Should I Stay or Should I Go: Investor Protection, Firm Selection and Aggregate Productivity

Bernardo Morais*
UCLA

January 23, 2011

Abstract

The large cross-country variation in income per capita and aggregate productivity is highly correlated with various measures of investor protection. To account for this pattern I propose a theory of aggregate productivity based on firm selection. According to this theory, improvements in investor protection reduce the cost of credit of young firms, increasing the value of starting a project anew. This increase in the outside option of an incumbent raises the exit rate of low quality firms. A consequence of this selection mechanism is that the average productivity of old firms relative to young ones is increasing in the quality of investor protection. Using a comprehensive cross-country panel of firms, I show that this prediction holds in the data. What is more, quantitative simulations indicate that this selection mechanism accounts for around 25 percent of the cross-country differences in aggregate productivity. The model provides additional testable predictions reenforcing the plausibility of the theory. I show that in countries with better investor protection there is a larger share of young firms, these firms start larger, grow more slowly and deleverage more quickly. Finally, I analyze the effect that an exclusion from credit markets for an insolvent debtor has on the cost of credit and on aggregate outcomes. I find that the length of the exclusion that maximizes income per capita, is decreasing in the quality of investor protection.

---

*bernardo.cruz.morais@anderson.ucla.edu - I thank Daniel Dias, Sebastian Edwards, Paola Giuliano, Mark Grinblatt, Hugo Hopenhayn, Edward Leamer, Lee Ohanian, Claudia Ruiz, Nico Voigtlander, Romain Wacziarg, and Mark Wright, for their comments and guidance. I also thank seminar participants at the Anderson School and the Economics department at UCLA, and at the EconCon at Princeton. Financial support from the Fundação para a Ciência e Tecnologia and Fundação Gulbenkian are grateful acknowledged. Please download the latest version at sites.google.com/site/bcruzmorais/research.

1 Introduction

The large cross-country variation in income per capita reflects differences in aggregate productivity, and is highly correlated with various measures of investor protection.[1,2] In this paper, I analyze, both theoretically and empirically, the impact that investor protection has on the cost of debt and on firm dynamics, and propose a theory of aggregate productivity based on firm selection.

The main questions in the paper are: How does the quality of investor protection impact the cost of credit, and consequently firm policies such as leverage, growth and entry/exit? What is the quantitative importance of this impact in explaining the large cross-country differences in aggregate productivity and income per capita? To answer these questions I create a general equilibrium model in an incomplete markets setting with default risk. In the model economy, the savings accumulated by households are invested by entrepreneurs in projects with uncertain returns. Moreover, in the event of default the amount of outstanding debt that can be recouped by the creditor is positively related with the quality of investor protection. Consequently, the cost of credit depends not only on the probability of default but also on the quality of investor protection.

The main insight of this paper is that improvements in the quality of investor protection increase the selection of firms on productivity. In economies with better investor protection the cost of credit is lower, thereby increasing the value of starting a firm. This increase in the outside option of incumbents, induces managers of low quality firms to drop their projects and start anew. Consequently, the turnover rate of low quality firms is higher in countries with better investor protection. This increase in the exit rate of low quality firms augments aggregate productivity and implies that in these countries older firms are relatively more productive than younger ones.[3] Using the recovery rate of creditors as an indicator of investor protection, I show that the posited selection mechanism is verified in the data.[4] For example, in an economy with a recovery rate in the top-quartile, firms with more than ten years have a median productivity 15 percentage points higher than that of firms with less than three years. Conversely, in countries where the recovery rate is in the bottom quartile this difference is only 1.5 percentage points. What is more, quantitative simulations of the economy indicate that this increase in firm-turnover has important aggregate effects and can explain around 25

[1] Klenow and Rodriguez-Claire 1997; Prescott 1998; Hall and Jones 1999.

[2] Following Djankov et al. (2008), I relate the quality of investor protection with the efficiency of the set of legal mechanisms and institutions that allow lenders to pursue a defaulting borrower's income and assets.

[3] In the model, the low quality firms are endogenously selected out of the market. Consequently, older cohorts will be on average more productive, with this effect being stronger in countries with better investor protection.

[4] Recovery rate is measured as the average of the present value recouped by creditors for every dollar lent to an insolvent firm.

percent of the cross-country differences in aggregate productivity and income per capita. Additionally, the model provides several concurrent predictions on firm dynamics (i.e. leverage, growth) and on aggregate outcomes (i.e. share of young firms, income per capita) that reenforce the plausibility of this theory. More concretely, from the model we obtain that an improvement in investor protection: increases the entry size and lowers firm growth; increases the share of young firms; and increases the speed of deleverage. Using a comprehensive cross-country panel of firms, I show that these novel predictions hold in the data.[5]

The impact of investor protection on the aggregate economy is large. Consequently, I analyze possible actions to be taken by public policy agents aimed at reducing the consequences of this impact. In particular, I analyze the length exclusion from credit markets after default, that maximizes aggregate income.[6] Using a calibrated version of the model, I find that the optimal length of exclusion is decreasing in the quality of investor protection.[7] For example, in an economy with a Russian recovery rate of 25 cents a defaulter should be excluded from credit markets for an average of six years. Conversely, in a country with a recovery rate of 85 cents, as in the U.K., an entrepreneur who defaults on his debt should be excluded for an average of four years.[8]

The outline of the model is as follows: The economy is populated by three sets of agents, a representative consumer, financial intermediaries and entrepreneurs. The representative consumer supplies labor inelastically and invests his savings with financial intermediaries. Financial intermediaries are competitive, risk-neutral and lend funds to entrepreneurs at a risk-adjusted interest rate. An novel feature of this model is that in the event of default the lender only recovers a fraction of the outstanding debt. The magnitude of this fraction represents the quality of investor protection and impacts the interest rate demanded. Finally, there is a fixed mass of entrepreneurs who can run one firm at a time and using labor and capital as inputs. An important assumption of this model, which is consistent with the data, is that the uncertainty on the returns of a firm is decreasing with age.[9] This

[5]These results are robust when controlling for size, sector and country characteristics.

