International Trade: Linking Micro and Macro[1]

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Abstract

Standard models of international trade with heterogeneous firms treat the set of available firms as a continuum. The advantage is that relationships among macroeconomic variables can be specified independently of shocks to individual firms, facilitating the derivation of closed-form solutions to equilibrium outcomes, the estimation of trade equations, and the calculation of counterfactuals. The cost is that the models cannot account for the small (sometimes zero) number of firms engaged in selling from one country to another. We show how a standard heterogeneous-firm trade model can be amended to allow for only an integer number of firms. Estimating the model using data on bilateral trade in manufactures among 92 countries and bilateral exports per firm for a much narrower sample shows that it accounts for zeros in the data very well while maintaining the good fit of the standard gravity equation among country pairs with thick trade volumes.

1 Introduction

The field of international trade has advanced in the past decade through a healthy exchange between new observations on firms in export markets and new theories that have introduced producer heterogeneity into trade models. As a result, we now have general equilibrium theories of trade that are also consistent with various dimensions of the micro data. Furthermore, we have a much better sense of the magnitudes of key parameters underlying these theories. This work is surveyed in Bernard, Jensen, Redding, and Schott (2007) and more recently Redding (2010).

Despite this flurry of activity, the core aggregate relationships between trade, factor costs, and welfare have remained largely untouched. While we now have much better micro foundations for aggregate trade models, their predictions are much like those of the Armington model, for years a workhorse of quantitative international trade. Arkolakis, Costinot, and Rodríguez-Clare (2010) emphasize this (lack of) implication of the recent literature for aggregate trade.

What then are the lessons from the micro data for how we conduct quantitative analyses of trade relationships at the aggregate level? In this paper we explore the implications of the fact that only a finite number (sometimes zero) of firms are involved in trade. While participation of a small number of firms in some export markets is an obvious implication of the micro evidence, previous models (including our own) have ignored its consequences for aggregates by employing the modeling device of a continuum of goods and firms. Here we break with that tradition, initiated by Dornbusch, Fischer, and Samuelson (1977), and explicitly aggregate over a finite number of goods (each produced by a distinct firm).

We use this finite-good-finite-firm model to address an issue that can plague quantitative general equilibrium trade models, zero trade flows. While not a serious issue for trade between large economies within broad sectors, zeros are quite common between smaller countries, or within particular industries. Table 1 shows the frequency of zero bilateral trade flows for manufactured goods in a large sample of countries. Zeros are likely to be an increasingly important feature of general equilibrium analyses as models are pushed to incorporate greater geographic and industrial detail.

Without arbitrary bounds on the support of the distribution of firm efficiency, there are at least two facets of the zero trade problem for a model in which there is no aggregate uncertainty. First, the zeros have extreme implications for parameter values, requiring an infinite trade cost. Second, zeros lead to strong restrictions when used to calibrate a trade model for counterfactual analysis, as a zero can never switch to being a positive trade flow under any exogenous change in parameters. By developing a model with a finite number of heterogeneous firms, we can deal with both these issues.

Our paper deals with a particular situation in which an aggregate relationship (here bilateral trade flows) is modelled as the outcome of heterogeneous decisions of individual agents (here of firms about whether and how much to export to a destination). But the issues it raises apply to any aggregate variable whose magnitude is the summation of what a diverse set of individuals choose to do, which may include nothing.

The paper proceeds as follows. We begin with a review of related literature followed by an overview of the data. Next, we introduce our finite-firm model that motivates the estimation approach that follows. Finally, we examine the ability of the model and estimates to account for observations of zero trade.

2 Related Literature

The literature on zeros in the bilateral trade data includes Eaton and Tamura (1994), Santos Silva and Tenreyro (2006), Armenter and Koren (2008), Helpman, Melitz, and Rubinstein (2008), Martin and Pham (2008) and Baldwin and Harrigan (2009). Our estimation approach builds on Santos Silva and Tenreyro (2006), showing how their Poisson estimator arises from a structural model of trade. We then extend their econometric analysis to fit the variance in trade flows by incorporating structural disturbances in trade costs. Our underlying model of trade is close to that of Helpman, Melitz, and Rubinstein (2008), but instead of obtaining zeros by truncating a continuous Pareto distribution of efficiencies from above, zeros arise in our model because, as in reality, the number of firms is finite. Like us, Armenter and Koren (2008) assume a finite number of firms, stressing, as we do, the importance of the sparsity of the trade data in explaining zeros. Theirs, however, is a purely probabilistic rather than economic model.[1]

Another literature has emphasized the importance of individual firms in aggregate models. Gabaix (2010) uses such a structure to explain aggregate fluctuations due to shocks to very large firms in the economy. This analysis is extended to a model of international trade by di Giovanni and Levchenko (2009), again highlighting the role of very large firms in generating aggregate fluctuations.

[1] Mariscal (2010) shows that Armenter and Koren approach also goes a long way in explaining multinational expansion patterns.

Our work also touches on Balistreri, Hillberry, and Rutherford (2009). That paper discusses both estimation and general equilibrium simulation of a heterogeneous firm model similar to the one we consider here. It does not, however, draw out the implications of a finite number of firms, which is our main contribution.

3 The Data

We use macro and micro data on bilateral trade among 92 countries. The macro data are aggregate bilateral trade flows (in U.S. Dollars) of manufactures X_{ni} from source country i to destination country n in 1992, from Feenstra, Lipsey, and Bowen (1997). The micro data are firm-level exports to destination n for four exporting countries i. The efforts of many researchers, exploiting customs records, are making such data more widely available. We were generously provided micro data for exports from Brazil, France, Denmark, and Uruguay.[2] The micro data allow us to measure the number K_{ni} of firms from i selling in n as well as mean sales per firm $\overline{X}_{ni}$ when K_{ni} is reported as positive.[3] In merging the data, we chose our 92 countries for the macro-level analysis in order to have observations at the firm level from at least two of our four sources.

[2]The French data for manufacturing firms in 1992 are from Eaton, Kortum, and Kramarz (2010). The Danish data for all exporting firms in 1993 are from Pedersen (2009). The Brazilian data for manufactured exports in 1992 are from Arkolakis and Muendler (2010). The Uruguayan data for 1992 were compiled by Raul Sampognaro.

[3]We cannot always tell in the micro export data if the lack of any reported exporter to a particular destination means zero exports there or that the particular destination was not in the dataset. Hence our approach, which exploits the micro data only when $K_{ni} > 0$, leaves the interpretation open.

Table 1 lists our 92 countries and each country's total exports and imports to the other 91. The last two columns display the number of zero trade observations at the aggregate level, indicating for each country how many of the other 91 it does not export to and how many it does not import from. Not surprisingly, zeros become less common as a country trades more. Overall, zeros make up over one-third of the 8372 bilateral observations.

The average number of zeros per country, either as an exporter or as an importer, is 31.4. The variance of zeros for countries as exporters, however, is 652.5 while the variance of zeros for countries as importers is only 283.6. As discussed below, our analysis provides an explanation for the large deviation between the variances.

For country pairs for which $K_{ni} > 0$ Figure 1 plots K_{ni} against X_{ni} on log scales, with source countries labeled by the first letter of the country name. The data cluster around a positively-sloped line through the origin, with no apparent differences across the four source countries.

4 A Finite-Firm Model of Trade

Our framework relates closely to work on trade with heterogeneous firms such as Bernard, Eaton, Jensen, and Kortum (BEJK, 2003), Melitz (2003), Chaney (2008), and Eaton, Kortum, and Kramarz (EKK, 2010). The key difference is that we treat the range of potential technologies for these firms not as a continuum but as an integer. An implication is that zeros can naturally emerge simply because the number of technologies can be sparse. While some results from the existing work survive, others do not. We show the difficulties introduced by dropping the continuum and an approach to overcoming them.

4.1 Technology

As in the recent literature (but also as in the basic Ricardian model of international trade), our basic unit of analysis is a technology for producing a good. We represent technology by the quantity Z of output produced by a unit of labor.[4] A higher Z can mean: (1) more of a product, (2) the same amount of a better product, or (3) any combination of the first two that renders the output of the good produced by a unit of inputs more valuable. For the results here the different interpretations have isomorphic implications. We refer to Z as the efficiency of the technology.

A standard building block in modeling firm heterogeneity is the Pareto distribution. We follow this tradition in assuming that Z is drawn from a Pareto distribution with parameter $\theta > 0$:

$$\Pr[Z > z] = (z/\underline{z})^{-\theta}, \tag{1}$$

for any z above a lower bound $\underline{z} > 0$. The Pareto distribution has a number of properties that make it analytically very tractable.[5] Moreover, for reasons that have been discussed by

[4] Here "labor" can be interpreted to mean an arbitrary bundle of inputs and the "wage" the price of that input bundle. EK (2002) and EKK (2010) make the input bundle a Cobb-Douglas combination of labor and intermediates.

[5] To list a few of them: (i) Integrating across functions weighted by the Pareto distribution often yields simple closed form solutions. Hence, for example, if a continuum of firms are charging prices that are distributed Pareto, under standard assumptions about preferences, a closed-form solution for the price index emerges. (ii) Trunctating the a Pareto distribution from below yields a Pareto distribution with the same shape parameter θ. Hence, as is the case here, if entry is subject to an endogenous cutoff, the distribution of the technologies that make the cut remains Pareto. (iii) A Pareto random variable taken to a power is also Pareto. Hence, if individual prices have a Pareto distribution, with a constant elasticity of demand, so do sales. (iv) The order

Simon and Bonini (1958), Gabaix (1999), and Luttmer (2010), the relevant data (e.g., firm size distributions) often exhibit Pareto properties, at least in the upper tail.

In contrast with previous work, however, we don't treat each country as having a continuum of firms. Instead, we assume that each country i has access to an integer number of technologies, with the number having $Z \geq z$ the realization of a Poisson random variable with parameter $T_i z^{-\theta}$.[6] It will be useful to rank these technologies according to their efficiency, i.e., $Z_i^{(1)} > Z_i^{(2)} > Z_i^{(3)} ... > Z_i^{(k)} > ....$ Selling a unit of a good to market n from source i requires exporting $d_{ni} \geq 1$ units, where we set $d_{ii} = 1$ for all i. It also requires hiring a fixed number F_n workers in market n, which we allow to vary by n but, for simplicity, keep independent of i.[7]

statistics generated by multiple draws from the Pareto distribution have closed-form solutions. For example, if one makes D draws from a Pareto distribution, where D is distributed Poisson with parameter $T\underline{z}^{-\theta}$, then the distribution of the largest Z (call it $Z^{(1)}$) is distributed:

$$\Pr[Z^{(1)} \leq z] = \exp(-Tz^{-\theta}),$$

the type II extreme value (Fréchet) distribution.

[6] The level of T_i may reflect a history of innovation and diffusion, as discussed in Eaton and Kortum (2010, Chapter 4). There we show how the lower bound $\underline{z}$ of the support of z can be made arbitrarily close to zero.

[7] As we discuss below, the data handle a cost that is common across sources with relative equanimity, but balk at the imposition of an entry cost that is common across destinations. Since assuming a cost that is the same for all entrants in a market yields some simplification, we take that route here. Chaney (2008) and EKK (2010) show how to relax it.

4.2 The Aggregate Economy

The goods produced with the sequence of technologies described above combine into a single manufacturing aggregate according to a constant elasticity of substitution (CES) function, with elasticity of substitution $\sigma > 1$. Country i's total spending on this manufacturing aggregate X_i is taken as exogenous. We also take the wage there, w_i, as exogenous.

The price index P_n of the manufacturing aggregate is an equilibrium outcome. We assume, however, that no firm operating in a market has enough influence to bother taking into account the consequences of its own decisions on the price index.

Associated, then, with a technology $Z_i^{(k)}$ in market i is a unit cost to deliver in market n of

$$C_{ni}^{(k)} = w_i d_{ni} / Z_i^{(k)}.$$

Since we assume that any seller in a market ignores the effect of its own price on aggregate outcomes, it charges the Dixit-Stiglitz markup $\overline{m} = \sigma/(\sigma - 1)$ over its unit cost. Its price in market n is therefore $P_{ni}^{(k)} = \overline{m} C_{ni}^{(k)}$. Since the markup is constant we will work with unit costs rather than prices. The aggregate analog is the price index relative to the markup which we refer to as the price level and denote by:

$$\widetilde{P}_n = \frac{P_n}{\overline{m}}.$$

4.2.1 Entry

A firm with unit cost C in delivering to market n would earn a profit there, net of the fixed cost, of:

$$\Pi_n(C) = \left(\frac{C}{\widetilde{P}_n} \right)^{-(\sigma-1)} \frac{X_n}{\sigma} - w_n F_n.$$

To simplify notation in what follows we define:

$$E_n = \sigma w_n F_n$$

as the relevant measure of entry cost. We thus establish a cutoff unit cost:

$$\bar{c}_n = \widetilde{P}_n \left(\frac{X_n}{E_n} \right)^{1/(\sigma-1)}, \tag{2}$$

such that $\Pi_n(\bar{c}_n) = 0$. Since we assume the same E_n for sellers from anywhere, this cutoff is the same for all sources i.

