Information about Sellers' Past Behavior in the Market for Lemons*

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Abstract

In markets under adverse selection, buyers' inferences on the quality of a good rely on the information they have about the seller' past behavior. This paper examines the roles of difference pieces of information about sellers' past behavior. Agents match randomly and bilaterally, and buyers make take-it-or-leave-it offers to sellers. It is shown that when market frictions are small (low discounting or fast matching), the observability of time-on-the-market improves efficiency, while that of number-of-previous-match deteriorates it. When market frictions are not small, the latter may improve efficiency. The results suggest that market efficiency is not monotone in the amount of information available to buyers but crucially depends on what information is available under what market conditions.

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1 Introduction

Consider a prospective home buyer who is about to make an offer to a seller. He understands that the seller knows better about her own house and thus there is an adverse selection problem. A low price may be rejected by the seller, while a high price runs the risk of overpaying for a *lemon*. Having this uncertainty, the buyer can rely on the information he has about the seller's past behavior. If the seller has rejected good prices in the past, it would indicate that (she believes) her house is even more worthwhile. Access to sellers' past behavior, however, can be limited. Regulations or market practices may not allow it, or relevant records simply may not exist. There are several possibilities. The buyer may not get any information or observe only how long the house has been up for sale. Or, the broker may hint how many buyers have shown interest before. The buyer's inference will crucially depend on what information he has. What makes the problem more intriguing is the seller's strategic behavior. The seller has an incentive to reject an acceptable price today if, by

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doing so, she can extract an even better offer tomorrow. The buyer must take into account that the seller has strategically behaved in the past and will strategically respond to his offer. Now it is far from clear what effects buyers' having more information about sellers' past behavior would have on agents' payoffs and market efficiency.

This paper examines the roles of different pieces of information about sellers' past behavior in the market for lemons. In particular, it investigates the relationship between efficiency and the amount of information available to buyers: Does more (or less) information improve efficiency? Intuitively, there are two opposing arguments. On the one hand, as in reputation games, information about sellers' past behavior provides information about their intrinsic types, and thus more information would enable buyers to better tailor their actions. On the other hand, as in signaling games, if sellers' current behavior is better observable to future buyers, sellers have a stronger incentive to signal their types, which may cause efficiency losses.

The model is a dynamic decentralized version of Akerlof's market for lemons (1970). In each unit of time, unit measures of buyers and sellers enter the market. Buyers are homogeneous, while there are two types of sellers. Some sellers possess a unit of low quality, while the others own a unit of high quality. A high-quality unit is more valuable to both buyers and sellers. There are always gains from trade, but type (quality) is each seller's private information. Agents match randomly and bilaterally. In a match, the buyer makes a take-it-or-leave-it offer to the seller. If an offer is accepted, trade takes place and the pair leave the market. Otherwise, they stay and wait for next trading opportunities.

The following three information regimes are considered:

1. Regime 1 (no information) : Buyers do not receive any information about their partners' past behavior.

2. Regime 2 (time on the market) : Buyers observe how long their partners have stayed on the market.

3. Regime 3 (number of previous matches): Buyers observe how many times their partners have matched before, that is, how many offers they have rejected before.

Ex ante, the regimes are partially ordered in the amount of information available to buyers. The order between Regime 2 and Regime 3 is not obvious. On the one hand, in dynamic environments, time-on-the-market enables informed players to credibly signal their types and/or uninformed players to effectively screen informed players. Therefore, the role of number-of-previous-match might be just to provide an estimate of time-on-the-market. On the other hand, number-of-previous-match purely reflects the outcomes of informed players' decisions, while time-on-the-market is compounded with search frictions. It will be shown later that number-of-previous-match dominates time-on-the-market in the sense that Regime 3 outcome is essentially independent of whether time-on-the-market is jointly observable or not, that is, Regime 3 outcome obtains if both time-on-the-market and number-of-previous-match are observable. Consequently, the three regimes are de facto fully ordered.

Hörner and Vieille (2009) (HV, hereafter) studied the role of previously rejected prices in a closely related model. They considered a game in which a single seller faces a sequence of buyers and compared the case where past offers are observable to future buyers (public offers) and the case where they are not (private offers). Their setup is in discrete time without search frictions. Therefore, time-on-the-market and number-of-previous-match are indistinguishable and always observable to buyers. Their private case corresponds to the regime in which both time-on-the-market and number-of-previous-match are observable (so, effectively to Regime 3), while past offers are additionally observable in their public case.

HV found that more information may reduce efficiency. Precisely, when adverse selection is severe and discounting is low, bargaining impasse necessarily occurs with a high probability in the public case, while agreement is always reached in the private case. In their game, the two regimes are not Pareto ranked, as the low-type seller is strictly better off in the private case, while buyers are better off in the public case.[1] One can show, however, that if their game is embedded into a market setting as in this paper, then the private case weakly Pareto dominates the public case.[2]

I show that if market frictions are small (agents are patient or matching is fast), the observability of number-of-previous-match also reduces efficiency. There is a cutoff level of market frictions below which Regime 1 Pareto dominates Regime 3: buyers weakly prefer Regime 1 to Regime 3, while (low-type) sellers strictly prefer Regime 1 to Regime 3. The intuition for this result is as follows. In any regime, low-type sellers, due to their lower reservation value, leave the market faster than high-type sellers. This implies that the proportion of high-type sellers in the market would be larger than the corresponding proportion in the entry population. In Regime 1, all sellers look the same to buyers, that is, all cohorts of sellers are completely mixed. Therefore, the incentive constraint for buyers to offer a high price is relaxed. In Regime 3, however, the observability of number-of-previous-match prevents mixing of different cohorts of sellers. Therefore, a higher proportion of high-type sellers in the market does not facilitate trade. This difference yields, for example, the following consequence: in Regime 3 high-type sellers can never trade in their first matches (as buyers offer only a low price to them), while in regime 1 they do trade with a positive probability (as buyers offer a high price with a positive probability). This result is in line with HV's finding and seems to suggest that less information could be always preferable.

The observability of time-on-the-market, however, enhances efficiency. I show that if market frictions are sufficiently small, realized market surplus is strictly higher in Regime 2 than in Regime 1. To understand this, consider a new seller who quickly met a buyer. In Regime 3, the opportunity cost of accepting a current offer is rather large, as the seller's rejection would be observable to future buyers, who would update their beliefs accordingly. In Regime 2, the corresponding opportunity

[1] Precisely, no buyer obtains a positive expected payoff in the private case, while the first buyer obtains a strictly positive expected payoff in the public case.

[2] This is because the distribution of sellers matters in the market. In the public case, a buyer would get a positive expected payoff if and only if he meets a new seller. However, some low-type sellers and all high-type sellers stay in the market forever, and thus the probability of a buyer meeting a new seller would be negligible. Steady state would not be well-defined in this case. However, one can introduce the probability of exogenous exit and consider the limit of steady-state equilibria as the probability vanishes.

cost is smaller, as future buyers would not know that the seller rejected a price. This different yields the following consequence: in Regime 2 low-type sellers who quickly meet a buyer trade with probability 1, while in Regime 3 they do not. In Regime 1, the opportunity cost of accepting a current offer is independent of sellers' private histories, and thus low-type sellers reject a low price with a positive probability in their first matches. Consequently, in a cohort, the proportion of high-type sellers increases faster in Regime 2 than in Regime 1, which induces buyers to offer a high price relatively quickly in Regime 2. In Regime 2, as in Regime 3, different cohorts of sellers are not mixed, which is negative for efficiency. When market frictions are small, it turns out that the former positive effect more than offsets the latter negative effect, and thus Regime 2 is more efficient than Regime 1.

This result demonstrates that efficiency is not monotone in the amount of information available to buyers. It also shows that the results on number-of-previous-match and rejected prices stem from the nature of such information, not from any general relationship between efficiency and information.

In addition, depending on market conditions, the observability of number-of-previous-match may contribute to efficiency. I show that if market frictions are not small, there are cases where Regime 3 is more efficient than the other two regimes. This reinforces the argument that efficiency is not monotone in the amount of information. It also points out that what matters is what information is available under what market conditions.

The remainder of the paper proceeds as follows. The next section links the paper to the literature. Section 3 introduces the model. The following three sections analyze each regime. Section 7 compares the regimes and Section 8 concludes.

2 Related Literature

This paper contributes to the literature mainly in three ways. First, a few papers investigated the consequences of different assumptions on information flow in dynamic games with asymmetric information. Second, there is a fairly large literature on dynamic markets under adverse selection. Last, a few papers analyzed bargaining with interdependent values.

In a dynamic version of Spence's signalling model, Nöldeke and Van Damme (1990) showed that, although there are multiple sequential equilibria, there is an essentially unique sequential equilibrium outcome that satisfies the never a weak best response requirement (Kohlberg and Mertens (1986)). As the offer interval tends to zero, the unique equilibrium outcome converges to the Riley outcome. Swinkels (1999) pointed out that the result crucially depends on the assumption on information flow. He showed that if offers are not observable to future uninformed players (which are observable in Nöldeke and van Damme), then the unique equilibrium outcome is complete pooling with no delay.[3] Using the terminology of this paper, full histories of informed player's past behavior are observable to future uninformed players in Nöldeke and van Damme, while only

[3]He also showed that if education is productive, then there may be a separating equilibrium.

time-on-the-market and number-of-previous-match are observable in Swinkels.

In a closely related two-period model, Taylor (1999) showed that the observability of previous reservation price and inspection outcome is efficiency-improving, that is, more information about past trading outcome is preferable. The key difference is that buyer herding, rather than sellers' signaling or buyer's screening, is the main concern in his paper. In his model, buyers have no incentive to trade with low-type sellers ($v_L - c_L = 0$) and a winner in an auction conducts an inspection prior to exchange. These cause buyers to get more pessimistic over time, that is, the probability that a seller owns a high-quality unit is lower in the second period than in the first period. The better observability of past outcome improves efficiency by weakening negative buyer herding.

