International Trade Dynamics with Intermediate Inputs

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Abstract

This paper develops a model of trade in intermediate inputs with heterogeneous producers to analyze the dynamics of aggregate trade flows in response to movements in the relative price of imported to domestic goods. In aggregate data, trade volumes adjust slowly in response to relative price changes, an observation at odds with standard theories. The main feature of the model is a plant-level irreversibility in the structure of intermediate inputs used in production. When calibrated to match cross-section data on plant-level heterogeneity in the use of imported intermediates, the model generates a slow response of the volume of trade in response to relative price changes. Relative price movements induce immediate changes in aggregate imported relative to domestic purchases through adjustment within importing producers, and through the reallocation of resources between non-importing and importing producers. The magnitudes of these margins predicted by the model are broadly in line with those in plant-level data. Additionally, trade volumes adjust slowly through gradual changes in the fraction of importers in the economy. This slow adjustment in aggregate trade flows significantly reduces the measured welfare gains from trade policy reform.

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1 Introduction

This paper builds a model of international trade in intermediate inputs with heterogeneous producers, in which the producer-level decision to use imported inputs is irreversible. The model is used to analyze the dynamic behavior of aggregate and producer-level trade flows in response to movements in the relative price of imported to domestically produced goods. Aggregate trade data show that imports relative to domestic purchases move slowly in response to changes in the relative price of imports. Long-term growth in trade is much larger than the immediate response to trade reform. The model presented here accounts for the slow-moving dynamic behavior of aggregate trade flows, as a result of the irreversiblity in the decision to import intermediate inputs at the micro-level.

Intermediate goods comprise about forty to sixty percent of total international merchandise trade for many of the world's industrial economies.[1] At the micro level, producers are heterogeneous in their use of imported relative to domestically produced intermediate inputs. Namely, relatively few producers use imports, and importers are larger in size than non-importers. For example, in both the US and Chile, only about one quarter of manufacturing plants use imported intermediate inputs. In addition, these importing plants employ two to three times as many workers, on average, as their non-importing counterparts.[2] Many empirical studies have documented analogous facts for exporting producers, and most of the theory developed so far incorporating heterogeneity in producer-level participation in international trade has focused on exporting behavior.[3]

This paper instead focuses on the producer-level importing decision to study trade in intermediate inputs, in light of the evidence of the importance of heterogeneity in importing behavior. The importing decision is modeled at the plant level as an irreversible technology choice: a plant can choose a production technology that uses intermediate inputs of only domestically produced goods, or a technology that combines imported and domestic intermediates. The technology that a plant chooses when it is built is fixed for the life of the plant, so the decision to import or not is permanent. This feature of the model is motivated by plant-level evidence. In the data, the plant-level responses to changes in the relative price of imports over time indicate that there is substantial irreversibility in the composition of intermediate inputs that plants use; importing is a relatively irreversible choice.[4]

[1] See Table 1 for details.

[2] See Kurz (2006) for the US, and Section 2 below for Chile. Similar findings are reported in Amiti and Konings (2007) for Indonesia; Biscourp and Kramarz (2007) for France; and Halpern, Koren, and Szeidl (2006) for Hungary.

[3] Empirical studies of exporting behavior include Bernard and Jensen (1995) and Clerides, Lach and Tybout (1998). Theoretical models of exporting behavior include Melitz (2003) and Bernard, Eaton, Jensen and Kortum (2003).

[4] Kasahara (2004), using Chilean plant data, also finds that a large change in the ratio of imports relative

With plants divided into importers and non-importers based on their initial investment decisions, movements in the relative price of imported to domestic goods affect the volume of aggregate trade through three mechanisms. The first is the within-plant ratio of imports relative to domestic inputs. The second mechanism is the equilibrium allocation of factors of production across existing importing and non-importing plants at any point in time. The third is the dynamic allocation of investment in importing across newly established plants. A decrease in the price of imports relative to domestic goods makes importers relatively more profitable than non-importers. The static effects associated with this change are that importing plants use imports more intensively, and importing plants expand relative to non-importing plants. In addition, if it is expected to persist, the dynamic effect of a price decrease is that newly established plants expect a higher gain in profit from using imports; thus more plants undertake the investment required to import. These two effects determine the response over time of aggregate trade flows to the change in the relative price of imported to domestic goods. Because the dynamic behavior of aggregate imports relative to domestic goods are linked to the rate at which new plants are created, aggregate trade flows respond slowly to changes in the relative price of imports.

The model is calibrated so that both the fraction of plants importing and their size relative to non-importers match the plant-level statistics previously mentioned. The calibrated model is used to measure the contributions of the static and dynamic reallocation effects to the short-run and long-run dynamics of aggregate trade flows. When the model is subjected to aggregate technology shocks of standard business cycle magnitudes, the static effect is predominant. This is because new plants are a small fraction of the total. The model predicts fluctuations in aggregate trade flows that are characterized by a low elasticity of substitution between imported and domestic intermediate goods. A permanent trade liberalization, however, is followed by a large, gradual increase in the volume of trade over several years following the policy change. The number of importing plants relative to non-importing plants increases over time. In response to a trade reform of reasonable magnitude, the model predicts a long-run doubling in the volume of trade relative to GDP, with about half the growth in trade occurring within ten years.

This paper is related to recent work on dynamic models of producer-level exporting decisions. These include Ruhl (2008), Ghironi and Melitz (2005), Alessandria and Choi (2007a and 2007b), and Atkeson and Burstein (2009). As in Ruhl (2008), this paper isolates different effects that influence the short-run and long-run response of trade flows to relative price changes. Ghironi and Melitz (2005) and Alessandria and Choi (2007a) examine the busi-

to domestic inputs within a plant is associated with a large concurrent investment in physical capital, interpreted as the adoption of a new technology.

ness cycle properties of models with fixed costs of exporting. Alessandria and Choi (2007b) and Atkeson and Burstein (2009) study the transition path following trade liberalization in models in which producer-level efficiency evolves over time.[5] In contrast, in the model of importing behavior presented here, cyclical fluctuations in trade flows and gradual growth in trade depend on the irreversibility of the choice between importing and non-importing technologies. The models of exporting in previous studies differ in the extent to which the decision to export is irreversible.[6] However, they all share the feature that the decision made at any time to *not* export can be undone. The essential difference between the model in this paper and previous models of dynamic exporting decisions is that, in this paper, *either* of the choices available to producers - to not import or to import - is a permanent decision.

The assumption of irreversibility in technology choice is similar to that in models of "putty-clay" capital, recent examples of which include Atkeson and Kehoe (1999) and Gilchrist and Williams (2000). In these models, investing in capital requires an irreversible choice of the amount of another variable input that will be combined with the capital in the future. (The variable input is energy in Atkeson and Kehoe (1999) and labor in Gilchrist and Williams (2000)). The application of this type of irreversibility to production with imported and domestic intermediate inputs in this paper is motivated by Kasahara (2004), who finds evidence of the putty-clay nature of a producer's choice between imported and domestic intermediate goods.

A recent paper on producer-level importing decisions is Kasahara and Lapham (2007), who consider a producer's joint import and export decisions in a stationary model derived from that of Melitz (2003). Their model incorporates fixed costs of importing to generate cross-sectional differences in the use of imports by plants. This paper analyzes an environment with aggregate dynamics, and finds that the irreversibility in individual plant technology and the cross-section heterogeneity associated with fixed costs of importing can account well for the dynamic behavior of trade flows observed in the data.

The rest of the paper is organized as follows. Section 2 presents data for the aggregate and plant-level facts mentioned in this introduction. Section 3 presents the model and characerizes the plant-level and aggregate implications of relative price movements. Section 4 provides a calibration and quantitative analysis of the model, and Section 5 concludes.

[5] Chaney (2005) also considers the transition path following trade reform in a model with producer-level exporting decisions, but focuses on the average productivity of operating plants rather than the behavior of trade flows.

[6] In Ruhl (2008), the decision to export is completely irreversible. In Ghironi and Melitz (2005) the decision is made independently each period. Alessandria and Choi (2007) incorporate both irreversible and independent per-period dimensions in the decision to export.

2 Data

This section presents two sets of facts from the data that motivate the paper. The first set of facts, from aggregate trade data, establishes that the response of trade flows at the aggregate level responds slowly to changes in relative prices across countries. The second set of facts provides plant-level evidence that on the costly and irreversible aspects of the decision to use imported intermediate inputs, and therefore motivates the approach taken in this paper in accounting for the observations in the aggregate data.

2.1 Aggregate Facts

Sudden changes in the price of imported goods have gradual effects on a country's imports. Figure 1 depicts the total imports by Mexico from the United States, relative to US GDP, over the period 1982-2000, along with average Mexican tariffs on US goods.[7] During this period, there were two episodes in which tariffs were reduced by a large amount within a single year: Mexico's unilateral trade liberalization in 1988, and the regional North American Free Trade Agreement with the US and Canada in 1994. There was substantial growth in trade over this period, with imports from the US relative to US GDP growing four-fold from 1987-1993 and nearly doubling again from 1993-2000.

Attributing the growth in Mexico's trade with the US to the large tariff cuts in 1987 and 1993 implies that changes in the price of imported relative to domestic goods generate large changes in trade flows. However, the growth in trade from a one-time tariff reduction is gradual, slowly accumulating over several years.

Another way to depict the gradual response of trade flows to price changes is the "elasticity puzzle" described in Ruhl (2008). Researchers estimating the elasticity of substitution between imported and domestic goods rely on either business cycle fluctuations, or on single trade liberalization events, to generate variation in the price of imports realtive to domestic goods. The estimates from cyclical fluctuations in prices imply small elasticities, mostly in the range of 1-2, while estimates from the growth in trade several years following trade liberalizations imply large elasticities, generally above 6. Therefore, the response in trade growth to a price change takes time to develop.

[7] Trade and GDP data are from International Monetary Fund, *International Financial Statistics* CD-ROM. Mexican tariffs are from Hinojosa-Ojeda et al. (2000) for 1982-1994, and from Office of US Trade Representative, *Trade Policy Agenda* and *Report on Foreign Trade Barriers* (various years).

2.2 Plant-level Facts

This section describes data from a panel survey of Chilean manufacturing plants, from Chile's *Instituto Nacional de Estadistica* (*INE*). The period covered is 1979-1986. Each plant reports its imported and total intermediate input purchases. If imports are positive, I consider the plant an importer.

2.2.1 Cross-section

I first describe the cross-section characteristics of plants. Statistics are computed for all plants existing in the sample in each year, then averaged across years.

Few manufacturing plants in Chile use imported intermediate inputs, and they tend to be much larger than the plants that do not use any imported inputs. Table 2 shows that only about 24 percent of plants, on average, use a positive amount of imported intermediate inputs. These plants employ about three times as many workers, on average, as the plants that do not use imported inputs.

For comparison, Kurz (2006) reports that in 1992, about the same proportion of US manufacturing plants use imported inputs, and they are on average about twice the size of the plants that do not.

These figures imply that using imported inputs along with domestic inputs is disproportionately more costly than using domestic inputs alone. In addition, Kasahara and Rodrigue (2007), using the same sample of Chilean plants, find that using imported along with domestic inputs brings with it a significant gain in plant productivity, so that plants operating at a larger scale would benefit the most from using imports. Therefore, only large plants find it worthwhile to pay the additional costs of using imported inputs.

2.2.2 Panel

The allocation of resources across plants over time provides evidence that the decision to use imported inputs or domestic inputs alone is not easily reversed. Over the period 1979-1986, the aggregate quantity of imported relative to total intermediate inputs purchased by Chilean manufacturing plants declined by 18 percent per year, on average.

In light of the cross-section heterogeneity among plants' use of imports highlighted in the previous subsection, this aggregate decline can be attributed at the plant-level to several different channels. If some plants import and some do not, and plants can enter and exit the economy, aggregate imports relative to total intermediate inputs can fall because: (i) importing plants import relatively less of their inputs; (ii) importing plants shrink relative to non-importing plants; (iii) importing plants stop importing and become non-importing plants; or (iv) importing plants that exit the economy are replaced by entering plants that do not import.

