Credit risk and disaster risk

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Abstract

Standard macroeconomic models imply that credit spreads directly reflect expected losses (the probability of default and the loss in the event of default). In contrast, in the data credit spreads are significantly larger than expected losses, suggestive of an aggregate risk premium. Building on the idea that corporate debt, while safe in normal times, is exposed to economic depressions, this paper embeds a trade-off theory of capital structure into a real business cycle model with a small, time-varying risk of economic disaster. The model replicates the level, volatility and cyclicality of credit spreads, and variation in the corporate bond premium amplifies macroeconomic fluctuations in investment, employment and GDP.

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1 Introduction

The widening of credit spreads during the recent financial crisis has drawn attention to their important allocative role: for many large corporations, the bond market, much more than the equity market, is the "marginal source of finance". Consistent with this view, credit spreads are significantly correlated with investment or GDP.[1] This underscores the need for a framework linking macroeconomic aggregates and bond prices. Existing frameworks, such as the financial accelerator model of Bernanke, Gertler and Gilchrist (1999), are validated in some dimensions by recent estimation exercises,[2] but are also at odds with the "credit spread puzzle" documented in the empirical finance literature:[3] in the model, the credit spread roughly equals expected losses, i.e. the product of the probability of default and the loss in the

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[1] See Philippon (2008), Gilchrist, Yankov and Zakrajek (2009), Mueller (2009).

[2] See Christiano, Motto and Rostagno (2009), and Gilchrist, Ortiz and Zakrajek (2009).

[3] See Huang and Huang (2003), Hackbarth, Miao and Morellec (2006), Chen (2008), Chen, Collin-Dufresne and Goldstein (2009), among others.

event of default. As a result, the average return on a portfolio of corporate bonds is nearly equal to the risk-free rate. In contrast, in the data the probability of default of an investment grade bond is small, around 0.4% per year, and there is substantial recovery upon default, around 50%, but spreads average around 100bps.[4] A natural explanation is that the credit spread reflects an aggregate risk premium: corporate bonds returns are lower when aggregate consumption is low, and hence offer higher expected returns.[5]

This paper introduces a framework that reproduces the key features of credit spreads, and studies its implications for business cycles. By their very nature, corporate bonds are safe in normal times, with limited default during ordinary recessions, but are exposed to the risk of a very large downturn. Building on this idea, I embed a simple trade-off model of capital structure, where the choice of defaultable debt is driven by taxes and bankruptcy costs, into a real business cycle (RBC) model, and assume that there is a small, exogenously time-varying risk of large shock – an economic "disaster", following the work of Rietz (1988), Barro (2006), Gabaix (2007), and Gourio (2010). The risk of disaster captures the possibility of a very large recession such as the Great Depression.[6] The capital structure choice modifies the standard RBC model equilibrium in two ways. First, the standard Euler equation is adjusted to reflect that investment is financed using both debt and equity, and the user cost of capital hence takes into account expected discounted bankruptcy costs as well as the tax savings generated by debt finance. Second, an additional equation determines the optimal leverage choice, by equating the marginal expected discounted (tax) benefits and (bankruptcy) costs of debt. The model remains highly tractable and intuitive, which allows to evaluate the role of defaultable debt and leverage choice on quantities and prices in a transparent fashion. In particular, the model encompasses the standard real business cycle model as a special limiting case.

The first result is that time-varying disaster risk generates large, volatile and countercyclical credit spreads, which are significantly larger than expected losses. The second main result is that financial frictions amplify substantially – by a factor of about two – the response of the economy to a shock to the disaster probability. Consistent with the extant literature, this amplification effect does not arise if the economy is subjected to TFP shocks. Hence, it is the interaction between the trade-off model, a staple of corporate finance, and time-varying disaster risk which generates novel, quantitatively appealing implications for both asset prices and quantities.

[4] This is the spread of a BAA-rated corporate bond over a AAA-rated corporate bond (rather than a Treasury), so as to net out differences in liquidity.

[5] Some researchers argue that the variation in credit spreads during the 2008 financial crisis is driven by the deteriorating balance sheets of banks and other financial institutions, which may be the marginal investors in these markets. Under this interpretation, the credit spread reflects a time-varying intermediation (or liquidity) wedge rather than an aggregation risk premium. While more research is needed to disentangle the importance of each factor, the risk premium explanation is attractive *a priori* because corporate bonds are not exotic assets: any household can buy directly a mutual fund or an ETF of corporate bonds. Moreover, the simultaneous appearance of large spreads (low prices) in many different markets is suggestive of a risk premium.

[6] The probability of economic disaster can be interpreted either as a rational, objective belief, but an alternative "behavioral" interpretation is that the probability of disaster reflects time-varying pessimism ("animal spirits"). This simple modeling device captures the idea that aggregate uncertainty is sometimes high, and that some asset price changes are not obviously related to current or future productivity ("bubbles").

The key mechanism is as follows. When the probability of economic disaster exogenously increases, the probability of default rises (holding constant the leverage policy). A higher probability of default directly raises expected discounted bankruptcy costs. However, expected discounted bankruptcy costs also rise through a second channel: agents anticipate that defaults are now more systematic, i.e. more likely to be triggered by a bad aggregate shock rather than a bad idiosyncratic shock. This higher systematic default risk increases the risk premium on corporate debt, making it more expensive ex-ante to raise funds for investment. (The effect of disaster risk is not a Peso problem, such as a large increase in default probability that is unrealized in short sample, but rather by the increase in this bond premium.) Overall, higher expected discounted bankruptcy costs increase the user cost of capital, leading to a reduction in investment. In equilibrium, firms also cut back on debt and substitute for equity, but since debt is cheaper due to the tax advantage, the user cost of capital has to rise. To summarize, higher disaster risk worsens financial frictions because debt is not efficient when disaster risk is high.

The model has several implications. First, eliminating the deductibility of interest expenses from taxable corporate income leads to a reduction in macroeconomic volatility and hence to significant welfare gains. Second, making debt payments contingent on disaster realizations (as has been recently suggested by several commentators) reduces volatility substantially, by eliminating the amplification effect of financial frictions. Third, a high level outstanding debt makes the economy more fragile, as any negative shock is likely to lead a significant share of firms into default, which is inefficient. A consequence is that a low perceived risk of economic disaster, which leads to higher leverage, makes the economy less resilient to shocks – consistent with a widely held view regarding the past decade.

In contrast to most of the literature, which focuses on small entrepreneurial firms which cannot raise equity easily and rely on bank or debt finance, the model is designed to capture the richer margins that large US corporations use to raise capital. In my model, firms always pay dividends (unless they default), and no borrowing constraint binds. The relative attractiveness of debt and equity finance varies over time, leading to variation in the user cost of capital. My model thus is not subject to a standard critique of financial frictions models, that most firms do pay dividends (or hold cash) and are "thus" unconstrained. Nor does my model rely on a significant heterogeneity between small, productive, constrained firms on the one hand, and large, unproductive, unconstrained firms on the other hand. Incorporating these realistic elements is likely important, but the model mechanism holds more generally. My model is also at least qualitatively consistent with stylized facts on the correlation of corporate defaults: first, the "excess clustering" documented by Das et al. (2007), and second the significant probability of large default losses on portfolios of corporate bonds estimated by Duffie et al. (2009). Last, while many firms do not access the corporate bond market directly and instead rely on bank loans, many of these loans are securitized and trade on a market similar to that of corporate bonds.

Organization of the paper

The rest of the introduction discusses the related literature. Section 2 sets up the model. Section 3 studies its quantitative implications. Section 4 considers some implications and extensions of the baseline model. Section 5 concludes. An online appendix provides additional robustness results and details the numerical method.

Related literature

This paper is related to four different branches of literature. First, the paper draws from the recent literature on "disasters" or rare events (Rietz (1988), Barro (2006), Gabaix (2007), Wachter (2008), and the criticisms of Julliard and Ghosh (2008) and Backus, Chernov and Martin (2009)). In particular, the model is a direct, but significant, extension of Gourio (2010), who studied a frictionless real business cycle model with time-varying disaster risk.

Second, the paper builds on the large macroeconomic literature studying general equilibrium business cycle models with financing constraints. Some recent studies in this vein are Gomes and Schmid (2008), Jermann and Quadrini (2008), Mendoza (2010), Miao and Wang (2010), and Liu, Wang and Zha (2010). Amdur (2010), Covas and Den Haan (2009), and Hennessy and Levy (2007) study the business cycle behavior of capital structure. Several of these papers analyze linearized DSGE models, where asset prices are much less volatile than in the data, and aggregate risk premia are small and nearly constant. Because the economic mechanism of these models often features asset prices, it seems important to examine the effect of financial frictions in a model where asset prices more closely mimic the data. The paper is also closely related to Philippon (2009), who demonstrates how to link bond prices an real investment,[7] and to Gilchrist and Zakrajsek (2011), who construct an "excess bond premium" that contains significant macroeconomic information.

Third, the paper considers the real effects of a particular shock to uncertainty – a change in the probability of disaster. The negative effect of uncertainty on output has been studied most recently by Bloom (2009), who emphasizes the "wait-and-see" effect driven by lumpy hiring and investment behavior. My model focuses on changes in *aggregate* uncertainty and the mechanism is different: higher uncertainty lowers desired investment by increasing the risk premium on capital and by exacerbating financial frictions. A related mechanism has recently been explored in the studies of Arellano, Bai and Kehoe (2010), Chugh (2010), and Gilchrist, Sim and Zakrajek (2010), who consider changes in idiosyncratic uncertainty as in Bloom (2009), in a setup with credit frictions. I compare this mechanism and my mechanism in more detail in section 4.8.

Fourth, the paper relates to the vast finance literature on the "credit spread puzzle" (e.g. Leland (1994), Huang and Huang (2003), Hackbardt, Miao and Morellec (2006), Chen (2010), Chen, Collin Dufresne and Goldstein (2009), Collin-Dufresne, Goldstein and Martin (2001), Collin-Dufresne and Goldstein (2001), Bhamra, Kuehn and Strebulaev (2009a, 2009b)). As discussed in the introduction, this literature documents that the prices of corporate bonds are "too low". Perhaps surprisingly, there is, to my knowledge, no study measuring the contribution of disaster risk to the credit spread puzzle.[8] Moreover, this literature has exogenous cash flows, no investment, and is not set in general equilibrium, making it difficult to evaluate the macroeconomic impact of the financial frictions. On the other hand, this literature studies the asset pricing implications in more detail and incorporates long-term debt. Of direct interest is the study by Giesecke et al. (2011) documenting, using long-term U.S. data, a series of large corporate default waves, including the Great Depression.

[7] Philippon's results, which hold under the Modigliani and Miller theorem (given an exogenous leverage policy) do not require him to specify a full general equilibrium model.

[8] See however the work in progress of Bhamra and Strebulaev (2011).

2 Model

I first present the firm problem, then the household problem, and finally define the equilibrium.

2.1 Firms

The section first describes the general structure of the firm problem, then fills in the details.

2.1.1 Summary

There is a continuum of mass one of perfectly competitive firms, which are all identical ex-ante and differ ex-post only in their realization of an idiosyncratic shock. For simplicity, we assume that firms live only for two periods. Firms purchase capital at the end of period t in a competitive market, for use in period $t+1$. This investment is financed through a mix of equity and debt. In period $t+1$, the aggregate shocks and the idiosyncratic shock are revealed, firms decide on employment and production, and then sell back their capital. Two cases arise at this point: (1) the firm value is larger than outstanding debt: the debt is then repaid in full and the residual value goes to shareholders as dividends; or (2) the firm value is smaller than outstanding debt: in this case the firm declares default, equityholders receive nothing, and bondholders capture the firm's value, net of some bankruptcy costs. In all cases, the firms disappear after production in period $t+1$ and new firms are created, which will raise funds and invest in period $t+1$, and operate in period $t+2$.[9]

The timing assumption clarifies the mechanism, because a default *realization* does not affect employment, output and profits. Ex-ante however, default *risk* affects the cost of capital to the firm and hence its investment decision. This investment decision in turns affects employment and output, and in general equilibrium all quantities and prices. In section 4.1, we consider an extension where default affects employment and production.

Since firms are ex-ante identical, they will all make the same choices. Because both production and financing technologies exhibit constant return to scales, the size distribution of firms is indeterminate, and has no effect on aggregate outcomes.

2.1.2 Production

All firms operate the same constant returns to scale Cobb-Douglas production function using capital and labor. The output of firm i is

$$Y_{it} = K_{it}^{\alpha}(z_t N_{it})^{1-\alpha},$$

where z_t is aggregate total factor productivity (TFP), K_{it} is the individual firm capital stock, and N_{it} is labor. Both input and output markets are competitive and frictionless.

[9] The assumption that firms live two periods, while obviously unrealistic, leads to substantial simplification of the analysis, which is useful to solve the model but also to clarify its implications. An important direction of future research is to incoporate long-lived firms and long-term debt in the model. Based on section 3.1 below, I conjecture that the model mechanism would remain quantitatively relevant. Because equity issuance is frictionless, retained earnings would not be advantageous.

2.1.3 Productivity shocks

To model the possibility of large recessions, I assume that the aggregate TFP process in this economy is driven not only by the usual "small" normally distributed shocks standard in RBC theory, but also by rare large negative shocks.[10] Formally,

$$\log z_{t+1} = \log z_t + \mu + \sigma e_{t+1} + \log(1 - x_{t+1} b_{tfp}),$$

where $\{e_{t+1}\}$ is *i.i.d.* $N(0,1)$, and x_{t+1} is an indicator equal to 1 if a disaster happens, and 0 otherwise. Hence, a disaster realization leads total factor productivity to fall permanently by a factor b_{tfp}. The realization of disaster also directly affects the capital stock, see the next paragraph. The probability of a disaster at time $t+1$ is denoted p_t, and it follows itself a Markov chain with transition matrix Q. The *three* aggregate shocks $\{e_{t+1}, x_{t+1}, p_{t+1}\}$ are assumed to be independent, conditional on p_t.

