Time Preference and the Distributions of Wealth and Income

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Abstract

This paper presents a dynamic competitive equilibrium model with heterogeneous time preferences that can account for the observed patterns of wealth and income inequality in the United States. This model generalizes the standard neoclassical growth model by including (i) a demand for status by the consumers and (ii) human capital formation. The first feature prevents the wealth distribution from collapsing into a degenerate distribution. The second feature generates a strong positive correlation between earnings and wealth across agents. A calibrated version of this model succeeds in replicating the wealth and income distributions of the United States.

Keywords: Inequality, Heterogeneity, Time Preference, Human Capital

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1 Introduction

Empirical studies show that individuals do not discount future values at the same rate.[1] Since individuals' investment decisions are strongly affected by the way they discount the future, this type of heterogeneity would naturally lead to inequality in wealth and income. In this paper, we present a dynamic competitive equilibrium model with heterogeneous time preferences that can account for the observed patterns of wealth and income inequality in the United States.

It is well known that standard dynamic competitive equilibrium models have difficulty in generating realistic wealth distribution based on heterogeneous time preferences alone. Specifically, when consumers have time-additive separable preferences and different constant discount factors, all the wealth in the model economy will eventually be concentrated in the hands of the most patient consumers. This classic result is first conjectured by Ramsey (1928) and formally proved by Becker (1980). The current study begins by showing that Becker's result cannot be extended to the case where consumers derive utility from both consumption and wealth. In our baseline model, we adopt the same economic environment as in Becker (1980), which features a neoclassical production technology, a complete set of competitive markets, consumers with heterogeneous time preferences and a borrowing constraint. The only modification we make is the inclusion of wealth in consumers' preferences. It is formally shown that the baseline model possesses a unique stationary equilibrium in which every consumer owns a positive amount of wealth. A calibrated version of the baseline model is able to replicate some key features of the wealth distribution in the United States. In particular, it is able to generate a large group of wealth-poor consumers and a very small group of extremely wealthy ones. However, the baseline model falls short in explaining income inequality. This problem remains even if we allow for endogenous labor supply. To overcome this problem, we extend the baseline model by introducing human capital formation. The main idea of this extension is that more patient consumers are more willing to invest in their wealth and human capital than less patient ones. A higher level of human capital then leads to a higher level of future earnings for the more patient consumers. This gives rise to a strong positive correlation between wealth and earnings which is essential in accounting for wealth and income inequality simultaneously. A calibrated version of this model succeeds in replicating the distributions of wealth and income in

[1] See, for instance, Hausman (1979), Lawrance (1991), Samwick (1998), and Warner and Pleeter (2001). A detailed review of this literature can be found in Frederick, *et al.* (2002) Section 6.

the United States.

In the current study, we assume that consumers value wealth directly in their preferences. This assumption has long been used in economic studies. In an early paper, Kurz (1968) introduces this type of preferences into the optimal growth model and explores the long-run properties of the model. Zou (1994) interprets this type of preferences as reflecting the “capitalist spirit,” which refers to the tendency of treating wealth acquisition as an end in itself rather than a means of satisfying material needs. Cole, *et al.* (1992) suggest that this type of preferences can serve as a reduced-form specification to capture people’s concern for their relative wealth position or status within society. Subsequent studies have followed these traditions and interpreted this type of preferences as embodying the spirit of capitalism or reflecting the demand for wealth-induced social status. There is now a growing literature that explores the implications of capitalist spirit on a wide range of issues, including asset pricing, economic growth, the effects of monetary policy and wealth inequality.[2] Among the existing studies, Luo and Young (2009) is most relevant to this paper. These authors consider an economy in which consumers share the same time preference, concern about their status in the economy and face uninsurable idiosyncratic labor income risk. They find that the demand for social status is a force that tends to *reduce* wealth inequality. This result is also observed in our model. First, the equilibrium wealth distribution is no longer degenerate once we introduce the demand for social status into Becker’s model. Second, in the quantitative analysis, we find that wealth inequality decreases as we increase the coefficient that controls the demand for status.

Our baseline model can yield a non-degenerate wealth distribution because adding a demand for status fundamentally changes consumers’ investment behavior. In the original Becker (1980) model, a consumer facing a constant interest rate invests according to the following rules: accumulate wealth indefinitely if the interest rate exceeds his rate of time preference, deplete his wealth until it reaches zero if the opposite is true, and maintain a constant positive level of wealth if the two rates coincide. Since no one can accumulate wealth indefinitely in a stationary equilibrium, the equilibrium interest rate is determined by the lowest rate of time preference among the consumers. It follows from the

[2] Studies that explore the implications of capitalist spirit on asset pricing include Bakshi and Chen (1996), and Boileau and Braeu (2007) among others. Studies on economic growth include Zou (1994) and Smith (1999) among others. Gong and Zou (2001), Chang and Tsai (2003) and Chen and Guo (2009) examine the effects of monetary policy on capital accumulation and economic growth in the presence of social-status concern. Finally, Luo and Young (2009) explore the implications of capitalist spirit on wealth inequality.

above rules that consumers with a higher rate of time preference would end up having zero wealth. In contrast, a status-seeking consumer is willing to hold a constant positive amount of wealth even if the equilibrium interest rate is lower than his rate of time preference. The consumer is willing to do so because holding a positive amount of wealth also satisfies his need for status. The demand for status in effect creates some additional rewards from investment other than the market rate of return. These additional rewards keep consumers from depleting their wealth to zero.

How important is the demand for status to our quantitative results? First, the demand for status prevents the wealth distribution from collapsing into a degenerate distribution. This allows us to explore the implications of heterogeneous time preferences on wealth and income inequality. Our quantitative results show that the extent of wealth inequality is strongly influenced by the coefficient that controls the demand for status. However, this is not the only decisive factor: the distribution of discount factor plays an equally important role in determining wealth inequality. Our baseline results also show that the demand for status cannot generate a substantial degree of income inequality. In the model with human capital, the demand for status does not play any role in determining the distributions of hours and labor earnings. These distributions are completely determined by two factors: (i) the distribution of discount factor, and (ii) the parameters in the human capital accumulation process which are chosen based on empirical findings.

This paper is complementary to two different groups of studies. The first group of studies attempt to explain the observed patterns of income and wealth inequality by considering different versions of the heterogeneous-agent model à la Huggett (1993) and Aiyagari (1994). In this type of models, income inequality is driven by the exogenous labor income risk, and wealth inequality is largely determined by consumers' precautionary saving motive. It is well-documented that the standard Aiyagari-Huggett model has difficulty in generating realistic wealth inequality.[3] Krusell and Smith (1998) show that introducing heterogeneous time preferences can significantly improve the Aiyagari-Huggett model in this regard. To obtain this result, these authors assume that consumers' subjective discount factors are stochastic in nature. The second group of studies are mainly theoretical studies that establish a non-degenerate wealth distribution in the presence of heterogeneous time preferences. Lucas and Stokey (1984) and Boyd (1990) show that Becker's result is no longer valid when consumers have recursive preferences. Sarte (1997) establishes the existence of a

[3] See Quadrini and Ríos-Rull (1997), and Castañeda, *et al.* (2003) for detailed discussions of this problem.

non-degenerate wealth distribution by introducing a progressive tax structure into Becker's model. More recently, Espino (2005) establishes this result by assuming that consumers have private information over an idiosyncratic preference shock. Sorger (2002, 2008) show that Becker's result cannot be extended to the case where consumers are strategic players, rather than price-takers, in the capital market.

The rest of this paper is organized as follows. Section 2 describes the baseline model environment, presents the main theoretical results and evaluates the quantitative relevance of this model. Section 3 extends the baseline model by including endogenous labor supply. Section 4 presents the extension with human capital formation. Section 5 concludes.

2 The Baseline Model

2.1 Preferences

Consider an economy inhabited by a large number of infinitely lived agents. The size of population is constant over time and is given by N. Each agent is indexed by a subjective discount factor β_i, for $i \in \{1, 2, ..., N\}$. The discount factors are ranked according to $1 > \beta_1 \geq \beta_2 \geq \ldots \geq \beta_N > 0$. There is a single commodity in this economy which can be used for consumption and investment. All agents have preferences over streams of consumption and social status, which can be represented by

$$\sum_{t=0}^{\infty} \beta_i^t u\left(c_{it}, s_{it}\right), \tag{1}$$

where c_{it} and s_{it} denote the consumption and social status of agent i at time t. The period utility function $u : \mathbb{R}_+^2 \rightarrow \mathbb{R}$ is assumed to be identical for all agents and have the following properties:

Assumption A1 The function $u\left(c, s\right)$ is twice continuously differentiable, strictly increasing and strictly concave in $\left(c, s\right)$. It also satisfies the Inada condition for consumption, i.e., $\lim_{c \rightarrow 0} u_c\left(c, s\right) = \infty$, where $u_c\left(c, s\right)$ is the partial derivative with respect to c.

Assumption A2 The function $u\left(c, s\right)$ is homogeneous of degree $1 - \sigma$, with $\sigma > 0$.