Magnitudes can be assigned to these channels through decomposing the aggregate ratio of imported to total intermediate inputs as follows. Let $M_t = \sum_{i \in I_{mt}} m_t^i$ be the aggregate quantity, in year t, of imported inputs used at importing plants, where i denotes a plant, m_t^i denotes imported inputs used by plant i in year t, and I_{mt} is the set of plants that uses imports in year t. Similarly, let $X_t = \sum_{i \in I_t} x_t^i$ be the aggregate quantity of total intermediate inputs (imported plus domestic) used by all plants, with x_t^i denoting all the intermediate inputs purchased by plant i in year t, and I_t denoting the entire set of plants operating in period t.[8] Then, the change at the aggregate level in imports relative to total intermediate goods can be decomposed as follows:[9]

$$
\begin{aligned}
\frac{M_{t+1}}{X_{t+1}} - \frac{M_t}{X_t} \quad = \quad & \sum_{i \in I_{mt+1} \cap I_{mt}} \frac{x_t^i}{X_t} \left(\frac{m_{t+1}^i}{x_{t+1}^i} - \frac{m_t^i}{x_t^i} \right) \\
& + \sum_{i \in I_{mt+1} \cap I_{mt}} \left(\frac{x_{t+1}^i}{X_{t+1}} - \frac{x_t^i}{X_t} \right) \frac{m_t^i}{x_t^i} \\
& + \sum_{i \in I_{mt+1} \cap I_{mt}} \left(\frac{x_{t+1}^i}{X_{t+1}} - \frac{x_t^i}{X_t} \right) \left(\frac{m_{t+1}^i}{x_{t+1}^i} - \frac{m_t^i}{x_t^i} \right) \\
& + \sum_{i \in (I_{mt+1} \setminus I_{mt}) \cap (I_t \cap I_{t+1})} \frac{x_{t+1}^i}{X_{t+1}} \frac{m_{t+1}^i}{x_{t+1}^i} - \sum_{i \in (I_{mt} \setminus I_{mt+1}) \cap (I_t \cap I_{t+1})} \frac{x_t^i}{X_t} \frac{m_t^i}{x_t^i} \\
& + \sum_{i \in (I_{mt+1} \setminus I_{mt}) \cap (I_{t+1} \setminus I_t)} \frac{x_{t+1}^i}{X_{t+1}} \frac{m_{t+1}^i}{x_{t+1}^i} - \sum_{i \in I_{mt} \setminus I_{mt+1} \cap (I_t \setminus I_{t+1})} \frac{x_t^i}{X_t} \frac{m_t^i}{x_t^i}
\end{aligned} \tag{1}
$$

The first line in the sum above gives the total effect of each plant that imports in both years t and $t+1$ adjusting its ratio of imported to domestic inputs (m/x), weighted by its initial share in the aggregate economy (x/X). This is adjustment within the plant. The second line is the sum of changes in these continuously importing plants' share of the economy, holding fixed the intensity with which each plant uses imports. This is adjustment by reallocating between plants. The third line gives the effect of the plants' ratios m/x and their shares of the economy x/X changing together. The fourth line is the contribution of continuing plants that start to import in year $t+1$, net of the loss due to continuing plants that no longer import in year $t+1$. Finally, the fifth line is the contribution of new entrants that import less the loss due to importing plants that exit the economy. Table 3 gives

[8] Total intermediate inputs are deflated with industry-specific input price indices, and imported intermediate inputs are deflated with an economy-wide import price index.

[9] This is similar to the methodologies used by many authors to decompose aggregate productivity growth into its plant-level components. See, for example, Baily, Hulten and Campbell (1992).

the contributions of each of these five components, labeled "within", "between", "cross", "switch" and "entry", respectively, as a percentage of the aggregate change $M_{t+1}/X_{t+1} - M_t/X_t$ (so that the components sum to one hundred). Two sets of figures are reported: the average across one-year changes, and the 7-year change.

The figures in the first row of Table 3 show that, on average, each year, 78 percent of the decline in imports at the aggregate level is accounted for by each importing plant adjusting the ratio of imports relative to total intermediate inputs it uses. About 26 percent is accounted for by importing plants shrinking in scale relative to non-importing plants. Two percent of the aggregate change is accounted for by new entrants using less imports than exiting plants, and about three percent is attributed to importing plants switching to becoming non-importers more often than non-importing plants switching to importing. The fact that the "between" component is substantial provides evidence that there is some irreversibility in the nature of the decision to import: not all the adjustment at the aggregate level comes from each plant changing the composition of goods it uses. In addition, the year-to-year net effects of entry and exit and of plants switching importing status are very small. In contrast, over the entire 7-year period, the effects of entry and exit accumulate, and contribute five times more to the aggregate change in imports than they do on average each year.

In the model presented in the next section, plants face a costly, irreversible decision to use imported intermediate inputs. This generates both the cross-sectional properties of plant heterogeneity discussed in the previous subsection, and generates trade growth at the aggregate level through the "within", "between", and "entry" plant-level margins discussed here. When calibrated to match the cross-sectional properties of the plant data, the model generates aggregate implications for the dynamic behavior of trade flows that mimic the aggregate facts discussed earlier in this section.

3 Model

3.1 Outline

The model economy consists of two countries, referred to as *home* and *foreign*. There are two goods in the economy, and each good is produced in only one country and can be traded internationally. Production in each country is carried out in plants that can operate one of two available technologies to produce their country's good. The first technology combines labor with intermediate inputs of the domestically-produced good. The second technology uses, in addition, intermediate inputs of the imported good. Plants that operate each technology are referred to as non-importing and importing plants, respectively. Plants in the economy are distinguished by the technology they use (denoted d using only domestic goods and m using imports) and the idiosyncratic efficiency, denoted z, with which they operate the technology. All plants are subject to country-wide shocks to aggregate efficiency, denoted A in the home country and A^* in the foreign country. (Throughout, all foreign variables are indexed with an asterisk (*).)

Each period, all plants face a constant probability of death. New plants continually enter the economy and choose the technology, importing or not, with which they will operate. This is an irreversible decision, fixed over the life of each plant. The entry and technology choices of a plant require fixed investment costs that cannot be recovered.

Each country is populated by a continuum of mass one of identical infinitely-lived consumers who are each endowed with 1 unit of time to be allocated between labor and leisure, and an equal share of ownership of the all the plants in the country. The consumers' labor is used for production in all existing domestic plants.

Consumers in each country do not value consumption of the good produced abroad, so there is no trade in goods for final consumption. Output produced in each country is allocated to final domestic consumption, intermediate consumption of domestic and foreign plants, and investment in new plants.

3.2 Time and Uncertainty

Time is discrete and indexed $t = 0, 1, \ldots$. At each date t, an event s_t occurs, which is drawn from a Markov process with transition function $\phi(s_t|s_{t-1})$. The state of the economy at any date t is the complete history of events up to and including date t, denoted $s^t = (s_0, s_1, \ldots, s_t)$. The probability of state s^t as of period 0 is denoted $\tilde{\phi}(s^t)$. Commodities and prices are functions of the state s^t.

3.3 Consumers

The preferences of a representative consumer in the home country are represented by the expected discounted present value of utility from consumption and leisure,

$$\sum_{t=0}^{\infty} \sum_{s^t} \beta^t \tilde{\phi}(s^t) U\left(C(s^t), 1 - N(s^t)\right) \tag{2}$$

The consumer faces the following budget constraint in every state s^t:

$$C(s^t) + \sum_{s_{t+1}} Q(s^t, s_{t+1})B(s^t, s_{t+1}) \leq w(s^t)N(s^t) + B(s^t) + \Pi(s^t) + T(s^t) \tag{3}$$

where C denotes consumption and N is the fraction of time spent working. $Q(s^t, s_{t+1})$ is the price, in units of home country output at state s^t, of an internationally traded claim to a unit of home country output in state (s^t, s_{t+1}) and B is the quantity of these claims purchased. The wage rate, in units of domestic output, is w, and the aggregate profits Π of plants are rebated equally to all consumers. T is tariff duty collected on total imports, also rebated equally to all consumers.

Consumers have access to complete asset markets, as evident by the dependence of Q and B on the future event s_{t+1}. The consumer's ownership of the plants is modeled as passive, in that they take the profit rebate Π as given. Below, the plants' problems are specified so that their operating, entry, and technology choices are the same as those the consumer would choose for them.

The consumer's problem is to choose $C(s^t)$, $N(s^t)$ and $B(s^t, s_{t+1})$ to maximize (2) subject to (3). The first order conditions of this problem include

$$\frac{U_2(s^t)}{U_1(s^t)} = w(s^t)$$

$$Q(s^t, s_{t+1}) = \beta\phi(s^{t+1}|s^t)\frac{U_1(s^{t+1})}{U_1(s^t)} \tag{4}$$

where $U_j(s^t)$ is the partial derivative of U with respect to its j'th argument.

Consumers in the foreign country have the following utility function:

$$\sum_{t=0}^{\infty}\sum_{s^t} \beta^t\tilde{\phi}(s^t)U\left(C^*(s^t), 1 - N^*(s^t)\right)$$

and face the the budget constraint:

$$C^*(s^t) + \sum_{s_{t+1}} Q(s^t, s_{t+1})\frac{B^*(s^t, s_{t+1})}{p(s^t)} \leq w^*(s^t)N^*(s^t) + \frac{B^*(s^t)}{p(s^t)} + \Pi^*(s^t) + T^*(s^t)$$

Here, the foreign budget constraint is written in units of foreign country output, and $p(s^t)$ is the price of foreign goods in units of home-country goods. The first order conditions for the foreign consumer's problem are:

$$\frac{U_2^*(s^t)}{U_1^*(s^t)} = w^*(s^t)$$

and

$$Q(s^t, s_{t+1}) = \beta\phi(s^{t+1}|s^t)\frac{U_1^*(s^{t+1})}{U_1^*(s^t)}\frac{p(s^t)}{p(s^{t+1})}$$

3.4 Plants

Plants in the economy face two types of decisions: those made at the time of establishment, and those made each period thereafter. I start with the decisions made by existing plants each period. The plant's dynamic decision at the time of establishment then anticipates the profits generated each period by the static decisions each period.

At any state s^t, a plant is distinguished by its efficiency z and its technology, importing or not. In particular, the age of a plant, reflecting the date at which it entered the economy, is irrelevant for describing its current production possibilities and decision problem, so I do not distinguish existing plants by age.

Plants operate each period under perfect competition, with decreasing returns to scale technologies. They are subject to country-specific aggregate shocks to efficiency each period, denoted $A(s^t)$ in the home country and $A^*(s^t)$ in the foreign country. These shocks are the only exogenous source of uncertainty in the economy.

3.4.1 Non-importing plants

The technology used by a non-importing plant with efficiency z at state s^t combines labor n and intermediate inputs d to produce output y according to:

$$y = A(s^t)z^{1-\alpha-\theta}d^{\alpha}n^{\theta}$$

where $\alpha + \theta < 1$.

The plant's static profit from operating is denoted $\pi_d(z, s^t)$, and is given by the following maximization problem:

$$\pi_d(z, s^t) = \max_{n,d\geq 0} A(s^t)z^{1-\alpha-\theta}d^{\alpha}n^{\theta} - d - w(s^t)n$$

The plant takes as given the prices of inputs in units of its output: the wage w and the price for intermediate inputs, equal to 1.

The decreasing-returns technology yields an optimal scale of production for each plant, which depends on its idiosyncratic efficiency z, and on the aggregate state s^t (through dependence on both $A(s^t)$ and the wage $w(s^t)$).

The plant's optimal input and output decisions are summarized by

$$\begin{aligned} y_d(z, s^t) &= h_d(s^t)^{1/(1-\alpha-\theta)} z \\ n_d(z, s^t) &= \frac{\theta}{w(s^t)} y_d(z, s^t) \\ d_d(z, s^t) &= \alpha y_d(z, s^t) \end{aligned} \tag{5}$$

where

$$h_d(s^t) = A(s^t)\alpha^\alpha \theta^\theta w(s^t)^{-\theta} \tag{6}$$

Plant input and output decisions are homogeneous in z. That is, for $\psi > 0$, if $z_1 = \psi z_2$, then

$$y_d(z_1, s^t) = \psi y_d(z_2, s^t)$$

and similarly for the input demands n_d and d_d.

This property of plant decisions is exploited in characterizing the model's aggregate properties below.

Maximized profits are given by

$$\pi_d(z, s^t) = (1 - \alpha - \theta) y_d(z, s^t)$$

3.4.2 Importing plants

An importing plant with efficiency z at state s^t produces according to:

$$y = A(s^t) z^{1-\alpha-\theta} (\gamma d^\omega m^{1-\omega})^\alpha n^\theta$$

Here $n, d, m,$ and y denote labor, domestic and imported intermediates, and output, respectively.

Importing plants combine intermediate inputs of domestic and imported goods to create a composite intermediate input, defined as $\gamma d^\omega m^{1-\omega}$, that is combined with labor. The parameter ω reflects the relative importance of domestic goods; if it is greater than $\frac{1}{2}$, then there is a technological bias within the plant towards intermediate inputs of the domestically produced good.

The parameter γ measures the efficiency advantage of the importing technology relative to the non-importing technology, discussed further in the next subsection. An efficiency advantage associated with using imported and domestic intermediate goods relative to using domestic intermediate goods alone is related to feature of "increasing returns to specialization" in the models of Ethier (1982) and Romer (1987). In these papers, production technologies are defined so that using a larger number of inputs yields higher output than using fewer inputs, in the same total quantity. Increasing returns to specialization is captured here by the parameter γ, which is calibrated in the quantitative experiments to match statistics in cross-section plant data.