2.1.4 Depreciation shocks

Firms decide on investment at time t, but the actual quantity of capital that they will have to operate at time $t+1$ is random, and is affected both by realizations of aggregate disasters x_{t+1} as well as an idiosyncratic shock ε_{it+1}. Specifically, if a firm i picks K^w_{it+1} at time t (where w stands for wish), it actually has $K_{it+1} = K^w_{it+1}(1 - x_{t+1} b_k)\varepsilon_{it+1}$ to operate in period $t+1$, and $(1-\delta)K_{it+1}$ units of capital to resell. The idiosyncratic shock ε_{it+1} is *i.i.d.* across firms and across time, and drawn from a cumulative distribution function H, with mean unity.

2.1.5 Discussion of the assumptions regarding disasters

Barro (2006) and Barro and Ursua (2008) identify numerous large negative macroeconomic shocks in a cross-section of countries, which are usually caused by wars or economic depressions. In a standard neoclassical model there are two simple ways to model macroeconomic disasters − as destruction of the capital stock, or as a reduction in total factor productivity. My formulation allows for both.

TFP appears to play an important role during economic depressions (Kehoe and Prescott, 2007). While economists do not understand well the sources of fluctuations in total factor productivity, large and persistent declines in TFP may be linked to poor government policies, such as expropriation, confiscatory taxes, or trade policies. They may also be caused by disruptions in financial intermediation, if these lead to inefficient capital allocation.

Capital destruction is clearly realistic for wars or natural disasters, but it can also be interpreted more broadly. Perhaps it is not the physical capital but the intangible capital (customer and employee value) that is destroyed during prolonged economic depressions.

At the heart, the model mechanism requires two ingredients: (1) that disasters are clearly bad events, with high marginal utility of consumption; (2) that the return on capital is low during disasters. These assumptions are certainly realistic. Introducing a large TFP shock is the simplest way to obtain

[10] For parsimony and tractability, these rare disasters are modeled as one-time permanent jump in TFP; Gourio (2011) considers various extensions and shows that the key results are largely unaffected if disasters are modeled as smaller shocks that are persistent, and are followed by recoveries, provided that risk aversion is increased somewhat.

(1) in a neoclassical model, and introducing a depreciation shock is the simplest way to obtain (2). An alternative to depreciation shocks is to introduce steep adjustment costs: since investment falls significantly during disasters, the price of capital would also fall, generating endogenously a low return on capital during disasters. I do not pursue this strategy in the paper in the interest of simplicity.

2.1.6 Capital structure choice

The choice of equity versus debt is driven by a standard trade-off between default (bankruptcy) costs and the tax advantage of debt: bondholders recover a fraction θ of the firm value upon default, where $0 < \theta < 1$; moreover, a firm which issues debt at a price q receives χq, where $\chi > 1$. That is, for each dollar that the firm raises in the bond market, the government gives a subsidy $\chi - 1$ dollar. For simplicity, I assume that the subsidy takes place at issuance.[11]

The bond price q is determined at time of issuance, taking into account default risk, and hence depends on the firm's choice of debt and capital as well as the economy's state variables. Equity issuance is assumed to be costless. When $\chi = \theta = 1$, the capital structure is indeterminate and the Modigliani-Miller theorem holds. When $\chi = 1$, the firm finances only through equity, since debt has no advantage. As a result, there is no default, and we obtain the standard RBC model. When $\theta = 1$, or more generally $\theta\chi \geq 1$, the firm finances only through debt, since default is not costly enough. I will assume $\chi\theta < 1$, a necessary assumption to generate an interior choice for the capital structure.

2.1.7 Employment, Output, Profits, and Firm Value

To solve the optimal financing choice, we first need to determine the profits and the firm value. (The distribution of firm value determines the probability of default and hence the lending terms the firm can obtain ex-ante.) The labor choice is determined through static profit maximization, given the realized values of both productivity and capital stock, and given the aggregate wage:

$$\pi\left(K_{it}, z_t; W_t\right) = \max_{N_{it} \geq 0} \left\{ K_{it}^{\alpha} (z_t N_{it})^{1-\alpha} - W_t N_{it} \right\},$$

which leads to the labor demand

$$N_{it} = K_{it} \left(\frac{z_t^{1-\alpha}(1-\alpha)}{W_t} \right)^{\frac{1}{\alpha}}, \tag{1}$$

and the output supply

$$Y_{it} = K_{it}^{\alpha} (z_t N_{it})^{1-\alpha} = K_{it} \left(\frac{z_t (1-\alpha)}{W_t} \right)^{\frac{1-\alpha}{\alpha}}.$$

These equations can then be aggregated. Define aggregates through $K_t = \int_0^1 K_{it} di$, $Y_t = \int_0^1 Y_{it} di$, etc., we obtain that $Y_t = K_t^{\alpha}(z_t N_t)^{1-\alpha}$, i.e. an aggregate production function exists, and it has exactly the same shape as the microeconomic production function. Aggregating equation (1) shows that the wage satisfies the usual condition $W_t = (1-\alpha)\frac{Y_t}{N_t}$. The law of motion for capital is obtained by summing over i the equation $K_{it+1} = K_{it+1}^{w}(1 - x_{t+1} b_k)\varepsilon_{it+1}$. Since all firms are identical ex-ante, and they will

[11] In reality, interest on corporate debt is deductible from the corporate income tax, hence the implicit subsidy takes place when firms' earnings are taxed.

make the same investment choice $K^w_{it+1} = K^w_{t+1}$, and since ε_{it+1} has mean unity, idiosyncratic shocks average out and the aggregate capital is

$$K_{t+1} = K^w_{t+1}(1 - x_{t+1}b_k).$$

Profits at time $t+1$ are given by

$$\pi_{it+1} = Y_{it+1} - W_{t+1}N_{it+1} = \alpha Y_{it+1} = \alpha K_{it+1}\left(\frac{z_{t+1}(1-\alpha)}{W_{t+1}}\right)^{\frac{1-\alpha}{\alpha}} = K_{it+1}\alpha\frac{Y_{t+1}}{K_{t+1}},$$

i.e. each firm receives factor payments proportional to the quantity of capital it has, and to the aggregate marginal product of capital $\alpha\frac{Y_{t+1}}{K_{t+1}}$. The total firm value at the end of the period is

$$V_{it+1} = \pi_{it+1} + (1-\delta)K_{it+1} = K_{it+1}\left(1 - \delta + \alpha\frac{Y_{t+1}}{K_{t+1}}\right). \tag{2}$$

Define the aggregate return on capital as $R^K_{t+1} = (1 - x_{t+1}b_k)\left(1 - \delta + \alpha\frac{Y_{t+1}}{K_{t+1}}\right)$. The individual return on capital is $R^K_{it+1} = \varepsilon_{it+1}R^K_{t+1}$. The firm value is thus

$$V_{it+1} = R^K_{it+1}K^w_{t+1} = \varepsilon_{it+1}R^K_{t+1}K^w_{t+1}.$$

From ease of notation, I will from now on abstract from the firm subscript i, since all firms are identical and differ only ex-post in their realization of ε.

An important remark is that it is assumed here that the firm is always run to maximize total value rather than equity value — however, when equity holders know that default will happen at the end of the period, they have zero incentive in profit maximization. This is an example of debt overhang. Within the model, it has little importance because equity holders do not have any alternative: they are simply indifferent to the profits in the event of default.[12]

2.1.8 Investment and Financing Decisions

All firms make the same choices for capital, debt, and hence equity issuance, which are linked through the budget constraint $\chi q_t B_{t+1} + S_t = K^w_{t+1}$. To find the optimal choice of investment and financing, we first need to find the likelihood of default, and the loss-upon-default, for any possible choice of investment and financing. This determines the price of corporate debt. Taking as given this bond price schedule, the firm can then decide on optimal investment and financing.

More precisely, the firm will default if its realized value V_{t+1}, which is the sum of profits and the proceeds from the sale of undepreciated capital, is too low to repay the debt B_{t+1}. This will occur if the firm's idiosyncratic shock ε is smaller than a cutoff value, which itself depends on the realization of aggregate states $(e_{t+1}, p_{t+1}, x_{t+1})$. Mathematically, at time $t+1$, the value of firms which finish operating is $V_{t+1} = \varepsilon_{t+1}R^K_{t+1}K^w_{t+1}$, hence default occurs if and only if

$$\varepsilon_{t+1} < \frac{B_{t+1}}{R^K_{t+1}K^w_{t+1}} \stackrel{def}{=} \varepsilon^*_{t+1}.$$

[12] To allow debt overhang to have real effects, one must adopt a sequential set-up where the firm first decides on debt, and then makes decisions (investment and hiring), before the shocks are revealed (see e.g., Occhino and Pescatori, 2010). In contrast, in a model where the firm decides on investment and short-term debt simultaneously, as in the current paper, debt overhang does not arise.

If a disaster is realized ($x_{t+1} = 1$), the return on capital is lower and the default threshold ε^*_{t+1} is higher, and more firms default. We can define desired leverage $L_{t+1} = B_{t+1}/K^w_{t+1}$, which is decided at time t. The firm defaults if $\varepsilon R^K_{t+1} < L_{t+1}$ i.e. if the return on capital is low relative to the leverage. Given this default rule, the bond issue is priced ex-ante using the representative agent's stochastic discount factor:

$$q_t = E_t\left(M_{t+1}\left(\int_{\varepsilon^*_{t+1}}^{\infty} dH(\varepsilon) + \frac{\theta}{B_{t+1}}\int_0^{\varepsilon^*_{t+1}} \varepsilon R^K_{t+1}K^w_{t+1}dH(\varepsilon)\right)\right).$$

In this equation, the first integral gives the value of the debt in the full repayment states. These states depend on the realization of shocks occurring at time $t+1$, notably disasters, through the threshold for default ε^*_{t+1}. The second term gives the average recovery in default states, divided among all the bondholders and net of bankruptcy costs. The bond price can be rewritten as

$$q_t = E_t\left(M_{t+1}\left(1 - H\left(\varepsilon^*_{t+1}\right) + \frac{\theta R^K_{t+1}K^w_{t+1}}{B_{t+1}}\Omega\left(\varepsilon^*_{t+1}\right)\right)\right), \tag{3}$$

where $\Omega(x) = \int_0^x sdH(s)$. Note the following properties of Ω, which follow from the fact that H is a c.d.f. with mean unity: (i) $\Omega(x) = 1 - \int_x^\infty sdH(s)$; (ii) $\lim_{x\to\infty}\Omega(x) = 1$; (iii) $\Omega'(x) = xh(x)$.

We can now set up the firm's problem at time t : it must decide how much to invest, how much debt to issue (and hence how much of the investment is financed through equity), so as to maximize the expected discounted equity value:

$$\max_{B_{t+1},K^w_{t+1},S_t} E_t\left(M_{t+1}\max\left(V_{t+1} - B_{t+1}, 0\right)\right) - S_t, \tag{4}$$

subject to:

$$\chi q_t B_{t+1} + S_t = K^w_{t+1}, \tag{5}$$

$$V_{t+1} = \varepsilon_{t+1}R^K_{t+1}K^w_{t+1}. \tag{6}$$

Equation (5) is the funding constraint: investment must come out of equity S_t, or the sale of bonds (including the subsidy) $\chi q_t B_{t+1}$. The objective function (4) takes into account the option of default for equityholders. Note that, given constant return to scale and no equity issuance costs, this net equity value will equal zero in equilibrium, reflecting free entry.

Given that the firm defaults if $\varepsilon_{t+1} < \varepsilon^*_{t+1}$, we can rewrite this problem as:

$$\max_{B_{t+1},K^w_{t+1}} E_t\left(M_{t+1}\left(\begin{array}{c} R^K_{t+1}K^w_{t+1} + (\chi\theta - 1)\, R^K_{t+1}K^w_{t+1}\Omega(\varepsilon^*_{t+1}) \\ +\,(\chi - 1)\, B_{t+1}\left(1 - H\left(\varepsilon^*_{t+1}\right)\right)\end{array}\right)\right) - K^w_{t+1}, \tag{7}$$

$$s.t. \quad : \qquad \varepsilon^*_{t+1} = \frac{B_{t+1}}{R^K_{t+1}K^w_{t+1}}.$$

In this expression, the first term is the expected discounted firm value, $E_t\left(M_{t+1}R^K_{t+1}K^w_{t+1}\right)$; the second term (which is negative since $\chi\theta < 1$) is expected discounted bankruptcy costs; and the third term is the expected discounted tax shield. The last term K^w_{t+1} is simply the cost of investment. By contrast, in a frictionless model, the firm would simply maximize $E_t\left(M_{t+1}R^K_{t+1}K^w_{t+1}\right) - K^w_{t+1}$. The difference is that the firm also takes into account the value of tax subsidies and default costs in making its decisions. Default costs are born by debt holders ex-post, but expected default costs are passed on into debt prices ex-ante, implying that equity holders actually bear the costs of default.

The first-order condition with respect to K^w_{t+1} yields,

$$E_t\left(M_{t+1}R^K_{t+1}\left(1+\left(\chi\theta-1\right)\Omega(\varepsilon^*_{t+1})+\left(\chi-1\right)\varepsilon^*_{t+1}\left(1-H\left(\varepsilon^*_{t+1}\right)\right)\right)\right)=1. \tag{8}$$

Recall that $R^K_{t+1}=\left(1-x_{t+1}b_k\right)\left(1-\delta+\alpha\frac{Y_{t+1}}{K_{t+1}}\right)$ is the familiar expression for the unlevered physical return on capital, adjusted to reflect the possibility of disasters. In a model without financial frictions, the standard Euler equation implies $E_t\left(M_{t+1}R^K_{t+1}\right)=1$; here, equation (8) is modified to take into account the bankruptcy costs (the second term), which raise the cost of capital, and the tax shield (the third term), which reduces it. When $\chi=\theta=1$, we return to the standard equation, corresponding to the case of an unlevered firm. Overall the firm has always access to cheaper financing than in the frictionless (all-equity financed) model, since it always has the possibility to not take any debt. As a result, the steady-state capital stock is always higher when $\chi>1$ than in the frictionless version.