Assumption A2 is imposed to ensure the existence of balanced growth equilibria. Under this assumption, the partial derivatives $u_c\left(c, s\right)$ and $u_s\left(c, s\right)$ are both homogeneous of degree $-\sigma$. We can then define a function $h : \mathbb{R}_+ \to \mathbb{R}$ according to

$$h(z) \equiv \frac{u_s(z,1)}{u_c(z,1)}. \tag{2}$$

Under Assumption A1, the function $h(z)$ is continuously differentiable and non-negative. We now impose some additional assumptions on this function.

Assumption A3 The function $h(z)$ defined by (2) is strictly increasing and satisfies $h(0) = 0$ and $\lim_{z\to\infty} h(z) = \infty$.

It is straightforward to check that if $u_{cs}(c,s) \geq 0$ then $h(z)$ is strictly increasing. The converse, however, is not true in general. In other words, Assumption A3 does not preclude the possibility of having a negative cross-derivative for some values of c and s.

All three assumptions stated above are satisfied by the following functional forms which are commonly used in the existing literature,

$$u(c,s) = \frac{1}{1-\sigma}\left(c^{1-\sigma} + \theta s^{1-\sigma}\right), \tag{3}$$

with $\sigma > 0$ and $\theta > 0$, and

$$u(c,s) = \frac{1}{1-\sigma}\left[\phi c^{\psi} + (1-\phi) s^{\psi}\right]^{\frac{1-\sigma}{\psi}}, \tag{4}$$

with $\sigma > 0$, $\phi \in (0,1)$ and $\psi < 1$.[4]

2.2 The Agents' Problem

In each period, each agent is endowed with one unit of time which is supplied inelastically to the market. The agents receive labor income from work and interest income from previous savings. All savings are held in the form of physical capital, which is the only asset in this economy. As in Becker (1980), the agents are not allowed to borrow so that capital holdings must be non-negative

[4] The additively separable utility function is used in Zou (1994), Gong and Zou (2001), and Luo and Young (2009) among others. The non-separable utility function is used in Boileau and Braeu (2007). Luo and Young (2009) also consider a Stone-Geary type of preferences over status. We do not consider this in here because it is inconsistent with balanced growth equilibria.

in each period. An agent's social status is measured by the level of wealth owned by the agent at the beginning of the current period. Specifically, this means

$$s_{it} = k_{it}, \qquad \text{for all } t \geq 0,$$

where k_{it} is the stock of capital owned by agent i at the beginning of time t. The same specification of status is also used in Zou (1994), Bakshi and Chen (1996) and Luo and Young (2009) among others.

Given a sequence of wages and rental rates, the agents' problem is to choose sequences of consumption and capital so as to maximize their discounted lifetime utility, subject to sequences of budget constraints and borrowing constraints. Let w_t and r_t be the market wage rate and the rental rate of capital at time t. Formally, agent i's problem is given by

$$\max_{\{c_{it}, k_{it+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta_i^t u\left(c_{it}, s_{it}\right)$$

subject to

$$c_{it} + k_{it+1} - (1-\delta) k_{it} = w_t + r_t k_{it}, \tag{5}$$

$$k_{it+1} \geq 0,$$

$$s_{it} = k_{it},$$

and $k_{i0} > 0$ given.[5] The parameter $\delta \in (0,1)$ is the depreciation rate of capital.

The agents' optimal choices are completely characterized by the sequential budget constraint in (5), and the Euler equation

$$u_c\left(c_{it}, k_{it}\right) \geq \beta_i \left[u_s\left(c_{it+1}, k_{it+1}\right) + \left(1 + r_{t+1} - \delta\right) u_c\left(c_{it+1}, k_{it+1}\right)\right], \tag{6}$$

which holds with equality if $k_{it+1} > 0$. For a status-seeking agent, additional investment in the current period raises future utility by (i) generating more resources for future consumption, and (ii) promoting his future status. The second effect is captured by the term $u_s\left(c_{it+1}, k_{it+1}\right)$ in the

[5] In the theoretical and quantitative analyses, we focus on balanced-growth equilibria which are independent of the initial conditions. Thus, the initial distribution of capital across agents is irrelevant to our analyses.

Euler equation. If status is not valued, then $u_s(c_{it+1,}k_{it+1}) = 0$ and the Euler equation in (6) will be identical to the one in Becker (1980).

2.3 Production

Output is produced according to a standard neoclassical production function:

$$Y_t = F(K_t, X_t L_t),$$

where Y_t denote aggregate output at time t, K_t is aggregate capital, L_t is aggregate labor and X_t is the level of labor-augmenting technology. We will refer to $\widehat{L}_t \equiv X_t L_t$ as the effective unit of labor. The technological factor is assumed to grow at a constant exogenous rate so that $X_t \equiv \gamma^t$ for all t, where $\gamma \geq 1$ is the exogenous growth factor and X_0 is normalized to one. The production function $F : \mathbb{R}_+^2 \to \mathbb{R}_+$ is assumed to have all the usual properties which are summarized below.

Assumption A4 The production function $F\left(K, \widehat{L}\right)$ is twice continuously differentiable, strictly increasing and strictly concave in each argument. It exhibits constant returns to scale and satisfies the following conditions: $F\left(0, \widehat{L}\right) = 0$ for all $\widehat{L} \geq 0$, $F(K, 0) = 0$ for all $K \geq 0$, $\lim_{K \to 0} F_K\left(K, \widehat{L}\right) = \infty$ and $\lim_{K \to \infty} F_K\left(K, \widehat{L}\right) = 0$.

Because of the constant-returns-to-scale assumption, we can focus on a representative firm whose problem is given by

$$\max_{K_t, L_t} \{F(K_t, X_t L_t) - w_t L_t - r_t K_t\}.$$

The solution of this problem is completely characterized by the first-order conditions:

$$w_t = X_t F_{\widehat{L}}(K_t, X_t L_t) = X_t F_{\widehat{L}}\left(\widehat{k}_t, 1\right)$$

and

$$r_t = F_K(K_t, X_t L_t) = F_K\left(\widehat{k}_t, 1\right),$$

where $\widehat{k}_t \equiv K_t / (X_t L_t)$ is the amount of capital per effective unit of labor at time t.

2.4 Competitive Equilibrium

Let $\mathbf{c}_t = (c_{1t}, c_{2t}, ..., c_{Nt})$ denote a distribution of consumption across agents at time t and $\mathbf{k}_t = (k_{1t}, k_{2t}, ..., k_{Nt})$ be a distribution of capital at time t. A competitive equilibrium consists of sequences of distributions of consumption and capital, $\{\mathbf{c}_t, \mathbf{k}_t\}_{t=0}^{\infty}$, sequences of aggregate inputs, $\{K_t, L_t\}_{t=0}^{\infty}$, and sequences of prices, $\{w_t, r_t\}_{t=0}^{\infty}$, so that

(i) Given the prices $\{w_t, r_t\}_{t=0}^{\infty}$, the sequences $\{c_{it}, k_{it}\}_{t=0}^{\infty}$ solve agent i's problem.

(ii) In each period $t \geq 0$, given the prices w_t and r_t, the aggregate inputs K_t and L_t solve the representative firm's problem.

(iii) All markets clear in every period, so that for each $t \geq 0$,

$$K_t = \sum_{i=1}^{N} k_{it},$$

and

$$\sum_{i=1}^{N} c_{it} + K_{t+1} - (1-\delta) K_t = F(K_t, X_t N).$$

In this paper, we confine our attention to balanced-growth equilibria. Formally, a set of sequences $\mathcal{S} = \{\mathbf{c}_t, \mathbf{k}_t, K_t, L_t, w_t, r_t\}_{t=0}^{\infty}$ is called a balanced-growth equilibrium if the following conditions are satisfied:

(i) $\mathcal{S}$ is a competitive equilibrium as defined above.

(ii) The rental rate of capital is stationary over time, i.e., $r_t = r^*$ for all t.

(iii) Individual consumption and capital, aggregate capital and the wage rate are all growing at the same constant rate. In particular, the common growth factor is $\gamma \geq 1$.

2.5 Theoretical Results

The main objective of this subsection is to show that, under certain conditions, the baseline model possesses a unique balanced-growth equilibrium in which all agents hold a strictly positive amount of capital. Before stating the formal theorem, we need to introduce some additional notations.

A balanced-growth equilibrium for this economy is characterized by a constant rental rate r^* which clears the capital market. Once the equilibrium rental rate is determined, all other variables in a balanced-growth equilibrium can be uniquely determined. Thus it suffices to establish the existence and uniqueness of r^*. To achieve this, we first formulate the supply and demand for capital as a function of r.