Various dynamic versions of Akerlof's market for lemons have been developed. Janssen and Karamychev (2002) and Janssen and Roy (2002, 2004) examined the settings, with constant inflow of agents or one-time entry, where a single price clears each spot market. Inderst and Müller (2002) studied competitive search equilibrium with constant inflow of agents. Wolinsky (1990), Serrano and Yosha (1993, 1996), Blouin and Serrano (2001), and Blouin (2003) considered various settings, with constant inflow of agents or one-time entry and with two-sided uncertainty or one-sided uncertainty, in which agents meet bilaterally and play a simple bargaining game with only two possible transaction prices. Hendel and Lizzeri (1999, 2002) and Hendel, Lizzeri, and Siniscalchi (2005) studied dynamic durable goods markets where units are classified according to their vintages.

Moreno and Wooders (2010) considered a discrete-time version of Regime 1 and compared the outcome to the static competitive benchmark. They argued that if agents are sufficiently but not perfectly patient, realized market surplus is greater in the dynamic decentralized market than in the static competitive benchmark, but the difference vanishes as agents get more patient. The new findings of this paper regarding Regime 1 are as follows. First, they focused on the case where agents are sufficiently patient, while this paper provides a complete characterization under a mild assumption on the discount factor (Assumption 2). A consequence of this difference is the inequality in Proposition 1, which is absent in their paper. Second, this paper shows that their welfare result that social welfare is higher in the dynamic decentralized market than in the static benchmark is an artifact of their discrete-time formulation. In the continuous-time setting of this paper, as long as the inequality in Proposition 1 holds, social welfare in Regime 1 is exactly the same as that of the static competitive benchmark, independently of the discount rate and the matching rate. This illustrates that the driving force for their result is not agents' impatience but the fact that matching occurs only at the beginning of each period in the discrete-time setting.

The setting of this paper can be interpreted as bargaining taking place in a market. In this regard, this paper is also related to the literature on bargaining with interdependent values. Evans (1989) and Vincent (1989) offered early results and insightful examples. Deneckere and Liang (2006) provided a general characterization for the finite type case and explained the source and mechanics of bargaining delay due to adverse selection. They found an equilibrium structure that is quite similar to the ones in this paper. I will explain similarities and differences in Section 8. Fuchs and Skrzypacz (2010) considered an incomplete information bargaining game with a continuum of types and "no gap". In their model, values are not inherently interdependent but are endogenously interdependent because of random arrival of events that end the game with payoffs that depend on the informed player's type.

3 The Model

3.1 Setup

The model is set in continuous time. In each unit of time, unit measures of buyers and sellers enter the market for an indivisible good. Buyers are homogeneous, while there are two types of sellers. A measure $\widehat{q}$ of sellers possess a unit of low quality (low type) and the others own a unit of high quality (high type). A unit of low (high) quality costs c_L (c_H) to a seller and yields utility v_L (v_H) to a buyer. A high-quality unit is more costly to sellers ($c_H > c_L \geq 0$) and more valuable to buyers ($v_H > v_L$). There are always gains from trade ($v_H > c_H$ and $v_L > c_L$), but the quality of each unit is private information to each seller.

Agents match randomly and bilaterally according to a Poisson rate $\lambda > 0$. In a match, the buyer offers a price and then the seller decides whether to accept it or not. If an offer is accepted, then they receive utilities and leave the market. If a price p is accepted by a low-type (high-type) seller, then the buyer's utility is $v_L - p$ $(v_H - p)$ and the seller's is $p - c_L$ $(p - c_H)$. Otherwise, they stay in the market and wait for next trading opportunities. All agents are risk neutral. The common discount rate is $r > 0$. It is convenient to define $\delta \equiv \lambda / (r + \lambda)$. This is the effective discount factor in this environment that account for search frictions as well as discounting.

I focus on steady-state equilibrium in which agents of each type employ an identical strategy. Formally, let Ξ be the information set of buyers, that is, the set of distinct seller types from buyers' viewpoints. The set Ξ is a singleton in Regime 1. It is isomorphic to $\mathcal{R}_+$ in Regime 2 and $\mathcal{N}_0$ in Regime 3. Buyers' pure strategy is a function $B : \Xi \rightarrow \mathcal{R}_+$ where $B(\xi)$ represents their offer to type ξ sellers. Denote by σ_B buyers' mixed strategy. Buyers' beliefs are represented by a function $q : \Xi \rightarrow [0, 1]$ where $q(\xi)$ is the probability that a type ξ seller is the low type. Sellers' pure strategy is a function $S : \{L, H\} \times \Xi \times \mathcal{R}_+ \rightarrow \{A, R\}$ where L and H represent sellers' intrinsic types (low and high, respectively), an element in $\mathcal{R}_+$ represents a current offer, and A and R represent acceptance and rejection, respectively. Denote by σ_S sellers' mixed strategy where $\sigma_S(t, \xi, p)$ is the probability that a type (t, ξ) seller accepts price p. Buyers' offers are independent of their own histories. They are conditioned only on sellers' observable types, that is, only on Ξ. Sellers' actions depend on their own histories, but only through Ξ. Suppose, for example, there are two sellers who have met different numbers of buyers or been offered different prices. In Regime 2, if they have stayed on the market for the same length of time, then they are assumed to behave identically.

Let χ be the Borel measure over Ξ. Denote by $\emptyset$ the type (in Ξ) new sellers belong to. Given a strategy profile and the measure over Ξ, define a stochastic transition function $\phi : \{L, H\} \times (\Xi \cup \{n\}) \rightarrow \Delta(\Xi \cup \{e\})$ where n represents new sellers, e represents "exit the market", and $\Delta(X)$

is a Borel σ-algebra over a set X. Denote by $\phi(t,\xi,E)$ the probability that a type (t,ξ) seller belongs to the set $E \subseteq \Xi$ an instant later. In addition, let $\psi_s(\xi) \in \Xi$ denote the type of a type ξ seller after s length of time, conditional on the event that she has not matched for the period.

Given a strategy profile and the corresponding steady-state measure and transition function, agents' expected continuation payoffs can be calculated. Denote by V_B the expected continuation payoff of buyers and by $V_S(t,\xi)$ the expected continuation payoff of type (t,ξ) sellers.

Definition 1 *A collection $(\sigma_B, \sigma_S, q, \chi, V_B, V_S)$ is a symmetric steady-state equilibrium if*

(1) (buyer optimality) for each $\xi \in \Xi$,

$$\begin{aligned} supp\{\sigma_B(\xi)\} \;\subset\; & \arg\max_p q(\xi)[\sigma_S(L,\xi,p)(v_L-p)+(1-\sigma_S(L,\xi,p))V_B] \\ & +(1-q(\xi))[\sigma_S(H,\xi,p)(v_H-p)+(1-\sigma_S(H,\xi,p))V_B], \end{aligned}$$

(2) (seller optimality) for all $t \in \{L,H\}, \xi \in \Xi$, and p,

$$\sigma_S(t,\xi,p)\begin{cases} =1, & \text{if } p-c_t > V_S(t,\xi), \\ \in[0,1], & \text{if } p-c_t = V_S(t,\xi), \\ =0, & \text{if } p-c_t < V_S(t,\xi), \end{cases}$$

(3) (consistent beliefs) for almost all $E \in \Delta(\Xi)$,

$$q(E)=\frac{\int_{\Xi\cup\{n\}} q(\xi)\phi(L,\xi,E)\,d\chi}{\int_{\Xi\cup\{n\}}(q(\xi)\phi(L,\xi,E)+(1-q(\xi))\phi(H,\xi,E))\,d\chi},$$

(4) (steady-state condition) for almost all $E \in \Delta(\Xi)$,

$$\chi\{E\}=\int_{\Xi\cup\{n\}}(q(\xi)\phi(L,\xi,E)+(1-q(\xi))\phi(H,\xi,E))\,d\chi,$$

(5) (buyers' expected payoff)

$$V_B=\delta\int U_B(\xi)\frac{d\chi}{\chi\{\Xi\}},$$

where

$$\begin{aligned} U_B(\xi) \;=\; & q(\xi)\sigma_S(L,\xi,p)(v_L-p)+(1-q(\xi))\sigma_S(H,\xi,p)(v_H-p) \\ & +(q(\xi)(1-\sigma_S(L,\xi,p))+(1-q(\xi))(1-\sigma_S(H,\xi,p)))V_B \end{aligned}$$

for $p \in supp\{\sigma_B(\xi)\}$,

(6) (sellers' expected continuation payoffs) for each $t \in \{L,H\}, \xi \in \Xi$, and p,

$$V_S(t,\xi)=\delta\int_{p'} U_S(t,\psi_s(\xi),p')\,d\sigma_B(\psi_s(\xi))(p'),$$

where

$$U_S\left(t,\xi',p'\right)=\sigma_S\left(t,\xi',p'\right)\left(p'-c_t\right)+\left(1-\sigma_S\left(t,\xi',p'\right)\right)V_S\left(t,\xi',p'\right).$$

3.2 Assumptions

I focus on the case where (1) adverse selection is severe (so high-type units cannot trade in the static benchmark) and (2) market frictions are small (so agents have non-trivial intertemporal considerations). Formally, I make the following two assumptions.

