Competition in Public School Districts: Charter School Entry, Student Sorting, and School Input Determination

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Abstract

I develop and estimate an equilibrium model of charter school entry, student sorting, and endogenous school inputs in public school markets using administrative student- and school-level data from North Carolina for 1998-2001. In the model, students differ by ability, and both charter and public schools make input decisions to test score production functions to affect the ability distribution of attendant students. The model successfully fits key endogenous outcomes as observed in the data: 1) charter schools enter larger markets and markets where they would have higher per-pupil resources, 2) charter schools and public schools in markets in which charter schools are present both choose higher input levels than public schools in markets where there are no charters, and 3) charter schools have the highest average test scores, followed by public schools in markets with charter school competition, followed by public schools in monopoly markets. I use the estimated model to simulate changes in the test score distribution for three counterfactual scenarios: 1) ban charter schools, 2) lift the currently binding statewide cap on the number of charter schools, and 3) equate charter and public school per-pupil resources. In the first and second counterfactuals, charter school entry increases test scores for students who would attend charters by one fifth of a standard deviation. Test scores for public school students in markets with charter schools increase marginally. Equating charter and public school capital triples the fraction of markets with charters and increases the test scores of students attending charters over the monopoly outcome by an even larger amount.

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1 Introduction

The provision of school choice is often proposed as a way to improve educational outcomes for students in poorly performing public schools.[1] The entry of a school that competes with a public school for students is perceived to potentially both directly effect attending students and exert pressure on the public school to improve its quality. Much of the recent debate and policy innovation concerning school choice has focused on charter schools, such as President Obama's education reform initiative, "Race to the Top,"[2] which rewards states that lift restrictions on charter school growth. Charter schools are publicly funded schools that compete with traditional public schools for students and which, like public schools, cannot selectively admit students.[3] Charter schools typically have considerably more autonomy than public schools regarding personnel decisions, curricula, school hours, and pedagogical methods, but they often have lower per-pupil resources due to a lack of separate capital funding streams. All students have access to a public school, but not all students have access to charter schools because charter schools enter certain markets and not others.[4]

About 1 million students attended over 3,000 charter schools in 40 states in 2008.[5] Nationally, there has been rapid growth in the number of charter schools since their inception in 1991, but this growth has been constrained by legislative caps on the number of charter schools in two-thirds of the states with charters. This paper focuses on North Carolina, where 30,000 students are enrolled in about 100 charter schools. North Carolina is a policy-relevant context because the statewide cap on the total number of charter schools has been binding since 2002, and policymakers are currently debating whether to increase or eliminate this cap.[6] The growth in charter school popularity is in part motivated by the perception that charter schools improve student academic outcomes. Charter school advocates argue that charter schools improve the performance of both students attending charters ("direct effect") and also have a positive spillover effect on students attending competing public schools ("competitive effect").[7] Critics of charter schools argue that charter schools "cream-skim" – that is, the better outcomes at charter schools represents student selection, not test score gains.

[1] "School choice" is a collective term which refers to charter schools, magnet and alternative public schools, and private schools coupled with voucher schemes.

[2] White (2009)

[3] Although charter schools are technically a type of public school, I refer to them as "charter schools" and traditional public schools as "public schools" for brevity.

[4] As will be made clear later, "markets" are defined as public school attendance zones.

[5] National statistics are from Snyder et al. (2009)

[6] Wilder (2010)

[7] This can also be phrased in terms of the treatment effects literature. If we consider charter schools to be the treatment, then the average test score difference for students attending charter schools from the no-entry scenario is the effect of "treatment-on-the-treated," while the average difference for students attending public schools in markets with charter schools, compared with the no-entry scenario, is the effect of "treatment-on-the-untreated."

This theoretically ambiguous effect on student achievement has contributed to a national debate over charter school policy. At the most basic level, policymakers would like to know how the presence of charter schools has affected the test scores of students attending charters and nearby public schools. As I discuss below, there is a sizable economics literature devoted to answering this question. Policymakers are also interested in the effects of promoting charter schools. Two often discussed policies are increasing or eliminating caps in states where caps currently exist and providing more resources to charter schools. To answer the last two questions, in addition to knowing how many more charter schools would enter under the new policies, we need to know the potential effect of charter schools on student test scores in markets currently without charter schools and how the effect of charter schools on test scores might differ were all charter schools given more resources.

Accordingly, I consider three key questions relevant to this policy debate in this paper:

1. How has the presence of charter schools affected the test scores of students attending both charter and public schools?

2. Given that charter school entry is often capped, what would be the effect of allowing charter school entry in all public school markets on both the number of charter schools and student test scores?

3. Given that charter schools typically have lower levels of resources than public schools, what would be the effect of equalizing per-pupil capital levels across charter and public schools on both the number of charter schools and student test scores?

To answer these questions, I build and estimate an equilibrium model of charter school entry, school input provision, and student sorting. The following motivates why it is necessary to endogenize each of these variables in an equilibrium framework:

Charter School Entry

In order to know the effect of lifting caps on charter school entry a priori, in addition to knowing what the effect of entry would be in new markets, we also need a model of *which markets* charter schools would enter, were they allowed to do so. By endogenizing charter school entry decisions, I can say how probable charter school entry is for any market and therefore, how many charter schools would enter if caps on entry were lifted. A model of entry also allows me to see how changing market characteristics would affect charter school entry patterns, as I do when I increase per-pupil capital levels of charter schools.

School Input Provision

Estimates of the effect of charter schools on student test scores may be underestimated if they do not take into account changes in public school inputs induced by charter school entry. For the sake of argument, say public schools drastically increased their provision of inputs in response to the presence of a charter school and that students were randomly assigned to charter schools. A simple comparison of the mean test scores of charter and public school students may make it appear that charter schools have a negative effect on student test scores, while in reality all students may be performing better compared to the monopoly outcomes, where there is only a public school in the market. By endogenizing school inputs, I can predict what inputs for the public school *would have been* in the absence of the charter school, which is necessary to calculate the change in test scores for students attending the charter school. In addition to allowing me to take changes in school inputs into account when determining the effect of the presence of a charter school in a market, endogenizing school effort choices is a key step when considering the effect of charter schools in new markets charter schools have not yet entered, or predicting how the effect of charter schools would change if charter schools were given higher levels of per-pupil capital.

Student Sorting

I explicitly model student school choices to take into account the possibility that unobservable student characteristics may influence both student school choices and test scores. The previous literature on charter school effectiveness has focused on estimating the direct effect of charter school entry, which may derive from the above-mentioned differences between how charter and public schools operate.[8] The authors have various non-structural ways of mitigating the inferential problem posed by the potential non-random selection of students into charter schools (a discussion of this literature comes later in this section).[9]

In addition to addressing selection on unobservable student ability, my approach of explicitly modeling student school choices has advantages over the less-structured approach used by the existing literature. First, this approach allows schools in the model to take into account student sorting on ability when choosing inputs. This allows me to calculate the equilibrium probability any student would attend a charter school in markets where charters have not yet entered, or when charter schools

[8] In addition, Sass (2006) and Bifulco and Ladd (2006) examine at the spillover effect on students at nearby public schools. I discuss these papers later in this section.

[9] In some of these papers, authors estimate production functions, which are inherently structural, but do not have structural models of school choice.

are given higher per-pupil capital. Second, it provides a theory for *why* certain students, and not others, attend charter schools. Finally, because students differ by ability and choose schools based on what test score they would receive were they to attend, my framework allows for heterogeneous effects of charter school entry.

The model is structured as a one-period game played in each market and time period. A market is the attendance zone for a public school, which captures the idea that students are typically assigned to attend a public school based on where they live. There are three types of players: students, a public school, and a potential charter school entrant. Each market is endowed with a measure of students. Students differ by ability and their location within the market, which are drawn from independent market-level distributions. Every market is also endowed with a public school and a potential charter school entrant: each has its own exogenous level of per-pupil capital and location within the market.[10]

The game has three stages. At the beginning of the period, the charter school decides whether to enter the market. If the charter school enters the market, the public school and the charter school simultaneously choose effort levels. Otherwise, the public school is a monopolist and chooses the monopoly level of effort. In the last stage, if the charter school has entered, students choose between schools. Otherwise, all students are assigned to the public school.

A key feature of the model is the endogenous school input choices. I start from the literature on the effectiveness of school inputs in test score production[11] and allow schools to have two inputs: one exogenous index of per-pupil resources, "capital," and one endogenous variable representing the assigned school workload, "effort." School inputs and student ability are assumed complements in test score production. Endogenous effort plays a central role in the model because if students differ by ability and can choose schools, schools may choose higher effort levels in order to attract more capable students. Proponents of charter schools argue that such changes in effort levels are an important feature of competition between charter and public schools.[12]

On the supply side, schools' objective functions depend on test scores, school size, and the cost incurred by the school from exerting effort.[13] Charter and public schools differ in their production technologies and the parameterization of their objectives; that is, how they weigh test scores versus school size and the parameters of their effort cost functions. In markets where charter schools have entered, both public and

[10]Neither charter nor public schools are assumed to be restricted in size because capital is a per-pupil measure.
[11]See Hanushek (1986) for an example.
[12]For example, Knowledge Is Power Program (KIPP) schools require parents to sign contracts to do homework with their children (KIPP (2010)).
[13]Different effort costs are incurred by students and both types of schools.

charter schools choose effort from a continuous interval to maximize their objectives given the strategies of the other school and of the students. This chosen effort level is required of all students at a given school.

On the demand side, students differ by ability and location within the market.[14] Their utility is a function of their own test score, the non-pecuniary cost of attending the school they choose, and a preference shock. The cost of attending a school consists of the cost of exerting the effort level *chosen by the school* and a distance cost of commuting to the school.[15] Students of all abilities face the same effort cost per unit of effort, and all students attending a particular school pay the same effort cost; that is, effort is a school-level, not student-level, variable.

I prove existence of a subgame perfect Nash equilibrium of the period game. In the equilibrium of the charter school entry subgame, high-ability students are more likely to attend the school with higher-effort workloads than low-ability students, ceteris paribus. This means that, even though neither charter nor public schools are allowed to explicitly admit the best students, they may be able to induce differences in the ability distributions of their students by choosing higher effort levels. Moreover, because the relative importance schools place on average test scores, school size, and the cost of exerting effort are parameters I estimate, the model can accommodate either a positive or negative spillover effect of the presence of a charter school on the test scores of public school students within the market. This is important because much of the debate about charter school expansion is focused on the spillover effect onto public school students.

I use maximum likelihood to estimate the model using administrative student- and school-level data from North Carolina. The model is estimated on middle school students (grades 6-8) for the years 1998 to 2001, from the first year of charter school entry until the cap was nearly reached.[16] The data are quite rich—they contain the universe of schools and students in the North Carolina public school system (including charter schools) from 1997 to 2005.[17] In particular, they contain variables necessary to fit both school and student outcomes for all schools and students in the North Carolina public education system. School attendance and standardized test scores are recorded for each student in each year. The student-level data also contain student locations, which enter the model through the distance cost of attending a school and exogenously shifts the probability a student will attend a charter school.[18] The school-level panel contains public and

[14] By "ability" I mean the student-specific, time-invariant component of student test scores.

[15] I assume that households with multiple children make school attendance decisions independently for each child. This means I do not allow for correlations in their shocks or scale effects on the distance cost.

[16] I stop using data on years well before the cap was reached to obviate modeling the interactions between different charter schools that would arise when the cap was close to binding.

[17] The 1997 data identify market ability distributions and 2002-2005 data enter the counterfactual where I lift the statewide cap on charter schools.

[18] In Cullen et al. (2005) the authors use distance from schools as an instrument for the probability of choosing a particular school.

charter school locations and detailed per-pupil school resources, which enter the model as a per-pupil capital index.[19]

The results of the estimation are that charter schools have higher productivity parameters than public schools for both the capital and effort inputs, and both types of schools have higher productivities of effort than capital. After estimating the parameters of the model, I first quantify the effect of the presence of charter schools on test scores and then examine two policy-relevant counterfactual scenarios that would increase the role charters play in the public education system. I quantify the effect of the presence of charter schools on test scores (Question 1) by first simulating the model assuming that charters were not allowed to enter any markets and then comparing those results with those of the model under the status quo in which charter schools were allowed to enter markets, over the period 1998-2001. Students attending charters in markets with charter schools have 19% of a standard deviation higher test scores than they would under the no-entry scenario, and students attending public schools in entry markets have slightly higher (1% of a standard deviation) test scores, due to the competitive effect of charter school entry on public schools. The slightly higher test scores suggest that the competitive response to the presence of charter schools slightly outweighs the effect of "cream-skimming."

After quantifying the effect of the presence of charter schools on student test scores, I examine the effect of allowing unrestricted charter school entry on the number of markets with charter schools and student test scores (Question 2). I find that the cap on charter school entry was binding: the average fraction of markets with charter schools increases from 11.1% to 12.4% over the years 2002-2005, which corresponds to an increase from an average of 55 to 62 charter schools over the period.[20] In the long run, the stationary distribution has charter schools in about 60% of markets, which would correspond to about 300 charter schools statewide. Students attending charters in markets simulated to have new charter schools have 20% of a standard deviation higher test scores than under the monopoly scenario. Students attending public schools in those markets have basically the same test scores.

Finally, in response to the policy debate about whether to increase the level of capital funding for charter schools to that of public schools, I ask what the effect of equating per-pupil charter and public school capital would be on the number and size of charter schools and student test scores (Question 3). Under the capital-equalization policy, charter schools enter three times as many markets and have 60% larger enrollment.

[19] Per-pupil capital comprises three variables: number of computers per pupil, teachers per pupil, and number of teachers with 4 or more years of experience per pupil. I define school effort as the average hours of homework done by students at a school.

[20] Due to my sample restriction of only middle schools, this is not the same as the 100-school statewide cap, which applies to all charter schools.

