Auctions in Markets: Common Outside Options and the Continuation Value Effect*

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Abstract

We study auctions with outside options that are determined through bargaining in an external market. In contrast to the case of exogenous outside options, auctions with less information revelation may yield higher revenues, and second-price auctions may outperform English-auctions. Effects that favor non-transparent auctions include a small payoff difference between different states, a great value of information in the continuation problem, and imprecise signals of the bidders. The timing of information revelation is important: it is never optimal to reveal information after the auction, while it may be optimal before the auction.

1 Introduction

Information transmission in decentralized markets has long been of central interest to economists.[1] The speed and extent of information transmission depend on trading institutions, the rules and norms that govern the interactions of traders. In many decentralized markets, these trading institutions arise endogenously through the uncoordinated choices

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[1] The literature on markets and information dates back to Hayek (1945) and Arrow (1974). Both emphasized the important role that prices play in revealing aggregate scarcity to ensure allocative efficiency of markets. A recent literature initiated by Duffie and Manso (2007) has analyzed the speed and extent of information aggregation.

by individual economics actors. Therefore, understanding how information is trasmitted requires an understanding of the individual traders' incentives to reveal their information to each other. In this paper, we investigate whether an individual seller prefers to run "transparent" auctions (which enhance information percolation in markets) or "opaque" auctions, when the auction is embedded in a larger market. We find that a seller has the strongest incentives to run an opaque auction when (i) buyers have very imprecise information about the outside option provided by the market and (ii) when the value of the outside option is very sensitive to the actions taken by the buyers in the market interaction. The first result implies that the auctioneer has incentives to hide his information when the bidders have imprecise information on their own. Consequently, in a dynamic interaction information will "percolate" only if it is learned reasonably well by the buyers already before arriving on the market.

The outline of our model is as follows. A fixed number of buyers participate in an auction where a single, indivisible good is for sale. A losing bidder is paired with another seller who offers the same good as the original auctioneer. To model bargaining between the two sides, we assume that the buyer makes a take-it-or-leave-it offer to the seller. The seller's cost is unknown to the buyer, and the distribution of the cost depends on the unknown state of the world, which is either *High* or *Low*. In the high state the seller is more likely to have higher costs, so the buyer's continuation payoffs are lower. Before submitting their bids in the auction, buyers receive noisy signals about the state, inducing differences in beliefs among buyers who are otherwise identical. Higher signals are indicative of the high state, and thus lower continuation values. Therefore, bidders with higher signals bid more in the auction.

We show that in the unique equilibrium each bidder places a bid that is equal to his valuation *minus* the continuation utility that arises from the bargaining problem described above. Then, we ask whether the seller has an incentive to commit ex ante to reveal his informative signal about the state before the auction. If the state is revealed, then the revenue decreases if the bidders' own signals are uninformative (see Proposition 4), and it increases if payoffs in the bargaining game depend more on the state than on the price that is offered (see Proposition 3). We also show that the auctioneer always prefers to not disclose the bids after the auction to the losing bidders. Finally, the second price auction always yields higher revenues than the first price auction, and that the comparison between the English auction and the second price auction depends on the informativeness of the bidders' signals, and the payoffs of the bargaining game.

To gain intuition, we decompose the revenue impact of an increase in transparency into two opposing effects. The first of the two effects is the *"linkage effect"*, which is reminiscent from the common value auction literature.[2] This effect favors information revelation, as revealing extra information alleviates worries that a bidder only won because all other bidders had low signals and thus winning may not actually be profitable. The *"continuation value effect",* which favors opaque auctions, is a newly identified effect. A transparent auction provides a losing bidder with precise information about market conditions, which improves his continuation payoffs. The improvement of the continuation payoffs lead to lower bids, decreasing the seller's revenue.[3] The continuation value effect is strong when bidders enter the auction with imprecise signals or when information has a high value in the continuation problem, that is, the optimal action is very sensitive to the state of the economy. In these cases revealing any information about the state reduces the revenues of the auctioneer. When the two states provide intrinsically different continuation payoffs (regardless of the action taken in the market subsequent to the auction), then the linkage effect is strong, since bidders need to make sure to avoid overbidding in the low state where continuation values are high. Our decomposition can also illustrate that revealing any information after the auction (like the winning bid) decreases revenues. When information is revealed after the auction the linkage effect is absent since the bidders cannot incorporate such information into their bids. On the other hand, such information revelation improves outside options, and thus lowers revenues.

The insights from our analysis extend to other settings with dynamic market interaction. Lauermann, Merzyn, Virag (2010), consider a large dynamic matching and bargaining game where many buyers and sellers are matched every period and sellers run auctions. Each individual buyer participates in a sequence of auctions and information learned in one auction allows refining bids in future auctions. Individual sellers who need to choose whether to reveal information face a problem that is similar to the problem of the auctioneer in our model. In Duffie and Manso (2007) traders learn about aggregate market conditions in a dynamic matching market, which is interpreted as a decentralized over-the-counter

[2] The fact that common outside options introduce common value elements into bidding has been noted before, so our setup is naturally related to the literature on common value auctions. In such auctions, Milgrom and Weber (1982) observed that the seller prefers to reveal his information to alleviate the "winner's curse" phenomenon. This rule is called the Linkage Principle in the auction literature.

[3] The continuation value effect is related to the convexity of the continuation payoffs in buyers' posteriors. Revealing information leads to a mean-preserving spread of the posteriors. Therefore, when continuation payoffs are (strictly) convex, revealing information increases ex ante expected continuation payoffs.

asset market. Agents meet each other in small groups and exchange information each period. They study how fast information spreads if agents never hide any information from the others they meet. Interestingly, real world over-the-counter asset market do not always support information percolation. In fact, the opacity of such markets has raised regulator's concerns, motivating a number of recent regulatory interventions to increase the market's transparency.[4] Our analysis provides a possible explanation why opaque trading institutions may develop in decentralized markets.

The literatures on auctions with resale[5] and multi unit auctions[6] also model post auction market interaction between bidders. The main difference between those works and ours is twofold. On one hand, instead of studying strategic interaction between the original bidders in the aftermarket, we concentrate on the case where such an interaction is absent. On the other hand, we allow post auction market interaction to take any form, and still obtain important results for auction design. Our work also abstracts from the possibility of selling information, which may not be feasible in many auction settings.[7]

The rest of the analysis is organized as follows. Section 2 describes the main model, Section 3 provides the main results for the second-price format, and Section 4 contains revenue comparison between first-price, second-price and English-auctions. Section 5 provides some robustness checks, while Section 6 concludes. Most of the proofs are contained in the Appendix.

2 Model and preliminary analysis

2.1 Setup

The interaction unfolds in three stages. First, the auctioneer and the N bidders receive signals about the state of the world. Second, the auctioneer runs an auction for an indivisible object. Third, each losing bidder chooses a price offer in a bilateral bargaining problem.

Information. There are two states of the world, $w \in \{H, L\}$, and the realization is not observed by the bidders. The probability of the high state is ρ_0. The state of the world is interpreted as the aggregate market condition. The bidders receive private signals

[4] The over-the-counter market for coporate bond was traditionally opaque. In 2002, the market underwent a fundamental change when the Transaction Reporting and Compliance Engine (TRACE) was introduced; see Bessembinder and Maxwell (2008).

[5] See Cheng and Tan (2009), Garrat and Troger (2007), Haile (2001) and Hafalir and Krishna (2008).

[6] See for example Mezzetti et.al. (2008).

[7] For an analysis of selling information see Gershkov (2009) and Hörner and Skrzypacz (2010).

that are correlated with the state, and these signals are denoted by $s_1, s_2, ..., s_N$. In state w, the bidders' signals are distributed independently and identically according to G_w on support $[\underline{s}, \overline{s}]$. We assume that G_w admits a differentiable density function g_w. With a signal s, the Bayesian posterior probability of the high state H is denoted by $\rho(s) = \rho_0 g_H(s) / ((1-\rho_0) g_L(s) + \rho_0 g_H(s))$. We assume that g_H/g_L is strictly increasing, and thus the posterior $\rho(s)$ is a strictly increasing function of s.[8]

Auction. All bidders participate in an auction where a single indivisible object is for sale. We analyze bidding in standard auction formats, including the first-price, the second-price, and the ascending (English) auction.