[6]In the United States federal law allows credit bureaus to report past bankruptcies up to ten years from the day of the filing. Musto (1998) documented that this bankruptcy flag has an important impact on credit access. Therefore, this exclusion from credits has real effects on the actions of creditors and consequently on the interest rates demanded by lenders. In this paper, I analyze the impact that variations on the length of exclusion has on the overal economy.

[7]From the model we have that an exclusion from credit markets presents a tradeoff. On the one hand, an increase in the length of exclusion reduces the incentive to default, and consequently the risk-premium on debt. On the other hand, this harsher punishment reduces the incentives for risk-taking, and may lead entrepreneurs into forgoing good investment opportunities. Taking this tradeoff into account we have that the optimal punishment is decreasing in the level of investor protection.

[8]I chose the U.K. and Russia as my benchmark economies for three reasons: First, my database only includes information on European firms. Second, the quality of data for the U.K. and Russia is relatively high. Third, their recovery rates place them in the top and bottom quartile respectively.

[9]This empirical finding was documented empirically for the U.K. manufacturing sector by Evans (1987). In this paper I

reduction in uncertainty makes the cost of credit dependent on firm age. I model this decrease in uncertainty by assuming that the idiosyncratic component of the productivity of firms has a variance decreasing with age.[10,11] Each period, and after production is complete, the entrepreneur decides whether to continue the project or to exit. This decision, depends on the current state of the firm and on the value of the outside option, and will affect the aggregate productivity in the economy.

The model is solved using two parameterizations. The first one simplifies the productivity process and provides an illustration of the basic selection mechanism. Furthermore, this parameterization uncovers several analytical predictions of the theory. For example, the model predicts that improvements in the recovery rate lower the cost of credit, increasing the outside option of starting a firm. This larger outside option causes higher firm turnover of low quality firms, leading us to the second prediction that countries with better recovery rate should have a larger share of entrant firms. This prediction is supported empirically. In countries with a quality of investor protection in the top quartile, firms with less than four years old represent 25 percent of all firms in the economy. Conversely, in countries with a recovery rate in the bottom quartile only 10 percent of all firms have less than four years old. A third prediction of the model is that in countries with better recovery rate the lower cost of credit allows firms to start larger and closer to their optimal size. This larger initial size implies that in these countries younger firms will grow more slowly with this cross-country difference dissipating with older age-cohorts. Again, this prediction holds in the data. For instance, firms less than four years old, in a country with a recovery rate similar to that of Russia, have an average growth rate 20 percentage points higher than a young firm in the United Kingdom. This difference is reduced to 4 percentage points when we analyze firms with more than ten years.[12] Finally, the model predicts that deleverage is faster in countries with higher recovery rate. In countries with a lower credit cost, firms start at a larger scale and obtain higher profits, which allow them to repay their debt and achieve the optimal long-term leverage relatively faster. I find that a decrease in recovery rate from the one in the U.K. to the one in Russia decreases the deleverage rates between firms with one and ten years by 4 percentage points.

In the second parameterization, I use the model to quantitatively analyze the cross-country patterns in aggregate productivity. To discipline the analysis I calibrate the model requiring that the benchmark model with a high quality of investor protection matches the data on firm size distribu-

provide further evidence that the uncertainty on the returns of the firm is in fact decreasing with age.

[10]This assumption tries to capture in a reduced form, the learning mechanism posited by Jovanovic (1982) and Jovanovic and Nyarko (1996).

[11]This implies that the risk of the firm, and consequently the cost of credit, is age dependent.

[12]These predictions are robust to size and sectoral controls.

tion, firm growth and firm leverage from the U.K.. I then use the cross-country data on recovery rate to analyze the impact that this measure of investor protection on aggregate economy. This exercise also allows me to perform counterfactual policy experiments.

I find that investor protection has sizable effects on the distribution of firms across age-cohorts as well as in the aggregate economy. For example, I find that variations in the recovery rate can explain roughly 80 percent of the cross-country differences in the share of young firms. As discussed above, these variations in recovery rate can explain around 25 percent of the differences in aggregate productivity and income per capita.

Finally, and for policy purposes, I analyze alternative instruments aimed at reducing the impact of low investor protection on the cost of credit. As the interest rate is mainly affected by the probability of default, I study the consequences of varying the length of exclusion from credit markets after insolvency.[13] Quantitative simulations of the model indicate that the optimal punishment is decreasing with the level of recovery rate. This indicates that in countries with worse investor protection the reduction of the default-premium is relatively more important than reducing the relative risk-aversion of entrepreneurs.

Related Literature - This work bridges two large strands of the literature. The first, relates investor protection and financial frictions with economic development, while the second relates financial frictions to firm dynamics. Additionally, this paper contributes to the literature on empirical firm dynamics by uncovering a series of cross-country patterns on the productivity, growth and financing of firms.

The literature relating investor protection and financial frictions to economic development is vast. It includes empirical work by King and Levine (1993), La Porta et al. (1996, 1997) and theoretical and quantitative work by Amaral and Quintin (2007), Erosa and Cabrillana (2008), Castro, Clementi and MacDonald (2009), Buera, Shin and Kaboski (2010), and Greenwood, Sanchez and Wang (2010). The literature relating financial frictions with firm dynamics includes: Hopenhayn (1992), Cooley and Quadrini (2001), Albuquerque and Hopenhayn (2004), Clementi and Hopenhayn (2006) and Luttmer (2007).