The profit maximization problem of an importing plant is:

$$\pi_m(z, s^t) = \max_{n,d,m \geq 0} A(s^t) z^{1-\alpha-\theta} (\gamma d^{\omega} m^{1-\omega})^{\alpha} n^{\theta} - d - p(s^t)(1+\tau)m - w(s^t)n$$

where $p(s^t)$ is the price of foreign country goods in units of home country goods, and τ is the *ad valorem* tariff rate. These are both taken as given by the plant, in addition to the wage $w(s^t)$.

The optimal decisions are:

$$\begin{aligned} y_m(z, s^t) &= h_m(s^t)^{1/(1-\alpha-\theta)} z \\ n_m(z, s^t) &= \frac{\theta}{w(s^t)} y_m(z, s^t) \\ d_m(z, s^t) &= \alpha \omega y_m(z, s^t) \\ m(z, s^t) &= \frac{\alpha(1-\omega)}{p(s^t)(1+\tau)} y_m(z, s^t) \end{aligned} \tag{7}$$

where

$$h_m(s^t) = A(s^t) \left(\gamma \alpha \omega^{\omega} \left(\frac{1-\omega}{p(s^t)(1+\tau)} \right)^{1-\omega} \right)^{\alpha} \left(\frac{\theta}{w(s^t)} \right)^{\theta} \tag{8}$$

Maximized profit for an importing plant is

$$\pi_m(z, s^t) = (1-\alpha-\theta) y_m(z, s^t)$$

3.4.3 Difference between non-importers and importers

This section considers the differences in both *potential* production possibilities and *observed* behavior between operating the importing and non-importing technologies, for a given plant with $z = 1$. Within a plant, these differences determine the realized difference in profit between importing and not, and thus impact the dynamic choice discussed in the next section.

The non-importing and importing production functions are defined over different sets of inputs. This means they cannot be meaningfully used, by themselves, to compare production possibilities, in the sense of how much output a plant gets from a given set of inputs. An alternative is to compare the total cost of production across different levels of output, measured in units of domestic goods, given that the composition of inputs is chosen to minimize total cost when using either technology.

The total (variable) cost of producing y units of output using the non-importing technology with efficiency $z = 1$ in state s^t is:

$$
\begin{aligned}
c_d(y, s^t) \quad = \quad & \min_{d,n \geq 0} d + w(s^t)n \\
& \text{subject to} \\
& A(s^t)d^\alpha n^\theta \geq y
\end{aligned}
$$

The analogue for the importing technology is:

$$
\begin{aligned}
c_m\left(y, s^t\right) \quad = \quad & \min_{d,m,n \geq 0} d + p(s^t)(1+\tau)m + w(s^t)n \\
& \text{subject to} \\
& A(s^t)(\gamma d^\omega m^{1-\omega})^\alpha n^\theta \geq y
\end{aligned}
$$

When minimized, these costs as functions of y are increasing and convex, and satisfy:

$$
c_m(y, s^t) = \frac{c_d(y, s^t)}{\varrho(s^t)} \tag{9}
$$

where $\varrho(s^t) = \left(\frac{\gamma}{\omega^{-\omega}(p(s^t)(1+\tau))^{1-\omega}(1-\omega)^{\omega-1}}\right)^{\alpha/(\alpha+\theta)}$. It follows that if $\varrho(s^t) > 1$, that is, if

$$
\gamma > \omega^{-\omega}(p(s^t)(1+\tau))^{1-\omega}\left(1-\omega\right)^{\omega-1} \tag{10}
$$

then producing with the importing technology is more cost-efficient than producing with the non-importing technology, in the sense that any level of output can be produced at lower cost. Essentially, the inequality (10) states that the gain in efficiency from importing (γ), is greater than the ratio of the unit price paid for intermediate goods if importing to the unit price paid for intermediate goods if only using domestic goods - the former is given by the price index of the composite of imported and domestic goods, $\omega^{-\omega}\left(1-\omega\right)^{\omega-1}\left(p(s^t)(1+\tau)\right)^{1-\omega}$, and the latter is 1.

Under perfect competition, a plant's optimal scale of production sets marginal cost equal to the price of output. Denote these optimal scales $\tilde{y}_d(s^t)$ for the non-importing technology and $\tilde{y}_m(s^t)$ for the importing technology.[10] Plants operating either technology produce the same good, so the price of the output produced using either technology is the same. Therefore, these optimal levels of output must satisfy

$$\frac{\partial c_m}{\partial y}(\tilde{y}_m(s^t), s^t) = \frac{\partial c_d}{\partial y}(\tilde{y}_d(s^t), s^t) \tag{11}$$

Now, (9) holds for all y, and thus, in particular, at the optimal scale with the importing technology, $\tilde{y}_m(s^t)$. If $\varrho(s^t) > 1$, then

$$\begin{aligned} \frac{\partial c_m}{\partial y}(\tilde{y}_m(s^t), s^t) &= \frac{1}{\varrho(s^t)} \frac{\partial c_d}{\partial y}(\tilde{y}_m(s^t), s^t) \\ &< \frac{\partial c_d}{\partial y}(\tilde{y}_m(s^t), s^t) \end{aligned} \tag{12}$$

Since c_d and c_m are convex, $\frac{\partial c_d}{\partial y}$ and $\frac{\partial c_m}{\partial y}$ are increasing. Thus in order for (11) to hold, in light of (12), it must be that

$$\tilde{y}_m(s^t) > \tilde{y}_d(s^t)$$

Therefore, if $\gamma > \omega^{-\omega}(p(s^t)(1+\tau))^{1-\omega}(1-\omega)^{\omega-1}$, so that $\varrho(s^t) > 1$, then any plant produces at a higher scale using the importing technology than with the non-importing technology. In addition, average costs (which are proportional to marginal costs) are equal at the optimal scale using either technology, so profit is higher using the importing technology.[11] The difference in profit from using either technology is one side of the tradeoff considered by an entering plant in choosing its technology. The other side is measured by the sunk costs of either technology incurred at entry.

3.4.4 Entering Plant's Problem

The timing of the decisions facing a plant within the period it enters (and one period before it starts production) is as follows. An entering plant first invests κ_e to receive an efficiency z. The efficiency z is drawn independently for each entrant from a distribution with support $[z_L, \infty)$ and probability density function g. After z is revealed, a plant may decide to shut down and incur no further costs. Alternatively, it may choose to continue with future production using either of the two technologies available; the non-importing technology

[10] All plant level variables with a tilde ($\sim$) and without dependence on z denote the relevant quantity for a plant with $z = 1$.

[11] If $\gamma < \omega^{-\omega}(p(s^t)(1+\tau))^{1-\omega}(1-\omega)^{\omega-1}$, then all the inequalities are reversed, so importers have less cost-efficient production technologies, are smaller in size, and have lower maximized profit than non-importers. This would contradict one fact in the data mentioned in the introduction: importing plants are, on average, larger than non-importing plants.

comes at a cost κ_c, and the importing technology at a cost κ_m. All the sunk costs of production are paid in units of domestic output.

Each plant faces uncertainty over future profits after learning its efficiency z and choosing its production technology, due to the aggregate technology shocks $A(s^t)$ and $A^*(s^t)$. Plants are also subject to a constant exogenous probability δ of exiting the economy. The timing of events is depicted in Figure 2.

Entrants maximize the expected present discounted value of profits from future production, less the sunk costs associated with the entry decisions. Let $V_d(z, s^t)$ denote the expected present discounted value of future profits of a plant that enters at state s^t, to begin production at date $t+1$, using the non-importing technology, with efficiency z. That is,

$$V_d(z, s^t) = \sum_{r=t+1}^{\infty} \sum_{s^r|s^t} P(s^r, s^t)(1-\delta)^{r-t-1}\pi_d(z, s^r)$$

where summation over $s^r|s^t$ refers to summation over states with histories of the form $s^r = (s^t, s_{t+1}, s_{t+2}, \ldots, s_r)$. The static profit $\pi_d(z, s^t)$ is as defined in the static maximizations of the previous section. $P(s^r, s^t)$ denotes the price of output at state s^r in units of output at state s^t, and δ is the probability that a plant dies each period. Plant death occurs at the end of the period, after production, and entering plants cannot die before they start production.

The price at which plants value future profit, $P(s^r, s^t)$ is given by

$$P(s^r, s^t) = Q(s^t, s_{t+1})Q(s^{t+1}, s_{t+2}) \cdots Q(s^{r-1}, s_r)$$

with the Q's defined as in the consumer's problem. Using the consumer's first order condition (4),

$$P(s^r, s^t) = \beta^{r-t}\phi(s^r|s^t)\frac{U_1(s^r)}{U_1(s^t)}$$

That is, plants value profits at future possible states with the consumer's marginal rate of substitution.

Similarly, define $V_m(z, s^t)$ as the expected present value of profits using the importing technology:

$$V_m(z, s^t) = \sum_{r=t+1}^{\infty} \sum_{s^r|s^t} P(s^r, s^t)(1-\delta)^{r-t-1}\pi_m(z, s^r)$$

Now, the plant's decisions at entry can be characterized as follows, working backwards from the technology decision. The expected present discounted value of a plant with efficiency z that has paid the cost of entry κ_e, and has the options to exit or continue with either technology, is

$$V(z,s^t) = \max\left\{0, -\kappa_c + V_d(z,s^t), -\kappa_m + V_m(z,s^t)\right\} \tag{13}$$

Exiting immediately after learning z brings no additional benefits or costs, so the value of exiting is zero.

Potential entrants do not know their efficiency z before payment of the cost κ_e. The expected present discounted value for a potential entrant is then

$$V_e(s^t) = -\kappa_e + \int_{z_L}^{\infty} V(z,s^t)g(z)dz \tag{14}$$

An entrant's decisions are summarized by discrete decision rules determining the choice of an entrant of efficiency z at state s^t. Let $\varepsilon_d(z,s^t)$ record the decision of entrants who continue production using the non-importing technology, and let $\varepsilon_m(z,s^t)$ be the analogue for entrants who use imports. That is,

$$\varepsilon_d(z,s^t) = \begin{cases} 1 \text{ if } V(z,s^t) = -\kappa_c + V_d(z,s^t) \\ 0 \text{ otherwise} \end{cases} \tag{15}$$

$$\varepsilon_m(z,s^t) = \begin{cases} 1 \text{ if } V(z,s^t) = -\kappa_m + V_m(z,s^t) \\ 0 \text{ otherwise} \end{cases} \tag{16}$$

3.4.5 Aggregate Plant Dynamics

The set of plants in the economy at any date is characterized by distributions of efficiencies across plants operating each type of technology. Denote $\mu_d(z,s^{t-1})$ as the density of plants that enter a state (s^{t-1}, s_t) using the non-importing technology, with efficiency z. Similarly, $\mu_m(z,s^{t-1})$ is for importers. The mass of plants that pay the cost of entry κ_e at state s^t is denoted $X(s^t)$.

The evolution of the plant distributions follows:[12]

$$\begin{aligned} \mu_d(z,s^t) &= (1-\delta)\mu_d(z,s^{t-1}) + X(s^t)\varepsilon_d(z,s^t)g(z) \\ \mu_m(z,s^t) &= (1-\delta)\mu_m(z,s^{t-1}) + X(s^t)\varepsilon_m(z,s^t)g(z) \end{aligned} \tag{17}$$

That is, the set of operating plants is determined by previously existing plants that survive into the current period, along with the decisions of new entrants. For example, the mass $X(s^t)g(z)$ of new entrants with efficiency z that choose $\varepsilon_d(z,s^t) = 1$ enter the mass

[12] μ_d and μ_m are not necessarily *probability* distributions, because they are not normalized by the total mass of non-importing and importing plants, respectively.

$\mu_d(z,s^t)$ in a manner identical to any surviving plant in $\mu_d(z,s^{t-1})$. The dependence of the distributions μ on s^{t-1} emphasizes that the set of plants in the economy at any state s^t depends only on events prior to the current period. Current decisions of new entrants affect the set of plants operating in the next period.

3.4.6 Aggregate Feasibility

Feasibility in the goods markets requires that the sum of demands for final and intermediate consumption, plus total goods required for investment by new plants, equal the total output produced by all plants. Plant input demands and output supplies are defined by (5) and (7) and aggregated using the distributions defined by (17). The total amount of goods required for $X(s^t)$ entrants is determined by the decisions in (15) and (16).