Preferences and payoffs. The winning bidder receives the object and pays a price p, while the losing bidders do not make payments in all the auctions studied. The valuation for the object, v, is the same for all bidders and publicly known. The utility of the winner is equal to $v - p$, while that of the losers' is equal to their continuation payoffs (as defined below).

Outside Option After the auction each losing bidder proceeds to a "market" which is modeled as follows. After losing, the bidder is matched with another seller with probability μ_ω. If matched, the buyer can make a take-it-or-leave it offer to the seller. The seller accepts the offer, whenever it is below his costs. The buyer does not know the cost of the seller but believes that, conditional on state ω, the costs are distributed on some interval $[\underline{c}, \overline{c}]$ according to a distribution function F_ω which is atomless and has a differentiable density. Therefore, the expected utility of a buyer who makes an offer x is

$$u_w(x) = \mu_w (v - x) F_w(x).$$

Let $a(\rho) = \arg\max \rho u_H(x) + (1-\rho) u_L(x)$ be the optimal action (price offer) given belief ρ and $V(\rho)$ be the value function. Let $U_w(\rho)$ be the payoff in state w of a buyer who takes action $a(\rho)$.[9]

When only the matching probabilities—but not the distribution of the seller's costs—differ across states, the optimal action is independent of the state. In this case, $V(\rho)$ is

[8] To simplify exposition, we assume that $\rho(\overline{s}) = 1$ and $\rho(\underline{s}) = 0$, that is, there are some perfectly informative signals.

[9] Whenever there are multiple actions that are optimal and that deliver different payoffs conditional on the realized state, let $U_H(\rho) = \max_{x \in \alpha(\rho)} u_H(x)$ and let $U_L(\rho) = \min_{x \in \alpha(\rho)} u_L(x)$. In the Appendix we show that for almost all ρ all optimal actions provide the same payoffs in the two states, that is, $U_w(\rho) = u_w(x)$ for all $x \in \alpha(\rho)$.

simply a linear function of the probability of the high state, $V(\rho) = \rho V(1) + (1-\rho) V(0)$. If the distribution does depend on the state, the optimal action depends on the belief ρ. In the latter case, it follows from standard arguments from the economics of information that the value function V is convex in beliefs.

Example 1: A continuation value problem. Let $\mu_H = \mu_L = 1$ and let $v \geq 1$, $k \geq 0$ with $F_L(x) = \frac{d-0.5kx^2+t}{v-x}$ and $F_H(x) = \frac{k(x-0.5x^2-0.5)+t}{v-x}$.[10] We constructed the example so that the optimal price offer is identical to the belief, that is, $\alpha(\rho) = \rho$. Therefore, the value function is

$$V(\rho) = t + (1-\rho)(d - 0.5k) + 0.5k\rho^2.$$

The value function is strictly convex if $k > 0$, highlighting the value of information. If $k = 0$, the value function is linear and the payoff depends only on the state and not on the action taken.

Relation to dynamic matching models

The continuation value function of our model may be a reduced form representation of the continuation utility in a dynamic matching model where losing bidders are matched with other sellers. In Lauermann, Merzyn and Virag (2010) we provide such a model, and analyze how bidders learn about the state of the economy, and how it affects their bidding behavior. We also study auction design questions in the context of our infinite horizon matching model, and show that similar insights are available as here.

3 Second Price Auction

To start our analysis of the effects of the information policy on revenues we consider a sealed-bid second price auction. This auction format lends itself to a very tractable analysis, since bidding incentives are relatively simple. We characterize equilibrium bidding behavior and compare revenues for three different information policies. In Section 3.1 we consider the case where no information (other than who won) is released, then in Section 3.2 the case where the auctioneer has perfect information about the state and reveals his information, and in Section 3.3 the case where the winning bid is revealed.

[10] To ensure that F_L, F_H are proper distribution functions, we assume that $k \leq 1$, $t \geq k/2$, and that $0.5 + \frac{d}{k} + \frac{t}{k} \geq v \geq d + t - 0.5k + 1$. This region is non-empty if $k \leq 1$. To ensure (strict) monotonicity of V we assume that $d \geq k/2$.

3.1 Equilibrium without information revelation

We study symmetric equilibrium where each bidder's bid is strictly monotone in his signal. We build on the insight of Milgrom and Weber (1982) who show that in the standard common value setup each bidder bids his valuation assuming that he ties at the top spot. Let $\rho_{tie}(s)$ denote an agent's belief conditional on being tied at the top, and let $\rho_{lose}(s)$ denote the probability of the high state conditional on losing with a signal s and being matched with a seller.[11] The relevant continuation value is assessed conditional on tieing (ρ_{tie}) as in Milgrom and Weber (1982). However, in our setup the continuation value also depends on the action taken in the continuation problem, and this action is the optimal action for the belief upon losing (ρ_{lose}). The following Proposition summarizes our findings, providing our existence and uniqueness result:

Proposition 1 *If V is strictly decreasing, then there exist monotone and symmetric equilibria. In every such equilibrium for almost all s each bidder bids according to*

$$b(s) = v - [\rho_{tie}(s)U_H(\rho_{lose}(s)) + (1 - \rho_{tie}(s))U_L(\rho_{lose}(s))]. \tag{1}$$

If V is not strictly monotone decreasing, then a monotone equilibrium does not exist. A monotone and symmetric equilibrium exists if and only if $U_H(\rho) < U_L(\rho)$ for all $\rho < 1$.

The proof in Appendix 1 establishes that (1) follows from necessary first order conditions for the bidders' problems, and that the global optimality conditions are also satisfied when V is monotone. For the rest of the paper, except when it is stated otherwise (see Example 3), we assume that V is monotone, and concentrate on the monotone equilibrium in the game with no information revelation.[12]

If V is linear, then information has no value and U_H, U_L are constant functions of ρ. Consider the following common value auction in the framework of Milgrom and Weber (1982). There are two states, the value of the object is $v - U_H$ in the high, and $v - U_L$

[11] The posterior upon tieing at the top is $\rho_{tie}(s) = \frac{\rho_0 g_H^2(s) G_H^{N-2}(s)}{\rho_0 g_H^2(s) G_H^{N-2}(s) + (1-\rho_0) g_L^2(s) G_L^{N-2}(s)}$. The posterior upon losing (and being matched) is

$$\rho_{lose}(s) = \frac{\rho_0 g_H (1 - G_H^{N-1}) \mu_H}{\rho_0 g_H (1 - G_H^{N-1}) \mu_H + (1 - \rho_0)\, g_L (1 - G_L^{N-1}) \mu_L}.$$

[12] Later we provide an example where V is non-monotone (Example 3), and show that a non-monotone bidding equilibrium exists, and analyze revenue comparisons for that case.

in the low state, and bidders receive signals that are i.i.d. conditional on the state. This common value auction is formally equivalent to our setup when V is linear, and in both models the equilibrium bid function is $b = v - V(\rho_{tie})$. This shows that one can model auctions with common outside options as common value auctions if and only if the value of the outside option depends on the state, but not on an action taken in the continuation problem.

3.2 Revenue comparison when the state may be revealed

We now derive the ex-ante expected revenue when the auctioneer reveals the state before the auction (ER^{Before}), and compare it to the ex-ante expected revenue when the state is not revealed (ER^{None}). In order to obtain insights into the main trade-offs involved we calculate the difference in revenues ($ER^{Before} - ER^{None}$) as the difference of two opposing effects. To formally introduce those two effects, let ER^{After} denote the ex-ante expected revenue in the second price auction when the state is revealed *after* the auction. We propose the following decomposition for the change in revenue from revealing the state before the auction:

$$ER^{Before} - ER^{None} = \underbrace{\left(ER^{Before} - ER^{After}\right)}_{\text{Linkage Effect}} - \underbrace{\left(ER^{None} - ER^{After}\right)}_{\text{Continuation Value Effect}}.$$

The first of the two opposing effects is the *linkage effect*. This effect measures the increase in revenues when the state is revealed as the result of eliminating the winner's curse by making the auction more transparent.[13] The second effect, the *continuation value effect* measures the decrease in revenue when the state is revealed as the result of improving outside options, and lower willingness to pay in the current auction. We show that the two revenue differences work in the opposite direction, that is, they are both positive.