Given aggregate magnitudes, then, a firm from i will enter n if its unit cost there satisfies $C_{ni} \leq \bar{c}_n$, and not otherwise. The number of firms that enter, K_{ni}, satisfies:

$$C_{ni}^{(K_{ni})} \leq \bar{c}_n < C_{ni}^{(K_{ni}+1)}. \tag{3}$$

The set of entrants from i selling in n have costs $\left\{ C_{ni}^{(k)} \right\}_{k=1}^{K_{ni}}$. [8] Given $\bar{c}_n$ and w_i, our assumptions about the distribution of efficiencies implies that the number K_{ni} of firms with $C_{ni}^{(k)} \leq \bar{c}_n$ is the realization of a Poisson random variable with parameter:

$$\lambda_{ni} = \Phi_{ni} \bar{c}_n^{\theta} \tag{4}$$

where:

$$\Phi_{ni} = T_i (w_i d_{ni})^{-\theta}. \tag{5}$$

Note that these magnitudes depend on the parameters T_i and d_{ni} as well as w_i, and, through $\bar{c}_n^{\theta}$, on $\widetilde{P}_n$ and X_n.

[8] With a finite number of firms a potential for multiple equilibria arises. Consider two firms with nearly the same unit cost in a market very close close to the cutoff. Entry by either one might drive the price index down to the point where entry by the other is no longer profitable. We eliminate such multiplicity simply by assuming that a lower unit cost firm would enter before a higher unit cost firm, as would naturally be the case if there were a continuum of firms.

4.2.2 Equilibrium

Having determined the K_{ni} conditional on $\widetilde{P}_n$ we now solve for the $\widetilde{P}_n$ given the K_{ni}. The price level is simply:

$$\widetilde{P}_n = \left[\sum_{i=1}^{N} \sum_{k=1}^{K_{ni}} \left(C_{ni}^{(k)} \right)^{-(\sigma-1)} \right]^{-1/(\sigma-1)}. \tag{6}$$

In this version of the model, with the wage exogenous, equilibrium is a set of price levels $\left\{ \widetilde{P}_n \right\}_{n=1}^{N}$, cost cutoffs $\{\bar{c}_n\}_{n=1}^{N}$ and firm entry $\{K_{ni}\}_{i,n=1}^{N}$ satisfying (2), (3), and (6).

To relate the model results back to trade, note that the firm with rank $k \leq K_{ni}$ from country i active in market n will sell:

$$X_{ni}^{(k)} = \left(\frac{C_{ni}^{(k)}}{\widetilde{P}_n} \right)^{-(\sigma-1)} X_n$$

in that market. Thus country n's total imports from n are:

$$X_{ni} = \sum_{k=1}^{K_{ni}} X_{ni}^{(k)}. \tag{7}$$

Hence our model relates aggregate bilateral trade X_{ni}, a measure that has been the subject of countless gravity studies, to the decisions of a finite number of sellers. We now turn to what our derivation implies for the specification and estimation of a gravity equation.

5 Estimating the Micro-Based Gravity Equation

In the equilibrium specified above the outcomes of individual firms in terms of their efficiency draws Z together determine the aggregate price levels $\widetilde{P}_n$ and the cutoffs $\bar{c}_n$. While in principle "everything depends on everything," we can get some insight, which we exploit in the estimation section that follows, by asking about the outcomes for exports to various countries taking these price levels and cost cutoffs as given.

Our strategy is to decompose aggregate exports from i to n, X_{ni}, into the product of the number of sellers K_{ni} and, where $K_{ni} > 0$, mean sales per firm $\overline{X}_{ni} = X_{ni}/K_{ni}$. That is, we work with the identity:

$$X_{ni} = K_{ni}\overline{X}_{ni}. \tag{8}$$

To implement our estimation procedure we need to know various moments of these components, to which we now turn.

5.1 Mean Sales per Firm

How much a firm sells depends on its unit cost of supplying a market. The distribution of unit cost for a seller from i selling in n is simply:

$$H_n(c) = \Pr[C \leq c | C \leq \overline{c}_n] = \left(\frac{c}{\overline{c}_n}\right)^{\theta}, \tag{9}$$

for any $c \leq \overline{c}_n$, which is independent of i. Since the distribution of costs of supplying n is the same from any source, expected sales per firm will be the same from any source selling in a given destination.

We can compute expected mean sales, given that $K_{ni} = K > 0$, as:[9]

$$E\left[\overline{X}_{ni}|K_{ni}=K\right] = \frac{1}{K}\sum_{k=1}^{K} E[X_{ni}(C)|C \leq \overline{c}_n] = \frac{\widetilde{\theta}}{\widetilde{\theta}-1}E_n, \tag{10}$$

where:

$$\widetilde{\theta} = \frac{\theta}{\sigma - 1},$$

a term we introduce since, in what follows, θ and σ always appear together in this form.

Hence expected sales per firm are proportional to the entry cost. Note that for expected sales to be finite we need $\widetilde{\theta} > 1$. We will assume $\widetilde{\theta} > 2$, which, as we show next, keeps the variance of firm sales finite as well.

We will also make use of the variance of mean sales, which for $K_{ni} = K > 0$, is:

$$V\left[\overline{X}_{ni}|K_{ni}=K\right] = \frac{1}{K^2}\sum_{k=1}^{K} V[X_{ni}(C)|C \leq \overline{c}_n] = \frac{\widetilde{\theta}}{\left(\widetilde{\theta}-1\right)^2\left(\widetilde{\theta}-2\right)}\frac{(E_n)^2}{K}, \tag{11}$$

[9] The derivation is as follows:

$$\begin{aligned} E[X_{ni}(C)|C &\leq \overline{c}_n] = \int_0^{\overline{c}_n}\left(\frac{c}{\widetilde{P}_n}\right)^{-(\sigma-1)} X_n dH_n(c) \\ &= X_n\left(\widetilde{P}_n\right)^{\sigma-1}\frac{\theta}{\theta-(\sigma-1)}\left(\overline{c}_n\right)^{-(\sigma-1)} \\ &= \frac{\widetilde{\theta}}{\widetilde{\theta}-1}E_n \end{aligned}$$

which, not surprisingly, is inversely proportional to K.[10]

5.2 Number of Firms

We take X_n, $\widetilde{P}_n$, and, consequently, $\bar{c}_n$ as given. Also taking w_i as given, we can treat λ_{ni} defined in (4) as a parameter. Doing so, the number of sellers from i selling in market n, K_{ni}, is the realization of a Poisson random variable with parameter λ_{ni}, so that:

$$\Pr[K_{ni} = k] = \frac{e^{-\lambda_{ni}} \left(\lambda_{ni}\right)^k}{k!}. \tag{12}$$

Since the number of firms from i selling in n is distributed Poisson, a zero is a possible outcome, which becomes more likely the lower λ_{ni}.

A well known property of the Poisson is that:

$$E[K_{ni}] = V[K_{ni}] = \lambda_{ni}. \tag{13}$$

5.3 Bilateral Trade

Having derived the first and second moments of the two pieces of the bilateral trade flows, mean sales per firm $\overline{X}_{ni}$ and number of firms K_{ni}, we now turn to the moments of the total

[10] The derivation is as follows:

$$\begin{aligned} V\left[X_{ni}(C)|C \leq \bar{c}_n\right] &= E[(X_{ni}(C))^2 \,|C \leq \bar{c}_n] - (E[X_{ni}(C)|C \leq \bar{c}_n])^2 \\ &= \int_0^{\bar{c}_n} \left[\left(\frac{c}{\widetilde{P}_n}\right)^{-(\sigma-1)} X_n\right]^2 dH_n(c) - \left(\frac{\widetilde{\theta}}{\widetilde{\theta}-1} E_n\right)^2 \\ &= \frac{\widetilde{\theta}}{\widetilde{\theta}-2}\left[X_n \left(\widetilde{P}_n\right)^{\sigma-1}\right]^2 (\bar{c}_n)^{-2(\sigma-1)} - \left(\frac{\widetilde{\theta}}{\widetilde{\theta}-1} E_n\right)^2 \\ &= \frac{\widetilde{\theta}}{\left(\widetilde{\theta}-1\right)^2\left(\widetilde{\theta}-2\right)} (E_n)^2 . \end{aligned}$$

sales in n of firms from i, X_{ni}.

Taking expectations over the decomposition (8), since X_{ni} is necessarily zero if no firm from i sells in n, we only need to consider $K_{ni} > 0$:

$$
\begin{aligned}
E\left[X_{ni}\right] &= \sum_{K=1}^{\infty} \Pr[K_{ni} = K] E[K_{ni}\overline{X}_{ni} | K_{ni} = K] \\
&= \sum_{K=1}^{\infty} K \Pr[K_{ni} = K] E\left[\overline{X}_{ni} | K_{ni} = K\right] \\
&= \lambda_{ni} \frac{\widetilde{\theta}}{\widetilde{\theta} - 1} E_n, \qquad (14)
\end{aligned}
$$

where we have exploited (10) and (13).

To obtain more efficiency in our estimation, we want to use the model's implications for the variance of bilateral trade as well. Using (13), (10), (11), and (14), this variance is:[11]

$$
V[X_{ni}] = \lambda_{ni} \left(E_n\right)^2 \frac{\widetilde{\theta}}{\left(\widetilde{\theta} - 2\right)}. \qquad (16)
$$

We would like to work with a transformation of bilateral trade that inherits properties of the Poisson distribution. In that way we can exploit econometric procedures developed out

[11] The calculation is:

$$
\begin{aligned}
V[X_{ni}] &= E[(X_{ni})^2] - E[X_{ni}]^2 \\
&= \sum_{K=1}^{\infty} \Pr[K_{ni} = K] K^2 E[\left(\overline{X}_{ni}\right)^2 | K_{ni} = K] - (\lambda_{ni})^2 \left(\frac{\widetilde{\theta}}{\widetilde{\theta} - 1} E_n\right)^2 \\
&= \sum_{K=1}^{\infty} \Pr[K_{ni} = K] K^2 \left\{V\left[\overline{X}_{ni} | K_{ni} = K\right] + E\left[\overline{X}_{ni} | K_{ni} = K\right]^2\right\} - (\lambda_{ni})^2 \left(\frac{\widetilde{\theta}}{\widetilde{\theta} - 1} E_n\right)^2 \\
&= \lambda_{ni} \left(E_n\right)^2 \frac{\widetilde{\theta}}{\left(\widetilde{\theta} - 1\right)^2 \left(\widetilde{\theta} - 2\right)} + \lambda_{ni} \left(\frac{\widetilde{\theta}}{\widetilde{\theta} - 1} E_n\right)^2 \\
&= \lambda_{ni} \left(E_n\right)^2 \frac{\widetilde{\theta}}{\left(\widetilde{\theta} - 2\right)} \qquad (15)
\end{aligned}
$$

of the analysis of count data. By analogy to $K_{ni} = X_{ni}/\overline{X}_{ni}$, which is distributed Poisson, it is natural to work with:

$$\widetilde{K}_{ni} = \frac{X_{ni}}{E\left[\overline{X}_{ni}\right]} = \frac{(\widetilde{\theta}-1)}{\widetilde{\theta}}\frac{X_{ni}}{E_n}.$$

Applying (14) we get:

$$E\left[\widetilde{K}_{ni}\right] = \lambda_{ni},$$

while from (16) we get:

$$V[\widetilde{K}_{ni}] = \frac{\lambda_{ni}\left(E_n\right)^2 \frac{\widetilde{\theta}}{(\widetilde{\theta}-2)}}{\left(\frac{\widetilde{\theta}}{\widetilde{\theta}-1}E_n\right)^2} = \frac{1}{\gamma}\lambda_{ni},$$

where

$$\gamma = \frac{(\widetilde{\theta}-2)\widetilde{\theta}}{\left(\widetilde{\theta}-1\right)^2} = \frac{\left(\widetilde{\theta}-1\right)^2 - 1}{\left(\widetilde{\theta}-1\right)^2}. \tag{17}$$

Since $0 < \gamma < 1$, we have $V[\widetilde{K}_{ni}] > E\left[\widetilde{K}_{ni}\right]$, so that $\widetilde{K}_{ni}$ lacks a key property of the Poisson.[12]

We can easily correct this deficiency by working with a closely related variable which we call **scaled bilateral trade**:

$$\widetilde{X}_{ni} = \gamma\widetilde{K}_{ni} = \frac{X_{ni}E[X_{ni}]}{V[X_{ni}]}. \tag{18}$$

Like a Poisson random variable, scaled bilateral trade has mean equal to variance:

$$E[\widetilde{X}_{ni}] = V[\widetilde{X}_{ni}] = \gamma\lambda_{ni}. \tag{19}$$

Note that scaled bilateral trade requires data not only on bilateral trade X_{ni}, which we have, but on E_n, which we don't. Furthermore, we need a value for γ. For that we impose $\widetilde{\theta} = 2.46$, the estimate obtained from micro data in EKK (2010), to obtain $\gamma = 0.53$.