In the home country,

$$\begin{aligned} & C(s^t) + X(s^t)\left(\kappa_e + \kappa_c \int \varepsilon_d(z,s^t)g(z)dz + \kappa_m \int \varepsilon_m(z,s^t)g(z)dz\right) \\ & + \int d_d(z,s^t)\mu_d(z,s^{t-1})dz + \int d_m(z,s^t)\mu_m(z,s^{t-1})dz + \int m^*(z,s^t)\mu_m^*(z,s^{t-1})dz \\ = & \int y_d(z,s^t)\mu_d(z,s^{t-1})dz + \int y_m(z,s^t)\mu_m(z,s^{t-1})dz \end{aligned} \tag{18}$$

In addition, plant demands for labor must sum to total domestic labor supply:

$$\int n_d(z,s^t)\mu_d(z,s^{t-1})dz + \int n_m(z,s^t)\mu_m(z,s^{t-1})dz = N(s^t) \tag{19}$$

The rebates of profits and tariff revenue in the consumer's budget constraint (3) are defined by

$$\begin{aligned} \Pi(s^t) &= \int \pi_d(z,s^t)\mu_d(z,s^{t-1})dz + \int \pi_m(z,s^t)\mu_m(z,s^{t-1})dz \\ &\quad -X(s^t)\left(\kappa_e + \kappa_c \int \varepsilon_d(z,s^t)g(z)dz + \kappa_m \int \varepsilon_m(z,s^t)g(z)dz\right) \end{aligned} \tag{20}$$

$$T(s^t) = \tau p(s^t) \int m(z,s^t)\mu_m(z,s^{t-1})dz \tag{21}$$

Analogues of conditions (18) through (21) hold for the foreign country.

The international asset market clearing condition is

$$B(s^t,s_{t+1}) + B^*(s^t,s_{t+1}) = 0 \tag{22}$$

3.5 Equilibrium

An equilibrium for this economy consists of state-contingent sequences of prices, allocations of goods and labor, decisions of entering plants, and distributions over efficiency levels of existing plants that solve consumers' and plants' problems and satisfy the home country and froeign country versions of the laws of motion (17) and feasibility conditions (18) through (21), as well as the international asset market clearing condition (22). In addition, the mass of entrants $X(s^t)$ must be such that

$$V_e(s^t) \leq 0, \;=\; \text{if } X(s^t) > 0$$

with $V_e(s^t)$ defined in (14).

3.6 Characterization of Equilibrium

As presented here, an equilibrium of this economy is a complicated by two things: (1) the discrete decision rules for plant technology choices at entry ε_d and ε_m; and (2) the distributions μ as equilibrium objects. The first issue can be resolved by restricting attention to equilibrium paths that satisfy a certain monotonicity condition on the difference in profits between importers and non-importers. The second issue is resolved through an explicit aggregation of plant distributions into moments relevant for the equilibrium feasibility conditions (18) through (21). Each of these issues are discussed in turn.

3.6.1 Plant Entry Decisions

The decision of a plant at entry involves comparing the value of the two expected discounted infinite sums in the definitions of V_d and V_m in the plant dynamic decisions. In general, it is not straightforward to determine which of these is larger for any given plant. The expected static profit difference between importing and not, discussed above, depends on future values of the endogenous price p.

To resolve this, I restrict attention to equilibrium paths that satisfy the following condition:

$$\gamma > \omega^{-\omega}(p(s^t)(1+\tau))^{1-\omega}\,(1-\omega)^{\omega-1} \;\text{ for all } s^t$$

This is not an assumption on parameters of the economy, since it involves the equilibrium price p, the relative price of foreign to home output. Rather, I compute an equilibrium path under the conjecture that this condition always holds for a given set of parameters, and then check that it does in fact hold in equilibrium, verifying the conjecture.

The reason for imposing this condition is that analysis of the plant's technology choice at entry can then be characterized by a simple rule that depends on the current state. If $\gamma > \omega^{-\omega}(p(s^r)(1+\tau))^{1-\omega}(1-\omega)^{\omega-1}$ for all s^r following s^t, then a plant entering at s^t expects to make higher profit every period it operates if it chooses the importing technology over the non-importing technology. The difference in profit is

$$\pi_m(z, s^r) - \pi_d(z, s^r) = (1-\alpha-\theta)(h_m(s^r)^{1/(1-\alpha-\theta)} - h_d(s^r)^{1/(1-\alpha-\theta)})z$$

If $\gamma > \omega^{-\omega}(p(s^r)(1+\tau))^{1-\omega}(1-\omega)^{\omega-1}$, then, from (6) and (8), the difference in profit, $\pi_m(z, s^r) - \pi_d(z, s^r)$, is increasing in z. Under the conjecture that $\gamma > \omega^{-\omega}(p(s^r)(1+\tau))^{1-\omega}(1-\omega)^{\omega-1}$ for all s^t, the difference in the present values $V_m(z, s^t) - V_d(z, s^t)$ is also increasing in z, and therefore is high enough to cover the additional sunk cost κ_m over κ_c only if z is large enough. Similar reasoning shows that $V_d(z, s^t)$ is high enough to cover the first sunk cost κ_c only for sufficiently large z as well, though for a lower range of z than for the importing decision.

Therefore, a plant's decision at entry in state s^t is characterized by two cutoff levels of its efficiency draw, denoted $\hat{z}_d(s^t)$ and $\hat{z}_m(s^t)$, with $\hat{z}_d(s^t) < \hat{z}_m(s^t)$. If a plant draws a $z \in [\hat{z}_d(s^t), \hat{z}_m(s^t)]$, it produces with the non-importing technology; if $z > \hat{z}_m(s^t)$, the plant uses the importing technology; and if $z < \hat{z}_d(s^t)$, the plant chooses not to continue producing. These cutoff rules are depicted in Figure 3. Across the mass of plants entering in a given period, efficiency levels z are distributed according to the fixed density g, and potential entrants along this distribution are partitioned into importers, non-importers, and exiting plants that shut down before production.

The decision rules ε_d and ε_m in (15) and (16) are replaced by

$$\varepsilon_d(z, s^t) = \begin{cases} 1 \text{ if } z \in [\hat{z}_d(s^t), \hat{z}_m(s^t)] \\ 0 \text{ otherwise} \end{cases}$$

$$\varepsilon_m(z, s^t) = \begin{cases} 1 \text{ if } z > \hat{z}_m(s^t) \\ 0 \text{ otherwise} \end{cases}$$

Therefore, an equilibrium of this economy displays two selection effects: only relatively efficient plants (those with $z \geq \hat{z}_d(s^t)$) continue beyond entry. Furthermore, only the most inherently efficient plants, those with $z > \hat{z}_m(s^t) > \hat{z}_d(s^t)$, will be profitable enough to afford the technology that uses imported intermediate inputs. These effects of sunk costs of production and importing are similar to the selection effects in Melitz (2003), in a model with sunk costs of production and exporting.

3.6.2 Aggregation

The endogenous state-dependent distributions $\mu_d(z, s^{t-1})$ and $\mu_m(z, s^{t-1})$ over plant efficiency can be aggregated into moments that summarize the information necessary for determining aggregate equilibrium quantities. Because the production technologies are homogeneous in efficiency z, different plants operating the same type of technology (e.g., non-importing) with different efficiencies choose inputs and outputs that are proportional to each other. So, for example, the labor demand of a non-importing plant of efficiency z at state s^t satisfies:

$$n_d(z, s^t) = \tilde{n}_d(s^t)z$$

where $\tilde{n}_d(s^t) = n_d(1, s^t)$ (the labor demand of a non-importing plant with $z = 1$) is a function of equilibrium prices, defined by (5). The aggregate feasibility condition (19) for labor at state s^t, can then be written

$$N(s^t) = \tilde{n}_d(s^t)Z_d(s^{t-1}) + \tilde{n}_m(s^t)Z_m(s^{t-1}) \tag{23}$$

where Z_d and Z_m are the the following aggregates of the distributions μ_d and μ_m.

$$\begin{aligned} Z_d(s^{t-1}) &= \int z\mu_d(z, s^{t-1})dz \\ Z_m(s^{t-1}) &= \int z\mu_m(z, s^{t-1})dz \end{aligned}$$

Using these aggregate variables in addition to the cutoff rules $\hat{z}_d(s^t)$ and $\hat{z}_m(s^t)$ for entrants, the (home) goods market clearing condition can be written[13]:

$$\begin{aligned} & C(s^t) + \tilde{d}_d(s^t)Z_d(s^{t-1}) + \tilde{d}_m(s^t)Z_m(s^{t-1}) + \tilde{m}^*(s^t)Z_m^*(s^{t-1}) \\ & +X(s^t)\left(\kappa_e + \kappa_c \int_{\hat{z}_d(s^t)}^{\infty} g(z)dz + \kappa_m \int_{\hat{z}_m(s^t)}^{\infty} g(z)dz\right) \\ = \; & \tilde{y}_d(s^t)Z_d(s^{t-1}) + \tilde{y}_m(s^t)Z_m(s^{t-1}) \end{aligned}$$

In order to replace the distributions μ in summarizing the distributions of plants in the economy with the aggregates Z, the endogenous laws of motion (17) must also be replaced. This is done using the plant entry cutoff rules again. The aggregated laws of motion are found by mutiplying (17) by z for each z, and integrating over the ranges defined by the

[13]All variables with a tilde ($\tilde{}$) and no dependence on z are defined analogously to $\tilde{n}_d(s^t)$ above.

entry cutoff rules:

$$Z_d(s^t) = (1-\delta)Z_d(s^{t-1}) + X(s^t)\int_{\hat{z}_d(s^t)}^{\hat{z}_m(s^t)} zg(z)dz \tag{24}$$
$$Z_m(s^t) = (1-\delta)Z_m(s^{t-1}) + X(s^t)\int_{\hat{z}_m(s^t)}^{\infty} zg(z)dz$$

As with the original distributions μ, the aggregates Z at date t depend only on events up to period $t-1$, included in s^{t-1}. The aggregates evolve through the death of plants and the decisions made by new entrants.

With the plant distributions thus aggregated, solving for the aggregate variables in an equilibrium reduces to solving an aggregated maximization problem with endogenous state variables Z_d, Z_m, Z_d^*, Z_m^*. The details are in the appendix. The aggregation of plant decisions as in (23) is similar to the characterization in Melitz (2003) and Ghironi and Melitz (2005). Replacing the dynamics of the distributions μ with aggregated state variables is related to the method used by Atkeson and Kehoe (1999) to solve a model with "putty-clay" capital embodying an irreversibility similar to that considered here.

3.7 Steady state and comparative statics

In the next section I quantitatively evaluate the model's implications for changes in a country's aggregate trade flows in response to two types of movements in the relative price of imported to domestic goods. The first type are cyclical changes in $p(s^t)$ due to exogenous fluctuations in $A(s^t)$ and $A^*(s^t)$. The second type are exogenous permanent changes in trade policy, as measured by the tariff rate τ.

In this subsection I first analyze the effects of a change in the tariff τ on a symmetric steady state of the economy: an equilibrium without fluctuations in A and A^* in which all aggregate variables are constant over time. All previously defined equilibrium variables without dependence on s^t refer to steady state values. The equilibrium value of p in a symmetric steady state is 1.

Although equilibrium aggregates are constant, there is continual turnover of plants in each country, as new entrants replace dying plants. The equilibrium plant efficiency distributions μ_d and μ_m (and efficiency aggregates Z_d and Z_m) are constant, but depend on the exogenous policy τ.

Therefore, a change in τ has three effects on aggregate trade flows, two that are static and one that is dynamic. The first static effect is on the allocation of resources (labor and intermediate inputs) across existing importing and non-importing plants in any period: a reduction in tariffs reallocates resources to importing plants. The second static effect is on the ratio of imported relative to domestic intermediate inputs used within each importing plant: when imports become cheaper, importing plants use relatively more imports. The dynamic effect is on the investment decisions of new plants: a tariff reduction causes more entering plants to pay the sunk cost of importing, and causes fewer plants to continue producing at all.

These effects can be seen in the steady state ratio of aggregate imports relative to aggregate purchases of domestic intermediate goods, which is:

$$\frac{M}{D} = \frac{\int m(z)\mu_m(z)dz}{\int d_d(z)\mu_d(z)dz + \int d_m(z)\mu_m(z)dz}$$

Using the homogeneity of plant decisions in z from (5) and (7), with the definition of the aggregates Z_d and Z_m in (24),

$$\begin{aligned} \frac{M}{D} &= \frac{\tilde{m}Z_m}{\tilde{d}_d Z_d + \tilde{d}_m Z_m} \\ &= \frac{\tilde{m}}{\tilde{d}_m}\left(\frac{\tilde{d}_d}{\tilde{d}_m}\frac{Z_d}{Z_m} + 1\right)^{-1} \end{aligned} \tag{25}$$

The three effects of a drop in tariffs can be seen in the ratios $\tilde{m}/\tilde{d}_m$, $\tilde{d}_d/\tilde{d}_m$, Z_d/Z_m.

First, at importing plants, $m(z) = \frac{1-\omega}{\omega(1+\tau)}d_m(z)$, so $\tilde{m}/\tilde{d}_m = \frac{1-\omega}{\omega(1+\tau)}$: a lower tariff rate, τ, increases the ratio of imported to domestic inputs used at importing plants.

Second, using the input demand functions in (5) and (7), the ratio $\tilde{d}_d/\tilde{d}_m$ is:

$$\frac{\tilde{d}_d}{\tilde{d}_m} = \left(\frac{\omega^{-\omega}(1+\tau)^{1-\omega}\,(1-\omega)^{\omega-1}}{\gamma}\right)^{\alpha/(1-\alpha-\theta)}$$

This is increasing in τ. Therefore, a decrease in τ causes less inputs to be allocated to non-importing plants relative to importing plants, as measured by the ratio $\tilde{d}_d/\tilde{d}_m$.