The first order condition with B_{t+1} is

$$\left(1-\theta\right)E_t\left(M_{t+1}\varepsilon^*_{t+1}h\left(\varepsilon^*_{t+1}\right)\right)=\left(1-\frac{1}{\chi}\right)E_t\left(M_{t+1}\left(1-H\left(\varepsilon^*_{t+1}\right)\right)\right). \tag{9}$$

This equation determines the optimal financing choice between debt and equity.[13] The left-hand side is the marginal cost of debt, i.e. an extra dollar of debt will increase the likelihood of default, and the associated bankruptcy costs. The right-hand side is the marginal benefit of debt, i.e. the higher tax shield in non-default states. Importantly, both the marginal cost and the marginal benefit are discounted using the stochastic discount factor M_{t+1}. The importance of this risk-adjustment is consistent with the empirical work by Almeida and Philippon (2007), who note that corporate defaults are more frequent in "bad times" and as a result the ex-ante marginal cost of debt is higher than a risk-neutral calculation would suggest. This risk-adjustment will play a substantial role in the analysis below: for a given debt level, an increase in the probability of disaster increases expected discounted default costs, not only because defaults become more likely, but also because they are more likely to occur during bad aggregate times.

2.2 Household

The representative household has recursive preferences over consumption and leisure, following Epstein and Zin (1989):

$$U_t=\left((1-\beta)(C_t^{\upsilon}(1-N_t)^{1-\upsilon})^{1-\psi}+\beta E_t\left(U_{t+1}^{1-\gamma}\right)^{\frac{1-\psi}{1-\gamma}}\right)^{\frac{1}{1-\psi}}. \tag{10}$$

Here ψ is the inverse of the intertemporal elasticity of substitution (IES) over the consumption-leisure bundle, and γ measures risk aversion towards static gambles over the bundle. When $\psi=\gamma$, the model collapses to expected utility. While the additional flexibility of recursive utility is useful in calibrating the model, the key qualitative results can be obtained with standard CRRA preferences (See section 4.7).

[13]A second order condition is required to ensure that this condition is sufficient. Some regularity condition must be imposed on the distribution H, e.g. the function $z\rightarrow\frac{zh(z)}{1-H(z)}$ is increasing. Bernanke, Gertler and Gilchrist (1999) make the same assumption in the context of a related model. Most distributions (such as the log-normal distribution) satisfy this assumption.

The household supplies labor in a competitive market, and trades stocks and bonds issued by the corporate sector.[14] The budget constraint reads

$$C_t + n_t^s P_t + q_t B_t \leq W_t N_t + \varrho_t B_{t-1} + n_{t-1}^s D_t - T_t, \tag{11}$$

where W_t is the real wage, B_{t-1} is the aggregate quantity of debt issued by the corporate sector in period $t-1$ at price q_{t-1}, each unit of which is redeemed in period t for ϱ_t, n_t^s is the quantity of equity shares, P_t is the price of equity, D_t is the payoff to equityholders,[15] and T_t is a lump-sum tax. The number of equity shares n_t^s is normalized to one. In the absence of default, $\varrho_t = 1$, but $\varrho_t < 1$ if some bonds are not repaid in full. The household takes the process of ϱ_t as given, but it is determined in equilibrium by default decisions of firms.

The labor supply decision is governed by the familiar condition:

$$W_t = \frac{1-\upsilon}{\upsilon} \frac{C_t}{1-N_t}. \tag{12}$$

Intertemporal choices are determined by the stochastic discount factor (a.k.a. marginal rate of substitution), which prices all assets:

$$M_{t+1} = \beta \left(\frac{C_{t+1}}{C_t} \right)^{\upsilon(1-\psi)-1} \left(\frac{1-N_{t+1}}{1-N_t} \right)^{(1-\upsilon)(1-\psi)} \frac{U_{t+1}^{\psi-\gamma}}{E_t \left(U_{t+1}^{1-\gamma} \right)^{\frac{\psi-\gamma}{1-\gamma}}}. \tag{13}$$

Hence, we have the following Euler equations

$$\begin{aligned} E_t \left(M_{t+1} R_{t+1}^c \right) &= 1, \\ E_t \left(M_{t+1} R_{t+1}^e \right) &= 1, \end{aligned}$$

where $R_{t+1}^c = \frac{\rho_{t+1}}{q_t}$ is the return on corporate bonds, and $R_{t+1}^e = \frac{D_{t+1}}{P_t}$ is the return on equity.[16] The expressions for the returns are determined by the firm problem studied in the previous section. Consistent with equation (3),

$$\varrho_{t+1} = 1 - H\left(\varepsilon_{t+1}^* \right) + \frac{\theta R_{t+1}^K K_{t+1}^w}{B_{t+1}} \Omega \left(\varepsilon_{t+1}^* \right), \tag{14}$$

and $q_t = E_t \left(M_{t+1} \varrho_{t+1} \right)$, and moreover the equity value satisfies (equation (7)):

$$R_{t+1}^e = \frac{R_{t+1}^K K_{t+1}^w \left(1 - \Omega \left(\varepsilon_{t+1}^* \right) \right) - B_{t+1} \left(1 - H \left(\varepsilon_{t+1}^* \right) \right)}{S_t}.$$

Last, we can obtain the price of the risk-free asset as the expectation of the stochastic discount factor, $P_t^{rf} = E_t \left(M_{t+1} \right)$. Following Barro (2006), the government bond is assumed to default by a factor Δ during disasters, and hence its price is $P_t^{gov} = E_t \left(M_{t+1} \left(1 - x_{t+1} \Delta \right) \right)$.

[14] It is possible to introduce government bonds as well. If the government finances this debt using lump-sum taxes and transfers, Ricardian equivalence holds, and government policy does not affect the equilibrium allocation and prices. This allows us to price risk-free or exogenously defaultable government bonds below.

[15] There is no capital gains term $n_{t-1}^s \left(D_t + P_t \right)$ since firms live only two periods.

[16] The Euler equations hold for any individual firm's equity return or bond return. Given that firms are ex-ante identical, they are written here only for the aggregate equity and bond returns.

2.3 Equilibrium

The equilibrium definition is standard. First, the labor market clears:

$$(1-\alpha)\frac{Y_t}{N_t} = W_t = \frac{(1-\upsilon)C_t}{\upsilon\,(1-N_t)}. \tag{15}$$

Second, the goods market clears, i.e. total consumption plus investment plus bankruptcy costs equals output,

$$C_t + I_t + (1-\theta)\Omega(\varepsilon_t^*)V_t = Y_t. \tag{16}$$

This equation implies that a wave of defaults leads to large bankruptcy costs and induces a negative wealth effect. In order to clarify the mechanism, I abstract from this effect, by assuming that the default cost is a tax, i.e. it is transferred to the government, which then rebates it to household using lump-sum transfers (T_t in equation 11); section 8 relaxes this assumption. Then, the resource constraint is simply

$$C_t + I_t = Y_t. \tag{17}$$

Under this simplification, equations (8) and (9) are the only departures of our model from the standard real business cycle model: first, the Euler equation needs to be adjusted to reflect the tax shield and bankruptcy costs; second, the optimal leverage is determined by the trade-off between costs and benefits of debt finance. To summarize, the equilibrium is characterized by the equations (15), (17), as well as (8) and (9) and the definition of the stochastic discount factor (10) and (13).

2.3.1 Recursive Representation

It is useful, both for conceptual clarity and to implement a numerical algorithm, to present a recursive formulation of this equilibrium. First, note that the equilibrium can be entirely characterized from time t onwards given the values of the realized aggregate capital stock K_t, the probability of disaster p_t, and the level of total factor productivity z_t, i.e. these are the three state variables.[17] Hence, the model has the same states as the frictionless real business cycle (RBC) model. Second, examination of the first-order conditions shows that they can be rewritten solely as a function of the detrended capital $k_t = K_t/z_t$ and p_t. This is a standard simplification in the stochastic growth model when technology follows a unit root, which also applies to our framework.

As a result the equilibrium policy functions can be expressed as functions of two state variables only, k and p. Compared to the standard RBC model, we have an additional equilibrium policy function to solve for, the desired leverage $L(k,p)$, and correspondingly, we have an additional first-order condition (equation (9)). Last, the first-order condition determining optimal investment, i.e. the standard Euler equation (equation 8)), is modified to take into account the marginal financing costs. The full list of equations of this recursive representation is in appendix.

[17] The level of outstanding debt B_t at the beginning of period is not a state variable, since it does not affect production or investment possibilities. It does affect default, but because defaults do not affect production, and bankruptcy costs are not in the resource constraint, the realization of default does not matter in itself – what matters is the possibility of default going forward. Here we rely on two assumptions: (1) the default cost is a tax; (2) default takes place after production.

3 Results

This section studies the implications of the model presented in the previous section. First, I present a combination of analytical results and numerical comparative statics to illustrate the workings of the model. Then, a parametrized version of the model is solved numerically so as to delineate its predictions for business cycle quantities, for asset returns, an in particular for the level and volatility of credit spreads, and their relation with investment and GDP.[18]

3.1 Steady-state comparative statics

To better understand the model, it is useful to perform a "steady-state" analysis, as is commonly done in macroeconomics, but one that takes into account the risk of disaster. The first step is the following result.

Proposition 1 *Assume that $b_k = b_{tfp}$, i.e. capital and productivity fall by the same factor in a disaster Then, a disaster leads consumption, investment, output to also drop by the same factor $b_k = b_{tfp}$, while hours do not change. The return on physical capital is reduced by the same factor. There is no further effect of the disaster on quantities or prices, i.e. all the effect is on impact.*

Proof. The equilibrium is characterized by the policy functions for detrended consumption and investment, hours, and leverage, $c(k,p), i(k,p), N(k,p), L(k,p)$ and detrended output $y(k,p) = k^{\alpha} N(k,p)^{1-\alpha}$ which express the solution as a function of the probability of disaster p (the exogenous state variable) and the detrended capital k (the endogenous state variable). The detrended capital evolves according to the shocks ε', x', p' through

$$k' = \frac{(1 - x' b_k)\left((1-\delta)k + i(k,p)\right)}{(1 - x' b_{tfp})\, e^{\mu + \sigma e'}}.$$

Since $b_k = b_{tfp}$,

$$k' = \frac{((1-\delta)k + i(k,p))}{e^{\mu + \sigma e'}},$$

is independent of the realization of disaster x'. As a result, the realization of a disaster does not affect c, i, N, y, L since k is unchanged, and hence it leads consumption $C = cz$, investment $I = iz$, and output $Y = yz$ to drop, like z, by a factor $b_k = b_{tfp}$ on impact. Furthermore, once the disaster has hit, it has no further effect since all the endogenous dynamics are captured by k, which is unaffected. The statement regarding returns follows from the expression $R^K_{t+1} = (1 - x_{t+1} b_k)\left(1 - \delta + \alpha \frac{y_{t+1}}{k_{t+1}}\right)$, and that y and k fall by the same amount. ■

To obtain further results, we consider a simplified version of the model, where we shut down the shocks to the probability of disaster and the TFP shocks e_{t+1}. As a result, the only source of shocks are disaster realizations, which makes it possible to solve for the path of quantities and returns.

Proposition 2 *Assume that the disaster probability is constant, that $b_k = b_{tfp}$, and that there are no TFP shocks ($\sigma = 0$). The economy has a balanced growth path where $k_t, c_t, i_t, y_t, L_t, N_t$, the risk-free rate,*

[18] Given the nonlinear form of the model, and the focus on risk premia, it is important to use a nonlinear solution method. The policy functions $c(k,p), N(k,p), g(k,p)$, and $L(k,p)$, are approximated using Chebychev polynomials and solved for using projection methods. The appendix details the computational method.

the expected return on capital, and the probability of default, and the credit spread are constant, equal to k^*, c^*, i^*, *etc. Along this balanced growth path, the level of capital, consumption, investment and output* K_t, C_t, I_t, Y_t, *are obtained by multiplying* k^*, c^*, i^*, y^* *by* z_t, *which is evolves as* $z_{t+1} = z_t e^{\mu + x_{t+1}\log(1-b_z)}$.

Proof. Given that $\sigma = 0$, and p is constant, we can conjecture an equilibrium of the form described in proposition, and it is easy to check that it satisfies the first-order conditions (listed in appendix). Along this balanced growth path, k is constant since it is unaffected by disaster realizations; the policy functions $c(k), i(k), N(k), L(k)$ then imply that these variables are also constant if $k = k^*$. As a result, consumption growth and other variables are iid, implying that expected returns and credit spreads are constant. ■

A graphical illustration of this result, is that macroeconomic quantities simply grow along constant trends, without any shocks except for occasional large downward jumps. During these jumps, realized returns on bonds and equity are low, but the dynamics of quantities are unaffected. The discount factor for this simplified version of the model depends only on the disaster realization:

$$M(x') = \frac{\beta e^{\mu((1-\psi)\upsilon-1)}(1-x'b)^{(1-\gamma)\upsilon-1}}{\left(1-p+p(1-b)^{(1-\gamma)\upsilon}\right)^{\frac{\psi-\gamma}{1-\gamma}}},$$

and the economy's steady-state capital-labor ratio k/N and leverage $L = B/K^w$ are determined by equations (8) and (9), which simplify in this case to

$$\begin{aligned} & \frac{\beta e^{\mu((1-\psi)\upsilon-1)}}{\left(1-p+p(1-b)^{(1-\gamma)\upsilon}\right)^{\frac{\psi-\gamma}{1-\gamma}}}\left(1-\delta+\alpha\left(\frac{k}{N}\right)^{\alpha-1}\right) \\ = \quad & (1-p)\left(1+(\chi\theta-1)\,\Omega\left(\varepsilon_{nd}^*\right)+(\chi-1)\,\varepsilon_{nd}^*\left(1-H\left(\varepsilon_{nd}^*\right)\right)\right) \\ & +p\,(1-b)^{\upsilon(1-\gamma)}\left(1+(\chi\theta-1)\,\Omega\left(\varepsilon_d^*\right)+(\chi-1)\,\varepsilon_d^*\left(1-H\left(\varepsilon_d^*\right)\right)\right). \end{aligned} \tag{18}$$

and

$$\begin{aligned} 0 \quad = \quad & (1-p)\left(\chi\left(\theta-1\right)\varepsilon_{nd}^* h\left(\varepsilon_{nd}^*\right)+(\chi-1)\left(1-H\left(\varepsilon_{nd}^*\right)\right)\right) \\ & +p\left(1-b_{tfp}\right)^{\upsilon(1-\gamma)-1}\left(\chi\left(\theta-1\right)\varepsilon_d^* h\left(\varepsilon_d^*\right)+(\chi-1)\left(1-H\left(\varepsilon_d^*\right)\right)\right), \end{aligned} \tag{19}$$

with $\varepsilon_d^* = \frac{L}{(1-b)\phi}$, $\varepsilon_{nd}^* = \frac{L}{\phi}$, and $\phi = 1-\delta+\alpha\left(\frac{k}{N}\right)^{\alpha-1}$ is the standard marginal product of capital.

