Deficits, Gifts, and Bequests

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Abstract

What is the response in aggregate consumption to a deficit-financed tax cut? In theory, gifts and bequests motivated by altruism play an important role, but quantitative studies on fiscal policy often rely on two standard workhorse models: (1) infinite-horizon models, and (2) overlapping-generations (OLG) models. The former implicitly assumes perfect altruism, implying frequent transfers among family members, thereby possibly understating the response in aggregate consumption, while the latter a priori excludes them, possibly overstating the response. A plausible alternative is to allow for imperfect altruism. I study an OLG model, which nests the two standard models, but allows for arbitrary degrees of altruism. The concept of Markov-perfect equilibrium is used in order to deal with the strategic considerations, which arise when altruism is imperfect. After studying the stationary equilibrium I study the transition path of the economy when a tax cut is financed through deficits. The size and the dynamics of aggregate consumption in the benchmark economy is surprisingly similar to the one in the standard OLG model. Welfare implications, however, lie half-way between the two standard workhorse models.

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1 Introduction

Budget deficits are a contentious issue in policy debates. Some policy makers argue that substituting a budget deficit for current taxation leads to an expansion in aggregate consumption, while others disagree and point to the ramifications an increased tax burden has on future generations. According to the Ricardian view on budget deficits, aggregate consumption does not change since a deficit-financed tax cut leaves the present value of government expenditure unchanged and merely acts to postpone the incidence of taxes. In juxtaposition, the Samuelson-Diamond overlapping generations (OLG) model predicts that budget deficits enrich current generations at the expense of future generations. As a result aggregate consumption of current generations increases.

In his seminal paper, Barro (1974) revives the Ricardian view: an individual becomes part of an extended household (dynasty) through intergenerational transfers motivated by altruism. Thanks to these intergenerational links the individual fully internalizes the future repercussions of the current policy. Hence, a substitution of budget deficits for taxation cannot enrich current generations at the expense of future generations and as a result aggregate consumption remains unchanged in response to a deficit-financed tax cut.[1] While Barro's (1974) result requires transfers to be motivated by altruism it does not stipulate a particular degree of altruism, in particular, it does not depend on altruism being *perfect*, by which is meant that an individual attributes the same importance to someone else's well-being as to its own. Ricardian equivalence holds for allocations for which it is true that equilibrium transfers are an interior solution for all current and future generations, a requirement Barro refers to as *universally operative transfer motives*.

In reality we would think that for some families transfer motives are operative and for some they are not and which families have operative transfer motives changes over time due to previous decisions, luck, or government actions. Thus, it seems imperative for the answer to the question of the paper, which importantly rests on operative transfer motives, to allow for heterogeneity. In a theoretical paper Laitner (1988) shows that when altruism is *imperfect* – an individual cares more about her own well-being than about the well-being of someone else – and there is heterogeneity in lifetime-earnings ability, transfer motives fail to be universally operative. Specifically, equilibrium transfers are an interior solution only when the recipient household is poor relative to the

[1] Consider a two-period OLG economy. Generation 1 gets a tax-cut in period 1 which has to be paid back in period 2 by generation 2. If generation 1 had planned to give a transfer motivated by altruism to generation 2 prior to the tax cut then generation 1 will increase the transfer to generation 2 by the amount of the tax-cut. As a result government-induced transfers are privately undone and there is no response in aggregate consumption. However, if altruism is absent the tax cut acts like an increase in generation 1's income so that aggregate consumption increases.

donor household but equilibrium transfers are a corner solution when the wealth distribution between households is relatively balanced.

My paper is related to Laitner (1988) but is of a quantitative nature. Specifically, I study deviations from Ricardian equivalence due to imperfect altruism and incomplete markets in the form of the following question: ”What is the response in aggregate consumption to a deficit-financed tax cut?” From a modeling perspective the most important difference to Laitner (1988) is that households overlap for *many* time periods. This allows me to study the short-term effects of a policy, indeed, the main focus of the paper, and not just focus on long-run steady-states. Furthermore, having many time periods allows decision-makers to reconsider their decisions frequently, which in a setting with imperfect altruism is particularly important since game-theoretic issues (e.g. commitment) arise. Finally, in addition to heterogeneity in lifetime-earnings ability, which is related to heritability of ability, additional relevant sources of heterogeneity can be introduced. In the economy I study, households face an uninsurable idiosyncratic income process. As a result of an income shock, private transfers may only flow temporarily, a characteristic evidenced by the data and an important feature to take into account for the question at hand.

I follow the quantitative literature on fiscal policy which assumes an exogenously incomplete market structure. In particular, there is a risk-free asset available but no state-contingent assets and households cannot borrow. This literature typically uses either an infinite-horizon model, as for example Heathcote (2005), or the Diamond-Samuelson OLG model, as for example Kitao (2010), as building blocks.[2] A shortcoming of using these two canonical models as building blocks for the current question at hand, is that they implicitly make strong assumptions about the degree of altruism, leading to counterfactual implications about transfer behavior. The infinite-horizon model implicitly assumes perfect altruism, (see Laitner (1992) for a careful study of a perfect altruism model).[3] Perfect altruism implies that family members fully share resources and a timing indeterminacy of transfers; two features which are strongly at odds with the data. The first implication has been strongly rejected by Altonji, Hayashi & Kotlikoff (1992) using the PSID. In terms of the second implication, panel data on

[2]Nishiyama (2002) is a notable exception in the quantitative literature in that he allows for imperfect altruism by the parent household. His main focus is on studying the wealth distribution of the United States in a heterogeneous-agent OLG model with bequests and inter-vivos transfers. Agents live for at most 4 periods and consumption is residually determined.

[3]This modeling approach has the advantage that the individual household problems can be pooled and solved as a joint-maximization problem, a significant simplification of perfect altruism. Another strand in the literature employs other forms of altruism such as the joy-of-giving type which is also referred to as impure altruism since the donor derives pleasure directly from the act of giving and does not depend on the well-being of the recipient; see for example Abel & Bernheim (1991) and Andreoni (1989).

intra-family transfers suggest that there are clear timing patterns. For example, McGarry & Schoeni (1995) and Berry (2008) find that the transfer behavior in the Health and Retirement Study (HRS) is suggestive that they are especially likely to occur when the recipient is liquidity constrained. Cox (1990), provides evidence that gifts flow to liquidity constrained family members. Contrasting the predictions of a perfect altruism model with the data one would suspect that a model with perfect altruism understates the effects deficit financing has on aggregate consumption. On the other hand, one would expect an OLG model to overstate the response in aggregate consumption since intended transfers are a priori excluded but are shown to occur at least in certain circumstances.

In order to allow for transfers motivated by altruism without having to restrict the degree of altruism a priori, I follow Barczyk & Kredler (2010a,b). In terms of altruism the authors mean Becker-type preferences, which are preferences that take the well-being of someone else into account, say with a factor α, which has the interpretation of being the degree of altruism. They study a dynamic game in continuos-time between two players, both of which have these Becker-type preferences, and are infinitely-lived. They characterize Markov perfect equilibria and provide a building-block for dynamic heterogeneous-agents models. When using their framework as a building-block for an OLG economy, the resulting model nests an infinite-horizon model, which I will refer to as dynastic household model, that is, an Aiyagari-type model, and a standard OLG model,[4] as special cases. The model can be thought of as being indexed by a pair of parameters (α_1, α_2). The interpretation of that pair being the degrees of altruism of household 1 and household 2, respectively, taking on values in the unit square. When both α's are zero the model reduces to a standard OLG model and if both are one, a dynastic household model arises. I will refer to the model for values of α between zero and one as an OLG economy with imperfect altruism. Crucially, when altruism is imperfect the nature of the model becomes very different since strategic considerations arise. Strategic considerations arise since the incidence of transfers depends on the relative wealth of the players.[5] In order to pin down reasonable degrees of altruism (α_1, α_2) I follow Nishiyama (2002) and calibrate the degrees of altruism to match U.S. aggregate data on intended transfers and bequests as reported by Gale & Scholz (1994). The rate-of-time preference in the dynastic-household model, the standard OLG model, and the OLG

[4]The classifier “standard” is meant to highlight that intergenerational links are entirely absent as in the canonical Diamond-Samuelson OLG model.

[5]Consider the following static game between households 1 and 2: in the first stage of the game the players decide on a non-negative transfer. In the second stage they consume what is left. Player 1 and 2 are endowed with wealth w_1 and w_2. With, for example, log-utility it is easy to see that transfers flow if $\alpha w_1 > w_2$. In a dynamic framework wealth is endogenous which is why strategic considerations arise.

economy with imperfect altruism are calibrated to match the same wealth-to-GNP ratio.

This paper studies the following environment: the economy is a small open endowment economy. Time is continuous. At each point in time there is a continuum of young households and a continuum of old households. A family consists of one young household and one old household. The old household faces a constant probability of death. Upon the death of the old household, the young household becomes an old household, and thereby a new family is formed. Households face an uninsurable idiosyncratic income process and are not allowed to die with debt. Insurance for idiosyncratic risks are absent and remaining wealth of a deceased household is passed on to the following household. Taxes are lump-sum. Young and old households play Markovian strategies, which are consumption policies and transfer policies. An equilibrium is confined to the class of Markov-perfect equilibria. In the computations I use the standard OLG model and the dynastic economy as benchmarks in order to put the effects from the deficit-financed tax cut on aggregate consumption into perspective.

The measure I use to quantify deviations from Ricardian equivalence is the fraction consumed out of a \$1 tax cut over-and-above steady-state consumption. If Ricardian equivalence holds this measure is 0 and in an economy with hand-to-mouth consumers it is 1. Keep in mind that in the dynastic economy Ricardian equivalence does not hold because of incomplete markets.

When I take the average of this measure, where the average is taken for the duration over which the tax cut is in place, I find that out of a \$1 tax cut 22 cents is consumed in the dynastic economy, 37 cents is consumed in the standard OLG economy, and 39 cents is consumed in the OLG economy with imperfect altruism. Furthermore, the path of aggregate consumption resembles the one in the standard OLG economy but welfare implications lie half-way between the one from the two standard models.

First, let me explain why there is such a strong similarity to the standard OLG economy. In the equilibrium there is a large number of households for which decisions resemble the decisions in the standard OLG economy. One reason is that in the data the size of transfers is not large enough to justify high α's. In the model this basically means that it doesn't pay to rely on someone else so you might as well just look out for yourself. Another reason is that in the equilibrium transfers only flow when the recipient household has exhausted its own wealth. Why? A potential donor has strong incentives to delay transfers since she worries that the recipient eats up a transfer too quickly (over-consumes in the eyes of the donor) and then asks for more. However, when the potential recipient is broke the potential donor has control over the recipients consumption behavior i.e. the donor essentially becomes the family dictator.

Second, a priori we would have expected that the measure which quantifies deviations from Ricardian equivalence should be between the 22 cents from the dynastic economy and the 37 cents from the standard OLG model. This highlights that the model with imperfect altruism is not just an "in- between" of the two standard models but has a very different nature. Specifically, after the tax cut has been in place and in the years before the tax hike takes place aggregate consumption starts to exceed aggregate consumption in the standard OLG economy. This happens because of strategic considerations in anticipation of the future tax hike. In the model there are more transfers after taxes increase. Transfers increase since savings increase in anticipation of the future tax-hike. Since transfers are increasing in the donors wealth the anticipation of higher transfers leads to higher consumption by current and future recipients of transfers. In short, a potential recipient knows that she will benefit from the increased savings from the other through higher transfers and therefore does not save, or not as much, as households in the standard OLG economy

Finally, the intuition for why the resemblance to the standard OLG economy in terms of welfare implications disappears is that the deficit-financed tax cut crowds out private transfers. This mechanism (the crowding out of private transfers) is entirely absent in the standard OLG economy.

In short, the behavior of aggregate variables resembles the one's of the standard OLG economy, but, welfare implications lie half-way between the two standard models. It is worthwhile to emphasize that despite the strong resemblance to the standard OLG model, the model with imperfect altruism does allow for Barro's (1974) mechanism to potentially undo government-induced transfers through gifts and bequests. The standard OLG model a priori excludes such a possibility.