Assumption 1 *(Severe adverse selection)*

$$\widehat{q}v_L+\left(1-\widehat{q}\right)v_H<c_H.$$

This inequality is a familiar condition in the adverse selection literature. The left-hand side is buyers' willingness-to-pay to a seller who is randomly selected from an entry population. The right-hand side is the minimal price high-type sellers may possibly accept. When the inequality holds, no price can yield nonnegative payoffs to both buyers and high-type sellers, and thus high-type units cannot trade.

For future use, let $\overline{q}$ be the value such that

$$\overline{q}v_L+(1-\overline{q})v_H=c_H,$$

that is, $\overline{q}=\left(v_H-c_H\right)/\left(v_H-v_L\right)$. A necessary condition for a buyer to be willing to offer c_H to a seller is that he believes that the probability that the seller is the low type is less than or equal to $\overline{q}$. Assumption 1 is equivalent to $\widehat{q}>\overline{q}$.

Assumption 2 *(Small market frictions)*

$$v_L-c_L<\delta\left(c_H-c_L\right)=\frac{\lambda}{r+\lambda}\left(c_H-c_L\right).$$

This assumption states that low-type sellers never accept any price that buyers may possibly offer to them (at most v_L) if they expect to receive an offer that high-type sellers are willing to accept (at least c_H) in their next matches. Given Assumption 1, this assumption is satisfied when δ is large (r is small or λ is large).

In addition, I assume that in any match the buyer offers either the reservation price of the low-type seller or that of the high-type seller.

Assumption 3 *For any $\xi\in\Xi$, if $p\in$supp$\{\sigma_B\left(\xi\right)\}$, then either $p=c_L+V_S\left(L,\xi\right)$ or $p=c_H+V_S\left(H,\xi\right)$.*

This assumption incurs no loss of generality. First, buyers never offer prices that are strictly higher than high-type sellers' reservation prices or between the two types' reservation prices. Second, future types of sellers do not depend on current prices in all three regimes.[4] Therefore, whenever buyers make losing offers in equilibrium (offers that will be rejected for sure), the offers and the corresponding acceptance probabilities can be set to be equal to the reservation prices of low-type sellers and 0, respectively.

The following result, which is a straightforward generalization of the Diamond paradox, greatly simplifies the subsequent analysis.

Lemma 1 *Buyers never offer strictly more than c_H, and thus high-type sellers' expected payoffs are always equal to 0, that is, $V_S(H,\xi) = 0$ for all $\xi \in \Xi$.*

From now on, abusing notations, let V_S denote the expected payoff of new low-type sellers and $V_S(\xi)$ denote the expected payoff of type (L,ξ) sellers.

4 Regime 1: No Information

In this section, the set Ξ is a singleton, that is, $\Xi = \{\emptyset\}$. Buyers do not obtain any information about sellers' past behavior and, therefore, cannot screen sellers. Similarly, sellers simply cannot signal their types.

Under severe adverse selection, one may think that no information flow (and the consequent impossibility of screening and signaling) may cause no trade of high-type units. Such intuition does not apply to the current dynamic setting. Suppose only low-type units trade. Then, due to constant inflow of agents, the proportion of high-type sellers would keep increasing over time. Eventually the market would be populated mostly by high-type sellers, and thus buyers would be willing to trade with high-type sellers. On the other hand, it cannot be that buyers always offer c_H and any match turns into trade. If so, the proportion of low-type sellers in the market would be equal to $\widehat{q}$. But then, due to Assumption 1, buyers' expected payoff would be negative.

In equilibrium, buyers randomize between a price $p^* (\leq v_L)$ and c_H, which are the reservation prices of low-type and high-type sellers, respectively. The equilibrium is sustained as follows. High-type sellers accept only c_H, while low-type sellers accept both c_H and p^*. High-type sellers stay relatively longer than low-type sellers. Then, the proportion of low-type sellers in the market will be smaller than $\widehat{q}$. This provides an incentive for buyers to offer c_H and trade also with high-type sellers. In equilibrium, buyers offer p^* and low-type sellers accept p^* with just enough probabilities so that, with the resulting proportion of low-type sellers in the market, buyers are indifferent between p^* and c_H.

To formally describe the equilibrium, let

- α^* be the probability that buyers offer p^*,
- β^* be the probability that low-type sellers accept p^*, and

[4] This claim does not hold if buyers observe previously rejected prices. In fact, the dependence of future offers (and, consequently, sellers' expected continuation payoffs) on current prices is the key to HV's bargaining impasse result in their public case.

- q^* be the proportion of low-type sellers in the market.

In equilibrium, the following three conditions must be satisfied.

1. Buyers' indifference:

$$q^*\beta^*(v_L - p^*) + (1 - q^*\beta^*)\,\delta\,(q^*v_L + (1 - q^*)\,v_H - c_H) = q^*v_L + (1 - q^*)\,v_H - c_H. \tag{1}$$

   The left-hand side is a buyer's expected payoff by offering p^*. The offer is accepted only when the seller is the low type and, conditional on that, with probability β^*. If the offer is not accepted, then the buyer can offer c_H in the next match. The right-hand is a buyer's expected payoff by offering c_H.

2. Low-type sellers' indifference (the reservation price of low-type sellers):

$$p^* - c_L = \delta\left((1 - \alpha^*)\,c_H + \alpha^* p^* - c_L\right). \tag{2}$$

   The left-hand side is a low-type seller's payoff by accepting p^*, while the right-hand side is her expected continuation payoff. If she rejects p^*, then in her next match she will receive p^*, which she is again indifferent between accepting and rejecting, with probability α^* and c_H with probability $1 - \alpha^*$.

3. Steady-state condition:

$$\frac{\widehat{q}}{1 - \widehat{q}} = \frac{q^*}{1 - q^*}\frac{\alpha^*\beta^* + 1 - \alpha^*}{1 - \alpha^*}. \tag{3}$$

   The left-hand side is the ratio of low-type sellers to high-type sellers among new sellers, while the right-hand side is the corresponding ratio among leaving sellers. The proportion of low-type sellers in the market is invariant only when the two ratios are identical.

The following proposition completely characterizes equilibrium in Regime 1.

Proposition 1 *In Regime 1, there is a unique equilibrium in which*
(1) buyers offer p^* *with probability* α^* *and* c_H *with probability* $1 - \alpha^*$*,*
(2) low-type sellers accept p^* *with probability* β^**,*
(3) the proportion of low-type sellers in the market is equal to q^**. If*

$$\delta \geq \frac{\widehat{q}\,(v_L - c_L)}{\widehat{q}\,(v_L - c_L) + (1 - \widehat{q})\,(v_H - c_H)}, \tag{4}$$

then $p^* = v_L$*, and* α^*, β^**, and* $q^*\,(= \overline{q})$ *solve Equations (1), (2), and (3). Otherwise,* $\beta^* = 1$*, and* $p^*\,(< v_L)\,, \alpha^*$*, and* q^* *solve Equations (1), (2), and (3).*

Proof. *Buyers know that only low-type sellers would accept* p^**, and thus* $p^* \leq v_L$*. One can show that if* $p^* = v_L$*, then both* α^* *and* β^* *are well-defined if and only if the inequality in (4) holds. If* $p^* < v_L$*, then* p^* *must be accepted by low-type sellers with probability* 1*, that is,* $\beta^* = 1$

(otherwise, buyers will deviate to a slightly higher offer). By applying the equations, it can be also shown that α^* and q^* $(< \overline{q})$ are well-defined if and only if the inequality in (4) is reversed. ■

If δ is large and so the inequality in Proposition 1 holds, then agents' expected payoffs are independent of the parameter values. Buyers' expected payoff is 0, while low-type sellers' expected payoff is $v_L - c_L$. Intuitively, when δ is large, low-type sellers are willing to accept p^* only when it is sufficiently high. However, the price p^* is bounded by v_L. In equilibrium the condition $p^* \leq v_L$ binds, and all other results follow from there.

If δ is rather small and so the inequality in Proposition 1 does not hold, agents' expected payoffs are not independent of the parameter values. Low-type sellers obtain less than $v_L - c_L$, and buyers obtain a positive expected payoff. Intuitively, when δ is small, low-type sellers are willing to accept a relatively low price and buyers can exploit such incentive. One can easily show that low-type sellers' expected payoff increases in δ, while buyers' expected payoff decreases in δ.

5 Regime 2: Time-on-the-market

In this section, the set Ξ is isomorphic to the set of non-negative real numbers, $\mathcal{R}_+$. A typical element $t \in \Xi$ represents the length of time a seller has stayed on the market.

As in other dynamic games with asymmetric information, time-on-the-market serves as a screening device. The reservation price of low-type sellers, due to their lower cost, is strictly smaller than that of high-type sellers. Therefore, low-type sellers leave the market relatively faster than high-type sellers. Buyers offer low prices to relatively new sellers and a high price to sellers who have stayed for a long time. Of course, in equilibrium, the length of time a seller must endure in order to receive the high price must be long enough. Otherwise, low-type sellers would mimic high-type sellers.

A Single Seller vs. A Sequence of Buyers

I first consider a game between a seller and a sequence of buyers. Buyers arrive stochastically according to the Poisson rate $\lambda > 0$. Refer to the buyer who arrives at time t and the seller who has stayed in the game for t length of time as time t buyer and time t seller, respectively. Assume that buyers' outside option is exogenously given as $V_B \in [0, \min\{v_L - c_L, v_H - c_H\})$.[5] I will endogenize V_B later by embedding this game into the market setting.

I start by describing two equilibria that are particularly simple and of special interest. In the first one, buyers play pure strategies. The second one is similar to the equilibrium in Regime 1. Figure 1 depicts the structures of the two equilibria.