The ex ante expected revenues of the auctioneer when the state is revealed before or after the auction are derived next. When the state is revealed *before* the auction takes place, all bidders bid $v - V(1)$ in the high state, and all bidders bid $v - V(0)$ in the low state. The ex-ante expected revenue can be written as

$$ER^{Before} = v - (1 - \rho_0)\, V(0) - \rho_0 V(1) = v - V(0) + (V(0) - V(1))\, \rho_0. \tag{2}$$

[13]This effect was introduced in Milgrom and Weber (1982) in the context of common value auctions.

When the state is revealed *after* the auction is run, the full information optimal action is taken in both states, yielding utilities $V(1)$ and $V(0)$. Therefore, the equilibrium bid is $v-(\rho_{tie}(s)V(1)+(1-\rho_{tie}(s))V(0))$. Let $g^{(2)}(s)$ denote the density function of the second largest signal of the N signals from an ex-ante perspective.[14] The bidder with the second highest signal determines the revenue in the second price auction, and the expected revenue is

$$ER^{After}=v-V(0)+(V(0)-V(1))\int_0^1 \rho_{tie}(s)g^{(2)}(s)ds.$$

Finally, we consider revenue in a second price auction without state revelation. We have derived the equilibrium bids before. Using (1), the ex-ante expected revenue of the seller can be written as

$$ER^{None}=\int_0^1 g^{(2)}(s)b(s)ds=v-V(0)+$$

$$+(V(0)-V(1))\int_0^1 g^{(2)}\rho_{tie}ds+\int_0^1 g^{(2)}[\rho_{tie}(V(1)-U_H(\rho_{lose}))+(1-\rho_{tie})(V(0)-U_L(\rho_{lose}))]ds.$$

The following result collects our findings:

Proposition 2 *The linkage effect and the continuation value effect are given by:*

$$LP=ER^{Before}-ER^{After}=(V(0)-V(1))\int_0^1 g^{(2)}(s)\left(\rho_0-\rho_{tie}(s)\right)ds, \tag{3}$$

$$CV=ER^{None}-ER^{After}=\int_0^1 g^{(2)}[\rho_{tie}(V(1)-U_H(\rho_{lose}))+(1-\rho_{tie})(V(0)-U_L(\rho_{lose}))]ds. \tag{4}$$

Suppose that information has no value, that is, the utility in the continuation problem depends only on the state, but not on the action. In this case for all beliefs $\rho\in[0,1]$ it holds that $V(1)-U_H(\rho)=V(0)-U_L(\rho)=0$. Inspection of the continuation value effect shows that is is zero. Since only the linkage effect remains active, revealing information is profitable as it was suggested by Milgrom and Weber (1982). This is not surprising, since (as we argued after Proposition 1) in this case our model reduces to the Milgrom and Weber (1982) setup.

Let us discuss more generally conditions under which revealing or hiding information may be profitable. We consider an example to show that revealing the state is profitable if

[14] Formally, $g^{(2)}(s)=\rho_0 N g_H(s)(1-G_H(s))G_H^{N-1}(s)+(1-\rho_0)\,N g_L(s)(1-G_L(s))G_L^{N-1}(s)$ holds.

the value of information is low, if the payoff differences between the states are high and if bidders have precise signals. General comparative statics results follow after the example.

Example 2. Assume that there are two bidders and the two states are equally likely ex-ante. The signal distribution function is $G_L(x) = 1 - (1-x)^\beta$ in the low state, and $G_H(x) = x^\beta$ in the high state for some $\beta \geq 1$. Importantly, the beliefs ρ_{lose} and ρ_{tie}[15] depend only on β, but not on the other parameters of the example. To measure the precision of signals, introduce $a = 1 - 1/\beta$ and note that when $a = 0$ the signals are uninformative as they have the same distribution in the two states. As a increases the signals become more precise, and in the limit when $a \to 1$ the signals in the low state converge to 0 in probability, and the signals in the high state converge to 1 in probability, that is, signals are perfectly informative in the limit. The value function is the same as in Example 1, that is $V = t + k\rho(\rho - 0.5\rho^2 - 0.5) + (1-\rho)(d - 0.5k\rho^2)$. The utility from taking the action that is optimal with belief ρ is $U_H(\rho) = k(\rho - 0.5\rho^2 - 0.5) + t$ if the state is high, and $U_L(\rho) = d - 0.5k\rho^2 + t$ if the state is low.[16] The difference of the expected utility in the two states is $U_H(\rho) - U_L(\rho) = d + k\rho - 0.5$, so d can be interpreted as a payoff difference between the two states. A higher value of k means that information is more valuable, because the utility losses from taking a suboptimal action in the low state ($U_L(0) - U_L(\rho) = \frac{k\rho^2}{2}$) and in the high state ($U_H(1) - U_H(\rho) = k(\frac{1}{2} - \rho + \frac{\rho^2}{2})$) are both increasing in k. Using that $V(0) = U_L(0)$ and $V(1) = U_H(1)$, straightforward algebra shows that

$$CV = k \int_0^1 g^{(2)}(s)(0.5\rho_{lose}^2(s) + 0.5\rho_{tie}(s) - \rho_{tie}(s)\rho_{lose}(s))ds. \tag{5}$$

The fact that $d = V(0) - V(1)$ implies that the linkage effect can be written as

$$LP = d(0.5 - \int_0^1 g^{(2)}(s)\rho_{tie}(s)ds). \tag{6}$$

The continuation value effect is increasing in k, since the larger the value of information is, the more it helps the losing bidders in the continuation problem. The linkage effect is increasing in d, since the greater the payoff differences between the states are, the stronger the impact of the winner's curse is, and the more it helps to reveal the state in overcoming the bid reduction due to the winner's curse. Figure 1 captures these comparative statics

[15] Formally, $\rho_{tie} = \frac{(g_H)^2}{(g_H)^2 + (g_L)^2}$ and $\rho_{lose} = \frac{g_H(1-G_H)}{g_H(1-G_H) + g_L(1-G_L)}$.

[16] As we introduced in Example 1, this value function is derived from a continuation problem where the losing buyer sets his price.

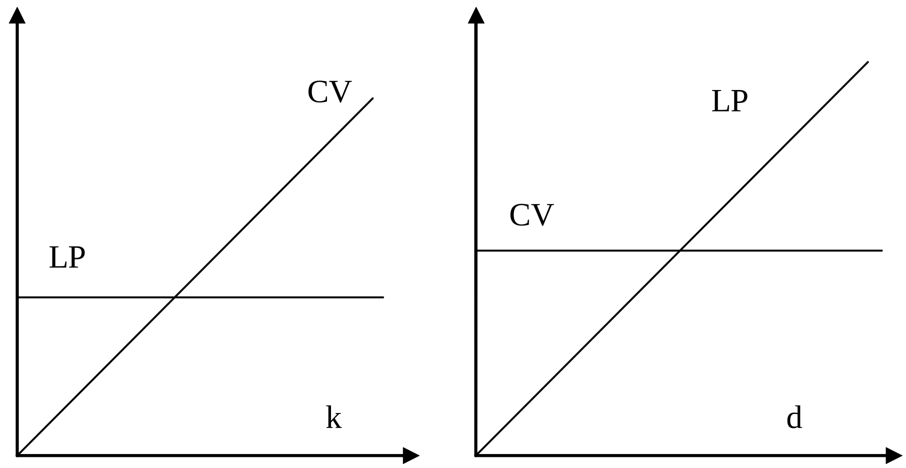

Figure 1: The CV and LP effects as functions of k and d.

results by depicting the continuation value effect and linkage effect as function as a function of k and d.