The remainder of the paper proceeds as follows. Section 2 describes the physical setting. After defining the equilibrium concept, intra- and inter-temporal optimality conditions are provided and discussed. The Euler equations are crucial since they provide insights into the strategic considerations that arise when altruism is imperfect. Furthermore, they highlight the differences in savings motives among the dynastic economy, the standard OLG economy, and the OLG economy with imperfect altruism. In section 3, I study the stationary Markov-perfect equilibrium. Figure (1) reveals differences and similarities in the savings behavior in the OLG economy with imperfect altruism and the standard OLG economy. This figure is of central importance in understanding why the dynamics of aggregate consumption in the OLG economy with imperfect altruism is similar to the one in the standard OLG economy in response to a deficit-financed tax cut. Having build an understanding of the stationary equilibrium, in which the government's budget is balanced, section 4 studies the response in aggregate consumption to a deficit-financed tax cut. I quantify, portray, and intuitively discuss the effects of deficit financing on aggregate consumption in the standard OLG economy, the dynastic economy, and the OLG economy with imperfect altruism. Section 5 analyzes the dynamic behavior of deficits, gifts, and bequests and provides welfare implications. Section 6 concludes.

2 Setting

2.1 Physical Environment

Time t is continuous. At each point in time, there is a continuum of old households and a continuum of young households. A family consists of one young and one old household. Variables for a young household are indexed with a 1 (first life-cycle stage) and variables referring to the old household are indexed with a 2 (second life-cycle stage). Roughly speaking, young corresponds to the age range 25 to 50 and old to the age range 50 and above. The old household faces a mortality hazard given by a Poisson rate δ. Upon the death of the old household, the young household becomes an old household, and a new young household enters the economy. Young and old households draw an income realization, y_i^1 and y_j^2, respectively, which are independent and follow a discrete-state Markov process. When the old household dies, wealth is accidentally bequeathed to the corresponding new old household. A new young household enters the economy with a (small) initial endowment which is proportional to her income realization with factor of proportionality denoted by m, i.e. $m \cdot y_i^1$. This is a modeling short-cut to account for initial wealth observed in the data without taking a stance on how this initial wealth comes about. Households can hold a non-negative amount w_t in an asset that pays a time-invariant rate of interest r. There is a given stream of government purchases G, which is financed through lump-sum taxes and deficits.

The per-period utility of a household is given by

$$U^s(c^s, c^{s'}) = u(c^s) + \alpha_s u(c^{s'}), \quad \alpha_s \in [0,1], \quad s \in \{1,2\}$$

where $u(\cdot)$ is a CRRA utility function, $s = 1$ is the first life-cycle stage, $s = 2$ is the second life-cycle stage, and α_s is the degree of altruism of a household in life-cycle stage s. An old household ranks consumption allocations with the criterion given by

$$J^2 = E_0 \left\{ \int_0^\tau e^{-\rho t} \left[u(c_t^2) + \alpha_2 u(c_t^1) \right] dt + \alpha_2 e^{-\rho\tau} V^e(Z_\tau, y^1) \right\}.$$

The rate-of-time preference is denoted by ρ. The random time of death is denoted by τ

and follows an exponential distribution.[6] The function V^e summarizes recursively the value of the consumption allocation of all related, future-old households. The value enters the current old household's payoff function weighted by α_2, due to the imperfect altruism, and discounted by ρ.[7] The variable $Z_\tau = w_\tau^1 + w_\tau^2$ is the sum of the life-cycle savings of the young household and the life-cycle savings of the old household, which become a bequest upon the death of the old household. Since the time of death is uncertain, Z is a random variable and therefore indexed by τ.

In order to define the function V^e, let V^2 be the value of being an old household with wealth Z, income realization y_i^2, being related to a young household with wealth $m \cdot y_j^1$ and income realization y_i^1. Let π_{ij} be the probability of the new young household entering with endowment realization j, given the new old household has endowment realization i; then, the function V^e is given by

$$V^e(Z, y_i^1) = \sum_j \pi_{ij} V^2 \left(m \cdot y_j^1, Z, y_j^1, y_i^2\right). \tag{1}$$

In words, V^e is the (instantaneous) expected value of a currently young household to become an old household if current wealth by the family is given by Z and the current income realization of the young household is y_i^1.

The young household ranks consumption allocations with the criterion given by

$$J^1 = E_0 \left\{ \int_0^\tau e^{-\rho t} \left[u(c_t^1) + \alpha_1 u(c_t^2)\right] dt + e^{-\rho\tau} V^e(Z_\tau, y^1) \right\}.$$

This criterion is almost mirror-symmetric to the one of the old household. This is because of the assumption that when the old household dies, the young household becomes itself an old household, and a new family is formed. It is important to note that the degree of altruism for the young household towards the old household is given by α_1 which can be larger, smaller, or equal to the old household's degree of altruism α_2. The special case of a dynastic household model arises when $\alpha_1 = 1 = \alpha_2$ and the standard OLG model when $\alpha_1 = 0 = \alpha_2$. Observe that in the former case the payoff function of the young and the old are identical so that a family has a joint criterion. In the latter case the criteria are entirely separate.

Since the mortality hazard is given by the Poisson rate δ, the probability of death over a short interval of time, Δt, is approximately $\delta \Delta t$. It follows that we can write the

[6]i.e. $F(t) = Pr(\tau < t) = 1 - e^{-\delta t}$ where δ is the Poisson hazard rate mentioned before.

[7]It is important to emphasize that while this function can be interpreted as a "bequest motive" it should not be confused with an ad hoc bequest function as, for example, in the case of warm-glow altruism. Just think of the case when $\alpha_2 = 1$; then this criterion is simply the one of an infinitely-lived household.

criterion for the old household, thereby eliminating τ, as follows

$$J^2 = \int_0^\infty e^{-(\rho+\delta)t} \left\{ u(c_t^2) + \alpha_2 u(c_t^1) + \alpha_2 \delta V^e(Z_t, y^2) \right\} dt \tag{2}$$

and for the young household by

$$J^1 = \int_0^\infty e^{-(\rho+\delta)t} \left\{ u(c_t^1) + \alpha_1 u(c_t^2) + \delta V^e(Z_t, y_t^2) \right\} dt. \tag{3}$$

As is common in this formulation, the discount rate ρ is increased by the mortality hazard rate δ (Yaari (1965)). Furthermore, a pair of households overlap an expected $1/\delta$ years.

Households play Markovian strategies and cannot commit.[8] The payoff-relevant state for a pair of households is given by $x \equiv (t, w^1, w^2, y^1, y^2)$, where w^1 denotes the wealth of the child household, w^2 denotes the wealth of the parent household, disposable income of the young is denoted by $y^1 = (1-\tau)Y^1$ and of the old by $y^2 = (1-\tau)Y^2$. Labor supply is inelastic, so that the tax is a lump-sum tax. Decision-making is simultaneous. Strategies are measurable with respect to the current state and are consumption $c^1(x) \geq 0, c^2(x) \geq 0$, and transfers $g^1(x) \geq 0, g^2(x) \geq 0$. They residually determine the laws of motion for wealth

$$\begin{aligned} \dot{w}^1(x) &= rw^1(x) + y^1 + g^2(x) - c^1(x) - g^1(x) \\ \dot{w}^2(x) &= rw^2(x) + y^2 + g^1(x) - c^2(x) - g^2(x). \end{aligned}$$

The laws of motion, as they are written here, describe the evolution of wealth for the two types of households during a life-cycle stage. When the old household dies, the young household obtains the remaining assets from the old household, which is a discrete change in its state.

The government finances a constant expenditure stream G using a combination of deficits and lump-sum taxes. The flow version of the government budget constraint is

[8] There is a substantial mathematical literature on dynamic games in continuous time referred to as differential games. Historically, mathematicians have studied zero-sum differential games but in the past two decades non-zero sum differential games have become prominent especially in operations research, management science, and industrial organization. In the jargon of that literature the game setting employed here would be referred to as a closed or feedback loop.

given by[9]

$$\dot{D}_t = rD_t + G - \tau_t Y, \quad \text{given} \quad D_t = D.$$

2.2 Equilibrium Definition

An equilibrium is given by feasible policies for young households, $\{c^1(x), g^1(x)\}$, for old households, $\{c^2(x), g^2(x)\}$, and distributions $\{\lambda_t\}$ of households over the state-space, such that, given the world interest rate r, the government policy rules $\{\tau_t, D_t, G_t\}$, and an initial distribution λ_0,

1. $\{c^1(x), g^1(x)\}$ maximizes J^1, given the old household's strategy $\{c^2(x), g^2(x)\}$, for any x, and $\{c^2(x), g^2(x)\}$ maximizes J^2, given the young households strategy $\{c^1(x), g^1(x)\}$, for any x,

2. the governments budget constraint holds $\dot{D}_t = rD_t + G - \tau_t Y, D_0 = \bar{D}$ for each time t, and

3. the probability measure λ follows the law of motion induced by the policies.

2.3 Best Response: Hamilton-Jacobi-Bellman Equation

Let $V^1(x)$ and $V^2(x)$ be the value for a young household and an old household, respectively, when the state is given by x. Denote a partial derivative with respect to a continuous state, i.e. w^1 or w^2, by subscripts; for example, $V^1_{w^1}$ is the partial derivative of the young household with respect to its own wealth and $V^1_{w^2}$ is the partial derivative of the young household with respect to the related old household's wealth. The function V^e is given by (1).

The value function V^1 for the young household, given the strategy $\{c^2(x), g^2(x)\}$ of the old household, satisfies the following partial differential equation, known as the

[9]The government budget constraint over a short time interval Δt is

$$G_t\Delta t + rD_{t-\Delta t}\Delta t = \tau Y\Delta t + (D_t - D_{t-\Delta t}) \Rightarrow \Delta D = (G - \tau Y + rD)\Delta t$$

Hamilton-Jacobi-Bellman equation (HJB)

$$\begin{aligned}\rho V^1 = & \underbrace{\alpha_1 u(c^2) + \left(rw^2 + y^2 - c^2 - g^2\right) V^1_{w^2}}_{\text{old household}} + \underbrace{(rw^1 + y^1) V^1_{w^1}}_{\text{young household}} + \\ & + \underbrace{\kappa_1 \left(V^{1'} - V^1\right)}_{\text{income uncertainty}} + \underbrace{\kappa_2 \left(V^{1'} - V^1\right)}_{\text{income uncertainty}} + \underbrace{\delta \left(V^e - V^1\right)}_{\text{aging uncertainty}} + \\ & + \underbrace{\max_{g^1 \geq 0} \left\{g^1 \left(V^1_{w^2} - V^1_{w^1}\right)\right\}}_{\text{transfer choice}} + \underbrace{\max_{c^1 \geq 0} \left\{u(c^1) - c^1 V^1_{w^1}\right\}}_{\text{consumption choice}}\end{aligned} \tag{4}$$

where κ_1 and κ_2 denote the income hazard rates for the young and the old household, respectively. The mortality hazard rate is denoted by δ.

The value function V^2 for the old household, given the strategy $\{c^1(x), g^1(x)\}$ of the young household, satisfies the following HJB

$$\begin{aligned}\rho V^2 = & \underbrace{\alpha_2 u(c^1) + \left(rw^1 + y^1 - c^1 - g^1\right) V^2_{w^1}}_{\text{young household}} + \underbrace{(rw^2 + y^2) V^2_{w^2}}_{\text{old household}} + \\ & + \underbrace{\kappa_2 \left(V^{2'} - V^2\right)}_{\text{income uncertainty}} + \underbrace{\kappa_1 \left(V^{2'} - V^2\right)}_{\text{income uncertainty}} + \underbrace{\delta \left(\alpha_2 V^e - V^2\right)}_{\text{mortality uncertainty}} + \\ & + \underbrace{\max_{g^2 \geq 0} \left\{g^2 \left(V^2_{w^1} - V^2_{w^2}\right)\right\}}_{\text{transfer choice}} + \underbrace{\max_{c^2 \geq 0} \left\{u(c^2) - c^2 V^2_{w^2}\right\}}_{\text{consumption choice}}.\end{aligned} \tag{5}$$

The left-hand side of equations (4) and (5) is the flow value of the optimal program. For a given state x, the first two lines on the right-hand side of the HJBs are fixed. The terms "income uncertainty", "aging uncertainty", and "mortality uncertainty" capture the sources of uncertainty present in the economy. Over a small interval of time Δt, the probabilities that the income of the young or the old households change are given by $\kappa_1 \Delta t$ and $\kappa_2 \Delta t$, respectively. The prime in the term "income uncertainty" indicates an income realization different from the current one. The difference in the value functions is the change in value which would occur if the current income realization jumps to an adjacent level. The interpretation of the terms "aging/mortality uncertainty" is similar, except that the value which arises conditional on the mortality hazard is given by the function V^e, which enters the old household's HJB discounted with α_2, due to the imperfect altruism.