[5]There is no loss of generality in restricting attention to the interval $[0, \min\{v_L - c_L, v_H - c_H\})$. If $V_B \geq v_H - c_H$, then high-type sellers would never trade. But then in the long run the market would be populated mostly by high-type sellers, and then buyers' expected payoff would not be materialized. If $V_B \geq v_L - c_L$, then either high-type sellers never trade or buyers offer only c_H. If it were the former, then a similar argument to the above would hold. If it were the latter, then buyers' expected payoff would be negative due to Assumption 1.

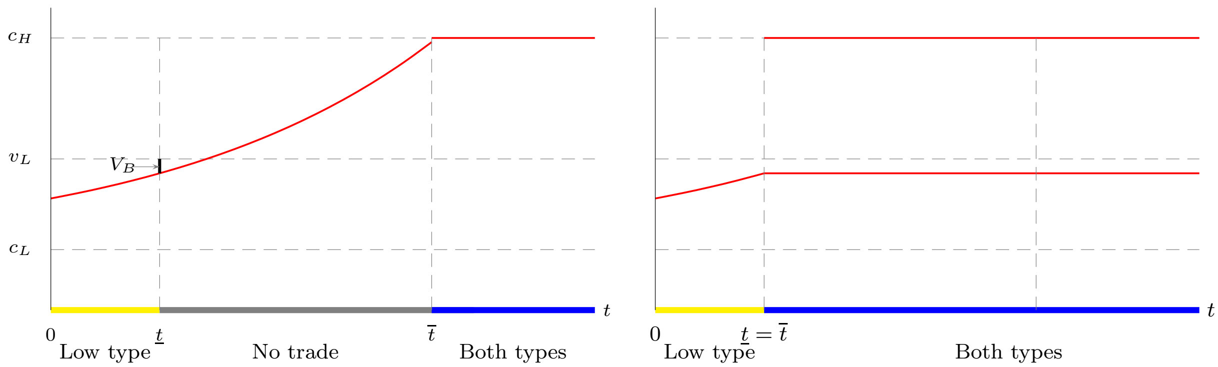

Figure 1: The left (right) panel shows the equilibrium structure in Example 1 (2).

Example 1 *Let $\overline{t}$ be the first time buyers offer c_H. Assume that all buyers who arrive after $\overline{t}$ would offer c_H. Then the probability that the seller is the low type does not change after $\overline{t}$, that is, $q(t) = q(\overline{t})$ for all $t \geq \overline{t}$. In addition, it must be that*

$$V_B = q(\overline{t})\, v_L + \left(1 - q(\overline{t})\right) v_H - c_H.$$

Obviously, $V_B \leq q(\overline{t})\, v_L + \left(1 - q(\overline{t})\right) v_H - c_H$ for buyers to be willing to offer c_H. If $V_B > q(\overline{t})\, v_L + \left(1 - q(\overline{t})\right) v_H - c_H$, then buyers who arrive right before $\overline{t}$ would be also willing to offer c_H, which contradicts the definition of $\overline{t}$.

Denote by $p(t)$ time t buyer's offer. If $t \leq \overline{t}$, then, by Assumption 3, $p(t)$ must be equal to the reservation price of time t low-type seller. Therefore,

$$\begin{aligned} p(t) - c_L &= e^{-r(\overline{t}-t)} \left(\int_{\overline{t}}^{\infty} e^{-r(s-\overline{t})} d\left(1 - e^{-\lambda(s-\overline{t})}\right) \right) (c_H - c_L) \\ &= e^{-r(\overline{t}-t)} \delta\, (c_H - c_L). \end{aligned}$$

Time t low-type seller can wait until time $\overline{t}$, from which point all buyers offer c_H. She may match between t and $\overline{t}$ but can reject all the offers during the period, because each offer will be equal to her reservation price.

Let $\underline{t}$ be the time such that $v_L - p(\underline{t}) = V_B$. Such $\underline{t}$ uniquely exists because $V_B < v_L - c_L$ and $p(\cdot)$ is strictly increasing. Then trade must occur whenever $t \leq \underline{t}$ and the seller is the low type. Otherwise, time t buyer would offer slightly above $p(t)$. If $t \in \left(\underline{t}, \overline{t}\right)$ or the seller is the high type, there must be no trade.

Buyers' beliefs evolve as follows:

$$q(t) = \max\left\{ \frac{\widehat{q}e^{-\lambda t}}{\widehat{q}e^{-\lambda t} + (1 - \widehat{q})}, \frac{\widehat{q}e^{-\lambda \underline{t}}}{\widehat{q}e^{-\lambda \underline{t}} + (1 - \widehat{q})} \right\}.$$

The low-type seller finishes the game whenever she matches before time $\underline{t}$, and thus the probability *that the seller is the low type decreases according to the matching rate. The decrease stops once it reaches $\underline{t}$. From this point, the seller waits for c_H, while buyers are indifferent between taking the outside option and offering c_H. For the incentive compatibility of the low-type seller (that she must accept low prices before $\underline{t}$), buyers must make only losing offers between $\underline{t}$ and $\overline{t}$.*

To sum up, an equilibrium is characterized by time $\underline{t}$ and time $\overline{t}$ such that

$$V_B = q\left(\underline{t}\right) v_L + \left(1 - q\left(\underline{t}\right)\right) v_H - c_H,$$

and

$$p\left(\underline{t}\right) - c_L = v_L - c_L - V_B = e^{-r\left(\overline{t}-\underline{t}\right)}\delta\left(c_H - c_L\right).$$

Example 2 *Using the same notations as in the previous example, suppose $\underline{t} = \overline{t}$ and buyers offer c_H with a constant probability, say κ, after $\underline{t}$. Then an equilibrium is characterized by time $\underline{t}$ and probability κ such that*

$$V_B = q\left(\underline{t}\right) v_L + \left(1 - q\left(\underline{t}\right)\right) v_H - c_H,$$

and

$$p\left(\underline{t}\right) - c_L = v_L - c_L - V_B = \left(\int_{\underline{t}}^{\infty} e^{-r(s-\underline{t})} d\left(1 - e^{-\lambda\kappa(s-\underline{t})}\right)\right)\left(c_H - c_L\right).$$

In this equilibrium, the seller expects to receive c_H from $\underline{t}$. The low-type seller still has an incentive to accept low prices before $\underline{t}$, because buyers offer c_H with probability less than 1. After $\underline{t}$, buyers mix between c_H and $v_L - V_B$. The latter is the reservation price of the low-type seller. In equilibrium, the low-type seller never accepts the price, and thus offering the price is equivalent to taking the outside option.

The following proposition characterizes the set of equilibria. There is a continuum of equilibria. As shown in the examples, the multiplicity stems from buyers' indifference after $\underline{t}$ and the resulting latitude in specifying buyers' behavior.

Proposition 2 *(Partial equilibrium in Regime 2) Given $V_B \in [0, \min\{v_L - c_L, v_H - c_H\})$, any equilibrium between a single seller and a sequence of buyers is characterized by time $\underline{t}\,(>0)$, time $\overline{t}\,(\geq \underline{t})$, and a Borel measurable function $\gamma : \left[\overline{t}, \infty\right) \rightarrow [0,1]$ such that*

$$V_B = q\left(\underline{t}\right) v_L + \left(1 - q\left(\underline{t}\right)\right) v_H - c_H, \tag{5}$$

$$v_L - V_B - c_L = e^{-r\left(\overline{t}-\underline{t}\right)}\left(\int_{\overline{t}}^{\infty} e^{-r\left(s-\overline{t}\right)} d\gamma\left(s\right)\right)\left(c_H - c_L\right), \tag{6}$$

$$v_L - V_B - c_L \leq \left(\int_{t}^{\infty} e^{-r(s-t)} \frac{d\gamma\left(s\right)}{1-\gamma\left(t\right)}\right)\left(c_H - c_L\right), \text{ for any } t \geq \overline{t}, \tag{7}$$

where

$$q\left(\underline{t}\right) = \frac{\widehat{q}e^{-\lambda\underline{t}}}{\widehat{q}e^{-\lambda\underline{t}} + \left(1-\widehat{q}\right)}. \tag{8}$$

In equilibrium,

(1) if $t \leq \underline{t}$ then time t buyer offers the reservation price of the low-type seller, that is,

$$p(t) = c_L + e^{-r(\bar{t}-t)} \left(\int_{\bar{t}}^{\infty} e^{-r(t-\bar{t})} d\gamma(t) \right) (c_H - c_L).$$

The low-type seller accepts this price with probability 1.

(2) If $t \in (\underline{t}, \bar{t})$ then time t buyer makes a losing offer.

(3) If $t \geq \bar{t}$ then time t buyer offers c_H with a positive probability, so that $\gamma(t)$ is the cumulative probability that the seller receives c_H by time t.

Proof. See the Appendix. ■

The roles of $\underline{t}$ and $\bar{t}$ must be clear from the examples above. Given V_B, Equations (5) and (8) pin down $\underline{t}$, and thus it is constant across all equilibria. Obviously, $\bar{t}$ takes different values in different equilibria.

The function $\gamma(\cdot)$ corresponds to $1 - e^{-\lambda(t-\bar{t})}$ in Example 1 and to $1 - e^{-\lambda\kappa(t-\underline{t})}$ in Example 2. Although the function $\gamma(\cdot)$ can take many forms, Condition (7) imposes a restriction on its behavior. To better understand the effect of the constraint, extend the domain of $\gamma(\cdot)$ to $[\underline{t}, \infty]$ by letting $\gamma(t) = 0$ for any $t \in [\underline{t}, \bar{t}]$ and consider Example 2. In this case, the inequality binds for any $t \geq \underline{t}$. This implies that for t close to $\underline{t}$ the function $\gamma(\cdot)$ cannot increase faster than in Example 2. Otherwise, at some $t > \underline{t}$, the low type would be willing to accept a lower price than $v_L - V_B$. If so, time t buyer would deviate and offer slightly below $v_L - V_B$. Similarly, for t sufficiently large, the function $\gamma(\cdot)$ must increase at least as fast as in Example 2. In other words, buyers must offer c_H with increasing probability over time.[6] The equilibrium in Example 1 most effectively satisfies this constraint, as it is the one in which buyers offer c_H as late as possible. Indeed, the inequality binds only at $t = \underline{t}$ and the difference between the two sides increases in t.