Let us now provide general counterparts to the comparative statics result depicted in Figure 1. Consider an auction where the continuation value function is indexed by parameters δ and κ, while the signal distributions are indexed by a parameter α. The interpretation of these variables are the same as in Example 2: δ measures the payoff differences between states, κ measures the value of information and α measures the precision of the signals of the bidders. In what follows we formalize these three measures for a general environment. First, to measure intrinsic payoff differences between states, let $V_{\delta}^{\kappa}(\rho) = \delta(1-\rho) + V_0^{\kappa}(\rho)$ that is an increase in δ implies a boost of the utility in the low state that does not depend on the action taken. Second, to measure an increase in the value of information, let an increase in κ induce changes in $U_L(\rho), U_H(\rho)$ such that $U_L(\rho), U_H(\rho)$ are decreasing in κ for all $\rho \in (0,1)$ and $U_H(1)$ $(= V(1))$ and $U_L(0)$ $(= V(0))$ are not affected by κ. Such a change then implies that the utility from not being able to make the optimal action in the high state, that is $V(1) - U_H(\rho)$, is increasing in κ, and a similar statement is true for the loss in the low state. In other words, the value of information is increasing in κ.

We are ready to state our general comparative statics result as the value of information (κ) and the payoff differences between states (δ) change such that a monotone equilibrium exists:

Proposition 3 ***(Incentives for Information-Revelation and the Shape of V.)*** *The continuation value effect is strictly increasing in the value of information κ, and is independent of the payoff difference between states δ. The linkage effect is strictly increasing in the payoff difference between states, and is independent of the value of information.*

Proof. First, note that for any fixed κ it holds that $V_\delta^\kappa(0) - V_\delta^\kappa(1) = \delta + V_0^\kappa(0) - V_0^\kappa(1)$ and thus the linkage effect can be written as

$$LP = (\delta + V_0^\kappa(0) - V_0^\kappa(1)) \int_0^1 g^{(2)}(s)\,(\rho_0 - \rho_{tie}(s))\,ds, \tag{7}$$

which is clearly increasing in δ and is independent of κ. Second, by construction for all ρ, $V(1) - U_H(\rho_{lose})$ and $V(0) - U_L(\rho_{lose})$ are increasing in κ once δ and α are fixed. Therefore, the continuation value effect

$$CV = \int_0^1 g^{(2)}[\rho_{tie}(V(1) - U_H(\rho_{lose})) + (1 - \rho_{tie})(V(0) - U_L(\rho_{lose}))]ds \tag{8}$$

is increasing in κ too. Moreover, the continuation value effect is independent of δ, because utilities in state H are not affected by d, while utilities in state L are increased by d in the same way regardless of the action taken in the continuation problem. **Q.E.D.**

Let us return to Example 2 and study the effect of signal precision (a) on revenues. Taking any d and k such that a monotone equilibrium exists (that is, $d \geq k/2$), let us vary a ranging from full precision ($a = 1$) to being completely uninformative ($a = 0$). We depict the resulting graph in Figure 2.

If signals are uninformative, then the linkage principle effects is absent, because bidders cannot fall victim to the winner's curse. However, the continuation value effect is strong, because bidders value any extra information about the state a lot when they do not have precise information to begin with. When signals are perfectly informative both effects disappear, because the bidders know the state, so it does not make a difference whether the auctioneer reveals his information or not. More interestingly, when signals are *almost* perfectly informative the linkage principle dominates. The reason is that when the losing bidders have very precise information about the state, any increased precision has only a

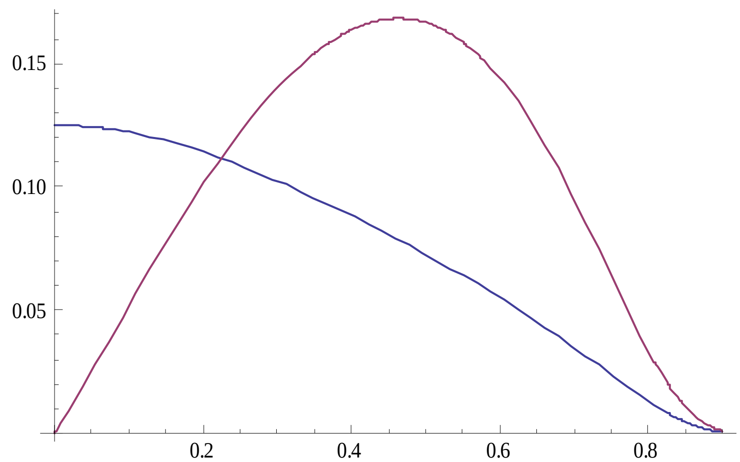

Figure 2: The CV (blue) and LP (red) effects when $d = k = 1$ and α varies from 0 to 1

second order effect on their continuation utilities. The figure also shows that for some a^*

$$LP \geq CV \Longleftrightarrow a \geq a^* \in (0, 1),$$

that is, the linkage effect wins over if and only if signals are precise enough. Consequently, the auctioneer has incentives to reveal his information only if the bidders have precise information already when entering the auction.

Since the linkage effect tends to dominate when signals are precise, the auctioneer has more incentives to reveal the state when the bidders have fairly good signals on their own. On the flip side, when bidders are less well informed, the auctioneer prefers to hide information to avoid increasing the continuation utilities of the bidders. This outcome is unfavorable to bidders, since the auctioneer hides his information exactly when the bidders need it the most to make better decision on the market unfolding after the auction. The result has implications for the information percolation literature as well. Suppose that bidders are matched with sellers in a dynamic game, and ask whether the seller has incentives to reveal the maximum amount of information about market conditions. Our result shows that unless bidders arrive at the transaction place with precise information about the market already, they may not be able to learn the state quickly, since the auctioneer has no incentive to reveal it to them. In other words, information will "percolate" only if it was learned reasonably well already before bidders arrive on the market.

We now show that the above results hold more generally. Let the signal distribution functions G_H^α, G_L^α be parameterized by α and let $\rho^\alpha(s) = \frac{\rho_0 g_H^\alpha(s)}{\rho_0 g_H^\alpha(s)+(1-\rho_0)g_L^\alpha(s)}$ denote the posterior upon receiving signal s. We adopt the convention that signals are uninformative when $\alpha = 0$ holds, and perfectly informative when $\alpha = 1$. In particular, we assume that for all s it holds that $G_H^\alpha(s) = G_L^\alpha(s)$ when $\alpha = 0$, so each signal realization s has the same meaning. To capture that signals become perfectly informative when α is close to 1 we assume that for any $y > 0$ it holds that

$$\lim_{\alpha\to 1} \Pr(\rho^\alpha(s) \geq y \mid \omega = L) = \lim_{\alpha\to 0} \Pr(\rho^\alpha(s) \leq 1 - y \mid \omega = H) = 0.$$

Our next result shows that even in the general setup hiding the state is revenue enhancing when bidders have uninformative signals:

Proposition 4 ***(Incentives for Information-Revelation when Signals are Uninformative.)*** *The continuation value effect is stronger than the linkage effect if signals are uninformative ($\alpha = 0$), and information has value, that is, V is not linear.*

Proof. When $\alpha = 0$ for all s it holds that $g_H^\alpha(s) = g_L^\alpha(s)$ and thus $\rho_{tie}^\alpha(s) = \rho_0$ holds[17], and thus $\int_0^1 g^{(2)}(s)\,(\rho_0 - \rho_{tie}(s))\,ds = 0$. This then implies that when signals are not informative than the linkage effect disappears, that is

$$\alpha = 0 \Rightarrow LP = 0.$$

However, as long as V is not linear it holds for a set of beliefs $\rho \in (\underline{\rho}, \overline{\rho})$ that $V(1) - U_H(\rho)$, $V(0) - U_L(\rho) > 0$, and the continuation value effect is strictly positive (see formula (4). **Q.E.D.**

Finally, we would like to confirm that it continues to hold in the general framework that when signals become *almost* perfectly informative, then the linkage effect dominates the continuation value effect. To establish this result we need to make a further assumption on the exact way signals become perfectly informative. As the information becomes almost perfectly informative the (integrand of the) linkage effect (see formula (3)) dominates the (integrand of the) continuation value effect (see formula (4)) except for signals, which indicate that the high state is very likely. Moreover, the continuation value effect is only

[17] To see this, note that in this case $\rho^\alpha(s) = \frac{\rho_0 g_H^\alpha(s)}{\rho_0 g_H^\alpha(s)+(1-\rho_0)g_L^\alpha(s)} = \rho_0$, and $\rho_{tie}^\alpha(s) = \frac{\rho^\alpha(s)g_H^\alpha(s)(1-G_H^\alpha)^{N-2}}{\rho^\alpha(s)g_H^\alpha(s)(1-G_H^\alpha)^{N-2}+(1-\rho^\alpha(s))g_L^\alpha(s)(1-G_L^\alpha)^{N-2}} = \rho_0$ for all s, since $G_H^\alpha = G_L^\alpha$ also holds.