Students attending charters have 23% of a standard deviation higher test scores than students attending charters under the old charter capital levels, and students attending public schools in those markets also have slightly higher test scores.

This paper is directly related to a literature where authors estimate the effect of charter schools on student test scores. Authors in this literature typically adopt one of two strategies of addressing potential non-random attendance of charter schools: 1) compare children who attend against those who are randomized out of over-subscribed charter schools by lotteries, to determine the effect of attending oversubscribed charter schools on students who apply; or 2) estimate student fixed effects using panel data to compare test score growth for students attending charter and public schools in markets with charters. Both strategies pose difficulties when we try to generalize them outside the estimation sample. Hoxby and Rockoff (2004) and Angrist et al. (2010) find that charter school students who attend over-subscribed schools do have larger test score growth than students who were randomized to attend local public schools, but their results do not account for potential input responses by public schools and may not be generalizable to charter schools that are not in such high demand as to be oversubscribed. This paper, on the other hand, explicitly models how public schools may change their behavior in response to charter school entry. In addition, the model explains *why* students would receive higher test scores at charter schools, because it makes explicit the effects of student sorting and school input choices; and uses detailed information on school inputs, as opposed to treating both charter and public schools as black boxes.

Hanushek et al. (2007), Sass (2006), and Bifulco and Ladd (2006) estimate value-added models of test score growth using statewide student panel data for Texas, Florida, and North Carolina, respectively.[21] All three papers find that the charter school dummy variable in a fixed-effect regression of student test score growth on school type has a negative sign, which authors interpret as a negative effect of charter schools on test score growth. Sass (2006) and Bifulco and Ladd (2006) also estimate the competitive effect of charter school entry by examining how school fixed effects change in public schools close to and far from new charter schools. Sass (2006) finds a positive and significant spillover effect on test scores while Bifulco and Ladd (2006) find a positive but insignificant effect. The fixed-effects estimation strategy poses problems for generalizability, however, because it requires several years of data on students in charter schools, two years of which they must attend a public school. In the North Carolina data, the distribution of charter school test scores for students who are only observed in charter schools has a higher mean than that for students who ever attend a charter and at some point attend a public school. This may be in part due to the fact

[21] I use the same dataset used by Bifulco and Ladd (2006).

that switching schools is disruptive and, most likely, non-random. Like the lottery studies, the value-added papers do not explain what causes test score growth at either charter or public schools.[22] Moreover, the value-added papers, like the lottery papers, do not take into account public school responses to charter school entry, so the lack of changes in test score growth may be illusory. In my paper, I explicitly model school choice as a function of ability, which allows me to assess the impact of all charter schools (including those that have not yet entered) on test scores and allows for the impacts of schools to be heterogeneous between students. I also account for equilibrium responses of public schools to charter school entry, which, as I discussed earlier, if not accounted for may understate the effect of charter school entry. The model of endogenous inputs also allows for charter school impacts to be heterogeneous between markets. Finally, I estimate the spillover effect of charter schools on public school students in a manner generalizable to new markets.

This paper is the first to model charter school entry and, as far as I know, the first to build and structurally estimate an equilibrium model of endogenous school inputs and student sorting. It fits into a related literature, where authors build and calibrate or estimate equilibrium models of competition between public and private schools, in which peer effects play a large role in student sorting. Epple and Romano (1998) develop and calibrate an equilibrium model of public and private school interactions to examine the effects of vouchers using aggregate data. Ferreyra (2007) builds on and structurally estimates Nechyba's calibrated general equilibrium model of household sorting, public good provision, private schools, and housing prices[23] in an equilibrium framework. The focus of all these papers is on student sorting due to peer effects, the effectiveness of private school vouchers, and competition between public schools and private schools: none of them addresses competition between public schools, which they assume are monolithic and do not make input choices or any other decisions.[24] Additionally, school quality is a function of the average household income for students attending the school, as their focus is on residential location decisions and/or provision of public goods more than on specific educational outcomes. By contrast, I use student-level data on school attendance and test scores while modeling school inputs in order to see what the effect of different charter school policies would be on policy-relevant outcomes like the test scores of students attending charter and public schools.

[22] Hanushek et al. (2007) regresses the probability of charter school attendance on a school-specific fixed effect and finds that students are more likely to attend charter schools with higher fixed effects but does not say *why* these fixed effects may differ.

[23] Nechyba (2000)

[24] One exception is Chakrabarti (2008), where the author models competition between public schools and private voucher schools, and estimates implications of the model using a difference-and-differences approach.

2 Model

The model is structured as a one-period game played in each market and time period. A market is the attendance zone for a public school, which captures the idea that students are typically assigned to attend a public school based on where they live. There are three types of players: students, a public school, and a potential charter school entrant. Each market is endowed with a measure of students. Students differ by ability and their location within the market, which are drawn from independent market-level distributions. Every market is also endowed with a public school and a potential charter school entrant: each has its own exogenous level of per-pupil capital and location within the market.[25]

The game has three stages. At the beginning of the period, the charter school decides whether to enter the market. If the charter school enters the market, the public school and the charter school simultaneously choose effort levels. Otherwise, the public school is a monopolist and chooses the monopoly level of effort. In the last stage, if the charter school has entered, students choose between schools. Otherwise, all students are assigned to the public school.

On the supply side, schools' objective functions depend on test scores, school size, and the cost incurred by the school from exerting effort.[26] Charter and public schools differ in their production technologies and the parameterization of their objectives; that is, how they weigh test scores versus school size and the parameters of their effort cost functions. In markets where charter schools have entered, both public and charter schools choose effort from a continuous interval to maximize their objectives given the strategies of the other school and of the students. This chosen effort level is required of all students at a given school.

On the demand side, students differ by ability and location within the market.[27] Their utility is a function of their own test score, the non-pecuniary cost of attending the school they choose, and a preference shock. The cost of attending a school consists of the cost of exerting the effort level *chosen by the school* and a distance cost of commuting to the school.[28] Students of all abilities face the same effort cost per unit of effort, and all students attending a particular school pay the same effort cost; that is, effort is a school-level, not student-level, variable.

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schools are allowed to explicitly admit the best students, they may be able to induce differences in the ability distributions of their students by choosing higher effort levels. Moreover, because the relative importance schools place on average test scores, school size, and the cost of exerting effort are parameters I estimate, the model can accommodate either a positive or negative spillover effect of the presence of a charter school on the test scores of public school students within the market. This is important because much of the debate about charter school expansion is focused on the spillover effect onto public school students.

2.1 Notation

A market is the attendance zone for a public school, meaning there is one public school in each market, denoted tps (for "traditional public school"). There is one potential charter school entrant in each market, denoted ch. Schools are indexed by s, and students are indexed by i. The model exposition and analysis will be for schools and students in one market and period. I start with students because they take school actions as given and it is most natural to solve the game backwards.

Variables in bold denote the pair of variables for both schools in a market, for example $\mathbf{k} = (k_{ch}, k_{tps})$ is the vector of school capital levels for the market. Also, $\tilde{\cdot}$ denotes the natural logarithm of a variable.

2.2 Students

There is a continuum of students of measure μ in the market. A student $i \in I$ has ability a_i, where $a_i \sim F(a_i)$ with density $f(a_i)$. Students have only one decision each period – they choose a school $s \in S_i$, their school choice set. If there is a charter school in student i's market then $S_i = \{tps, ch\}$, otherwise $S_i = \{tps\}$. Denote student i's school choice $s_i = 1$ if i chooses to attend the charter school.

Students care about the test score they would receive at a school s, y_{is}, and the non-pecuniary cost of attending a school c_{is}.[29] They also receive a choice-specific preference shock $\eta_{is} \sim \mathrm{N}\left(0, \sigma_\eta^2\right)$. Student i's choice-specific utility takes the form

$$u_{is} = y_{is} - c_{is} + \eta_{is}. \tag{1}$$

The test score y_{is} is a function of ability, school inputs e_s^o and k_s, and a productivity shock ν_{is}^y. The student takes school inputs — realized school-wide effort, e_s^o, and per-pupil capital, k_s — as given.[30]

$$y_{is} = a_i (e_s^o)^{\beta_{es}} k_s^{\beta_{ks}} + \nu_{is}^y, \tag{2}$$

[29] Neither traditional public schools nor charter schools may charge admission.

[30] I distinguish between "realized" or "observed" effort, e_s^o, and that chosen by the school, e_s, because there is a productivity shock on the chosen level of school effort. I show how this enters in the next subsection.

where $\nu^y_{is} \sim i.i.d.\, \mathrm{N}\left(0, \sigma^2_{\nu^y}\right)$.

Student i's cost of attending school s is composed of the realized school effort, e^o_s, distance from the student to the school, r_{is}, and a fixed cost of attending the charter school, c_{ch}, according to

$$c_{is} = c_e e^o_s + c_r r_{is} + c_{ch} \mathbf{1}_{\{s=ch\}}. \tag{3}$$

None of the parameters c_e, c_r, or c_{ch} are assumed to be "costs" in the estimation of the model, as their signs are unrestricted. A student knows its own ability and the preference shocks, productivity shocks, capital, realized effort, and distance for the two schools before making its school choice

$$\gamma_i = \arg\max_{s \in S_i} \{u_{is}\}. \tag{4}$$

Note that ability enters the model only through the test score, which implies that the student utility function satisfies the single-crossing property in ability: given the same set of shocks and distances to the two schools, a high-ability student has higher utility than a low-ability student from attending a school with high effort.

2.3 Schools

Because each market is endowed with a public school, public schools have no entry decision. Denote the potential charter school entrant's entry decision $z \in Z = \{0, 1\}$. Each school is endowed with a capital level and location within the market, and the potential charter school entrant knows both schools' capital levels and locations before making its entry decision.

If the charter school has entered the market, both types of schools make an effort decision, e_s. The school's objective is a weighted average of four elements: average test score of students at the school ($\bar{y}_s$), total test score of students at the school ($\bar{y}_s \mu_s$), school size (μ_s), and the cost of exerting the realized (i.e. observed) level of effort (c_{es}[31]):

$$v_s(e_s, e_{-s} | \nu^{\mathbf{e}}) = \delta_{ys1} \bar{y}_s + \left(\delta_{ys2} \bar{y}_s + 1\right) \mu_s - c_{es}, \tag{5}$$

where ν^e_s is an independently distributed effort productivity shock that is realized after the school chooses

[31] I suppress the dependence of the cost function on school effort choices and the effort productivity shocks.

its effort level, and which determines observed effort according to

$$e_s^o = e_s \nu_s^e,$$

where $\widetilde{\nu}_s^e \sim \mathrm{N}\left(0, \sigma_{\nu^e}^2\right)$. Although I suppress dependence on the last of these, the pair of observed school efforts, $\mathbf{e^o}$, pair of effort productivity shocks, $\nu^{\mathbf{e}}$, and capital levels, $\mathbf{k}$, enter each school's objective because students take both pairs into account when choosing schools – which in turn enters the school's average test score and size.[32] The school's cost of exerting effort is a convex function of expended effort that allows for interactions between effort and both capital and school size

$$c_{es} = \psi_{s1} e_s^o + (e_s^o)^{\psi_{s2}} + \psi_{s3} e_s^o \mu_s + \psi_{s4} e_s^o \mu_s^2 + \psi_{s5} e_s^o k_s + \psi_{s6} e_s^o k_s^2. \tag{6}$$

School size, μ_s, enters the school objective with a coefficient of 1, which means that the remaining school parameters are enumerated in terms of the school's valuation of size. The school's incentives may be thought of from the perspective of the school principal – school size enters the principal's objective because if no students attend the school she may be fired. The schools' direct preferences for average test score, δ_{ys1}, and total test score, δ_{ys2}, capture the idea that, in addition to caring about the size of their school, principals want the students to do well.[33] If the average test score did not directly enter the school's objective function ($\delta_{ys1} = \delta_{ys2} = 0$), then the model would predict that monopoly public schools would exert no effort if the cost of effort is positive, because they would draw all students in their market and could avoid paying any effort exertion cost by doing so.

The charter school enters if and only if the charter is in expectation viable, that is, if

$$\mathrm{E}_{\nu^{\mathbf{e}}} \left[v_{ch}^* | \nu^{\mathbf{e}} \right] \geq \underline{v}, \tag{7}$$

where v_{ch}^* is the value of entry for the charter school in the entry subgame equilibrium, given a chosen effort pair (please see Section 2.4 for equilibrium definition and derivation) and $\underline{v}$ is a random variable, known to the charter school, which denotes an exogenous fixed cost of entry and operating.[34]

The monopoly objective does not allow for students to choose the charter school, so the public school

[32] I also suppress the dependence of average test score and school size on $(\mathbf{e^o}, \mathbf{k})$ for notional ease.

[33] There are some institutional features this captures as well, such as No Child Left Behind, which punishes schools for failing to meet certain levels of test score growth.

[34] This is actually drawn per market and time period $\underline{v}_{tm}$, but I am suppressing the t and m subscripts to simplify exposition.

serves all students in the market

$$v_{tps}(e_{tps}|\nu^e_{tps})^{mono} = \delta_{y,tps,1}\bar{y}_{tps} + \left(\delta_{y,tps,2}\bar{y}_{tps} + 1\right)\mu - c_{e,tps}. \tag{8}$$

Note that because students in monopoly markets have no school choice, the average ability for students attending the monopoly public school is the market average, $\bar{a}$, and the measure of students attending is μ, the market size.