All equilibria are payoff-equivalent. The low-type seller's expected payoff is $p(0) - c_L$. Time t buyer obtains $q(t)(v_L - p(t)) + (1 - q(t)) V_B$ if $t \leq \underline{t}$ and V_B otherwise. However, as will be shown shortly, different equilibria have different payoff implications, once they are embedded in the market.

Endogenizing Buyers' Outside Option

In order to endogenize V_B, fix an equilibrium in Proposition 2. Let M be the total measure of sellers in the market and $G : R_+ \to [0, 1]$ be the distribution function of (observable) seller types with density g. Then

$$M = \int_0^{\underline{t}} \left(\widehat{q} e^{-\lambda t} + 1 - \widehat{q} \right) dt + \int_{\underline{t}}^{\infty} \left(\widehat{q} e^{-\lambda \underline{t}} + 1 - \widehat{q} \right) (1 - \gamma(t))\, dt,$$

[6] Recall that the probability that each buyer offers c_H is constant over time in Example 2.

and

$$M \cdot g(t) = \begin{cases} \widehat{q}e^{-\lambda t} + 1 - \widehat{q}, & \text{if } t \leq \underline{t}, \\ \left(\widehat{q}e^{-\lambda \underline{t}} + 1 - \widehat{q}\right)(1 - \gamma(t)), & \text{if } t > \underline{t}. \end{cases}$$

Sellers who have stayed on the market for $t(\leq \underline{t})$ length of time are either low-type sellers who have not matched yet or high-type sellers. If $t > \underline{t}$, then trade occurs only at c_H, and thus both types of sellers leave the market at the same rate.

In equilibrium, V_B must satisfy

$$V_B = \delta \left[\int_0^{\underline{t}} \left(q(t)(v_L - p(t)) + (1 - q(t)) V_B \right) dG(t) + \int_{\underline{t}}^{\infty} V_B dG(t) \right].$$

If a buyer meets a seller who has stayed shorter than $\underline{t}$, trade occurs, at price $p(t)$, if and only if the seller is the low type, whose probability is $q(t)$. In all other cases, buyers obtain V_B, whether trade occurs or not. Using the fact that $V_B = v_L - p(\underline{t})$,

$$V_B = \frac{\delta}{1-\delta} \frac{\widehat{q}}{M} \int_0^{\underline{t}} e^{-\lambda t} \left(p(\underline{t}) - p(t) \right) dt.$$

Define a correspondence $\Phi : [0, \min\{v_L - c_L, v_H - c_H\}) \Rightarrow [0, \min\{v_L - c_L, v_H - c_H\})$ so that

$$\Phi(V_B) = \frac{\delta}{1-\delta} \frac{\widehat{q}}{M} \int_0^{\underline{t}} e^{-\lambda t} \left(p(\underline{t}) - p(t) \right) dt.$$

Then it is a market equilibrium if and only if V_B is a fixed point of the correspondence Φ.

The mapping Φ is a correspondence rather than a function. This is because different partial equilibria produce different values of M. Let $\Psi(V_B)$ be the set of possible values of M. Then the set $\Psi(V_B)$ is an interval and its extreme values are provided by the equilibria in Examples 1 and 2. To be precise, let $\gamma_1, \gamma_2 : [\underline{t}, \infty) \rightarrow [0, 1]$ be the distribution functions that correspond to the equilibria in Examples 1 and 2, respectively. On the one hand, in any equilibrium, the function $\gamma(\cdot)$ must be between the functions $\gamma_1(\cdot)$ and $\gamma_2(\cdot)$. Otherwise, either Condition (6) or Condition (7) would be violated. On the other hand, any convex combination of $\gamma_1(\cdot)$ and $\gamma_2(\cdot)$ results in a partial equilibrium. Therefore, any value between the two extremes can be obtained.

The existence of equilibrium follows from the fact that the correspondence Φ is nonempty, convex-valued, compact-valued and continuous. Since each partial equilibrium structure has a corresponding general equilibrium,[7] there is a continuum of market equilibria with different agent payoffs.

[7]This is because $\Phi(0) > 0$ and $\Phi(\min\{v_L - c_L, v_H - c_H\}) < \min\{v_L - c_L, v_H - c_H\}$, independent of the value of M.

6 Regime 3: Number of Previous Matches

In this section, the set Ξ is isomorphic to the set of all non-negative integers. A typical element $n \in \Xi$ represents the number of matches a seller has gone through in the market.

The equilibrium structure is similar to that of Regime 2. Low-type sellers leave the market relatively faster than high-type sellers. Buyers offer low prices to relatively new sellers and offer c_H to old enough sellers. The difference is now sellers are classified according to number-of-previous-match, instead of time-on-the-market.

As in the previous section, first consider the game between a single seller and a sequence of buyers in which buyers' outside option is exogenously given as $V_B \in [0, \min\{v_L - c_L, v_H - c_H\})$. The following lemma characterizes buyers' expected payoffs in the reduced-form game.

Lemma 2 *All buyers obtain V_B, whether they trade or not.*

Proof. *It suffices to show that no buyer can extract more than V_B from the seller. Let p_n be the price that the $(n+1)$-th buyer offers to the seller and q_n be the probability that the seller is the low type, conditional on the event that she has matched n times before.*

Suppose the statement is not true. Let n be the minimal number of previous matches such that the next buyer ($(n+1)$-th buyer) obtains more than V_B. Then it must be that (1) $p_n - c_L = V_S(n+1)$ (the buyer offers the low type's reservation price) and $v_L - p_n > V_B$ (the buyer obtains more than V_B when the seller is the low type) or (2) $p_n = c_H$ (the buyer offers the high type's reservation price) and $q_n v_L + (1 - q_n) v_H - c_H > V_B$.

Suppose (1) is the case. Then the low-type seller must accept the offer for sure (otherwise, the buyer would slightly increase his offer). Given this, the next buyer ($(n+2)$-th buyer) is certain that the seller is the high type and, therefore, will offer c_H. But then, due to Assumption 2, the low-type seller in her $(n+1)$-th match would not accept p_n, which is a contradiction.

Now suppose (2) is the case. Due to Assumption 1, certainly $n \geq 1$. For (2) to be true, trade cannot occur at the n-th match of the seller, and thus the expected payoff of the n-th buyer must be equal to V_B. But then the n-th buyer could obtain more than V_B by offering c_H, which is a contradiction. ■

This lemma implies that trade occurs only at either $v_L - V_B$ or c_H. The following lemma shows that the first case occurs only at the seller's first match and must occur with a positive probability.

Lemma 3 *Trade at $v_L - V_B$ occurs only at the first match and occurs with a positive probability.*

Proof. *Suppose trade occurs with a positive probability only after n matches for some $n \geq 1$. At the $(n+1)$-th match, the price must be $v_L - V_B$, because of Assumption 1 and the previous lemma. If the first buyer offers slightly more than $c_L + \delta^n (v_L - V_B - c_L)$, the low-type seller would accept it for sure. Furthermore, the first buyer could obtain more than V_B, which is a contradiction. This establishes the second part of the lemma.*

Suppose trade occurs at $v_L - V_B$ with a positive probability also at the $(n+1)$-th match for some $n \geq 1$. In this case, it cannot be that the buyer offers $v_L - V_B$ with probability 1. If so, *the low-type seller must have accepted the same price for sure in her first match. This implies that* $q_n v_L + (1-q_n) v_H - c_H = V_B$. *Now since trade occurs with a positive probability at* $v_L - V_B$, $q_{n+1} < q_n$, *which implies that the* $(n+2)$*-th buyer would offer* c_H *for sure. This leads to a contradiction, because, by Assumption 2, the low-type seller would not accept* $v_L - V_B$ *in her* $(n+1)$*-th match.* ■

Let $\alpha(n)$ be the probability that the $(n+1)$-th buyer does not offer c_H. The following proposition characterizes the set of all partial equilibria in the game.