relevant even for such signals if the precision of such signals are not too high in the limit, otherwise upon receiving a high signal, one is almost sure that it is the high state and thus the state revelation by the auctioneer would not change actions in the continuation problem by much. Therefore, if the signal precision converges not much slower in the high state than in the low state (an equal rate is more than sufficient), then the linkage effect is stronger than the continuation value effect. To formalize this, we need the following assumption:

Assumption CR: There exists $\underline{\varepsilon}, \overline{\varepsilon} < 1$, $K > 0$, $\overline{\alpha} < 1$ *such that if* $\alpha \geq \overline{\alpha}$ *then for all* $s \in [0, 1]$

$\rho(s) > \overline{\varepsilon} \Longrightarrow \exists A_\alpha \subset [0, 1]$ *such that* $\Pr(s \in A_\alpha \mid L) \geq 2K$ *and for all* $\widetilde{s} \in A_\alpha$ *it holds that* $\rho(\widetilde{s}) \leq \underline{\varepsilon}$, *and*

$$\frac{(1 - \rho(s))^2}{\rho(\widetilde{s})} \leq \frac{-V'(1)}{4(V(0) - V(1))} K^2.$$

To interpret this assumption imagine that $\underline{\varepsilon}$ is close to zero, while $\overline{\varepsilon}$ is close to 1, the leading case for our purposes. Then the assumption is fairly weak as it states that there exist some low signals such that the convergence of the posterior upon receiving those signals is not too much faster (to 0) than the convergence of the posterior (to 1) upon receiving some high signals. This condition holds if the convergence rates of posteriors in the two states are equal, or even if the convergence of the posterior in the high state is somewhat slower than the convergence of the posterior in the low state.[18]

Proposition 5 ***(Incentives for Information-Revelation when Signals are very Informative.)*** *Assume that a monotone equilibrium exist in the auction without state revelation. Under Assumption CR the continuation value effect is weaker than the linkage effect if the signals are informative, that is when* $\alpha \in (\widehat{\alpha}, 1)$ *for some* $\widehat{\alpha} < 1$*.*

Proof. See the online Appendix.

Some assumption on the convergence rates is necessary for the result, since in the online Appendix we provide an example where Assumption CR is violated and the continuation value effect dominates the linkage effect for any precision level α.

[18] If convergence is at the same rate x then the relevant ratio becomes $\lim_{x \to 0} \frac{(1-(1-x))^2}{x} = 0$.

3.3 Revenue when the winning bid is revealed

Another important question in auction design is whether any *bid* information should be revealed by the seller. In the previous Section we showed that revealing the state after the auction decreases revenues. The argument there relied on the fact that he linkage effect is absent when information is revealed after the auction, while the continuation value effect is present. This suggests that revealing any information after the auction should hurt the revenues of the auctioneer. We show that this is indeed the case when the winning bid is revealed by the auctioneer. Let ER^{Bid} denote the expected revenue when the winning bid is revealed, assuming that V is decreasing.

Proposition 6 *Revealing the winning bid after the auction decreases revenue relative to not revealing anything, $ER^{Bid} < ER^{None}$.*

Proof. When the seller reveals the winning bid the bid function is

$$b_b(s) = v - [\rho_{tie}(s)U_H(\rho_{tie}(s)) + (1 - \rho_{tie}(s))U_L(\rho_{tie}(s))].$$

Comparing it with the case of no such bid revelation yields

$$b_b(s) < v - [\rho_{tie}(s)U_H(\rho_{lose}(s)) + (1 - \rho_{tie}(s))U_L(\rho_{lose}(s))] = b(s),$$

which follows from the fact that $\rho \in \underset{q\in[0,1]}{\arg\max} \rho U_H(q) + (1-\rho)U_L(q)$. That is the bid of a type s if the winning bid is revealed is lower than in the benchmark case of no information revelation. Therefore,

$$ER^{Bid} = \int_0^1 g^{(2)}(s)b_b(s)ds < \int_0^1 g^{(2)}(s)b(s)ds = ER^{None}.$$

So, revealing the bids after the auction decreases revenues. **Q.E.D.**

4 Comparing first-price, second-price and English-auctions

In the auction design literature, the linkage principle implies that the more an auction format links the payments to the types of the other agents, the higher the expected revenue is. In a first price auction the expected payment conditional on winning does not depend on the types of the other bidders, while in a second price auction and in an English-auction it does. Therefore, the linkage principle implies that the second price and English-auctions yield higher expected revenues than the first price auction. Similarly, an English-auction links payments to others' bids (types) even more, since all the types except for the two highest are revealed by the bids at which those bidders are dropping out. Therefore, the English-auction yields a higher expected revenue than the second-price auction.

In our model with endogenous outside options the linkage effect is counteracted by the continuation value effect, when we analyze whether the auctioneer should reveal his exogenous information about the state of the world. It is natural to ask whether the same is true when one compares the three standard auction formats fixing the information policy. Assume that the auctioneer does not reveal any information and runs a first-price, second-price or English-auction. First, we observe that the second price auction still revenue dominates the first price auction.[19] The key is that in both auctions the losers learn the same information; they only learn that there was a bidder with a higher signal than theirs. This implies that they take the same actions in the continuation decision problems, and therefore the presence of endogenous outside options does not change the comparison between the two formats.

The important novelty when comparing second price and English-auctions is that the English-auction reveals more information, so it allows the losers to take better decisions in the aftermarket, and thus the continuation value effect favors the second price auction over the English auction. Since the continuation value and the linkage effects work in opposite directions, one needs to assess whether the second-price or the English-auction raises higher revenues. To present our result, we concentrate on a three bidder example, where a monotone equilibrium exists and the sellers' revenue is higher in the second price auction than in the ascending auction.[20]

Proposition 7 *Assume that* $\rho_0 = 1/2$, $N = 3$, $g_L = 2(1-s)$ *and* $g_H = 2s$, *and*

$$U_H(\rho) = \rho^{n-1} - \alpha_H \rho^n (n-1),$$

and

$$U_L(\rho) = d - \alpha_L \rho^n (n-1)$$

with $\alpha_H = \alpha_L = 1/n$, $n = 1.1$, $d = 1$. *Then the expected revenue of the English auction is*

[19] This can be done by modifying the analysis of Krishna (2008), Section 7, pages 105-108. The formal argument is provided in our online Appendix.

[20] The calculations are in Appendix 2.

less than the expected revenue of the second-price auction.

In this specification the continuation value effect is stronger than the linkage effect, and revealing information via holding a more open auction decreases revenues. If one considers variations in the parameter values, then the revenue comparison has the same qualitative features as in the case of state revelation. For example, if the two states provide very different utility values (that is d is high), then the linkage effect dominates, and the auctioneer prefers to run an English auction. If α_H decreases or α_L increases, then information is more valuable, and the continuation value effect becomes larger, which favors the less transparent second-price auction.

5 Discussion

In this Section we consider three extensions to inspect the robustness of our results. First, we discuss revenue comparison results when a monotone equilibrium does not exist. Let us revisit Example 2 and assume that the signals of the bidders are precise, that is, $a \geq \overline{a}$ for some appropriate value of $\overline{a}$. Then any monotone equilibrium is such that the linkage effect is stronger than the continuation value effect, regardless of the values of k and d,[21] which suggests that the linkage effect tends to be stronger in general. However, as the next Example shows, this is an artifact of the assumptions needed for a monotone equilibrium to exist.