[12] The reason is that variation in X_{ni} is positively correlated with variation in mean sales per firm, $\overline{X}_{ni}$. Dividing X_{ni} by the random variable $\overline{X}_{ni}$ (as in K_{ni}) therefore results in a smaller variance than dividing by the constant $E\left[\overline{X}_{ni}\right]$ (as in $\widetilde{K}_{ni}$).

We proceed in two steps. We first use our micro level data to infer the E_n. We use these estimates, and our value of γ, to scale bilateral trade as in (18) before proceeding to the estimation of our bilateral trade equation.

5.4 Estimating the Mean Sales Equation

For source countries $i \in \Omega = \{\text{Brazil, Denmark, France, Uruguay}\}$, we can measure $\overline{X}_{ni}$ for a large set of destination countries n. Let $\Omega_n \subset \Omega$ be the subset of source countries for which we can calculate mean sales in country n. As described above, we restrict the set of destinations n to those for which Ω_n has at least 2 elements.[13]

We estimate (10) simply by averaging over the sources for which we have data. Our variance result (11) suggests calculating a weighted average, using data on K_{ni} as the weights. Hence we compute:

$$\frac{\widetilde{\theta}}{\widetilde{\theta}-1}\hat{E}_n = \frac{\sum_{i \in \Omega_n} K_{ni}\overline{X}_{ni}}{\sum_{i' \in \Omega_n} K_{ni'}}, \tag{20}$$

which is equivalent simply to pooling the data from the available sources. Values of the right hand side of (20) are shown in Table 2. We use our value of $\widetilde{\theta} = 2.46$ to retrieve $\hat{E}_n$.[14]

[13] We drop the home-country observations (when available), since the universe of firms selling in the home market is measured very differently. The customs data tell us the number of exporters and their sales in a foreign market. The total number of active firms in a country is more difficult to tie down since many may not be counted.

[14] Our restriction that $E_{ni} = E_n$ is essential in allowing us to make use of limited firm-level data for an analysis of trade among a vast number of countries. To gauge the plausibility of this restriction, we examine whether our four source countries, which are diverse in economic size and development, differ among each other in a systematic way. We run a weighted regression of the unbalanced panel $\overline{X}_{ni}$ on a full set of destination country effects and source country effects. The weights, $K_{ni}/\left(\widehat{E}_n\right)^2$, undo the heteroscedasticity implied

With the estimates $\widehat{E}_n$ we can construct scaled bilateral trade according to (18). This variable is the basis for estimating the bilateral trade equation.

5.5 Estimating the Bilateral Trade Equation

Our estimation procedure exploits (19), which we rewrite as:

$$E[\widetilde{X}_{ni}|\lambda_{ni}] = V[\widetilde{X}_{ni}|\lambda_{ni}] = \gamma\lambda_{ni}. \tag{21}$$

From (4) and (5), we can write:

$$\lambda_{ni} = T_i w_i^{-\theta} d_{ni}^{-\theta} \bar{c}_n^{\theta},$$

which we connect to the data as follows:

First, as in EK (2002), we use source-country fixed effects S_i to capture $T_i\,(w_i)^{-\theta}$, reflecting country i's technological sophistication relative to it's factor cost, which applies across all destinations where it sells.

Second, as in EK (2002), we relate bilateral trade costs (adjusted for θ) $d_{ni}^{-\theta}$ to a vector of observable bilateral variables g_{ni} standard in the gravity literature: the distance between n and i and whether they share a common language and border. We also allow for destination-specific differences in trade costs m_n.[15]

Third, as in EK (2002), we capture the unobservable component of $d_{ni}^{-\theta}$ with a disturbance

by (11). Our null hypothesis is that the source-country effects should all be the same. The estimates of source-country effects (presented as source-country-specific intercepts) are shown in Table 3. They imply little variation across sources, although we can easily reject the joint hypothesis of equal coefficients.

[15] We arbitrarily associate differences in openness with imports rather than exports. Exploiting data on prices Waugh (2010) shows that they actually relate more to exports. For our purposes here, however, it doesn't matter which we do.

ν_{ni} that is i.i.d. across foreign country pairs. In contrast to EK (2002), however, we specify the trade equation in levels rather than in logs. Hence we require $E[\nu_{ni}] = 1$ and $V[\nu_{ni}] = \eta^2$.

Our estimation procedure does not require further restrictions on the distribution $g(\nu)$. Our simulations below require us to take a stand, and there we assume that ν is distributed gamma, which has density:

$$g(\nu) = \frac{\delta^\delta}{\Gamma(\delta)} v^{\delta-1} e^{-v\delta}, \tag{22}$$

for which $E(\nu) = 1$ and $\eta^2 = 1/\delta$.

Combining the observables and the disturbance we set:

$$(d_{ni})^{-\theta} = m_n \exp(g'_{ni}\alpha)\, \nu_{ni}, \tag{23}$$

for $n \neq i$, where α is a vector of parameters associated with the gravity variables.

Substituting these specifications into (21) yields:

$$\lambda_{ni} = S_i m_n \exp(g'_{ni}\alpha)\, \nu_{ni} \left(\bar{c}_n\right)^\theta, \tag{24}$$

Finally, we capture both the cost cutoffs and the destination-specific trade costs with destination-country fixed effects D_n where:

$$D_n = \left(\bar{c}_n\right)^\theta m_n.$$

Combining these steps gives us:

$$\lambda_{ni} = S_i D_n \exp(g'_{ni}\alpha)\, \nu_{ni}. \tag{25}$$

For $n = i$ we continue to impose $d_{nn} = 1$ so that:[16]

$$\lambda_{nn} = S_n \left(\bar{c}_n\right)^\theta. \tag{26}$$

[16]With a continuum of firms there would be no Poisson disturbance, hence we would have $\widetilde{X}_{ni} = \gamma\lambda_{ni}$ and

When it comes to simulating the model, we will use (26) to isolate the two terms in the destination effects. For estimation, we use only the observations for which $n \neq i$.

For compactness, we define the vector z_{ni} to include a constant, source-country dummy variables for all but one i, destination-country dummy variables for all but one n, and the bilateral variables g_{ni}, with the vector β their coefficients. We can then write:

$$\lambda_{ni} = \mu_{ni}\nu_{ni} \tag{27}$$

where:

$$\gamma\mu_{ni} = \exp\left(z'_{ni}\beta\right). \tag{28}$$

Note that γ is subsumed in the constant term of $z'_{ni}\beta$.

Expression (19) gives us the first two moments of $\widetilde{X}_{ni}$ conditional on the product of μ_{ni} and ν_{ni}:

$$E[\widetilde{X}_{ni}|\mu_{ni}, \nu_{ni}] = V[\widetilde{X}_{ni}|\mu_{ni}, \nu_{ni}] = \gamma\mu_{ni}\nu_{ni}. \tag{29}$$

But we can only condition on the component μ_{ni} that relates to observables. The first two moments of $\widetilde{X}_{ni}$ conditional just on μ_{ni} are:

$$\begin{aligned} E[\widetilde{X}_{ni}|\mu_{ni}] &= E\left[E[\widetilde{X}_{ni}|\mu_{ni}, \nu_{ni}]\right] = E[\gamma\mu_{ni}\nu_{ni}] \\ &= \gamma\mu_{ni}E[\nu_{ni}] = \gamma\mu_{ni} \end{aligned} \tag{30}$$

$\widetilde{X}_{nn} = \gamma\lambda_{nn}$. In that case we could simply divide (24) by (26), so that for $n \neq i$:

$$\frac{\widetilde{X}_{ni}}{\widetilde{X}_{nn}} = \frac{S_i}{S_n} m_n \exp\left(g'_{ni}\alpha\right)\nu_{ni},$$

with destination-country effects capturing the m_n. Taking logs of both sides, the equation could then be estimated as a linear regression with error term $\ln \nu_{ni}$, almost exactly as in EK(2002). We cannot follow that approach here.

and

$$\begin{aligned} V[\widetilde{X}_{ni}|\mu_{ni}] &= E\left[V[\widetilde{X}_{ni}|\mu_{ni},\nu_{ni}]\right] + V[E[\widetilde{X}_{ni}|\mu_{ni},\nu_{ni}]] \\ &= E[\gamma\mu_{ni}\nu_{ni}] + V[\gamma\mu_{ni}\nu_{ni}] \\ &= \gamma\mu_{ni} + (\gamma\mu_{ni})^2 V[\nu_{ni}] \\ &= \gamma\mu_{ni}(1+\eta^2\gamma\mu_{ni}). \end{aligned} \tag{31}$$

The mean and variance are thus as if $\widetilde{X}_{ni}$ were distributed negative binomial.[17]

5.6 Estimation Procedure

Our goal is to estimate the parameters β. If $\widetilde{X}_{ni}$ were distributed negative binomial then negative binomial maximum likelihood would offer an obvious procedure for estimating β as well as η^2.

Since $\widetilde{X}_{ni}$ is not restricted to integers, however, it is not distributed negative binomial. Gourieroux, Monfort, and Trognon (henceforth GMT, 1984) show that a consistent estimate of β, denoted $\widehat{\beta}_0$, satisfying (30) and (31), can be obtained by pseudo-maximum likelihood (PML) with either the Poisson likelihood or the negative binomial likelihood with η^2 set to an arbitrary value.[18] GMT (1984) propose using such a $\widehat{\beta}_0$ to obtain a consistent estimate of

[17] As shown in Greenwood and Yule (1920) and in Hausman, Hall, and Griliches (1984), under the assumption that ν_{ni} is distributed gamma (22), the distribution of K_{ni} given μ_{ni} is negative binomial. (The derivation is in footnote 23.) Scaled bilateral trade $\widetilde{X}_{ni}$ is not distributed negative binomial (as it is not even integer valued) but is obviously closely related to K_{ni}.

[18] Note from above that negative binomial PML with $\eta^2 = 0$ is simply Poisson PML.

η^2 by a simple regression.[19] From (31), we have:

$$E\left[\left(\widetilde{X}_{ni}-\exp\left(z_{ni}'\beta\right)\right)^2\right]-\exp\left(z_{ni}'\beta\right)=\eta^2\exp\left(z_{ni}'\beta\right)^2.$$

Thus, replacing β with a $\hat{\beta}_0$ we can estimate η^2 as the regression slope (with the intercept constrained to be 0):

$$\hat{\eta}^2=\frac{\sum_{n=1}^{N}\sum_{i\neq n}\left\{\left[\widetilde{X}_{ni}-\exp\left(z_{ni}'\hat{\beta}_0\right)\right]^2-\exp\left(z_{ni}'\hat{\beta}_0\right)\right\}\exp\left(z_{ni}'\hat{\beta}_0\right)^2}{\sum_{n'=1}^{N}\sum_{i'\neq n'}\exp\left(z_{n'i'}'\hat{\beta}_0\right)^4} \tag{32}$$

GMT (1984) propose a second-stage estimation of β, which we denote $\widehat{\beta}_1$, to maximize the negative binomial likelihood function, with η^2 set equal to a consistent first-stage estimate, $\hat{\eta}^2$. In the present context this estimator, called quasi-generalized pseudo-maximum likelihood (QGPML), is more efficient than the first-stage PML estimators.

Thus our estimation involves the following steps:

1. We use PML, using either the Poisson likelihood or negative binomial likelihood, setting η at various values, to obtain consistent estimate $\hat{\beta}_0$ of β using (30) and (31).

2. Using $\hat{\beta}_0$ we obtain an estimate of $\hat{\eta}^2$ using (32).

3. We use QGPML (which fixes η at $\hat{\eta}^2$) to obtain an estimate $\hat{\beta}_1$ of β using (30) and (31).

With our different estimates of β, denoted $\widehat{\beta}$, we can construct an estimate of the nonstochastic component of the Poisson parameter:

$$\hat{\mu}_{ni}=\frac{1}{\gamma}\exp\left(z_{ni}'\hat{\beta}\right).$$

[19] See Cameron and Trivedi (1986) for a further discussion.