The maximization problem with respect to transfers is a linear maximization problem in transfers. It instructs the decision maker, for example the old household, to provide transfers if $V^2_{w^2} > V^2_{w^1}$. This is intuitive since the inequality indicates that the net marginal benefit of transferring a unit of wealth from the old household to the young household is positive.[10] Since the old household loses the value $-V^2_{w^2}$ from a small decrease in her wealth but gains the value $V^2_{w^1}$, which is how she values a small addition to the young's household wealth, adding up these two values is the net marginal benefit of a transfer. Since this is a linear maximization, and time is continuous, the maximizer is a mass point, i.e. $g^2 = \infty$. The interpretation of this mass point is that the donor can induce an instantaneous jump in the state in order to ensure $V^2_{w^1} = V^2_{w^2}$.

The expression in the second maximization problem is the Hamiltonian of a standard consumption-savings problem. Given the current state, the choice of consumption influences the direction of the state through $\dot{w}^s$. An increase in consumption increases current utility but enters negatively into $\dot{w}^s$, an effect which is priced at $-V^s_{w^s}$. Thus, this maximization problem instructs the household to choose consumption to the point where $u_c(c_s) = V^s_{w_s}$. A notable feature about this maximization problem is that it is independent of current actions of the other player. It follows that at a given point in time, the best consumption response is constant; a significant simplification of continuous time, which is particularly advantageous when computing a solution of the model.[11] The technical reason for this is that second-order effects vanish as time becomes continuous, and, thus, $V^s_{w^s}$ is a function of the *current* wealth levels of the households and not the wealth levels in the next time-period as is the case in discrete time.[12]

Finally, if one of the agents has no wealth, for example the young household, and if the young household is constrained, i.e. $u'(c^1) > V^1_{w^1}$, the old household can dictate the consumption of the young household, i.e. $c^1 = y^1 + g^2$. If

$$V^2_{w^2} = u'(c^2) > \alpha_2 u'(y^1)$$

then $g^2 = 0$. Otherwise the old household equalizes her c^2- and g^2-margins[13]

$$V^2_{w^2} = u'(c^2) = \alpha_2 u'(y^1 + g^2).$$

[10]In mathematical terms, this is the directional derivative in (w^1, w^2)-space of moving in the direction $(1, -1)$ when w^1 is on the horizontal axis.

[11]This circumvents the need to compute a fixed-point, thereby eases the computational burden, and circumvents the difficulty of multiple Nash equilibria for that point in the state-space.

[12]For a more extensive discussion on the technical issues related to the maximization with respect to transfers and consumption see Barczyk & Kredler (2010a).

[13]In appendix (A.1) a more careful description is provided on how transfers are determined when one of the households in the family has no wealth.

2.4 Saving Motives: Euler Equations

An intuitive way of understanding the different saving motives in the economies with imperfect, perfect, and no altruism is to study the Euler equations. For example, differentiating the old household's HJB (5) with respect to w^2 results in the old household's Euler equation[14]

$$\frac{d}{dt}u'(c^2) = \underbrace{(\rho - r)}_{\infty\text{-lived}} u'(c^2) - \kappa \underbrace{\left[u'(c^{2'}) - u'(c^2)\right]}_{\text{precautionary}} - \delta \underbrace{\left[\alpha_2 V_Z^e - u'(c^2)\right]}_{\text{bequest motive}} + \underbrace{\left[V_{w^1}^2 - \alpha_2 u'(c^1)\right] \frac{\partial c^1}{\partial w^2}}_{\text{strategic}} - \underbrace{\left(V_{w^1}^2 - V_{w^2}^2\right)}_{\text{gift motive}} \left(\frac{\partial g^2}{\partial w^2} - \frac{\partial g^1}{\partial w^2}\right)$$

Suppose for now that the young household is currently not a donor; then $\partial g^1/\partial w^2 = 0$. If the old household provides transfers $V_{w^1}^2 = V_{w^2}^2$, and if it does not, $\partial g^2/\partial w^2 = 0$. Thus, we write

$$\frac{d}{dt}u'(c^2) = \underbrace{(\rho - r)}_{\infty\text{-lived}} u'(c^2) - \kappa \underbrace{\left[u'(c^{2'}) - u'(c^2)\right]}_{\text{precautionary}} - \delta \underbrace{\left[\alpha_2 V_Z^e - u'(c^2)\right]}_{\text{bequest motive}} + \underbrace{\left[V_{w^1}^2 - \alpha_2 u'(c^1)\right] \frac{\partial c^1}{\partial w^2}}_{\text{strategic}} \tag{6}$$

The main difference between (6) and the Euler equations in the economies with perfect, and no altruism is the "strategic" term. This term has first been discussed by Barczyk & Kredler (2010a). It is of central importance since it provides insight into the strategic considerations, which arise in savings decisions, when altruism is imperfect and commitment is absent. In order to obtain a better understanding for all later results, it is imperative to provide an interpretation for this term.

Suppose the old household considers a hypothetical deviation from the equilibrium path by increasing savings. According to standard Euler logic, this deviation has to be not profitable along an equilibrium path. Consider the consequences of the hypothetical deviation in three discrete time periods. In the first period, the increase in saving leads to a reduction in consumption, which comes at a cost $-u'(c_1^2)$. In the second period the old household has a higher level of wealth. Following the logic of Euler this addition is consumed by the old household in order to revert back to the equilibrium path. As in the standard Euler equation this yields a marginal benefit of

[14] In classical optimal control theory the resulting equations would be known as the co-state equations.

$\beta R u'(c_2^2)$. But now there are additional effects since the policy function of the young household is also a function of the old household's wealth. When the young household observes the higher level of wealth of the old household the young household response by changing her consumption by $\partial c_2^1/\partial w_2^2$. For the sake of the example, suppose that the response in the young household's consumption is to increase consumption, i.e. $\partial c_2^1/\partial w_2^2 > 0$.[15] As a consequence, the old household obtains additional utility from the young household's consumption given by $\alpha_2 \beta u'(c_2^1)\partial c_2^1/\partial w_2^2$ which provides an additional incentive to save, i.e. this term enters the Euler equation with the same sign as does the interest rate. On the other hand, an increase in consumption of the young household's consumption in the second period, means that in the third period it will have fewer resources. Since the old household values the wealth of the young household with $V_{w^1}^2$, the increase in the young household's consumption comes at a cost of $-\beta^2 R V_{k^1}^2 \partial c_2^1/\partial w_2^2$. This term captures information which is off the equilibrium path, a hallmark, of subgame perfection. Furthermore, it provides a disincentive to save, i.e. this term enters the Euler equation with the same sign as the rate-of-time preference does. Adding up the different pieces and imposing that the contemplated deviation is not profitable yields

$$u'(c_2^2) = \underbrace{(R\beta)^{-1}}_{\infty\text{-lived}} u'(c_1^2) + R^{-1} \underbrace{\left[\beta R V_{k^1}^2 - \alpha_2 u'(c_2^1)\right] \frac{\partial c_2^1}{\partial w_2^2}}_{\text{strategic}}.$$

The other three terms in the Euler equation (6) are more standard. The interpretation of the term "bequest motive" is fairly straightforward. If $\alpha_2 V_Z^\tau > u_c(c^2)$ this term acts in the same way as the interest rate. It provides an additional incentive to save since the marginal value of \$1 is valued higher when given to the young household in the form of bequests than consuming it. The interpretation of the term "precautionary" signifies the motive to save due to precautionary reasons related to the uncertainty of income. It provides an incentive to save when the marginal utility of consumption in the adjacent income realization, the one denoted by a prime, is larger than currently is the case. This term enters then with the same sign as does the interest rate.

The dynastic case follows when setting $\alpha_1 = 1 = \alpha_2$ in (2) and (3), respectively. The two individual decision problems can then be pooled into a single joint-maximization problem. The dynasty's payoff relevant state variables are given by their joint capital stock $K_t = w_t^1 + w_t^2$ and the endowment realization of the young household w_t. Thus,

[15]This is not unreasonable, since a higher wealth level by the old household implies, ceteris paribus, that transfers are more likely and larger on the one hand, and on the other, reduces the likelihood that the young household has to provide transfers to the old household.

consumption and saving decisions depend only on total family resources but not on how they are distributed. When altruism is perfect, $V^2_{w^1} = V^2_{w^2} = V^1_{w^1} = u'(c^1)$, and the Euler equation (6) becomes

$$\frac{d}{dt}u'(c^2) = \underbrace{(\rho - r)}_{\infty\text{-lived}}u'(c^2) - \kappa \underbrace{\left[u'(c^{2'}) - u'(c^2)\right]}_{\text{precautionary}} - \delta \underbrace{\left[V^e_Z - u'(c^2)\right]}_{\text{bequest motive}} \tag{7}$$

Euler equation (7) is like (6) without the strategic term.

The case without altruism follows when setting $\alpha_1 = 0 = \alpha_2$ in (2) and (3), respectively. Evidently, the payoff relevant state for the old household is its own wealth. For the young household the payoff relevant state also includes the wealth of the old, since in the event of their death they are accidentally bequeathed. Due to the absence of altruism, there is no transfer motive, $\partial c^1/\partial w^2 = 0$, and the Euler equation (6) becomes

$$\frac{d}{dt}u'(c^2) = \left[\underbrace{(\rho + \delta)}_{\text{Yaari (1965)}} - r\right] u'(c^2) - \kappa \underbrace{\left[u'(c^{2'}) - u'(c^2)\right]}_{\text{precautionary}} \tag{8}$$

Here, the effect of uncertain lifetimes is clearly seen in that the rate-of-time preference includes the hazard rate of death (Yaari (1965)). Since altruism is absent there are no terms which take into account anything beyond one's own lifetime.

A final remark concerns the issue of commitment. If commitment is assumed it follows that $\partial c^1/\partial w^2 = \partial g^2/\partial w^2 = \partial g^1/\partial w^2 = 0$ in (6). The Euler equation would then be "in-between" the one with perfect altruism and no altruism. Furthermore, in this case, the Euler equation is an ordinary-differential equation which does not take into account what would happen off the equilibrium path, just like the Euler equations (7) and (8). In the absence of commitment, however, the Euler equation is a partial-differential equation, which signifies that information off-equilibrium enters the equilibrium path, a manifestation of subgame perfection.

3 Stationary Equilibrium

It is straightforward, but useful, to exclude the policy functions from the dynastic economy and the standard OLG economy as possible candidate equilibrium policies for the OLG economy with imperfect altruism. Transfer policies in the standard OLG economy are not Markov-perfect since they stipulate that for any given state of the world no transfers are provided. However, the policy of not providing transfers is not credible due to the altruistic preferences. Suppose a shock leaves a household without any economic resources so that consumption would be zero in the absence of transfers. Then it is easy to see that with CRRA utility it would be optimal for the related household to provide transfers. Thus, policies from the standard OLG economy are not Markov-perfect. Dynastic policies are such that what matters for household consumption is only total dynasty resources but not how they are distributed between the two households. In particular, the ownership of resources between the two households becomes irrelevant. But if one of the households owns all the wealth it is optimal for that household to provide the recipient with a consumption level lower than her own due to imperfect altruism. Thus, policies from the dynastic economy are not Markov-perfect.