This paper is closely related and complementary to two others papers. Djankov et al. and Arel-

[13] Increasing the length of the punishment has two opposing effects. On the one hand, a harsher punishment decreases the incentive for default, diminishing its probability and consequently the risk-premium on debt. On the other hand, longer exclusions increase risk aversion, leading entrepreneur to potentially too conservative policies. Furthermore, a higher punishment on default induces leveraged firms with relative low quality to stay in the market until they are able to pay out their debt.

lano, Bai and Zhang (2010). Djankov *et al*, create a cross-country index of investor protection that measures the recovery rate of lenders in case of firm insolvency. I use this measure of investor protection to compute the cost of credit of firms across countries. The model in this paper is most closely related to that of Arellano *et al*. Using an incomplete markets setting with default risk, they analyze the impact of financial frictions on the financing and growth of firms across countries. My work differs from their paper in two ways: First, I use a reduced form of the learning mechanism proposed by Jovanovic (1982) and assume that the uncertainty on firms' returns is decreasing with age.[14] This reduction in the risk of firms creates an age-dependence on the cost of credit, which has important effects on the exit decision of entrepreneurs. Second, the general equilibrium nature of my model allows me to do welfare analysis and run policy experiments studying their effects on aggregate outcomes.

Finally, I provide novel empirical results on the cross-sectional productivity, growth and financing of firms. To the best of my knowledge, I believe I am the first in analyzing the average productivity of firms across age-cohorts and across countries. I show that older firms are relatively more productive in countries with lower cost of credit. In regard to growth and financing of firms only Arellano *et al* have studied these patterns across countries.[15] Nevertheless, their focus is on the importance of size, as collateral, in the growth and leverage policies of firms. In my model, I focus on age as an important predictor on these patterns across countries.

The rest of the paper is organized as follows. Section 2 describes the firm-level dataset and documents the facts around which the model will be build. Section 3 introduces and characterizes the model, which is solved using two parameterizations. The first allowing for analytical solutions, while the second calibrates it to match several moments of the data. Section 4 tests empirically the main predictions of the model. Section 5 concludes.

2 Data and Empirical Facts

In this section I describe in detail the two main data sources and document several relevant summary statistics. The first dataset is provided by the *Doing Business* and presents data at a country level on different outcomes of the insolvency process. In particular, it provides data on the recovery rate recouped by creditors of an insolvent firm, the time to resolve the insolvency process and the sum

[14]This decrease of firm uncertainty with age was documented by Evans (1987). I provide further evidence in Section 2 supporting it.

[15]Rossi-Hansberg and Wright (2007) analyze the growth patterns of firms only for the U.S., whereas Booth, Aivazian, Demirguc-Kunt and Maksimovic (2001) analyze, the non-representative, public firms in 10 developing countries.

of all legal costs involved. The second dataset, provided by the *Bureau Van Dijk*, contains financial information on firms in 40 European countries. This dataset is extremely valuable, as it not only provides detailed information in a panel format on firms across different European countries, but it is also representative of the universe of firms within each country.

2.1 Investor Protection

In this paper I relate the costs of the insolvency process with the quality of investor protection. The information on the outcomes of the bankruptcy procedures is provided by Doing Business, and follows the methodology developed in Djankov *et al* (2008). In that work, they presented a case study of an insolvent firm to legal practitioners in 88 countries. The case, developed jointly with the Committee on Bankruptcy of the International Bar Association, was representative of the insolvency process of a midsized firm. The firm had a given number of employees, capital and ownership structure whose value as a going concern exceeded its value if sold piecemeal. The case was identical across countries, except that the economic values were all normalized by the country's per capita income. Using several outcomes of the insolvency process, such as legal costs, nominal interest rate and length of the procedure, they computed the recovery rate of creditors. This recovery rate is an indicator that summarizes the efficiency of the bankruptcy proceedings. It is measured as the present value, net of all costs, recouped by the creditors of a defaulting firm. More concretely, the recovery rate R of a representative firm in country j is

$$R_j = \frac{100GC_j + 70\,(1 - GC_j) - 12\,(P-1) - 100c_j}{(1+r_j)^{t_j}}$$

where t represents the time in years to recover the funds, r is the nominal interest rate, c is the cost to complete the insolvency procedure expressed as a percentage of the bankruptcy estate, GC is a dummy variable that takes the value 1 if continuing operations is the going concern of the insolvency process, and takes the value 0 otherwise. Finally P represents the priority ($P \in \{1, 2, 3, 4\}$) that is assigned to the secured creditor. For example, if the secured creditor is paid first then $P = 1$, if the secured creditor is ranked after three other claimant groups $P = 4$. The results of the recovery rate are present in the second column of Table 1.[16] The third column presents the length of time, in years, to complete the insolvency process while the fourth column presents all the legal costs, in cents, associated with it.[17]

[16] In Table 1, I only present results on the financial development for the countries that I have firm level data on.

[17] The results present in Table 1 are obtained using data for the years 2004 and 2005.