5.7 Estimation Results

We estimate the parameters β of the bilateral trade equation (28) using scaled bilateral trade $\widetilde{X}_{ni}$ among our sample of 92 countries, giving us 8372 country pairs, since we do not include home observations. Our gravity variables g_{ni} are: (i) the distance from n to i, (ii) a dummy variable equal to 1 if n and i are not contiguous (otherwise 0), and (iii) a dummy variable equal to 1 if n and i do not share a common language (otherwise 0). To these geography variables we add (i) a constant term, (ii) a dummy variable for each destination country n (dropping the one for the UK), and (iii) a dummy variable for each source country i (again, dropping the one for the UK) to form the vector z_{ni}.

Table 4 shows the results of various estimation approaches for the parameters α corresponding to the three gravity variables. The interpretation of the coefficients in terms of their implications for the conditional mean μ_{ni} is the same in each.

For comparison purposes, Column 1 shows Ordinary Least Squares (OLS) estimates obtained by dropping observations for which $X_{ni} = 0$, ignoring the Poisson error, and taking logs of each side of (27) so that $\ln \nu_{ni}$ becomes the error term. The estimates are typical for such gravity equations, with distance, lack of contiguity, and lack of a common language all stifling trade, distance with an elasticity above one (in absolute value).

The second column shows the Poisson PML estimates, the approach advocated in Santos Silva and Tenreyro (2006). In fact, the results in our first two columns are very consistent with those reported in their Table 5, which is based on the specification most like ours. As in their results, the elasticity of trade with respect to distance is substantially reduced in going from the OLS to the Poisson PML.

The next four columns report estimates based on the negative binomial likelihood function, but with η^2 fixed at particular values. These sets of estimates are all versions of PML. The one in the third column sets η^2 to a very small number and so comes close to replicating Poisson PML. As η^2 is increased, however, the parameter estimates look more like those obtained from OLS.

The estimates in columns 2-5 all provide consistent estimates for β, allowing us to obtain consistent estimates of η^2 via (32). The estimates we obtain are shown in the penultimate row of the table. Poisson PML and negative binomial PML with a tiny value of η^2 (0.0001) imply small values of $\widehat{\eta}^2$. But if we start with η^2 set to 0.1 or higher the implied $\widehat{\eta}^2$'s are in the range 0.7-0.9. The last column of the table shows the QGPML estimates, as η^2 is fixed at a value equal to a consistent estimate. In fact, we chose to focus on a fixed point at which the value of η^2 we fixed for QGPML was the same as the value we obtained from (32) when using the QGPML estimates of β. As suggested by the results in the table, we found the estimates to be quite insensitive to the exact value of η^2 in the range of 0.5-1.

Santos Silva and Tenreyro (2006) provide intuition into their results, which also applies here. The OLS regression in logarithms implies an error whose variance is proportional to the amount of trade. PML estimation, formulated in levels rather than logarithms with $\widehat{\eta}^2 = 0$ or at a low value, implies an error whose variance does not increase in proportion with size. Hence more weight is placed on large countries since their observations are seen as having less variance relative to their size. As can be seen from (31), a higher value of $\widehat{\eta}^2$ implies that variance increases faster with μ_{ni}, bringing the PML weights more into line with those under OLS in logarithms. As a consequence, the weight of large countries is more as in the OLS procedure.[20]

The value of $\hat{\alpha}$ (and associated parameters composing $\hat{\beta}$) and $\hat{\eta}^2$ shown in the last column of Table 4 will be the values we use for simulating the implications of the model. In the end, these estimates of α obtained from QGPML are not far from those obtained from OLS, while they are quite different from those obtained from Poisson PML.

We can obtain further evidence on the size of η^2 by comparing how well the QGPML estimate predicts observations of zero trade compared with the Poisson PML estimate.

6 Accounting for Zeros

We now turn to the question that motivated our analysis: Can our finite-firm model account for the prevalence of zeros in the bilateral trade data?

Exports from i to n are zero when no firm in i exports to n. In our framework the number of firms from i selling in n is the realization of a Poisson random variable with parameter $\lambda_{ni} = \mu_{ni}\nu_{ni}$. Hence the question is how likely is the outcome zero. Randomness comes about from two sources. For one thing, given the Poisson parameter λ_{ni}, the realization is itself

[20] To examine the hypothesis that the relative weight of large countries versus small countries is at work we ran the OLS regression using only observations on trade among the 25 percent of our sample of countries with the largest home sales X_{nn}. The coefficient on the logarithm of distance is -0.849, more in line with the Poisson regression than the OLS regression with the full sample (-1.404). Fieler (2011) provides an explanation for the lower sensitivity to distance of trade among large countries. She develops a model in which there are two classes of goods with different values of θ, and finds that rich countries tend to specialize in both the production and consumption of the class with the lower θ. As rich countries tend to be larger, the lower θ would explain why distance is more easily overcome in trade among the large countries in our sample. This explanation lies, of course, beyond our model but suggests an interesting topic for further exploration.

random. But the error term ν_{ni} creates randomness in the Poisson parameter itself. We need to account for both types of randomness.

6.1 A Distribution for the Trade-Cost Disturbance

Hence, to predict the likelihood of a zero, we need to take a stand on the distribution of the trade cost disturbance ν_{ni}. As indicated above, we assume that ν_{ni} is distributed gamma with the density given in (22). This distribution implies a simple closed-form distribution of the number of firms from i selling in n. In particular, conditional on μ_{ni}, the K_{ni} are distributed negative binomial:[21]

$$\Pr[K_{ni} = k] = \frac{\Gamma(\frac{1}{\eta^2} + k)}{\Gamma(\frac{1}{\eta^2})\Gamma(k+1)} \left[\eta^2 \mu_{ni}\right]^k \left[1 + \eta^2 \mu_{ni}\right]^{-\left(\frac{1}{\eta^2}+k\right)}. \tag{33}$$

[21] The steps of the derivation are as follows:

$$\begin{aligned}
\Pr[K_{ni} &= k|\mu_{ni}] = \int_0^\infty \frac{e^{-\mu_{ni}\nu}\left(\mu_{ni}\nu\right)^k}{k!} \frac{\delta^\delta}{\Gamma(\delta)} \nu^{\delta-1} e^{-\nu\delta} d\nu \\
&= \frac{\delta^\delta}{\Gamma(\delta)k!} \int_0^\infty e^{-(\mu_{ni}+\delta)\nu} \left(\mu_{ni}\right)^k \nu^{k+\delta-1} d\nu \\
&= \frac{\delta^\delta \left(\mu_{ni}\right)^k}{\Gamma(\delta)k!} \left(\mu_{ni}+\delta\right)^{-(k+\delta)} \Gamma(\delta+k).
\end{aligned}$$

Replacing δ with $1/\eta^2$ and rearranging yields (33). The mean and variance are:

$$E\left[K_{ni}|\mu_{ni}\right] = \mu_{ni}$$

and

$$V\left[K_{ni}|\mu_{ni}\right] = \mu_{ni}(1 + \eta^2 \mu_{ni}).$$

As $\eta^2 \to 0$ we approach the Poisson distribution (12) in which $V[K_{ni}] = E[K_{ni}] = \mu_{ni} = \lambda_{ni}$.

6.2 The Probability of Zero Trade

We can calculate the probability of zero trade by evaluating (33) at $k = 0$ and replacing the parameters with our estimates, to get:

$$\hat{P}_{ni}^{NB}(0) = \left(1 + \hat{\eta}^2 \hat{\mu}_{ni}\right)^{-1/\hat{\eta}^2}. \tag{34}$$

This expression is decreasing in $\hat{\mu}_{ni}$ given $\widehat{\eta}^2$ and increasing in $\widehat{\eta}^2$ given $\hat{\mu}_{ni}$.[22] If $\hat{\eta}^2 = 0$ this expression reduces to the Poisson case:

$$\hat{P}_{ni}^{POI}(0) = e^{-\hat{\mu}_{ni}}. \tag{35}$$

We calculate the probabilities using our estimates of μ_{ni} and η^2 from QGPML in column 7 of Table 4 and from the Poisson PML in column 2 of Table 4. We compare these probabilities between cases in which $X_{ni} = 0$ and for those in which $X_{ni} > 0$ in the actual data.

Figure 2 displays the probabilities of zero for the 2889 observation in which trade is actually zero ($X_{ni} = 0$) for QGPML, as a histogram: The height gives the fraction of such observations

[22] The first result is immediate. To establish the second consider:

$$\ln \hat{P}_{ni}^{NB}(0) = -\frac{1}{\widehat{\eta}^2} \ln(1 + \hat{\eta}^2 \hat{\mu}_{ni}),$$

which is a monotonically increasing transformation of $\hat{P}_{ni}^{NB}(0)$. Taking the derivative:

$$\frac{d \ln \hat{P}_{ni}^{NB}(0)}{d\widehat{\eta}^2} = \frac{1}{\left(\widehat{\eta}^2\right)^2} \left[\ln(1 + \hat{\eta}^2 \hat{\mu}_{ni}) - \frac{1}{1 + \hat{\eta}^2 \hat{\mu}_{ni}}\right]$$

which, defining $x = \hat{\eta}^2 \hat{\mu}_{ni}$, has the sign of:

$$f(x) = \ln(1 + x) - \frac{x}{1 + x}.$$

Note that $f(0) = 0$ while:

$$f'(x) = \frac{x}{(1 + x)^2} > 0$$

for $x > 0$.

for which $\hat{P}_{ni}^{NB}(0)$ takes on a value in a given range (shown on the horizontal axis). The estimated probability of zero trade is above 0.9 for nearly one-fourth of the observations and is above 0.5 for nearly two-thirds of them. Figure 4 shows the equivalent histogram (again where $X_{ni} = 0$) for Poisson PML. It yields a probability above 0.9 for only 13 percent of the observations and above 0.5 for only 38 percent of them.

Figure 3 displays probabilities of zero for the 5483 observations in which trade is actually positive ($X_{ni} > 0$) for QGPML, again as a histogram. The estimated probability of zero $\hat{P}_{ni}^{NB}(0)$ is below 0.1 for nearly three-fourths of these observations, and is below 0.5 over 90 percent of the time. The equivalent histogram for Poisson PML, shown in Figure 5, indicates a probability below 0.1 nearly all the time.

In summary, the Poisson model rarely predicts a high probability of zero trade even when the actual observation is zero. Hence, it fares well for the observations in which trade is positive (Figure 5), but fails miserably when trade is in fact zero (Figure 4). The reason is that there is just so little variance that a zero value of trade is very unlikely even for relatively small values of μ_{ni}. An implication is that a large value of η^2 is needed to account for the frequency of zeros.

6.3 Simulating Zero Trade

In addition to predicting the probability of zero exports from a particular source to a particular destination, we would also like to simulate the analog of the zero trade observations across sources or destinations for an individual country, the equivalents of the numbers reported in the last two columns of Table 1. It might appear that we could simulate the number of zero-trade connections for a given country i by simply drawing independent Bernoulli random variables, with a success probability given by (34), for each of i's trading partners. That approach is legitimate when considering i as an importer, since firm technology is independent across the countries it buys from. But, when we consider i as an exporter, the model implies a positive correlation between i not selling to n and i not selling to some other country n'. The reason is that the same firm from i may be the only one selling to either n or n'. Hence we predict qualitatively the greater variance in the number of zeros among countries as exporters than as importers.[23]

To see how well we do quantitatively, we return to the ordering of firms by their unit cost. Whether or not country i sells to market n is completely determined by the lowest cost firm from i, whose cost of supplying its product to n is $C_{ni}^{(1)}$. In particular, no firm from i will sell in n if:

$$C_{ni}^{(1)} > \bar{c}_n. \tag{36}$$

Since $C_{ni}^{(1)} = d_{ni} C_{ii}^{(1)}$, the same firm from i is the lowest cost supplier to any market. Thus the draw for $C_{ii}^{(1)}$ affects the likelihood of i's entry into all destinations n.[24]

Exploiting a result in EK (2010) we can write:

$$C_{ni}^{(1)} = \left(\frac{U_i^{(1)}}{\Phi_{ni}} \right)^{1/\theta},$$

[23] The greater variance in the number of zeros arises because the source country effects are much more variable (with a variance of 8.41) than the destination country effects (with a variance of 1.75). Our model provides an explanation for this much greater variance in export effects than import effects..

[24] This extreme prediction of the model is attenuated in EKK (2010) by introducing a destination–country-specific shocks to demand and to entry costs.

where $\Pr[U_i^{(1)} \leq u] = 1 - e^{-u}$. Using this result, and rearranging, we can express (36) as:

$$U_i^{(1)} > \Phi_{ni} \left(\overline{c}_n\right)^{\theta} = \lambda_{ni} = \mu_{ni} \nu_{ni}. \tag{37}$$

Consider a given source country i. We can simulate zeros for its exports to each destination n simultaneously using (37) as follows: (1) We draw $U_i^{(1)}$ from the unit exponential distribution. (2) We draw ν_{ni} (independently for each n) from the gamma distribution with mean 1 and variance $\hat{\eta}^2$. (3) For each destination n we measure μ_{ni} with $\hat{\mu}_{ni}$.