Example 3:

Consider an example where the intrinsic payoff differences between the two states are completely absent, but it is very important to *know* the state when making the price offer. Let $v = 2$, $\mu_H = \mu_L = 1$ and $F_H(x) = \frac{1-a(1-x)^2}{2-x}$ and $F_L(x) = \frac{1-ax^2}{2-x}$ with $a \in (0, 0.5]$. Sellers have mostly intermediate costs in the high state, and more extreme costs in the low state, and as a result the revenue maximizing prices are very different in the two states.[22] Since the cost distributions F_H, F_L are not ranked by first order stochastic dominance,

[21] To see this, note that a monotone equilibrium exists if $d \geq k/2$, so to be able to choose d, k such that a monotone equilibrium exists *and* $CV > LP$ it has to hold by (5) and (6) that $r = \frac{\int_0^1 g^{(2)}(0.5\rho_{lose}^2 + 0.5\rho_{tie} - \rho_{tie}\rho_{lose})ds}{0.5 - \int_0^1 g^{(2)}\rho_{tie}ds} > \frac{1}{2}$. However, numerical calculations show that $r \leq 1/2$ when bidders have precise signals (that is, $a \geq \overline{a}$).

[22] The cost is 0 with probability $\frac{1}{2}$ $(\frac{1-a}{2})$ in the low (high) state, while the cost is less than 1 with probability 1 $(1-a)$ in the high (low) state. In the low state the cost is prohibitively high with probability a. In the high state a high price ($p = 1$) is optimal to capture all the sellers, but in the low state the most profitable offer is $p = 0$, since that already captures a large portion of the market.

the induced value function may not be monotone in the belief. In fact, the two states are symmetric in that $V(\rho) = V(1-\rho)$ for all $\rho \in [0,1]$, and the value function is U-shaped. The resulting utilities in the two states are $U_H(\rho) = 1 - a(1-\rho)^2$, and $U_L(\rho) = 1 - a\rho^2$. To fully specify the example, assume that $\rho_0 = 1/2$, $N = 2$, and $G_H(s) = s^2$, $G_L(s) = 2s - s^2$.

Since the two states are symmetric, we concentrate on a "state-symmetric" equilibrium where $b(s) = b(1-s)$ for all s. The posterior upon tieing is $\widetilde{\rho}_{tie}(s) = \Pr(H \mid s_1 = s, s_2 = s \text{ or } s_2 = 1-s) = s$, and the posterior upon losing is $\widetilde{\rho}_{lose}(s) = \Pr(H \mid s_1 = s, s_2 \in (s, 1-s)) = s$. When the state is not revealed the equilibrium bid function is $b = v - [\widetilde{\rho}_{tie} U_H(\widetilde{\rho}_{lose}) + (1 - \widetilde{\rho}_{tie}) U_L(\widetilde{\rho}_{lose})] = v - 1 + as(1-s)$. The bid with state revelation is $b = v - V(0) = v - 1$ in the low state, and $b = v - V(1) = v - 1$ in the high state as well. Consequently, the revenue comparison favors not revealing the state. In general, if the two states are similar (that is $V(0) = V(1)$), then the linkage principle loses its bite, and although a monotone equilibrium does not exist, it follows that revealing the state decreases revenues.

Second, we show that the assumption of two states can be relaxed without changing the results. We focus on comparing the revenues from the second price auction with and without the revelation of the winning bid, the question addressed in Section 3.3 for the case of two states. Let $t \in [0,1]$ denote the state of the world, and let $U_t(a)$ denote the continuation value when action a is taken in state t. Let g_t denote the conditional distribution of signals in state t, and let h the density function for the state of the world. Assuming that for all $U_t(a)$ is decreasing in t implies that there is an equilibrium with monotone bidding. Let $\rho_{tiet}(s)$ be the density of state t if one ties at the top with signal s, and $\rho_{loset}(s)$ be the density of state t if one lost with signal s.[23] Then the optimal action after losing with signal s satisfies $a(s) = \arg\max_{x \in X} \int_0^1 \rho^t_{lose}(s) U_t(x) dt$. Without bid revelation the equilibrium bid is $b_n(s) = v - \int_0^1 \rho^t_{tie}(s) U_t(a(s)) dt$. With bid revelation the tieing loser learns that he in fact tied with the winner and takes an action

$$a^*(s) = \arg\max_{x \in X} \int_0^1 \rho^t_{tie}(s) U_t(x) dt. \tag{9}$$

Therefore, the equilibrium bid becomes $b_y(s) = v - \int_0^1 \rho^t_{tie}(s) U_t(a^*(s)) dt$. By (9) it follows that $b_n(s) > b_y(s)$, so the revenue comparison is as in the two-state model.

Third, suppose that the winning bidder takes an action too, and his continuation utility functions are U^w_H, U^w_L, which have similar properties to U_H, U_L, the continuation utility

[23] Formally, $\eta_t(s) = \frac{h(t) g_t^2(s) G_t^{N-2}(s)}{\int_0^1 h(z) g_z^2(s) G_z^{N-2}(s) dz}$ and $\nu_t(s) = \frac{h(t) g_t(s)(1 - G_t^{N-1}(s))}{\int_0^1 h(z) g_z(s)(1 - G_z^{N-1}(s)) dz}$.

of the losers. We keep the assumption that the winner obtains a utility v from the object and that there are two states. Let us now concentrate on the question whether in the second-price format state revelation enhances or reduces revenues; the other questions can be studied similarly. Following similar argument as in the benchmark case, the equilibrium bid function without state revelation is

$$v + [\rho_{tie}(s)(U^w_H(\rho_{lose}(s)) - U_H(\rho_{lose}(s))) + (1 - \rho_{tie}(s))(U^w_L(\rho_{lose}(s) - U_L(\rho_{lose}(s))].$$

When the state is revealed, in the high state all bidders bid $v + V^w(1) - V(1)$, and in the low state all bid $v + V^w(0) - V(0)$. From these observations it is obvious that if the winner's continuation values are not very sensitive to the state of the world (that is $U^w_H - U^w_L$ is uniformly close to zero and thus V^w is close to being a constant function), then the revenue comparison is similar to the benchmark case where the winner's continuation problem was omitted. However, if the winner's continuation problem is important (compared to the losers'), then the continuation value effect *favors* information revelation[24], and thus transparent auctions are revenue enhancing.

6 Conclusion

We study auctions with endogenous outside options determined through actions taken in the aftermarket. In contrast to the case of exogenous outside options, auctions with less information revelation may yield higher revenues. Opaque auctions decrease the information available to losing bidders, which leads to worse decisions in the aftermarket. This leads to worse outside options, and thus more aggressive bidding in the original auction. Effects that favor non-transparent auctions include a small payoff difference between the two states, a great value of information in the continuation problem, and imprecise signals of the bidders. The timing of information revelation is important: it is never optimal to reveal information after the auction, while it may be optimal to reveal information before the auction. We also show that a less transparent auction format, the second price auction can yield higher revenues than an English auction, as it fosters less learning and provides lower continuation values for the bidders. The model is robust to introducing several states, and with respect to the winner's having state dependent continuation functions.

[24] If more information is available after the auction, then the winner can make a better decision, which then makes bidders more aggressive since the winning prize has become more valuable.

7 Appendix

7.1 Appendix 1

In this Appendix we prove Proposition 1, and several useful results about the continuation problem. We start by establishing a monotonicity result:

Lemma 1 *For all $\rho' > \rho$ and $U_H(a') - U_L(a') > U_H(a) - U_L(a)$ it holds that if type ρ weakly prefers a' over a, then type ρ' strictly prefers a' over a. Therefore, for all $b' \in \alpha(\rho')$ and $b \in \alpha(\rho)$ it holds that $U_H(b') - U_L(b') \geq U_H(b) - U_L(b)$. If for some $\rho' > \rho$ it holds that $e \in \alpha(\rho')$ and $f \in \alpha(\rho)$, then $U_H(e) \geq U_H(f)$ and $U_L(e) \leq U_L(f)$; and for almost all $\rho \in [0,1]$ if $c, d \in \alpha(\rho)$ then $U_H(c) = U_H(d)$ and $U_L(c) = U_L(d)$. Function U_H is monotone increasing, while U_L is monotone decreasing.*

Proof of Lemma 1:

Proof. Suppose that $\rho' > \rho$ and $U_H(a') - U_L(a') > U_H(a) - U_L(a)$, and type ρ weakly prefers a' over a, that is

$$\rho U_H(a') + (1-\rho)U_L(a') \geq \rho U_H(a) + (1-\rho)U_L(a). \tag{10}$$

Then $U_H(a') - U_L(a') \geq U_H(a) - U_L(a)$ implies that

$$\left(\rho' - \rho\right)\left(U_H(a') - U_L(a')\right) > \left(\rho' - \rho\right)\left(U_H(a) - U_L(a)\right).$$