Indicators of Investor Protection and Financial Development

Country	Recovery Rate	Years	Cost	Private	Depth
Fin	0.88	0.9	0.04	0.70	0.54
Ire	0.88	0.4	0.09	1.42	0.88
Ned	0.87	1.1	0.04	1.61	1.10
Nor	0.94	0.9	0.01	0.76	0.54
Bel	0.86	0.9	0.04	0.72	1.03
UK	0.86	1	0.06	1.52	1.22
Ice	0.82	1	0.04	1.97	0.61
Spn	0.77	1	0.15	1.30	1.00
Prt	0.73	2	0.09	1.40	0.96
Swe	0.81	2	0.09	1.04	0.47
Ita	0.43	1.2	0.18	0.86	0.59
Den	0.63	3.3	0.04	1.62	0.58
Ger	0.56	1.2	0.01	1.11	1.05
Fra	0.46	1.9	0.09	0.90	0.73
Gre	0.45	2	0.09	0.71	0.83
Lit	0.34	2	0.22	0.33	0.36
Slk	0.40	4.8	0.18	0.31	0.57
Est	0.37	3	0.09	0.62	0.41
Hun	0.39	2	0.15	0.48	0.47
Lat	0.36	3	0.13	0.55	0.39
Bul	0.34	3.3	0.09	0.38	0.54
Cro	0.29	3.1	0.15	0.56	0.64
Pol	0.26	3	0.22	0.28	0.42
Rus	0.25	3.8	0.09	0.23	0.29
Srb	0.21	2.7	0.23	0.23	0.22
Czh	0.15	9.2	0.18	0.33	0.68
Ukr	0.08	2.9	0.42		
Rom	0.07	4.6	0.09	0.17	0.30
$Corr(RR, i)$	1	-0.69	-0.66	0.75	0.59

Table 1: *Recovery Rate* is the amount recouped by creditors of an insolvent firm for each dollar lent. *Years* represents the average length of time necessary to resolve an insolvency process. *Cost* is the average legal expense disbursed in the bankruptcy procedure for each dollar of outstanding credit. *Private* is the ratio of credit to domestic firms over GDP. *Depth* is measured as the ratio of liquid liabilities of the financial system to GDP

In this paper, I relate the quality of investor protection with the cost of debt and consequently with the level of financial development. Next, I analyze the relationship between recovery rate and traditional measures of financial development. King and Levine (1993) propose measures of financial development such as *Private* and *Depth*. *Private* is the ratio of credit to private domestic firms over GDP. The assumption underlying this measure is that the larger the credit to private firms the more engaged financial intermediaries are in researching those same firms, in providing risk-management services and in facilitating transactions, rather than simply funneling credit to public enterprises. *Depth* is an indicator of the size of financial intermediaries and it is measured as the ratio of the liquid liabilities of the financial system to GDP. The data on these variables is present in the last two columns of Table 1 and was collected by Beck, Demirguc-Kunt, Levine (2007). These indicators of financial development are highly correlated with the recovery rate, as shown in the last row of Table 1.

2.2 Firm-level Data

The cross-country firm level data comes from Analyze Major Database from European Sources (Amadeus). Amadeus is a comprehensive, pan-European database provided by Bureau van Dijk. It is a highly useful database as it not only covers a large fraction of new and small firms across all industries but it is also representative of the universe of firms at national level.[18] The database started in 1997 and collects standardized data from 50 vendors across Europe, with the local source of the data generally the office of the Registrar of Companies. Amadeus presents standardized annual (for up to 10 years) on financial ratios, activities and ownership for approximately 5 million companies per year. The database includes accounting data in standardized financial format for balance sheets, income statements, and financial ratios. The accounts are transformed into a universal format to enhance comparison across countries though the coverage of these items varies across countries. I use period average exchange rates from the International Monetary Fund's International Financial Statistics to convert all accounting data into U.S. dollars. In addition to financial information, the dataset provides other information such as the age of the firm, ownership and legal status. Amadeus, assigns companies a three-digit NACE code, the European standard of industry classification, which can be used to classify firms and construct industry dummy variables.[19] For the empirical study and for the numerical exercises I use the financial information presented for the 2002 to 2005 years. I chose these consecutive years, as they are the ones with the most complete coverage. Given that not all firms present the necessary information for the analysis I need to impose a number of restrictions in order to clean the data.[20] I exclude firms that do not provide data on assets, liabilities, sales, year of incorporation, or that provide negative sales or assets. Furthermore, I exclude countries with less than

[18] In section 7.3 of the appendix I provide a comparison between the Amadeus sample and the Universe of firms present in the Eurostat. The Eurostat data, available at http://epp.eurostat.ec.europa.eu, uses the full national business registers as data sources.

[19] The NACE codes follow the NACE Revision 1 classification.

[20] A more detailed explanation on the the cleaning of the data is documented in section 7.1 of the appendix.

1000 firms.[21] These criteria leave us with 11 million observations in 28 countries: Belgium, Bulgaria, Croatia, Czech Republic, Denmark, Estonia, Finland, France, Germany, Greece, Hungary, Iceland, Ireland, Italy, Latvia, Lithuania, the Netherlands, Norway, Portugal, Romania, Russian Federation, Serbia, Slovakia, Spain, Sweden, Ukraine and the United Kingdom. In the appendix, and following Arellano *et al,* I show that the data for these countries is representative of the universe of firms, by comparing it with data reported by the European Commission. Small firms are slightly underrepresented in Amadeus and its fraction varies across countries, but this variation is not correlated neither with income level nor with the recovery rate.

The main focus of this paper is on the effect of investor protection on firm dynamics. Therefore, and for exposition purposes I divided the firms within each country into five age-bins, the amplitude of these bins being such that distribution of firms across each group is relatively equitative. In Table 2, I present the quality of the recovery rate along with the total number of firms per country, and the distribution of firms per age cohort.

In this work, I am interested in firm dynamics such as growth and leverage across age-cohort. Consequently, in Table 3 I present the median leverage and sales growth (CPI-deflated) for different age cohorts.[22,23] From the table it is clear that firms are highly dependent on external financing.[24] The median leverage for all firms in the sample is 72 percent. Furthermore, it is also clear that younger firms are more leveraged than their older counterparts irrespective of the recovery rate. The median leverage across countries of the youngest and oldest cohort is 81 and 57 percent respectively and in *all* countries firms deleverage with age. What is more, in 22 out of the 28 countries in the data, the mean leverage of the youngest cohort is larger than 75 percent. Regarding sales growth, there is also a pattern of larger growth for younger firms. Additionally, this pattern appears to be stronger in countries with lower recovery rate. For instance the mean growth (CPI-adjusted) in the youngest age-bin in Bulgaria, Croatia and Lithuania is 70, 58 and 63 percent whereas it is only 19, 21 and 31 percent in Belgium, Denmark and Finland respectively. Furthermore, growth is decreasing across age-cohorts in all countries in the sample. Whereas the median growth for younger cohorts is larger than 18 percent across all countries, the growth rate for the older cohorts is smaller than 10 percent

[21] I exclude from this analysis Austria, Belarus, Cyprus, Liechtenstein, Luxembourg, Macedonia, Moldova, Monaco and Switzerland.