In order to solve for a Markov-perfect equilibrium in the OLG economy with imperfect altruism numerical methods need to be used.[16] I first solve for a *stationary* Markov-perfect equilibrium by using backward iteration on the system of HJBs given by equations (1), (4), and (5). The equilibrium definition for a stationary Markov-perfect equilibrium is as defined above except that time does not play a role and $\dot{D} = 0$. Backward iteration on the system of HJBs starts with a guess for value functions V^1 and V^2. A guess for V^e can then be obtained from V^2. The guess I use for V^1 and V^2 has the interpretation of them being the value functions of a "final" pair of overlapping generations. Obtaining these value functions reduces to solving the value functions of infinitely-lived households with an increased rate-of-time preference due to the probability of death. An algorithm to solve the value functions for infinitely-lived households when altruism is imperfect is provided and discussed by Barczyk & Kredler (2010b).

In the remainder of section (3) I briefly discuss the calibration of the model, study a stationary Markov-perfect equilibrium in the OLG economy with imperfect altruism, and highlight differences and similarities in savings behavior of this economy in contrast to the standard OLG economy.

3.1 Calibration

A young household can be thought of belonging to the age group 25 to 50 and an old household belonging to the age group 50 and above. The mortality hazard δ is a Poisson rate and set to 4%. This implies an expected duration of each life-cycle stage of 25 years. A young household enters the economy with an initial endowment, which is proportional to her income realization, with factor of proportionality denoted by m, i.e. $m \cdot y_i^1$. This is a modeling short-cut to account for initial wealth observed in the data without taking a stance on how this initial wealth comes about. The income process follows a

[16]Methods to compute stochastic continuous-time problems are especially well-studied by Dupuis & Kushner (2001); for the purposes of the current paper see chapter 5.

Table 1: *Parameters set outside of the model. Common across three economies.*

γ	r	δ	κ	m	G/GNP
2	4%	4%	10%	1	20%

three-state Markov chain calibrated to the U.S. income distribution for households of ages 25 to 65. An income realization can be low, medium, or high. The income for an old household is a weighted average of the realization of the three-state Markov chain and the realization of social security income corresponding to the income realization. The weights capture the average duration an old household spends earning an income from work and obtaining social security. Since the average duration of life-cycle stage 2 is 25 years the weight on work is 3/5 (15 years out of 25 years) and on social security 2/5 (10 years out of 25 years). Finally, note that while there are only three income realizations, there are 9 different family types, which differ in terms of income profiles, since each family consists of a young and an old household. I will refer to a family of type 1 to consist of a young household with a low income realization and an old household with a low income realization. A family of type 2 to consist of a young household with a medium income realization and an old household with a low income realization, and so on up to a family of type 9, which consists of a young household with a high income realization and an old household with a high income realization.

Table (1) lists the parameter values which are set outside of the model for the three economies. As already mentioned the mortality hazard δ is a Poisson rate and set to 4%. The coefficient of relative risk aversion γ is assumed to be 2 and the world interest rate r to be 4%. There is only one hazard rate for a jump in income, denoted by κ and given by 10%, since I assume that income can only jump to directly neighboring levels with the same probability. The multiple m, which together with the income realization determines the initial wealth endowment of a young household, is 1. Government consumption is 20% of GNP.

The parameters ρ, the rate-of-time preference, α_1, the degree of altruism of a young household, and α_2, the degree of altruism of an old household are calibrated within the model. In particular, α_1 and α_2 are chosen to match aggregate measures of intergenerational transfers in the U.S. economy reported by Gale and Scholz (1994). The authors estimate the importance of *intended* intergenerational transfers as a source of capital accumulation using the 1983-86 Survey of Consumer Finances. Intended transfers are defined to include financial support given to other households, trust accumulations, and life insurance payments to children. Bequests are excluded since they are not necessarily intentional. The annual flow of intended transfers as a percentage of aggregate

Table 2: *Calibration targets.*

Target	Standard OLG	OLG with α	Dynasty
Wealth/GNP	3.4	3.4	3.4
Transfers/Wealth	n.a.	0.35%	n.a.
Bequests/Wealth	n.a.	1.06%	n.a.

Table 3: *Calibrated parameters.*

Parameter	Standard OLG	OLG with α	Dynasty
α_1	0	0.15	1
α_2	0	0.28	1
ρ	3.65%	3.6%	5%

net worth is 0.53% [17] and is made up of 0.35% of support given to adult family members, 0.12% of trusts, and 0.05% of life insurance. The annual flow of bequests as a percentage of aggregate net worth is 0.88% .

The 0.35% of support given to adult family members is an appropriate counterpart in the data to the flow of annual gifts generated by the model economy. The transfers-to-wealth ratio helps to pin down α_1 and α_2 since transfers are increasing in the α's. The bequests-to-wealth ratio helps to separately identify α_2 since bequests increase in α_2 but not in α_1. I group the annual flows, all as a percentage of aggregate net worth, of trust accumulations, 0.12%, of life insurance payments, 0.05%, together with the 0.88% annual flow of bequests. This measure is a good counterpart to the flow of aggregate bequests generated by the model economy since bequests in the model are also of an accidental and intentional nature. Values for ρ in the standard OLG economy, the dynastic economy, and the OLG economy with imperfect altruism are obtained by choosing ρ to match a wealth-to-GNP ratio of 3.4. The values of the calibration targets are summarized in table (2) and the calibrated values are shown in table (3).[18]

[17] This number appears to be very small which is because it is an annual flow. Converting this flow into a stock, Gale and Scholz argue that intended transfers are the source of at least 20% of aggregate net worth.

[18] It is worthwhile to note that the interpretation of the size of α is only meaningful in conjunction with the magnitude of the coefficient of relative risk aversion (here $\gamma = 2$). This point is explained in section (A.2) of the appendix.

3.2 Equilibrium Characteristics

There are three key features the computed stationary Markov-perfect equilibrium of the OLG economy with imperfect altruism displays: (1) A household, which is a recipient of an intended transfer, has exhausted its own wealth, that is, $w = 0$. (2) The realized consumption path of a recipient displays a downward jump upon entering a transfer region. (3) Policies for families with a relatively balanced intra-family wealth distribution are increasingly similar to the policies of families in the standard OLG economy.[19] The features of the equilibrium are qualitatively similar to Barczyk & Kredler (2010b), Laitner (1988), and to the well-known equilibrium from two-period models which display, what has been referred to as, a *Samaritan's dilemma* as for example studied by Lindbeck & Weibull (1988) or Bruce & Waldman (1990).

In order to explain the economics behind result (2) it is useful to explain the Samaritan's dilemma in discrete time. The Samaritan's dilemma describes the possibility that an individual consumes more in period 1 in anticipation of period 2 transfers than is optimal from the perspective of the donor. Consider a deterministic two-period game with two players. In order to solve the model we require an equilibrium to be subgame-perfect. It is easy to solve for the transfer function in the second period. This function is decreasing in the wealth of the recipient. If in the first period the individual anticipates to obtain transfers then an additional unit saved for period 2 is "taxed" by the donor by a reduction in transfers. As a result, the future recipient of transfers over-consumes in the eyes of the future donor in period 1. But subgame perfection renders a threat by a donor to not provide transfers as not credible, since ex-post it is nonetheless optimal to provide transfers. Thus, altruism puts the donor into a dilemma.[20] On the contrary, when the household does not expect to receive transfers, then the marginal value of saving equals the marginal utility of consumption and consumption in period 1 is as in the standard selfish case. In continuous time over-consumption by the future recipient manifests itself through a downward jump in the realized consumption path.

Feature (1) says that if a household receives an intended transfer it has no wealth on its own. Mathematically, the reason that there are no intended transfers within the state-space, i.e. $g^1 = 0$ and $g^2 = 0$ for all $w^1 > 0$ and $w^2 > 0$, is that the inequalities

[19] Recall from before that the policies from the standard OLG economy are never the actual equilibrium policies. It may still be the case that in certain parts of the state-space the Markov-perfect policy coincides with the policy in the standard OLG economy. This, however, can only be a knife-edge case since in the Euler equations there is the additional strategic term. While the strategic term becomes very small it only vanishes as a knife-edge case.

[20] This is also related to a resource extraction problem (fully share resources): The problem is over-extraction of the common resource since the costs of one's own consumption on the other's resources are not fully internalized. Similarly, there is over-consumption if altruism is less than perfect, since the costs of consumption out of common family resources in period 2 are not fully internalized.

$V^1_{w_2} < V^1_{w_1}$ and $V^2_{w_1} < V^2_{w_2}$ hold throughout. Recall the discussion on the interpretation in section (2.3) and the HJBs (4) and (5) which tell us that transfers do not occur within the state-space as long as these two inequalities hold. Simply speaking this means that either household prefers to do its own saving as opposed to reducing her own wealth and contributing to the savings of the other. The economic reason is that a household fears that resources transferred will be consumed at a rate faster than desired by the donor, after which the recipient household comes back and asks for more (consider the extreme case when a big transfer is made at once by an altruistic household to an entirely selfish household in the hope that the recipient household uses it to smooth its consumption). Since the donor household would prefer the recipient household to save but cannot force the household to do so it prefers to do its own saving and provide transfers only when the other household is without wealth. At this stage, the donor has substantial control over the consumption behavior of the recipient household since it can ensure a consumption rate by the recipient which is to the liking of the donor. The donor essentially becomes the "family dictator" and can implement her preferred allocation.

Another incentive to delay transfers is due to uncertainty in income and longevity. Suppose the degree of altruism of the old household is one and the young household is entirely selfish. If the old household provides a transfer "too early" it may wind up being impoverished due to low realizations of income or living longer than expected. In either case it cannot count on support from the selfish young household and therefore prefers to save herself. To summarize, strategic considerations and risks lead to strong incentives to delay transfers until the household dies or the other household is broke. In terms of the timing of intended transfers, the equilibrium feature (1) is qualitatively in line with documented facts from panel data on intra-family transfers, as pointed out in the introduction. [21]

Feature (3) of the equilibrium becomes apparent from figure (1). It portrays the savings behavior for the standard OLG economy and the OLG economy with imperfect

[21] An interesting implication of this result is that it suggests a failure of neutrality to certain types of unexpected redistributive policies. Suppose there is a one-time unexpected lump-sum transfer of wealth, say Tr, from the young generation to the old generation. Since the value function of the old is increasing in the direction $(k_1 - Tr, k_2 + Tr)$ the old household would have no reason to privately undo the government's induced transfer. In contrast, in the dynastic economy this redistribution would be neutral since only total dynasty resources matter but not how they are distributed. Furthermore, we would expect the welfare consequences of this redistribution to differ vis-á-vis the standard OLG model. While the value functions in the OLG economy with imperfect altruism do change, the change is attenuated since households care about both capital stocks, i.e. $V^2_{w_2} > 0$ and $V^2_{w_1} > 0$, whereas households in the standard OLG economy do not attribute any value to someone else's resources. Thus, while the redistribution in neither economy would be neutral the welfare consequences would differ. Neutrality to this redistribution in terms of allocations and welfare only holds in the dynastic economy.

altruism for families of type 1 (low,low), 3 (high,low), 7 (low,high), and 9 (high,high). Black represents the standard OLG economy and red the OLG economy with imperfect altruism. Along the horizontal axis is the level of wealth in $000's of the old household and along the vertical axis is the level of wealth in $000's of the young household. A particular point in the figure constitutes a particular state $x = (y^1, y^2, w^1, w^2)$ in the state-space X. The arrow emanating from that point represents the direction and the magnitude of the continuous state, i.e. $\dot{w}^1$ and $\dot{w}^2$, at that particular state x. The 4 different graphs represent jumps in the discrete state, i.e. y^1 and y^2.

Consider what happens if we take a ray emanating from the origin and start to rotate it beginning from the horizontal axis: we are tracing out points such that the wealth distribution between the old household and the young household goes from one in which the old household owns all the wealth to an increasingly balanced one until the ray reaches the 45-degree line at which point both households own the same fraction of wealth. Afterwards the wealth distribution starts to be tilted in favor of the young household until the ray coincides with the vertical axis and the young household owns all the wealth. What happens is that savings behavior in the standard OLG economy increasingly resembles the one in the OLG economy with imperfect altruism when families have a relatively balanced intra-family wealth distribution, that is, feature (3). The intuition is that the informal safety-net, which is provided by private transfers, becomes less important for consumption-savings decisions since the possibility of obtaining transfers is remote. In terms of the Euler equations this means that the strategic term becomes less important and (6) becomes increasingly similar to (8). On the flip side, if the intra-family wealth distribution is relatively unequal, the informal safety-net plays an important role and as a result savings policies differ starkly. Consider, for example, the graph for family type 1. Choose a point at which the old household has a high level of wealth and the young household a low level of wealth. The savings policy of the imperfectly altruistic young household points to the region where she is broke but the savings policy in the standard OLG economy does not. The former household has additional incentives to consume since the old household provides transfers when the young household has no wealth.