Adding the last inequality to (10) implies that

$$\rho' U_H(a') + (1-\rho')U_L(a') > \rho' U_H(a) + (1-\rho')U_L(a),$$

which establishes the first claim. To prove the second statement, suppose that $U_H(b) - U_L(b) > U_H(b') - U_L(b')$. Then the first statement implies that type ρ' strictly prefers b over b', which contradicts with the assumption that $b' \in \alpha(\rho')$. The second statement implies that $U_H(e) - U_L(e) \geq U_H(f) - U_L(f)$. Then $U_H(e) < U_H(f) \Longrightarrow U_L(e) < U_L(f)$, which implies that e is worse than f for any beliefs, and thus $e \in \alpha(\rho')$ could not hold. This contradiction establishes the third claim. To prove the last claim let $\tau(\rho) = \max_{x \in \alpha(\rho)} U_H(x) - U_L(x)$. The second statement implies that τ is weakly increasing, and thus it is almost everywhere continuous. Moreover, at every continuity point ρ of τ it holds that for all

$c, d \in \alpha(\rho)$, $U_H(c) - U_L(c) = U_H(d) - U_L(d)$.[25] Then suppose that $U_H(c) > U_H(d)$. In this case, it would follow that $U_H(c) > U_H(d)$, implying that c dominates d and contradicting $d \in \alpha(\rho)$. This contradiction establishes the last result about functions U_H, U_L. The monotonicity claim about U_H, U_L then follows by construction. **Q.E.D.**

Next we prove that a useful envelope condition holds for almost all ρ. In particular they hold at every continuity point of U_H, U_L. Let us formally state our claim first:

Lemma 2 *For almost every ρ it holds that*

$$a \in \alpha(\rho) \Longrightarrow V'(\rho) = U_H(a) - U_L(a) \tag{11}$$

and

$$V'(\rho) = U_H(\rho) - U_L(\rho). \tag{12}$$

Proof. Take any ρ and let $a \in \arg\max\limits_{x \in \alpha(\rho)} U_H(x)$, $b \in \arg\min\limits_{x \in \alpha(\rho)} U_H(x)$. Note, that by definition of $\alpha(\rho)$ it must hold that $a \in \arg\min\limits_{x \in \alpha(\rho)} U_L(x)$, $b \in \arg\max\limits_{x \in \alpha(\rho)} U_L(x)$ and thus for all $x \in \alpha(\rho)$

$$U_H(a) - U_L(a) \geq U_H(x) - U_L(x) \geq U_H(b) - U_L(b).$$

Next, note that for all $\rho' > p$ it holds that $V(\rho') \geq U(\rho', a)$. Therefore,

$$V(\rho') - V(\rho) \geq (\rho' - \rho)(U_H(a) - U_L(a)).$$

Also, the right hand derivative of a convex function exists everywhere, therefore the right hand derivative at ρ satisfies

$$V'_+(\rho) \geq U_H(a) - U_L(a).$$

A similar argument yields that the left hand derivative satisfies

$$V'_-(\rho) \leq U_H(b) - U_L(b).$$

[25] Suppose that $c, d \in \alpha(\rho)$, and $U_H(c) - U_L(c) > U_H(d) - U_L(d)$. Then for all $\rho' > \rho$ it holds by the second claim that for any $c' \in \alpha(\rho')$, $U_H(c') - U_L(c') \geq U_H(c) - U_L(c)$, and thus $\tau(\rho') \geq U_H(c) - U_L(c)$. Similarly, for any ρ'' and $d' \in \alpha(\rho'')$, $U_H(d') - U_L(d') \leq U_H(d) - U_L(d)$, and thus $\tau(\rho'') \leq U_H(d) - U_L(d)$. Therefore, the function τ must have a jump at such a ρ.

Since $V'(\rho)$ exists almost everywhere, therefore for almost every ρ it must hold that $U_H(a) - U_L(a) = U_H(b) - U_L(b)$. Therefore, wherever a derivative exists (which is almost everywhere) it holds that $V'(\rho) = U_H(a) - U_L(a)$ for all $a \in \alpha(\rho)$, which establishes that *(11)* for almost all ρ. To establish that (*(12)* holds for all continuity points of U_H, U_L (which is almost everywhere) note that the proof of Lemma 1 implies that for any such continuity point ρ it holds that if $c, d \in \alpha(\rho)$ then $U_H(c) = U_H(d)$ and $U_L(c) = U_L(d)$. Therefore, the argument establishing (*11*) applies to show that $V'(\rho) = U_H(c) - U_L(c) = U_H(\rho) - U_L(\rho)$, concluding the proof. **Q.E.D.**

We are ready to prove the existence of a monotone equilibrium stated in Proposition 1.

Proof of Proposition 1:

Proof: In the proof below, we use that U_H is weakly increasing, while U_L is weakly decreasing in ρ for any continuation value problem (see Lemma 1), that is in the high (low) state it is better to take actions that are optimal when the high (low) state is more likely. Then we prove that at all continuity points of U_H, U_L any optimal action provides the same utility in the continuation problem, thus there is a unique optimal action in terms of payoff consequences. Using this observation the first order condition (1) is shown to be necessary for optimal bidding for almost all s (that is for all s such that $\rho_{lose}(s)$ is a continuity point of U_H, U_L). To check that global sufficiency conditions and strict monotonicity of b also hold for the bidders' problem we use basic properties of affiliated random variables, extending the analysis of Milgrom and Weber (1982).[26]

From Lemma 1, we know that for almost all ρ all the optimal actions induce the same utilities in both states. We first concentrate on such values of ρ and then by construction the induced utilities by the optimal action(s) are equal to $U_H(\rho), U_L(\rho)$. We discuss what happens at other values of ρ at the end of the proof.

First, we show that the above defined bid function constitutes an ex-post equilibrium. Symmetry of b is immediate, while monotonicity follows from the facts that ρ_{tie}, ρ_{lose} are increasing, $\rho_{tie} < \rho_{lose}$ and that the monotonicity of V implies that $U_H(x) \leq U_L(x)$ for all $x \in A$. To see this, note first that the function $t(s) = \rho_{tie}(s)U_H(\rho_{lose}(s)) + (1 - \rho_{tie}(s))U_L(\rho_{lose}(s))$ may have a jump. However, $t = V(\rho_{lose}) + (\rho_{tie} - \rho_{lose})(U_H(\rho_{lose}) - U_L(\rho_{lose}))$, and since V and $\rho_{tie} - \rho_{lose}$ are continuous, $\rho_{tie} - \rho_{lose} < 0$ and any jump of $U_H - U_L$ is upward, therefore any jump of t must be downward. Second, the derivative of

[26] To prove that b is a strictly increasing function if V is monotone we use two observations. First, the tieing and the losing posterior are monotone in the signal. Second, the tieing posterior is lower than the losing posterior, which holds if bids are monotone.

t exists almost everywhere (where V'' exists), and

$$\frac{d}{ds}[\rho_{tie}(s)U_H(\rho_{lose}(s)) + (1-\rho_{tie}(s))U_L(\rho_{lose}(s))] =$$

$$\rho'_{tie}(s)(U_H(\rho_{lose}(s)) - U_L(\rho_{lose}(s))) + \rho'_{lose}(s)V''(\rho_{lose})(\rho_{tie}(1-\rho_{lose}) - (1-\rho_{tie})\rho_{lose}) < 0$$

follows from the observations above. But this is equivalent to $t'(s) < 0$. Thus $b = v - t$ has either a jump upward, or is (almost everywhere) differentiable with a positive derivative, which implies that b is monotone increasing.

Next, we show that if it is known that $s_1 = s_2 = s$, then winning with $b(s)$ yields the same utility as losing and acting in the future as if the probability of the high state was $\rho_{lose}(s)$. Losing yields a continuation utility that is equal to $\rho_{tie}(s)U_H(\rho_{lose}(s)) + (1-\rho_{tie}(s))U_L(\rho_{lose}(s))$ by construction, while winning with bid $b(s)$ yields a utility $v - b(s)$, which is equal to the continuation utility upon losing.