Denote by $\widehat{k}^d(r)$ the amount of capital per effective unit of labor that the representative firm desires when the rental rate is r. The function $\widehat{k}^d(r)$ is implicitly defined by the condition:

$$r = F_K\left(\widehat{k}^d, 1\right). \tag{7}$$

Under Assumption A4, the function $\widehat{k}^d : \mathbb{R}_{++} \to \mathbb{R}_+$ is continuously differentiable and strictly decreasing. Moreover, $\widehat{k}^d(r)$ approaches infinity as r tends to zero from the right and approaches zero as r tends to infinity. If r is an equilibrium rental rate, then the equilibrium wage rate at time t is uniquely determined by $w_t = \gamma^t \widehat{w}(r)$, where

$$\widehat{w}(r) = F_{\widehat{L}}\left(\widehat{k}^d(r), 1\right). \tag{8}$$

Next, we consider the supply side of the capital market. Along any balanced-growth equilibrium path, individual consumption and capital can be expressed as $c_{it} = \gamma^t \widehat{c}_i$ and $k_{it} = \gamma^t \widehat{k}_i$, where $\widehat{c}_i$ and $\widehat{k}_i$ are stationary over time. The values of $\widehat{c}_i$ and $\widehat{k}_i$ are determined by agent i's budget constraint and the Euler equation for consumption. In a balanced-growth equilibrium, the agent's budget constraint becomes

$$\widehat{c}_i = \widehat{w}(r) + \left(r - \widehat{\delta}\right)\widehat{k}_i, \tag{9}$$

where $\widehat{\delta} \equiv \gamma - 1 + \delta \geq \delta$. Under Assumptions A2 and A3, the Euler equation can be expressed as

$$\frac{\gamma^\sigma}{\beta_i} - (1-\delta) - r \geq h\left(\frac{\widehat{c}_i}{\widehat{k}_i}\right), \tag{10}$$

which holds with equality if $\widehat{k}_i > 0$. By Assumption A3, we have $h(z) \geq 0$ for all $z \geq 0$. In the above equation, z is the consumption-capital ratio for agent i, which must be non-negative in equilibrium. Thus, the Euler equation is valid only for $r \leq \widehat{r}_i$, where $\widehat{r}_i \equiv \gamma^\sigma/\beta_i - (1-\delta) > 0$. This essentially imposes an upper bound on the equilibrium rental rate, which is $\min_i \{\widehat{r}_i\} = \widehat{r}_1$.[6] For any $r \in (0, \widehat{r}_1)$, it is never optimal for any agent i to choose a zero value for $\widehat{k}_i$.[7] It follows that the Euler equation for consumption will always hold with equality in a balanced-growth equilibrium. Combining equations (9) and (10) gives

$$\frac{\gamma^\sigma}{\beta_i} - (1-\delta) - r = h\left(\frac{\widehat{w}(r)}{\widehat{k}_i} + r - \widehat{\delta}\right), \tag{11}$$

which determines the relationship between $\widehat{k}_i$ and r. Formally, this can be expressed as

$$\widehat{k}_i = g_i(r),$$

where $g_i : (0, \widehat{r}_i) \to \mathbb{R}_+$ is a continuously differentiable function implicitly defined by (11).

Denote by $\widehat{k}^s(r)$ the aggregate supply of capital per effective unit of labor when the rental rate is $r \in (0, \widehat{r}_1)$. Formally, this is defined as

$$\widehat{k}^s(r) = \frac{1}{N}\sum_{i=1}^{N} g_i(r).$$

Since each $g_i(r)$ is continuous on $(0, \widehat{r}_1)$, the function $\widehat{k}^s(r)$ is also continuous on this range. A balanced-growth equilibrium exists if there exists at least one value r^* within the range $(0, \widehat{r}_1)$ that solves the capital market equilibrium condition:

$$\widehat{k}^d(r) = \widehat{k}^s(r). \tag{12}$$

Once r^* is determined, all other variables, including the cross-sectional distributions of consumption and capital $(\mathbf{c}_t, \mathbf{k}_t)$, the aggregate capital K_t and the wage rate w_t, can be uniquely determined. If there exists at most one such value of r^*, then the balanced-growth equilibrium is unique.

We are now ready to state the main result of this section. Theorem 1 states the conditions under which a unique balanced-growth equilibrium exists. The formal proof of this result can be found in Appendix A.

[6] If r exceeds $\widehat{r}_1$, then the Euler equation will not be satisfied for some agents and so r cannot be an equilibrium rental rate.

[7] To see this, suppose the contrary that some agent i chooses to have $\widehat{k}_i = 0$ in a balanced-growth equilibrium. Then the right-hand side of (10) would become infinite as $\lim_{z\to\infty} h(z) = \infty$ under Assumption A3. This clearly exceeds the left-hand side of the inequality for any $r \in (0, \widehat{r}_1)$ and hence gives rise to a contradiction.

Theorem 1 *Suppose Assumptions A1-A4 are satisfied. Suppose the following condition holds*

$$\widehat{k}^d\left(\widehat{\delta}\right) > \widehat{k}^s\left(\widehat{\delta}\right). \tag{13}$$

Then there exists a unique balanced-growth equilibrium in which all agents hold a strictly positive amount of capital. In addition, more patient agents would have more consumption and hold more capital than less patient ones, i.e., $\beta_i > \beta_j$ implies $\widehat{c}_i > \widehat{c}_j$ and $\widehat{k}_i > \widehat{k}_j$.

We now explain the intuitions behind Theorem 1. To facilitate comparison with the original result in Becker (1980), we set $\gamma = 1$ for the moment. For each $i \in \{1, 2, ..., N\}$, the parameter $\rho_i \equiv 1/\beta_i - 1$ is the rate of time preference for agent i. In a world where status is not valued, an agent in a stationary equilibrium will invest according to the following rules: accumulate capital indefinitely if the effective return from investment $(r^* - \delta)$ exceeds his rate of time preference, deplete capital until it reaches zero if the effective return is lower than his rate of time preference, and maintain a constant positive capital stock if the two are equal. Since there is only one effective return from investment, it is not possible for agents with different rates of time preference to maintain a constant capital stock simultaneously. At the same time, no one can accumulate capital indefinitely in a stationary equilibrium. Thus the effective return must be equated to the lowest rate of time preference among the agents. It follows that only the most patient agents will hold a positive level of capital in the steady state, and that all other agents with a higher rate of time preference will deplete their capital until it reaches zero.

Introducing a demand for status breaks this spell by creating some additional benefits of holding capital. These additional benefits induce a change in consumers' investment behavior. In particular, a status-seeking agent is willing to maintain a constant positive capital stock even if the effective return from investment is lower than his rate of time preference. This is made clear by the Euler equation

$$\rho_i - (r^* - \delta) = \frac{u_s\left(\widehat{c}_i, \widehat{k}_i\right)}{u_c\left(\widehat{c}_i, \widehat{k}_i\right)} > 0,$$

which implies $\rho_i > (r^* - \delta)$ for all i. The term on the right-hand side of the equation captures the additional benefits of holding capital due to the demand for status. It is now possible to obtain a non-degenerate capital distribution because agents with different rates of time preference can choose a different value of $\widehat{k}_i$ based on the above equation. For impatient agents, they are willing to hold a constant capital stock only if they are compensated by large benefits from status. Under the stated assumptions, these benefits are diminishing in $\widehat{k}_i$. Thus, less patient agents would choose a smaller value of $\widehat{k}_i$ than more patient agents.

Condition (13) in Theorem 1 is imposed to ensure that the equilibrium rental rate r^* is greater than $\widehat{\delta}$. According to (9), $r^* > \delta$ is both necessary and sufficient to guarantee that individual consumption and capital holdings are positively correlated in the balanced-growth equilibrium.

2.6 Calibration

Our goal here is to evaluate the ability of the baseline model to replicate the observed patterns of inequality in the United States. To achieve this, we have to first specify the form of the utility function and the production function, and assign specific values to the model parameters. Some of these values are chosen based on empirical findings. Others are chosen to match some real-world targets. This details of this procedure are explained below.

Functional Forms and Parameters

In the numerical exercise, the production function is given by

$$F(K, XL) = K^{\alpha} (XL)^{1-\alpha},$$

with $\alpha \in (0,1)$. The period utility function is assumed to be additively separable as in (3). In this functional form, the parameter θ captures the importance of social status in consumers' preferences. The original Becker model corresponds to the case in which $\theta = 0$. The additively separable specification is chosen for the following reasons. In the current model, individuals' investment decisions are completely characterized by equation (11). Under the additively separable utility function, this equation can be expressed as

$$\frac{\gamma^{\sigma}}{\beta_i} - (1-\delta) - r = \theta \left[\frac{\widehat{w}(r)}{\widehat{k}_i} + r - \widehat{\delta} \right]^{\sigma}. \tag{14}$$

Under the non-separable functional form in (4), this equation becomes

$$\frac{\gamma^{\sigma}}{\beta_i} - (1-\delta) - r = \frac{1-\alpha}{\alpha}\left[\frac{\widehat{w}(r)}{\widehat{k}_i} + r - \widehat{\delta}\right]^{1-\psi}. \tag{15}$$

A direct comparison of these equations suggests that they can be made identical by a suitable choice of parameter values. When this is imposed, all agents will have the same optimal investment rule $g_i(r)$ under the two specifications of $u(c,s)$. It follows that the equilibrium rental rate r^* and the wealth distribution will also be identical. This result can be stated formally as follows. Let $\widehat{\mathbf{c}} = (\widehat{c}_1, ..., \widehat{c}_N)$ and $\widehat{\mathbf{k}} = \left(\widehat{k}_1, ..., \widehat{k}_N\right)$ be the distributions of consumption and capital obtained under the non-separable specification in (4) with a common growth factor γ. Then the same distributions can be obtained under an additively separable utility function with $\sigma = 1-\psi$, $\theta = (1-\alpha)/\alpha$, and a common growth factor $\widetilde{\gamma} = \gamma^{\frac{\sigma}{1-\psi}}$.[8] This result suggests that these two forms of utility function are likely to yield quantitatively similar results *in the balanced-growth equilibrium.*[9] We choose the additively separable form because it involves fewer parameters.