4 Aggregate Consumption

4.1 Deficit-Financed Tax-Cut

The Ricardian insight is that a government cannot indefinitely finance a given expenditure stream through borrowing and reduced taxes. Eventually taxes have to be raised in

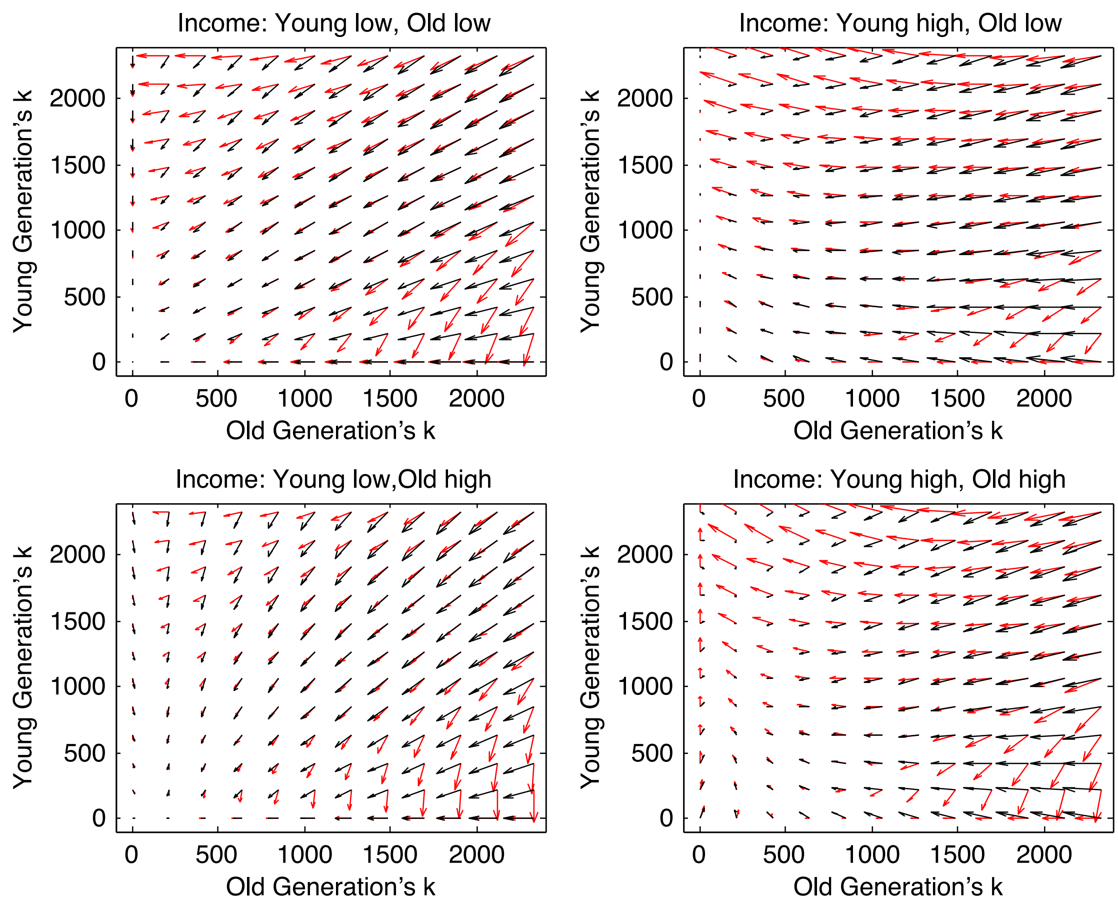
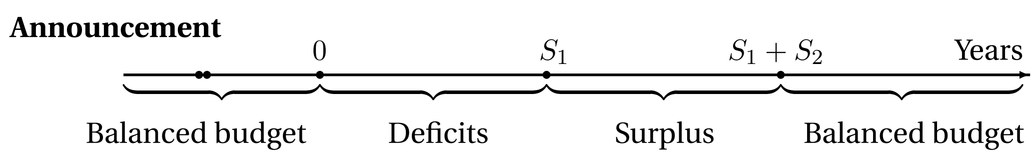


Figure 1: *Wealth evolution in the standard OLG economy (black) and the OLG economy with imperfect altruism (red). Each box corresponds to a certain realization of income for the young household and the old household; there are nine such combinations but only four are shown. Along a horizontal axis is wealth (k) for the old household and along a vertical axis is wealth (k) for the young household. At a particular point, the arrows indicate the direction and the "speed" at which the young's wealth and old wealth change.*

order for the government's inter-temporal budget constraint to hold. The costs of the resulting tax increase might then be borne by individuals other than the one's who benefited from the reduction in taxes. The Ricardian equivalence proposition tells us that the timing of taxes has no effect on the equilibrium allocation since private transfers undo the publicly induced transfers.

The time line illustrates the timing of taxes for the deficit-financed tax cut experiment. The new financing method is announced, the date is indicated by the two dots, prior to its implementation. Before the new financing policy is announced the economy is assumed to be in the stationary equilibrium discussed above. Recall that in the stationary equilibrium the government balances its budget by financing a constant expenditure stream, $\{G\}$, by levying a lump-sum tax on households. At time zero, the new method to finance government consumption is to reduce taxes for S_1 years, cover the shortfall by borrowing, and eventually reduce the accumulated debt to the level it was in the stationary equilibrium. The reduction in debt takes place over the following S_2 years.



The numerical values for the financing experiment are as follows. The announcement of the new financing policy is made one year in advance to its implementation. It calls for financing 3% of government consumption with deficits and cutting the tax rate accordingly. The length of the deficit regime is assumed to be $S_1 = 25$ year, the average duration of one stage of the life-cycle.[22] When the 25 years have passed, the debt which has been accumulated over and above the debt level in the steady-state, is repaid in equal payments over the following $S_2 = 25$ years.[23] From then on the financing policy reverts to the one in the initial stationary equilibrium. Thus, the economy eventually converges to its original stationary equilibrium. The corresponding tax rates required to balance the budget for the various financing regimes are summarized by table (4).

[22]The implied debt to (stationary) GNP ratio after 25 years is 25.8% when the economy starts off with a zero debt level.

[23]I have also computed the same experiment with durations shorter and longer as the one shown here. Qualitatively the results do not change.

Table 4: *Tax rates in the three different financing regimes to finance the constant stream of government consumption.*

Financing regime	Balanced	Deficit	Surplus
Tax rate	23.2%	22.52%	25.1%

4.2 Computing the Transition Path

It is instructive to briefly explain how the economy's transition path is computed. As of time $S_1 + S_2$ the optimal policies are known. Why? The payoff-relevant state for the households are back to the ones in the stationary equilibrium. Since the aggregate state is not a payoff-relevant state for the household it is irrelevant what the density of households over the state-space is after the deficit-financed tax cut has ended. That is, at time $S_1 + S_2$ there is no reason why the distribution should have converged to the stationary one, most likely it has not, but since the density is not a state in the household's decision problem the optimal decisions are as in the stationary equilibrium. The reason the aggregate state is not a relevant state for the household is because taxes are lump-sum and there is no aggregate uncertainty.

Since the stationary value functions at time $S_1 + S_2$ are known, the system of HJBs can be used to iterate backwards with time becoming a state-variable. The payoff-relevant state for a household throughout the transition is given by $x = (t, w^1, w^2, y^1, y^2)$. With this procedure policies $\{c^1(x), g^1(x)\}$ for the young households, and policies for the old households $\{c^2(x), g^2(x)\}$ are computed. Since the economy is assumed to be in the stationary equilibrium prior to the announcement of the new financing policy, the aggregate state of the economy is given by the stationary distribution of households over the state-space, i.e λ_0=$\bar{\lambda}$. Using the stationary distribution as an initial condition and the obtained non-stationary policies from the backward iteration we can iterate forward[24] to obtain the densities throughout the transition, i.e. $\{\lambda_t\}$. Given the world interest rate r and the government policy rules $\{\tau_t, D_t, G_t\}$ we have all the objects required by our definition of an equilibrium.

4.3 Propensity to Consume out of Income Taxes

What is the response in aggregate consumption to a deficit-financed tax cut? An intuitive measure which captures the effects is the "propensity to consume out of income taxes" defined by Heathcote (2005) . It quantifies the change in consumption between the periods characterized by different tax rates, relative to the change in tax revenue

[24]This is known as the Kolmogorov forward equation.

between the same two periods. In the current financing scheme this is the difference between aggregate consumption at time t, denoted by C_t, and aggregate consumption in the steady state, denoted by $\bar{C}$, divided by the financing shortfall, denoted by ψG, where in the current exercise $\psi = 3\%$. In particular, the PCT is given by

$$\mathrm{PCT}_t \equiv \frac{C_t - \bar{C}}{\psi G}, \quad C_t = \int_X c(t,x)n(t,x)dx, \quad \bar{C} = \int_X \bar{c}(x)\bar{n}(x)dx$$

where n is the density at time t of households over the state-space X and $\bar{n}$ is the stationary distribution. The following decomposition turns out to be instructive

$$\mathrm{PCT}_t \equiv \underbrace{\frac{\int \tilde{c}_t(x)\bar{n}(x)dx}{\psi G}}_{\text{optimality}} + \underbrace{\frac{\int \bar{c}(x)\tilde{n}_t(x)dx}{\psi G}}_{\text{distributional}} + \underbrace{\frac{\int \tilde{c}_t(x)\tilde{n}_t(x)dx}{\psi G}}_{\text{small}}. \tag{9}$$

The first term, which I refer to as the "optimality" term, is the component which captures the effects coming from the changes in the optimal consumption rules of the households relative to the optimal consumption policies in the steady state. The second term, here referred to as the "distributional" term, is the component which captures the changes in the distribution of households over the state-space relative to the stationary density. The last term is trivially small and can be neglected. The deviation of the object during the transition and its steady-state counterpart is denoted by

$$\tilde{c}_t(x) = c(t,x) - \bar{c}(x), \quad \tilde{n}_t(x) = n(t,x) - \bar{n}(x).$$

4.3.1 PCT: 2000 to 2011

Figure (2) shows the PCTs over time (left graph) as well as their decomposition into the optimality and distributional terms given by equation (9) (right graph) for the time period 2000 to 2011. Along the horizontal axis is calendar time and along the vertical axis is the proportion consumed out of the tax cut. In 2000 the financing policy of the deficit-financed tax cut is announced. It is implemented in 2001 with the tax reduction lasting until 2026. From then on the debt, which has been accumulated over and above the level of the steady-state level of debt, is paid off in equal payments over 25 years, that is, until 2051.

In an economy in which Ricardian equivalence holds the PCT is zero, whereas, it is one for an economy populated by hand-to-mouth consumers (not shown). Following the announcement in 2000, the PCTs increase in all three economies. The increase in aggregate consumption between 2000 and 2001 is solely due to the unconstrained households. The extent to which the tax cut is internalized as an increase in permanent income depends on the effective planning horizon. Holding everything else constant, the effective planning horizon is longest in the dynastic economy and shortest in the standard OLG economy. The reason that Ricardian equivalence does not hold in the dynastic economy is because capital markets are imperfect. The economy with perfect altruism is in the spirit as has been studied by Laitner (1992) and Heathcote (2005). The planning horizon for a dynasty household is effectively truncated due to the inevitability of binding borrowing constraints over time. Partitioning time into intervals, where the bounds are given by the date a borrowing constraint binds, the dynasty behaves across the intervals as if it is an economy without altruism. Thus, the effective planning horizon does not coincide with the planning horizon of an infinitely-lived household when markets are complete.