Proposition 3 *(Partial equilibrium in Regime 3) Given* $V_B \in [0, \min\{v_L - c_L, v_H - c_H\})$*, any equilibrium in the game between a single seller and a sequence of buyers is characterized by* $\alpha : \mathcal{N} \to [0,1]$ *and* $\beta \in (0,1)$ *such that*

$$v_L - V_B - c_L = \sum_{n=1}^{\infty} \delta^n \left(\prod_{k=1}^{n-1} \alpha(k) \right) (1 - \alpha(n)) (c_H - c_L), \tag{9}$$

$$v_L - V_B - c_L \leq \sum_{n=1}^{\infty} \delta^n \left(\prod_{k=1}^{n-1} \alpha(l+k) \right) (1 - \alpha(l+k)) (c_H - c_L), \text{ for any } l \geq 1. \tag{10}$$

and

$$q^* v_L + (1 - q^*) v_H - c_H = V_B, \tag{11}$$

where

$$q^* = \frac{\widehat{q}\beta}{\widehat{q}\beta + (1 - \widehat{q})}.$$

In equilibrium,

(1) the first buyer offers $v_L - V_B$*, and the low-type seller accepts the offer with probability* $1 - \beta$ *(so that after the first match the probability that the seller is the low type becomes equal to* q^**),*

(2) the n*-th buyer makes a losing offer (the reservation price of the low-type seller) with probability* $\alpha(n)$ *and offers* c_H *with probability* $1 - \alpha(n)$*,*

(3) the seller accepts c_H *for sure.*

Proof. *Equation (9) is the low-type seller's indifference between accepting and rejecting* $v_L - V_B$ *in her first match. Condition (10) plays the same role as Condition (7) in Regime 2. It ensures that the low-type seller's reservation price never falls below* $v_L - V_B$ *(so after the seller's first match trade occurs only at* c_H*). Equation (11) is buyers' indifference between* c_H *and losing offers after the seller's first match.*

The previous lemmas imply that these conditions are necessary for equilibrium. It is also straightforward that, conversely, any strategy profile that satisfies the three conditions is an equilibrium. ■

As in Regime 2, there are many equilibria. Again, it is because of buyers' indifference between c_H and losing offers and the resulting latitude in specifying the probabilities, $\alpha(\cdot)$. The following

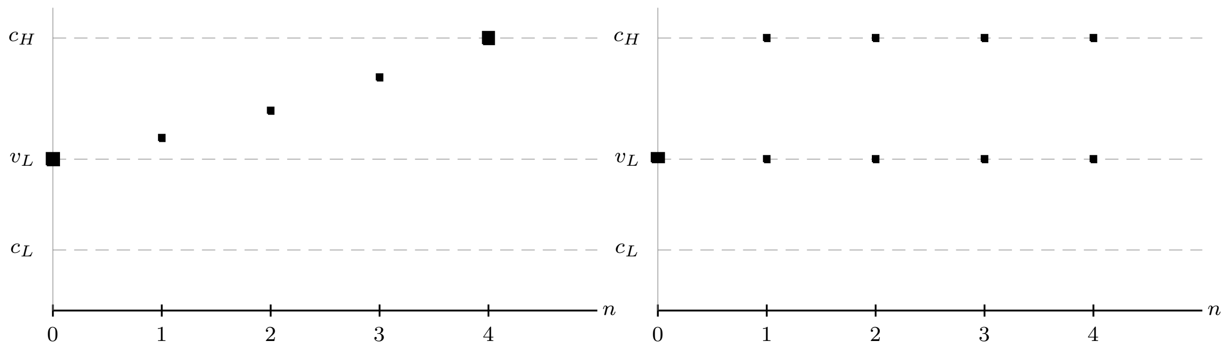

Figure 2: The left (right) panel shows the equilibrium structure in Example 3 (4).

two equilibria correspond to the equilibria in Examples 1 and 2. Their equilibrium structures are depicted in Figure 2.

Example 3 *There is an equilibrium in which after the first match trade never occurs for a while and then occurs within (at most) two matches for sure. More precisely, find n and $\alpha(n)$ that satisfy*

$$v_L - V_B - c_L = \left(\delta^n \left(1 - \alpha(n)\right) + \delta^{n+1}\alpha(n)\right)(c_H - c_L).$$

For such n and $\alpha(n)$, there exists an equilibrium in which trade never occurs from the second match to the n-th match. Trade occurs with probability $1-\alpha(n)$ at the $(n+1)$-th match and with probability 1 at the following match.

Example 4 *There is an equilibrium in which $\alpha(n)$ is independent of n. In this case, there is a unique solution to Equation (9), which is*

$$\alpha = \frac{\delta(c_H - c_L) - (v_L - V_B - c_L)}{\delta(c_H - v_L + V_B)}.$$

As in Regime 2, all agents obtain the same payoffs in all equilibria. The low-type seller is indifferent between accepting and rejecting $v_L - V_B$ at her first match, and thus her expected payoff is equal to $\delta(v_L - V_B - c_L)$. As shown in Lemma 2, all buyers obtain exactly as much as their outside option, V_B.

Embedding the game into the market setting, it immediately follows that V_B must be equal to 0. This is due to Lemma 2 and market frictions. Buyers obtain V_B in any match, but matching takes time.

Proposition 4 *Any market equilibrium in Regime 3 is characterized by $\alpha : \mathcal{N} \to [0,1]$ and $\beta \in (0,1)$ that satisfy the conditions in Proposition 3 with $V_B = 0$. The expected payoffs of low-type sellers and buyers are $\delta(v_L - c_L)$ and 0, respectively.*

Two remarks are in order. First, in the current two-type case, less information suffices to produce essentially the same outcome. The necessary information is whether sellers have matched before or not. In that case, the equilibrium in Example 4 is the unique equilibrium. Second, the results do not change even if time-on-the-market is also observable. In particular, Lemmas 2 and 3 are independent of the observability of time-on-the-market. The additional information may be used as a public randomization device to enlarge the set of equilibria but does not affect agents' payoffs.

7 Welfare Comparison

This section compares the welfare consequences of the regimes.

Regime 1 vs. Regime 3: the role of number-of-previous-match

Suppose market frictions are relatively small (r is small or λ is large). In particular, for simplicity, assume that Condition (4) in Proposition 1 holds.

It is immediate that Regime 1 weakly Pareto dominates Regime 3: Buyers are indifferent between the two regimes, while low-type sellers are strictly better off in Regime 1 ($v_L - c_L$) than in Regime 3 ($\delta(v_L - c_L)$).[8]

Why is Regime 1 more efficient than Regime 3? In Regime 1, different cohorts of sellers are completely mixed. Such mixing helps relax the incentive constraint, because low-type sellers, due to their lower cost, leave the market relatively faster, and thus the proportion of high-type sellers is larger in the market than among new sellers. In Regime 3, buyers' access to number-of-previous-match does not allow such mixing. Consequently, buyers never offer c_H to new sellers in Regime 3, while they do with a positive probability in Regime 1. One can check that this is the exact reason why low-type sellers are better off in Regime 1 than in Regime 3.

Regime 1 vs. Regime 2: the role of time-on-the-market

When market frictions are relatively small, Regime 1 and Regime 2 are not Pareto ranked: Buyers always obtain a positive expected payoff in Regime 2 and, therefore, strictly prefer Regime 2 to Regime 1. To the contrary, low-type sellers strictly prefer Regime 1 to Regime 2: Their expected payoff in Regime 2 is $p(0) - c_L$, which is strictly smaller than $v_L - c_L$.

The two regimes still can be compared in terms of realized market surplus, as the model is of transferable utility. Furthermore, the environment is stationary and I have focused on steady-state equilibrium. Therefore, I can further restrict attention to total market surplus of a cohort, that is, $\widehat{q}V_S + V_B$.[9]

[8] There is another cutoff value, say $\underline{\delta}$, such that if δ falls between $\underline{\delta}$ and the minimum value that satisfies Condition (4) in Proposition 1, then Regime 1 strictly Pareto dominates Regime 3.

[9] Recall that high-type sellers' expected payoff is always zero and the measures of low-type sellers and buyers of each cohort are $\widehat{q}$ and 1, respectively.

The following proposition shows that (at least) when market frictions are sufficiently small, Regime 2 outperforms Regime 1.[10]

Proposition 5 *When market frictions are sufficiently small, total market surplus of a cohort is greater in any equilibrium of Regime 2 than in the equilibrium of Regime 1.*

Proof. *When market frictions are small,* $V_S = v_L - c_L$ *and* $V_B = 0$ *in Regime 1, and thus* $\widehat{q}V_S + V_B = \widehat{q}\,(v_L - c_L)$*. In the Appendix, I prove that in Regime 2* $\widehat{q}V_S + V_B > \widehat{q}\,(v_L - c_L)$ *when* λ *is sufficiently large or* r *is sufficiently small. The proof proceeds in two steps. First, I show that* V_S *and* V_B *approach* $v_L - c_L$ *and* 0*, respectively, as market frictions vanish. Then, I show that* $\widehat{q}V_S + V_B$ *decreases in the limit as* λ *tends to infinity or* r *tends to* 0*.* ■

Why does time-on-the-market improve efficiency, while number-of-previous-match deteriorates it? As with number-of-previous-match, the observability of time-on-the-market prevents mixing of different cohorts of sellers, which is negative for efficiency. There is, however, an offsetting effect. To see this, consider a new seller who quickly met a buyer. In Regime 3, the seller has an incentive to reject a low price, because doing so will convince future buyers that she is the high type with a high probability. In Regime 2, the same incentive is present but weaker than in Regime 3. The length of time a seller has to endure to receive a high price is independent of whether, and how many times, the seller has rejected offers. Therefore, the opportunity cost of accepting a current offer is smaller in Regime 2 than in Regime 3 (recall that low-type sellers accept low prices with probability 1 if they match before $\underline{t}$ in Regime 2, while they do not in their first matches in Regime 3). Consequently, the proportion of high-type sellers in a cohort increases fast, which induces buyers to offer c_H relatively quickly. In Regime 1, sellers cannot signal their types by rejecting offers or waiting for a certain length of time. Still, sellers have an incentive to wait for a high price. This incentive is constant in Regime 1, while the same incentive increases as sellers stay on the market longer in Regime 2 (and Regime 3). For sellers who are quickly matched, this incentive is stronger in Regime 1 than in Regime 2. Consequently, low-type sellers often reject their reservation price in Regime 1 (recall that $\beta^* < 1$). This has the effect of increasing the proportion of low-type sellers in the market and, therefore, discouraging buyers to offer c_H. When market frictions are small, this offsetting effect turns out to dominate, and thus Regime 2 outperforms Regime 1.

Number-of-previous-match with not so small market frictions

Does the observability of number-of-previous-match always reduce efficiency? The following result shows that it depends on market conditions.

