) \right\} \succeq \left\{ \int \beta \rho_{RS}(d\beta) \right\}$.

To see that there must be some menu where the comparison is strict if \succeq exhibits temptation, consider any α and β that satisfy $\{\alpha\} \succ \{\beta\}$ and $\{\alpha\} \succ \{\alpha, \beta\}$. It is not hard to show that this implies that there is $v \in \text{supp}(\nu)$ with $u(\alpha) > u(\beta)$ and $v(\alpha) < v(\beta)$ and to use this to show that for the menu $x = \{\alpha, \beta\}$, we must have $V_{RGP}(x) < V_{RGP}(\left\{ \int \beta \rho_{RGP}(d\beta) \right\})$. Hence the inequality above is strict for such a menu.

Similarly, if we equate the choices from menus of the random Strotz and random GP agents, the random GP agent's preference over menus will show more concern about temptation. To be precise, fix random choice correspondences C_{RS} and C_{RGP} where C_{RS} has an RS rationalization (u, μ) and C_{RGP} has an RGP rationalization $(u, \hat{\nu})$. Note that we take the same u function for the two models. It is not hard to show that this hypothesis is unrestrictive in the sense that if a choice correspondence has an RGP rationalization, we can find such a rationalization with any u function we choose.¹⁹ Suppose these choice correspondences are (essentially) the same in the sense that for every x , $C_{RS}(x) \subseteq C_{RGP}(x)$. (Obviously, the argument to follow also works if we require these correspondences to be the same.) Let \succeq_{RS} and \succeq_{RGP} be the preferences over menus represented by the random Strotz and random GP rationalizations respectively. Then we claim that \succeq_{RGP} is more temptation averse than \succeq_{RS} , strictly so if \succeq_{RGP} exhibits temptation. Note that to show this result requires the hypothesis that both rationalizations use the same u since preferences can be compared under our notion of "more temptation averse" only if they have the same u .

¹⁹To see this, consider for simplicity an RGP rationalization (u, ν) where the support of ν is $\{v_1, \dots, v_J\}$. Fix any \hat{u} . For $j = 1, \dots, J$, let $\hat{v}_j = u - \hat{u} + v_j$, so that $\hat{u} + \hat{v}_j = u + v_j$. Define $\hat{\nu}$ by $\hat{\nu}(\hat{v}_j) = \nu(v_j)$. The RGP rationalization (u, ν) says that given menu x , the choice is the one which maximizes $u + v_j$ with probability $\nu(v_j)$. Clearly, this is the same thing as saying it is the choice which maximizes $\hat{u} + \hat{v}_j$ with probability $\hat{\nu}(\hat{v}_j)$. Hence $(\hat{u}, \hat{\nu})$ is also an RGP rationalization.

To show that the comparison holds, we show that $\{\alpha\} \succeq_{RS} x$ implies $\{\alpha\} \succeq_{RGP} x$. To see this, note that for any $\rho \in C_{RS}(x)$, $x \sim_{RS} \{\int \beta \rho(d\beta)\}$. Hence $\alpha \succeq_{RS} x$ implies $\alpha \succeq_{RS} \{\int \beta \rho(d\beta)\}$. Since both rationalizations use the same u , they rank singletons the same way, so $\{\alpha\} \succeq_{RGP} \{\int \beta \rho(d\beta)\}$. But exactly the same reasoning as above then shows that $\{\int \beta \rho(d\beta)\} \succeq_{RGP} x$. Hence $\{\alpha\} \succeq_{RGP} \{\int \beta \rho(d\beta)\} \succeq_{RGP} x$, establishing the desired conclusion. The strictness of the comparison when \succeq_{RGP} exhibits temptation follows from a similar argument to that used above.

Both these results are versions of the statement that random Strotz choices from menus are “worse” from the point of view of commitment utility than random GP choices. The first comparison says this directly since it states that if preferences over menus are the same, then commitment to the expected choice under random Strotz is worse than commitment to the expected choice under random GP. The second comparison says the same thing indirectly by saying that if we change the choices to make the choice behavior (essentially) the same, we must have “improved” the Strotzian agent’s behavior relative to the GP agent in the sense that the Strotzian now evaluates menus with less concern about temptation than the GP agent.