If there is a charter school in the market, each school solves[35]

$$\gamma_s = \arg\max_{e_s} \; \mathrm{E}_{\nu^{\mathbf{e}}}\left[v_s(e_s, e_{-s}|\nu^{\mathbf{e}})\right], \tag{9}$$

which says that each school chooses its own effort, e_s, to maximize its expected objective, given the action of the other school, e_{-s} and the pair of effort productivity shocks, $\nu^{\mathbf{e}}$. If there is only a public school, the school solves

$$\gamma^{mono}_{tps} = \arg\max_{e_{tps}} \; \mathrm{E}_{\nu^e_{tps}}\left[v^{mono}_{tps}(e_{tps}|\nu^e_{tps})\right]. \tag{10}$$

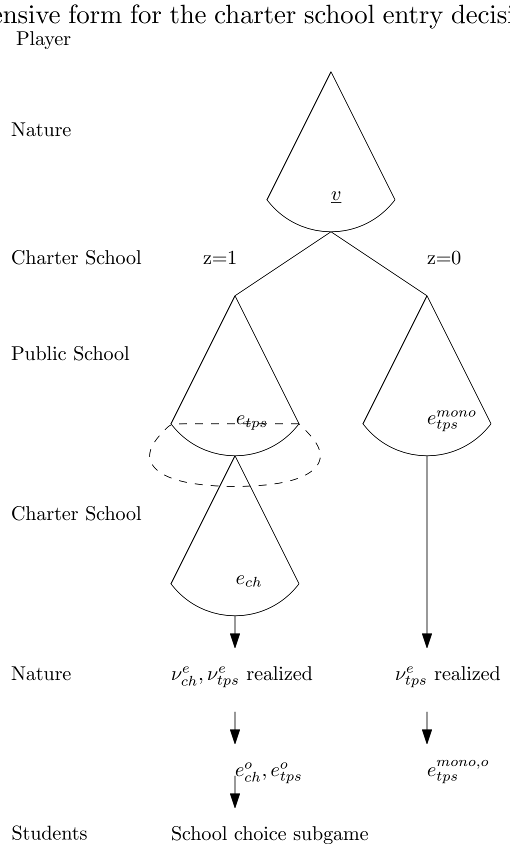
2.4 Equilibrium

2.4.1 Equilibrium Characterization

In this section I characterize the charter school entry decision and show how I solve for the equilibrium of the charter school entry subgame. Figure 1 shows the timing of school decisions. Schools take into account the market size, ability and distance distributions in a market in a time period, as well as capital levels and per-pupil revenues at both schools when choosing their own effort levels. A student knows his own ability and distance from either school, and sees effort, capital, and preference and productivity shocks for both schools, and makes a school attendance decision. If there is no charter school, all students choose the public school, which chooses the monopoly level of effort. The solution concept is subgame perfect Nash equilibrium, where the potential charter school entrant makes an entry decision in the first stage based on the expected payoff in the entry subgame. The Law of Large Numbers implies that the measure of students attending each school and each school's average test score are known exactly, given the productivity shocks $\nu^{\mathbf{e}}$.

[35] In the estimation, I solve a slightly modified version of the school's problem where I do not integrate over the distribution of effort productivity shocks in solving for effort and entry decisions to ease the computational burden. I have verified that the solutions are similar with and without this integration.

Figure 1: Extensive form for the charter school entry decision and subsequent school effort choice subgame.



$\underline{v}$	entry cost shock
z	charter entry ($z = 1$ is entry)
e_{ch}	charter effort
e_{tps}	public effort
e_{tps}^{mono}	public monopoly effort
ν_{ch}^e	charter effort prod. shock
ν_{tps}^e	public effort prod. shock

Let $\mathcal{M}$ denote the set of measures of students, $\mathcal{F}_a$ denote the set of ability distributions, $\mathcal{R}$ denote the set of student distance distributions, K be the set of school capital levels, Δ_μ denote the set of per-pupil revenues, E the set of effort levels chosen by schools, A denote the set of abilities, R the set of student distances, E^o denote the set of observed effort levels for schools, NU be the set of productivity shocks, ETA be the set of preference shocks, and S be the set of schools.

The strategies of students and schools in the entry subgame are

$$\begin{aligned} &\text{Schools:} && \gamma_s : \mathcal{M} \times \mathcal{F}_a \times \mathcal{R}^2 \times (K)^2 \times \Delta_\mu^2 \times E \mapsto E \\ &\text{Students:} && \gamma_i : A \times R^2 \times K^2 \times (E^o)^2 \times NU^2 \times ETA^2 \mapsto S \end{aligned}$$

Definition 1. *An Entry Subgame Equilibrium is a vector of student choices s_i^* and school effort levels $\mathbf{e}^*$ such that*

1. $s_i^* = \gamma_i\left(a_i, \mathbf{r_i}, \mathbf{k}, (\mathbf{e^o})^*, \nu_\mathbf{i}, \eta_\mathbf{i}\right)$ *maximizes student utility for students $i \in I$ and*

2. $e_s^* = \gamma_s\left(\mu_t, F_a, \mathbf{R}, \mathbf{k}, \boldsymbol{\Delta}_\mu, e_{-s}^*\right)$ *is the best response for schools $s \in \{tps, ch\}$.*

Proposition 1. *Existence of Entry Subgame Equilibrium*

Proof. Please see Appendix A. □

Definition 2. *A Monopoly Subgame Equilibrium is a vector of student choices s_i^{*mono} and public school effort level e_{tps}^{*mono} such that*

1. $s_i^{*mono} = 0$ *for students $i \in I$ and*

2. $e_{tps}^{*mono} = \gamma_{tps}^{mono}$

Definition 3. *A subgame perfect Nash equilibrium for the charter school entry game is an entry decision z^* and student and school decisions $(s_i^*, \mathbf{e}^*, s_i^{*mono}, e_{tps}^{*mono})$ such that*

1. *Given an entry cost shock $\underline{v}$ and entry subgame decisions $(s_i^*, \mathbf{e}^*)$, $z^* = 1$ if $\mathrm{E}_{\nu^\mathbf{e}}\left[v_{ch}^* | \nu^\mathbf{e}\right] \geq \underline{v}$ and 0 otherwise,*

2. *for $z^* = 1$, the subgame equilibrium is an Entry Subgame Equilibrium, and*

3. *for $z^* = 0$, the subgame equilibrium is a Monopoly Subgame Equilibrium.*

I solve the model by first computing the market equilibrium for the entry subgame and then comparing the equilibrium value to the charter school with its entry cost shock to determine the entry decision. In the subgame, I first solve for student demand as a function of school effort choices, given capital levels, ability, and distances both schools. Note that, because I need to know how far a student is from both schools to compute student school choice probabilities, I solve the model, given school effort choices, for students of each distance bin and then compute school size and average abilities at each school by summing over all bins in the market using to the market distance distribution. I then plug the school size and average ability at each school into the school objectives (they enter through both the measure of students attending each school μ_s and the average test scores at each school $\bar{y}_s$) and solve for the Nash equilibrium in school effort, given charter school entry.

A student with ability a_i knows realized effort levels, capital, and distances for both schools and all other shocks and chooses a charter school if and only if

$$
\begin{aligned}
y_{i,ch} - c_{i,ch} + \eta_{i,ch} &\geq y_{i,tps} - c_{i,tps} + \eta_{i,tps} \\
a_i (e^o_{ch})^{\beta_{e,ch}} k_{ch}^{\beta_{k,ch}} - c_e e^o_{ch} - c_r r_{i,ch} - c_{ch} + \underbrace{\nu^y_{i,ch} + \eta_{i,ch}}_{\epsilon_{i,ch}} &\geq a_i (e^o_{tps})^{\beta_{e,tps}} k_{tps}^{\beta_{k,tps}} - c_e e_{tps} - c_r r_{i,tps} + \underbrace{\nu^y_{i,tps} + \eta_{i,tps}}_{\epsilon_{i,tps}} \\
a_i \underbrace{\left((e^o_{ch})^{\beta_{e,ch}} k_{ch}^{\beta_{k,ch}} - (e^o_{tps})^{\beta_{e,tps}} k_{tps}^{\beta_{k,tps}} \right)}_{\Delta(x\beta)} + \underbrace{(\epsilon_{i,ch} - \epsilon_{i,tps})}_{\Delta\epsilon_i} &\geq \underbrace{c_e (e^o_{ch} - e^o_{tps})}_{\Delta c_e} + \underbrace{c_r \left(r_{i,ch} - r_{i,tps} \right)}_{\Delta c_{ri}} + \underbrace{c_{ch}}_{\Delta c_{ch}} \\
a_i \Delta(x\beta) + \Delta\epsilon_i &\geq \underbrace{\Delta c_e + \Delta c_{ri} + \Delta c_{ch}}_{\Delta c_i}, \qquad (11)
\end{aligned}
$$

where $\Delta\epsilon_i \sim \mathrm{N}\left(0, \sigma^2_{\Delta\epsilon}\right)$ and $\sigma^2_{\Delta\epsilon} = \sigma^2_{\nu^y_{ch}} + \sigma^2_{\nu^y_{tps}} + 2\sigma^2_\eta$. Equation (11) says that if the charter school has higher net effective school inputs, $\Delta(x\beta) > 0$, higher ability students are more likely to attend it than low ability students, because students of all abilities pay the same non-pecuniary cost of attending either school.

Ability is assumed to be distributed normally within the market according to $\mathrm{N}\left(\mu_a, \sigma^2_a\right)$, so the left hand side of (11) is the sum of two independent normals which is distributed according to

$$
a_i \Delta(x\beta) + \Delta\epsilon_i \sim \mathrm{N}\left(\mu_a \Delta(x\beta), \sigma^2_a \Delta(x\beta)^2 + \sigma^2_{\Delta\epsilon}\right). \qquad (12)
$$

This provides a simple expression for the measure of students attending the charter school, given a distance difference Δc_{ri}, $\mu_{r,ch}$:

$$
\mu_{r,ch}(\Delta c_{ri}) = 1 - \Phi\left(\frac{\Delta c_e + \Delta c_{ri} + \Delta c_{ch} - \mu_a \Delta(x\beta)}{\sqrt{\sigma^2_a \Delta(x\beta)^2 + \sigma^2_{\Delta\epsilon}}} \right), \qquad (13)
$$

where Φ denotes the standard cumulative normal distribution.

There are $\rho \in 1, \ldots, R$ separate distance pairs in the market, each with a measure μ_ρ. Therefore, the total measure of students at the charter school is the sum of the measures of students of each distance, weighted by the measure of each distance level μ_ρ

$$\mu_{ch} = \sum_{\rho=1}^{R} \mu_\rho \mu_{\rho,ch}(\Delta c_{\rho i}). \tag{14}$$

A student with ability a_i and relative charter distance cost Δc_{ri} will choose the charter if and only if

$$\Delta \epsilon_i \geq \Delta c_i - a_i \Delta(x_t \beta),$$

which happens with probability $\Phi\left(\frac{a_i \Delta(x_t\beta) - \Delta c_i}{\sigma_{\Delta\epsilon}}\right)$. By Bayes' Rule, the average ability of student attending the charter school is

$$\bar{a}_{r,ch}(\Delta c_{ri}) = \int_{a_i} a_i f_r(a_i | s_i = 1) da_i, \tag{15}$$

where

$$f_r(a_i | s_i = 1) = \frac{\Phi\left(\frac{a_i \Delta(x_t\beta) - \Delta c_i}{\sigma_{\Delta\epsilon}}\right) f(a_i)}{\mu_{r,ch}(\Delta c_{ri})} \tag{16}$$

is the density of the ability of students at the charter school, which takes into account their probability of selecting it. As with the measure of students attending the charter, the average ability of students attending the charter school is the weighted average of the average abilities of students attending the charter school from each bin:

$$\bar{a}_{ch} = \sum_{\rho=1}^{R} \mu_\rho \bar{a}_{r,ch}(\Delta c_{ri}). \tag{17}$$

I solve for μ_{tps} and $\bar{a}_{tps}$ analogously. After solving for $\bar{a}_s$, the average test score at school s is

$$\bar{y}_s = \bar{a}_s (e_s^o)^{\beta_{es}} k_s^{\beta_{ks}},$$

which, along with μ_s, is plugged into the school objectives when I solve for the optimal effort level.

I cannot obtain an analytical expression for the equilibrium because I am integrating over the probability of charter school attendance for each student in (15).[36] I numerically solve for the entry subgame equilibrium

[36]Even if I characterize the equilibrium with school first-order conditions I still need to integrate over probabilities computed from the normal CDF because the average test score interacts with the measure of students at each school.

by iterating the best response functions for charter and public schools.[37] The assumption that there is only one potential charter school per market per time period eliminates multiple equilibria where more than one charter may open in a market in a period. From the positive perspective, I do not believe multiple equilibria help explain variation in the data because the story is about public and charter school competition and the determinants of entry, not competition between charter schools.[38] I do not have a proof that the equilibrium in the entry subgame is unique but do not believe it poses a problem. Please see Appendix B for a discussion of uniqueness.

2.4.2 Properties of the Equilibrium

Students make decisions stochastically from the schools' perspectives, but students of the same ability and pair of distances from the schools have the same *probability* of choosing the charter school. Figure 2.4.2 demonstrates the two benefits a school receives from increasing its own effort levels, given the other school's effort: there is a direct effect from the increase in the average test score from higher effort and an equilibrium effect of higher average ability at the school. The latter effect follows from the single-crossing property in ability in the student utility function. As charter school effort increases from very low values, the gain from higher average ability outweighs diminishing marginal returns on effort in the test score production function. At higher values of effort, however, diminishing returns to additional increases in effort outweigh additional gains in mean ability because most of the high ability students already attend the charter school. Therefore, at higher charter school effort levels, the average test score at the charter school exhibits decreasing returns in effort, given the effort level of the public school.

The entry subgame equilibrium allows for a negative or positive effect of charter school entry on student test scores. If the charter school is much more productive per unit of effort exerted than the public school, high ability students will attend it with very high probability. Depending on the weights public schools place on test scores and effort exertion, relative to school size, a public school may not find it advantageous to try and retain these students but rather may decrease its equilibrium effort from the monopoly level and cater to below-average-ability students.