In 2001 the deficit-financed tax cut is implemented. The PCTs jump up since constrained households increase their consumption. The deficit-financed tax cut eases borrowing constraints providing additional opportunities to smooth consumption, which were previously unavailable. This jump is largest for the standard OLG economy, somewhat smaller for the OLG economy with imperfect altruism, and smallest for the dynastic economy. One may conjecture that this is because there are more households in the standard OLG economy without wealth than in the OLG OLG economy with imperfect altruism. But this turns out not to be the case. In the OLG economy with imperfect altruism there are additional incentives to consume since being without wealth means potentially obtaining private transfers. As a result, 29.5% of households have no wealth in the mixture economy, whereas this number is 23.5% and 14.3% for the standard OLG economy and the dynastic economy, respectively. The reason the response in aggregate consumption is somewhat muted in the OLG economy with imperfect altruism is since households without wealth are especially likely to obtain transfers which are scaled back when the tax cut is actually implemented.

In order to understand the driving forces behind the aggregate consumption dynamics, the graph on the right hand side shows the decomposition of the PCT as given by (9). The optimality terms are the solid lines and the distributional terms the dashed lines. This graph shows that the effects in aggregate consumption between 2000 and 2001 are almost entirely due to changes in the optimal policies, which is a forward looking component. Changes to aggregate wealth occur slower since it is a stock variable while consumption is a flow variable and therefore responds more rapidly. The distributional terms turn slightly negative. This is because unconstrained households consume more by eating out of their wealth in anticipation of the tax reduction. When the deficit-financed tax cut is implemented the distributional terms in the three economies turn positive since savings increase to buffer for the possibility to be responsible to pay back the accumulated debt.

Over time an increasing part of the effect captured by the PCT is due to changes in the distribution. As wealth increases consumption increases, simply because wealth is larger, and not because the decisions have changed relative to the ones in the steady-state equilibrium. The difference between the PCT and the distributional term is coming from changes in the optimal policies. That is, for a given state, the decision rule at time t is different from the one in the stationary equilibrium. This component becomes negative in 2009 for the dynastic economy and for the other two economies in 2013. An important take-away from figure (2) is that the way aggregate consumption changes, as captured by the PCT, is surprisingly similar in the standard OLG economy and the OLG economy with imperfect altruism for the time period 2000 to 2011. A caveat is that, despite the similarity of the standard OLG economy and the OLG economy with imperfect altruism, the latter does have implications for private transfers. It does allow for private undoing of publicly induced transfers but households *choose* not to due to strategic considerations and imperfect altruism.

4.3.2 PCT: 2000 to 2026

The left-hand side graph of figure (3) portrays the time path of the PCT from 2000 to 2026. An interesting feature which stands out is that the trajectory of the PCT in the standard OLG economy decreases below the one from the OLG economy with imperfect altruism as of 2011. A priori one would expect the PCT of the mixed economy to be between the one from the standard OLG economy and the dynastic economy. This, however, is not the case since the “strategic” motive enters the savings decision when altruism is imperfect as the Euler equation (6) shows. The right-hand side graph of the figure shows the number of households without wealth during the financing experiment relative to the number of households without wealth in the steady-state. It reveals that the trajectory of this number dives below the one from the mixture economy by 2012. These two graphs are of course closely related since consuming relatively more implies saving relatively less. In the OLG economy with imperfect altruism there are additional incentives to consume, which are absent from the standard OLG economy, which accounts for this unexpected behavior.

Table (5) summarizes the important differences in the economies which in part account for the various values of the PCT. First consider the column entitled “Barro”. There is a question mark in terms of specifying the degree of altruism. The reason is that Barro (1974) does not require a specific degree of altruism in order to obtain Ricardian equivalence but that transfers are motivated by altruism and transfer mo-

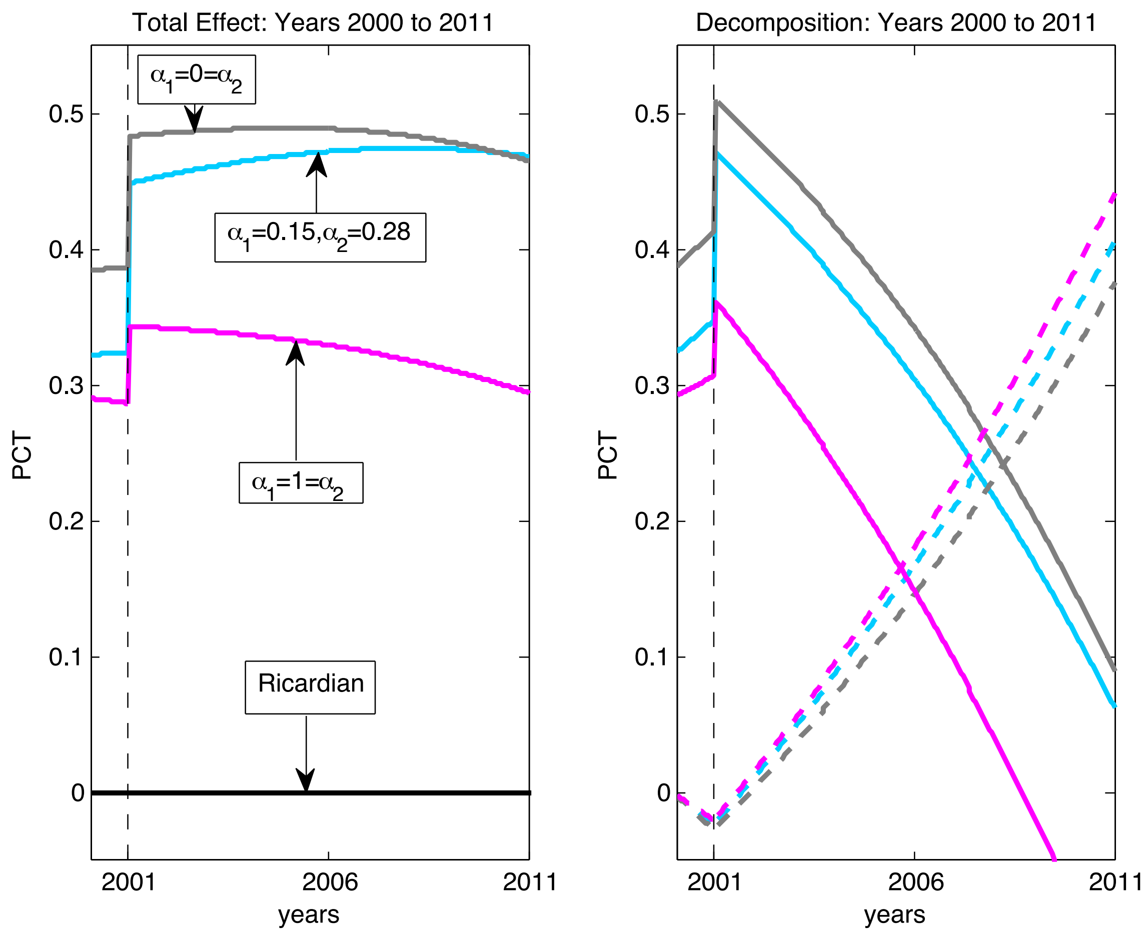


Figure 2: *Trajectories (left graph) and decompositions (right graph) of the PCTs starting at the announcement date 2000 and for 2001 to 2011. The dashed lines in the right figure are the distributional components and the solid lines are the optimality components; their sum add up to the PCTs shown on the left side.*

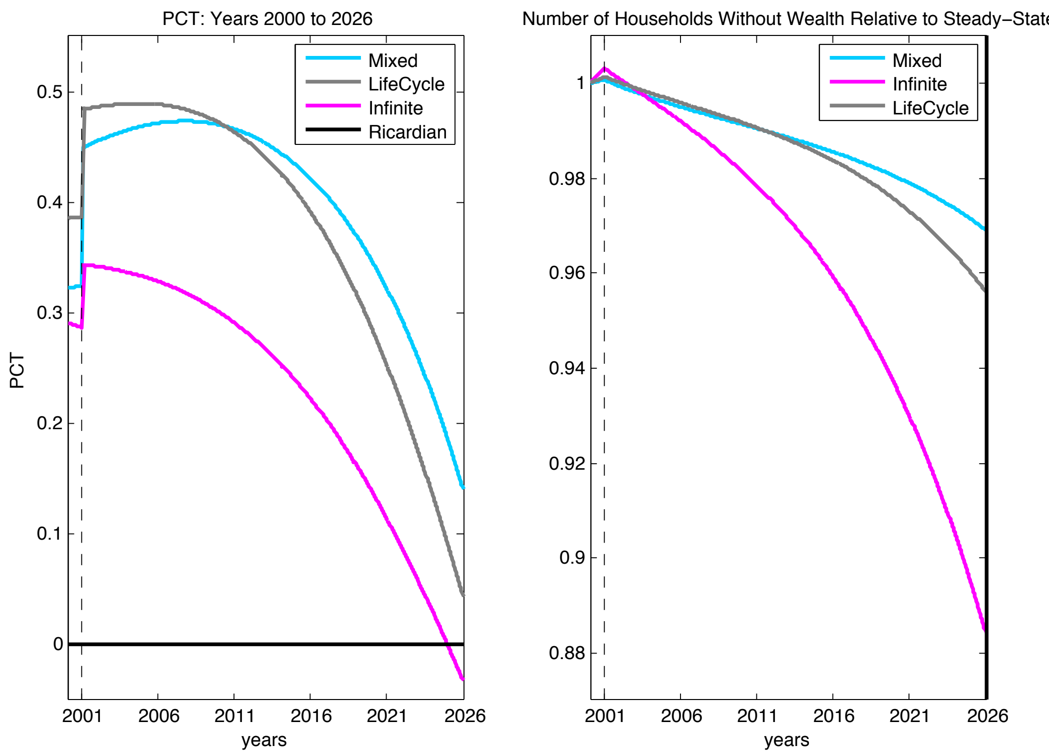


Figure 3: *Left: Trajectories of the PCTs starting at the announcement date 2000 and for the duration of the deficit regime, 2001 to 2026. Right: Proportion of households with zero wealth relative to that number in steady-state, 2000 to 2026. In the steady-state the percentage of households without wealth in the mixed, life-cycle, and dynastic economies is 29.5%, 23.5%, and 14.3% respectively.*

tives are *universally operative*, that is, current and all future generations intend to leave altruistically-motivated transfers. Thus, in theory it is possible to have Ricardian equivalence with an imperfect degree of altruism as long as the economy is engineered in such way that transfer motives are universally operative. The row "pooling" refers to the property of whether the objectives of the young household and the old household can be considered a joint problem with a pooled budget constraint. The row "strategic" asks whether strategic considerations arise between the young and the old households. The last row is the average PCT computed over the length of the deficit regime. In the Barro economy pooling occurs, even when altruism is imperfect, since the universally operative transfer motives provide a link among households. Strategic considerations are absent, perhaps because of strong commitment mechanisms, and aggregate consumption does not change since borrowing by the government is exactly offset by an increase in aggregate private saving.

For the dynastic economy, transfer motives are operative by construction for currently alive families, but not for all future generations. Family members equalize consumption at each point in time and state of nature and effectively pool their resources. Strategic considerations are absent since the resources of the other household are valued in the same way as one's own resources. The average fraction of income consumed out of income taxes is 22.7, substantially above zero, due to the incompleteness of markets. In the standard OLG economy the implicit degree of altruism is zero. Thus, by construction, transfer motives are never operative. Pooling and strategic considerations do not arise between the young and the old households. The average PCT is given by 37.6, which is 65% above the PCT in the dynastic economy.