While these expressions initially appear complicated, looking at some special cases provides some intuition. First, note that they are recursive: equation (19) first determines the ratio of leverage to the marginal product $\frac{L}{\phi}$, and equation (18) then determines the marginal product of capital ϕ and hence $\frac{k}{N}$.[19] When there is neither disaster risk nor financial frictions, i.e. $p = 0$ and $\chi = \theta = 1$, the first equation collapses to the standard user cost equation,

$$\beta e^{\mu((1-\psi)\upsilon-1)}\left(1-\delta+\alpha\left(\frac{k}{N}\right)^{\alpha-1}\right) = 1.$$

[19] Labor supply and the scale of the economy are then determined by preferences in the standard way. First, note that

$$c = k^\alpha N^{1-\alpha} - \delta k = N\left(\left(\frac{k}{N}\right)^\alpha - \delta\frac{k}{N}\right),$$

and second the MRS = MPL condition implies $\frac{1-\upsilon}{\upsilon}\frac{c}{1-N} = (1-\alpha)\left(\frac{k}{N}\right)^\alpha$.

When there is disaster risk but no financial frictions (as in Gourio (2010)), the steady-state capital is determined as

$$\beta e^{\mu((1-\psi)\upsilon-1)}\left(1-\delta+\alpha\left(\frac{k}{N}\right)^{\alpha-1}\right)\left(1-p+p\left(1-b_{tfp}\right)^{\upsilon(1-\gamma)}\right)^{\frac{1-\psi}{1-\gamma}}=1.$$

Simple algebra shows that a higher probability of disaster p induces to a lower capital stock provided that the IES is greater than unity: agents are reluctant to invest in the more risky capital stock. Consider now the case of financial frictions but no disaster risk, equation (19) reflects simply the trade-off between the default costs and tax benefits of leverage:

$$\chi\left(1-\theta\right)\varepsilon^{*}h\left(\varepsilon^{*}\right)=\left(\chi-1\right)\left(1-H\left(\varepsilon^{*}\right)\right).$$

Last, in the full model, disaster risk affects the amount of desired leverage for two reasons. First, it changes the distribution of payoffs to the investment. Second, it changes the discount rates which multiply this distribution of payoffs, as reflected in the term $\left(1-b_{tfp}\right)^{\upsilon(1-\gamma)-1}$ in equation (19).

3.1.1 The determinants of optimal leverage and investment

Figure 1 uses this simplified version of the model to illustrate the effect of several key parameters on the steady-state values of capital, leverage, default probability and credit spreads. Each column of this figure corresponds to one parameter; the first column shows the effect of idiosyncratic volatility σ_{ε}. Holding debt policy constant, higher idiosyncratic risk leads to more default and hence higher credit spreads, increasing the user cost of capital. This leads firms to reduce investment. In equilibrium, firms also endogenously reduce leverage, which mitigates the increase in default and in credit spreads, but makes firms rely more heavily on equity issuance, which is more costly.

The second column shows the effect of the tax subsidy χ. A higher χ directly reduces the user cost of capital, since holding debt policy constant, the firm is able to raise more capital. Second, a higher χ makes debt relatively more attractive than equity, leading firms to take on more debt and increase leverage. This higher leverage leads to a higher probability of default and higher credit spreads.

Finally, the third column shows the effect of increasing the recovery rate parameter θ. Since the expected cost of bankruptcy falls, the user cost of investment falls and investment rises. Holding debt policy constant, a higher θ leads to a lower credit spread, since the recovery value is higher. However, since firms take on more debt, the probability of default and credit spreads go up.

3.1.2 User cost, financial frictions and probability of disaster

Turning now to the effect of the probability of disaster, figure 2 displays the effect of a rise in p on capital, leverage, credit spreads and the user cost $\alpha\left(\frac{k}{N}\right)^{\alpha-1}$, which is $r+\delta$ in the standard neoclassical model. Higher disaster risk leads to a reduction in leverage in equation (19), and hence an increase in the user cost (adjusted for the tax shield and bankruptcy costs) in equation (18) and a lower capital-labor ratio. The figure compares the frictionless model ($\chi=\theta=1$, i.e. the firm is only equity-financed) and the model with the friction ($\chi>1$). The percentage response of the steady-state capital stock to a change in the probability of disaster is substantially larger in the model with the financial friction, reflecting that the user cost is much more affected by an increase in disaster risk. An increase in disaster risk in itself increases the probability of default, but also makes the risk of default more likely to be driven by a negative *aggregate* realization, hence increases the cost of debt significantly, as reflected by the credit spread.[20] Overall, the probability of disaster p has an effect similar to that of σ_ε, which is the shock considered by Arellano, Bai and Kehoe (2010) or Gilchrist, Sim and Zakrajek (2010) in very recent studies. I return to this comparison in section 4.8.

3.2 Parametrization

The model is solved and simulated at the annual frequency, using the parameters listed in Table 1. A first set of parameters follows the business cycle literature (Cooley and Prescott (1995)) and is fairly uncontroversial $(\alpha, \delta, \upsilon, \beta, \mu, \sigma)$. Next, the intertemporal elasticity of substitution of consumption (IES) is set at 2. We discuss below in detail (section 4.7) the role of this parameter, and why it is difficult to reconcile the model with the evidence if the IES is low.[21]

A critical part of the calibration regards the size and average probability of disasters. While the model description assumed a single disaster size for simplicity, it is realistic and computationally useful to use a smooth distribution of disaster sizes. (This does not alter the model: an additional expectation is taken over the disaster size b.) Following Barro and Ursua (2008) and Barro and Jin (2011), the average probability of disaster is 3.8% per year, and the size distribution of disaster is approximated with a combination of two power laws. Let $z = 1/(1-b)$, then the p.d.f. of z is given by

$$\begin{aligned} f(z) &= Az^{-\alpha_s}, \text{ for } z > \delta_s, \\ f(z) &= Bz^{-\beta_s}, \text{ for } z_0 \leq z \leq \delta_s, \\ f(z) &= 0 \text{ for } z < z_0, \end{aligned}$$

with $z_0 = 1.105$ (corresponding to a minimal disaster size of 10%), $\delta_s = 1.38$, $\alpha_s = 5.16$, and $\beta_s = 11.1$, and A and B are picked such that the density is continuous at δ_s, and integrates to one.[22] This implies an average disaster size of 21%. One potential criticism of these estimates is that they are obtained using international data for a variety of countries, and disaster risk may be milder in the United States. Section 4.6 shows how the results are affected when the disaster size or probability is assumed to be lower. Another important assumption is that the capital destruction and the TFP reduction are equal, i.e. $b_k = b_{tfp}$. This is a natural, parsimonious benchmark; Gourio (2010) discusses the effect of relaxing this assumption.

[20] For high values of the probability of disaster p, the credit spread is decreasing in p. This counterintuitive result simply reflects that for very high p, firms reduce debt significantly to avoid bankruptcy and associated costs.

[21] Of course, there is a large debate regarding the value of the IES. Most direct estimates using aggregate data find low numbers (e.g. Hall (1988)), but this view has been challenged by several authors (see among others Bansal and Yaron (2004), Gruber (2006), Mulligan (2004), Vissing-Jorgensen (2002)). As emphasized by Bansal and Yaron (2004), a low IES has the counterintuitive effects that higher expected growth lowers asset prices, and higher uncertainty increases asset prices. Moreover, in this model, a simple regression of consumption growth on interest rates implies an IES estimate around 0.3, much lower than the true IES, because states with high probability of disaster have low interest rates and high expected consumption growth.

[22] In practice, this p.d.f. is approximated using a 21-point discrete distribution.

A second critical element is the volatility of the disaster probability. I assume that the log of the probability follows an AR(1) process:

$$\log p_{t+1} = \rho_p \log p_t + (1 - \rho_p) \log \overline{p} + \sigma_p \varepsilon_{p,t+1},$$

where $\varepsilon_{p,t+1}$ is *i.i.d.* $N(0,1)$.[23] The parameter σ_p is picked to reproduce the volatility of credit spreads. The persistence of the probability of disaster is $\rho_p = .75$; this parameter is relatively unimportant as shown in the appendix.

The risk aversion parameter γ is picked to reproduce approximately the level of equity premia and of credit spreads. Note that γ is the risk aversion over the consumption-hours bundle. Since $\gamma = 6$ and the share of consumption in the utility index is .3, the effective risk aversion to a consumption gamble is 1.8 (Swanson (2010)), a low value by the standards of the asset pricing literature.

The last important element of the calibration pertains to the capital structure choice, with three parameters: the tax shield χ, the recovery rate on assets θ, and the standard deviation of idiosyncratic shocks σ_ε. I use a log-normal distribution for H, the distribution of idiosyncratic shocks, with mean unity. To obtain realistic recovery rates, I set $\theta = .7$. The implied recovery rate on debt in the event of default is $\theta \frac{1}{\varepsilon_t^* H(\varepsilon_t^*)} \int_0^{\varepsilon_t^*} \varepsilon dH(\varepsilon) < \theta$. For our parametrization, this recovery rate is 65%, i.e. the "loss given default" is 35%. This figure holds both in disasters and in normal times, and is roughly consistent with the data. Section 4.10 discusses how the results are affected by a change in θ.[24]

The parameters σ_ε and χ are then picked to match targets for the probability of default (average default rate) and leverage, in normal times. The target for the probability of default is 0.4% per year, and the target for leverage is 0.55.[25] This implies a volatility σ_ε of 21%, and a tax subsidy χ of 3.3 cents per dollar of debt issued. This measure can be compared to the deductibility of interest expenses from taxable corporate income. With a corporate income tax rate of 35%, a nominal interest rate of 7% would imply a tax subsidy of 2.45 cents on the dollar, which is lower than what I find. One interpretation is that this reflects my assumption that equity issuance is costless: as a result, I must make debt quite cheap to match my target leverage ratio. Another possibility is that there are other advantages to debt than the tax shield, as shown in the corporate finance literature (e.g., debt as a disciplining device for managers).

3.3 Impulse response functions

I first illustrate the dynamics of the model in response to the three aggregate shocks: the standard TFP shock, the disaster realization, and a shock to the probability of disaster. I next discuss how the model fits both quantities and price data.

[23] This equation allows the probability to be greater than one, however I will approximate this process with a 6 point Markov chain, which ensures that $0 < p_t < 1$.

[24] An interesting extension, studied in the appendix, is to allow the recovery rate θ or the volatility of idiosyncratic shocks σ_ε to be lower, or higher, in disaster states.

[25] In the data leverage is somewhat smaller, perhaps 0.45. It is difficult to replicate all the features of the data with a leverage equal to 0.45. I interpret this as reflecting the low volatility of firm profits and value in this model. In reality firms face fixed costs, and some production factors are hard to adjust, which imply a higher objective leverage than the financial leverage. This motivates my higher target for leverage. An alternative would be to allow for a richer distribution of idiosyncratic shocks.

3.3.1 The effect of a TFP shock

Figure 3 displays the response of quantities and returns to a one standard-deviation shock to the level of total factor productivity. (For clarity, this picture, as well as the ones following, assumes that no other shock is realized.) The response of quantities is similar to that of the standard real business cycle model: investment rises as firms desire to accumulate more capital, employment rises because of the higher labor demand, and consumption adjusts gradually, leading to temporarily high interest rates. The equity return is high on impact, reflecting the sensitivity of firms' dividends to TFP shocks due to leverage, but corporate bonds are largely immune to small TFP shocks - the default rate is barely affected.[26] As a result, corporate bond returns move in lockstep with the risk-free return. Leverage and credit spreads are constant, since the trade-off determining optimal leverage is hardly affected by the slightly higher TFP.

3.3.2 The effect of a disaster

Figure 4 shows the response of quantities and returns to a disaster which hits at $t = 5$. The disaster realization leads capital and TFP to fall by the factors b_k and b_{tfp} respectively. The calibration assumes that these parameters are equal, and in this simulation are large: $b_k = b_{tfp} = 34\%$. As a result, the transitional dynamics are very simple, as seen in the figure, and as proved in proposition 1: output, consumption and investment drop on impact by the same factor, and hours do not change. The return on capital is also -34%. These losses are divided between equity and debt, but they are further increased by default, which leads to losses since $\theta < 1$. In this simulation, approximately 27% of firms are in default, the realized market return is roughly -70% and the realized return on a diversified portfolio of corporate bonds is -10%. Both equity and corporate debt are risky assets, since their returns are low precisely in the states (disasters) when consumption is low, i.e. marginal utility is high. Consistent with proposition 1 a disaster does not generate any transitional dynamics in quantities, leverage, credit spreads, interest rates, or risk premia.