We repeat this simulation procedure 10,000 times to get the frequency distribution of zeros for each country's imports and exports. Table 5 and Figures 6 and 7 show the results. Starting with the table, the first column and fifth columns repeat (from Table 1) the number of zero exports to different destinations and the number of zero imports from different destinations. The second and sixth columns report the mean number of zeros across our simulations. The correlation between the zeros in exports and our mean predictions is 0.92 and between zeros in imports and our mean predictions is 0.85. We predict an average number of zeros of 27 as compared with 31.4 in the data. Remarkably, the variance of simulated zeros among countries as exporters is 683 (compared with 653 in the data) and the variance of simulated zeros among countries as importers is 97 (compared with 284 in the data). Our model thus accounts for the big discrepancy between the two variances quite successfully.

The third and fourth columns of Table 5 report the 25th and 75th percentile of the number of zeros across the 10,000 draws for countries as exporters while the seventh and eighth do the same for countries as importers. Note that the second pair are usually close to each other while the first are typically far apart.

Figure 6 displays the whole distribution for Denmark as both an importer and exporter.

As a small but rich country, Denmark actually imports from all but 8 of the other 91 countries. Our simulations over-predict this number of import-zeros, generating a "normal-like" probability distribution, centered around 18 countries, with nearly all the mass between 10 and 27. The results are quite different for Denmark as an exporter. As is typical of advanced countries in our sample, Denmark actually exports to all of the other 91 countries. Our simulation captures this fact, yielding an "exponential-like" distribution with over 45 percent of the mass on zero, less than 20 percent on one, and declining monotonically with essentially no probability of 10 or more export-zeros.

Figure 7 displays the distributions for Nepal, a small and poor country. Nepal does not import from 55 of the 91 other countries and does not export to 65 of them. The simulated distribution of the number of import-zeros for Nepal appears similar Denmark's except shifted to the right. It is centered at 47-48 with nearly all the mass between 40 and 55. The distribution of the number of Nepal's export-zeros is quite different: left-skewed with a median of about 70 and substantial mass over all possible outcomes. These simulation results illustrate the large variance induced by the model's implication that export-zeros hinge on the efficiency of a single firm.

7 Simulating Equilibrium and Monte Carlo Analysis

In order to apply a standard estimation technique we needed to make some simplifying assumptions along the way. For one thing, our analysis took as given the price indices $\widetilde{P}_n$ and cutoffs $\bar{c}_n$ in each destination, even though in principle the $\widetilde{P}_n$'s, and the hence $\bar{c}_n$'s, depend on the cost realization of individual firms potentially supplying n.

To get a sense of how our procedure performs we use a parameterized version of the complete model laid out in Section 4 to generate not only firm level data, but using these simulated data to calculate price levels $\widetilde{P}_n$ and cost-cutoffs $\bar{c}_n$ as equilibrium outcomes. We continue to fix total manufacturing absorption X_n at the levels we measure in 1992.[25]

7.1 Parameterizing the Model for Simulation

Our simulation procedure requires values for $\Phi_{ni} = T_i\left(w_i d_{ni}\right)^{-\theta}$. We use our estimates in the last column of Table 4, together with some additional information, to quantify the Φ_{ni}. The additional information is from (26), which can be written as:

$$\gamma\left(\bar{c}_n\right)^{\theta} = \frac{\gamma\lambda_{nn}}{S_n} = \frac{E[\widetilde{X}_{nn}|\lambda_{nn}]}{S_n}.$$

Thus:

$$\Phi_{ni} = \begin{cases} \frac{\gamma\mu_{ni}\nu_{ni}}{\gamma(\bar{c}_n)^{\theta}} = \frac{E[\widetilde{X}_{ni}|\mu_{ni}]}{E[\widetilde{X}_{nn}|\lambda_{nn}]} S_n\nu_{ni} & i \neq n \\ S_n & i = n \end{cases}. \tag{38}$$

To evaluate (38) we proceed as follows:

[25] We now require two new pieces of data: (i) manufacturing absorption X_n and (ii) home sales X_{nn}. We construct X_n from X_{nn}. We begin with manufacturing gross production y_n in 1992, which we construct following the procedure described in footnote 1 or in EKK (2004). We calculate home sales as

$$X_{nn} = y_n - e_n,$$

where e_n is total manufacturing exports to all other countries (not just the other 91 in our sample), which is available from Feenstra et. al. (1997). We then construct absorption as

$$X_n = \sum_{i=1}^{92} X_{ni}.$$

This procedure ignores exports to and imports from outside our sample of 92 countries.

1. We construct $\widetilde{X}_{nn}$ in parallel to how we constructed $\widetilde{X}_{ni}$, using the estimates $\hat{E}_n$ and data on home sales X_{nn}.

2. We measure $E[\widetilde{X}_{nn}|\lambda_{nn}]$ with $\widetilde{X}_{nn}$.

3. We measure $E[\widetilde{X}_{ni}|\mu_{ni}]$ with our estimates $\exp(z'_{ni}\hat{\beta})$, with elements of $\hat{\beta}$ coming from the last column of Table 4 together with the estimated source and destination-country effects.

4. We measure S_n with the associated estimate of the source-country effect $\widehat{S}_n$.

5. We simulate ν_{ni} from a gamma distribution with mean one and variance $\hat{\eta}^2$.

With our estimates of Φ_{ni} in hand we simulate as described next.

7.2 Simulation Procedure

The first step of our procedure is to use the Φ_{ni} to simulate the cost draws of individual firms in each country. We define a transformation of costs:

$$U_i^{(k)} = \Phi_{ni} \left[C_{ni}^{(k)} \right]^{\theta}. \tag{39}$$

We can draw the $U_i^{(k)}$, without knowledge of any parameters, independently across source countries i, based on the following result from EK (2010):

$$\Pr\left[U_i^{(1)} \leq u \right] = 1 - e^{-u}$$

and, for any $k \geq 1$:

$$\Pr\left[U_i^{(k+1)} - U_i^{(k)} \leq u \right] = 1 - e^{-u}.$$

Thus we can build up the sequence $U_i^{(k)}$ from a set of independent exponential random variables, each with parameter 1. We can then calculate the cost draws by inverting U as:

$$C_{ni}^{(k)} = (U_i^{(k)}/\Phi_{ni})^{1/\theta}$$

The second step is to use these cost draws $C_{ni}^{(k)}$ to construct $\left\{\widetilde{P}_n\right\}$, $\{\bar{c}_n\}$, and $\{K_{ni}\}$ that satisfy:

$$\widetilde{P}_n = \left[\sum_{i=1}^{N}\sum_{k=1}^{K_{ni}}\left(C_{ni}^{(k)}\right)^{-(\sigma-1)}\right]^{-1/(\sigma-1)}, \tag{40}$$

$$\bar{c}_n = \widetilde{P}_n\left(\frac{X_n}{E_n}\right)^{1/(\sigma-1)}, \tag{41}$$

and

$$C_{ni}^{(K_{ni})} \leq \bar{c}_n < C_{ni}^{(K_{ni}+1)}. \tag{42}$$

If $k < K_{ni}$ the k'th best firm from i sells:

$$X_{ni}^{(k)} = \left(\frac{C_{ni}^{(k)}}{\widetilde{P}_n}\right)^{-(\sigma-1)} X_n$$

in country n. By inspection, it is clear that firm-level sales will satisfy the adding up restriction:

$$X_n = \sum_{i=1}^{N}\sum_{k=1}^{K_{ni}} X_{ni}^{(k)}.$$

To make clear that the only parameters we require $\widetilde{\theta}$, $\{\Phi_{ni}\}$, $\{X_n\}$, and $\{E_n\}$, we introduce the terms:

$$A_{ni}^{(k)} = \left(C_{ni}^{(k)}\right)^{-(\sigma-1)} = \left(U_i^{(k)}\right)^{-1/\widetilde{\theta}}(\Phi_{ni})^{1/\widetilde{\theta}}, \tag{43}$$

$$\bar{a}_n = (\bar{c}_n)^{-(\sigma-1)}$$

and

$$\widetilde{A}_n = \left(\widetilde{P}_n\right)^{-(\sigma-1)}.$$

Using this notation we can express (40), (41), and (42) as:

$$\widetilde{A}_n = \sum_{i=1}^{N} \sum_{k=1}^{K_{ni}} A_{ni}^{(k)}, \tag{44}$$

$$\overline{a}_n = \widetilde{A}_n \frac{E_n}{X_n} \tag{45}$$

and

$$A_{ni}^{(K_{ni})} \geq \overline{a}_n > A_{ni}^{(K_{ni}+1)}. \tag{46}$$

The solution to (44), (45), and (46) yields $\left\{\widetilde{A}_n\right\}$, $\{\overline{a}_n\}$, and $\{K_{ni}\}$. These equations can be solved by a simple numerical procedure. We recover the sales in n of the k'th best firm from i (for $k = 1, 2, ..., K_{ni}$) as:

$$X_{ni}^{(k)} = \frac{A_{ni}^{(k)}}{\widetilde{A}_n} X_n.$$

We recover the cost cutoffs as:

$$(\overline{c}_n)^{\theta} = \left(\widetilde{A}_n\right)^{-\widetilde{\theta}} \left(\frac{X_n}{E_n}\right)^{\widetilde{\theta}}. \tag{47}$$

We use these results to calculate total bilateral exports X_{ni} from each source i to each destination n, and the number of firms K_{ni} from each source i selling in each destination n, exactly the data we used in our estimation. To accurately represent the constraints of the actual data, we only retain K_{ni} for $i \in$ {Brazil, Denmark, France, Uruguay}.

7.3 Simulation Results

In the cases where $X_{ni} > 0$ and $K_{ni} > 0$ we can plot our simulated exactly as we did the real data in Figure 1. Figure 8 shows the results for a particular simulation of the model. The simulated data show a striking resemblance to the actual data.

We use simulations of the model to perform Monte Carlo tests of our econometric procedures. In particular, we can examine how well our estimation technique, when applied to simulated data exactly as it was applied to the actual data, uncovers the true parameters used in the simulation. Table 6 illustrates a typical run. This table is just like Table 4 except that the first column of Table 6 shows the parameters used for the simulation (i.e. those from the last column of Table 4). All the procedures are quite successful at recovering the true parameters, with a slight edge going to QGPML over OLS and Poisson PML. We severely underestimate η^2 when using residuals from Poisson PML. As with the actual data, starting from a moderate value of η^2 using negative binomial PML, we estimate a value of around 0.8. That said, our estimate of η^2 displays the most variation across different simulations of the model. QGPML recovers other parameters quite precisely across the simulations.

8 Conclusion

We have combined firm-level export data, aggregate trade data, and a finite-firm model to investigate the prevalence of zeros in the trade data, but have only scratched the surface of what a parameterized model of this sort could be used for. We hope that future work will examine and relax some assumptions here. We have stuck with the standard Dixit-Stiglitz markup. We have sidestepped the equilibration of labor markets to determine wages. We have ignored interindustry interaction.

We see our analysis as a step in furthering two quite separate research agendas. One is the econometrics of estimating macroeconomic relationships, such as a bilateral trade equation, when sparseness in the number of agents underlying the relationship is an issue. A second is the formulation of economic models which can be taken to both individual and aggregate data.