The following parameter values are used in the quantitative exercise. The share of capital income in total output (α) is 0.33. The growth rate of per-capita variables ($\gamma - 1$) is 2.2 percent, which is the average annual growth rate of real per-capita GDP in the United States over the period 1950-2000. The parameter σ in the utility function is set to one, which is the same as in Luo and Young (2009). The range of subjective discount factors is chosen based on the estimates in Lawrance (1991). Using data from the Panel Study of Income Dynamics over the period 1974-1982, Lawrance (1991) estimates that the average rate of time preference for households in the bottom fifth percentile of the income distribution is 3.5 percent, after controlling for differences in age, educational level and race. This implies an average discount factor of 1/(1+0.035)=0.966 for these households. The estimated rate of time preference for the richest five percent is 0.8 percent, which corresponds to a discount factor of 0.992. In the benchmark scenario, we consider a hypothetical population of 1,000 agents with discount factors uniformly distributed between 0.966 and 0.992. This implies an average discount factor of 0.979. After presenting the benchmark results, we will examine the effects of changing the distribution of discount factors.

[8] In the expression $\gamma^{\frac{\sigma}{1-\psi}}$, the parameter σ is the one that appears in the non-separable utility function.

[9] We stress that this "equivalence" result is valid only in the balanced-growth equilibrium. The two specifications are likely to yield very different results along any transition path.

Table 1 Benchmark Parameters

σ	Inverse of intertemporal elasticity of substitution	1
α	Share of capital income in total output	0.33
γ	Common growth factor	1.022
$\beta_{\min}$	Minimum value of subjective discount factor	0.966
$\beta_{\max}$	Maximum value of subjective discount factor	0.992

Our aim here is to illustrate the relationship between θ and the degree of inequality in the wealth and income distributions. To achieve this, we consider different values of θ ranging from 0.005 to 0.5. For each value of θ, the depreciation rate (δ) is chosen so that the capital-output ratio is 3.0. Table 1 summarizes the parameter values used in the benchmark economy.

Findings

Table 2 summarizes the extent of inequality in the benchmark economy under different values of θ. The reported results include the Gini coefficients for wealth and income, the coefficients of variation for wealth and income, and the shares of wealth held by the bottom and top percentiles of the wealth distribution. The data of these inequality measures are taken from Rodríguez, *et al.* (2002).

Wealth and Income Inequality The results in Table 2 show a strong negative relationship between wealth inequality and the value of θ. This can also be seen from Figure 1, which shows the Lorenz curves for wealth under different values of θ. As θ approaches zero, both the Gini coefficient for wealth and the share of wealth held by the top one percent of the wealth distribution increase towards unity. This means the wealth distribution becomes more and more concentrated when the importance of status in consumers' preferences diminishes. This result is in agreement with theoretical predictions as $\theta = 0$ corresponds to the original Becker (1980) model.

When the value of θ is small, the baseline model is able to replicate some key features of the wealth distribution in the United States. In particular, it is able to generate a large group of wealth-poor agents and a small group of extremely wealthy agents. For instance, when $\theta = 0.0177$, the wealthiest five percent own 56.8 percent of total wealth in the model economy, while the wealthiest one percent own 34.4 percent. These figures are very close to the actual values reported in Rodríguez, *et al.* (2002). Under the same value of θ, the bottom 40 percent of the wealth distribution own 7.3 percent of total wealth. This value turns out to be higher than its real-world counterpart. Consequently, the model generates a slightly more equal wealth distribution than that observed in the United States.

As the value of θ increases, wealth becomes more and more uniformly distributed across the agents. This can be explained as follows. Holding other things constant, an increase in θ raises the marginal utility of status. In other words, the same increase in capital holdings can now generate a larger gain in utility. This effectively diminishes the differences in discount factor across agents. To see this formally, set $\sigma = 1$ and rewrite equation (14) as

$$\frac{1}{\theta}\left[\frac{\gamma}{\beta_i} - (1-\delta) - r\right] = \frac{\widehat{w}(r)}{\widehat{k}_i} + r - \widehat{\delta}.$$

Totally differentiate this with respect to β_i and $\widehat{k}_i$ gives

$$\frac{d\widehat{k}_i}{d\beta_i} = \frac{1}{\theta}\frac{\gamma}{\widehat{w}(r)}\left(\frac{\widehat{k}_i}{\beta_i}\right)^2 > 0.$$

This expression tells us how the variations in discount factor across agents are transformed into variations in $\widehat{k}_i$ under a given value of r. According to this expression, the variations in capital holdings diminish as the value of θ increases. In the limiting case where θ converges to infinity, the wealth distribution converges to a uniform distribution. This means the effects of heterogeneous discount factors would disappear when θ is sufficiently large.

The results in Table 2 also show that the current model tends to generate a relatively low degree of income inequality. This is true even when there is substantial inequality in wealth. For instance, when $\theta = 0.0177$, the Gini coefficient for income is 0.235, as compared to 0.713 for wealth. This occurs because labor income represents a sizable portion of total income for most of the agents in this economy. Table 3 shows the share of total income from labor income for different wealth groups. When θ is 0.0177 or less, labor income accounts for more than 80 percent of total income for the majority of the agents. Since there is no variation in labor income across agents, the extent of income inequality is thus low.

In sum, our quantitative results show that the baseline model is able to replicate some key features of the wealth distribution in the United States. However, it falls short of explaining income inequality. This is partly because there is no variation in labor income across agents. The two extensions considered in Sections 3 and 4 are intended to change this feature of the baseline model.

Changing the Range of Discount Factors In the benchmark scenario, the minimum and the maximum values of discount factor are 0.966 and 0.992, respectively. We now consider five different variations of these values. We maintain the uniform distribution assumption in each case. In the first variation, the benchmark values are both reduced by 0.01 so that $\beta_{\min} = 0.956$ and $\beta_{\max} = 0.982$. In the second variation, the benchmark values are both reduced by 0.02. In these two experiments, the difference between the minimum and the maximum values, $\triangle\beta \equiv |\beta_{\max} - \beta_{\min}|$, is the same as in the benchmark case. In the third and fourth experiments, this difference is reduced by half. Specifically, we consider the upper half of the benchmark interval in the third experiment, so that $\beta_{\min} = 0.979$ and $\beta_{\max} = 0.992$, and the lower half in the fourth one. In the final experiment, we extend the benchmark interval to the left by 50 percent, so that $\beta_{\min} = 0.953$ and $\beta_{\max} = 0.992$.

Table 4 reports the results of these experiments under three different values of θ. To facilitate comparison, we also show the benchmark results in each case. Two observations can be made from these results. First, shifting the range of discount factors while leaving the difference $\triangle\beta$ unchanged only has a small impact on wealth inequality. This is true for all three values of θ considered. This shows that the current model does not rely on large discount factors to generate a substantial degree of wealth inequality. Second, wealth inequality is positively related to the size of $\triangle\beta$. This is evident from the results of the last three experiments. For instance, reducing the difference by half lowers the Gini coefficient for wealth by about 32 percent when $\theta = 0.0177$. Meanwhile, extending the benchmark interval by 50 percent generates a 16-percent increase in the Gini coefficient under the same value of θ. These results show that the distribution of discount factor is another important factor in determining wealth inequality in this model. Note that the third and fourth experiments only involve a shift in the range of discount factors. The results obtained in these experiments are very similar, which is consistent with the first observation.

In sum, these experiments show that wealth inequality in the baseline model is sensitive to changes in the difference between $\beta_{\max}$ and $\beta_{\min}$ but not so sensitive to changes in the actual values of $\beta_{\max}$ and $\beta_{\min}$.

3 Endogenous Labor Supply

In this section, we extend the baseline model to include endogenous labor supply decisions. The agent's period utility function is now given by

$$u(c,s,l)=\frac{c^{1-\sigma}}{1-\sigma}+\theta\frac{s^{1-\sigma}}{1-\sigma}-\mu\frac{l^{1+1/\eta}}{1+1/\eta},$$

where l denote the amount of time spent on working, μ is a positive parameter and $\eta > 0$ is the intertemporal elasticity of substitution (IES) of labor. The agents' labor income is now endogenously determined by their choice of working hours. The rest of the model is the same as in Section 2.

A balanced-growth equilibrium for this economy can be defined similarly as in Section 2.4. This type of equilibrium now includes, among other things, a stationary distribution of labor which is represent by $\mathbf{l}=(l_1,l_2,...,l_N)$. Let $\widehat{k}^d(r)$ and $\widehat{w}(r)$ be the functions defined in (7) and (8). The equilibrium values of $\left\{\widehat{c}_i,\widehat{k}_i,l_i\right\}_{i=1}^N$ and the equilibrium rental rate r^* are determined by

$$\frac{1}{\theta}\left[\frac{\gamma^\sigma}{\beta_i}-(1-\delta)-r\right]=\left(\frac{\widehat{c}_i}{\widehat{k}_i}\right)^\sigma, \tag{16}$$

$$\frac{\widehat{w}(r)}{\widehat{c}_i}=\mu(l_i)^{\frac{1}{\eta}}, \tag{17}$$

$$\widehat{c}_i=\widehat{w}(r)\,l_i+\left(r-\widehat{\delta}\right)\widehat{k}_i, \tag{18}$$

$$\sum_{i=1}^N\widehat{k}_i=\left(\sum_{i=1}^N l_i\right)\widehat{k}^d(r), \tag{19}$$

where $\widehat{\delta}\equiv\gamma-1+\delta$. Equation (16) is the Euler equation for consumption evaluated along a balanced-growth path. Equation (17) is the first-order condition with respect to labor. Equation (18) is derived from the agent's budget constraint. Equation (19) is the capital market equilibrium condition.