To see the reasoning behind this result most simply, suppose \succeq has a GP representation and hence also a random Strotz representation. Suppose this preference has $\{\alpha\} \succ \{\alpha, \beta\} \succ \{\beta\}$. In the GP case, this is rationalized by having $u(\alpha) > u(\beta)$, $v(\beta) > v(\alpha)$, and $u(\alpha) + v(\alpha) > u(\beta) + v(\beta)$. These rankings imply that

$$\begin{aligned} V_{GP}(\{\alpha, \beta\}) &= \max\{u(\alpha) + v(\alpha), u(\beta) + v(\beta)\} - \max\{v(\alpha), v(\beta)\} \\ &= u(\alpha) - [v(\beta) - v(\alpha)]. \end{aligned}$$

Thus the predicted choice is α , the same as the “choice” from the menu $\{\alpha\}$, but the menu is ranked lower than $\{\alpha\}$ because of the self-control cost of $v(\beta) - v(\alpha)$. By contrast, the random Strotz representation would have $V_{RS}(\alpha, \beta) = pu(\alpha) + (1 - p)u(\beta)$ for some $p \in (0, 1)$. Thus the random Strotz representation “explains” the fact that $\{\alpha\} \succ \{\alpha, \beta\}$ not by self-control costs but by a nonzero probability of “bad” behavior under the latter menu.

In other words, the random Strotz model explains the desire for commitment entirely in terms of a fear of succumbing to temptation, while the random GP model explains it in part by this but in part by a desire to avoid self-control costs. Hence the choice from menus in the random Strotz model must involve succumbing to temptation more frequently.

6 Conclusion

In summary, we have shown that the random Strotz and random GP models are, in general, indistinguishable in terms of commitment behavior or in terms of subsequent choices. Only with both kinds of data do they have different predictions. In addition, we have shown that the random Strotz representation is unique up to an appropriate normalization, provided a characterization of a notion of “more temptation averse than,” and axiomatically characterized the Lipschitz continuous version of the representation.

There are many interesting directions for further research. First, it would be natural to consider dynamic versions of the random Strotz model analogous to the way Gul and Pesendorfer (2004) extend the static GP model to dynamic environments. It is easy to see that the dynamic version of GP can also be rewritten as a dynamic random Strotz model. This fact shows that there are at least some interesting recursive random Strotz models, suggesting that a broader exploration of such models may be fruitful.

Second, the results here may have other interpretations of interest. For example, Olszewski (2007) and Ahn (2007) suggest models of ambiguity where an act is viewed not as a function from states to consequences but as a set of lotteries, where this is interpreted as a set of consequences. (See also related work by Gajdos, Hayashi, Tallon, and Vergnaud (2008).²⁰ In other words, we interpret a menu not as a set of options that the agent will choose from later but as a set of possible outcomes that “Nature” will choose from later. Under this interpretation, the random Strotz model represents the agent as forming various theories about what guides Nature’s choices. The weak set betweenness axiom which characterizes random Strotz is arguably even more plausible in this context than in the context of temptation.

Finally, while numerous versions of the Strotz model have been used in applications, particularly the special case of quasi-hyperbolic discounting (e.g., Laibson (1997), O’Donoghue and Rabin (1999), or Benabou and Tirole (2002)), we think exploration of applications of the random Strotz model may be of great interest. As noted above and in Caplin and Leahy (2006), the commitment behavior of random Strotz is, in general, better behaved than that of the nonrandom version of the model as one can avoid problems with discontinuities. In addition, the choices from menus in random Strotz exhibit more temptation than the behavior in the GP or random GP model, suggesting that there may be more interesting results from this model.

²⁰The Steiner point, which plays a significant role in the analysis of Gajdos, Hayashi, Tallon, and Vergnaud, provides an interesting connection between their work and random Strotz. One definition of the Steiner point of a set of lotteries is that it is the expected value of the lottery chosen by an expected utility preference which is drawn at random from a uniform distribution. Thus it is the expected choice by a particular random Strotz agent.