It also has to be established that if $s_i = s$ then winning against a type $y > s$ with bid $b(y)$ is unprofitable, while if $y < s$ then winning against type y with bid $b(y)$ is profitable.[27] Let us just inspect the $y > s$ case, the other one is similar. In this case winning, upon tieing, yields a utility of

$$v - b(y) = \rho_{tie}(y)U_H(\rho_{lose}(y)) + (1-\rho_{tie}(y))U_L(\rho_{lose}(y)).$$

To calculate the utility from losing, upon tieing, let us introduce the relevant tieing posterior when one bids $b(y)$ and has type s as $h(s,y) = \frac{\rho_0 g_H(s)g_H(y)G_H^{N-2}(y)}{\rho_0 g_H(s)g_H(y)G_H^{N-2}(y)+(1-\rho_0)g_L(s)g_L(y)G_L^{N-2}(y)}$. By the fact that g_H and g_L satisfy the MLRP, it follows that $\rho_{tie}(y) > h(s,y)$. One can similarly define the losing posterior as $n(s,y) = \frac{\rho_0 g_H(s)(1-G_H^{N-1}(y))}{\rho_0 g_H(s)(1-G_H^{N-1}(y))+(1-\rho_0)g_L(s)(1-G_L^{N-1}(y))}$. Again, the MLRP condition implies that $\rho_{lose}(y) > n(s,y)$. Then the utility upon losing (and tieing) can be written as $h(s,y)U_H(n(s,y)) + (1-h(s,y))U_L(n(s,y))$.

Next, we show that

$$h(s,y)U_H(n(s,y)) + (1-h(s,y))U_L(n(s,y)) \geq$$

$$\geq h(s,y)U_H(\rho_{lose}(y))) + (1-h(s,y))U_L(\rho_{lose}(y)). \tag{13}$$

[27] Again, we need to use the relevant tieing belief $\Pr(H \mid s_1 = x, s_2 = y)$ and the relevant action inducing belief upon losing $\Pr(H \mid s_1 = x, s_2 \geq y)$.

To see this, note that by construction

$$n(s,y)U_H(n(s,y)) + (1-n(s,y))U_L(n(s,y)) \geq$$

$$\geq n(s,y)U_H(\rho_{lose}(y))) + (1-n(s,y))U_L(\rho_{lose}(y)). \tag{14}$$

Also, $\rho_{lose}(y) > n(s,y) > h(s,y)$ and the monotonicity of V implies that $U_H(n(s,y)) - U_L(n(s,y) \leq U_H(\rho_{lose}(y)) - U_L(\rho_{lose}(y))$. Thus it follows that

$$(h-n)\left(U_H(n(s,y)) - U_L(n(s,y))\right) \geq (h-n)\left(U_H(\rho_{lose}(y)) - U_L(\rho_{lose}(y))\right). \tag{15}$$

Adding up (14) and (15) implies (13). Then using (13), the utility difference between losing and winning satisfies

$$\triangle = hU_H(n) + (1-h)U_L(n) - (\rho_{tie}(y)U_H(\rho_{lose}(y)) + (1-\rho_{tie}(y))U_L(\rho_{lose}(y))) \geq$$

$$\geq hU_H(\rho_{lose}(y)) + (1-h)U_L(\rho_{lose}(y)) - (\rho_{tie}U_H(\rho_{lose}(y)) + (1-\rho_{tie})U_L(\rho_{lose}(y))) =$$

$$= (h-\rho_{tie})(U_H(\rho_{lose}(y)) - U_L(\rho_{lose}(y))) \geq 0,$$

where the last inequality follows, because $\rho_{tie} > h$ and $U_H(\rho_{lose}(y)) \leq U_L(\rho_{lose}(y))$ by monotonicity of V. Therefore, it is indeed more profitable to lose against a type y than to win if one's type is $s < y$. This concludes the proof of global optimality for the bidders' problem.

Uniqueness of b as in (1) follows from the above argument as well, since upon tieing indifference has to hold in an ex-post equilibrium which yields exactly (1) after taking it into account that the equilibrium is symmetric and monotone. The only caveat is that the bid function is not determined at the (at most countably many) discontinuity points of U_H, U_L. At such a belief ρ, the optimal action in the continuation problem is not unique which introduces multiple optimal bids when the belief is ρ. However, there are at most countably many such jump points, so this multiplicity arises only for a small set of types, and for all other beliefs the equilibrium bid is pinned down by formula (1).[28] If V is not (strictly) monotone, then it is easy to show that the candidate bid function (1) is not strictly increasing.

[28] When the value function is smooth such discontinuty of W_H, W_L cannot occur and the equilibrium bid is unique for all x. Moreover, the function b is continuous in this case.

For the last result it is sufficient to prove that V is (strictly) monotone if and only if $U_H(\rho) < U_L(\rho)$ for all $\rho < 1$. First, if $U_H(\rho) < U_L(\rho)$ for all $\rho < 1$, then by Lemma 2 $V' < 0$ for all $\rho < 1$ whenever the derivative exists, which implies that V is indeed monotone decreasing. Second, suppose that for some $\rho^* < 1$ it holds that $U_H(\rho^*) \geq U_L(\rho^*)$. Then Lemma 1 implies that U_H is increasing, while U_L is decreasing in ρ, and thus for all $\rho \geq \rho^*$ it holds that $U_H(\rho) \geq U_L(\rho)$. Therefore, using the envelope formula (12) that holds for almost all ρ implies that for all $\rho > \rho^*$ it holds that $V(\rho) \geq V(\rho^*)$ and thus V is not (strictly) monotone decreasing.**Q.E.D.**

7.2 Appendix 2

Proof of the revenue comparison result for the English- and the second-price auctions considered in Proposition 7:

Proof. In the second price auction the middle bidder's bid is the revenue, and the bid function can be written as

$$b^{II} = v - [\widehat{\rho}_{tie}(s)U_H(\widehat{\rho}_{lose}(s)) + (1 - \widehat{\rho}_{tie}(s))U_L(\widehat{\rho}_{lose}(s))],$$

where $\widehat{\rho}_{tie}, \widehat{\rho}_{lose}$ are the relevant tieing and losing posteriors. These beliefs can be written as $\widehat{\rho}_{tie}(s) = \Pr(H \mid s_1 = s_2 = s > s_3) = \frac{s^2\rho_{tie}}{s^2\rho_{tie}+(2s-s^2)(1-\rho_{tie})}$, and $\widehat{\rho}_{lose}(s) = \frac{s^2\rho_{lose}}{s^2\rho_{lose}+(2s-s^2)(1-\rho_{lose})}$

Let us now calculate the revenue in the English auction. Let z be lowest of the three types, and s be the medium one. Then the revenue is equal to

$$b^E(s,z) = v - [\widehat{\rho}_{tie}(s,z)U_H(\widehat{\rho}_{lose}(s,z)) + (1 - \widehat{\rho}_{tie}(s,z))U_L(\widehat{\rho}_{lose}(s,z))],$$

where $\widehat{\rho}_{tie}(s,z) = \Pr(H \mid s_1 = s_2 = s > s_3 = z) = \frac{z\rho_{tie}}{z\rho_{tie}+(1-z)(1-\rho_{tie})}$, and $\widehat{\rho}_{lose}(s,z) = \Pr(H \mid s_1 > s_2 = s > s_3 = z) = \frac{z\rho_{lose}}{z\rho_{lose}+(1-z)(1-\rho_{lose})}$. To calculate the expected revenues let $g_{mid}(s) = 12s - 42s^2 + 60s^3 - 30s^4$ denote the density of the medium type, and let $h(z \mid s) = \frac{2\rho_{lose}(s)z+2(1-\rho_{lose}(s))(1-z)}{\rho_{lose}(s)s^2+(1-\rho_{lose}(s))(2s-s^2)}$ be the density of the low type given the medium types. Then the expected revenues from the two auctions can be written as $ER^{II} = \int_0^1 g_{mid}(s)b^{II}(s)ds$ and $ER^E = \int_0^1 g_{mid}(s)\int_0^s h(z \mid s)b^E(s,z)dzds$. After substituting in the relevant functional form assumptions about U_H, U_L, and the parameter values from the Proposition, we can use the above bid functions to show that indeed $ER^{II} > ER^E$.

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