[22] Leverage is defined as the ratio of the total debt to total assets.

[23] I define total debt as the sum of total liabilities, current and non-current. Current liabilities include all loans as well as all short-term loans from suppliers. Non-current liabilities includes long-term debt as well as other non-current liabilities such as provisions. Arellano *et al.* use a different measure of debt comprising short and long-term debt as well as short-term loans from suppliers. Using their definition does not alter the pattern of decreasing leverage with age.

[24] External financing comprehends *all* liabilities of the firm. From short-term credit from suppliers to longer term credit from banks.

Number of Firms per Age-Cohort

			Share of Firms				
			Age-Cohorts				
Country	Recovery Rate	No. Firms	<4	[4,7]	[8,11]	[12,18]	18<
Fin	0.883	216,773	0.12	0.18	0.21	0.28	0.21
Ire	0.877	38,204	0.25	0.29	0.17	0.17	0.12
Ned	0.874	45,787	0.10	0.14	0.12	0.18	0.45
Nor	0.944	432,793	0.22	0.25	0.16	0.38	
Bel	0.86	974,593	0.18	0.20	0.16	0.26	0.20
UK	0.86	1,139,416	0.34	0.23	0.12	0.13	0.18
Ice	0.82	39,271	0.36	0.27	0.13	0.11	0.12
Spn	0.77	1,807,117	0.22	0.26	0.21	0.20	0.11
Prt	0.73	173,222	0.19	0.22	0.15	0.20	0.24
Swe	0.81	565,565	0.15	0.15	0.16	0.29	0.24
Ita	0.71	1,209,776	0.21	0.22	0.14	0.19	0.24
Den	0.63	165,227	0.30	0.28	0.10	0.17	0.16
Ger	0.56	148,490	0.15	0.19	0.15	0.21	0.30
Lit	0.34	15,998	0.20	0.29	0.36	0.14	
Fra	0.46	2,436,583	0.23	0.21	0.17	0.20	0.19
Gre	0.45	71,390	0.19	0.22	0.18	0.22	0.19
Slk	0.40	14,615	0.16	0.31	0.35	0.16	0.02
Hun	0.39	350,581	0.33	0.25	0.26	0.15	
Est	0.37	139,259	0.34	0.31	0.24	0.11	0.01
Lat	0.36	13,298	0.17	0.29	0.38	0.16	
Bul	0.34	15,475	0.19	0.25	0.27	0.17	0.12
Cro	0.29	62,885	0.04	0.13	0.48	0.30	0.05
Pol	0.26	68,339	0.13	0.24	0.24	0.24	0.15
Rus	0.25	377,692	0.32	0.28	0.32	0.04	0.03
Srb	0.21	49,513	0.06	0.16	0.39	0.33	0.06
Czh	0.15	158,346	0.20	0.31	0.33	0.16	0.01
Ukr	0.08	39,259	0.13	0.37	0.38	0.06	0.06
Rom	0.07	774,217	0.30	0.20	0.33	0.17	
Total	0.52	11,543,684	0.21	0.24	0.24	0.19	0.14

Table 2: *Recovery rate* is the average amount recouped by creditors of an insolvent firm, for each dollar of outstanding credit. The *Number of firms* is the total number of firm-year observations per country for 2002-2005. The *Share of firms* represents the fraction of firm-year observations per age-cohort. The share of firms older than 18 years-old in Norway, Hungary, Latvia, Bulgaria and Romania is lower than 0.5 percent.

for all countries with the exception of Iceland.

Leverage and Asset Growth

	Leverage					Sales Growth				
	Age-Cohorts					Age-Cohorts				
Ctry	<4	[4,7]	[8,12]	[13,16]	16<	<4	[4,7]	[8,12]	[13,16]	16<
Fin	0.65	0.59	0.56	0.54	0.52	0.31	0.18	0.14	0.13	0.08
Ire	0.81	0.71	0.65	0.61	0.55	0.45	0.18	0.13	0.11	0.03
Ned	0.82	0.76	0.74	0.71	0.69	0.27	0.14	0.10	0.07	0.04
Nor	0.81	0.79	0.77	0.73		0.21	0.12	0.09	0.06	
Bel	0.76	0.73	0.69	0.64	0.60	0.19	0.12	0.09	0.06	0.04
UK	0.81	0.72	0.68	0.64	0.56	0.39	0.17	0.11	0.08	0.05
Ice	0.78	0.82	0.78	0.75	0.68	0.29	0.24	0.17	0.20	0.16
Spn	0.86	0.80	0.74	0.64	0.58	0.37	0.15	0.08	0.07	0.04
Prt	0.82	0.78	0.73	0.70	0.64	0.23	0.12	0.05	0.03	0.00
Swe	0.69	0.65	0.62	0.59	0.59	0.30	0.17	0.12	0.10	0.08
Ita	0.87	0.84	0.82	0.79	0.74	0.22	0.09	0.04	0.04	0.02
Den	0.67	0.63	0.63	0.61	0.56	0.21	0.13	0.07	0.07	0.04
Ger	0.74	0.75	0.76	0.73	0.73	0.34	0.17	0.09	0.07	0.06
Lit	0.73	0.63	0.53	0.51		0.63	0.34	0.22	0.18	
Fra	0.78	0.73	0.68	0.66	0.62	0.24	0.10	0.06	0.04	0.02
Gre	0.68	0.67	0.65	0.62	0.58	0.33	0.16	0.09	0.05	0.05
Slk	0.70	0.66	0.61	0.57	0.44	0.39	0.19	0.11	0.09	0.05
Hun	0.71	0.67	0.60	0.56	0.46	0.37	0.20	0.12	0.08	0.05
Est	0.59	0.53	0.49	0.47	0.47	0.44	0.24	0.12	0.13	0.08
Lat	0.82	0.76	0.66	0.62		0.73	0.34	0.17	0.15	
Bul	0.78	0.74	0.71	0.64	0.48	0.70	0.33	0.22	0.18	0.09
Cro	0.80	0.74	0.70	0.66	0.50	0.58	0.15	0.06	0.04	0.05
Pol	0.69	0.65	0.60	0.57	0.49	0.29	0.15	0.10	0.07	0.05
Rus	0.78	0.71	0.62	0.61	0.63	0.50	0.24	0.17	0.10	0.06
Srb	0.74	0.65	0.62	0.60	0.41	0.71	0.27	0.28	0.20	0.06
Czh	0.76	0.74	0.67	0.60	0.33	0.45	0.21	0.14	0.08	0.03
Ukr	0.76	0.60	0.51	0.50	0.46	0.68	0.23	0.11	0.02	0.05
Rom	0.85	0.86	0.84	0.82		0.41	0.23	0.18	0.13	