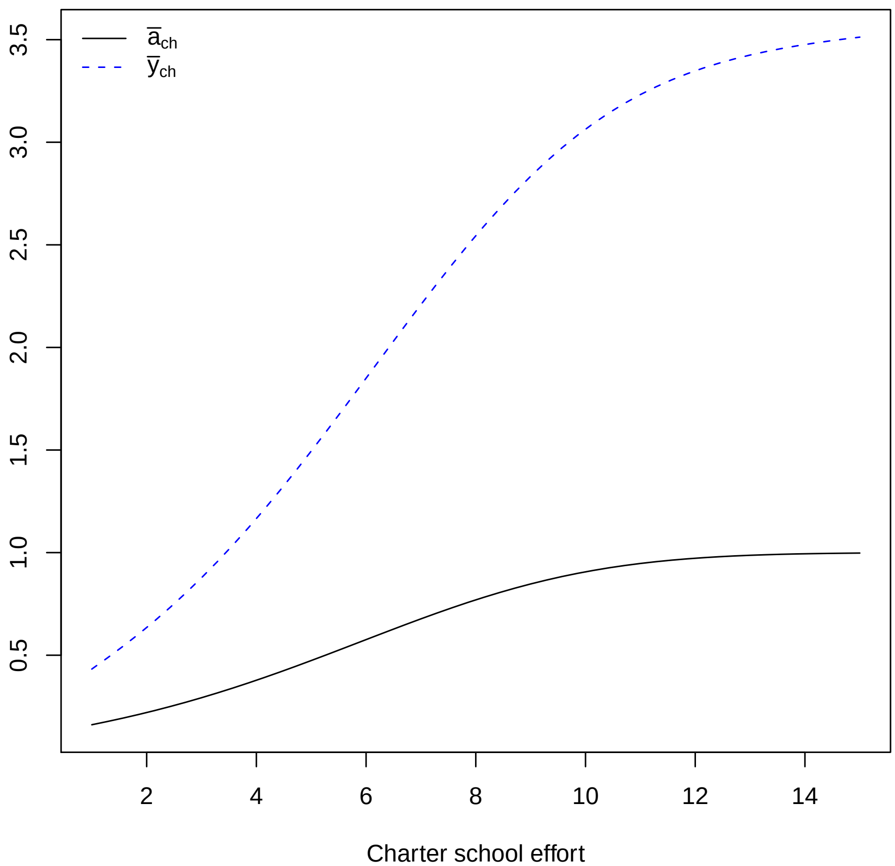
2.5 Discussion of Modeling Assumptions

In this section I discuss some of the modeling decisions I made in the model regarding the model's primitives, test score production, and school objective functions.

[37] Solving the system of two first-order conditions (one for each school) gives the same answer.

[38] Most middle schools that compete with at least one charter school compete with exactly one charter school.

Figure 2: Average ability and test score at charter increase with charter school effort



2.5.1 Primitives

The model is structured as a static model in one period, which precludes public schools from investing in capital to deter charter school entry. This assumption is not as restrictive as it may seem because individual schools have little control over per-pupil funding, which is determined by property tax rates at the district level in North Carolina.[39]

For tractability of both the student and school problems, I only allow one charter school entrant per market. This assumption is supported by observed competition patterns in the data. Please see Section 3.1 for details and discussion of how I construct markets.

2.5.2 Test Score Production

The test score production function captures the idea that unobservable student ability may play an important role in test score production, and allows public and charter schools to differ in the productivity of their inputs. When combined with the student school decision, it allows students to sort on their own ability, given school inputs. This is a prime inferential problem typically addressed in the literature on the effectiveness of school inputs on student achievement. By explicitly modeling student decisions, I address this problem with a parametric selection model where students choose schools based on their own ability, school inputs, and productivity and preference shocks.

I adopt a non-conventional specification for the test score production function (2) in order to allow for an interaction between individual ability and school inputs while maintaining the single-crossing property for ability and tractability of the model.[40] The test score production function also implies that only the ability of a student, current inputs, and the current productivity shock (not the previous test score) determine the test score. This assumption not only makes the student problem easier to solve[41] but, more importantly, makes the school's problem tractable, as otherwise I would have to record the entire distribution of previous test scores at the school and take it into account when considering the current effort choice.[42]

[39] If schools were able to invest in building a reputation over time, and this entered household demand, there may still be a role for preemptive investments by public schools. This is beyond the scope of this paper, but is possibly interesting for future research.

[40] With a Cobb-Douglas specification with multiplicative productivity shocks, a student with ability a_i attends the charter school iff

$$a_i(e^o_{ch})^{\beta_{e,ch}} k_{ch}^{\beta_{k,ch}} \epsilon_{i,ch} - c_{i,ch} \geq a_i(e^o_{tps})^{\beta_{e,tps}} k_{tps}^{\beta_{k,tps}} \epsilon_{i,tps} - c_{i,tps}.$$

The costs enter additively to allow for sorting on ability. Because the costs enter additively, I can't separate the productivity shocks from ability and inputs, which means that I would have to calculate a double integral for the shocks in order to obtain the probability of that event for a student with ability a_i. This would be my student demand function, which then needs to be integrated over the ability distribution for every evaluation of each school's objective.

[41] I do not have to record the previous test score in the student's state.

[42] Although forward-looking behavior may be more realistic, estimates of the ability distribution and capital and effort effectiveness from test score production functions with student fixed effects do not qualitatively change when lagged test scores

Both capital and effort inputs are assumed to be the same for all students at the same school. The assumption is innocuous for capital, because most school capital inputs are applied fairly evenly to students at the schools,[43] and even if they were, I do not have data that would allow me to distinguish otherwise. By contrast, I observe effort choices for individual students. Assuming that there is only one effort level per school per year, in addition to being typical in this literature, allows me to avoid solving for each student's effort choice. In doing so, I lose information on the variation on effort at a school that my data allow me to pick up, which means that I may end up overestimating the variance of the ability distributions and test score production shocks at schools.

2.5.3 School Objective Functions

The effort productivity shock, ν_s^e, makes the model estimable. Its interpretation is that the school chooses a school-wide effort level but, being a reasonably large entity with several teachers, etc., the realized effort level may differ slightly from that chosen by the school in the beginning of the period.

3 Data

I use administrative data on school- and student-level panels for North Carolina to estimate the model. The data are taken from the universe of all North Carolina public and charter schools, and were provided by the North Carolina Education Research Data Center (NCERDC),[44] which collects and processes data on the North Carolina public school system from the North Carolina Department of Public Education and National Center for Education Statistics. The NCERDC data contain variables vital to my strategy of estimating student-level test score production functions based on school-level inputs, and include detailed panels on teachers, students, and schools in the North Carolina public school system – including data on charter schools. For teachers, the data contain years of experience and the school in which they work. For students, the data contain demographic characteristics, which school they attend, grade in school, standardized reading and math test scores for students in grades 3-8 and grade 10, self-reported weekly hours of homework done, and student household locations. School-level data contain variables that can be used to compute computers per pupil and district per-pupil revenues.[45]

are included.

[43]Special education and gifted and talented student programs are notable exceptions. Charter schools tend to have much smaller fractions of both types of students.

[44]The NCERDC's website is `http://www.childandfamilypolicy.duke.edu/project_detail.php?id=35`

[45]Charter schools are considered to be their own school districts in North Carolina.

3.1 Definition of Markets

In the model, markets partition the state of North Carolina. Each student lives in a market, which is defined as the attendance zone for a public school. The schools within a student's market constitute a student's choice set. I need to know each student's choice set to estimate the model, but I do not observe the public school a charter school student would have gone to or which charter school(s) the student chose among, because North Carolina charter schools do not have geographic cut-offs for attendance.[46] I address this problem by assigning each charter school the public school closest in distance as its competitor – that is, the charter school is in that public school's market.[47] This ensures that every charter school is competing with exactly one public school.

In theory, a middle school could be competing with several charter schools, each of which in turn could be competing with several middle schools. The former (one middle school competing with several charters) would mean that a student's choice set could include more than two schools and would fundamentally change how I solve for student decisions in the model.[48] In the data, four middle schools are the closest public school for more than one charter, which would mean that students at these middle schools are choosing from more than two schools (the middle school and more than one charter school). In these cases, I choose the charter school that is closer to the public school as the charter school competitor and do not use the other charter school (i.e. it is dropped from the analysis).

It is tempting, but not appropriate, to create markets based on observed competition patterns. For example, say that a charter school has many students in attendance who transferred from two nearby public schools. In this case, it would seem natural to combine the two public schools in one market. This method poses a problem in counterfactual scenarios, however, since it does not tell me how to combine public schools in areas where there are no charter schools, which is necessary in order to compute charter school entry probabilities for all markets. I chose the above method, which is based on geographical restrictions, because it provides a consistent definition of markets.

[46] I could use elementary school attendance to see which middle schools compete with charters by taking elementary schools which send students to charters, and seeing which public middle schools those elementary schools also send students to. However, more than half of charter school students in my data are never in public schools, so it is difficult to say which public middle school they would have gone to based on their elementary school attendance.

[47] I found the neighbors using the STATA module GEONEAR (Picard (2010)). The module gives the closest geodesic distance school for each charter school. Five charter schools were missing latitude and longitude, so I looked them up manually using their addresses from the NCES (NCES (n.d.)) and Google Earth (Google (2010)).

[48] There is a simple expression for the probability that a student of a certain ability will choose the charter school when there are two schools in the student's choice set because productivity shocks are assumed to be normally distributed and the normal distribution is closed under subtraction. I could use a multinomial logit, but the errors would no longer be conjugate with the productivity shocks.

3.2 Estimation Sample

The NCERDC data contain information on elementary, middle, and high schools for the years 1995-2006. I restrict my analysis to middle schools, which I take to include grades 6-8. I do this for three reasons: 1) the three school types may have different test score production functions, 2) I only observe standardized test scores for the 10th grade for high school students and 3) middle school provides a natural decision-point for students because most students switch schools between grades 5 and 6. I restrict my analysis to the years 1998-2001 because 1998 is the first year charter schools were allowed in North Carolina, and the 100-school cap on charter schools was clearly not binding in 2001 (it comes close to binding in 2002 and 2003).[49] By using years well before the 100-school cap started binding, I avoid having to model the interdependence of charter school entry decisions that would be induced by the cap.

The NCERDC data contain 1,128,935 observations (an observation is per student and year) for students in public schools and 10,165 observations for students in charter schools in grades 6-8 during the years 1998-2001. I exclude markets where public schools (and the associated charter schools) open or close during the observation period (leaving me with 1,007,917 and 7,594 public and charter school students, respectively). I also exclude students who are observed attending public schools outside their market, as I define it, because they are choosing schools outside their choice sets (leaving me with 1,005,966 public and 5,574 charter school students[50]). I further exclude students observed for only one year, or observed attending more than 2 schools in the 3 years of middle school because the latter students are moving, which is also not part of the student's choice set (leaving me with 956,509 public school and 5,073 charter school students). Finally, I exclude students missing standardized test scores, leaving me with a final estimation sample of 912,748 observations of public school students and 4,941 observations for students in charter schools.

As noted above, I restrict the sample to markets that exist throughout the estimation period 1998-2001.[51] This restricts the initial sample of 2,703 public school observations and 126 charter school observations with at least one grade in the grades 6-8 to 2,366 public school observations and 108 charter school observations, after removing charter schools associated with new or dead markets. Finally, I removed public schools that had no children attending, after removing children from the sample as detailed above in the student sample restrictions. The final estimation sample contains 496 public school markets per year over the period 1998-2001.

[49] This is only for the likelihood for school and student outcomes. As I discuss in Section 4.2.1, I also use data for the year 1997 to identify market ability distributions.

[50] Students who attend charter schools outside their markets have on average lower test scores and are more likely to be black.

[51] A small number of public schools either open or close during the observation period.

3.3 Test Scores, Capital, and Effort

The NCERDC data contain standardized reading and math test scores for grades 6-8.[52] The test score used in the model is the average of the reading and math test scores, which is then normalized to have mean 0 and standard deviation 1 by grade so they are comparable across grades.[53]

The school inputs used for the per-pupil capital index are computers per pupil, teachers per pupil, and fraction of teachers at a school with high experience (4 or more years). I treat experience in this way based on Rivkin et al. (2005), which argues that after the first three years of experience, later years of teacher experience have little, if any, effect on student achievement. I combine these into a *per-pupil* index.[54] That capital is a per-pupil measure in the model implies that there are no scale effects with respect to capital for either charter or public schools.[55] A school makes its decisions knowing the per-pupil level of capital it would receive, and the same level of per-pupil capital is applied to all students in the school. The measure of capital used in estimation is not the simple average of the three measures, but rather a predicted value based on per-pupil revenue and school type (please see Appendix C for details). I treat per-pupil capital in this manner because I need to know what it would have been for charter schools in markets charters have not entered, where computers, teachers, and experienced teachers per pupil are not observed. By using the predicted per-pupil capital levels as inputs for both types of schools, I treat per-pupil capital for public and charter schools similarly.

I measure effort with self-reported data on the hours of homework students say they typically do per week. I then average these data within each school, to create a school-wide effort variable per year. Recall that effort in my model is a school-wide, not individual, choice. This is meant to capture broad differences in workloads between charter schools and public schools. As with capital, effort is also assumed to be a per-pupil measure applied evenly to all children in the school.

3.4 Distance Between Students and Schools

The distance between a student and each school in his choice set plays an important role in identification of the test score production function parameters because it shifts the probability a student will attend the charter school without directly affecting the test score (unlike student ability). Moreover, each school takes

[52] The tests are vertically scaled, which means that students in the 7th grade have average test scores that are higher than the average of those in the 6rd grade and lower than the average for those in the 8th grade.

[53] The estimation sample has a mean test score of 0.05, since some observations are lost while making sample restrictions. The standard deviation of the test score in the estimation sample is 0.95.