We now consider the same numerical exercise as in Section 2.6. The production function again takes the Cobb-Douglas form and the parameter values in Table 1 are used. The intertemporal elasticity of substitution of labor is set to 0.4.[10] To check the robustness of our findings, we also consider two other values of this elasticity, which are 0.2 and 1.0. As in Section 2.6, we focus on the

[10] The same value is used in Chang and Kim (2006, 2007) among others.

relationship between θ and the degree of inequality in wealth and income. We consider the same set of values for θ as in Table 2. In each case, the preference parameter μ is chosen so that the average amount of time spent on working is one-third and the depreciation rate δ is chosen so that the capital-output ratio is 3.0.

Table 5 shows the inequality measures obtained under $\eta = 0.4$. When comparing these to the baseline results in Table 2, it is immediate to see that the two sets of results are almost identical. Introducing endogenous labor supply decisions does not change the fundamental mechanism in the baseline model. In particular, the model continues to generate a high degree of wealth inequality when θ is small and a relatively low degree of income inequality in general. Our numerical results show that allowing for endogenous labor supply actually *lowers* the Gini coefficient for income. This can be explained by Figure 2, which shows the relationship between discount factor and labor supply. Most of the agents in this economy, except those who are very patient, choose to have the same amount of labor. Consequently, the distribution of labor is close to uniform with a long left tail.[11] This explains why the extended model generates a similar degree of income inequality as the baseline model. Since labor supply decreases as the discount factor increases, an impatient agent has less capital income but more labor income than a (very) patient agent. In other words, the two sources of income are negatively correlated. This negative correlation in effect reduces the degree of income inequality in the extended model.

Finally, Table 6 shows that the results in Table 5 are robust to changes in the intertemporal elasticity of substitution of labor. Specifically, increasing the elasticity from 0.2 to 1.0 only marginally affects the Gini coefficients for wealth and income.

4 Human Capital Formation

4.1 The Model

In this section, we extend the baseline model to include human capital formation. The agents' period utility function is now given by

$$u(c, s) = \log c + \theta \log s, \qquad \theta > 0.$$

[11] In all of our examples, the Gini coefficient and the coefficient of variaion for labor (and hence labor income) are close to zero. These results are not reported in the paper but are available from the author upon request.

In each period, all agents are endowed with one unit of time which they can divide between market work and on-the-job training. Denote by h_{it} the stock of human capital of agent i at time t. If this agent chooses to spend a fraction $l_{it} \in [0,1]$ of time on market work at time t, then his human capital at time $t+1$ is given by

$$h_{it+1} = \phi \left(1 - l_{it}\right)^{\eta} h_{it}^{\mu} + \left(1 - \delta_h\right) h_{it}, \tag{20}$$

where $\phi > 0$, $\eta \in (0,1)$, $\mu \in (0,1)$, and $\delta_h \in (0,1)$ is the depreciation rate of human capital. The agent's labor income at time t is given by $w_t l_{it} h_{it}$. We refer to $l_{it} h_{it}$ as the effective unit of labor and w_t as the market wage rate for effective unit of labor.

Let r_t be the rental rate of physical capital at time t. Agent i's problem is now given by

$$\max_{\{c_{it}, l_{it}, k_{it+1}, h_{it+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta_i^t u\left(c_{it}, s_{it}\right)$$

subject to

$$c_{it} + k_{it+1} - \left(1 - \delta_k\right) k_{it} = w_t l_{it} h_{it} + r_t k_{it},$$

$$k_{it+1} \geq 0, \qquad l_{it} \in [0,1],$$

$$s_{it} = k_{it},$$

the human capital accumulation equation in (20), and the initial conditions: $k_{i0} > 0$ and $h_{i0} > 0$. The parameter $\delta_k \in (0,1)$ is the depreciation rate of physical capital. The rest of the model economy is the same as in Section 2. In particular, long-term growth in per-capita variables is again fueled by an exogenous improvement in labor-augmenting technology.[12] The exogenous growth factor is again given by $\gamma \geq 1$.

A balanced-growth equilibrium for this economy can be defined similarly as in Section 2.4. In here we only present the key equations that characterize this type of equilibrium. A formal definition can be found in Appendix B. A balanced-growth equilibrium now includes, among other things, a stationary distribution of labor, $\mathbf{l} = (l_1, l_2, ..., l_N)$, and a stationary distribution of human capital,

[12] Unlike the endogenous growth model considered in Lucas (1988), human capital accumulation does not serve as the engine of growth in here. This is implicitly implied by the condition $\mu < 1$. The main idea of introducing human capital in this model is to increase the variation in labor income across agents.

$\mathbf{h} = (h_1, h_2, ..., h_N)$. The equilibrium values of $\left\{\widehat{c}_i, \widehat{k}_i, l_i, h_i\right\}_{i=1}^N$ and the equilibrium rental rate r^* are determined by

$$\frac{\gamma}{\beta_i} - (1 - \delta_k) - r = \theta\left(\frac{\widehat{c}_i}{\widehat{k}_i}\right), \tag{21}$$

$$\widehat{c}_i = \widehat{w}(r) l_i h_i + \left(r - \widehat{\delta}\right) \widehat{k}_i, \tag{22}$$

$$\frac{l_i}{1 - l_i} = \frac{1}{\eta}\left\{\frac{1}{\delta_h}\left[\frac{1}{\beta_i} - (1 - \delta_h)\right] - \mu\right\}, \tag{23}$$

$$h_i = \left[\frac{\phi}{\delta_h}(1 - l_i)^\eta\right]^{\frac{1}{1-\mu}}, \tag{24}$$

and

$$\sum_{i=1}^N \widehat{k}_i = \left(\sum_{i=1}^N l_i h_i\right) \widehat{k}^d(r), \tag{25}$$

where $\widehat{\delta} \equiv \gamma - 1 + \delta_k$. Equations (21) and (22) are derived from the Euler equation for consumption and the agent's budget constraint. Equations (23) and (24) are derived from the first-order conditions with respect to l_{it} and h_{it+1}, and the human capital accumulation equation. Equation (19) is the capital market equilibrium condition. The mathematical derivations of these can be found in Appendix B.

According to (23) and (24), the distributions of labor and human capital are completely determined by two factors: (i) the distribution of subjective discount factor and (ii) the parameters in the human capital accumulation process. In particular, these two distributions are independent of the period utility function $u(c, s)$, and thus the demand for status. If social status is not valued, i.e., $u_s(c, s) \equiv 0$, then the distribution of capital is degenerate but the distributions of labor and human capital are non-degenerate.

4.2 Calibration

Parameters

In the quantitative exercise, we use the same specification for production technology, and the same distribution of discount factor as before. Specifically, the production function for goods takes the Cobb-Douglas form with $\alpha = 0.33$. The population contains 1,000 agents with subjective discount factors uniformly distributed between 0.966 and 0.992. As for the parameter values in the human capital production function, we normalize ϕ to unity and set the values of η and μ according to the estimates reported in Heckman, *et al.* (1998). Using data from the National Longitudinal Survey of Youth for the period 1979-1993, these authors find that the values of η and μ for high school graduates are 0.945 and 0.832, respectively. The corresponding values for college graduates are 0.939 and 0.871, respectively. The results generated by these two sets of values turn out to be almost identical. In the following section, we only report the results for $\eta = 0.939$ and $\mu = 0.871$.[13] As for the depreciation rate of human capital, Heckman, *et al.* (1998) assume that it is zero. Other studies in the existing literature find that this rate is usually small and close to zero.[14] We use a depreciation rate of 3 percent, which is consistent with the estimates reported in Haley (1976).

It is now clear that the choice of θ is key to explaining wealth inequality. In here we choose the value of θ so as to match the Gini coefficient for wealth as reported in Rodríguez, *et al.* (2002). Specifically, we target a value of 0.803 for the Gini coefficient of wealth. The required value of θ is 0.0139. As explained above, the distributions of labor and human capital are independent of θ. Thus the distribution of earnings reported below is not influenced by this parameter. Finally, the depreciation rate of physical capital is chosen so that the capital-output ratio is 3.0. The parameter values used in the quantitative exercise are summarized in Table 7.

Findings

Table 8 summarizes the characteristics of the earnings, income and wealth distributions generated by the model. The first three columns of the table show the Gini coefficients, the coefficients of variation and the mean-to-median ratios for the three variables. The mean-to-median ratio is intended to measure the degree of skewness in these distributions. The rest of Table 8 shows the share of earnings, income and wealth held by agents in different percentiles of the corresponding distribution.

[13] The results for $\eta = 0.945$ and $\mu = 0.832$ are available from the author upon request.

[14] See Browning, *et al.* (1999) Table 2.3 for a summary of this literature.