Table 3: *Leverage* is defined as the sum of total liabilities over total assets. *Growth* is the average real growth rate of total sales (CPI-deflated). Countries are ordered by decreasing level of recovery rate

2.3 Productivity Process and Firm Age

Several studies have shown that both the growth rate and the volatility of growth are negatively related both with size and with age.[25] Using the results of previous literature, Cooley and Quadrini (2001) have concluded that *"Conditional on size, the dynamics of firms (growth, volatility of growth, job creation, job destruction, and exit) are negtively related to the age of the firms"*.

In this section I analyze whether these results also hold for firm productivity. To calculate the

[25]Evans (1987), Hall (1987) and Davis (1996).

productivity of firms, I assume that technology is of the type

$$q_i = A_i k_i^{\alpha_j} l_i^{\beta_j}$$

where q_i, k_i and l_i represent the production, capital and labor used by firm j while the input shares α and β are at the sector-j level.[26] Although I have data on total sales, I do not observe neither physical production nor prices. Since I cannot disentangle price p_i from physical production q_i, I use revenue-productivity TFP_i^S as the benchmark measure of productivity such that[27]

$$\begin{aligned} sales_i &= p_i q_i \\ TFP_i^S &\equiv p_i A_i = \frac{sales}{k_i^{\alpha_j} l_i^{\beta}} \end{aligned}$$

To check whether uncertainty decreases with age, I will conduct two tests. For the first one, I assume that the productivity process of firms follows a first-order autoregressive process. For this test, and for each country, I estimate

$$\ln TFP_{j,t}^S = \delta_{j,t} + \rho_\tau \ln TFP_{\tau,j,t-1}^S + \varepsilon_{\tau,j,t}$$

where $TFP_{j,t}^S$ represents the productivity of firm j at time t, while $\delta_{j,t}$ represents a sector-year fixed effect.[28] The results of this estimation for the different age-cohorts and for different countries are present in Table 4

	France		Spain		Italy	
Age-Cohort (τ)	ρ_τ	$\sigma^2_{\varepsilon,\tau}$	ρ_τ	$\sigma^2_{\varepsilon,\tau}$	ρ_τ	$\sigma^2_{\varepsilon,\tau}$
$\tau \leq 4$	0.67	0.23	0.69	0.40	0.80	0.32
$5 \leq \tau \leq 12$	0.72	0.19	0.75	0.33	0.82	0.25
$\tau \geq 13$	0.77	0.16	0.78	0.29	0.83	0.20

Table 4: This Table presents the results of the persistence and variance of the productivity process. These countries were chosen as they have the largest number of firms.

For the countries in the sample, the persistence of the productivity process increases with age, while its variance decreases with age.

[26] I calculate the input shares at the 2-digit NACE level. These shares are calculated for French firms, which have the most comprehensive data, and the factor intensity is assumed to be equal across countries.

[27] I also estimate productivity using value added instead of sales. I chose TFP^R as the benchmark on productivity given the relative unavailability of data on value added. For further details see Appendix.

[28] Sectors are defined at the 1-digit NACE (rev 1.2).

For the second test, I estimate for each age-cohort τ

$$\Delta \ln TFP = \delta_{j,t} + \varepsilon_{\tau,j,t}$$

where and collect the residuals. The expected value of these squared residuals is represented in Table 5

	France	Spain	Italy
Age-Cohort (τ)	$E\widehat{\varepsilon}^2_{\tau,j,t}$	$E\widehat{\varepsilon}^2_{\tau,j,t}$	$E\widehat{\varepsilon}^2_{\tau,j,t}$
$\tau \leq 4$	0.31	0.32	0.35
$5 \leq \tau \leq 12$	0.25	0.24	0.25
$\tau \geq 13$	0.20	0.19	0.20

Table 5

3 Model

In this section, I present a general equilibrium model to analyze the impact of the quality of investor protection on the cost of credit, and consequently, on firm dynamics and on aggregate outcomes. In particular, I focus on the recovery rate of creditors as a measure of the quality of investor protection.[29]