[54] The details of how I construct the index are in Appendix C.

[55] State funding is also per-pupil in North Carolina.

into account the distance distribution (that is, the fraction of students that are r_{ch} km away from the charter school and r_{tps} km from the public school) of students in the market when choosing its effort level, and the charter school must take it into account when making its entry decision. The NCERDC data contain both student and school latitude and longitude, which I convert to compute geodesic distances using the Stata module VICENTY (Nichols (2003).)

Because I need a distance distribution for every market, including markets where I never observe charter school entry and therefore lack any data on student distances from charter schools, I discretize the distance distribution for each market and model the relationship between the distance distribution for public schools and charter schools, assuming that public school, charter schools, and students are all endowed with locations within markets, and that all locations are observed by both types of schools and students (the details of the discretization are in Appendix D.) This market distance distribution, in addition to closing the model from the school's perspective, also allows me to deal with missing student distances, which is of particular importance for students who only ever attend charter schools because charter schools typically do not report locations for their students.[56] If I observe a student's location at least once and it is missing in another year I assume the student did not move and assign him the previous location, so long as he attends a school in the same city as before.

3.5 Facts about Charter Schools in North Carolina

Charter schools in North Carolina are much smaller than public schools: they on average comprise 7% of a market (the rest of the students attend the public school). Charter schools also have about three-quarters of the per-pupil capital levels of public schools (0.41 versus 0.54). The following facts are pertinent to my questions, as they show patterns for charter school entry, endogenous school effort, and student outcomes.

1. Charter schools enter larger markets and markets in which where they would have more resources (Table 1).

2. The amount of time spent doing homework is higher in markets in charters and increases when a charter school enters a market (Tables 2 and 3).

3. Student choices suggest sorting on ability. In the year before a charter school enters a market, students who attend charters in the following year have 5% of a standard deviation higher test scores than those who do not attend the charter in the following year.

[56] Telephone conversations with NCDPI indicate that charter schools often fill in less paperwork because they are understaffed relative to public schools.

4. Students in charter schools have the highest test scores, followed by students in public schools in markets charters have entered, followed by students in public schools in markets without charters (Table 4).

Table 1: Charter school entry probabilities by market characteristics

	Fraction of Markets with Charter Schools
Total	0.07
Per-pupil resources	
Below median	0.02
Above median	0.12
Market size	
Below median	0.06
Above median	0.09

Table 2: Average hours of homework done in a week

	No Charter in Market	Charter in Market
Public School	2.39* hours	2.69 hours
Charter School	–	2.60 hours

* different from public schools and charter schools in entry markets (p-value < 0.001)

Table 3: Hours of homework done in a week for public schools in markets with charter school in market

Before Charter Entered Market	After Charter Entered Market
2.58 hours	2.68 hours

4 Estimation

4.1 Likelihood

The one-period game is played in every market $m \in 1, \ldots, M$ and time period $t \in 1, \ldots, T$. In every time period t in a market m, there is a new measure of students with abilities and locations within the markets and a new public school and potential charter school entrant with exogenous per-pupil capital levels

Table 4: Average test scores of students by school type

	No Charter in Market	Charter in Market
Public School	0.03*	0.06*
Charter School	–	0.13*

* all means significantly different (p-value < 0.0001)

and locations within the market. The only links between two periods in the same market are the ability distribution and whether a charter entered in the previous period in the market, which affects the entry cost shock distribution.

The likelihood includes probability (or likelihood) statements for charter school entry decisions, school effort levels, student school choices, and student test scores. Entry cost shocks $\underline{v}_{tm}$, effort productivity shocks ν^e_{stm}, test score productivity shocks ν^y_{istm}, and preference shocks η_{istm} are all assumed to be distributed independently.

4.1.1 Timing of the game:

1. Potential charter school entrant observes capital levels[57] for both schools $\mathbf{k_{tm}}$,[58] the market ability distribution $F_m(a_i)$, and its entry cost $\underline{v}_{tm}$.
    - Potential charter school entrant makes entry decision z_{tm} based on expected payoff to entry $\mathrm{E}_{\nu^{\mathbf{e}}}\left[v^*_{ch,tm}\right]$ and entry cost $\underline{v}_{tm}$.
2. Effort productivity shocks $\nu^{\mathbf{e}}_{\mathbf{tm}}$ are realized.
    - $\mathbf{k_{tm}}, \mathbf{e^o_{tm}}$ are now observable to students.
3. Students receive test score productivity shocks $\nu_{\mathbf{itm}}$ and preference shocks $\eta_{\mathbf{itm}}$.
4. Students choose schools s_{itm}.
    - Test scores y^o_{istm} are assigned to students.

4.1.2 School Contribution to Likelihood

Charter school entry in markets with no charter:

[57]Capital levels k_{stm} are not estimated within the model, but calculated according to the functions $\phi^k_s\delta^\mu_{tps,t,m}$, where ϕ^k_s are given outside the model. See Appendix C for details.

[58]Variables in bold font represent the vector of that variable for the two schools in a district.

The potential charter school entrant in each market compares the expected value of entry $\mathrm{E}_{\nu^{\mathbf{e}}}\left[v^*_{ch,tm}\right]$, which is a function of both school capital levels $\mathbf{k_{tm}}$ and the market ability distribution $F_m(a_i)$, with the period-specific entry cost shock $\underline{v}_{\text{entry}}$. In the first period of the model, or if the potential charter school entrant did not enter in the market in the previous period, the potential charter school entrant enters ($z_{tm} = 1$) with probability

$$\Pr\{z_{tm} = 1 | z_{t-1,m} = 0, \mathbf{k_{tm}}, F_m(a_i)\} = \Phi\left(\frac{\mathrm{E}_{\nu^{\mathbf{e}}}\left[v^*_{ch,tm}\right] - \mu_{\underline{v}_{\text{entry}}}}{\sigma_{\underline{v}_{\text{entry}}}}\right). \tag{18}$$

Charter school entry in market where charter entered last period:

The potential charter school entrant in a market where there was previously a charter school entrant has a similar problem, but the entry cost shock is drawn from a different distribution. Note that the assumption is that a different potential entrant makes an entry decision in each year – the previous entrant's decision enters only through the parameters of the entry cost shock distribution. Its probability of entry is

$$\Pr\{z_{tm} = 1 | z_{t-1,m} = 1, \mathbf{k_{tm}}, F_m(a_i)\} = \Phi\left(\frac{\mathrm{E}_{\nu^{\mathbf{e}}}\left[v^*_{ch,tm}\right] - \mu_{\underline{v}_{\text{operating}}}}{\sigma_{\underline{v}_{\text{operating}}}}\right). \tag{19}$$

Effort level at school s given a charter school in the market:

The likelihood of the observed effort in markets where charter schools have entered is simply the density of the difference between effort predicted by the model (i.e., that chosen in equilibrium by each school) and the observed effort (i.e., chosen effort augmented by the effort productivity shock):

$$L\{e^o_{stm} | z_{tm} = 1, \mathbf{k_{tm}}, F_m(a_i)\} = \frac{1}{\sigma_{\nu^e}} \phi\left(\frac{\tilde{e}^o_{stm} - \tilde{e}^*_{stm}}{\sigma_{\nu^e}}\right) \tag{20}$$

where $\tilde{\cdot}$ denotes the natural logarithm of a variable and e^*_{stm} is the duopoly equilibrium effort level.

Effort level at public school given no charter school in the market:

$$L\{e^o_{tps,tm} | z_{tm} = 0, \mathbf{k_{tm}}, F_m(a_i)\} = \frac{1}{\sigma_{\nu^e}} \phi\left(\frac{\tilde{e}^o_{tps,tm} - \tilde{e}^{*\text{monopoly}}_{tps,tm}}{\sigma_{\nu^e}}\right) \tag{21}$$

4.1.3 Student Contribution to Likelihood

Student likelihood statements are all conditional on ability a_i, which is integrated out in the likelihood function according to the market ability distribution.

School choice for student i with ability a_i given a charter school in the market:

The probability student i attends the charter school ($s^o_{itm} = 1$) is a function of observed school inputs ($\mathbf{k_{tm}}, \mathbf{e^o_{tm}}$), the pair of distances from the student to both schools $\mathbf{r_{itm}}$, and own ability a_i. This was derived in equation (11) in Section 2.4.

$$\Pr\{s^o_{itm} = 1 | z_{tm} = 1, \mathbf{k_{tm}}, \mathbf{e^o_{tm}}, \mathbf{r_{itm}}, a_i\} = \Phi\left(\frac{a_i \Delta(x_{tm}\beta_x) - \Delta c_{itm}}{\sigma_{\Delta\epsilon}}\right) \tag{22}$$

Test score for student i with ability a_i at the charter school, given a charter school in the market:

The observed test score of a student attending the charter school $y^o_{i,ch,tm}$ is a function of student ability a_i, charter school test score production function parameters $\beta_{e,ch}$ and $\beta_{k,ch}$, observed school inputs $e^o_{ch,tm}$ and $k_{ch,tm}$, and a test score productivity shock $\nu^y_{i,ch,tm}$. The test score production function is repeated here for convenience. The test score y_{istm} is a function of ability, school inputs e^o_{stm} and k_{stm}, and a productivity shock ν^y_{istm}:

$$y_{istm} = a_i (e^o_{stm})^{\beta_{es}} k^{\beta_{ks}}_{stm} + \nu^y_{istm}.$$

From the test score production function, the *unconditional* distribution of the observed test score of a student with ability a_i attending the charter school is

$$y^0_{i,ch,tm} \sim \mathrm{N}\left(a_i (e^o_{ch,tm})^{\beta_{e,ch}} (k_{ch,tm})^{\beta_{k,ch}}, \sigma^2_{\nu^y}\right),$$

but this does not control for the fact that the student is choosing the charter school based on information about the productivity shocks at both schools. The distribution of the observed test score of the student, conditional on attending the charter school must take into account the conditional mean $\mu_{\nu^y|s=ch}$ and standard deviation $\sigma_{\nu^y|s=ch}$ of the test score productivity shock. In particular, student i chooses the charter school if and only if

$$\Delta\epsilon_{itm} \geq \Delta c_{itm} - a_i \Delta(x_{tm}\beta_x), \tag{23}$$

so the conditional distribution of ν^y_{ch} is the density of a bivariate conditional normal distribution, where there is truncation in $\Delta\epsilon_{itm}$ according to (23).[59]

$$L\{y^o_{i,ch,tm} | z_{tm} = 1, \mathbf{k_{tm}}, \mathbf{e^o_{tm}}, \mathbf{r_{itm}}, a_i, s_{itm} = 1\} = f_{\nu^y|s=ch}(y^0_{i,ch,tm} | z_{tm} = 1, \mathbf{k_{tm}}, \mathbf{e^o_{tm}}, \mathbf{r_{itm}}, \mathbf{a_i}).$$

[59] Please see Appendix E for the actual form and derivation of $f_{\nu^y|s=ch}$ and $f_{\nu^y|s=tps}$.

Test score for student i with ability a_i at the public school, given a charter school in the market:

The test score for a student attending the public school in a market with charter school entry is similar, but the density of the test score productivity shock ν^y_{tps} is conditional on the opposite truncation because the student is attending the public school:

$$L\{y^o_{i,tps,tm}|z_{tm}=1,\mathbf{k_{tm}},\mathbf{e^o_{tm}},\mathbf{r_{itm}},a_i,s_{itm}=0\}=f_{\nu^y|s=tps}(y^0_{i,tps,tm}|z_{tm}=1,\mathbf{k_{tm}},\mathbf{e^o_{tm}},\mathbf{r_{itm}},\mathbf{a_i}).$$

Test score for student i with ability a_i given no charter school in the market:

This is simply the unconditional density of the test score, as there is no selection of schools by students in markets without charter school entry.

$$L\{y^o_{i,tps,tm}|z_{tm}=0,\mathbf{k_{tm}},\mathbf{e^o_{tm}},a_i,s_{itm}=0\}=\frac{1}{\sigma_{\nu^y}}\phi\left(\frac{a_i(e^o_{tps,tm})^{\beta_{e,tps}}(k_{tps,tm})^{\beta_{k,tps}}-y^o_{i,tps,tm}}{\sigma_{\nu^y}}\right) \tag{24}$$

The likelihood function combines the previous probability and likelihood statements for markets and students and integrates over the ability distribution in a market, given all the data X and parameters θ (X and θ are suppressed in the right-hand-side):

$$\begin{aligned}
L(\theta|X)=(\prod_{m\in M}\prod_{t\in 2,\ldots,T}&\Pr\{z^o_{tm}=1|z^o_{t-1,m}\}^{z^o_{tm}}(1-\Pr\{z^o_{tm}=1|z^o_{t-1,m}\})^{(1-z^o_{tm})})\\
&\Pr\{z^o_{1m}=1|z^o_{0m}=0\}^{z^o_{1m}}(1-\Pr\{z^o_{1m}=1|z^o_{0m}\})^{(1-z^o_{1m})}\cdot\\
(\prod_{m\in M}\prod_{s\in S_{tm}}\prod_{t\in T}&\left(L(e^o_{ch,tm}|z^o_{tm}=1)L(e^o_{tps,tm}|z^o_{tm}=1)\right)^{z^o_{tm}}L(e^o_{tps,tm}|z^o_{tm}=0)^{1-z^o_{tm}})\cdot\\
(\prod_{m\in M}\int_{a_i\in A_m}(\prod_{i\in I_{tm}}\prod_{t\in T}&((\Pr\{s^o_{itm}=1|a_i\}L(y^o_{i,ch,tm}|z^o_{tm}=1,s^o_{itm}=1,a_i))^{s^o_{itm}}\cdot\\
&((1-\Pr\{s^o_{itm}=1|a_i\})L(y^o_{i,tps,tm}|z^o_{tm}=1,s^o_{itm}=0,a_i))^{1-s^o_{itm}})^{z^o_{tm}}\cdot\\
&L(y^o_{i,tps,tm}|z^o_{tm}=0,a_i)^{1-z^o_{tm}})dF_m(a_i))
\end{aligned} \tag{25}$$

I maximize the likelihood using APPSPACK (Gray and Kolda (2006)), which is a derivative-free optimization program that is designed for easy parallelization.