3.3.3 The effect of an increase in the probability of a disaster

The important shock in this paper is the shock to the probability of disaster – i.e. an increase in perceived risk. Figure 5 presents the responses to an unexpected increase in the probability of disaster at time $t = 5$. The higher risk leads to a sharp reduction in investment. Simultaneously, the higher risk pushes down the risk-free interest rate, as demand for precautionary savings increases. This lower interest rate decreases employment through intertemporal substitution. Hence, output decreases because employment decreases, even though there is no change in current or future total factor productivity, and even though the capital stock adjusts slowly. Intuitively, there is less demand for investment and this reduces the need for production.

Consumption increases on impact since households want to invest less in the now more risky capital. Consumption then falls over time. Qualitatively, these dynamics are similar to that in the frictionless

[26] The default rate is defined as the share of firms in default. Because some of the capital is recovered in defaults, this is not the realized loss for debholders.

version, but the quantitative results are quite different. To illustrate this clearly, figure 6 superimposes the responses to a shock to the probability of disaster for the frictionless model ($\chi = \theta = 1$) and for the benchmark model. The response of macro quantities on impact is approximately twice than in the frictionless model: the model amplifies significantly the effect of risk shocks.

As argued in section 3.1, the mechanism through which disaster risk affects the economy is by increasing expected discounted bankruptcy costs. Holding debt fixed, default is (i) more likely and (ii) more likely to be systematic, i.e. default is more likely to occur in "bad times". Higher expected bankruptcy costs increase the user cost for a given financial policy, leading firms to cut back more on investment than in the frictionless model. Moreover, firms also adjust their financial policy, reducing debt and leverage.

Because risk increases, risk premia rise as the economy enters this recession: the difference between equity returns and risk-free returns becomes larger, and the spread of corporate bonds over risk-free bonds also rises (see the bottom panel of figure 5). The model hence generates the required negative correlation between credit spreads and investment output.[27] More generally, the model implies that risk premia are larger in recessions, consistent with the data. Last, and perhaps counterintuitively, the default probability $falls$ slightly as disaster risk goes up, because firms cut back on debt and no disaster is realized. Including the possibility of disasters, the probability of default remains approximately constant. Hence, *the observed increase in spreads reflects an increase in the corporate bond risk premium, rather than an increase in the probability of default*, a result consistent with the empirical findings of Gilchrist and Zakrajsek (2011).

3.4 Business cycle and financial statistics

Tables 2, 3 and 4 report standard business cycle and asset return statistics as well as default rates and leverage ratios.[28] To illustrate the role of disaster risk and time-varying disaster risk, the tables report results for three different assumptions about the structure of shocks hitting the economy: (i) no disaster risk, i.e. only TFP shocks, (ii) TFP shocks and a positive, but constant probability of disaster; (iii) TFP shocks and time-varying disaster risk. I also report results for the frictionless RBC model ($\chi = \theta = 1$).

TFP shocks alone (rows 1 and 4) generate a decent match for quantity dynamics, as is well known from the business cycle literature. This model, however, generates rather small spreads for corporate bonds (28bps), and these spreads simply account for the average default of corporate bonds: the excess return on corporate bonds is close to zero. By definition, the spread y is the sum of the (physical) compensation for default risk, plus a risk premium:

$$E(y) = \Pr(Default) \times E(LGD) + E(R_c - R_f),$$

[27]The equilibrium level of credit spreads depends on the *endogenous* leverage that firms decide to take on. For certain parameter values, leverage endogenously falls as disaster risk increases, leading paradoxically to lower credit spreads in response to a higher probability of disaster. However, for the parameter values that we use, firms do not decide to cut back on debt too much, and spreads rise with the probability of disaster.

[28]The leverage and default probability data are taken from Chen, Collin-Dufresne, and Goldstein (2009). The other data (GDP, consumption, investment, and credit spreads) are from FRED. I use BAA-AAA as the credit spread measure, and obtain similar results as Chen, Collin-Dufresne, and Goldstein. All series are annualized.

and here the last term is nil : spreads are completely accounted for by the (high) probability of default ($0.28 = 0.81 \times 0.35 + 0$). Moreover, these spreads are essentially constant. The risk premium for equity is also very small. Note that except for investment, which is somewhat less volatile in the model with capital structure, the quantity moments are largely unchanged as we go from row 1 to row 3. Hence, financial frictions do not amplify the response to TFP shocks.[29] The smaller volatility of investment in the model with capital structure is apparently driven by the higher steady-state capital stock (as in Santoro and Wei (2010)).

When constant disaster risk is added to the model (rows 2 and 5), the quantity dynamics are essentially unaffected (table 2). Table 3 reveals that credit spreads are significantly larger however, because defaults are much more likely during disasters, when marginal utility is high. The model generates a plausible credit spread of 135bps, much higher than the probability of default (34bps). The equity premium is also high, and higher in the model with capital structure, reflecting the leverage effect. However, the volatility of spreads is still close to zero. This motivates turning to the model with time-varying risk of disaster.

Rows 3 and 6 display the results for the models with time-varying disaster risk. The variation in the disaster risk does indeed lead to volatile credit spreads, roughly in line with the data. The equity premium is a bit too low, but it is sizeable, and similar to that of the model with constant probability of disaster. Introducing the time-varying risk of disaster also generates new quantity dynamics: output, and especially investment and employment become more volatile, consistent with the impulse response functions (figure 5). Moreover, credit spreads are countercyclical: the model reproduces well the relation between investment or output and credit spreads emphasized in the introduction.

Table 8 decomposes the variation of the spread into the expected loss and the risk premium in the model. Not only is the largest share of the spread driven by the risk premium on average, but so is its variation over the business cycle, and the covariance with investment are almost entirely driven by variation in the risk premium.

The amplification effect of disaster risk shock through financial frictions is visible in table 2: while the financial friction model exhibits less volatility than the RBC model when disaster risk is constant, it has more volatility than the RBC model when disaster risk is added. This is especially true for investment volatility, which goes from 3.19% in the RBC model without disaster risk, to 5.56% in the RBC model with disaster risk, to 7.81% in the capital structure model with disaster risk.

Finally, the model implies some volatility of leverage, but it falls somewhat short of the data. However, the one-period nature of firms in this model makes it difficult to interpret this statistic: the flow and stock of debt are equal in the model, while they behave differently in the data (Jermann and Quadrini (2009), Covas and Den Haan (2009)). The model prediction that leverage is procyclical is reasonable, when applied to the flow of new debt.

It is interesting to illustrate the increase in *systematic* risk that occurs when the disaster probability rises. Figure 7 presents the correlation of defaults that is expected given the probability of disaster today, i.e. $Corr_t\left(def_{i,t+1}, def_{j,t+1}\right)$ for any two firms i and j in the model economy. In normal times,

[29] The appendix presents a comparison of the impulse response functions to a TFP shock for the different models, which confirms this result.

the probability of disaster is low, and defaults are largely idiosyncratic since aggregate TFP shocks do not create much variation in default rates. Hence, this correlation is low. The correlation becomes much higher, however, when the probability of disaster rises. This is because defaults are now much more likely to be simultaneously triggered by the realization of a disaster. This higher correlation would show up in some asset prices such as CDO or CLO (collateralized debt or loan obligations). This higher correlation stems directly from the increase in aggregate uncertainty, holding idiosyncratic uncertainty constant. This correlation is affected by firms' choices, however, since they decide on how much debt to take which affects their default likelihood: for certain parameter values firms cut back on debt when disaster risk, leading this correlation to eventually fall.

Because of its highly stylized nature, the model also has certain shortcoming. In particular, the correlation of consumption and output is too low, around 0.2. I abstract from many ingredients such as habits or sticky prices which may help with consumption comovement. Finally, the equity return is quite smooth in this model outside disasters. Equities are a one-period asset here, implying that the conditional volatility of equity returns equals the conditional volatility of dividends (i.e. there is only a cash flow effect and no discount rate effect).

4 Extensions and Robustness

This section considers some implications and extensions of the baseline model, and the sensitivity of the quantitative results to parameter changes.

4.1 Default crises and time-varying resilience of the economy

For the purpose of analytical clarity, the benchmark model assumes that default does not affect output: (i) bankruptcy costs are a tax rather than a real resource cost, and (ii) a firm in default is as productive as a firm in good standing. This section relaxes these two assumptions: (i) in reality, bankruptcies are costly: costs include legal fees as well as the loss of intangible capital such as customer goodwill and specific human capital; (ii) firms in default are likely less productive as they need to reorganize and are constrained in their relations with suppliers and customers. Relaxing either of these assumptions implies that an economy with a high level of outstanding debt is prone to "default crises": any negative shock may drive many firms into default, which further degrades the economy. The exact effect of (i) and (ii) is however different: (i) is a pure wealth effect, while (ii) reduces productivity and hence labor demand.

Formally, we make the following two changes to the model. The first is to assume that a share $\omega \in [0,1]$ of the bankruptcy costs is a real resource cost. The second is that firms in default have lower productivity, by a factor ζ. These two changes do not affect the expression for the default threshold $\varepsilon^*_{t+1} = \frac{B_{t+1}}{K^w_{t+1} R^K_{t+1}}$.Total output, taking into account the lower productivity of firms in default, is now

$$Y_t = (K_t)^\alpha (z_t N_t)^{1-\alpha} \left(\left(1 - \zeta^{\frac{1}{\alpha}} \Omega(\varepsilon^*_{t+1})\right)\right)^\alpha,$$

and recall that $\Omega(\varepsilon^*_{t+1})$ is the weighted default rate. The resource constraint now reads

$$C_t + I_t + (1-\theta)\omega\Omega\left(\varepsilon^*_{t+1}\right) R^K_t K_t = Y_t.$$

We also need to modify consequently the firm value and bond price equations and the associated first order conditions; these equations are available in the appendix. As a result of this change, the quantity of debt B is now an additional state variable.

Figure 8 illustrates the negative effect of outstanding debt on the economy for the case $\zeta^{\frac{1}{\alpha}} = 0.5$ and $\omega = 0$, i.e. firms in default are more productive. (The appendix presents an examples for the case of $\zeta = 0$ and $\omega = 0.5$, i.e. bankruptcies have real resource costs.) Ceteris paribus, a larger amount of debt increases default rates, and reduces output, employment, investment and consumption.

An important further implication of this model extension is that the economy's sensitivity to shocks is time-varying. For instance, as discussed in the previous section, a low probability of disaster leads firms to pick a high leverage. This makes the economy less resilient, i.e. its investment and output will fall more should a bad shock occur. This is consistent with a widely held view that during the 2000s, perception of risk fell, leading firms to increase leverage and making the 2008 recession more severe. Figure 9 shows that a higher outstanding debt makes investment and output more response to an innovation to TFP: for low levels of debt, a one-percent increase in TFP leads to an increase of output around 1.2%, but for high debt levels, the effect goes to 1.35%. For investment, the effect is more dramatic, with a response less than 4% in "normal times" but close to 7% when debt is high.[30]

4.2 State-contingent debt

In the aftermath of the 2008 financial crisis, several economists have proposed that private sector borrowers, rather than using standard debt contracts, issue state-contingent debt with payments conditional on large aggregate shocks (e.g., "contingent convertibles" or CoCos). This section evaluates this proposal by allowing firms in the model to issue debt contingent on the disaster realization x'. The model is easily modified; first, the budget constraint now reads,

$$K^w_{t+1} = S_t + \chi q^{nd}_t B^{nd}_{t+1} + \chi q^d_t B^d_{t+1},$$

where B^{nd}_{t+1} (resp. B^d_{t+1}) is the face value of the debt to be repaid in non-disaster (resp. disaster) states, and q^{nd}_t (resp. q^d_t) the associated price:

$$q^{nd}_t = E_t\left((1-x_{t+1})M_{t+1}\left(\int_{\varepsilon^*_{t+1}}^{\infty} dH(\varepsilon) + \frac{\theta}{B_{t+1}}\int_0^{\varepsilon^*_{t+1}} \varepsilon R^K_{t+1} K^w_{t+1} dH(\varepsilon)\right)\right),$$

where $(1-x_{t+1})$ is a dummy equal to 1 if no disaster happens, and similarly for q^d_t. Taking first-order conditions leads to the following characterization of the equilibrium: first, the Euler equation is

$$E_t\left(M_{t+1}R^K_{t+1}\left(\begin{array}{c} 1+(\chi-1)\,L^{nd}_{t+1}(1-x_{t+1})\left(1-H\left(\varepsilon^*_{t+1}\right)\right) \\ +(\chi-1)\,L^d_{t+1}x_{t+1}\left(1-H\left(\varepsilon^*_{t+1}\right)\right) \\ +(\theta\chi-1)\,\Omega(\varepsilon^*_{t+1}) \end{array}\right)\right) = 1,$$

and second, optimal debt is determined through the two equations:

$$\frac{\chi-1}{\chi}E_t\left((1-x_{t+1})M_{t+1}\left(1-H\left(\varepsilon^*_{t+1}\right)\right)\right) = (1-\theta)\,E_t\left(M_{t+1}\Omega'(\varepsilon^*_{t+1})\,(1-x_{t+1})\right), \quad (20)$$

$$\frac{\chi-1}{\chi}E_t\left(x_{t+1}M_{t+1}\left(1-H\left(\varepsilon^*_{t+1}\right)\right)\right) = (1-\theta)\,E_t\left(M_{t+1}\Omega'(\varepsilon^*_{t+1})x_{t+1}\right). \quad (21)$$

[30] A similar result holds for shocks to p or disaster realizations: the effect of a given shock is more pronounced when the economy has a higher outstanding debt.

The Euler equation interpretation is similar to that of the benchmark model; the investor takes into account the total user cost of debt, which now must take into account the different leverage in disaster and non-disaster states. The optimal leverage condition simply says that, rather than equating expected discounted marginal costs and benefits of debt over all the states together, the firm can now equate these expected marginal costs and benefits *conditional on the disaster happening or not.* This added flexibility will lead the firm to issue little debt that is payable in disaster states, since bankruptcy is much more likely and costly in these states (given the state prices M_{t+1}). The following proposition demonstrates this for a special case.