The main difference in the OLG economy with imperfect altruism is that the transfer motive for currently alive families is neither engineered to be operative nor to be entirely absent but depends on choices which have been made. Households do not pool their resources as was discussed at the beginning of section 3. Since altruism is imperfect, strategic considerations in the savings and consumption decisions arise. In this particular calibration the average PCT of the mixture economy exceeds the one in the economy without altruism. This may be surprising, since a priori one would expect it to be strictly between the dynasty and the life-cycle economies. The reason that the PCT in the OLG economy with imperfect altruism may be above the one in the standard OLG economy is because of the strategic motive. As the economy moves closer to the date at which taxes are increased it becomes increasingly likely that current households are increasingly responsible for paying back the accumulated debt through higher taxes. Savings increase in anticipation but the increase in savings is lowest in the OLG economy with imperfect altruism. The reason is that there is a fraction of households which

Table 5: *Average PCT over duration of deficit regime.*

	Barro	Dynasty	Standard OLG	OLG with α
Altruism	?	$\alpha_1 = 1 = \alpha_2$	$\alpha_1 = 0 = \alpha_2$	$\alpha_1 = 0.15, \alpha_2 = 0.28$
Operative	Universally	Currently	Never	Endogenous
Pooling	✓	✓	✗	✗
Strategic	✗	✗	✗	✓
Average PCT	0	22.7	37.6	**39.7**

count on being "bailed out" with private transfers when taxes increase, since then they are sufficiently poor to obtain transfers, and therefore have a disincentive to save. In other words, if they would have wealth at the time of the tax hike they would not receive transfers. But this is not optimal for them since they would rather have no wealth and obtain private transfers. The households without altruism do not have this type of "moral hazard" and therefore increase their savings at an earlier point in time.[25]

4.3.3 PCT by Income: 2000 to 2026

So far the analysis has focused on aggregate consumption disregarding the heterogeneity in earnings profiles. We have learned that the response in aggregate consumption in the OLG economy with imperfect altruism is similar to the one in the life cycle economy and that the path of the PCT is not necessarily between the ones from the standard OLG economy and the dynastic economy due to the strategic motive. Here we will consider how the PCTs look like for families with various earning profiles.

At each point in time a family can have one of nine possible earnings profiles, e.g. the young household has a low income realization and the old household has a high income realization etc. In figure (4) an income realization is fixed for the old household and an average is taken of the PCT over the three possible income realizations of the young household. An interesting pattern emerges. For the dynastic economy (the pink line) the trajectory of the PCTs decrease as income for the old household increases. In the OLG economy with imperfect altruism (the blue line) the PCT is below the PCT from the dynastic economy when the old household has a low income realization. The PCTs for the economies without altruism and imperfect altruism increase as the income of the old household increases. When the old household has the middle income realization, the PCT in the mixed economy is weakly below the one of the life-cycle. But the

[25]Section (A.3) in the appendix shows a related point through the current account which directly reflects savings. It also considers the case of symmetric altruism.

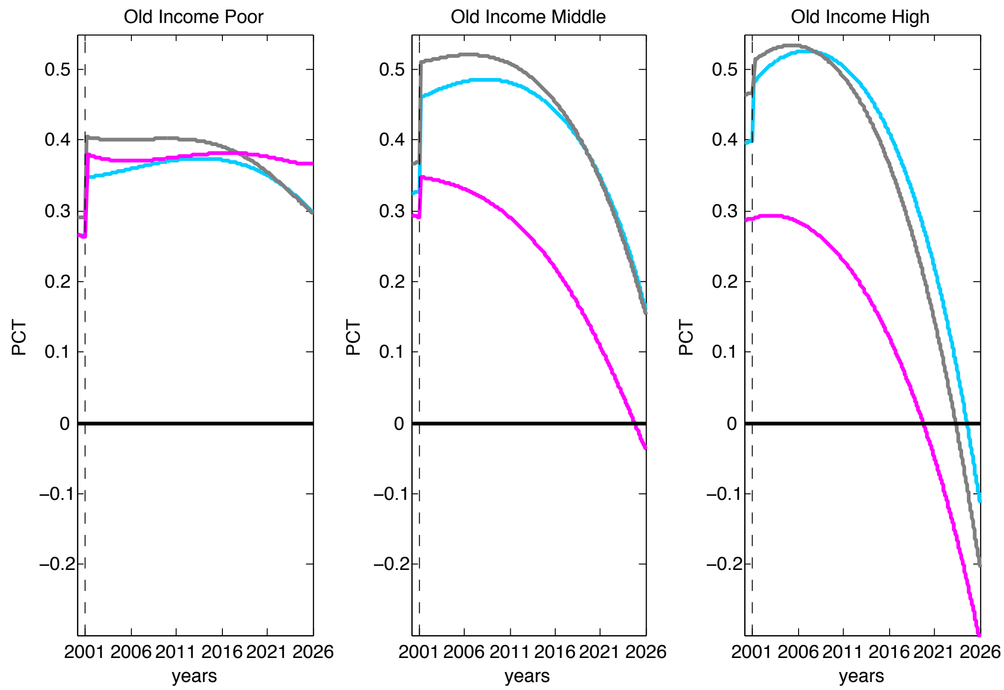


Figure 4: *PCTs by income realization of the old household, averaged over the three possible income realizations of the young household. Pink is the dynastic economy, gray the standard OLG economy, blue the OLG economy with imperfect altruism, and black is the economy in which Ricardian equivalence holds.*

PCT in the OLG economy with imperfect altruism when the old household has a high income realization is above the one in the standard OLG economy for the majority of the duration of the deficit regime. It is here that sufficiently poor young households can expect to obtain gifts and therefore consume relatively more.

5 Deficits, Gifts, and Bequests

An important strength of the model with imperfect altruism is that it has precise predictions on the timing and size of intended transfers. In the economy with perfect altruism the timing of intended transfers is indeterminate and in the one without altruism Barro's mechanism to potentially undo some of the effects deficit financing has is a priori excluded. Thus, only the model with imperfect altruism has something meaningful to say about the behavior of aggregate gifts.

Figure (5) shows the differences between deficits, gifts, and bequests during the transition and their steady-state values normalized by the stationary level of wealth. Along the horizontal axis is calendar time in years. The vertical line in 2001 indicates

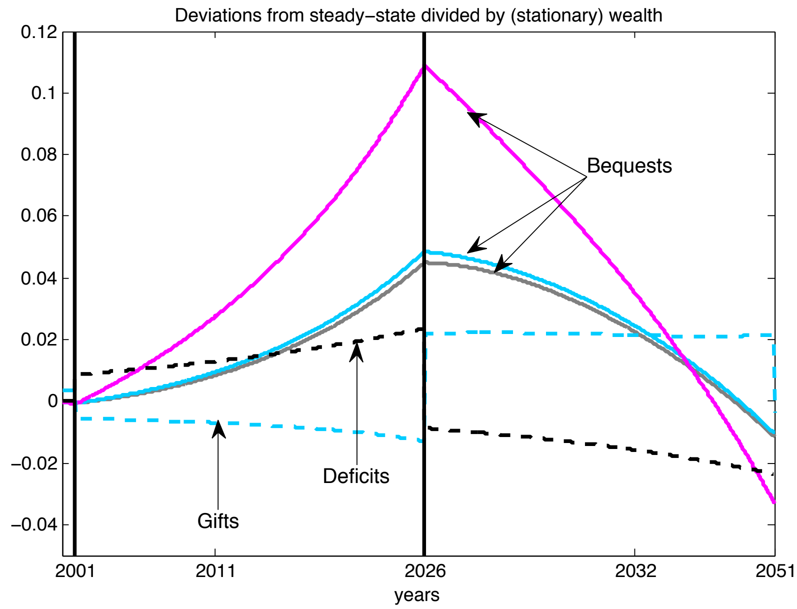


Figure 5: *Deficits, Gifts, and Bequests: The difference between the values of deficits, gifts, and bequests during the transition relative to their steady-state values normalized by the stationary level of wealth over the length of the alternative financing method, 2000 to 2051.*

the onset of the deficit regime, and the one in 2026 indicates the onset of the tax-hike regime. The deficit is depicted by the dashed black line and the dashed baby-blue line depicts gifts in the OLG economy with imperfect altruism. Bequests are the tent-shaped curves where pink represents the dynasty, baby-blue the mixture economy, and gray the standard OLG economy.

We see that the behavior of intended transfers is the opposite of deficits. The reason is that the number of households without wealth decreases, see the right graph of figure (3). Since these are the ones who are especially likely to be the recipient of private transfers and the donor households have strong incentives to delay making transfers donors cut back their transfers. The increase in transfers as of the year 2026 is larger than the drop since households have accumulated additional wealth. Thus, transfers increase in magnitude and there are additional incentives for young households to become broke so that the number of young, broke households increases.

The behavior of bequests in the mixed economy and the standard OLG economy is very similar. Since we know that in the standard OLG economy all bequests are accidental we can conclude that the majority of the bequests in the OLG economy with imperfect altruism are accidental. Bequests in the life-cycle model is wealth accumulated by the old households only for precautionary reasons. Thus, the primary motive to save of the old households in the mixture economy is not to undo the effects of the

Table 6: *Consumption equivalent variation "under the veil of ignorance".*

Generation	Barro	Standard OLG	OLG with α	Dynasty
Both	0	0.25%	**0.17%**	0.1%
Young	0	0.27%	0.15%	0.1%
Old	0	0.224%	0.19%	0.1%

deficit-financed tax cut but for own consumption-smoothing reasons. Using Barro's language, bequest motives are operative for a rather small fraction of the population; the motive to smooth consumption beyond ones own lifetime does not appear to play a significant role. The accumulation of wealth by the dynastic households is much larger and therefore the size of bequests is larger. Wealth, and therefore bequests, of the old generation goes below steady-state wealth since they expect a permanent increase in their income as of 2052. This is especially true for the dynastic households since they pool their resources with young households which are most affected by the financing scheme.

Table (6) provides the percentage change of annual consumption required in order to compensate a household in the stationary equilibrium in such way that she becomes indifferent to the equilibrium in which the alternative financing method is used. This welfare criterion is computed without conditioning on any characteristics and is therefore sometimes referred to as "under the veil of ignorance". A larger number indicates a higher desirability of the deficit financing scheme. Unsurprisingly, households entering a life-cycle economy would like the financing scheme the most, while in a Barro-economy households would be indifferent. A more interesting welfare implication is that the model with imperfect altruism lies half-way between the dynastic economy and the standard OLG economy. The reason the resemblance to the life-cycle model disappears is that deficits crowd out gifts. Since gifts are free, but now are replaced with deficits which have to be paid back, recipients of private transfers lose most, an effect which plays no role in the life-cycle model. The second and the third rows reveal that it is especially the young households who move closer to the dynastic welfare implication since they are more likely to be borrowing-constrained, and therefore more likely to receive gifts, than the old. Again we can see that even for households entering the dynastic economy there is a positive welfare effect coming from the deficit financing scheme. The intuition is as in the discussion on the PCT. The effective planning horizon of a dynastic household is truncated due to the inevitability of binding borrowing constraints.

Figure (6) shows the average compensating equivalent variation by family type. Along

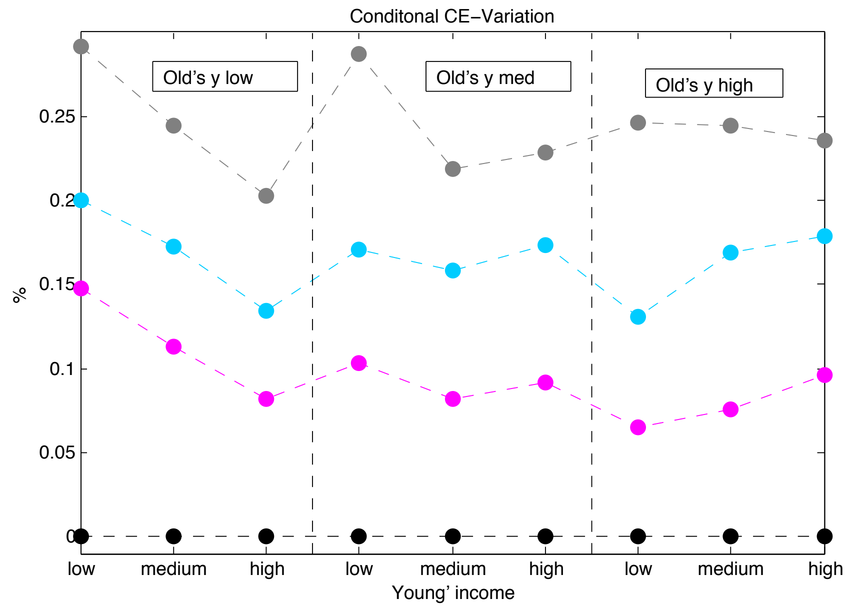


Figure 6: *Consumption equivalent variation by family type.*

the horizontal axis is the family type and along the vertical axis is the annual percentage compensation in consumption required in order for a household to be indifferent between the steady-state financing scheme and the alternative one. The ordering of family types is such that for family types 1-3 the old household has low income and the young goes from low to high, for family types 4-6 the old household has medium income and the young household goes from low to high, and for types 7-9 the old household has high income and the young households income goes from low to high. In all three economies the family with the low-low income profile benefits the most from the alternative financing scheme. As the youngs income goes from low to high the attractiveness of the alternative financing scheme decreases. The interpretation becomes a bit murkier for the other family types. The figure shows that the conditional welfare implications of the OLG economy with imperfect altruism lies also half-way between the economies with and without altruism.