The two central features of the model are the link between recovery rate of creditors and interest rate demanded, and on the assumption that the volatility of firms decreases as they grow older. The main mechanism is that the reduction in the cost of credit, resulting from an improvement in the recovery rate, increases the outside option of starting a new firm leading incumbent entrepreneurs with worse firms to drop them and start anew. This selection effect increases aggregate productivity and has an important impact on aggregate income. For exposition purposes, and leading up the quantitative analysis, I present an example of the model with closed form solutions that uncovers novel and testable predictions. Afterwards, I provide the quantitative results of the model calibrated to match several relevant moments of a benchmark economy. The quantitative nature of this calibration allows me not only to evaluate the impact of investor protection on firm selection and aggregate productivity, but also to perform counterfactual experiments. In particular, I analyze the optimal punishment for default, defined as length of exclusion from credit markets, and I show that this

[29]Recovery rate is the present value recouped by a creditor of an insolvent firm for each dollar of outstanding credit. It has a direct impact on the cost of credit given that, everything else constant, a decrease in the recovery rate increases the default premium demanded.

optimal punishment is decreasing in the recovery rate.[30]

3.1 Outline

The setup of the model is as follows: The economy is populated by a representative consumer, by financial intermediaries and by entrepreneurs. The representative consumer supplies a fixed amount of labor at a competitive wage and deposits his savings with financial intermediaries at a risk-free rate. Financial intermediaries are competitive, borrow their funds from the representative consumer and lend them to entrepreneurs at a risk-adjusted interest rate. Finally, there is a fixed mass risk-neutral entrepreneurs who can only run one firm at a time and who have in management their only possible occupation. The productivity process of firms has a transitory component whose variance is decreasing with age. Each period, after production is complete and before the transitory shock hits the firm, the entrepreneur decides whether to continue the project or to quit it and start a new one. In the latter scenario he must also decide whether to repay any outstanding debt or to default partially on it. In case of default he is penalized with a temporary exclusion from credit markets. This possibility of default induces financial intermediaries to demand a risk-premium which is dependent on the quality of investor protection of the economy as well as on the probability of default.

Representative Consumer - The economy is populated by an infinitely lived representative consumer with mass $\overline{L}$ and a discount rate β_c. He supplies labor inelastically for a wage w and deposits his savings S with a financial intermediary at a risk-free interest rate R_f. Each period, the consumer must decide how much to consume c and how much to save. Formally, the problem of the representative consumer is

$$Max \sum_{t=0}^{\infty} \beta_c^t U(c_t)$$

$$s.t.\ c_t + S_{t+1} = w_t \overline{L} + R_f S_t$$

Financial Intermediaries - The industry of financial intermediation is competitive as its members have free entry and exit. Financial intermediaries are risk-neutral, collect savings from consumers at rate R_f and lend them to entrepreneurs at a risk-adjusted rate $R_{j,t}$. This spread is due to the possibility of default. In this eventuality, the financial intermediary only recovers part of his credit, with the magnitude of this recovery rate ξ representing the quality of investor protection. All debt contracts with firm j have a one-period maturity where the lender receives $\frac{B_{j,t}}{R_{j,t}}$ units of capital for the promise to repay $B_{j,t}$ units the following period. The interest rate $R_{j,t}$ incorporates both the

[30] Punishment is measured as the expected number of periods of exclusion from credit markets after default.

probability of default $\pi_{j,t}$ as well as the recovery rate in case of default. More concretely, in the event of default the lender receives

$$\xi \min \{C_{j,t}, B_{j,t}\}, \ \xi \in [0,1]$$

where ξ represents the recovery rate of the economy, $C_{j,t}$ the residual value of the firm j at time t, and $B_{j,t}$ the outstanding debt.[31] This setup implies that in case of default the returns of the lender are

$$R_{j,t}\xi \min \left\{ \frac{C_{j,t}}{B_{j,t}}, 1 \right\}$$

Given the risk-neutrality and competitiveness of financial intermediaries, the risk-premium demanded is such that the deposit rate is equal to the expected lending return.

Entrepreneurs - Entrepreneurs have a mass 1, are risk neutral and have a probability ϕ of dying each period.[32] They maximize the present value of dividends at a discount rate β_e, and manage one firm at a time. Firms have neither an entry nor an exit cost but managers can only run one at a time, which implies that if they decide to start a new project they must close their previous one.

Regarding production, firms use capital and labor as inputs with their sales y being

$$\begin{aligned} y_{j,t} &= z_{j,t} k_{j,t}^{\alpha} l_{j,t}^{\gamma} \\ 0 &\leq \alpha, \gamma, \alpha + \gamma \leq 1 \end{aligned}$$

where $z_{j,t}$, $k_{j,t}$ and $l_{j,t}$ represent revenue productivity, capital and labor of firm j at time t. The good produced by the firm can be either used as consumption c or as investment i.[33] The capital accumulated by the firm depreciates at the rate δ, and is accumulated through periodical investments j which are made before the shock hits the firm. Consequently, the assets of a firm evolve according to

$$k_{j,t+1} = (1 - \delta) k_{j,t} + i_{j,t} \tag{1}$$

The productivity of the project is firm specific and has both a persistent and a transitory component. As documented in section 2, the persistence and variance of the productivity process increases and decreases with age respectively. Consequently, the productivity process z of a firm j with age τ

[31]The residual value of the firm includes the current net profits as well as the net value of the assets.
[32]For each death a new entrepreneur is born.
[33]There are no capital adjustment costs. Capital can be transformed into output at a constant one-to-one rate.

is such that

$$
\begin{aligned}
\ln z_{j,t} &= \rho_\tau \ln z_{j,t-1} + \ln \varepsilon_{i,\tau,t} && (2)\\
\ln \varepsilon_{i,\tau} &\sim i.i.d.\left(0, \sigma_\tau^2\right) &&\\
\rho_\tau &\leq \rho_{\tau+1}, \forall \tau \geq 1 && (3)\\
\sigma_\tau^2 &\geq \sigma_{\tau+1}^2, \forall \tau \geq 1 &&
\end{aligned}
$$

To reduce the state-space in the economy, I assume that the possible number of age-states τ is such that $\tau \in \{1, 2, ..., T\}$. Where each period, the firm has a probability $pr(\tau + 1|\tau)$ of evolving into the next phase of their lifecycle.