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Table 1. Descriptive Statistics

		Total Trade (Million USD)		No. of Zeros in Sample	
	Country	Total Exports	Total Imports	Exports to	Imports from
1	Algeria	262.02	6230.41	57	44
2	Angola	48.04	2149.29	71	53
3	Argentina	7111.71	12284.37	8	27
4	Australia	15566.94	30132.72	5	19
5	Austria	22085.23	21720.69	0	6
6	Bangladesh	1446.20	1188.85	19	43
7	Benin	15.96	448.10	74	55
8	Bolivia	305.03	1111.53	50	37
9	Brazil	27212.22	13626.56	0	21
10	Bulgaria	1341.33	1283.07	31	38
11	Burkina Faso	26.11	232.03	70	57
12	Burundi	5.08	88.01	70	56
13	Cameroon	390.73	877.53	53	46
14	Canada	106421.63	106100.68	0	7
15	Central African Republic	17.02	87.79	74	60
16	Chad	2.69	110.86	72	64
17	Chile	7067.69	7613.92	16	23
18	China	31071.30	39042.04	0	17
19	Colombia	2557.45	6204.99	21	22
20	Costa Rica	639.36	2363.57	44	36
21	Côte d'Ivoire	675.01	1457.22	46	44
22	Denmark	23624.13	19651.31	0	8
23	Dominican Republic	2294.14	2882.82	49	42
24	Ecuador	876.57	2565.07	48	36
25	Egypt	995.60	6324.02	15	26
26	El Salvador	326.56	1291.13	49	39
27	Ethiopia	31.62	535.79	73	42
28	Finland	17197.93	11243.78	0	20
29	France	141492.66	130104.82	0	0
30	Ghana	723.87	1184.87	42	24
31	Greece	4535.57	13795.85	6	10
32	Guatemala	514.37	2201.65	51	38
33	Honduras	122.73	910.98	64	39
34	Hungary	4567.63	5024.21	3	24
35	India	12955.11	8470.82	0	18
36	Indonesia	16126.92	18685.77	7	19
37	Iran	640.27	12368.96	40	43
38	Ireland	21663.64	17493.05	0	14
39	Israel	9252.63	11270.82	27	32
40	Italy	117066.40	93372.11	0	1
41	Jamaica	1071.58	1172.92	46	45
42	Japan	273219.72	121513.38	0	1
43	Jordan	353.57	1974.08	39	40
44	Kenya	327.22	1031.39	35	22
45	Korea	59662.13	47027.97	0	16
46	Kuwait	274.11	4757.93	47	40

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		Total Trade (Million USD)		No. of Zeros in Sample	
	Country	Total Exports	Total Imports	Exports to	Imports from
47	Madagascar	74.45	289.07	63	44
48	Malawi	33.71	448.13	63	48
49	Malaysia	21881.53	25116.63	5	19
50	Mali	28.84	270.31	70	53
51	Mauritania	215.04	363.36	68	55
52	Mauritius	749.66	1122.83	36	32
53	Mexico	36481.61	56450.13	14	22
54	Morocco	2723.01	4864.38	18	24
55	Mozambique	129.24	702.29	58	53
56	Nepal	124.93	290.90	65	55
57	Netherlands	63075.79	63236.59	0	0
58	New Zealand	7167.16	6989.50	14	31
59	Nigeria	261.50	5915.16	48	35
60	Norway	14116.79	18442.85	0	20
61	Oman	440.42	2292.31	46	39
62	Pakistan	4808.01	5441.02	5	28
63	Panama	320.01	7850.87	48	35
64	Paraguay	295.52	1532.92	48	44
65	Peru	2422.71	2731.93	28	34
66	Philippines	4675.29	8433.17	22	31
67	Portugal	12726.92	19680.55	1	5
68	Romania	2182.08	2094.73	8	36
69	Rwanda	5.51	114.88	74	58
70	Saudi Arabia	3088.77	27632.93	36	30
71	Senegal	373.17	804.17	59	52
72	South Africa	6671.92	10369.34	3	9
73	Spain	46963.64	63036.14	0	1
74	Sri Lanka	1476.41	2182.93	32	37
75	Sweden	40954.33	29656.78	0	8
76	Switzerland	44029.96	36146.51	0	4
77	Syrian Arab Republic	141.13	2141.40	50	43
78	Taiwan	65581.95	50130.16	27	33
79	Tanzania, United Rep. of	72.00	842.68	51	45
80	Thailand	21645.97	27416.26	0	11
81	Togo	20.69	489.79	63	48
82	Trinidad and Tobago	481.03	1068.05	45	39
83	Tunisia	2230.96	4130.15	35	37
84	Turkey	6824.79	12386.31	3	24
85	Uganda	23.50	266.95	60	50
86	United Kingdom	128688.75	137566.47	0	0
87	United States of America	359292.84	395010.78	0	0
88	Uruguay	1324.24	1672.66	35	35
89	Venezuela	2819.75	11546.50	34	31
90	Viet Nam	833.21	1695.58	38	54
91	Zambia	912.95	768.91	55	48
92	Zimbabwe	555.31	1286.70	39	35
	Total			2889	2889

Table 2. Mean Sales Estimation

Country	No. of Source Countries	Mean Sales per Firm
Algeria	2	0.426
Angola	2	0.272
Argentina	4	0.638
Australia	4	0.324
Austria	4	0.334
Bangladesh	2	0.391
Benin	2	0.079
Bolivia	3	0.174
Brazil	3	0.493
Bulgaria	4	0.211
Burkina Faso	2	0.065
Burundi	2	0.065
Cameroon	2	0.096
Canada	4	0.301
Central African Republic	2	0.047
Chad	2	0.070
Chile	4	0.345
China	3	1.811
Colombia	3	0.351
Costa Rica	3	0.190
Côte d'Ivoire	2	0.134
Denmark	3	0.323
Dominican Republic	3	0.258
Ecuador	3	0.229
Egypt	4	0.486
El Salvador	3	0.118
Ethiopia	2	0.099
Finland	4	0.223
France	3	0.904
Ghana	2	0.194
Greece	4	0.354
Guatemala	3	0.151
Honduras	3	0.090
Hungary	4	0.226
India	4	0.452
Indonesia	3	1.162
Iran	4	1.121
Ireland	4	0.301
Israel	3	0.235
Italy	4	1.375
Jamaica	3	0.132
Japan	4	1.124
Jordan	3	0.171
Kenya	3	0.230
Korea	4	0.715
Kuwait	4	0.256

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Country	No. of Source Countries	Mean Sales per Firm
Madagascar	2	0.079
Malawi	2	0.126
Malaysia	3	0.435
Mali	2	0.082
Mauritania	2	0.107
Mauritius	2	0.101
Mexico	4	0.835
Morocco	3	0.258
Mozambique	2	0.519
Nepal	3	0.173
Netherlands	4	0.884
New Zealand	4	0.108
Nigeria	3	0.618
Norway	4	0.290
Oman	2	0.422
Pakistan	3	0.414
Panama	3	0.195
Paraguay	3	0.229
Peru	3	0.199
Philippines	4	0.502
Portugal	4	0.346
Romania	4	0.292
Rwanda	2	0.055
Saudi Arabia	4	0.536
Senegal	2	0.093
South Africa	3	0.238
Spain	4	0.992
Sri Lanka	3	0.291
Sweden	4	0.446
Switzerland	4	0.314
Syrian Arab Republic	2	0.341
Taiwan	4	0.607
Tanzania, United Rep. of	2	0.130
Thailand	4	0.692
Togo	3	0.077
Trinidad and Tobago	3	0.170
Tunisia	3	0.240
Turkey	4	0.497
Uganda	2	0.061
United Kingdom	4	1.311
United States of America	4	1.603
Uruguay	2	0.176
Venezuela	3	0.330
Viet Nam	3	0.548
Zambia	2	0.110
Zimbabwe	2	0.195

Table 3. Source Country Coefficients

	Mean Sales*
France	1.308***
	(0.110)
Denmark	1.280***
	(0.112)
Brazil	1.380***
	(0.111)
Uruguay	1.282***
	(0.131)
p-value for F test of joint significance	0.0011
Number of observations	282

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

*OLS Regression also includes all destination country effects as independent variables

Table 4. Bilateral Trade Regressions

	OLS	Poisson	$\eta^2 = 0.0001$	$\eta^2 = 0.1$	$\eta^2 = 1$	$\eta^2 = 2$	QGPML ($\eta^2 = 0.84$)
Distance	-1.404***	-0.741***	-0.821***	-1.178***	-1.350***	-1.407***	-1.335***
	(0.0374)	(0.0394)	(0.0383)	(0.0305)	(0.0359)	(0.0378)	(0.0355)
Lack of Contiguity	-0.500**	-0.599***	-0.550***	-0.486***	-0.289*	-0.228	-0.306*
	(0.154)	(0.111)	(0.109)	(0.108)	(0.124)	(0.130)	(0.122)
Lack of Common language	-0.907***	-0.328***	-0.447***	-0.920***	-1.013***	-1.045***	-1.005***
	(0.0721)	(0.0886)	(0.0819)	(0.0671)	(0.0713)	(0.0730)	(0.0709)
η^2		0.0134	0.260	0.734	0.846	0.878	0.837
Number of observations	5483	8372	8372	8372	8372	8372	8372

Standard errors in parentheses

$^{*}p < 0.05, ^{**}p < 0.01, ^{***}p < 0.001$

Table 5. Simulated Number of Zeros

	No. Zero Exports				No. Zero Imports			
			Quartiles				Quartiles	
Country	Actual	Mean	p25	p75	Actual	Mean	p25	p75
Algeria	57	37.2	22	52	44	27.7	26	30
Angola	71	63.1	51	80	53	24.7	23	27
Argentina	8	2.1	0	3	27	23.6	22	25
Australia	5	3.2	1	5	19	15.3	14	17
Austria	0	1.6	0	2	6	19.0	17	21
Bangladesh	19	16.4	6	25	43	40.1	38	42
Benin	74	80.7	80	88	55	28.3	26	30
Bolivia	50	51.2	40	65	37	36.9	35	39
Brazil	0	0.5	0	1	21	16.5	15	18
Bulgaria	31	18.1	7	28	38	33.7	32	36
Burkina Faso	70	77.6	75	87	57	44.7	42	47
Burundi	70	85.1	86	90	56	41.9	40	44
Cameroon	53	33.0	18	48	46	25.6	24	28
Canada	0	0.8	0	1	7	8.9	7	10
Central African Republic	74	78.1	75	88	60	43.0	41	45
Chad	72	86.0	87	91	64	50.6	48	53
Chile	16	4.9	1	7	23	21.7	20	23
China	0	0.6	0	1	17	23.5	22	25
Colombia	21	20.0	9	30	22	26.6	25	28
Costa Rica	44	36.8	24	50	36	28.0	26	30
Côte d'Ivoire	46	21.0	8	32	44	27.7	26	30
Denmark	0	1.3	0	2	8	18.5	17	20
Dominican Republic	49	40.9	28	55	42	32.7	31	35
Ecuador	48	41.1	29	55	36	29.7	28	32
Egypt	15	19.5	8	30	26	26.0	24	28
El Salvador	49	55.3	46	68	39	31.5	30	33
Ethiopia	73	76.3	71	88	42	24.0	22	26
Finland	0	1.9	0	3	20	20.4	19	22
France	0	0.1	0	0	0	6.4	5	8
Ghana	42	23.6	10	35	24	16.9	15	19
Greece	6	6.9	2	10	10	18.9	17	21
Guatemala	51	45.2	34	59	38	28.5	27	30
Honduras	64	69.1	64	79	39	32.3	30	34
Hungary	3	9.0	2	14	24	30.7	29	33
India	0	1.2	0	2	18	12.5	11	14
Indonesia	7	2.1	0	3	19	28.3	26	30
Iran	40	26.0	13	38	43	26.3	24	28
Ireland	0	2.3	0	3	14	19.0	17	21
Israel	27	3.5	1	5	32	17.1	15	19
Italy	0	0.1	0	0	1	10.6	9	12
Jamaica	46	20.3	10	29	45	32.5	30	34
Japan	0	0.1	0	0	1	9.4	8	11
Jordan	39	22.6	10	34	40	24.9	23	27
Kenya	35	23.7	11	35	22	27.1	25	29
Korea	0	0.2	0	0	16	18.2	16	20
Kuwait	47	41.7	28	57	40	26.9	25	29