Proposition 6 *If* $v_H - c_H$ *is sufficiently small, then realized market surplus of a cohort is close to zero in Regimes 1 and 2, while it is equal to* $\widehat{q}\delta\,(v_L - c_L)$ *in Regime 3.*

[10] Sufficiently small market frictions are only a sufficient condition. Numerical simulations show that Regime 2 outperforms Regime 1 even when market frictions are not so small. Unfortunately, it is not possible to do a more comprehensive welfare comparison. It is mainly due to the difficulty of characterizing agents' expected payoffs in Regime 2.

Proof. *Recall that in Regime 3* $V_S = \delta(v_L - c_L)$ *and* $V_B = 0$ *as long as Assumptions 1 and 2 are satisfied. It suffices to show that in Regimes 1 and 2 both* V_S *and* V_B *converge to zero as* v_H *tends to* c_H*. The result for Regime 1 is immediate from the characterization in Section 4. In Regime 2,* $V_B < v_H - c_H$*, and thus obviously* V_B *converges to* 0 *as* v_H *tens to* c_H*. For* V_S*, observe that when* v_H *is close to* c_H*,* $\overline{q}$ *is close to* 0*. Since in equilibrium* $q(\underline{t}) \leq \overline{q}$ *(otherwise, buyers would never offer* c_H*),* $\underline{t}$ *must be sufficiently large. This implies that* $p(0)$ *will be close to* c_L*.* ■

The intuition for this result is as follows. In any regime, buyers offer c_H to some sellers. Their benefit of offering c_H depends on $v_H - c_H$, while their opportunity cost is independent of $v_H - c_H$.[11] If buyers obtain a positive expected payoff, they are less willing to offer c_H as $v_H - c_H$ gets smaller. This is exactly what happens in Regimes 1 and 2.[12] In the limit, buyers offer c_H with probability 0. Then as in the Diamond paradox, buyers offer c_L with probability 1 and low-type sellers does not obtain a positive expected payoff. Buyers' expected payoff would be also zero. A buyer would obtain $v_L - c_L$ if he meets a low-type seller, but the probability of a buyer meeting a low-type seller would be zero, as the market would be populated mostly by high-type sellers. In Regime 3, buyers always obtain zero expected payoff. Therefore, their incentive to offer c_H is independent of $v_H - c_H$. This prevents all the surplus from disappearing even as $v_H - c_H$ approaches zero, unlike in the two other regimes.

Rather informally, number-of-previous-match serves as sellers' signalling device (while time-on-the-market is buyers' screening device). Using it for signalling purpose is socially wasteful in itself, as in the standard signalling game. However, it also enables informed players (sellers) to ensure a certain payoff. When market frictions are small, the former (negative) effect dominates and thus the observability of number-of-previous-match reduces efficiency. When market frictions are not small, however, the latter effect could be significant. In particular, when all agents' expected payoffs may be driven down close to zero, the observability of number-of-previous-match can contribute to market efficiency by preserving informed players' informational rents.

8 Discussion

More than Two Types

Equilibrium characterization can generalize beyond the simple two-type case, with significant technical difficulties but without any conceptual difficulties. In Regime 1, buyers will play a mixed bidding strategy over the set of sellers' reservation prices. The bidding strategy and sellers' acceptance strategies will be jointly determined so that the distribution of seller types in the market is invariant and buyers are indifferent over the prices. In Regimes 2 and 3, the equilibrium structures will extend just like Deneckere and Liang (2006). Welfare comparison, however, will be quite involved. Closed-form solutions, which greatly simplified the welfare comparison in the previous

[11] The opportunity cost depends on low-type sellers' willingness-to-wait, that is, δ.

[12] Buyers obtain a positive expected payoff in Regime 1 whenever Condition (4) does not hold.

section, will not be available in any regime, unless some strong restrictions are imposed on parameter values.

Comparison with Deneckere and Liang (2006)

The equilibrium structures of Regimes 2 and 3 resemble that of Deneckere and Liang (2006) (DL, hereafter). In particular, DL's limit outcome (as the offer interval tends to zero) exhibits the same qualitative properties as in Examples 1 and 3: trade occurs either at the beginning of the game or only after some real-time delay.

There are two important differences. First, it is only the limit outcomes that have the same qualitative properties. Away from the limit, trade occurs with a positive probability in every period in DL, while there is always an interval of time or a set of matches during which trade is not supposed to occur in Examples 1 and 3. Second, in the comparable limit case (λ is arbitrarily large in Regimes 2 and 3), the equilibrium delay is half as much as that of DL. To be more precise, let τ be the value that satisfies $(v_L - c_L) = e^{-r\tau}(c_H - c_L)$. The equilibrium delay is exactly equal to 2τ in DL, while it is equal to τ in Examples 1 and 3.[13]

Further Questions

The results of this paper raise several questions. A question particularly germane to this paper is what information flow would be most efficient. When market frictions are sufficiently small, is it possible to improve upon Regime 2? Also, what information flow is optimal for buyers?[14] More generally, one may ask what is the constrained-efficient benchmark in dynamic markets under adverse selection with constant inflow of agents (with or without search frictions). Different from static settings, with constant inflow of agents, as shown in Regime 1, subsidization across different cohorts of sellers is possible. Would the mechanism designer exploit such possibility or completely separate different cohorts of sellers? Would the constrained-efficient outcome be stationary, cyclical (for example, high-type units trade every n periods), or non-stationary?

One potentially interesting extension, which was also suggested by HV, is to allow buyers to conduct inspections before or after bargaining and with or without cost. With inspections, buyers' beliefs can evolve in any direction. When a unit remains on the market for a long time, it might be because the previous offers have been rejected by the seller, as in HV and this paper, or because the previous buyers have observed bad signals about the unit, as in Taylor (1999). The former inference shifts buyers' beliefs upward, while the latter does the opposite. The discrepancy between Taylor and HV (and this paper) suggests that it may have an importance consequence on the relationship between efficiency and information about past trading outcomes.

[13] For simplicity, consider Example 1 and suppose $V_B = 0$ (this is without loss of generality because V_B indeed converges to zero as λ tends to infinity). The result is straightforward because $v_L - c_L = e^{-r(\bar{t}-\underline{t})}\delta(c_H - c_L)$ for any λ and $\underline{t}$ converges to zero as λ tends to infinity.

[14] Low-type sellers' expected payoff is bounded by $v_L - c_L$ in any circumstance, and thus Regime 1 is the one (of possibly many) that is optimal for sellers, as long as market frictions are relatively small.

Appendix

Proof of Proposition 2

(1) The high type eventually trades.

Suppose the high type never trades. Then by the same reasoning as in the Diamond paradox, all buyers would offer c_L, and the low type would accept it in her first match. Then, buyers' beliefs would evolve as follows:

$$q(t) = \frac{\widehat{q}e^{-\lambda t}}{\widehat{q}e^{-\lambda t} + (1-\widehat{q})}.$$

The function $q(\cdot)$ approaches zero as t tends to ∞. Since $V_B < v_H - c_H$, for t sufficiently large,

$$q(t) v_L + (1 - q(t)) v_H - c_H > \max \{V_B, q(t)(v_L - c_L) + (1 - q(t)) \delta V_B\}.$$

Therefore, buyers would eventually offer c_H, which is a contradiction.

Now suppose the high type does not trade with probability 1. This can happen only when the low type also does not trade with a positive probability (otherwise, buyers will eventually offer c_H) and a positive measure of buyers make only losing offers (otherwise, either the low type trades or both types trade for sure). But then buyers who make losing offers could deviate to slightly above c_L and the low type would accept those offers, which is a contradiction.

(2) Let $\overline{t}$ denote the first time after which a positive measure of buyers offer c_H. The previous lemma implies that $\overline{t}$ is finite. In addition, let $\gamma : [\overline{t}, \infty) \to [0, 1]$ be a Borel-measurable function where $\gamma(t)$ represents the probability that the seller receives c_H by time t.

(3) After $\overline{t}$, buyers either offer c_H or make losing offers. Therefore, trade occurs only at c_H and $q(t)$ is constant after $\overline{t}$.

Suppose after time $\overline{t}$, trade occurs at prices below c_H with a positive probability. Let $\overline{t}'$ be the time such that $q\left(\overline{t}'\right) < q(\overline{t}) - \varepsilon$ for some $\varepsilon > 0$. Then any buyers that arrive after $\overline{t}'$ never make losing offers, because

$$\begin{aligned} q(t) v_L + (1 - q(t)) v_H - c_H &\geq q\left(\overline{t}'\right) v_L + \left(1 - q\left(\overline{t}'\right)\right) v_H - c_H \\ &> q(\overline{t}) v_L + (1 - q(\overline{t})) v_H - c_H \geq V_B. \end{aligned}$$

This implies that there exists $\widetilde{t} \geq \overline{t}'$ such that buyers that arrive after $\widetilde{t}$ offer only c_H with probability 1. Otherwise, $q(\cdot)$ approaches 0 as t tends to infinity, and so offering c_H and trading with both types will eventually dominate offering less than v_L and trading with only the low type. Let $\widetilde{t}$ be the infimum value of such time. Then the low-type seller at time close to $\widetilde{t}$ will never accept prices below v_L due to Assumption 2. Therefore, it must be that $\widetilde{t} = \overline{t}'$. Since this holds for arbitrary $\varepsilon > 0$, it must be that buyers either offer c_H or make losing offers after $\overline{t}$.

(4) After time $\overline{t}$, buyers are indifferent between offering c_H and making losing offers. Therefore, it must be that

$$q(\overline{t}) v_L + (1 - q(\overline{t})) v_H - c_H = V_B.$$

Suppose buyers strictly prefer offering c_H to making losing offer. Consider t that is slightly smaller than $\overline{t}$. Time t buyer strictly prefer offering c_H to making losing offers, because $q(t)$ is close to $q(\overline{t})$. In addition, he strictly prefers c_H to any offers that can be accepted only by the low type. This is because time t low-type seller knows that buyers will offer only c_H after $\overline{t}$ and, therefore, never accepts below v_L. But then $\overline{t}$ is not the first time buyers offer c_H, which is a contradiction.