Finally, The dynamic effect of a drop in τ works on the ratio M/D through the ratio of efficiency aggregates Z_d/Z_m. Evaluating the laws of motion (24) at a steady state give $\delta Z_d = X\int_{\hat{z}_d}^{\hat{z}_m} zg(z)dz$ and $\delta Z_m = X\int_{\hat{z}_m}^{\infty} zg(z)dz$, so the ratio is:

$$\frac{Z_d}{Z_m} = \frac{\int_{\hat{z}_d}^{\hat{z}_m} zg(z)dz}{\int_{\hat{z}_m}^{\infty} zg(z)dz}$$

I argue that the equilibrium value of this ratio decreases with a decrease in the tariff τ.

The cutoffs $\hat{z}_d$ and $\hat{z}_m$ are defined by the solutions to the steady state versions of entering plants' dynamic decision problems. The steady state versions of an entering plant's present discounted value of profits (from not importing and importing) are:

$$V_d(z) = \frac{\beta}{1-\beta(1-\delta)}\pi_d(z)$$

$$V_m(z) = \frac{\beta}{1-\beta(1-\delta)}\pi_m(z)$$

where β is the consumer's discount factor and δ is the plant's probability of death. The cutoffs $\hat{z}_d$ and $\hat{z}_m$ solve the maximization in (13), and therefore satisfy:

$$\frac{\beta}{1-\beta(1-\delta)}\pi_d(\hat{z}_d) = \kappa_c$$

and

$$\frac{\beta}{1-\beta(1-\delta)}\left(\pi_m(\hat{z}_m) - \pi_d(\hat{z}_m)\right) = \kappa_m$$

A plant with the cutoff efficiency level for each decision makes zero additional profit above the cost of the decision (the continuing cost κ_c for $\hat{z}_d$ and the importing cost κ_m for $\hat{z}_m$).

A decrease in τ raises the difference $\pi_m(z) - \pi_d(z)$ for any z. Since this difference is increasing as a function of z, $\hat{z}_m$ decreases, and thus more entering plants import. In addition, the equilibrium effect on $\hat{z}_d$ will typically be that, since a higher fraction of plants import, and importers hire more labor than non-importers, the equilibrium wage w increases so that fewer potential non-importing entrants are profitable enough to continue, and $\hat{z}_d$ increases.

Therefore, the integral $\int_{\hat{z}_d}^{\hat{z}_m} zg(z)dz$ decreases, and $\int_{\hat{z}_m}^{\infty} zg(z)dz$ increases, so Z_d/Z_m decreases. The dynamic effect of a tariff reduction is to increase the aggregate ratio M/D through a reduction in the mass (and aggregate efficiency, which determines aggregate intermediate demands) of non-importing plants relative to importing plants.

In the following sections, I show that these two effects interact in different ways to determine the dynamics of trade flows in response to aggregate fluctuations and in response to trade reform. Short-run fluctuations in the relative price of imports to domestic goods cause short-run fluctuations in the import/domestic ratio mainly through the static effects within and between existing plants - changes in the ratios $\tilde{m}/\tilde{d}_m$ and $\tilde{d}_d/\tilde{d}_m$ in (25). Trade liberalization increases trade through both the static effects and the dynamic effect of more new plants importing - a change in Z_d/Z_m. The latter effect is larger, and occurs gradually.

4 Quantitative Analysis

4.1 Parameter Values

I choose parameter values so that the steady state of the model under a tariff rate of 10% matches several aggregate statistics as well as key facts on plant-level importing behavior. The calibration is summarized in Table 4.

A model period corresponds to one quarter of a year. The discount factor β is set to 0.99, which implies an annual real interest rate of about 4%. The utility function is

$$U(C, 1-N) = \frac{\left(C^{\zeta}(1-N)^{\zeta}\right)^{1-\nu}}{1-\nu}$$

The parameter ζ is set to 0.34, implying that the steady state fraction of time supplied as labor, N, is 30%. The parameter ν is set to 2, a standard value in international real business cycle models (as in, for example, Backus, Kehoe and Kydland (1995)).

I set $\delta = 0.02$ based on interpreting plants as the model economy's capital stock. An accounting measure of capital in the model would cumulate investment expenditures in new plants to form a capital stock. Investment expenditures are

$$I(s^t) \equiv X(s^t)\left(\kappa_e + \kappa_c \int_{\hat{z}_d(s^t)}^{\infty} g(z)dz + \kappa_m \int_{\hat{z}_m(s^t)}^{\infty} g(z)dz\right)$$

$X(s^t)$ represents new plants entering at date s^t, a fraction δ of which will die at the end of period $t+1$. Therefore, additions to the capital stock in the form of investment expenditures I depreciate at the rate δ.

The parameters of the plant production functions that are common between non-importing plants and importing plants are α, the share of output spent on intermediate inputs, and θ, the share of output spent on labor compensation. I set $\alpha = 0.5$ and $\theta = 0.33$, so that expenditure on intermediates is the same fraction of gross output as is value added (gross output less intermediates), and labor compensation is two-thirds of value added.

In a steady state with $p = 1$, every importing plant spends a fraction $1 - \omega$ of total intermediate expenditures on imports. In US manufacturing plant data, Kurz (2006) reports an average across importing plants of 0.20 for this fraction. Kasahara and Lapham (2007), in Chilean manufacturing plant data, find an average of 0.29. Amiti and Konings (2007) find an even higher ratio of 0.46 for importing plants in Indonesia, and Halpern, Koren and Szeidl (2006) find variation in this ratio between 0.1 and 0.5 in importing Hungarian firms. I set $\omega = 0.8$ so that this fraction equals 20% for all importing plants.

The remaining parameters affect plant heterogeneity and the differences between importing plants and non-importing plants.

The parameter γ determines the advantage of using the importing technology. Several studies have attempted to measure the implicit within-plant output gain of importing intermediate inputs, given the total volume of inputs and controlling for other aspects of plant heterogeneity. The results are mixed. Kasahara and Rodrigue (2007) suggest that this gain is between 2 and 20%. Halpern, Koren and Szeidl (2006) estimate that an increase of 0.1 in a plant's import share of intermediates has a significantly positive effect on output on the order of $1-2\%$. Muendler (2004), however, reports no significant effect of importing on plant output among manufacturing plants in Brazil.

These three studies all use plant-level panel data to estimate a production function relating plant output to inputs (of labor, capital, and materials), augmented with a term relating to a plant's use of imported intermediate inputs. In the appendix, I construct a production function in logs, relating output to labor, total material expenditures, and a dummy variable indicating whether a plant is importing or not, for all plants. The coefficient multiplying this variable, which corresponds to the factor estimated by Kasahara and Rodrigue (2007) is:

$$\alpha \log\left(\frac{\gamma}{\omega^{-\omega}\left(1+\tau\right)^{1-\omega}\left(1-\omega\right)^{\omega-1}}\right)$$

I choose γ so that this factor is equal to 0.05. That is, any plant can produce 5% more output, given labor and total expenditures on intermediate inputs, every period (at the steady state) if it chooses the importing technology rather than the non-importing technology.

I choose the distribution over plant efficiency draws at entry to be Pareto, with probability density

$$g(z) = k(z_L)^k z^{-k-1}$$

The lower bound z_L is a normalization, so I set it equal to 3. The values of the sunk costs of entry, κ_e and continuing production, κ_c are also normalizations in that their sizes matter only relative to the sunk cost of importing, κ_m.

The cost κ_m and the shape parameter k in the distribution determine the fraction of plants in the steady state that import, and the average size difference between importers and non-importers. I turn again to the plant-level studies for these statistics. As reported in Table 2, about 24% of Chilean and US manufacturing plants import intermediate inputs. In Chile, these plants are over three times the size of their non-importing counterparts, and in the US they are about twice the size of non-importers. I choose the two parameters k and κ_m so that 24% of plants import and importers, on average, are 2.3 times the size of non-importers.

When simulating business cycle fluctuations, the aggregate shocks follow AR(1) processes in logs,

$$\begin{aligned}\log A(s^{t+1}) &= \rho \log A(s^t) + \varepsilon(s_{t+1}) \\ \log A^*(s^{t+1}) &= \rho \log A^*(s^t) + \varepsilon^*(s_{t+1})\end{aligned}$$

with $\rho = 0.90$ and $[\varepsilon, \varepsilon^*]$ jointly normally distributed with mean 0, standard deviation 0.005, and cross-correlation 0.25.

4.2 Aggregate fluctuations

In this section, I assess the model's predictions for fluctuations in the volume and balance of trade over the business cycle, and report standard business cycle statistics. First, I measure the degree to which, at the aggregate level, a country substitutes between purchases of imported and domestic goods when their relative price changes. Aggregate quantities of imported and domestic intermediate goods used in the home country at date t, denoted M_t and D_t are:

$$M_t = \int m_t(z)\mu_{mt}(z)dz$$

$$D_t = \int d_{dt}(z)\mu_{dt}(z)dz + \int d_{mt}(z)\mu_{mt}(z)dz$$

As in Ruhl (2008), I estimate the elasticity of substitution between imports and domestic intermediate goods - that is, the *Armington elasticity* - from model-generated time series of M_t, D_t, and the price p_t. To do this, I follow empirical studies such as Reinert and Roland-Holst (1992), who estimate this elasticity in US data, and estimate the following equation by least-squares regression:[14]

$$\log\left(\frac{M_t}{D_t}\right) = -\sigma \log(p_t) + b \tag{26}$$

The estimate of σ gives the percentage increase in the aggregate ratio M_t/D_t predicted by a one percent decrease in the price p_t. The model's time series give an estimate of σ equal to 1.96. At the aggregate level, a one percent decrease in the price of imports leads, on average, to a 1.96 percent increase in the quantity of imported interemediate goods relative

[14] In these studies, the equation is derived from the decision problem of a consumer with CES preferences over aggregate imports and domestic goods. Maximizing utility

$$U(M_t, D_t) = (\varpi D_t^{(\sigma-1)/\sigma} + (1-\varpi)M_t^{(\sigma-1)/\sigma})^{\sigma/(\sigma-1)}$$

subject to the budget constraint $D_t + p_t(1+\tau)M_t \leq E$ for any expenditure E, gives (26) as the first order condition for the optimal M_t/D_t ratio, with the constant b depending on ϖ and τ

to domestic intermediate goods consumed. Ruhl (2008) finds that a broad set of empirical estimates of this elasticity are in the range of about 0.2 to 3. Therefore, the model generates aggregate substitution between imported and domestic goods in line with empirical estimates.

At the model's micro level, the plant-specific ratio of imported to domestic intermediate goods is either *zero* if a plant is not an importer, or equal to $\frac{1-\omega}{\omega p_t(1+\tau)}$, if a plant is an importer. The import/domestic ratio for each importing plant responds proportionally to price changes for each plant; that is, the plant-level elasticity of substitution is equal to one. At the aggregate level, the model displays greater fluctuations in the imported-domestic goods ratio in response to price movements through the mechanisms discussed in the comparative statics exercise. Specifically, a decline in the price of imports relative to domestic goods leads existing importing plants to import more relative to their domestic inputs, and to expand in size relative to non-importing plants. In addition, the expected persistence of a price decrease leads more of the new plants entering to become importers.

Table 5 decomposes the model's aggregate fluctuations in imports using the decomposition performed earlier on the plant-level data, as detailed in equation (1). Roughly, the components of the decomposition can be matched up with pieces of the comparative statics discussion above as follows: the "within" margin corresponds to the effects of changes in $m/(d+m)$, the plant-level import ratio; the "between" margin corresponds to the effect of changes in $\tilde{d}_m/\tilde{d}_d$, the average size of importing plants relative to non-importing plants; and the "entry" margin correponds to the effect of changes in Z_m/Z_d, measuring the ratio of importing to non-importing plants in the economy. The figures in Table 5 show that essentially all of the cyclical fluctuations in imports is attributed to the "within" and "between" margins. When compared to the decomposition done on the Chilean plant-level data, the model correctly predicts that almost all of the aggregate fluctuations in imports is accounted for by the "within" and "between" margins, and that the within-plant adjustment accounts for more of the aggregate movements than the between-plant reallocation. The fraction of aggregate fluctuations in imports accounted for by the between-plant reallocation margin is, however, much higher in the model than in the data.