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	No. Zero Exports				No. Zero Imports			
			Quartiles				Quartiles	
Country	Actual	Mean	p25	p75	Actual	Mean	p25	p75
Madagascar	63	62.2	50	79	44	35.2	33	37
Malawi	63	69.6	62	82	48	38.0	36	40
Malaysia	5	1.9	0	3	19	19.8	18	22
Mali	70	54.9	42	72	53	39.8	38	42
Mauritania	68	48.6	33	66	55	37.1	35	39
Mauritius	36	26.5	13	39	32	24.7	23	27
Mexico	14	6.2	2	10	22	23.4	22	25
Morocco	18	9.2	3	14	24	23.5	22	25
Mozambique	58	43.9	28	61	53	42.0	40	44
Nepal	65	58.7	49	73	55	47.7	46	50
Netherlands	0	0.3	0	0	0	12.6	11	14
New Zealand	14	3.7	1	6	31	17.7	16	19
Nigeria	48	41.0	25	58	35	25.5	23	27
Norway	0	2.3	0	3	20	18.1	16	20
Oman	46	17.8	7	27	39	37.0	35	39
Pakistan	5	3.8	1	6	28	24.5	23	26
Panama	48	42.5	30	56	35	20.0	18	22
Paraguay	48	45.6	32	61	44	38.5	36	41
Peru	28	13.8	5	21	34	22.9	21	25
Philippines	22	15.8	6	24	31	27.5	26	29
Portugal	1	2.4	0	4	5	13.0	11	15
Romania	8	7.1	2	11	36	30.5	29	32
Rwanda	74	86.2	86	91	58	35.1	33	37
Saudi Arabia	36	7.4	2	11	30	17.1	15	19
Senegal	59	34.8	18	51	52	31.7	30	34
South Africa	3	1.4	0	2	9	13.9	12	16
Spain	0	0.4	0	1	1	10.6	9	12
Sri Lanka	32	23.2	10	34	37	30.8	29	33
Sweden	0	0.8	0	1	8	19.5	18	21
Switzerland	0	0.4	0	1	4	5.8	4	7
Syrian Arab Republic	50	38.1	24	53	43	32.1	30	34
Taiwan	27	0.6	0	1	33	17.8	16	20
Tanzania, United Rep. of	51	53.0	41	69	45	22.9	21	25
Thailand	0	0.7	0	1	11	18.6	17	20
Togo	63	72.1	68	84	48	24.7	23	27
Trinidad and Tobago	45	26.5	14	38	39	35.8	34	38
Tunisia	35	13.7	5	21	37	27.2	25	29
Turkey	3	4.3	1	6	24	23.5	22	25
Uganda	60	71.3	65	84	50	30.5	28	33
United Kingdom	0	0.1	0	0	0	9.9	8	11
United States of America	0	0.0	0	0	0	5.4	4	7
Uruguay	35	17.0	7	25	35	29.1	27	31
Venezuela	34	16.6	7	25	31	22.7	21	24
Viet Nam	38	16.6	6	25	54	46.5	45	48
Zambia	55	19.2	8	29	48	25.7	24	28
Zimbabwe	39	26.9	13	40	35	29.3	27	31

Table 6. Bilateral Trade Regressions on Artificial Data (single draw)

	Parameters	OLS	Poisson	$\eta^2 = 0.0001$	$\eta^2 = 0.1$	$\eta^2 = 1$	$\eta^2 = 2$	QGPML ($\eta^2 = 0.84$)
Distance	-1.335	-1.212***	-1.201***	-1.250***	-1.365***	-1.402***	-1.417***	-1.395***
		(0.0229)	(0.0521)	(0.0536)	(0.0225)	(0.0207)	(0.0208)	(0.0207)
Lack of Contiguity	-0.306	-0.341***	0.183	0.0143	-0.249**	-0.308***	-0.337***	-0.296***
		(0.0886)	(0.165)	(0.160)	(0.0794)	(0.0781)	(0.0807)	(0.0773)
Lack of Common language	-1.005	-0.868***	-1.181***	-1.128***	-1.043***	-1.046***	-1.053***	-1.044***
		(0.0425)	(0.103)	(0.0844)	(0.0403)	(0.0369)	(0.0370)	(0.0370)
η^2	0.837		0.0422	0.430	0.679	0.736	0.759	0.726
Number of observations	8372	5858	8372	8372	8372	8372	8372	8372

Standard errors in parentheses

$^{*}p < 0.05, ^{**}p < 0.01, ^{***}p < 0.001$

Figure 1. Micro and Macro Bilateral Trade

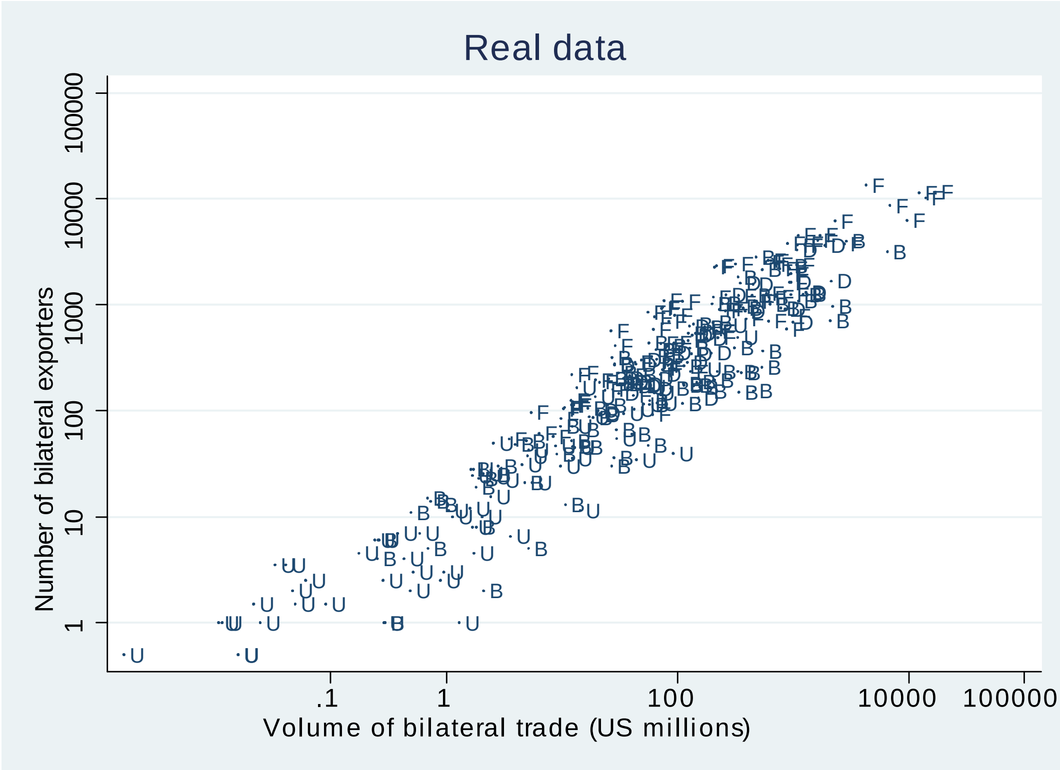


Figure 2. Probabilities of observing zero, given no trade (QGPML)

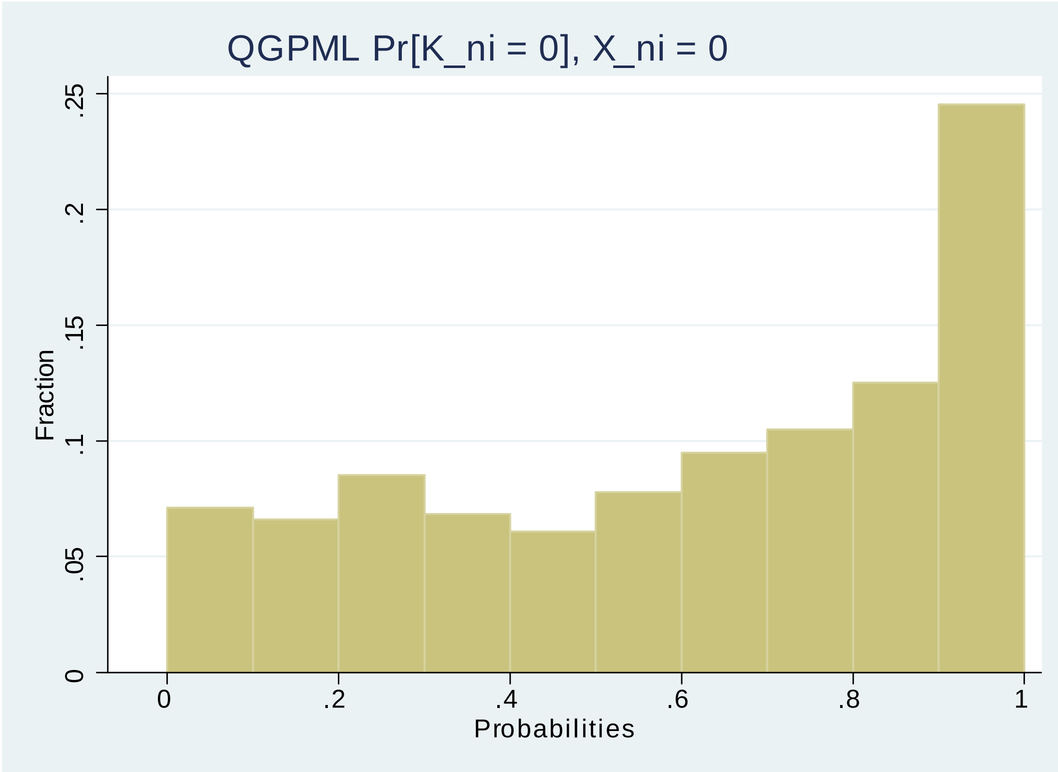


Figure 3. Probabilities of observing zero, given trade (QGPML)

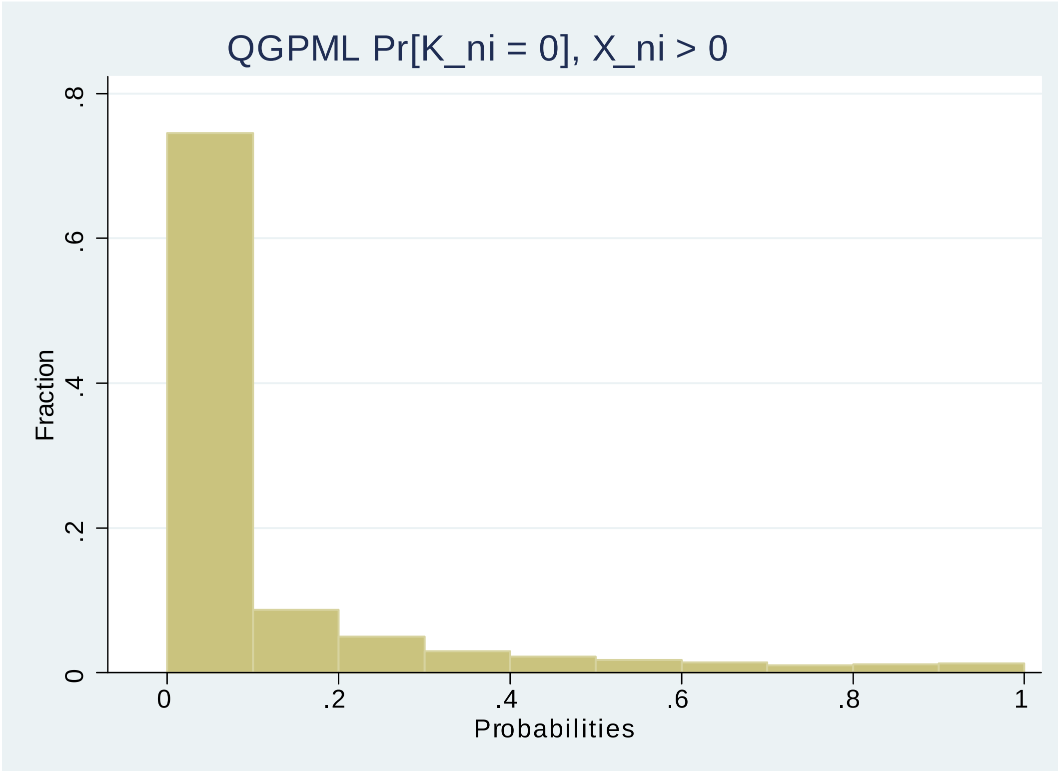


Figure 4. Probabilities of observing zero, given no trade (Poisson)

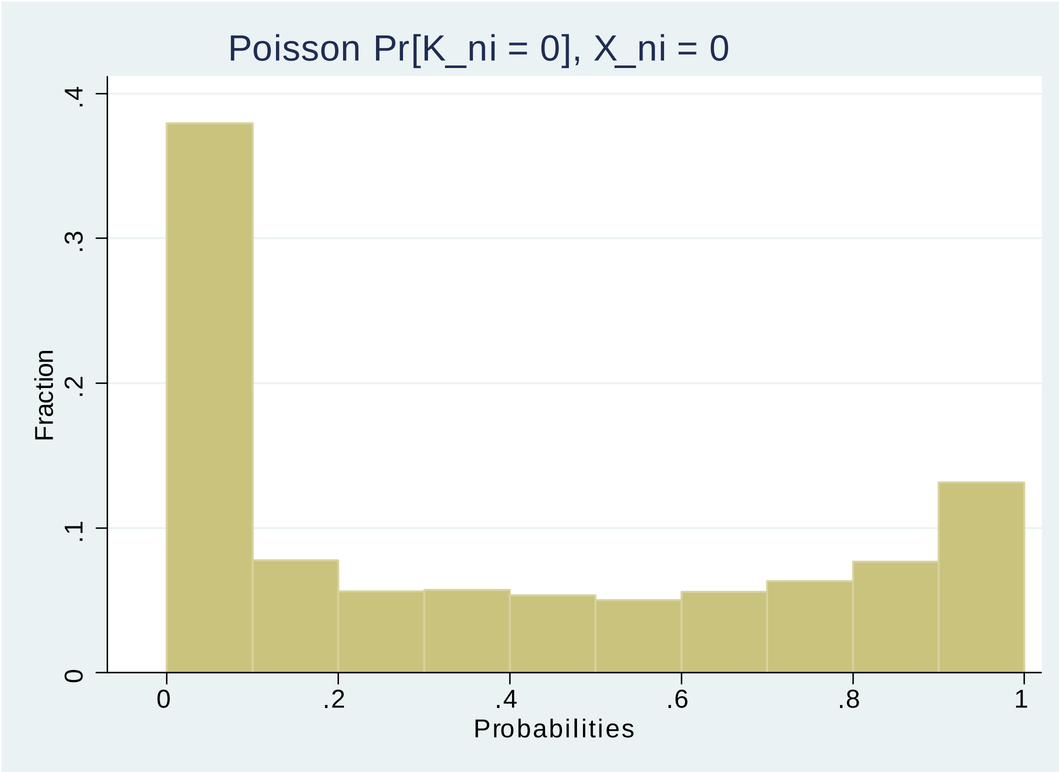


Figure 5. Probabilities of observing zero, given trade (Poisson)