Proposition 3 *Assume that p is constant, $b_k = b_{tfp}$, and that there are no TFP shocks ($\sigma = 0$). If corporations are allowed to issue disaster-contingent debt, they will structure their debt to make the probability of default equal in disaster states and non-disaster states. As a result, corporations issue a fraction b_k less of debt payable in disaster states.*

Proof. Given the assumptions, we can simplify the first-order conditions (20-**??**); the expectations are just expectations over the idiosyncratic shocks ε. Denoting default cutoffs in non-disaster states by ε_{t+1}^{nd*} and in disasters by ε_{t+1}^{d*}, we have

$$\frac{\chi - 1}{\chi}\left(1 - H\left(\varepsilon_{t+1}^{d*}\right)\right) = (\theta - 1)\,\Omega'(\varepsilon_{t+1}^{d*}),$$

$$\frac{\chi - 1}{\chi}\left(1 - H\left(\varepsilon_{t+1}^{nd*}\right)\right) = (\theta - 1)\,\Omega'(\varepsilon_{t+1}^{nd*}),$$

implying that $\varepsilon_{t+1}^{d*} = \varepsilon_{t+1}^{nd*}$, so that the probability of default $H\left(\varepsilon_{t+1}^{d*}\right)$ is the same. Moreover, $\varepsilon_{t+1}^{d*} = \varepsilon_{t+1}^{nd*}$ implies $\frac{B_{t+1}^{d}}{K_{t+1}^{w}(1-b_k)} = \frac{B_{t+1}^{nd}}{K_{t+1}^{w}}$ or $B_{t+1}^{d} = B_{t+1}^{nd}(1 - b_k)$. ■

Figure 10 compares the response of the model with state-contingent debt to an increase in disaster risk, with the response of the benchmark model. The amplification effect largely disappears, and the model with state-contingent debt implies now no more investment volatility than the frictionless RBC model. While the assumption that private contracts are not made contingent on aggregate realizations is common to many models, this result suggest that it is far from innocuous.[31]

The benefits of debt conditionality in reducing volatility in response to shocks to disaster risk, comes on top of the obvious advantage that, should a disaster happen, there will be fewer defaults, which are likely to be costly (as in the previous section). This suggests that debt conditionality is likely valuable, provided that disasters can be well defined in a contract, and that there are no expectations of bailout.

4.3 Welfare cost of the tax shield

Following a large literature in corporate finance, the model features as a prime determinant of capital structure the tax subsidy to debt, or tax shield. The tax shield is inefficient in the model for two reasons. First, the tax shield lowers the user cost of capital and hence encourages capital accumulation. However, the competitive equilibrium of the model without taxes is already Pareto optimal, hence the subsidy leads to *overaccumulation* of capital. Second, the tax shield also amplifies fluctuations in

[31] Krishnamurthy (2003) similarly found that allowing for conditionality in the Kiyotaki-Moore model reduces or eliminate the amplification effect of financial frictions.

aggregate quantities, including consumption, and hence reduces welfare. Table 9 illustrates this effect by displaying the level of leverage, and the volatility of output and investments, for various values of χ. Both in terms of steady-states and in terms of fluctuations then, the tax subsidy generates deadweight losses. For our benchmark calibration, removing the tax shield entirely would increase welfare significantly, equivalent to a permanent increase of consumption of 0.90%.[32]

4.4 Fixed Leverage

The model assumes that firms are able to adjust their leverage ratio each period costlessly. In reality, it may be costly to issue new securities or to repurchase existing securities. Tables 5, 6, and 7 show the effect of imposing a constant ratio of debt to its capital stock, i.e. $L_t = B_{t+1}/K^w_{t+1} = \overline{L}$ is fixed at its average value.[33] This version of the model implies more volatility of both macroeconomic quantities and credit spreads, as firms cannot react to an increase in disaster risk by deleveraging. This makes the amplification effect larger than in the benchmark model. Having a constant leverage ratio has little effect on the level of credit spreads.

4.5 Samples with disasters

So far the results reported are calculated in samples which do not include disasters. Measured excess returns arise both through a standard risk premium and through sample selection (a "Peso problem") since the sample does not include the lowest possible return realizations. To quantify the importance of the second effect, tables 5 through 7 report the model moments in the benchmark model for a long sample that includes disasters. The average excess returns on equities is lower, 2.68% vs. 4.26% in a sample without disasters, but remains relatively large. Similarly, the average return on corporate bonds is 0.61%, rather than 0.72%. Hence, the Peso problem does not account for the bulk of the corporate bond premium. Last, quantities and returns are of course more volatile since they include some large negative realizations, but the level and volatility of credit spreads and leverage are completely unaffected.

4.6 Smaller or less frequent disasters

One potential concern with the current calibration is that it uses international data to calibrate the size and frequency of disasters. This is necessary given the length of macroeconomic time series, but it may overstate the potential severity or probability of a disaster in the United States – the country to which our model is calibrated. To evaluate this concern, I solve the model after halving either the size of disaster, or the probability of disaster. Holding all the other parameters are left constant, a smaller probability of disaster leads to a higher leverage, and hence higher probability of default in "normal times". Because the risk premium becomes significantly smaller, the level of credit spreads falls to 30bps and the excess return on corporate bonds is only 6bps, while the excess return on equity is 1.42%. Moreover, variation in disaster risk has now a small effect on quantities and on spreads. However, as shown in the technical

[32] Glover et al. (2010) also study the effect of eliminating the tax shield. They find that, in the context of their model, this may not reduce volatility significantly.

[33] In this version of the model, there is no choice of capital structure, hence the first-order condition 9 is discarded.

appendix, if risk aversion is increased to reinstate a significant equity premium, and the volatility σ_ε and tax shield χ are recalibrated to match the leverage, the model implications are maintained. Similarly, halving the size of disasters has similar, though smaller, effect: higher leverage, lower credit spreads and excess return on corporate bonds, less volatile quantities. But similarly, increasing risk aversion reinstates the key conclusions of the analysis.

4.7 Role of the IES and risk aversion

While the households are assumed to have recursive utility, the model can also be solved in the special case of expected utility. When the elasticity of substitution is kept equal to 2, and the risk aversion is lowered to .5 to reach expected utility, the *qualitative* implications are largely unaffected. Tables 5 through 7 report the model moments with this specification. Because risk aversion is lower, all risk premia are lower, and the response of quantities to a probability of disaster shock is also smaller since agents care less about risk. For instance, the volatility of investment falls from 7.81% to 5.09%, the excess return on corporate bonds falls from 72bps to 37bps, and the credit spreads from 72bps to 29bps.[34]

In contrast, when the elasticity of substitution is small, a shock to the probability of disaster leads to starkly different *qualitative* effects. When the IES is low enough, investment, output and employment rise (rather than fall) as the probability of disaster rises. The intuition is that higher risk makes people save more, despite the fact that capital is more risky. In the frictionless model, the threshold value for the IES is exactly unity. In the model of this paper, higher uncertainty has a more negative effect on investment demand, and hence the threshold value for the IES is lower than unity. Hence, for a range of values of IES below unity, the financial friction model implies that higher disaster risk lowers economic activity, while the frictionless model implies the opposite – an extreme example of the potential importance of financial frictions.

4.8 Comparison with idiosyncratic uncertainty shocks

Following Bloom (2009), several recent studies consider the effect of an increase in idiosyncratic uncertainty, σ_ε in our notation. While Bloom (2009) focused on the transmission of this shock through adjustment costs frictions, Arellano, Bai and Kehoe (2009), and Gilchrist, Sim and Zakrajek (2010) use default risk frictions, similar to my model. The shock to disaster risk is also an increase in uncertainty, and hence has a qualitatively similar effect. For instance, comparing figures 1 and 2 shows that the two parameters p and σ_ε have similar effects on steady-states. However, the channel through which the mechanism operates is somewhat different in my model, because an increase in aggregate uncertainty makes defaults more systematic and hence affects the bond risk premium.

To illustrate the differences in the mechanism, we can think of three experiments. First, the response of the economy to a shock to σ_ε is essentially unaffected by the coefficient of risk aversion. In contrast,

[34] There is one qualitative change, but it is hard to discern in most statistics. A shock to the probability of disaster increases consumption, hence with expected utility it is a "good state", i.e. low marginal utility of consumption state. This is not the case with Epstein-Zin utility, since the future value is lower, making a high probability of disaster state a "bad state" (high marginal utility of consumption). This in turn implies that assets which pay off well in that state have higher risk premia rather than lower risk premia.

as discussed in section 4.7, the response to an increase in disaster risk in my model is stronger when risk aversion is larger. Second, in the frictionless version, an increase in disaster risk leads to a recession, whereas an increase in idiosyncratic risk has no effect on economic activity.[35] Finally, suppose that we consider a shock to disaster risk, such that high disaster risk states have low idiosyncratic volatility, making the total quantity of risk constant over time. In essence, we are changing only the relative importance of aggregate and idiosyncratic risk, and hence the *correlation* across firms. This shock reduces investment and output, if risk aversion is positive, even though total risk does not change at the microeconomic level. The appendix produces the impulse responses corresponding to these three experiments.

The aim of this discussion is obviously not to argue that idiosyncratic uncertainty shocks are unimportant, but that the channel through which they operate is different than the channel through which aggregate uncertainty shock operate, at least in this model. The two approaches have different strengths: my model connects well with the evidence on the behavior of credit spreads, correlation risk and aggregate risk premia. In contrast, the studies of Arellano et al. and Gilchrist et al. focus on more realistic microeconomic heterogeneity, and take into account the effect of uncertainty on reallocation and on the labor wedge among other issues.

4.9 Capital adjustment costs

While the benchmark model abstracts from adjustment costs in the interest of simplicity, introducing them is useful to generate further volatility in the value of capital. In particular, the model implies that an increase in the probability of disaster has essentially no effect on realized equity returns or bond returns.[36] This implication is overturned if there are adjustment costs, because the price of capital then falls following an increase in the probability of disaster, since investment and marginal Q fall. It is simplest to consider an external adjustment cost formulation. Suppose that capital goods are produced by a competitive investment sector which takes I_t consumption goods at time t, and K_t capital goods at time t, and generates $K_{t+1} = (1-\delta)K_t + \Phi\left(\frac{I_t}{K_t}\right)K_t$ capital goods next period. These capital goods are then sold in a competitive market to final goods producing firms at a price given by: $P_t^K = \frac{1}{\Phi'\left(\frac{I_t}{K_t}\right)}$.The same formulas as in the model then apply, with the proviso that the return on capital R_{t+1}^K is now

$$R_{t+1}^K = \left(\frac{(1-\delta)\,P_{t+1}^K + \alpha\frac{Y_{t+1}}{K_{t+1}}}{P_t^K}\right)(1 - x_{t+1}b_k),$$

and $V_t = K_t R_{t+1}^K P_t^K$, with $\varepsilon_{t+1}^* = \frac{B_{t+1}}{R_{t+1}^K K_{t+1}^w P_t^K} = \frac{L_{t+1}}{R_{t+1}^K P_t^K}$. The appendix reports results obtained using the $\Phi(x) = a_0 + a_1\frac{x^{1-\eta}}{1-\eta}$, where a_0 and a_1 are picked to make the steady-state investment rate and marginal Q independent of η, and a figure in appendix compares the impulse response function of the benchmark model (without adjustment costs) and the model with adjustment costs. As expected, adjustment costs smooth the response of investment and output. The qualitative dynamics, as well as

[35] In some models, an increase in uncertainty would lead to a boom by leading to labor reallocation among firms with decreasing return to scale. But in the model of this paper, idiosyncratic shocks literally wash out because of the combined assumptions of constant return to scale and frictionless labor market.

[36] Technically, the only effect is through a decrease in the supply for labor which pushes the wage up, leading to slightly lower profits and hence slightly higher default rates.

the asset prices, remain similar. When the probability of disaster rises, the return on equity is now lower, and the return on the corporate bond is also slightly lower, reflecting the fall in the resale value of capital and the ensuing higher default rate. Overall, adjustment costs have a limited effect in the model.[37]

4.10 Additional comparative statics

Tables 5 through 7 go through some additional comparative statics and show the effect of reducing the tax shield by 20% to 2.6%, increasing recovery rates by 20% to 0.85, or reducing the idiosyncratic volatility shock by 20% to 17%. Reducing the tax shield leads to lower leverage, and lower credit spreads as both the default probability and the risk premium on corporate bonds fall. Last, the effect of disaster risk on quantities is now smaller as firms rely less on debt financing, and hence the diminished attractiveness of debt financing when disaster risk is high has smaller real effects.

Increasing recovery rates leads to higher leverage as bankruptcy costs become smaller. The effect on spreads is theoretically unclear, since higher recovery rates in themselves work to reduce spreads; overall the effect is limited. Macroeconomic quantities are now less sensitive to disaster risk, as default during disasters are less costly. Last, diminishing idiosyncratic volatility leads to higher leverage, but here the sensitivity of macroeconomic quantities to disaster risk increases, as leverage is high and there are more firms that are close to default given the higher hazard rate.

These experiment underscore that leverage is not a sufficient statistic for the sensitivity of the economy to shocks. High leverage may be driven by idiosyncratic volatility or high risk aversion and aggregate volatility, and these economies behave differently.

5 Conclusion

The paper makes two main contributions: first, the model embeds a capital structure trade-off model in an equilibrium business cycle setup. The trade-off model is a well established theory in corporate finance, and is a promising approach to link bond prices and macroeconomic aggregates in a tractable way. In contrast to alternative financial frictions which emphasize binding borrowing constraints, the trade-off model applies to all firms, large and small, even if they pay dividends. Second, the paper studies the effect of time-varying disaster risk on the economy. The model replicates the empirical evidence outlined in the introduction: credit spreads are large, in particular larger than expected losses, and volatile. Moreover, credit spreads are countercyclical, and the movements in the credit spread reflect not a time-varying default probability, but a time-varying corporate bond risk premium. Finally, the trade-off friction substantially amplifies the response of macroeconomic aggregates to disaster risk. The key mechanism is that defaults are expected to be more systematic, increasing risk-adjusted bankruptcy costs, and hence the user cost of capital, leading to lower leverage and lower capital expenditures.