Table 7 Parameters in Model with Human Capital

θ	Importance of status in preferences	0.0139
$\beta_{\min}$	Minimum value of subjective discount factor	0.966
$\beta_{\max}$	Maximum value of subjective discount factor	0.992
α	Share of capital income in total output	0.33
γ	Common growth factor	1.022
δ_k	Depreciation rate of physical capital	0.08029
δ_h	Depreciation rate of human capital	0.03
ϕ	Parameter in human capital production	1
η	Parameter in human capital production	0.939
μ	Parameter in human capital production	0.871

First, consider the statistics of the wealth distribution. Similar to the baseline model, this extension is able to replicate some key features of the wealth distribution in the United States. In particular, the model continues to generate a large group of wealth-poor agents and a small group of extremely wealthy agents. For instance, the share of total wealth owned by the agents in the second quintile of the wealth distribution is merely 1.2 percent, whereas the share owned by the wealthiest five percent is 52.9 percent. These figures are very close to the actual value observed in the United States, which are 1.3 percent and 57.8 percent, respectively. The model is also to match quite closely the share of total wealth owned by agents in the other quintiles. As for the income distribution, the model is able to generate a Gini coefficient and a mean-to-median ratio that are close to the observed values. Except for the top one percent of the income distribution, the model is able to replicate almost exactly the share of aggregate income owned by agents in different income groups.

As for earnings, the model yields a more equal distribution than that observed in the data. In the model economy, the earnings-poor agents own a larger share of total earnings than their real-world counterparts, while the earnings-rich agents own a lower share than that observed in the data. For instance, agents in the second quintile of the earning distribution hold 7.3 percent of total earnings in the model economy, while those who are in the top five percent of the distribution own about 16 percent. The corresponding figures in the United States are 4.0 percent and 31.1 percent, respectively. This result is not surprising because the data take into account retirees who have zero earnings and the model does not. According to Rodríguez, *et al.* (2002), 22.5 percent of households in their sample have zero earnings and a large portion of these are retired people. If we consider only households headed by employed worker, then the Gini coefficients for earnings in the United States is 0.435. This value is very close to the one predicted by the model.

5 Summary and Conclusions

This paper presents a tractable dynamic competitive equilibrium model that can account for the observed patterns of wealth and income inequality in the United States. In our baseline model, consumers have different subjective discount factors and concern about their wealth-induced status in the economy. The demand for status prevents the wealth distribution from collapsing into a degenerate distribution. This allows the current study to explore the implications of heterogeneous time preferences on wealth and income inequality. The rest of the model is identical to the standard neoclassical growth model. A calibrated version of the baseline model is able to replicate some key features of the wealth distribution in the United States. In particular, it is able to generate a large group of wealth-poor consumers and a very small group of extremely wealthy ones. However, the baseline model falls short in explaining income inequality. This problem remains even if we allow for endogenous labor supply. To overcome this problem, we extend the baseline model by introducing human capital formation. A calibrated version of this extended model successfully replicates the distributions of wealth and income in the United States.

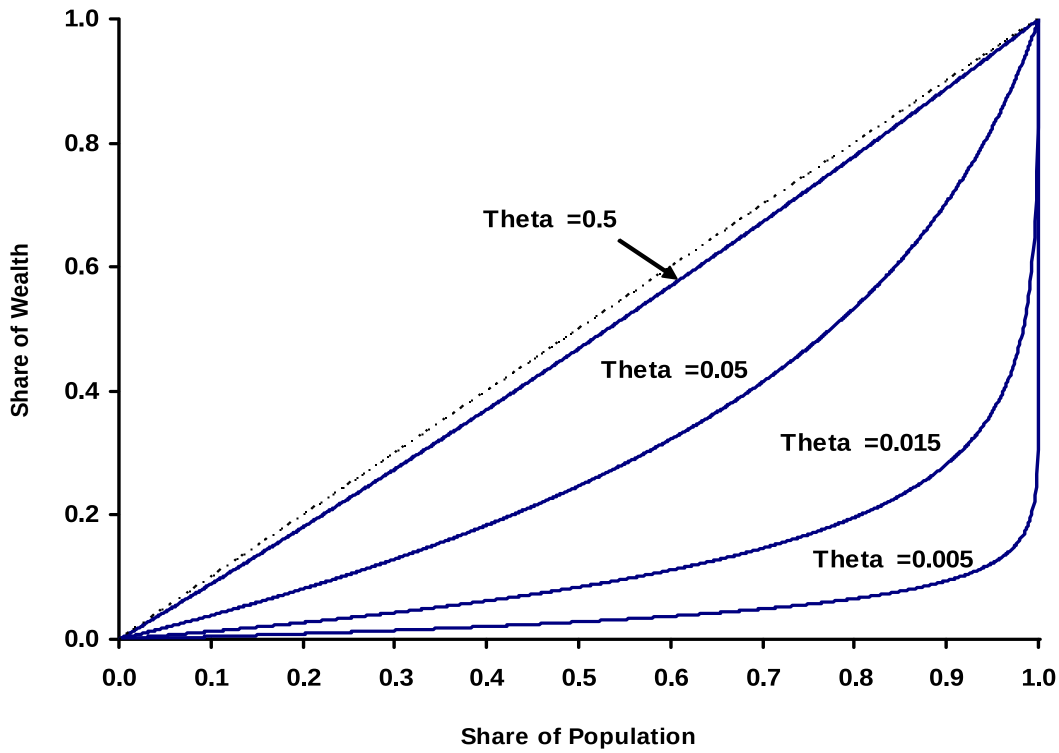

Figure 1 Lorenz Curves for the Wealth Distribution.

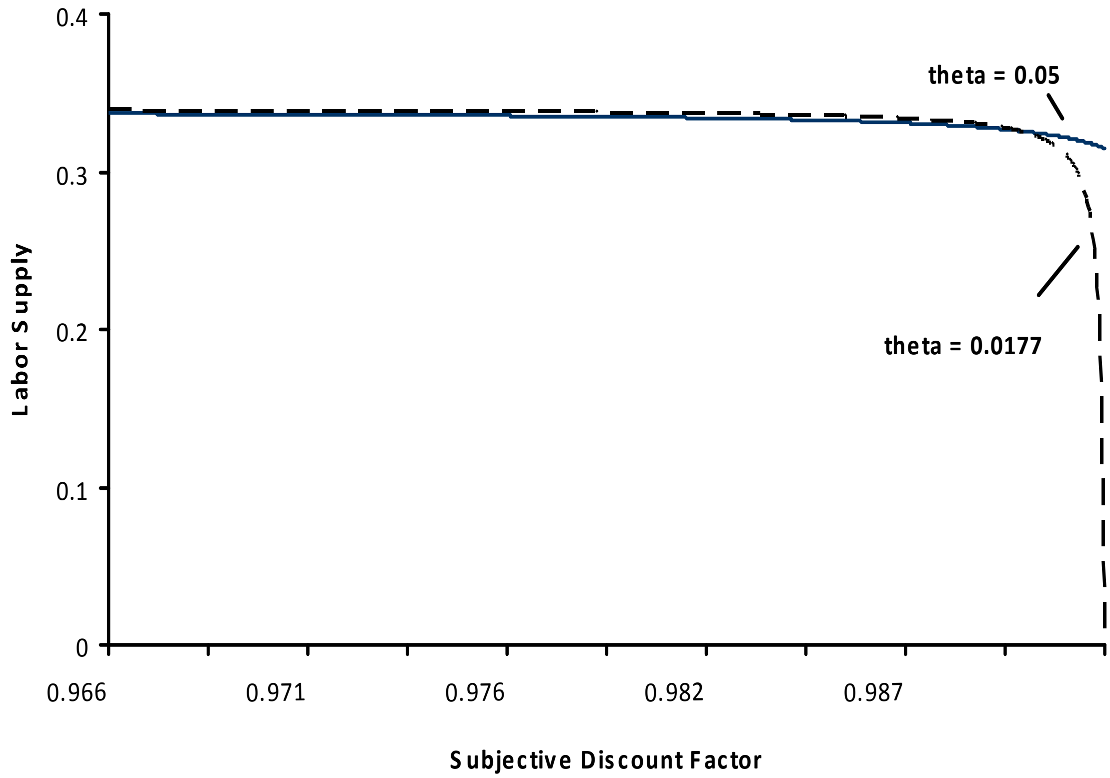

Figure 2: Relationship between Labor Supply and Subjective Discount Factor.

Table 2 Wealth and Income Inequality in Baseline Model

	Gini Coeff.		C.V.		Share of Wealth (%) Held by			
					Bottom			
θ	Wealth	Income	Wealth	Income	40%	Top 10%	Top 5%	Top 1%
0.005	0.918	0.303	22.04	7.27	2.1	90.6	87.7	80.9
0.010	0.836	0.276	13.19	4.35	4.1	81.2	75.4	61.9
0.012	**0.804**	0.265	10.11	3.34	4.9	77.4	70.5	54.4
0.015	0.756	0.249	6.43	2.12	6.2	71.8	63.2	43.5
0.0177	0.713	0.235	4.34	1.43	**7.3**	**66.8**	**56.8**	**34.4**
0.025	0.608	0.201	2.11	0.70	10.2	54.3	41.4	17.1
0.050	0.375	0.124	0.78	0.26	18.4	29.5	17.4	4.1
0.100	0.201	0.066	0.37	0.12	26.9	17.6	9.3	1.9
0.200	0.103	0.034	0.18	0.06	32.9	13.3	6.8	1.4
0.500	0.041	0.014	0.07	0.02	37.0	11.2	5.6	1.1
Data*	**0.803**	**0.553**	**6.53**	**3.57**	**1.0**	**69.1**	**57.8**	**34.7**

Note: C.V. refers to the coefficient of variation.