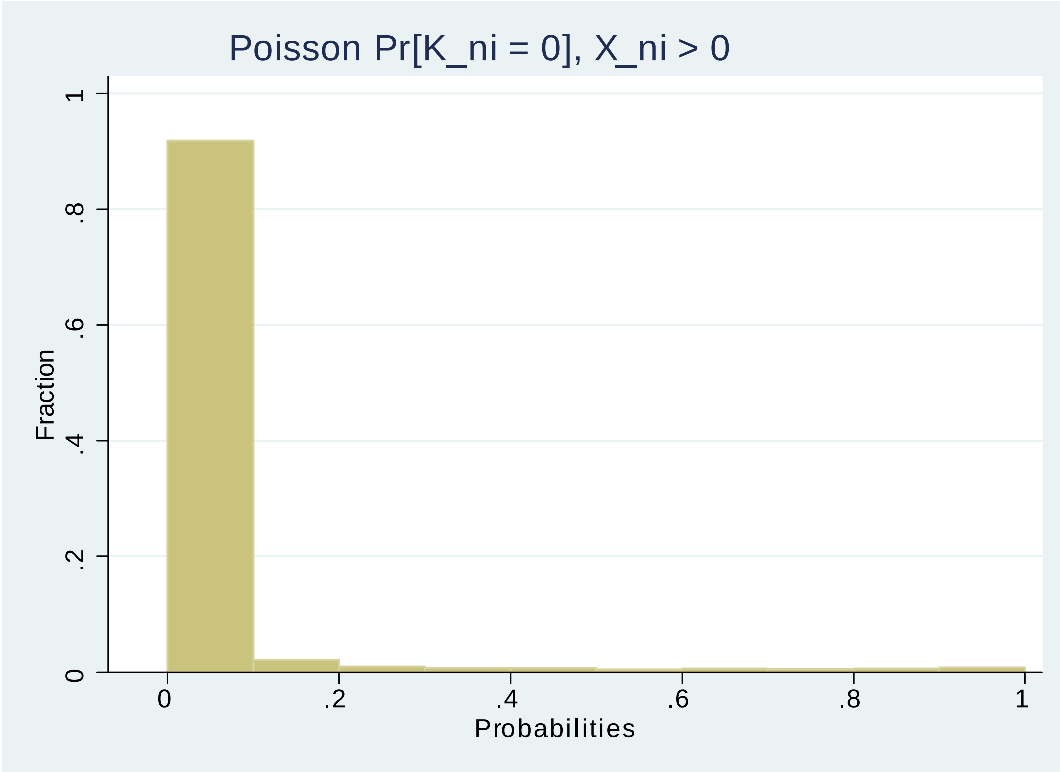
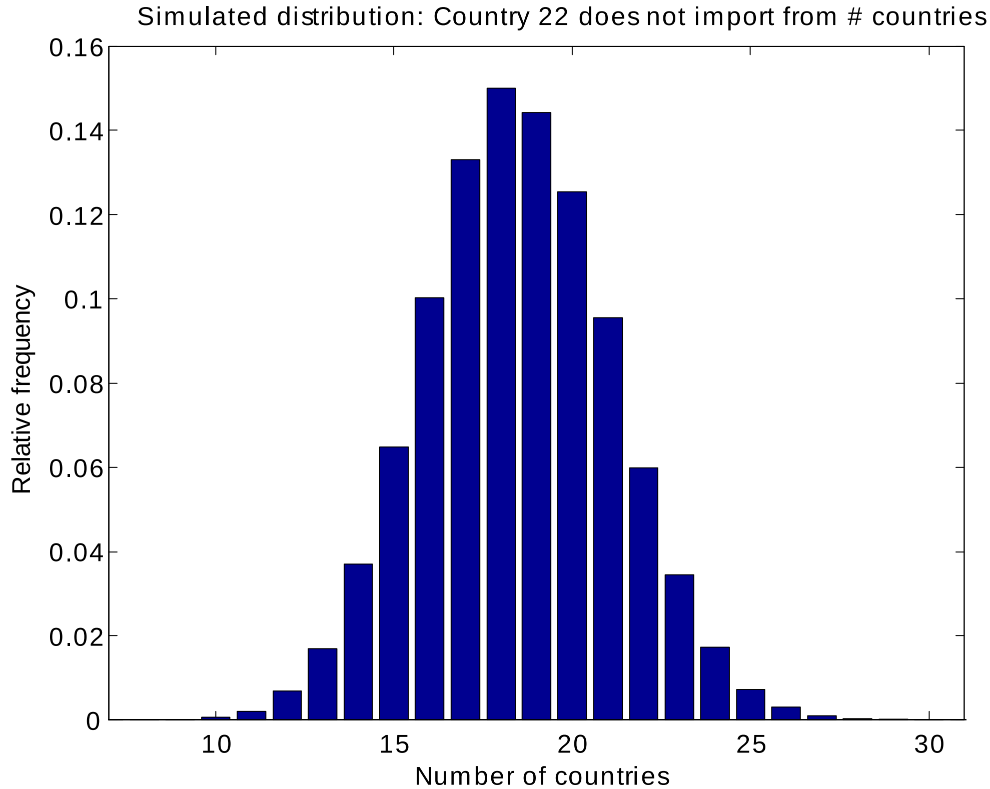


Figure 6. Simulated distributions for Denmark



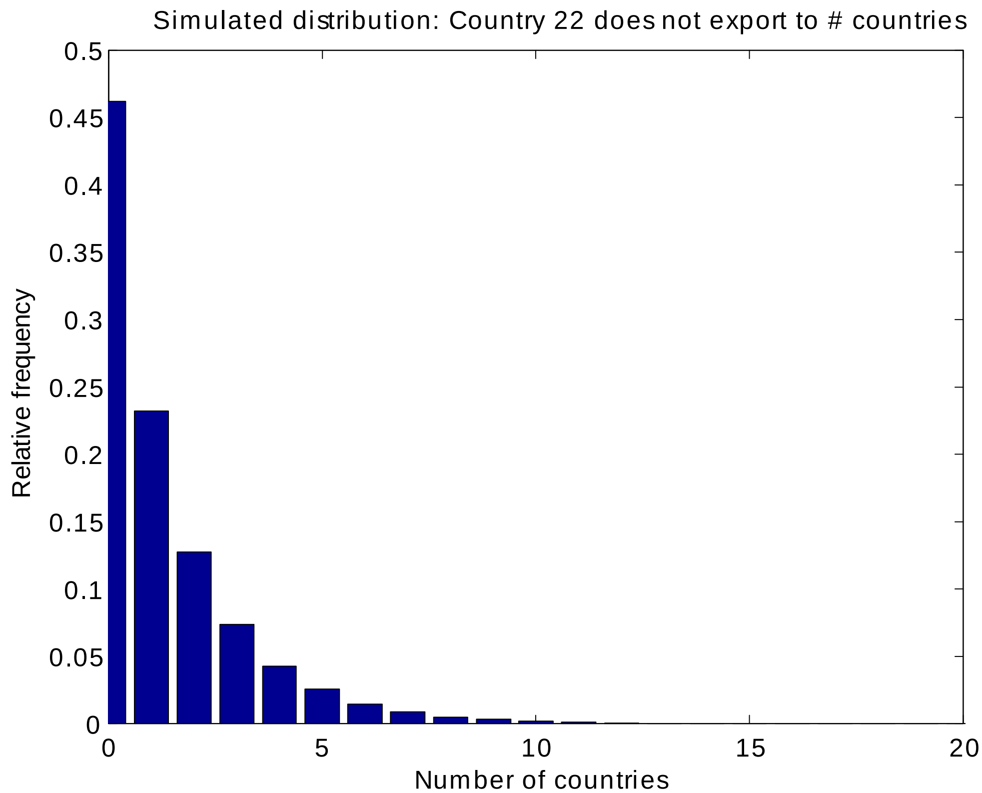


Figure 7. Simulated distributions for Nepal

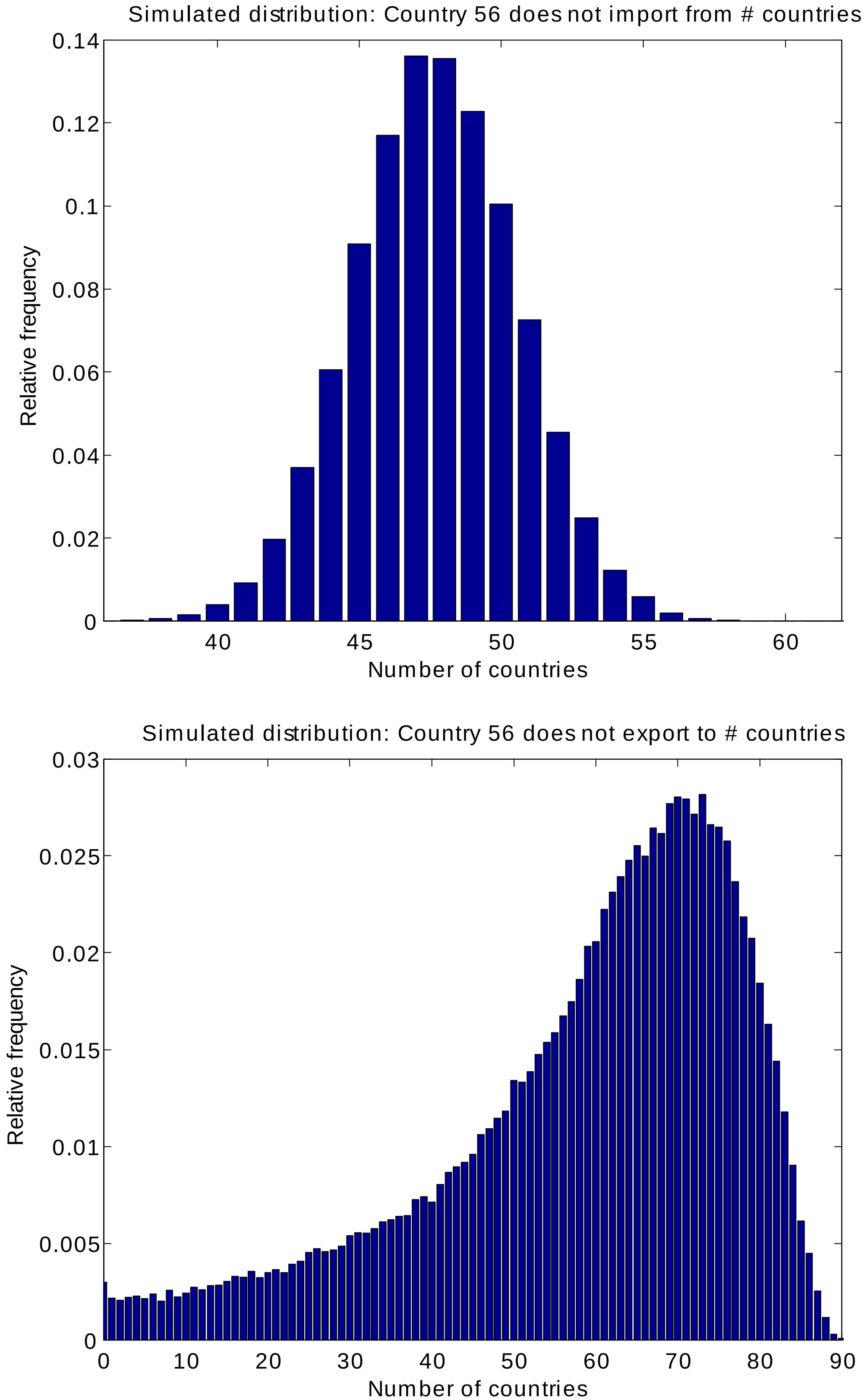


Figure 8. Micro and Macro Bilateral Trade