4.2 Identification

4.2.1 Market Ability Distributions

The ability distribution for each market is assumed to be normally distributed. The market ability distribution is, in principle, non-parametrically identified, but the it is normally distributed makes some parts of the school objective much easier to solve.[60] The market ability distribution enters both the school problem through the charter school entry decision and subsequent school effort choices and student likelihood statements. In this section I show how I recover the mean and variance of each market's ability distribution. The recovered market ability distributions are functions of the public school test score production function parameters, so they must be recovered jointly with the estimation of the model. Even though ability is inherently unobservable, by recovering it I can treat it as it were observed when I integrate over ability in solving the school maximization problems and student likelihood statements.

I use the public school production function for 1997, the year before charter school authorization in North Carolina. Using the production function for public schools (2), the mean test score for market m in 1997 $\bar{y}_{tps,1997,m}$ is

$$\begin{aligned}
\bar{y}_{tps,1997,m} &= \int\limits_a \int\limits_\epsilon y_{i,tps,1997,m} f_\epsilon(\epsilon) f_m(a) d\epsilon da \\
&= \int\limits_a a(e^o_{tps,1997,m})^{\beta_{e,tps}} k^{\beta_{k,tps}}_{tps,1997,m} f_m(a) da + \int\limits_\epsilon \epsilon f_\epsilon(\epsilon) d\epsilon \\
&= (e^o_{tps,1997,m})^{\beta_{e,tps}} k^{\beta_{k,tps}}_{tps,1997,m} \int\limits_a a f_m(a) da + \underbrace{\mathrm{E}\left[\epsilon_{i,tps,1997,m}\right]}_{0} \\
&= \bar{a}_m \left((e^o_{tps,1997,m})^{\beta_{e,tps}} k^{\beta_{k,tps}}_{tps,1997,m} \right) \qquad (26)
\end{aligned}$$

where f_m, the ability distribution of market m, is assumed to be invariant over time in each market. The key here is that I can use the test score distribution before charter school entry to solve for the parameters of the ability distribution because there was no choice of schools available in the year before entry, obviating controlling for selection on either ability or test score productivity shocks. Given data on the observed inputs in the market in 1997, $e^o_{tps,1997,m}$ and $k_{tps,1997,m}$, and the public school's test score production function parameters $\beta_{e,tps}$ and $\beta_{k,tps}$, I can recover the mean ability for the market $\bar{a}_m$ jointly while estimating the rest of the model. The variance of the test score $\sigma^2_{y,tps,1997}$ can be written as a function of the variance of

[60] This can be seen when I derive the school objective in Section 2.4, where the measure of students attending a particular school from a certain distance bin can be written in terms of the normal distribution function. The average ability of students from each bin still requires integration over the ability distribution, which I do numerically when solving the model.

ability in market m, $\sigma^2_{a,m}$, observed inputs, and the variance of the test score productivity shock $\sigma^2_{\nu^y}$

$$\sigma^2_{y,tps,1997,m} = \sigma^2_{a,m} \left((e^o_{tps,1997,m})^{\beta_{tps,e}} k^{\beta_{tps,k}}_{tps,1997,m} \right)^2 + \sigma^2_{\nu^y}.$$

If only data from 1997 are used for each market, the variance of the ability distribution cannot be separated from that of the test score productivity shock. I therefore fix $\sigma^2_{\nu^y}$ in the estimation. Note that, although the variance of the test score productivity shock may be recovered using only the 1997 data, it is in theory possible to use more than one year of data to recover it. Note that the chosen level of the variance of ν^y affects the importance attributed to the ability distribution, which may affect outcomes for counterfactual scenarios by over- or under-emphasizing its role relative to test score productivity shocks.

4.2.2 School Effort

In many models, effort taken by agents is unobservable. For example, unobservable effort plays a key role in moral hazard problems. In my model, schools commit to a chosen effort level at the beginning of the school year and then receive an effort productivity shock which augments it. The key is that I have data on effort, average weekly hours of homework done by students at each school, which means that it is trivially identified.

4.2.3 Test Score Production Functions

It is difficult to separate student ability from the productivity of school inputs in the presence of student sorting on ability, which the model predicts will typically occur. The productivity of inputs may be estimable due to restrictions imposed by functional form, but it is preferable to identify test score production function parameters using variation in the data. I use data on the distance from a student to the public and charter schools in estimating the probability a student will attend the charter school. Changes in these distances, so long as they do not imply a change in the ability distribution, shift the probability a household would attend the charter school without changing the test score. This approach has been taken by others, such as Cullen et al. (2005), where the authors use distance from a school as an instrument for the probability of school attendance.

4.3 Missing Data

I assume data are missing randomly. There are some charter school observations where homework was not reported for any students, so those schools did not contribute to the effort likelihood. As I discussed in the sample restriction section, about 5% of observations in charter and public schools are missing test score data, so these students also do not contribute to the likelihood as they are excluded.

More importantly, about two-thirds of the students in charter schools are never observed attending public schools, which, when combined with the fact that charter schools tend not to report student locations, means that it is unlikely that I observe their addresses. The assumption that these addresses are missing at random may be justified by the fact that an indicator for whether the address is missing is not significantly associated with a student's test score when controlling for student ethnicity. I integrate over students' market distance distributions for all students missing location data.

5 Estimation Results

5.1 Parameters

Table 5 shows the estimated parameters for the model. The first four rows are the test score production function parameters. Recall that all public schools share the same test score production technology, and all charter schools share (a different) test score production technology – that is, there is no heterogeneity within public schools or within charter schools. Charter schools are more productive than public schools in both capital ($\beta_{kc} = 0.08$ vs. $\beta_{kp} = 0.00$)[61] and effort ($\beta_{ec} = 0.19$ vs. $\beta_{ep} = 0.06$), and both schools are more productive in their effort input than capital input. The low marginal productivity of per-pupil capital at public schools is consistent with the general finding of Hanushek (2003), which argues that increased pupil-teacher ratios and per-pupil expenditures have not resulted in substantial increases in achievement for students at public schools.

The disutility of effort is *negative* ($c_e = -0.23$), which means students prefer attending the school where they have to work harder, even controlling for higher test scores. The per-kilometer distance cost is about one-third of a standard deviation ($c_r = 0.29$) of test scores. The disutility from attending a charter school is quite large ($c_{ch} = 3.75$), which is what allows the model to fit revealed student school choice probabilities given the higher productivity of charter schools and the relatively small distance cost of commuting to a

[61] β_{kp} is actually positive but quite small.

school.[62] The disutility from attending a charter school may also capture capacity constraints on charter schools, which are not explicitly modeled.

In the school effort cost functions, charter schools have much larger diseconomies of scale from exerting effort than public schools ($\psi_{ec3} = 39 > \psi_{ep3} = 4$). Both schools pay a cost of exerting effort that is mitigated by higher levels of capital ($\psi_{ec5} = -54, \psi_{ep5} = -16$). There may be an intuitive explanation for this: Higher per-pupil capital levels may make it easier for the school to create, assign, and grade homework because there are more computers per student or if there are smaller class sizes.

Finally, as expected, the mean of the entry cost shock distribution is much lower when there has already been a charter school in the market in the previous period ($\mu_{\underline{v},\text{given entry}} = -49 < \mu_{\underline{v}} = 112$). This captures the persistence of charter school entry in the data.

5.2 Model Fit

The model fits basic charter school entry, charter and public school effort, and student test score patterns for North Carolina. Table 6 shows the fraction of markets of certain characteristics with charter schools. The first row is the observed and predicted overall fraction of markets with charter schools over the estimation period 1998-2001. The model also captures the fact that charter schools are more likely to enter markets where they would receive higher per-pupil capital (data rows 2 and 3) and larger markets (data rows 4 and 5).[63] The model slightly over-predicts overall entry and entry in high-capital and large markets, which may be explained by the fact that I am not explicitly targeting these moments in the data, but rather am trying to fit observed entry patterns for all markets using maximum likelihood.

Table 7 shows that the model is capable of reproducing patterns of the observed school effort levels in the data: charter and public schools in markets where charter schools have entered exert higher levels of effort than public schools in monopoly markets. This is because charter schools enter markets with higher mean abilities, higher per-pupil capital levels, and the competitive effect of charter school entry. Again, the predicted means may be slightly off because I do not explicitly target these means in the estimation, but rather the relationship between observed and predicted effort in all markets. Table 8 shows that the model also fits the relationship between school capital and effort. Charter schools in markets where the charter schools have above median per-pupil capital exert more effort than they do in markets with below median per-pupil capital. The same is true for public schools both in markets with and without charter school entry.

[62] The distance cost is on average effectively larger for students to attend a charter school.

[63] Recall that per-pupil capital at charter schools is based on a prediction, so it exists even in markets where the charter school has not entered.

Table 5: Parameter Estimates*

Parameter	Estimated Value	Description
Test score production function		
β_{ec}	0.187	productivity of effort, charter school
β_{kc}	0.080	productivity of capital, charter school
β_{ep}	0.062	productivity of effort, public school
β_{kp}	0.000	productivity of capital, public school
Student cost		
c_e	-0.233	student effort cost
c_r	0.293	student distance cost
c_{ch}	3.752	student charter school attendance cost
School valuation of test scores		
δ_{yc1}	0.000	value of average test score, charter school
δ_{yp1}	6.138	value of average test score, public school
δ_{yc2}	20.950	value of total test score, charter school
δ_{yp2}	36.475	value of total test score, public school
School effort cost functions		
ψ_{ec1}	4.643	disutility of effort, charter school
ψ_{ep1}	0.914	disutility of effort, public school
ψ_{ec2}	2.948	convex disutility of effort, charter school
ψ_{ep2}	2.448	convex disutility of effort, public school
ψ_{ec3}	38.553	effort, school size interaction, charter school
ψ_{ep3}	3.903	effort, school size interaction, public school
ψ_{ec4}	-28.543	effort, school size squared interaction, charter school
ψ_{ep4}	-1.383	effort, school size squared interaction, public school
ψ_{ec5}	-53.860	effort, capital interaction, charter school
ψ_{ep5}	-16.606	effort, capital interaction, public school
ψ_{ec6}	-46.315	effort, capital squared interaction, charter school
ψ_{ep6}	0.366	effort, capital squared interaction, public school
Entry cost shock distributions		
$\mu_{\underline{v}}$	112.000	mean entry cost distribution, market without entry last period
$\sigma_{\underline{v}}$	37.250	st. dev. entry cost distribution, market without entry last period
$\mu_{\underline{v},\text{given entry}}$	-48.500	mean entry cost distribution, market with entry last period
$\sigma_{\underline{v},\text{given entry}}$	52.000	st. dev. entry cost distribution, market with entry last period
School preference shock		
σ_{η}	2.320	st. dev. student school preference shock

* Standard errors are still being calculated

Table 6: Fit: Charter school entry patterns by market characteristics

	Fraction of Markets with Charter Schools	
	Observed	**Predicted**
Total	0.072	0.092
Charter per-pupil capital		
above median k_{ch}	0.118	0.156
below median k_{ch}	0.025	0.028
Market size		
above median μ	0.086	0.121
below median μ	0.057	0.063

In general, schools exert higher effort in markets where they have higher per-pupil resources for two reasons: capital directly augments test score production and also makes effort exertion less costly for schools through the interaction between capital and effort in school effort cost functions. This latter effect is why public schools exert higher effort in markets where they have higher per-pupil capital in spite of the fact that the coefficient on per-pupil capital in the public school test score production function is small. Table 9 shows that the model also captures the fact that charter schools exert higher effort in smaller markets, while public schools exert higher effort in larger markets. This may be explained by the difference in scale effects at charter and public schools: charter schools are estimated to have diseconomies of scale in their effort cost functions.

Table 7: Fit: Mean Hours of Homework by School Type

Entry Markets	**Predicted**	**Observed**
Charter Schools	2.57 hours	2.66 hours
Public Schools	2.55 hours	2.69 hours
Monopoly Markets		
Public Schools	2.40 hours	2.43 hours

Table 10 shows the fraction of students choosing the charter school in markets where charters have entered. The model fits the pattern that charter schools are smaller than the public schools in markets they have entered. The first data column, 0.072, is the average for all markets with charter schools. The second data column presents the average fraction of students choosing the charter school for the full simulation of the model – that is, first simulating charter school entry decisions and then school effort choices, and simulating student school choices based on predicted school effort choices. The model over-predicts the size

Table 8: Fit: Mean Hours of Homework by School Type, Conditional on School Capital, k_s

Entry Markets	**Observed**	**Predicted**
Charter Schools		
above median k_{ch}	2.74 hours	2.63 hours
below median k_{ch}	2.20 hours	2.24 hours
Public Schools		
above median k_{tps}	2.78 hours	2.58 hours
below median k_{tps}	2.21 hours	2.39 hours
Monopoly Markets		
Public Schools		
above median k_{tps}	2.51 hours	2.45 hours
below median k_{tps}	2.35 hours	2.35 hours

Table 9: Fit: Mean Hours of Homework by School Type, Conditional on Market Size, μ

Entry Markets	**Observed**	**Predicted**
Charter Schools		
above median μ	2.51 hours	2.52 hours
below median μ	2.89 hours	2.64 hours
Public Schools		
above median μ	2.85 hours	2.7 hours
below median μ	2.46 hours	2.33 hours
Monopoly Markets		
Public Schools		
above median μ	2.49 hours	2.46 hours
below median μ	2.37 hours	2.33 hours

of charter schools in markets with entry. When I compute the fraction of students attending the charter school in markets where there was *observed* charter school entry and using *observed* school effort data, the fit is much closer (third data column). The third column does not exactly match the first because the estimation algorithm maximizes the sum of student and school likelihoods, as opposed to only the student contribution to the likelihood. Because the fit from the full simulation (data column 2) is a bit off, I present most of the counterfactual results conditional on school choice.