6 Conclusion

In this paper I have studied the response in aggregate consumption to a deficit-financed tax cut. Since transfers motivated by altruism is a central mechanism by which publicly-induced transfers can potentially be undone through private transfers the emphasis was on the degree of altruism. The effects in the economy in which altruism is allowed to be imperfect have been compared to the ones from the two standard workhorse models in which altruism is either stipulated to be perfect or entirely absent. I have found some important similarities and differences among the predictions of the three models. In particular, the behavior of the aggregate variables in the OLG economy with imperfect altruism is surprisingly similar to the behavior of the aggregate variables in the standard OLG economy. Nonetheless, while one would have expected the implications of imperfect altruism to lie between the model without altruism and perfect altruism this has been shown not to be the case in the current calibration. An important difference between no altruism and perfect altruism is that a strategic motive arises which is responsible for this counterintuitive effect. As a result, the average PCT taken over the length of the deficit-financing regime is largest in the OLG economy with imperfect altruism. Furthermore, the resemblance to the life-cycle model disappears in terms of welfare implications. These lie half-way between the models with perfect and no altruism.

In dynamic macroeconomics the quantitative implications of imperfect altruism is a largely unexplored issue, despite the frequent use of representative-agent economies and life-cycle economies, which make strong implicit assumptions on it. One reason why these strong assumptions are made is that modeling the decision-making is straightforward. Nonetheless, in this paper I build on Barczyk & Kredler (2010a,b) and show that with standard techniques interesting and important topics can be explored. While I have focused on the consequences of the effects a deficit-financed tax cut has, an array of other questions may be tackled, such as the effects of changes in the estate tax, changes in social security benefits, or allocation of resources within the household when commitment is absent. While restricting strategies to be Markov-perfect may be a limitation it enables a tractable model which is in spirit similar to standard dynamic programming applications. Furthermore, structural estimation of parameters may become feasible since the model economy can be computed relatively quickly despite the dynamic game aspect.

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A Appendix

A.1 Transfers When One Player Has No Wealth

This section is intended to provide an idea of how transfers are computed in the equilibrium in which transfers only flow when the recipient has no wealth. Another special case which arises is when both households have no wealth. For a more extensive version don't hesitate to contact the author.

Suppose, the young household has no wealth. We implicitly define the consumption strategies $c_{1,0}$ and $c_{2,0}$ by

$$\begin{aligned} u_c(c_{1,0}) &= V^1_{w^1}(0, w^2) \\ u_c(c_{2,0}) &= V^2_{w^2}(0, w^2). \end{aligned} \tag{10}$$

The strategies g_2 and $c_{1,0}$ are mapped into realized consumption c_1^* by

$$c_1^*(c_{1,0}, g_2) = \begin{cases} c_{1,0} & \text{if } y^1 + g_2 \geq c_{1,0} \\ y^1 + g_2 & \text{if } y^1 + g_2 < c_{1,0} \end{cases}$$

This says that, if the young household is unconstrained after receiving transfers, it chooses consumption according to the standard first-order condition. If it is constrained, it consumes what it has.

The problem for the old household is given by

$$\max_{c_2 \geq 0, g_2 \geq 0} H(c_2, g_2) =$$
$$\max_{c_2 \geq 0, g_2 \geq 0} \left\{ u(c_2) + \alpha u(c_1^*(c_{1,0}, g_2)) + (rw_2 + y_2 - c_2 - g_2)V^2_{w^2} + (y^1 - c_1^*(c_{1,0}, g_2) + g_2)V^2_{w^1} \right\} \tag{11}$$

We obviously have

$$\frac{\partial H}{\partial c_2} = u_c(c_2) - V^2_{w^2}, \quad \begin{cases} \frac{\partial H}{\partial c_2} \geq 0 & \text{if } c_2 \leq c_{2,0} \\ \frac{\partial H}{\partial c_2} < 0 & \text{if } c_2 > c_{2,0}. \end{cases}$$

The derivative in g_2 is

$$\frac{\partial H}{\partial g_2} = \begin{cases} \alpha_2 u_c(y^1 + g_2) - V^2_{k_2} & \text{if } g_2 < c_{1,0} - y_1 \\ V^2_{w^1} - V^2_{w^2} & \text{if } g_2 \geq c_{1,0} - y^1 \end{cases}$$

reflecting that transfers from the old household go directly into consumption of the young household until the satiation point $c_{1,0}$ is reached; from this point on the young household starts saving, which is valued by the old household with $V^2_{w^1}$.

Note that $\partial H / \partial g_2$ is strictly decreasing for $g_2 < c_{1,0} - y^1$. So since $V^2_{w^1} \leq V^2_{w^2}$, it will be optimal to set transfers below the satiation point so that he does not save. We now define the optimal transfer as

$$g_2 = \max \left\{ 0; \min\{c_{1,0} - y^1, \alpha_2^{1/\gamma} c_{1,0} - y^1\} \right\} \tag{12}$$

It is zero when $\partial H / \partial g_2 |_{g_2 = 0} \leq 0$; there may be an interior solution where $\partial H / \partial g_2 = 0$ or another corner solution where the old household stops giving transfers upon reaching the child household's satiation point. The maximizers for the problem (11) are thus $(c_2, g_2)_{eq} = (c_{2,0}, g_2)$, and the young household's consumption is given by

$$c^*_{2,eq} = \min\{c_{1,0}, y^1 + g_2\} \tag{13}$$

A.2 Altruism Parameter

The purpose of this section is to provide an interpretation of the altruism parameter and it's relationship with the coefficient of relative risk aversion.

Consider a typical per-period utility function of, say, old household i. It is assumed to be additively separable in its own consumption and consumption of the young household j, i.e.

$$U^i(c_i, c_j) = u(c_i) + \alpha u(c_j), \quad \alpha \in [0,1]$$

where $u(\cdot)$ is a CRRA utility. If $\alpha = 1$ we speak of perfect altruism and when $\alpha = 0$ we speak of no altruism. But what are reasonable values for this parameter? We can get a sense of what "reasonable" may mean by considering the following FOC, which in equilibrium, has to hold in the static model

$$u_c(c_i) \geq \alpha u_c(c_j) \quad \Rightarrow \quad \alpha \leq \left(\frac{c_i}{c_j}\right)^{-\gamma}$$

If the inequality points the other way, the old household would provide the young household with a transfer since the additional utility she obtains if the young household consumes is larger than from her own consumption. On the other hand if the old household does not have enough resources to equalize this margin her marginal utility is strictly larger than the marginal utility she would obtain from consumption by the young household, in which case there are no transfers.

When the margin is equalized we can think about parameter values for α as the answer to the following thought experiment: What degree of consumption inequality does an (imperfect) altruist tolerate before she decides to transfer resources, which can only be consumed, from her own consumption? If, for example, a reasonable answer appears to be 2 and $\gamma = 2$ then we can infer $\alpha = (2)^{-2} = 0.25$. With logarithmic utility the interpretation is particularly simple since $\alpha_i = c_j/c_i = 0.5$: Agents who have a degree of altruism of degree 0.5 provide voluntary transfers when consumption inequality exceeds 2.

Another interesting observation which becomes evident is that it is not the absolute value of α which in itself is meaningful, but its size in conjunction with the coefficient of relative risk-aversion. From the example we see that an agent with $\gamma = 2$ and $\alpha = 0.25$ tolerates the same degree of consumption inequality as an agent with $\gamma = 1$ and $\alpha = 0.5$. Thus, in order to speak about degrees of altruism we have to keep in mind that this is only meaningful in the context of specifying a degree of risk-aversion.

A.3 Current Account

M.Obstfeld & K.Rogoff (1996) provide suggestive evidence that the tax cuts in the U.S. in the early 80's have led to current account deficits as shown by figure (8). Ricardian equivalence would imply that budget deficits have no effect on the current account. Total savings in the economy are unchanged since private savings increase by the same amount as borrowing does by the government.

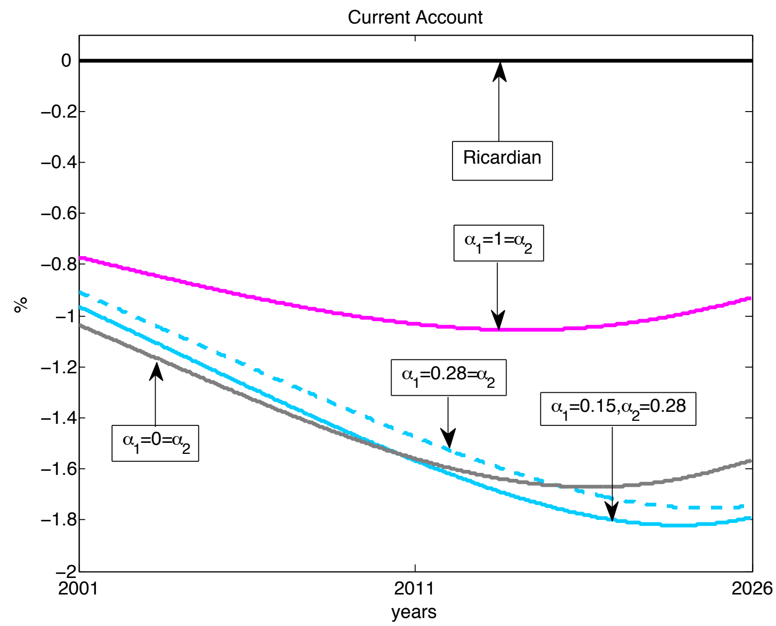

Figure 7: *Current account deficit over the length of the deficit financing regime 2001 to 2026.*

Figure (7) portrays the path of the current account deficit which is induced by the budget deficit. As with aggregate consumption we see that the behavior of the current account in the standard OLG economy and the OLG economy with imperfect altruism is very similar. Counter to what one would expect, the trajectory of the current account in the mixed economy is not between the dynastic economy and the economy without altruism. This is because the households know that taxes will increase in the future, but anticipate that they may receive transfers if they are sufficiently poor. The households without altruism do not have this type of "moral hazard" and therefore increase their savings at an earlier point in time.

In order to check that this is not an artifact of asymmetric altruism, the case with symmetric altruism is included. The dashed line in the figure is the trajectory of the current account deficit with symmetric altruism, where the degree of altruism is set to the one from the old household in the benchmark economy. The rate-of-time preference is chosen in order to obtain the same wealth-to-GNP ratio as in the other economies. As intuition would suggest, the current account deficit moves closer to the the economy with perfect altruism, but is qualitatively as before.

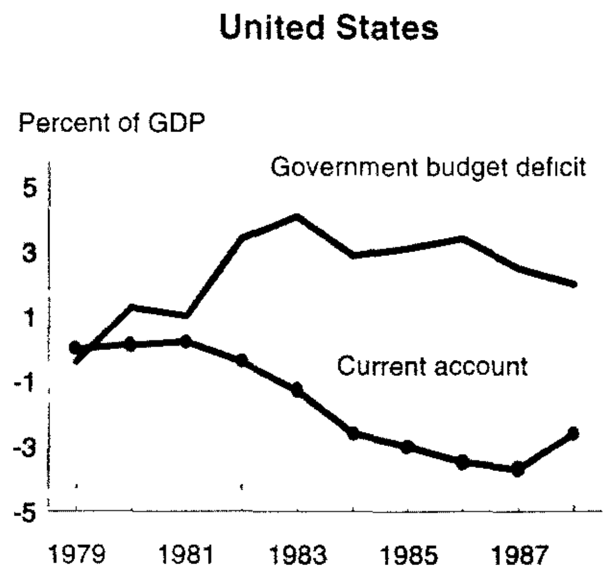

Figure 8: *An inverse relationship between budget deficits and current account deficits. M.Obstfeld & K.Rogoff (1996)*