*Source: Rodríguez, *et al.* (2002).

Table 3 Share of Total Income from Labor Income (%) in Each Wealth Group

	Percentiles in Wealth Distribution						
θ	Bottom 1%	1-5%	5-10%	40-60%	90-95%	95-99%	Top 1%
0.005	98.0	98.0	97.9	96.1	78.1	56.8	17.2
0.010	96.2	96.1	95.9	92.5	64.2	40.6	10.0
0.012	95.4	95.3	95.1	91.1	60.0	36.5	8.7
0.015	94.4	94.2	94.0	89.1	54.7	32.0	7.7
0.0177	93.4	93.3	92.9	87.4	50.9	29.2	7.6
0.025	91.0	90.8	90.4	83.3	44.6	26.6	11.1
0.050	84.6	84.3	83.7	74.6	45.6	38.1	33.1
0.100	78.0	77.7	77.1	69.4	54.9	52.5	51.1
0.200	73.2	72.9	72.5	67.6	61.0	60.1	59.6
0.500	69.6	69.5	69.3	67.1	64.6	64.4	64.2
Data*	**98.9**	**95.9**	**98.1**	**94.0**	**69.8**	**52.3**	**33.6**

*Source: Rodríguez, *et al.* (2002) Table 7, excluding transfers.

Table 4 Wealth Inequality under Different Ranges of Discount Factor

					Share of Wealth (%) Held by			
θ	$\beta_{\min}$	$\beta_{\max}$	Gini	C.V.	Bottom 40%	Top 10%	Top 5%	Top 1%
0.0177	**0.966**	**0.992**	**0.713**	**4.34**	**7.3**	**66.8**	**56.8**	**34.4**
	0.956	0.982	0.719	4.55	7.1	67.5	57.7	35.6
	0.946	0.972	0.724	4.78	7.0	68.1	58.5	36.7
	0.979	0.992	0.486	1.21	14.1	40.4	26.7	7.6
	0.966	0.979	0.494	1.26	13.8	41.4	27.7	8.1
	0.953	0.992	0.809	10.43	4.8	77.9	71.1	55.3
0.050	**0.966**	**0.992**	**0.375**	**0.78**	**18.4**	**29.5**	**17.4**	**4.1**
	0.956	0.982	0.381	0.80	18.1	30.1	17.8	4.2
	0.946	0.972	0.388	0.82	17.8	30.6	18.2	4.4
	0.979	0.992	0.199	0.36	27.0	17.6	9.2	1.9
	0.966	0.979	0.204	0.37	26.7	17.8	9.4	2.0
	0.953	0.992	0.512	1.34	13.2	43.2	29.3	8.9
0.100	**0.966**	**0.992**	**0.201**	**0.37**	**26.9**	**17.6**	**9.3**	**1.9**
	0.956	0.982	0.205	0.37	26.7	17.8	9.4	2.0
	0.946	0.972	0.209	0.38	26.4	18.1	9.5	2.0
	0.979	0.992	0.102	0.18	32.9	13.2	6.8	1.4
	0.966	0.979	0.104	0.18	32.8	13.3	6.8	1.4
	0.953	0.992	0.295	0.57	21.9	23.3	12.9	2.8

Note: C.V. refers to the coefficient of variation. Figures in bold are the benchmark results as shown in Table 2.

Table 5 Wealth and Income Inequality when $\eta = 0.4$

	Gini Coeff.		C.V.		Share of Wealth (%) Held by			
					Bottom			
θ	Wealth	Income	Wealth	Income	40%	Top 10%	Top 5%	Top 1%
0.005	0.918	0.299	23.11	7.61	2.1	90.5	87.7	81.2
0.010	0.836	0.270	15.80	5.19	4.2	81.0	75.4	62.9
0.012	0.803	0.258	13.06	4.28	5.0	77.2	70.5	55.7
0.015	0.754	0.241	9.12	2.98	6.3	71.5	63.1	45.1
0.0177	0.710	0.225	5.84	1.89	7.4	66.4	56.5	35.7
0.025	0.600	0.188	2.04	0.64	10.4	53.4	40.4	16.4
0.050	0.369	0.118	0.77	0.24	18.6	29.1	17.0	4.0
0.100	0.203	0.068	0.37	0.12	26.8	17.7	9.3	2.0
0.200	0.110	0.041	0.19	0.07	32.4	13.5	6.9	1.4
0.500	0.052	0.025	0.09	0.04	36.2	11.5	5.8	1.2
Data*	**0.803**	**0.553**	**6.53**	**3.57**	**1.0**	**69.1**	**57.8**	**34.7**

Note: C.V. refers to the coefficient of variation.

*Source: Rodríguez, *et al.* (2002).

Table 6 Gini Coefficients in Model with Endogenous Labor Supply

	$\eta = 0.2$		$\eta = 0.4$		$\eta = 1.0$	
θ	Wealth	Income	Wealth	Income	Wealth	Income
0.005	0.918	0.298	0.918	0.299	0.918	0.300
0.010	0.836	0.269	0.836	0.270	0.836	0.271
0.012	0.803	0.257	0.803	0.258	0.803	0.260
0.015	0.754	0.239	0.754	0.241	0.755	0.243
0.0177	0.710	0.224	0.710	0.225	0.711	0.228
0.025	0.599	0.186	0.600	0.188	0.603	0.192
0.050	0.369	0.117	0.369	0.118	0.371	0.120
0.100	0.203	0.069	0.203	0.068	0.202	0.068
0.200	0.111	0.042	0.110	0.041	0.108	0.039
0.500	0.054	0.026	0.052	0.025	0.049	0.021

Table 8 Main Results in Model with Human Capital

				Share (%) Held by Agents in Each Group										
			Mean-to-	Bottom			Quintiles					Top	Top	Top
	Gini	C.V.	Median	1%	1-5%	5-10%	1st	2nd	3rd	4th	5th	10%	5%	1%
Earnings														
Model	0.458	0.86	1.52	0.1	0.7	0.9	4.1	7.3	13.4	25.4	49.2	29.1	15.9	3.7
Data	0.611	2.65	1.57	-0.2	0.0	0.0	-0.2	4.0	13.0	22.9	60.2	42.9	31.1	15.3
Income														
Model	0.572	1.34	2.04	0.1	0.5	0.7	2.9	5.3	10.0	20.3	60.4	42.7	28.3	9.3
Data	0.553	3.57	1.61	-0.1	0.1	0.5	2.4	7.2	12.5	20.0	58.0	43.1	32.8	17.5
Wealth														
Model	0.803	2.61	6.90	0.0	0.0	0.1	0.5	1.2	3.0	9.8	82.9	70.3	52.9	19.2
Data	0.803	6.53	4.03	-0.2	-0.1	0.0	-0.3	1.3	5.0	12.2	81.7	69.1	57.8	34.7

Data source: Rodríguez, *et al.* (2002).

Appendix A

Proof of Theorem 1

The proof of this theorem is divided into three main steps. First, it is shown that there exists a rental price $\widetilde{r}_1 > \widehat{\delta}$ such that $\widehat{k}^s(r) \to \infty$ as r approaches $\widetilde{r}_1$ from the left. Since both $\widehat{k}^s(r)$ and $\widehat{k}^d(r)$ are continuous on $\left(\widehat{\delta}, \widetilde{r}_1\right)$ and $\widehat{k}^s(\widetilde{r}_1) < \infty$, this result, together with $\widehat{k}^d\left(\widehat{\delta}\right) > \widehat{k}^s\left(\widehat{\delta}\right)$, would ensure the existence of at least one value of $r \in \left(\widehat{\delta}, \widetilde{r}_1\right)$ that solves the equation

$$\widehat{k}^d(r) = \widehat{k}^s(r). \tag{26}$$

The second step is to show that there exists at most one solution on the interval $(0, \widetilde{r}_1)$. Together, these two steps show that a unique r^* exists in the interval $\left(\widehat{\delta}, \widetilde{r}_1\right)$. Finally, it is shown that $\beta_i > \beta_j$ implies $\widehat{c}_i > \widehat{c}_j$ and $\widehat{k}_i > \widehat{k}_j$.

Step 1 For each $i \in \{1, 2, ..., N\}$, one can show that there exists a unique value $\widetilde{r}_i > \widehat{\delta}$ that solves

$$\frac{\gamma^\sigma}{\beta_i} - (1-\delta) - r = h\left(r - \widehat{\delta}\right).$$

First, $h(0) = 0 < \frac{\gamma^\sigma}{\beta_i} - (1-\delta)$. Second, the left-hand side of the above expression is strictly decreasing in r, while the right-hand side is strictly increasing in r. Hence the two cross at most once. It is straightforward to show that $\widetilde{r}_i < \widehat{r}_i \equiv \gamma^\sigma/\beta_i - (1-\delta)$ and $\widetilde{r}_N \geq \widetilde{r}_{N-1} \geq \ldots \geq \widetilde{r}_1 > \widehat{\delta}$ given the ordering $1 > \beta_1 \geq \beta_2 \geq \ldots \geq \beta_N > 0$.