Figure 4 presents the dynamic responses in the aggregate ratio M_t/D_t, and the three components $\tilde{m}_t/\tilde{d}_{mt}$, $\tilde{d}_{mt}/\tilde{d}_{dt}$, and Z_{mt}/Z_{dt} following a single, one-standard-deviation shock to aggregate technology in the foreign country. The relative price of imports for the home country falls. On impact, all the growth in aggregate imports relative to domestic intermediate consumption is due to the changes in $\tilde{m}_t/\tilde{d}_{mt}$ and $\tilde{d}_{mt}/\tilde{d}_{dt}$, the static within- and between-plant effects. Over time, there is a large, persistent change in the set of importing relative to non-importing plants in the economy, as measured by Z_{mt}/Z_{dt}. This large change is reflected in the time path of aggregate imports relative to domestic intermediates,

M_t/D_t. Although this growth in Z_{mt}/Z_{dt} has the potential to be very large, it does not play a larger part than changes in $\tilde{d}_{mt}/\tilde{d}_{dt}$ in accounting for more of the time-series fluctuations in M_t/D_t because the growth does not have time to fully unfold when the economy is subject to recurrent fluctuations that tend to drive the relative price p_t back to its steady state value.

Table 6 presents business cycle statistics for the model economy and for a variation (labeled CES in the table) in which the plant-level importing decision is not present. In this variation, the sunk cost for using the importing is the same as for not importing ($\kappa_m = \kappa_c$), so all producing plants import. However, in order to make this comparable to the original model, I replace the production technology for all plants with one that features a constant elasticity of substitution between imported and domestic intermediate goods. Plants still differ by the efficiency z drawn at entry, but any plant with efficiency z produces according to the CES techology:

$$y = A(s^t)z(\upsilon d^{(\eta-1)/\eta} + (1-\upsilon)m^{(\eta-1)/\eta})^{\eta/(\eta-1)}n^{\theta}$$

The elasticity of substitution η is set equal to the estimated elasticity σ from the original model, 1.96, and the parameters υ and the sunk investment cost of production κ_c are re-calibrated so that equilibrium aggregates in the steady state are the same as in the original model. All other parameters are as in Table 4.

The statistics in Table 6 show that, in response to fluctuations at business cycle frequency, the model's aggregate predictions are extremely similar to one in which the technology for combining domestic and imported intermediate goods simply assumes substitability at the rate estimated in the original model. One exception is that investment is slightly more volatile and less correlated across countries in the original model than in the model with CES technology. This is because in the model with all plants importing, there is one less source of variability in investment (the sunk cost to import). The relative price p is slightly less volatile and more persistent in the original model, and the trade balance, measured as the ratio of net exports to GDP, is more volatile and more persistent, than in the CES model. These differences, however, are small. In addition, these predictions are generally very close to those of standard international real business cycle models with complete asset markets, as in, for example, Backus, Kehoe and Kydland (1995).

A final remark is that the conjecture that allowed a simple characterization of equilibrium plant entry decisions can be (approximately) verified from the model's time series. Recall that, if the model's equilibrium price of foreign country goods relative to home country goods, $p(s^t)$, satisfies the inequality

$$\gamma > \omega^{-\omega}(p(s^t)(1+\tau))^{1-\omega}(1-\omega)^{\omega-1} \tag{27}$$

for all s^t, then the plant decision at entry is characterized in terms of two cutoffs, $\hat{z}_d(s^t)$ and $\hat{z}_m(s^t)$, of idiosyncratic efficiency z. The value of γ required for importers to be 5% more productive than non-importers is equal to 1.8583. The term $\omega^{-\omega}(p(s^t)(1+\tau))^{1-\omega}(1-\omega)^{\omega-1}$ is equal to 1.7675 when $p(s^t) = 1$, its steady state value. With these parameters, the value of p would have to reach about 1.65 for the inequality (27) to be reversed. With the AR(1) shocks assumed here, there is no explicit bound that can be placed on the equilibrium value of $p(s^t)$, but an argument can be made that extreme values are sufficiently improbable. The maximum of the standard deviation of the price p across 1000 simulations is 3.83%. With this volatility, the price p required to violate the inequality (27) is about 17 standard deviations above the steady state value of 1. For the purposes of plants' evaluation of their expected profits V_d and V_m, the probability of such an extreme deviation from the steady state price is effectively zero.[15]

4.3 Dynamics of trade reform

I now consider the model's dynamic response to a sudden, permanent reduction in the import tariff, from 10% to 0%, when the aggregate technology shocks are constant at their mean values of 1.[16] In response to a one-time change in the price of imported relative to domestic intermediate goods in the form of a tariff reduction, the trade dynamics suggested in Figure 4 gradually develop, and there is a large increase in the volume of trade.[17]

Figure 5 displays the same trade variables as Figure 4, for the first five years following the trade liberalization. The variables are, again, the ratio of aggregate imported to domestic intermediate goods, M_t/D_t; the ratio of imported to domestic inputs used by importing plants, $\tilde{m}_t/\tilde{d}_{mt}$; the ratio of goods allocated to importing relative to non-importing plants, $\tilde{d}_{mt}/\tilde{d}_{dt}$; and the ratio of aggregate efficiency of importing plants relative to nonimporting plants, Z_{mt}/Z_{dt}. These ratios display similar dynamic patterns as in Figure 4, except that they do not eventually revert back to the original steady state. Both the static ratio of

[15] A similar argument is used by Atkeson and Kehoe (1999). However, their argument is regarding a price with an exogenous stochastic structure, and therefore applies to properties of a known distribution.

[16] I compute the equilibrium path assuming that the model reaches its new steady state 100 years after the tariff reduction. This time horizon is long enough that increasing it does not significantly affect the results.

[17] This experiment is concerned with the gradual effects of a *one-time policy change*. Some previous work on the dynamic effects of trade liberalization, including Kouparitsas (1997) and Albuquerque and Rebelo (2000), studied the timing of *gradual policy changes*.

imports to deomstic inputs used by importers, $\tilde{m}_t/\tilde{d}_{mt}$, and the allocation of goods across plants measured by $\tilde{d}_{mt}/\tilde{d}_{dt}$, adjust to their new steady state levels immediately, and this adjustment drives all of the growth in trade in the period immediately following the tariff reduction. Over time, the gradual change in the number of plants importing relative to those not importing, measured by Z_{mt}/Z_{dt}, accounts for the large, gradual growth in the ratio M_t/D_t.

Figures 6 and **??** present the dynamics of other aggregate variables along the transition following the trade reform. Figure 6 displays GDP and its aggregate expenditure components, consumption and investment. There is a large increase in investment, as a larger proportion of new plants invest in the importing technology. Part of this increase in investment is financed by an initial reduction in consumption. GDP also increases, so that the drop in consumption is small, and consumption begins to increase relative to the original steady state after only about one year.

The growth in GDP is further decomposed in Figure **??** into changes in aggregate labor input N_t and GDP per unit of labor input, or labor productivity. In the first few periods following trade liberalization, labor increases more than GDP, so labor productivity actually falls, and only begins to grow after about three years.

Table 7 presents detailed measures of the magnitude and speed of the transition following trade liberalization. The first panel shows, for the trade variables and macroeconomic aggregates depicted in Figures 5-**??**, growth rates across steady states, and growth rates one and ten years after the tariff reduction. Both the ratios of imports to GDP and imports to domestic intermediate goods reach about half their growth within ten years. The portion of this growth due to the static allocation of resources across importing and non-importing plants is small, and is exhausted immediately. Growth in the set of new importing plants is very large, and only about one third completed after ten years. Consumption and labor productivity initially fall and then rise in the long-run, mirrored by initial increases in labor and investment higher than their respective long-run increases.

The second part of Table 7 again relates to Ruhl (2008), in calculating the model's implied elasticity of substitution at three different horizons following trade liberalization. At each time $t = 1$, 10, and ∞, where ∞ denotes the new free-trade steady state, the elasticity is calculated as the percentage increase in the ratio M_t/D_t relative to the original steady state, divided by the change in the relative price, reflected in the tariff reduction. That is,

$$\sigma = \frac{\left(\frac{M_t/D_t}{M/D} - 1\right)}{\left(\frac{1}{1+\tau} - 1\right)}$$

where M/D is the original steady state ratio.

After one year, the growth in trade implies an elasticity of about 2.1, which is similar to that estimated in response to business cycle fluctuations. After 10 years, the measured elasticity is about 6, and across steady states, the implied elasticity is nearly 10.

Finally, the gradual adjustment in aggregate quantities following trade liberalization suggests that there could be significant consequences for the welfare gains from trade reform. In particular, as shown in Figures 6 and **??**, the initial response of the economy features a *decrease* in consumption with an *increase* in time spent working, with only a gradual increase in consumption. The welfare consequences of this can be assessed by comparing two measures of welfare gains from the trade reform.[18] The first measure compares lifetime utility across steady states, by calculating the percentage increase in the original steady state's consumption needed to attain the level of lifetime utility at the new steady state. This is the factor λ_1 that solves:

$$U(\lambda_1 C, 1-N) = U(\bar{C}, 1-\bar{N})$$

where C and N are consumption and labor supply in the original steady state, and $\bar{C}$ and $\bar{N}$ are for the free-trade steady state. The second measure of welfare gains computes an analogous consumption-variation measure, comparing lifetime utility the initial steady state to utility over the entire transition to the new steady state. That is, the second measure is the factor λ_2 that solves:

$$U(\lambda_2 C, 1-N) = \sum_{t=0}^{\infty} \beta^t U(C_t, 1-N_t)$$

where C_t and N_t are consumption and labor supply t periods following the trade liberalization.

The final panel of Table 4 shows the two measures λ_1 and λ_2. Although consumption in Figure 3 initially declines, its subsequent growth is large enough that the present value of discounted utility along the transition is larger than in the initial steady state: the consumption variation required in the initial steady state, given by $100 \times (\lambda_2 - 1)$, is 0.28%. However, this is substantially lower than the analogous measure implied by λ_1, 0.72%. The initial decline and slow growth of consumption following trade liberalization therefore have significant consequences for the welfare gains of trade policy reform.

[18] These calculations are similar to those in Kouparitsas (1997).

5 Conclusion

This paper has constructed a model of international trade in intermediate inputs used by heterogeneous plants. The model features a technological advantage for plants that use imported goods, but plants must make a costly, irreversible decision to do so. As a result, only more inherently efficient plants choose to import their intermediates.

The model is parametrized to match several features of plant-level importing behavior. When the model is subject to short-run fluctuations driven by aggregate technology shocks, it generates low volatility of trade flows. A low degree of aggregate substitution between imports and domestic goods in the short-run is acheived through shifts in the allocation of resources within and across across importing and non-importing plants.

In response to a sudden, permanent trade liberalization, the set of plants in the economy gradually changes. A higher proportion of new plants import intermediates. Existing plants cannot change their production technologies, but gradually die out. Over a very long time horizon, imports double as a fraction of GDP in response to the one-time removal of a 10% tariff; however, along the transition path, only about half of this increase is attained within 10 years. The welfare gain calculated from the transition following trade liberalization is significantly lower than that computed from comparing steady states.

The model provides a framework for analyzing the dynamic effects of trade policy through changes in producer-level importing decisions. With irreversibility in these decisions, changes in trade policy have both static and dynamic effects on the allocation of resources across plants that import and plants that do not. These contribute to very large effects on trade flows that occur gradually over time.

The model here has focused on the plant-level decision to import, motivated by recent emprical evidence of the importance of this decision. A large body of evidence exists as well for the importance of the plant-level exporting decision, and a useful extension would be to integrate the dynamic plant-level importing decisions introduced here with the exporting decisions analyzed in much of the recent trade literature.

6 Appendix

6.1 Aggregation

For any plant-level variable $j_q(z, s^t)$, with $q = m$ or d, define the corresponding equilibrium aggregate by $J_q(s^t) = \int j_q(z, s^t)\mu_q(z, s^{t-1})dz$. Aggregating the plant decision rules in (5) and (7) shows that

$$Y_d(s^t) = A(s^t)Z_d(s^{t-1})^{1-\alpha-\theta}D_d(s^t)^\alpha N_d(s^t)^\theta$$

$$Y_m(s^t) = A(s^t)Z_m(s^{t-1})^{1-\alpha-\theta}\left(\frac{\gamma}{\omega}\right)^\alpha D_m(s^t)^\alpha N_m(s^t)^\theta$$

where Z_d and Z_m are defined in (24).

The aggregated version of the feasibility conditions can be written as follows.