A direction in which to extend the analysis is to consider long-lived firms that hold both long-term

[37] Because of the large IES, increasing adjustment costs does not have strong effects: investment volatility falls as the adjustment cost curvature is increased.

debt and cash. This would likely generate additional volatility as well as endogenous persistence.

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Parameter	Symbol	Value
Capital share	α	.3
Depreciation rate	δ	.08
Share of consumption in utility	υ	.3
Discount factor	β	.987
Trend growth of TFP	μ	.01
Standard deviation of TFP shock	σ	.015
Intertemporal elasticity of substitution	$1/\psi$	2
Risk aversion	γ	6
Mean probability of disaster		.038
Distribution of $b_{tfp} = b_k$	α_{size}	5.16
	β_{size}	11.1
	δ_{size}	1.38
Persistence of $\log(p)$	ρ_p	.75
Unconditional std. dev. of $\log(p)$	$\frac{\sigma_p}{\sqrt{1-\rho_p^2}}$	1.4
Idiosyncratic shock volatility	σ_ε	0.21
Tax subsidy	$\chi - 1$	0.033
Recovery rate	θ	0.7

Table 1: **Parameter values for the benchmark model.** The time period is one year.

		$\sigma(\Delta \log Y)$	$\sigma(\Delta \log C)$	$\sigma(\Delta \log I)$	$\sigma(\Delta \log N)$
	Data	2.78	1.81	7.01	2.67
RBC model	No disaster risk	1.37	0.76	3.19	0.48
	Constant disaster risk	1.37	0.77	3.26	0.48
	Time-varying disaster risk	1.49	1.03	5.56	1.03
Benchmark model	No disaster risk	1.36	0.75	2.72	0.46
	Constant disaster risk	1.36	0.75	2.87	0.47
	Time-varying disaster risk	1.78	1.53	7.81	1.78

Table 2: **Business cycle statistics (annual).** Volatility of investment, consumption, employment and output growth. The model statistics are computed in a sample without disasters.

		$E(R_f)$	$E(R_c\text{-}R_f)$	$E(R_e\text{-}R_f)$	$E(y)$	$\sigma(y)$	$\rho_{y,gdp}$
	Data	0.80	0.80	6.80	0.94	0.41	-0.37
RBC model	No disaster risk	1.91	–	-0.02	–	–	–
	Constant disaster risk	-0.55	–	2.61	–	–	–
	Time-varying dis. risk	-0.32	–	2.37	–	–	–
Benchmark model	No disaster risk	1.86	-0.00	-0.01	0.28	0.00	0.05
	Constant disaster risk	-0.56	1.23	5.14	1.35	0.00	-0.66
	Time-varying dis. risk	-0.29	0.72	4.26	0.90	0.49	-0.48

Table 3: **Financial Statistics, 1.** Mean risk-free return, mean excess return on corporate bonds, mean excess return on equity, mean and volatility of the credit spread, and correlation between credit spread and HP-filtered GDP. The model statistics are calculated in a sample without disasters.

	$E(Leverage)$	$\sigma(Leverage)$	$E(Default)$	$\sigma(Default)$	$E(LGD)$
Data	45	9	50	30	45
No disaster risk	59.25	0.10	81.05	1.19	34.58
Constant disaster risk	55.87	0.10	33.76	0.58	34.23
Time-varying disaster risk	55.46	6.31	50.32	26.30	34.23

Table 4: **Financial Statistics, 2.** Mean and volatility of leverage and default rate, and mean of loss rate on debt given default (LGD). The model statistics are calculated in a sample without disasters. The default rate is in basis points, and leverage and loss given default are in percetage points.

	$\sigma(\Delta \log Y)$	$\sigma(\Delta \log C)$	$\sigma(\Delta \log I)$	$\sigma(\Delta \log N)$
Data	2.78	1.81	7.01	2.67
Benchmark	1.78	1.53	7.81	1.78
Constant Leverage	2.04	1.92	11.00	2.34
Sample with disasters	5.97	6.19	9.32	1.78
Lower probability of disaster	1.39	0.84	3.26	0.65
Smaller size of disaster	1.53	1.12	5.09	1.16
Lower tax shield $\chi = 1.026$	1.72	1.44	7.47	1.64
Higher recovery rate on assets $\theta = 0.85$	1.72	1.44	7.08	1.64
Lower idiosyncratic volatility $\sigma_\varepsilon = 0.17$	1.87	1.68	8.48	1.98
Expected utility (low risk aversion= 1/2)	1.53	1.13	5.09	1.16

Table 5: **Sensitivity analysis. Business cycle statistics(annual).**

	$E(R_f)$	$E(R_c\text{-}R_f)$	$E(R_e\text{-}R_f)$	$E(y)$	$\sigma(y)$	$\rho(y, GDP)$
Data	0.80	0.80	6.80	0.94	0.41	-0.37
Benchmark	-0.29	0.72	4.26	0.90	0.49	-0.48
Constant Leverage	-0.24	1.02	4.73	1.11	1.35	-0.80
Sample with disasters	-0.29	0.61	2.68	0.90	0.50	-0.49
Lower probability of disaster	1.30	0.06	1.42	0.30	0.02	-0.61
Smaller size of disaster	0.72	0.48	2.33	0.70	0.41	-0.62
Lower tax shield $\chi = 1.026$	-0.32	0.61	4.16	0.74	0.37	-0.42
Higher recovery rate on assets $\theta = 0.85$	-0.28	0.81	4.70	1.09	0.66	-0.62
Lower idiosyncratic volatility $\sigma_\varepsilon = 0.17$	-0.28	0.82	4.47	0.94	0.57	-0.49
Expected utility (low risk aversion= 1/2)	1.27	0.29	2.25	0.51	0.22	-0.71

Table 6: **Sensitivity analysis. Financial Statistics, 1.**

	$E(Leverage)$	$\sigma(Leverage)$	$E(Default)$	$\sigma(Default)$	$E(LGD)$
Data	45	9	50	30	45
Benchmark	55.46	6.31	50.32	26.30	34.23
Constant Leverage	55.00	0.00	26.75	1.27	34.15
Sample with disasters	55.46	6.25	76.06	262.84	34.29
Lower probability of disaster	58.51	1.09	69.49	13.31	34.50
Smaller size of disaster	57.37	4.01	61.31	21.02	34.40
Lower tax shield $\chi = 1.026$	53.69	7.25	36.88	21.08	34.07
Higher recovery rate on assets $\theta = 0.85$	60.87	4.30	131.80	47.98	20.76
Lower idiosyncratic volatility $\sigma_\varepsilon = 0.17$	60.41	7.68	36.44	20.83	33.27
Expected utility (low risk aversion= 1/2)	57.67	2.72	61.39	20.30	34.42

Table 7: **Sensitivity analysis. Financial Statistics, 2.**

	$Spread_t$	$E_t(R_{c,t+1} - R_{f,t+1})$	$E_t\left(Loss_{t+1}\right)$
Mean	0.89	0.61	0.28
Std Deviation	0.49	0.50	0.04
Covariance with Invt	-0.17	-0.20	0.02
Correlation with Invt	-0.45	-0.49	0.53

Table 8: **Time-varying corporate bond premium.** The table reports the mean, standard deviation, and covariance and correlation with investment of (i) the spread, (ii) the conditional expected return on the corporate bond, and (iii) the conditional expected loss (i.e. the product of the probability of default, and the loss given default). These statistics are identical in samples with or without disasters.

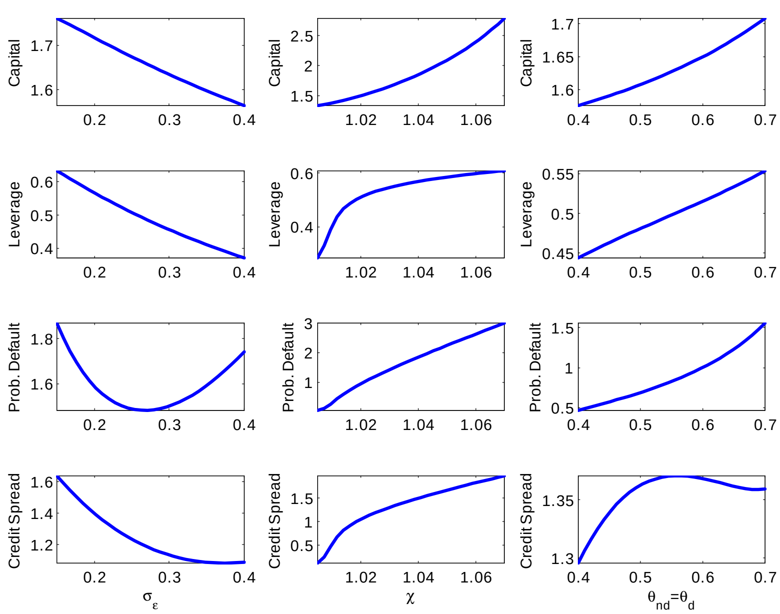

Figure 1: **Comparative statics on steady-state.** Effect of idiosyncratic volatility σ_ε, tax subsidy χ, and recovery rate θ, on capital, leverage, probability of default (in %), and credit spread (in %).

$100 \times (\chi - 1)$	$E(Leverage)$	$\sigma(\Delta \log Y)$	$\sigma(\Delta \log I)$	Welfare
5.00	58.03	1.95	8.37	2.89
4.50	57.46	1.88	8.12	2.13
4.00	56.77	1.84	7.98	1.53
3.50	55.95	1.79	7.79	1.06
3.00	54.82	1.74	7.57	0.70
2.50	53.31	1.70	7.37	0.43
2.00	51.44	1.65	7.06	0.24
1.50	49.44	1.60	6.60	0.12
1.00	46.37	1.56	6.22	0.04
0.50	39.69	1.51	5.74	0.01
0.30	36.12	1.51	5.66	0.00
0.10	27.76	1.50	5.60	0.00
0.05	24.44	1.49	5.54	0.00
0.01	19.34	1.49	5.49	0.00

Table 9: **Effect of tax shield parameter on mean leverage, on volatility of output and investment, and on welfare.**

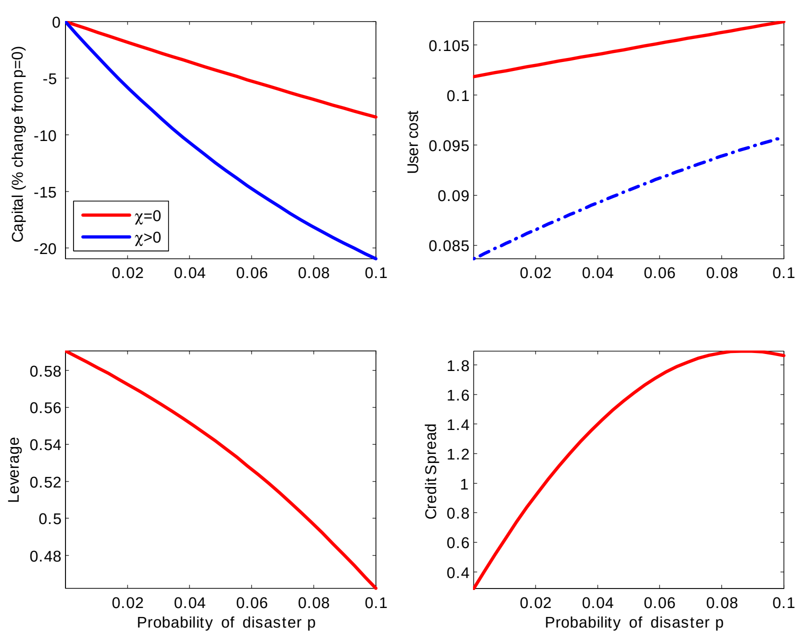

Figure 2: **Comparative statics on steady-state.** Effect of disaster probability on capital, leverage, credit spreads (in %), and the user cost of capital, for the frictionless model ($\chi = 0$, dot-dashed line) and the benchmark model ($\chi > 0$, full line).

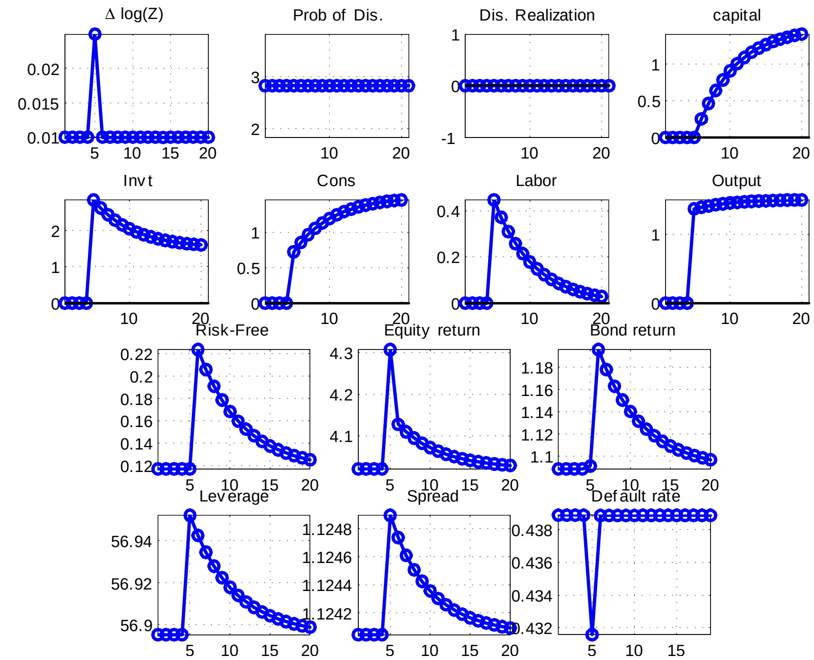

Figure 3: **Impulse response function of model quantities and returns to a one standard deviation shock to total factor productivity**. Quantity responses are shown in % deviation from balanced growth path. Returns, default rates, credit spreads, leverage and the probability of disaster are annual, in % per year.