By the definitions of $g_1(r)$ and $\widetilde{r}_1$, it must be the case that $g_1(r) \to \infty$ as r approaches $\widetilde{r}_1$ from the left. Since $\widetilde{r}_1 \leq \widetilde{r}_i < \widehat{r}_i$ for any $i \geq 2$, we have $g_i(r) > 0$ for all $i \geq 2$ when r is arbitrarily close to $\widetilde{r}_1$. Thus, as r approaches $\widetilde{r}_1$ from the left, we have

$$\widehat{k}^s(r) = \frac{1}{N}\sum_{i=1}^{N} g_i(r) \to \infty.$$

Step 2 To establish the uniqueness of r^*, we need to consider the derivative of $\widehat{k}^s(r)$. Using equation (11), one can derive the derivative of $g_i(r)$, which is given by

$$g_i'(r) = \frac{1}{\widehat{w}(r)}\left\{[g_i(r)]^2 + \widehat{w}'(r)\, g_i(r) + \frac{[g_i(r)]^2}{h'(z_i(r))}\right\},$$

where $z_i(r) \equiv \widehat{w}(r)/g_i(r) + r - \widehat{\delta}$ and $\widehat{w}'(r) = -\widehat{k}^d(r) < 0$. Hence the derivative of $\widehat{k}^s(r)$ is given by

$$\begin{aligned}\frac{d}{dr}\widehat{k}^s(r) &= \frac{1}{N}\sum_{i=1}^{N} g_i'(r) \\ &= \frac{1}{\widehat{w}(r)}\left\{\frac{1}{N}\sum_{i=1}^{N}[g_i(r)]^2 - \widehat{k}^d(r)\,\widehat{k}^s(r) + \frac{1}{N}\sum_{i=1}^{N}\frac{[g_i(r)]^2}{h'(z_i(r))}\right\}.\end{aligned}$$

Let r^* be any solution of (26). The derivative of $\widehat{k}^s(r)$ at $r = r^*$ is

$$\frac{1}{\widehat{w}(r^*)}\left\{\frac{1}{N}\sum_{i=1}^{N}[g_i(r^*)]^2 - \left[\widehat{k}^s(r^*)\right]^2 + \frac{1}{N}\sum_{i=1}^{N}\frac{[g_i(r^*)]^2}{h'(z_i(r^*))}\right\},$$

after we imposed the condition $\widehat{k}^d(r^*) = \widehat{k}^s(r^*)$. The above expression is strictly positive as

$$\frac{1}{N}\sum_{i=1}^{N}[g_i(r^*)]^2 \geq \left[\frac{1}{N}\sum_{i=1}^{N} g_i(r^*)\right]^2 = \left[\widehat{k}^s(r^*)\right]^2,$$

and $h'(z) > 0$. Since $\widehat{k}^d(r)$ is monotonically decreasing, this means $\widehat{k}^s(r)$ must be cutting $\widehat{k}^d(r)$ from below *at every intersection point*. Since both $\widehat{k}^d(r)$ and $\widehat{k}^s(r)$ are continuous, if there exists more than one solution of (26) then at least of them must have $\widehat{k}^s(r)$ cutting $\widehat{k}^d(r)$ from above. This gives rise to a contradiction and hence establishes the uniqueness of r^*.

Step 3 Totally differentiate the equation

$$\frac{\gamma^\sigma}{\beta} - (1-\delta) - r = h\left(\frac{\widehat{w}(r)}{\widehat{k}} + r - \widehat{\delta}\right)$$

with respect to β and $\widehat{k}$ yields

$$\frac{d\widehat{k}}{d\beta} = \gamma^{\sigma} \left(\frac{\widehat{k}}{\beta} \right)^2 \left[h' \left(\frac{\widehat{w}\,(r)}{\widehat{k}} + r - \widehat{\delta} \right) \right]^{-1} > 0.$$

Hence $\beta_i > \beta_j$ implies $\widehat{k}_i > \widehat{k}_j$. Since the equilibrium rental rate r^* is strictly greater than $\widehat{\delta}$, $\widehat{c}_i$ is positively related to $\widehat{k}_i$ according to (9).

This completes the proof of Theorem 1.

Appendix B

This section provides the technical details of the model in Section 4. First, we define a balanced-growth equilibrium for this economy. A competitive equilibrium consists of sequences of distributions of individual variables, $\{\mathbf{c}_{t,}\mathbf{k}_t, \mathbf{l}_t, \mathbf{h}_t\}_{t=0}^{\infty}$, sequences of aggregate inputs, $\{K_t, L_t\}_{t=0}^{\infty}$, and sequences of prices, $\{w_t, r_t\}_{t=0}^{\infty}$, so that

(i) Given the prices $\{w_t, r_t\}_{t=0}^{\infty}$, the sequences $\{c_{it}, k_{it}, l_{it}, h_{it}\}_{t=0}^{\infty}$ solve agent i's problem.

(ii) In each period $t \geq 0$, given the prices w_t and r_t, the aggregate inputs K_t and L_t solve the representative firm's problem.

(iii) All markets clear in every period, so that for each $t \geq 0$,

$$K_t = \sum_{i=1}^{N} k_{it} \quad \text{and} \quad L_t = \sum_{i=1}^{N} l_{it} h_{it}.$$

A set of sequences $\mathcal{S} = \{\mathbf{c}_{t,}\mathbf{k}_t, \mathbf{l}_t, \mathbf{h}_t, K_t, L_t, w_t, r_t\}_{t=0}^{\infty}$ is called a balanced-growth equilibrium if the following conditions are satisfied:

(i) $\mathcal{S}$ is a competitive equilibrium as defined above.

(ii) The rental rate of capital is stationary over time, i.e., $r_t = r^*$ for all t.

(iii) The distributions of labor and human capital are stationary over time.

(iv) Individual consumption and capital, aggregate capital and the wage rate are all growing at the same constant rate. In particular, the common growth factor is $\gamma \geq 1$.

We now provide the mathematical derivations of equations (21)-(24). Let λ_{it} and ψ_{it} be the multipliers for the budget constraint and the human capital accumulation equation, respectively. The first-order conditions for the agent's problem are given by

$$u_c\left(c_{it}, k_{it}\right) = \lambda_{it}, \tag{27}$$

$$\lambda_{it} w_t h_{it} = \psi_{it} \eta \phi \left(1 - l_{it}\right)^{\eta - 1} h_{it}^{\mu}, \tag{28}$$

$$\lambda_{it} = \beta_i \left[u_s \left(c_{it+1}, k_{it+1} \right) + \lambda_{it+1} \left(1 + r_{t+1} - \delta_k \right) \right], \tag{29}$$

$$\psi_{it} = \beta_i \left\{ \lambda_{it+1} w_{t+1} l_{it+1} + \psi_{it+1} \left[\mu \phi \left(1 - l_{it+1} \right)^{\eta} h_{it+1}^{\mu - 1} + \left(1 - \delta_h \right) \right] \right\}. \tag{30}$$

Combining (27) and (29) gives

$$\frac{u_c \left(c_{it}, k_{it} \right)}{u_c \left(c_{it+1}, k_{it+1} \right)} = \beta_i \left[\frac{u_s \left(c_{it+1}, k_{it+1} \right)}{u_c \left(c_{it+1}, k_{it+1} \right)} + 1 + r_{t+1} - \delta_k \right].$$

Equation (21) can be obtained from this after imposing the balanced-growth conditions: $c_{it} = \gamma^t \widehat{c}_i$ and $k_{it} = \gamma^t \widehat{k}_i$. The derivation of (22) is straightforward and is omitted. Along a balanced-growth equilibrium path, individual human capital is stationary. It follows from the human capital accumulation equation that

$$\delta_h h_i = \phi \left(1 - l_i \right)^{\eta} h_i^{\mu}.$$

Equation (24) follows immediately from this expression. Finally, combining (28) and (30) gives

$$\psi_{it} = \beta_i \psi_{it+1} \left\{ \phi \left(1 - l_{it+1} \right)^{\eta - 1} h_{it+1}^{\mu - 1} \left[\mu \left(1 - l_{it+1} \right) + \eta l_{it+1} \right] + \left(1 - \delta_h \right) \right\}.$$

In the balanced-growth equilibrium, the multiplier ψ_{it} is stationary over time. To see this, combine (27) and (28) to get

$$\frac{w_t}{c_{it}} = \psi_{it} \eta \phi \left(1 - l_{it} \right)^{\eta - 1} h_{it}^{\mu - 1}.$$

In a balanced-growth equilibrium, l_{it} and h_{it} are stationary while w_t and c_{it} are growing at the same rate. Thus ψ_{it} must be stationary over time. Thus, in this kind of equilibrium, we have

$$1 = \beta_i \left\{ \phi \left(1 - l_i \right)^{\eta - 1} h_i^{\mu - 1} \left[\mu \left(1 - l_i \right) + \eta l_i \right] + \left(1 - \delta_h \right) \right\}.$$

Equation (23) can be obtained by substituting (24) into this.

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