Home country goods feasibility:

$$\begin{aligned} & C(s^t) + D_d(s^t) + D_m(s^t) + (1+\tau)M^*(s^t) - T^*(s^t) \\ & +X(s^t)\left(\kappa_e + \kappa_c\int_{\hat{z}_d(s^t)}^{\infty} g(z)\,dz + \kappa_m\int_{\hat{z}_m(s^t)}^{\infty} g(z)\,dz\right) \\ = \;& A(s^t)Z_d(s^{t-1})^{1-\alpha-\theta}D_d(s^t)^\alpha N_d(s^t)^\theta + A(s^t)Z_m(s^{t-1})^{1-\alpha-\theta}\left(\frac{\gamma}{\omega}\right)^\alpha D_m(s^t)^\alpha N_m(s^t)^\theta \end{aligned} \tag{28}$$

Foreign country goods feasibility:

$$\begin{aligned} & C^*(s^t) + D^*_d(s^t) + D^*_m(s^t) + (1+\tau)M(s^t) - T(s^t) \\ & +X^*(s^t)\left(\kappa_e + \kappa_c\int_{\hat{z}^*_d(s^t)}^{\infty} g(z)\,dz + \kappa_m\int_{\hat{z}^*_m(s^t)}^{\infty} g(z)\,dz\right) \\ = \;& A^*(s^t)Z^*_d(s^{t-1})^{1-\alpha-\theta}D^*_d(s^t)^\alpha N^*_d(s^t)^\theta + A^*(s^t)Z^*_m(s^{t-1})^{1-\alpha-\theta}\left(\frac{\gamma}{\omega}\right)^\alpha D^*_m(s^t)^\alpha N^*_m(s^t)^\theta \end{aligned} \tag{29}$$

Home country labor feasibility:

$$N_d(s^t) + N_m(s^t) \leq N(s^t) \tag{30}$$

Foreign country labor feasibility:

$$N^*_d(s^t) + N^*_m(s^t) \leq N^*(s^t) \tag{31}$$

The aggregated laws of motion for the state variables are as follows.

For the home country:

$$Z_d(s^t) = (1-\delta)Z_d(s^{t-1}) + X(s^t)\int_{\hat{z}_d(s^t)}^{\hat{z}_m(s^t)} zg(z)dz \tag{32}$$

$$Z_m(s^t) = (1-\delta)Z_m(s^{t-1}) + X(s^t)\int_{\hat{z}_m(s^t)}^{\infty} zg(z)dz \tag{33}$$

For the foreign country:

$$Z_d^*(s^t) = (1-\delta)Z_d^*(s^{t-1}) + X^*(s^t)\int_{\hat{z}_d^*(s^t)}^{\hat{z}_m^*(s^t)} zg(z)dz \tag{34}$$

$$Z_m^*(s^t) = (1-\delta)Z_m^*(s^{t-1}) + X^*(s^t)\int_{\hat{z}_m^*(s^t)}^{\infty} zg(z)dz \tag{35}$$

The presence of the tariff τ along with the rebates T in the feasibility conditions allows the incorporation of the distortions arising from import tariffs in the aggregated planning problem.[19] The planning problem is, *given* sequences of $T(s^t)$ and $T^*(s^t)$ and initial values of $Z_d(s^0), Z_d^*(s^0), Z_m(s^0), Z_m^*(s^0)$, to maximize an equally-weighted sum of home and foreign consumers' utilities,

$$\sum_{t=0}^{\infty}\sum_{s^t}\beta^t\tilde{\phi}(s^t)\left[U(C(s^t),N(s^t)) + U(C^*(s^t),N^*(s^t))\right]$$

subject to (28) through (35) for all s^t, by choosing:

1. Consumption and labor for consumers, C, C^*, N, and N^*;
2. Allocations of inputs, $D_d, D_d^*, D_m, D_m^*, M, M^*, N_d, N_d^*, N_m$, and N_m^*;
3. Mass of new plants X and X^*;
4. Cutoffs $\hat{z}_d, \hat{z}_d^*, \hat{z}_m$, and $\hat{z}_m^*$; and
5. Future values of the state variables Z_d, Z_d^*, Z_m, and Z_m^*.

A "side condition" imposed on this problem is that the choices for M and M^* satisfy the following:

$$\begin{aligned} T(s^t) &= \tau M(s^t) \\ T^*(s^t) &= \tau M^*(s^t) \end{aligned}$$

The equivalence between this planning problem and an equilibrium of the original model is established through a comparison of the first order conditions of this problem and the

[19] This method follows Kehoe, Levine and Romer (1992).

equilibrium conditions from consumers' and plants' decisions in the original model.

6.2 Calibrating γ

Although the two production functions for importing and non-importing plants in the model are defined over different sets of inputs, a production function relating output to labor and *total expenditures* on intermediate inputs (which are in the same units for all plants) can be defined as follows. Let x_d and x_m denote total expenditures on intermediate inputs for a non-importing plant and an importing plant, respectively. For any non-importing plant,

$$x_d = d_d$$

where d_d is from the original production function. A plant with efficiency z, using intermediate inputs x and labor n produces output

$$y = z^{1-\alpha-\theta} x^{\alpha} n^{\theta}$$

For an importing plant,

$$x_m = d_m + (1+\tau)m$$

Now, for any importing plant, $m = \frac{1-\omega}{\omega(1+\tau)} d_m$. Therefore,

$$x_m = \frac{d_m}{\omega}$$

The output produced by a plant operating the importing technology with efficiency z is then

$$y = z^{1-\alpha-\theta} \left(\gamma \omega^{\omega} \left(\frac{1-\omega}{1+\tau} \right)^{1-\omega} x \right)^{\alpha} n^{\theta}$$

Across all plants, the production function is:

$$y = \begin{cases} z^{1-\alpha-\theta} x^{\alpha} n^{\theta} \text{ if a plant does not import} \\ z^{1-\alpha-\theta} \left(\frac{\gamma}{\omega^{-\omega}(1+\tau)^{1-\omega}(1-\omega)^{\omega-1}} \right)^{\alpha} x^{\alpha} n^{\theta} \text{ if it does} \end{cases}$$

Taking logs, the following production function with a dummy variable indicating importing status applies to all plants:

$$\log y = (1-\alpha-\theta)\log z + \alpha \log x + \theta \log n + \alpha \log \left(\frac{\gamma}{\omega^{-\omega}(1+\tau)^{1-\omega}(1-\omega)^{\omega-1}} \right) \chi$$

where $\chi = 1$ if the plant imports and $\chi = 0$ if not.

Therefore, the term $\alpha \log \left(\frac{\gamma}{\omega^{-\omega}(1+\tau)^{1-\omega}(1-\omega)^{\omega-1}} \right)$ measures the percentage increase in a given plant's output if it imports relative to if it does not. This is the analogue of the statistic estimated in Kasahara and Rodrigue (2007), and is related to the one measured in Halpern, Koren and Szeidl (2006) and Muendler (2004).

References

[1] Albuquerque, R. and S. Rebelo (2000): On the Dynamics of Trade Reform, Journal of International Economics, 51, 21-47.

[2] Alessandria, G. and H. Choi (2007a): Do Sunk Costs of Exporting Matter for Net Export Dynamics? Quarterly Journal of Economics, 122, 289-336.

[3] Alessandria, G. and H. Choi (2007b): Establishment Heterogeneity, Exporter Dynamics, and the Effects of Trade Liberalization, Federal Reserve Bank of Philadelphia and University of Auckland.

[4] Amiti, M. and J. Konings (2007): Trade Liberalization, Intermediate Inputs, and Productivity: Evidence from Indonesia, American Economic Review, 97, 1611-1638.

[5] Atkeson, A. and A. Burstein (2009): Innovation, Firm Dynamics, and International Trade, UCLA and Federal Reserve Bank of Minneapolis.

[6] Atkeson, A. and P. J. Kehoe (1999): Models of Energy Use: Putty-Putty versus Putty-Clay, American Economic Review, 89, 1028-1043.

[7] Backus, D. K., P. J. Kehoe, and F. E. Kydland (1995): International Business Cycles: Theory and Evidence, in Frontiers of Business Cycle Research, ed. by T. F. Cooley. Princeton, NJ: Princeton University Press, 331-356.

[8] Baily, M. N., C. Hulten, and D. Campbell (1992): Productivity Dynamics in Manufacturing Plants, Brookings Papers on Economic Activity. Microeconomics, 1992, 187-267.

[9] Bernard, A. B., J. Eaton, J. B. Jensen, and S. Kortum (2003): Plants and Productivity in International Trade, American Economic Review, 93, 1268-1290.

[10] Bernard, A. B. and J. B. Jensen (1995): Exporters, Jobs and Wages in U.S. Manufacturing: 1976-1987, Brookings Papers on Economic Activity. Microeconomics, 1995, 67-119.

[11] Bernard, A. B., J. B. Jensen, and P. K. Schott (2005): Importers, Exporters, and Multinationals: a Portrait of Firms in the U.S. that Trade Goods, NBER Working Paper 11404.

[12] Biscourp, P. and Kramarz, F. (2007): Employment, Skill Structure, and International Trade: Firm-level Evidence for France, Journal of International Economics, 72, 22-51.

[13] Burstein, A., C. J. Kurz, and L. Tesar (2008): Trade, Production Sharing, and the International Transmission of Business Cycles, Journal of Monetary Economics, 55, 775-795.

[14] Chaney, T. (2005): The Dynamic Impact of Trade Opening: Productivity Overshooting with Heterogeneous Firms, University of Chicago.

[15] Clerides, S. K., S. Lach, and J. R. Tybout (1998): Is Learning by Exporting Important? Micro-Dynamic Evidence from Colombia, Mexico, and Morocco, Quarterly Journal of Economics, 113, 903-947.

[16] Ethier, W. J. (1982): National and International Returns to Scale in the Modern Theory of International Trade, American Economic Review, 72, 389-405.

[17] Ghironi, F. and M. J. Melitz (2005): International Trade and Macroeconomic Dynamics with Heterogeneous Firms, Quarterly Journal of Economics, 120, 865-915.

[18] Gilchrist, S. and J. C. Williams (2000): Putty-Clay and Investment: A Business Cycle Analysis, Journal of Political Economy, 108, 928-960.

[19] Halpern, L., M. Koren, and A. Szeidl (2006): Imports and Productivity, Hungarian Academy of Sciences, Federal Reserve Bank of New York, and University of California, Berkeley.

[20] Heathcote, J. and F. Perri (2002): Financial Autarky and International Business Cycles, Journal of Monetary Economics, 49, 601-627.

[21] Hinojosa-Ojeda, R., D. Runsten, F. Depaolis, and N. Kamel, The U.S. Employment Impacts of North American Integration after NAFTA: A Partial Equilibrium Approach. North American Integration and Development Center, School of Public Policy and Social Research, University of California, Los Angeles.

[22] Kasahara, H. (2004): Technology Adoption Under Relative Factor Price Uncertainty: The Putty-Clay Investment Model, University of Western Ontario.

[23] Kasahara, H. and B. Lapham (2007): Productivity and the Decision to Import and Export: Theory and Evidence, University of Western Ontario and Queen's University.

[24] Kasahara, H. and J. Rodrigue (2007): Does the Use of Imported Intermediates Increase Productivity? Plant-Level Evidence, University of Western Ontario and Queen's University.

[25] Kehoe, T. J., D. K. Levine, and P. M. Romer (1992): On Characterizing Equilibria of Economies with Externalities and Taxes as Solutions to Optimization Problems, Economic Theory, 2, 43-68.

[26] Kouparitsas, M. A. (1997): Dynamic Trade Liberalization Analysis: Steady State, Transitional and Inter-Industry Effects, Federal Reserve Bank of Chicago.

[27] Kurz, C. J. (2006): Outstanding Outsourcers: A Firm- and Plant-Level Analysis of Production Sharing, Federal Reserve Board of Governors.

[28] Melitz (2003): The Impact of Trade on Intra-Industry Reallocations and Aggregate Industry Productivity, Econometrica, 71, 1695-1725.

[29] Muendler, M.-A. (2004): Trade, Technology, and Productivity: A Study of Brazilian Manufacturers, 1986-1998, University of California, San Diego.

[30] Office of the United States Trade Representative (various years): Trade Policy Agenda and Annual Report of the President of the United States on the Trade Agreements Program.

[31] Office of the United States Trade Representative (various years): National Trade Estimate Report on Foreign Trade Barriers.

[32] Reinert, K. A. and D. W. Roland-Holst (1992): Armington Elasticities for United States Manufacturing Sectors, Journal of Policy Modeling, 14, 631-639.

[33] Romer, P. M. (1987): Growth Based on Increasing Returns to Specialization, American Economic Review, 77, 56-62.

[34] Ruhl, K. J. (2008): The Elasticity Puzzle in International Economics, University of Texas, Austin.

Table 1: Imported Intermediate Inputs in World Trade

Country	$\frac{\text{Intermediates}}{\text{Merchandise Imports}}$	Year
Australia	0.35	1994-5
Brazil	0.52	1996
Canada	0.39	1997
China	0.62	1997
Czech Republic	0.49	1995
Denmark	0.35	1997
Finland	0.56	1995
France	0.47	1995
Germany	0.43	1995
Greece	0.27	1994
Hungary	0.57	1998
Italy	0.51	1992
Japan	0.50	1995
Korea	0.63	1995
Netherlands	0.34	1995
Norway	0.32	1997
Poland	0.49	1995
Spain	0.52	1995
United Kingdom	0.37	1998
United States	0.34	1997

Source: OECD Input-Output Tables. Ratio reported is the fraction of manufacturing, mining, and agricultural imports used as intermediate inputs by manufacturing, mining, and agricultural industries.