(5) (3) and (4) imply that for any $t \geq \bar{t}$, it must be that

$$v_L - V_B - c_L \leq \left(\int_t^{\infty} e^{-r(s-t)} \frac{d\gamma(s)}{1-\gamma(t)} \right) (c_H - c_L).$$

If this condition is violated, then buyers would deviate to slightly below $v_L - V_B$, which would be accepted by the low type.

(6) Let $\underline{t} \left(\leq \bar{t} \right)$ be the last time at which trade may occur at a price below c_H. By definition, $q(\underline{t}) = q\left(\bar{t}\right)$.

(7) The reservation price of time $\underline{t}$ low-type seller is $v_L - V_B$.

Suppose the reservation price of time $\underline{t}$ low-type seller is strictly greater (lower) than $v_L - V_B$. Then buyers who arrive just before (after) $\underline{t}$ would prefer making losing offers (offers slightly below $v_L - V_B$). This contradicts the definition of $\underline{t}$.

(8) Time $t \leq \bar{t}$ buyer offers $p(t)$ such that

$$p(t) - c_L = e^{-r\left(\bar{t}-t\right)} \left(\int_{\bar{t}}^{\infty} e^{-r\left(t-\bar{t}\right)} d\gamma(s) \right) (c_H - c_L).$$

For $t \leq \bar{t}$, due to Assumption 3, buyers offer only the reservation price of the low-type seller. Since this is true for all $t \leq \bar{t}$, the low-type seller is indifferent between accepting $p(t)$ and waiting until $\bar{t}$.

(9) In equilibrium, the low-type seller accepts $p(t)$ with probability 1 if $t \leq \underline{t}$ and rejects $p(t)$ with probability 1 if $t > \underline{t}$. Otherwise, buyers would offer slightly above (below) $p(t)$ if $t \leq (>) \underline{t}$. This implies that

$$q(t) = \frac{\widehat{q} e^{-\lambda t}}{\widehat{q} e^{-\lambda t} + (1-\widehat{q})}, \text{ for } t \leq \underline{t}.$$

Q.E.D.

Proof of Proposition 5:

Step 1: Given V_B,

$$\begin{aligned} \Phi(V_B) &= \frac{\delta}{1-\delta} \frac{\widehat{q}}{M} \int_0^{\underline{t}} e^{-\lambda t} \left(p(\underline{t}) - p(t) \right) dt \\ &\leq \frac{\lambda}{r} \frac{\widehat{q}}{M} \int_0^{\underline{t}} e^{-\lambda t} \left(p(\underline{t}) - p(0) \right) dt \\ &= \frac{\widehat{q}}{rM} \left(1 - e^{-\lambda \underline{t}} \right) \left(p(\underline{t}) - p(0) \right). \end{aligned} \tag{12}$$

(1) V_B approaches 0 as λ tends to infinity.

The proof differs depending on whether $v_L - c_L < v_H - c_H$ or not.

(i) $v_L - c_L < v_H - c_H$

Suppose λ is sufficiently large. I argue that in this case $\underline{t}$ will be close to 0, which immediately implies that V_B is close to 0 and V_S is close to $v_L - c_L$ (See Figure 1). Suppose $\underline{t}$ is bounded away from 0. Then $q(\underline{t})$ will be close to zero, and thus buyers will strictly prefer offering c_H to making losing offers after $\underline{t}$, because

$$q(\underline{t}) v_L + (1 - q(\underline{t})) v_H - c_H \simeq v_H - c_H > v_L - c_L \geq v_L - p(\underline{t}).$$

(ii) $v_L - c_L \geq v_H - c_H$

(ii-1) Given V_B, $\Phi(V_B)$ approaches 0 as λ tends to infinity (pointwise convergence).

Fix $V_B < v_H - c_H$. If λ is large, by the same reasoning as in (i), $\underline{t}$ must be sufficiently small. Since M is clearly bounded away from zero, Condition (12) then implies that $\Phi(V_B)$ is close to zero.

(ii-2) For a fixed λ, $\Phi(V_B)$ approaches 0 as V_B tends to $v_H - c_H$. Together with (i), this implies that if λ is sufficiently large, then equilibrium V_B is close to 0 (uniform convergence).

Fix λ and suppose V_B is sufficiently close to $v_H - c_H$. For $q(\underline{t})v_L + (1-q(\underline{t}))v_H - c_H = V_B$, $q(\underline{t})$ must be sufficiently small, and thus $\underline{t}$ must be sufficiently large. Since $M > (1-\widehat{q})\underline{t}$, Condition (12) implies that $\Phi(V_B)$ will be close to 0.

(2) V_B approaches 0 as r tends to zero.

Using the fact that

$$p(t) - c_L = e^{-r(\underline{t}-t)}(p(\underline{t}) - c_L) = e^{-r(\underline{t}-t)}(v_L - c_L - V_B),$$

Condition (12) is equivalent to

$$V_B \leq \frac{\widehat{q}}{M}\frac{\left(1-e^{-r\underline{t}}\right)}{r}\left(1-e^{-\lambda\underline{t}}\right)(v_L - c_L - V_B). \tag{13}$$

(i) $v_L - c_L < v_H - c_H$

Suppose r is sufficiently small. Then $\underline{t}$ must be bounded from above. Otherwise, $q(\underline{t})$ will be close to 0, and then the same contradiction as in (1-i) arises. In Condition (13), the term $\left(1-e^{-r\underline{t}}\right)/r$ is bounded from above (as r tends to zero, the term approaches $\underline{t}$). On the other hand, for low-type sellers' incentive compatibility (See Equation 7), the function γ must increase sufficiently slowly in any equilibrium (For example, in the equilibrium of Example 1, $\overline{t}$ must be sufficiently large). This implies that for r sufficiently small, M will be sufficiently large, and thus V_B must be close to zero.

(ii) $v_L - c_L \geq v_H - c_H$

The proof is essentially identical to the one in (1). First, use the argument in (2-i) to show that given V_B, $\Phi(V_B)$ approaches 0 as r tends to zero (pointwise convergence). Then, apply (ii-2).

Step 2: Recall that in Regime 2, $V_S = p(0) - c_L$ and $V_B = v_L - p(\underline{t})$. From

$$p(t) = c_L + e^{-r(\overline{t}-t)}\left(\int_{\overline{t}}^{\infty} e^{-r(t-\overline{t})}d\gamma(t)\right)(c_H - c_L),$$

I get that

$$p(\underline{t}) - p(0) = \left(1-e^{-r\underline{t}}\right)(p(\underline{t}) - c_L).$$

Hence

$$\frac{d(p(\underline{t}) - p(0))}{d\lambda} \simeq re^{-r\underline{t}}(p(\underline{t}) - c_L)\frac{d\underline{t}}{d\lambda} + \left(1-e^{-r\underline{t}}\right)\frac{dp(\underline{t})}{d\lambda}.$$

As shown in (1), for λ sufficiently large, $\underline{t}$ is close to 0, and thus the second-term does not have a first-order effect. Similarly, when λ is sufficiently large, from Equations (5) and (8),

$$\frac{dV_B}{d\lambda} \simeq -(v_H - v_L)\frac{dq(\underline{t})}{d\lambda},$$

and

$$\frac{d\underline{t}}{d\lambda} \simeq -\frac{\left(\widehat{q}e^{-\lambda\underline{t}} + (1-\widehat{q})\right)^2}{\lambda\widehat{q}(1-\widehat{q})e^{-\lambda\underline{t}}}\frac{dq(\underline{t})}{d\lambda}.$$

Using all the results,

$$\begin{aligned}
\frac{d\left(\widehat{q}V_S+V_B\right)}{d\lambda} &= \widehat{q}\frac{d\left(p\left(0\right)\right)}{d\lambda}-\widehat{q}\frac{d\left(p\left(\underline{t}\right)\right)}{d\lambda}+\left(1-\widehat{q}\right)\frac{dV_B}{d\lambda} \\
&= -\widehat{q}\frac{d\left(p\left(\underline{t}\right)-p\left(0\right)\right)}{d\lambda}+\left(1-\widehat{q}\right)\frac{dV_B}{d\lambda} \\
&\simeq \widehat{q}r\left(p\left(\underline{t}\right)-c_L\right)\frac{\left(\widehat{q}e^{-\lambda\underline{t}}+\left(1-\widehat{q}\right)\right)^2}{\lambda\widehat{q}\left(1-\widehat{q}\right)e^{-\lambda\underline{t}}}\frac{dq\left(\underline{t}\right)}{d\lambda}-\left(1-\widehat{q}\right)\left(v_H-v_L\right)\frac{dq\left(\underline{t}\right)}{d\lambda} \\
&= \left(\widehat{q}r\left(p\left(\underline{t}\right)-c_L\right)\frac{\left(\widehat{q}e^{-\lambda\underline{t}}+\left(1-\widehat{q}\right)\right)^2}{\lambda\widehat{q}\left(1-\widehat{q}\right)e^{-\lambda\underline{t}}}-\left(1-\widehat{q}\right)\left(v_H-v_L\right)\right)\frac{dq\left(\underline{t}\right)}{d\lambda}.
\end{aligned}$$

This is negative because for λ sufficiently large, the first term is negative, while the second one is positive (as λ tends to infinity, V_B approaches zero, and thus $q\left(\underline{t}\right)$ converges to $\overline{q}$ from the left). The proof for the case where r tends to zero is essentially the same. **Q.E.D.**

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