Table 10: Fit: Student School Choices in Markets with Charters

	Observed	**Model Sim. Effort, Entry**	**Model Obs. Effort, Entry**
Fraction of Students Attending Charters	0.072	0.236	0.104

Table 11 shows that the model captures the ranking average of test scores: students at charter schools have the highest average test scores, followed by students attending public schools in markets charter schools have entered, followed by students in public schools in markets without charters. The fact that public schools are monolithic in test score production may explain the difference between predicted and simulated test scores: one test score production function must explain test scores in public schools with and without charter school entry. Because most markets do not have charter schools, the test score fit for markets without charter schools is closer (.028 vs. .036). Charter school test scores must be even higher to rationalize the school choice patterns in the data.

Table 11: Fit: Mean Student Test Scores

	Observed	**Model Sim. Effort, Entry**
All Markets	0.046	0.047
Entry Markets		
Charter Schools	0.128	0.252
Public Schools	0.059	0.071
Monopoly Markets		
Public Schools	0.028	0.036

Finally, the model predicts that charter school entry is more likely in markets with higher mean abilities. This follows from the positive weight charter schools place on student test scores, δ_{yc2}. For example, a unit increase in the mean ability of students in a market is associated with an increase in entry probability of about 3% for markets where there was no charter school in the prior period.

6 Counterfactual Results

In this section I use the estimated model to investigate the three research questions posed in the introduction:

1. How has the presence of charter schools affected the test scores of students attending both charter and public schools?

2. Given that charter school entry is often capped, what would be the effect of allowing charter school entry in all public school markets on both the number of charter schools and student test scores?

3. Given that charter schools typically have lower levels of resources than public schools, what would be the effect of equalizing per-pupil capital levels across charter and public schools on both the number of charter schools and student test scores?

6.1 The Effect of the Presence of Charter Schools

In the first counterfactual (Question 1), I quantify the effect of charter schools in markets where charter schools are already present. Previous attempts to quantify the effect of charter schools on test scores have not taken into account the potential changes in inputs at public schools in response to charter school competition. By using my model of endogenous school effort provision, I can ask what test scores *would have been* in the absence of charter schools in all markets charter schools have entered. I implement this first by simulating student outcomes if there were no charter schools, so that all students attend the public schools in their markets, which act as monopolists. I then compare these student outcomes with those under the status quo regime, where charter schools were allowed to enter markets (i.e. that presented in Section 5.2).

Table 12 shows that the test scores of both charter and public school students would be lower, but in particular, students in charter schools have a larger drop in test scores when charters are not allowed. The direct effect of charter schools, i.e. the effect of charter schools on students attending charters, is 19% of a standard deviation in test scores. This is about 20 times larger than the competitive effect, that is, the spillover effect of charters onto public school students in the same market.[64] The slightly lower test scores that students in public schools suffer when charter schools are banned suggest that the competitive response to the presence of charter schools outweighs despite potential effects from "cream-skimming." Both charter

[64] I do not present the overall change in the test score distribution for any of the three counterfactuals because I overestimate the fraction of students attending charter schools.

and public schools exert higher effort (0.159 hours for charters and 0.213 hours for public schools) than monopoly public schools in markets where charter schools have entered, but the low productivity of effort at public schools mitigates the effect of this decrease in school effort.

Table 12: Students Have Lower Test Scores When Charters Are Banned

	Decrease Resulting From No-Charter Scenario	
	Average Test Score	**Average School Effort**
Students attending charter in status quo	0.190 sd	0.159 hours
Students in public school in status quo (market with new charter)	0.011 sd	0.213 hours

6.2 The Effect of Allowing Unrestricted Charter School Entry

The second counterfactual (Question 2) asks how lifting the cap on the total number of charter schools in North Carolina would affect the fraction of markets with charter schools and average test scores in those markets. I operationalize this by taking the set of markets without charter schools in 2001 (90% of all markets) and simulating the model in these markets for 2002-2005, allowing unrestricted charter school entry. Recall from Section 3.2 that I avoid explicitly modeling the cap on charter schools by estimating the model using data from years well before the statewide cap in North Carolina started binding. Therefore, I can use the model, as it was estimated, to ask what would be the effect of allowing unrestricted charter school entry would be.

The results suggest the cap on the total number of charter schools in North Carolina was binding. The average fraction of markets with charter schools increases from 11.1 to 12.5% during the period 2002-2005, were charters free to enter all markets, which corresponds to an increase from 55 to 62 charter schools, on average, per year. Recall that the probability of entry in a market is Markov, because it depends on whether last period's potential charter school entrant entered the market. When I use the average entry probabilities over all markets and all years from 1998-2005, these Markov entry probabilities result in a steady state distribution of charter schools in 60% of all markets. Table 13 shows that the impact of charter schools in the additional markets that are entered in the simulation is similar to the effect in Question 1.

6.3 Increase Charter School Per-Pupil Capital to Public School Levels

In the third counterfactual (Question 3), I set the means of the per-pupil capital distributions for charter and public schools equal, which amounts to increasing the per-pupil capital levels of all charter schools by

Table 13: Student Test Scores Increase in Markets with Newly-Allowed Charters

	Increase in Average Test Score Compared with No-Charter Scenario
Students in new charter	0.202 sd
Students in public school (market with new charter)	0.001 sd

approximately one third. I then quantify the effect of charter school entry over the period 1998-2001, as I did in Question 1, given the new charter school per-pupil capital levels.[65] The counterfactual is motivated by the policy debate over whether charter schools should be funded at the same levels as public schools. The policy may be considered budget-neutral when compared with a scenario with no charter schools, because in that case, all students would have the same per-pupil capital levels as in this policy.

Table 14 shows that test scores for students attending charter schools increase dramatically and are accompanied by an increase in the test scores for public school students in markets with new charter schools. Moreover, charter schools now enter 33% of all markets, up from 9% under the status quo from 1998-2001. The increased penetration comes from charter school expansion into new markets that have low mean ability when compared with markets charters previously entered. Also, charter schools are 60% larger in the simulations. This is because students value the increased capital available at the charter schools, which increases their test scores without affecting the effort cost associated with attending charters, ceteris paribus. In addition, charter school effort increases to 3.47 (up from 2.57). Test scores of charter school students are on average higher when charters have higher per-pupil capital levels, but some of the gains are mitigated by the fact that charters are now entering markets with lower mean abilities.

Table 14: Student Test Scores Increase More when Charters Receive More Capital

	Increase in Average Test Score Compared with No-Charter Scenario
Students in charter school	0.234 sd
Students in public school (market with charter school)	0.028 sd

[65] This also assumes there are no restrictions on the total number of charter schools.

7 Discussion

In this paper, I developed and estimated a structural model of charter school entry, student school choices, and endogenous school inputs. The model fits key patterns in the data. I demonstrate that both the direct effect and spillover effects of charter school entry are positive, although the direct effect is much larger than the spillover effect of charter schools on public school students.

The estimates of the direct effect of charter schools on test scores may differ from those found in the previous literature for three reasons. First, my estimates account for changes in public school inputs that may occur in response to charter school entry. Second, the panel test score production function literature typically uses different subsets of students than I did in order to estimate the effect of charter school entry: they estimate the effect of charter schools on test scores by using students who came from public schools. These students tend to have lower test scores than charter school students who are only observed in charter schools. The counterfactual in which I calculate the impact of the presence of charter schools suggests that charter schools do have a positive impact on the test scores of attendant students. The second counterfactual suggests that allowing unlimited charter school entry would not result in charter school penetration in all markets, and that the effect in new entry markets would be approximately the same as that in markets charters entered between 1998-2001. Finally, the third counterfactual suggests that increasing per-pupil capital at charter schools may be an inexpensive way to increase test scores of students because charter schools are more productive given the same inputs.

References

Angrist, J.D., S.M. Dynarski, T.J. Kane, P. Pathak, and C. Walters, "Who benefits from KIPP?," *NBER Working Paper*, 2010.

Bifulco, R. and H.F. Ladd, "The impacts of charter schools on student achievement: Evidence from North Carolina," *Education Finance and Policy*, 2006, *1* (1), 50–90.

Cameron, A.C. and P.K. Trivedi, *Microeconometrics: Methods and Applications*, Cambridge Univ Pr, 2005.

Chakrabarti, R., "Can increasing private school participation and monetary loss in a voucher program affect public school performance? Evidence from Milwaukee," *Journal of Public Economics*, 2008, *92* (5-6), 1371–1393.

Cullen, J.B., B.A. Jacob, and S.D. Levitt, "The impact of school choice on student outcomes: An analysis of the Chicago public schools," *Journal of Public Economics*, 2005, *89* (5-6), 729–760.

Epple, D. and R.E. Romano, "Competition between private and public schools, vouchers, and peer-group effects," *American Economic Review*, 1998, *88* (1), 33–62.

Ferreyra, M.M., "Estimating the effects of private school vouchers in multidistrict economies," *The American Economic Review*, 2007, pp. 789–817.

Google, "Google Earth (Version 5.1.3535.3218)," Accessed April 2010.

Gray, G.A. and T.G. Kolda, "Algorithm 856: APPSPACK 4.0: Asynchronous parallel pattern search for derivative-free optimization," *ACM Transactions on Mathematical Software (TOMS)*, 2006, *32* (3), 485–507.

Hanushek, E.A., "The economics of schooling: Production and efficiency in public schools," *Journal of Economic Literature*, 1986, *24* (3), 1141–1177.

_ , "The failure of input-based schooling policies," *The Economic Journal*, 2003, *113* (485), F64–F98.

Hanushek, Eric A., John F. Kain, Steven G. Rivkin, and Gregory F. Branch, "Charter school quality and parental decision making with school choice," *Journal of Public Economics*, 2007, *91* (5-6), 823 – 848.

Hoxby, C.M. and J.E. Rockoff, "The impact of charter schools on student achievement," *Harvard University, November*, 2004.

KIPP, "KIPP: Knowledge Is Power Program," Accessed 10 2010.

NCES, "Universe of Public Elementary and Secondary Education Agencies - Fall 1997."

Nechyba, T.J., "Mobility, targeting, and private-school vouchers," *American Economic Review*, 2000, *90* (1), 130–146.

Nichols, Austin, "VINCENTY: Stata module to calculate distances on the Earth's surface," Statistical Software Components, Boston College Department of Economics October 2003.

Picard, Robert, "GEONEAR: Stata module to find nearest neighbors using geodetic distances," Statistical Software Components, Boston College Department of Economics 2010.

Rivkin, S.G., E.A. Hanushek, and J.F. Kain, "Teachers, schools, and academic achievement," *Econometrica*, 2005, *73* (2), 417–458.

Sass, T.R., "Charter schools and student achievement in Florida," *Education Finance and Policy*, 2006, *1* (1), 91–122.

Snyder, T.D., S.A. Dillow, and C.M. Hoffman, *Digest of Education Statistics, 2008. NCES 2009-020.*, National Center for Education Statistics. Available from: ED Pubs. PO Box 1398, Jessup, MD 20794-1398. Tel: 877-433-7827; Web site: http://nces. ed. gov/help/orderinfo. asp, 2009.

Sundaram, R.K., *A first course in optimization theory*, Cambridge Univ Pr, 1996.

White, John, "States open to charters start fast in 'Race to Top'," web June 2009.

Wilder, Mike, "Candidates for legislature, school board talk education. The Times News," 10 2010.

A Existence of Equilibrium [66]

In order to use Brouwer's Fixed Point Theorem, the pair of best-response functions must be continuous self-map on a compact and convex set. First, I prove there is a unique best response of one school to another, then that this best response function is continuous, and finally apply Brouwer's Fixed Point Theorem. Note that for this I assume certain restrictions on the parameter space: Cobb-Douglas production function with decreasing returns and a convex effort cost.

Lemma 1. *$e^*_{ch} = \arg\max_{e \in E} v_{ch}(e|e_{tps}, x, \theta)$ is strictly positive.*

Proof. First, note that $v_{ch}(0|e_{tps}, x, \theta) = \delta_{\mu,ch}\mu_{ch}(0, e_{tps}|x, \theta)$, where $\mu_{ch}(\cdot) \geq 0$. Second, $\lim_{e_{ch}\to 0} \frac{\partial v_{ch}(0|e_{tps},x,\theta)}{\partial e_{ch}} = \infty$ due to the Inada conditions on the test score production function, because there will always be some measure of students attending the charter school due to the preference shocks. □

Call $v^+_{ch} = \max\{v_{ch}, 0\}$. Note that v^+_c is strictly quasi-concave, due to the strict concavity of v_{ch} when it is above 0.

Lemma 2. *The effort set $E = [\underline{e}, \bar{e}]$ is compact.*

Proof. Let $\underline{e} = 0$. Given any allowable vector of parameters θ there exists $\bar{e}_\theta$ such that $v_{ch}(\hat{e}) < 0$, all $\hat{e} > \bar{e}_\theta$. Let $\bar{e} = \max_\theta \bar{e}_\theta$. It exists, so the set is not empty. □

Lemma 3. *γ_{ch} is continuous*

Proof. Berge's Maximum Theorem (Sundaram (1996)) requires a continuous objective v^+_{ch}, and compact and upper-hemicontinuous constraint set. Note first that the constraint set, E, is a fixed connected interval, so it is trivially UHC. v^+_{ch} is continuous, so the Maximum Theorem says the resulting correspondence which is the argmax of v^+_{ch} is UHC. Because v^+_{ch} is strictly quasi-concave, there is a unique argmax to v^+_{ch}, which means that γ_{ch} is a continuous function. □

Lemma 4. *There exists an equilibrium to the entry subgame.*

Proof. Since $\Gamma(e_{ch}, e_{tps}|x, \theta) = (\gamma_{ch}(e_{tps}|x, \theta), \gamma_{tps}(e_{ch}|x, \theta))$ is a continuous self map on the compact and convex domain E^2, there exists an equilibrium by Brouwer's Fixed Point Theorem, □

[66] The proofs are for the case where there are no productivity shocks to the school's chosen effort level. The results go through in the case where there are shocks.