References

- [1] Ahn, D., “Ambiguity without a State Space,” *Review of Economic Studies*, **75**, January 2008, 3–28.
- [2] Benabou, R., and M. Pycia, “Dynamic Inconsistency and Self-Control: A Planner–Doer Interpretation,” *Economic Letters*, **77**, 2002, 419–424.
- [3] Benabou, R., and J. Tirole, “Self–Confidence and Personal Motivation,” *Quarterly Journal of Economics*, **117**, August 2002, 871–915.
- [4] Billingsley, P., *Probability and Measure*, Third Edition, New York: Wiley, 1995.
- [5] Caplin, A., and J. Leahy, “The Recursive Approach to Time Inconsistency,” *Journal of Economic Theory*, **131**, November 2006, 134–156.
- [6] Chatterjee, K., and R. V. Krishna, “Menu Choice, Environmental Cues and Temptation: A ‘Dual Self’ Approach to Self–Control,” Pennsylvania State University working paper, 2007.
- [7] Chatterjee, K., and R. V. Krishna, “A ‘Dual Self’ Approach to Stochastic Temptation,” *American Economic Journal: Microeconomics*, **1**, August 2009, 148–167.
- [8] Dekel, E., B. Lipman, and A. Rustichini, “Representing Preferences with a Unique Subjective State Space,” *Econometrica*, **69**, July 2001, 891–934.
- [9] Dekel, E., B. Lipman, and A. Rustichini, “Temptation–Driven Preferences,” *Review of Economic Studies*, **76**, July 2009, 937–971.
- [10] Dekel, E., B. Lipman, A. Rustichini, and T. Sarver, “Representing Preferences with a Unique Subjective State Space: Corrigendum,” *Econometrica*, **75**, March 2007a, 591–600.
- [11] Dekel, E., B. Lipman, A. Rustichini, and T. Sarver, “Supplement to ‘Representing Preferences with a Unique Subjective State Space: Corrigendum,’” March 2007b, available at <http://www.econometricsociety.org/ecta/supmat/ECTA6232SUPP.pdf>.
- [12] Epstein, L., “A Definition of Uncertainty Aversion,” *Review of Economic Studies*, **66**, July 1999, 579–608.
- [13] Gajdos, T., T. Hayashi, J.-M. Tallon, and J.-C. Vergnaud, “Attitude toward Imprecise Information,” *Journal of Economic Theory*, **140**, May 2008, 27–65.
- [14] Ghirardato, P., and M. Marinacci, “Ambiguity Made Precise: A Comparative Foundation,” *Journal of Economic Theory*, **102**, February 2002, 251–289.

- [15] Gilboa, I., and D. Schmeidler, “Additive Representations of Non-Additive Measures and the Choquet Integral,” *Annals of Operations Research*, **51**, Issue 1–4, 1994, 43–65.
- [16] Gul, F., and W. Pesendorfer, “Temptation and Self-Control,” *Econometrica*, **69**, November 2001, 1403–1435.
- [17] Gul, F., and W. Pesendorfer, “Self-Control and the Theory of Consumption,” *Econometrica*, **72**, January 2004, 119–158.
- [18] Gul, F., and W. Pesendorfer, “Random Expected Utility,” *Econometrica*, **74**, January 2006a, 121–146.
- [19] Gul, F., and W. Pesendorfer, “Supplement to ‘Random Expected Utility,’” January 2006b, available at <http://www.econometricsociety.org/ecta/supmat/4734ext.pdf>.
- [20] Higashi, Y., K. Hyogo, and N. Takeoka, “Subjective Random Discounting and Intertemporal Choice,” *Journal of Economic Theory*, **144**, May 2009, 1015–1053.
- [21] Kelly, J., *General Topology*, New York: Springer-Verlag, 1955.
- [22] Laibson, D., “Golden Eggs and Hyperbolic Discounting,” *Quarterly Journal of Economics*, **112**, May 1997, 443–477.
- [23] Mas-Colell, A., M. Whinston, and J. Green, *Microeconomic Theory*, New York: Oxford University Press, 1995.
- [24] Milgrom, P., and I. Segal, “Envelope Theorems for Arbitrary Choice Sets,” *Econometrica*, **70**, March 2002, 583–601.
- [25] O’Donoghue, Ted, and Matthew Rabin, “Doing It Now or Later,” *American Economic Review*, **89**, March 1999, 103–124.
- [26] Olszewki, W., “Preferences over Sets of Lotteries,” *Review of Economic Studies*, **74**, April 2007, 567–595.
- [27] Pollard, D., *A User’s Guide to Measure Theoretic Probability*, Cambridge Series in Statistical and Probabilistic Mathematics, Cambridge: Cambridge University Press, 2002.
- [28] Sarver, T., “Anticipating Regret: Why Fewer Options May Be Better,” *Econometrica*, **76**, March 2008, 263–305.
- [29] Stovall, J., “Multiple Temptations,” *Econometrica*, forthcoming.
- [30] Strotz, R., “Myopia and Inconsistency in Dynamic Utility Maximization,” *Review of Economic Studies*, **23**, 1955, 165–180.

- [31] Weymark, J., “A Reconsideration of the Harsanyi–Sen Debate on Utilitarianism,” in J. Elster and J. Roemer, eds., *Interpersonal Comparisons of Well-Being*, Cambridge: Cambridge University Press, 1991, 255–320.