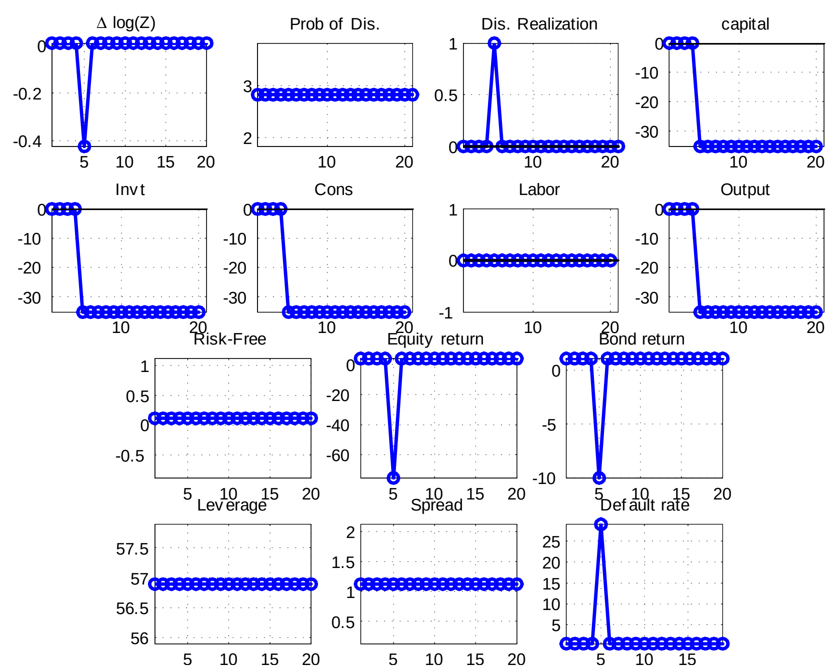

Figure 4: **Impulse response function of model quantities and returns to a disaster realization**. Quantity responses are shown in % deviation from balanced growth path. Returns, default rates, credit spreads, leverage and the probability of disaster are annual, in % per year.

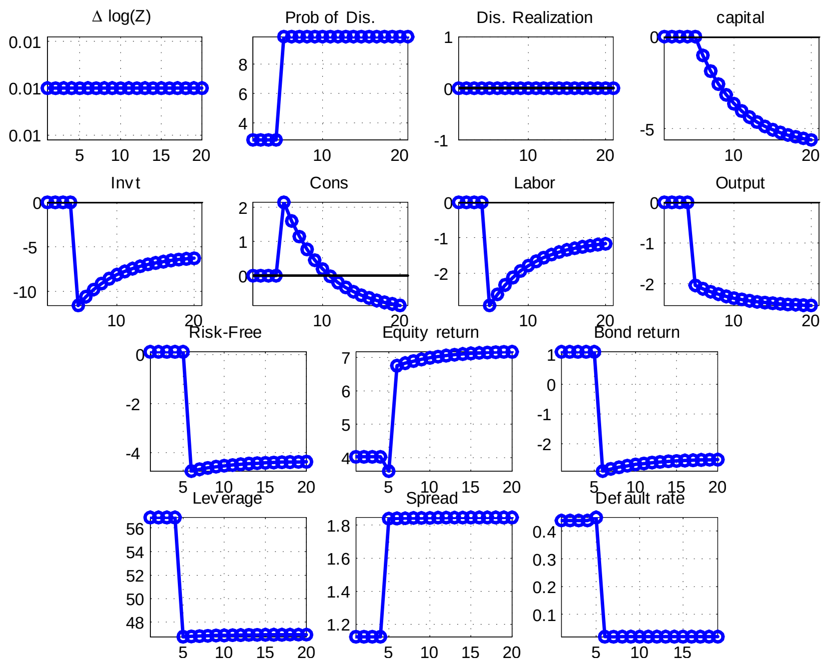

Figure 5: **Impulse response function of model quantities and returns to a shock to the probability of disaster**. Quantity responses are shown in % deviation from balanced growth path. Returns, default rates, credit spreads, leverage and the probability of disaster are annual, in % per year.

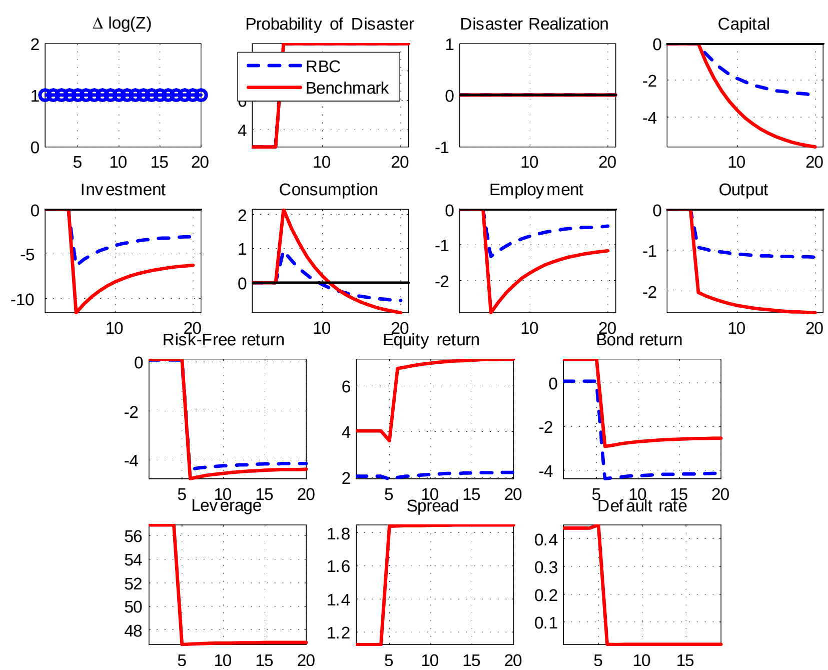

Figure 6: **Comparison of RBC model and benchmark model.** This figure compares the impulse response of the benchmark model (red full line), and the frictionless RBC model (blue dashed line) to an increase in disaster probability. Quantity responses are shown in % deviation from balanced growth path. Returns, default rates, credit spreads, leverage and the probability of disaster are annual, in % per year.

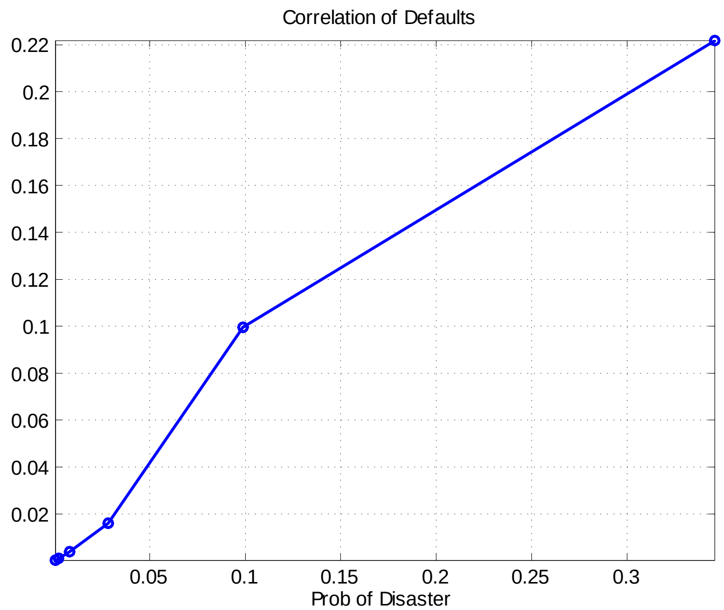

Figure 7: **Time-varying systematic risk: Correlation of defaults in the model.** This picture plots the correlation of default indicator between any two firms next period, i.e. $Corr_t(def_{it+1}, def_{jt+1})$, as a function of the disaster probability p.

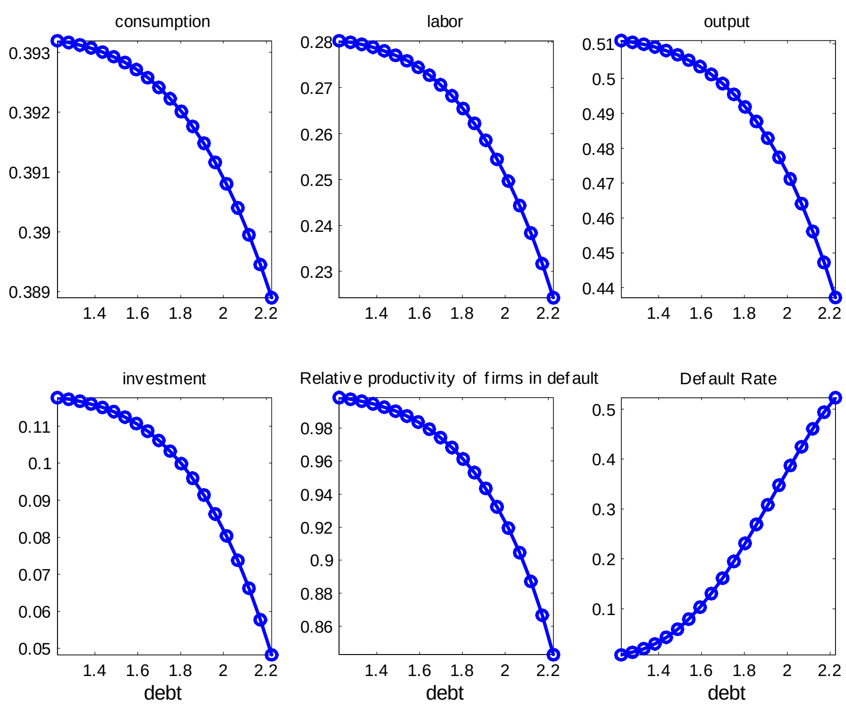

Figure 8: **Effect of outstanding debt on quantities, when firms in default are less productive.** The figure plots the policy functions for consumption, $c(k, b, p)$, employment $N(k, b, p)$, output $y(k, b, p)$, investment $i(k, b, p)$, the relative productivity of firms in default relative to firms not in default, and the share of firms in default, as a function of outstanding debt b (holding k and p fixed).

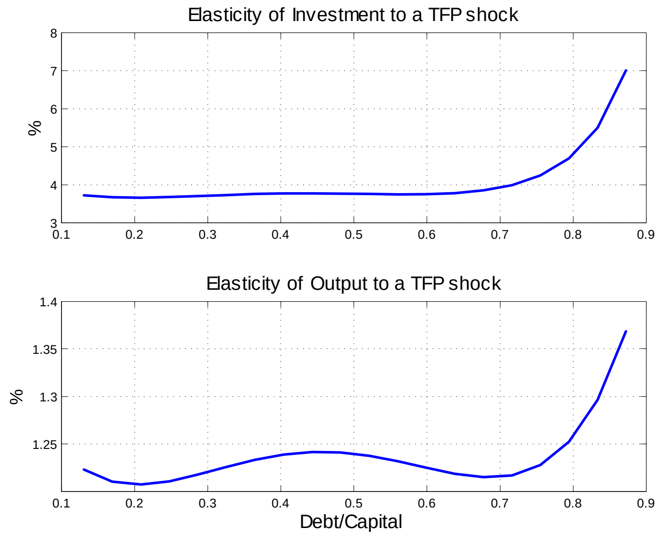

Figure 9: **Effect of oustanding debt on the sensitivity of investment and output to a TFP shock.** The figure plots, as a function of the debt-capital ratio, the percentage change of investment (top panel) and output (bottom panel) to a 1% TFP shock.

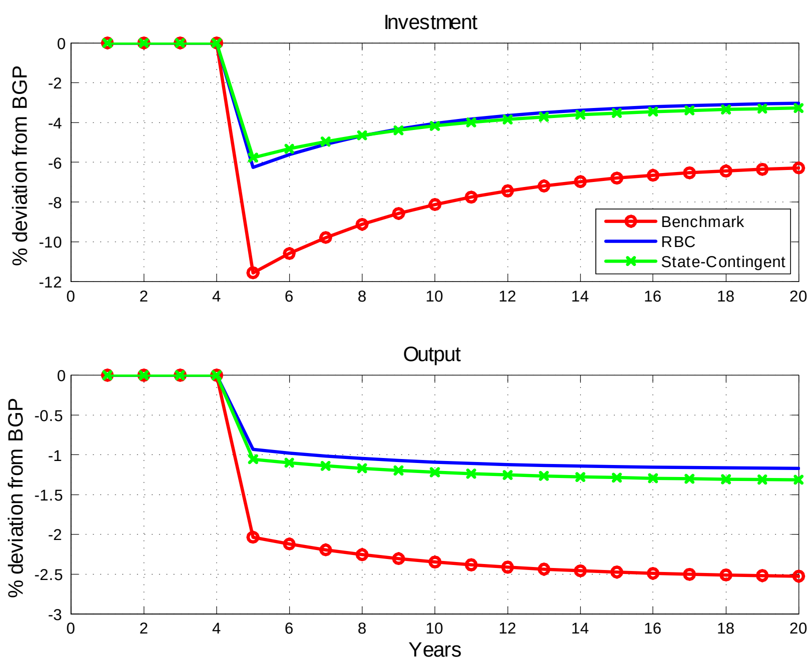

Figure 10: **Role of state-contingent debt.** The figure depicts the impulse response function of investment (top panel) and output (bottom panel) to a shock to the probability of disaster, in three different models. Green line = state-contingent debt (section 4.2), red line = benchmark model, blue = RBC frictionless model. Responses are shown in % deviation from balanced growth path.