Table 2: Cross-section Plant Characteristics

	Importers (%)	Size Ratio
Chile, 1979-86	24.1	3.4
US, 1992	23.8	2.3

Source: Chile, *INE* Survey; US, Kurz (2006). Size ratio is average employment of importing plants divided by average employment of non-importing plants.

Table 3: Decomposition of Aggregate Imports, Chile 1979-86

	% Change	% of Total				
Time period	Total	Within	Between	Cross	Switch	Entry
1 year†	-18	79	26	-10	3	2
7 years	-77	74	42	-30	5	10

Data from Chile's *INE* Survey. See text and equation (1) for column definitions.
†Average across 1-year changes.

Table 4: Calibration

Parameter	Role	Value	Chosen to Match
β	discount factor	0.99	annual $r = 0.04$
ζ	share on c in utility	0.34	$N = 0.3$
ν	intertemporal elasticity	2.00	standard value
α	intermediates / gross output	0.50	$\frac{INT}{GDP} = 1.00$
θ	wN / gross output	0.33	$\frac{wN}{GDP} = 0.66$
γ	advantage of importing	1.86	*see text*
ω	home bias	0.80	$\frac{m}{d_m} = 0.20$
δ	plant death rate	0.02	capital depreciation
z_L	distribution lower bound	3.00	normalization
κ_e	entry cost	0.05	normalization
κ_c	non-importing technology cost	0.25	normalization
κ_m	importing technology cost	0.38	*see text*
k	distribution shape parameter	3.75	*see text*
ρ	autocorrelation of shocks	0.90	$\mathrm{corr}(TFP_t, TFP_{t-1}) = 0.90$
σ_ε	std of shocks	0.005	$\sigma_{TFP} = 0.01$
$\mathrm{corr}(\varepsilon, \varepsilon^*)$	correlation of shocks	0.25	$\mathrm{corr}(TFP, TFP^*) = 0.25$

Table 5: Decomposition of Aggregate Imports, Model and Chilean Plant Data

	% of Total				
	Within	Between	Cross	Switch	Entry
Model	57	49	0	0	−6
Data	79	26	−10	3	2

Model: Medians of 1000 120-quarter simulations, annualized.
Data: Table 3.
See text and equation (1) for column definitions.

Table 6: Model Business Cycle Statistics

	$\mathrm{std}(x)^\dagger$		$\mathrm{corr}(x, GDP)$		$\mathrm{corr}(x, x^*)$		$\mathrm{corr}(x_t, x_{t-1})$	
Variable, x	Model	CES	Model	CES	Model	CES	Model	CES
GDP	1.88	1.88	1.00	1.00	0.23	0.20	0.67	0.67
Consumption	0.27	0.28	0.95	0.95	0.41	0.39	0.73	0.72
Investment	3.76	3.68	0.99	0.99	0.07	0.12	0.66	0.66
Labor	0.52	0.52	0.99	0.99	0.19	0.20	0.66	0.66
p	0.24	0.25	0.49	0.49			0.83	0.82
Net Exports / GDP	0.07	0.06	−0.50	−0.39			0.73	0.77

Means of statistics over 1000 simulations of 120 quarters each. CES variant of the model is described in the text. All variables except net exports are logged and Hodrick-Prescott filtered. †For GDP, percent standard deviation; for all other variables, ratio of standard deviation to that of GDP.

Table 7: Dynamics of Trade Liberalization

	Percent growth rate steady states	after 1 year	after 10 years
Imports / GDP	82.89	18.67	52.45
M / D	93.03	18.54	56.70
$\tilde{m}$ / $\tilde{d}_m$	10.01	10.01	10.01
$\tilde{d}_m$ / $\tilde{d}_d$	5.76	5.76	5.76
Z_m / Z_d	195.45	6.69	78.75
GDP	1.53	0.92	1.35
Consumption	1.31	0.02	0.92
Investment	2.31	4.02	2.83
Labor (N)	0.69	1.16	0.83
GDP / N	0.84	−0.24	0.51
	Implied elasticity of substitution		
	steady states	after 1 year	after 10 years
	9.73	2.10	6.05
	Percent welfare gain		
	$100(\lambda_1 - 1)$	$100(\lambda_2 - 1)$	
	0.72	0.28	

Figure 1: Mexico: Imports from US relative to US GDP and Average Tariff on US Goods

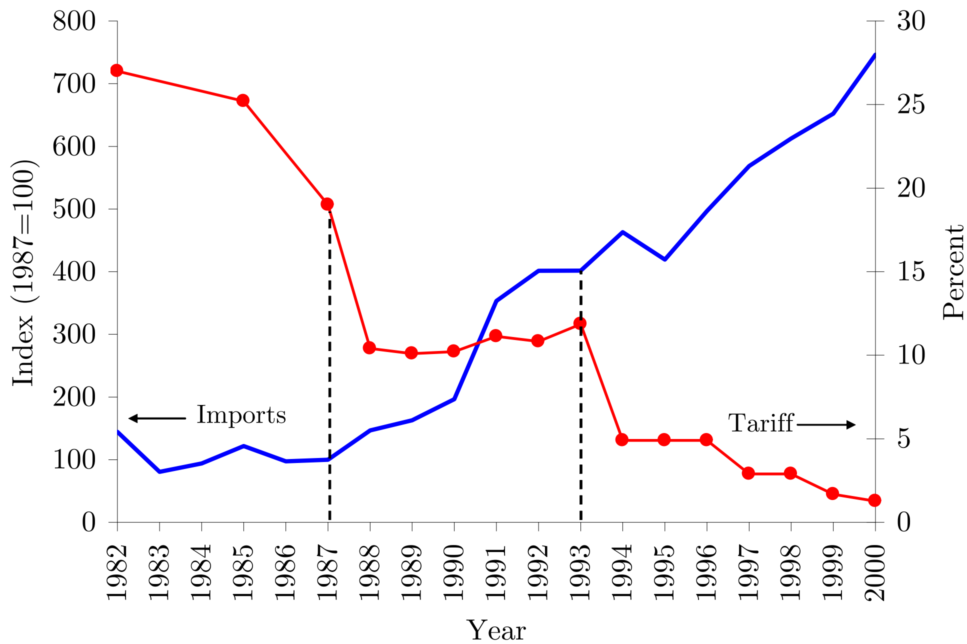

Figure 2: Dynamic decision of a plant entering at state s^t

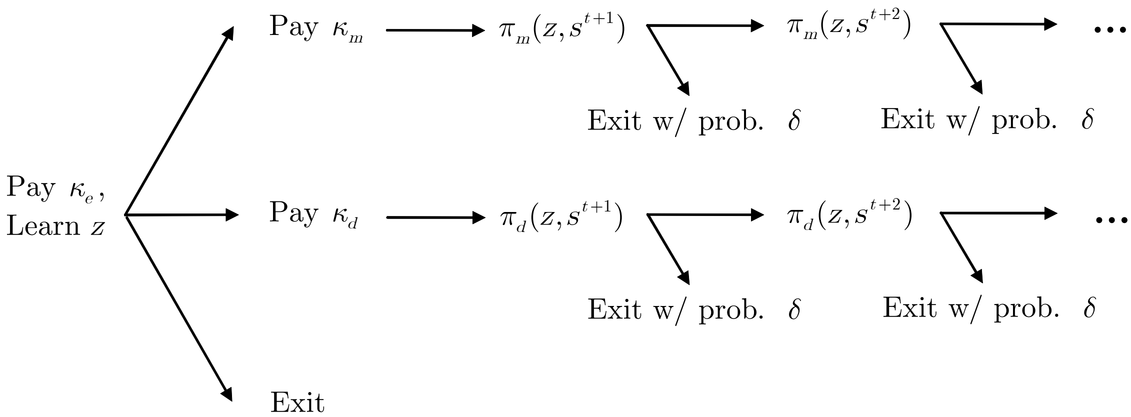

Figure 3: Technology choice cutoffs across entering plants

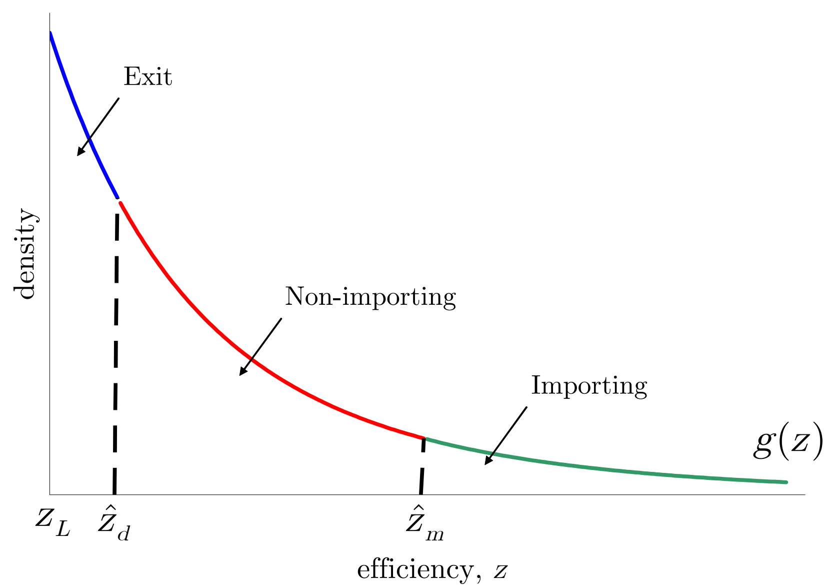

Figure 4: Dynamic responses to a one-standard-deviation shock to A^*

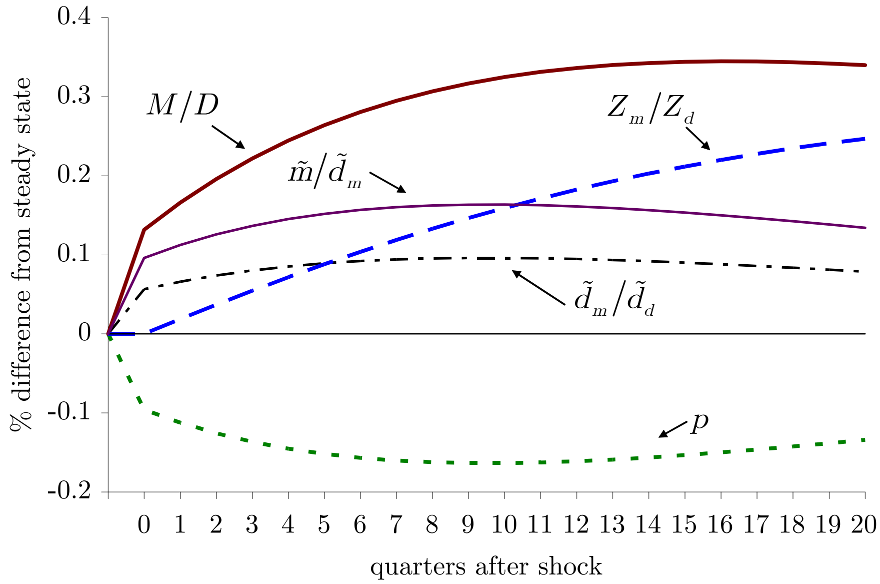

Figure 5: Dynamic responses following trade reform: Trade variables

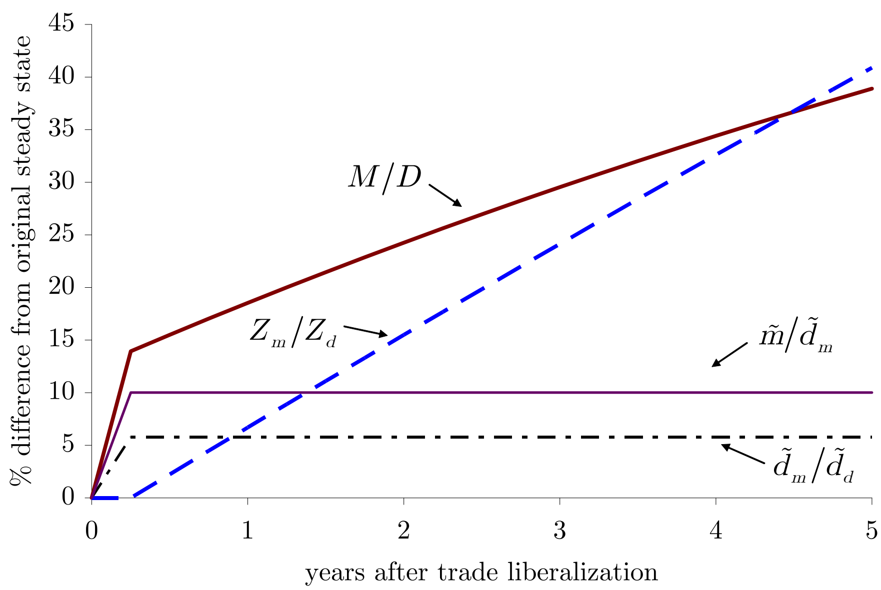

Figure 6: Dynamic responses following trade reform: GDP, Consumption, and Investment