B Uniqueness of Equilibrium

I cannot prove uniqueness equilibrium in the entry subgame but can rule out multiplicity of the charter school entry decision, *given a unique equilibrium in the ensuing entry subgame.*

Lemma 5. *There is no multiplicity in the charter school entry decision given uniqueness of equilibrium in the entry subgame.*

Proof. The charter school only receives one shock $\underline{v}_{tm}$, which it knows. It enters if and only if

$$\underline{v}_{tm} \leq \mathrm{E}_{\nu^e}\left[v^*_{ch,tm}\right]$$

where $\mathrm{E}_{\nu^e}\left[v^*_{ch,tm}\right]$ is known since under the assumption of the lemma there is a unique equilibrium in chosen effort levels of the entry subgame. □

Although I do not have a proof that the entry subgame has a unique equilibrium, I have searched for more than one equilibrium for a wide range of parameter values and have never found more than one equilibrium in a market. Intuitively, there will not be multiple equilibria in the entry subgame so long as schools are not too responsive to each other, which may be satisfied if the effort cost is sufficiently convex. I assume, for the sake of estimation, that both schools know which equilibrium they are in, and that they always play the same equilibrium.

C Construction of Capital Variable

There are several measures of school resources for charter and public schools in the NCERDC data, but many are missing for many schools – especially charters. Moreover, I do not observe any school-specific resources for charter schools in markets where there was no charter school entry, which are an input into the test score production function and are therefore necessary for calculating the probability of charter school entry in a particular market. Finally, it is not obvious how the different measures of school resources should enter into the test score production function. What I need is a way to compute the subjunctive level of capital for both charter and public schools given information that is always observable for the a market. Here is how capital is constructed:

1. Convert measures (computers/pupil, teachers/pupil, experienced teachers/pupil) to percentiles.

2. Average (unweighted) these percentiles into one index for each school.

3. Regress this index on inflation-adjusted per-pupil expenditures for the public school in each market, using separate regressions for charter and public schools.

4. Use the predicted value from the above regression as the capital measure for that school type in that market.

This measure always exists, so long as I have data on the per-pupil expenditures for the public school in that market. The last step obviates my having to integrate over the errors in the cost functions when solving the charter school's entry problem. Also, it precludes a role for charter schools making entry decisions based on unobservable information – that is, the predicted per-pupil capital levels are no different in expectation in entry and non-entry markets with the same level of per-pupil expenditures. Although such variation may play a role in charter school entry, I believe it is second order in addressing my question of charter school entry. Finally, note that since capital is percentile-based, the model would predict that rank-preserving changes in the capital distribution would have no effect in the economy.

D Construction of Distance Distribution

I need a distribution of distances for each market in order to solve for the equilibrium of the entry subgame. There are two steps involved: 1) I discretize the distance distribution and 2) I model what the distance distribution would be in a market where I do not observe charter school entry, which is key for evaluating the probability that a charter school will enter a market.

D.1 Discretization of Distance Distribution

The data provide me with a continuous distribution of student distances, but I discretize this distribution to avoid making an expensive two-dimensional integration over both ability and distance for students when computing the value of the school objective functions when solving for entry subgame equilibria. I allocate each student in every market-time unit to one of four bins, where each bin represents a different set of distances to charter and public schools. A student is allocated to a bin if its distance to each school falls within the distance cut-offs for that particular bin. For example, take a market with 3 students, j, k, and l, where the distances r_{is} are

Student	$r_{i,ch}$	$r_{i,tps}$
j	0.5	1.1
k	2	0.75
l	2.5	0.25

I use the the median distance to the public school, 0.75, as the cut-off, so the criteria for the four bins are

Bin	
1	$\{i: \quad r_{i,ch} \leq 0.75, r_{i,tps} \leq 0.75\}$
2	$\{i: \quad r_{i,ch} > 0.75, r_{i,tps} \leq 0.75\}$
3	$\{i: \quad r_{i,ch} \leq 0.75, r_{i,tps} > 0.75\}$
4	$\{i: \quad r_{i,ch} > 0.75, r_{i,tps} > 0.75\}$

The distance vectors and measure of the population in each bin are

$$\begin{aligned} \vec{r}_{ch} &= [\quad -, \quad 2.25, \quad 0.5, \quad - \quad] \\ \vec{r}_{tps} &= [\quad -, \quad 0.50, \quad 1.1, \quad - \quad] \\ \vec{\mu} &= [\quad 0, \quad \tfrac{2}{3}, \quad \tfrac{1}{3}, \quad 0 \quad] \end{aligned}$$

where I average over all the students in a bin to obtain the distance vector for that bin. For example, there are no students within 0.75 km from both the charter and public school, so the first entry in $\vec{\mu}$ is 0, and the first entries of $\vec{r}_{ch}$ and $\vec{r}_{tps}$ are undefined. There are two students (k and l) more than 0.75 km from the charter school and within 0.75 of the public school, so the second element of $\vec{\mu}$ is 2/3, and the average distance of students in the second bin from the charter school is 2.25, while the average distance for students in the second bin from the public school is 0.5 km.

D.2 Model for Distance Distribution

As with capital, I need to know what the distance distribution would be for all markets in order to calculate the value the charter school would expect to obtain upon entry, which then enters the expression for probability of entry. I first regress elements of $\vec{r}_{ch}$ and $\vec{r}_{tps}$ and $\vec{\mu}$ on a 2-bin distribution (fraction of students within median distance to the public school and further than median distance to the public school, and average the distance for within-median and beyond-median students) for public schools in markets where I observe charter school entry. I then normalize the elements of the predicted fraction of students in each bin to sum to 1 for each market. I use the relationship as the distance distribution for all markets because the 2-bin distribution for distance to the public school is available in all markets.

D.3 Discussion of My Assumptions about Distance

First, this in no way helps explain *why* charter schools locate where they do within a district. In the data, they are, on average, however, further than public schools for most students within a district. This may be so because districts are designed around public schools, meaning charters may be relegated to locations not at the center of population mass.

Second, I could have had the schools integrate over the pair of continuous distance distributions when solving their problems, but I discretize it when solving their problems because I have to solve for the ability distribution for every point in the distance distribution.

Third, note that I assign all students in markets where charters do no enter the same distance to the charter school, because otherwise this argument is missing. This simplification does not effect on my estimation, since within those districts, I do not observe students attending charters (since there are no charters). What matters in such districts is the probability of charter school entry, which is a function of the distribution of student distances for public schools and charters. I could alternatively have given the charter schools the same distance distribution as public schools when I do not observe charter school entry. Although simpler, this may introduce a bias in the estimation of the charter school operating cost distribution, because in the data charters are on average further from students than public schools.

Finally, note that, similar to my treatment of capital, these assumptions do not allow schools to select districts based on unobservable information about distance. This may be interesting, but it is not a first-order consideration because I am using distance as a demand shifter for student school choice, and my model picks up many other determinants of charter school entry.

E Derivation of Conditional Distributions of Test Score Productivity Shocks (for Likelihood)

I need the distributions of $\nu^y_{i,ch,tm}|s_{itm} = 1$ and $\nu^y_{i,tps,tm}|s_{itm} = 0$ in order to write the test score likelihood statements. The likelihood statements are

Test score for student i with ability a_i at the charter school, given a charter school in the market:

$$L\{y^o_{i,ch,tm}|z_{tm}=1,\mathbf{k_{tm}},\mathbf{e^o_{tm}},a_i,s_{itm}=1\}=\frac{1}{\sigma_{\nu^y|s=ch}}\phi\left(\frac{a_i(e^o_{ch,tm})^{\beta_{e,ch}}(k_{ch,tm})^{\beta_{k,ch}}+\frac{\sigma^2_{\nu^y}}{\sigma_{\Delta\epsilon}}\frac{\phi(\alpha_{itm})}{1-\Phi(\alpha_{itm})}-y^o_{i,ch,tm}}{\sigma_{\nu^y|s=ch}}\right) \tag{27}$$

where ϕ indicates the standard normal distribution and $\alpha_{itm}=\frac{-a_i\Delta(x_{tm}\beta_x)+\Delta c_{etm}}{\sigma_{\Delta\epsilon}}$ in the control function.

and

Test score for student i with ability a_i at the public school, given a charter school in the market:

$$L\{y^o_{i,tps,tm}|z_{tm}=1,\mathbf{k_{tm}},\mathbf{e^o_{tm}},a_i,s_{itm}=0\}=\frac{1}{\sigma_{\nu^y|s=tps}}\phi\left(\frac{a_i(e^o_{tps,tm})^{\beta_{e,tps}}(k_{tps,tm})^{\beta_{k,tps}}-\frac{\sigma^2_{\nu^y}}{\sigma_{\Delta\epsilon}}\frac{\phi(\alpha_{itm})}{\Phi(\alpha_{itm})}-y^o_{i,tps,tm}}{\sigma_{\nu^y|s=tps}}\right) \tag{28}$$

I will show the results for the charter school productivity shock since that for the public school can be derived from similar reasoning. First, note that $\nu^y_{i,ch,tm}$ and $\Delta\epsilon_{itm}$ are distributed bivariate normal according to

$$\begin{pmatrix}\nu^y_{i,ch,tm}\\ \Delta\epsilon_{itm}\end{pmatrix}\sim \mathrm{N}\left(\begin{pmatrix}0\\0\end{pmatrix},\begin{pmatrix}\sigma^2_{\nu^y} & \sigma^2_{\nu^y}\\ \sigma^2_{\nu^y} & \sigma^2_{\Delta\epsilon}\end{pmatrix}\right), \tag{29}$$

where the covariance is equal to the variance of the charter test score productivity shock since it is independent of both the other test score productivity shock and both preference shocks. Normalized, this becomes

$$\begin{pmatrix}\frac{\nu^y_{i,ch,tm}}{\sigma_{\nu^y}}\\ \frac{\Delta\epsilon_{itm}}{\sigma_{\Delta\epsilon}}\end{pmatrix}\sim \mathrm{N}\left(\begin{pmatrix}0\\0\end{pmatrix},\begin{pmatrix}1 & \rho_{\nu^y,\Delta\epsilon}\\ \rho_{\nu^y,\Delta\epsilon} & 1\end{pmatrix}\right), \tag{30}$$

where $\rho_{\nu^y,\Delta\epsilon}=\frac{\sigma^2_{\nu^y}}{\sigma_{\nu^y}\sigma_{\Delta\epsilon}}$ is the correlation coefficient between $\nu^y_{i,ch,tm}$ and $\Delta\epsilon_{itm}$. We can then use the following result from Cameron and Trivedi (2005): If

$$\begin{pmatrix}\epsilon_1\\ \epsilon_2\end{pmatrix}\sim \mathrm{N}\left(\begin{pmatrix}0\\0\end{pmatrix},\begin{pmatrix}1 & \rho\\ \rho & 1\end{pmatrix}\right), \tag{31}$$

then $\epsilon_2|\epsilon_1\geq x$ is distributed normal with mean

$$\mathrm{E}\left[\epsilon_2|\epsilon_1\geq x\right]=\rho IMR(-x),$$

where $IMR(-x) = \frac{\phi(-x)}{\Phi(-x)}$ is the inverse Mills ratio, and variance

$$\mathrm{Var}[\epsilon_2|\epsilon_1 \geq x] = 1 - \rho^2 IMR(-x)(-x + IMR(-x)).$$

(Intuition about result: If each $\nu^y_{i,ch,tm}$ is distributed normally given a $\Delta\epsilon_{itm}$, then adding them all together will result in a normal). This means that the mean and variance of $\frac{\nu^y_{i,ch,tm}}{\sigma_{\nu^y}}|\frac{\Delta\epsilon_{itm}}{\sigma_{\Delta\epsilon}} \geq \frac{-a_i\Delta(x_{tm}\beta)+\Delta c_{e,tm}}{\sigma_{\Delta\epsilon}}$ are

$$\mathrm{E}\left[\frac{\nu^y_{i,ch,tm}}{\sigma_{\nu^y}}|\frac{\Delta\epsilon_{itm}}{\sigma_{\Delta\epsilon}} \geq \frac{-a_i\Delta(x_{tm}\beta)+\Delta c_{e,tm}}{\sigma_{\Delta\epsilon}}\right] = \rho_{\nu^y,\Delta\epsilon} IMR\left(\frac{a_i\Delta(x_{tm}\beta)-\Delta c_{e,tm}}{\sigma_{\Delta\epsilon}}\right),$$

and

$$\sigma^2_{\nu^y|s=ch} = \mathrm{Var}[\frac{\nu^y_{i,ch,tm}}{\sigma_{\nu^y}}|\frac{\Delta\epsilon_{itm}}{\sigma_{\Delta\epsilon}} \geq \frac{-a_i\Delta(x_{tm}\beta)+\Delta c_{e,tm}}{\sigma_{\Delta\epsilon}}] =$$
$$1 - \left(\rho_{\nu^y,\Delta\epsilon}\right)^2 IMR(a_i\Delta(x_{tm}\beta) - \Delta c_{e,tm})(a_i\Delta(x_{tm}\beta) - \Delta c_{e,tm} + IMR(a_i\Delta(x_{tm}\beta) - \Delta c_{e,tm})).$$

The means and variance of $\nu^y|s = tps$ are derived in a similar manner.