After current productivity is known and production is complete, managers decide whether to continue their project or to start a new independent firm. If the entrepreneur continues the project he must choose how much to invest and how much to borrow with the remaining funds being distributed as dividends d such that

$$
d_{j,t} + i_{j,t} \leq y_{j,t} - wl_{j,t} - B_{j,t-1} + \frac{B_{j,t}}{R_{j,t}} \tag{4}
$$

where the left-hand side represents the use of funds and the right-hand side their origins. In the event of firm exit, entrepreneurs decide whether to fully repay their outstanding debt, or to default and appropriate a fraction of it. If default occurs, entrepreneurs face a temporary exclusion from credit markets. This exclusion is measured as the probability p of reaccessing the market once default has occurred.[34]

The timing of this model economy is as follows: At the beginning of each period an equal number ϕ of entrepreneurs is born and dies. Newborn entrepreneurs have no private endowment and require credit to finance their initial investments. Given their capital, debt, productivity, age and consequent set of interest rates, entrepreneurs make their investment and financing decisions. After these decisions are complete, firms are hit with an idiosyncratic shock. Given the idiosyncratic shock the entrepreneur hires labor and once production is complete he decides whether to continue with the project or to exit. Furthermore, if he decides to exit he must choose whether to default or to fully

[34] This probability of reaccessing the market p can also be interpreted as the average number of periods P for which the defaulter is excluded from the market where $P = \frac{1}{p}$.

repay. Therefore, the value of firm j with capital K, debt B, productivity z and age τ at time t is

$$V\left(k_{j,t}, B_{j,t}, z_{j,t}, \tau_{j,t}\right) = \max\left\{V^{c}, V^{nodef}, V^{def}\right\} \tag{5}$$

which represents maximum value between continuing the project V^c, exiting the project with full repayment V^{nodef} , or exiting the project by partially defaulting on debt V^{def}.

The value of continuing with current project is

$$\begin{aligned} V^{c}\left(k_{j,t}, B_{j,t}, z_{j,t}, \tau_{j,t}\right) &= \max_{\{d,B,i,l\}} d_{j,t} + \beta_{e}\left(1-\phi\right)\int^{\tau}\int^{Z} V\left(K_{j,t+1}, B_{j,t+1}, z_{j,t+1}, \tau_{j,t+1}|.\right) f\left(z'|.\right) f\left(\tau'|.\right) \partial z \partial \tau \\ s.t.\ d_{j,t} + i_{j,t} &\leq y_{j,t} - w_{t} l_{j,t} - B_{j,t} + \frac{B_{j,t+1}}{R_{j,t+1}} \\ k_{j,t+1} &= \left(1-\delta\right) k_{j,t} + i_{j,t} \end{aligned}$$

Conversely, if the entrepreneur closes the firm and decides to repay his outstanding debt then his value is

$$\begin{aligned} V^{nodef}\left(k_{j,t}, B_{j,t}, z_{j,t}, \tau_{j,t}\right) &= \max\ d_{j,t} + \beta_{e}\left(1-\phi\right)\int^{Z} V\left(k_{j,t+1}, B_{j,t+1}, z_{j,t+1}, \tau_{j,t+1}\right) \partial z \\ s.t.\ d_{j,t} + i_{j,t} &\leq y_{j,t} - w_{t} l_{j,t} - B_{j,t} + \frac{B_{j,t+1}}{R_{j,t+1}} \\ k_{j,t+1} &= i_{j,t} \\ \tau_{j,t+1} &= 1 \end{aligned}$$

where V^o represents the value of a starting a new firm. Finally, if the entrepreneur defaults, his value is

$$\begin{aligned} V_{t}^{def}\left(k_{j,t}, B_{j,t}, z_{j,t}, \tau_{j,t}\right) &= \max\ d_{j,t} + \frac{\beta_{e}\left(1-\phi\right)p}{1-\beta_{e}\left(1-\phi\right)\left(1-p\right)}\int^{Z} V\left(k_{j,t+1}, B_{j,t+1}, z_{j,t+1}, \tau_{j,t+1}\right) \partial z \\ s.t.\ d_{j,t} + i_{j,t} &\leq R_{j,t} + \frac{B_{j,t+1}}{R_{j,t+1}} \\ R_{j,t} &= \max\left\{\left(1-\xi\right) C_{j,t}, C_{j,t} - \xi B_{j,t}\right\} \\ C_{j,t} &\equiv y_{j,t} - w_{t} l_{j,t} + \left(1-\delta\right) k_{j,t} \\ k_{j,t+1} &= i_{j,t} \\ \tau_{j,t+1} &= 1 \end{aligned}$$

Stationary Competitive Equilibrium

From the problem of the consumer we know that the representative consumer will save up to the point where his returns on savings are equal to his discount rate. Therefore in equilibrium the gross deposit rate R_f is

$$R_f = \frac{1}{\beta_c}$$

The free-entry condition associated with the risk-neutrality of the financial intermediaries implies that their profits are zero. This leads us to the following condition

$$R_f = R_{j,t}\left(1-\pi_i\right) + R_{j,t}\xi \int_{\underline{z}}^{\widetilde{z}} \min\left\{\frac{y_{j,t} - w_t l_{j,t} + (1-\delta)\,k_{j,t}}{B_{j,t}}, 1\right\} f(z)dz \tag{6}$$

where the left-hand side represents the deposit rate, while the right-hand side represents the lending rate of financial intermediaries. This lending rate depends on π_i, the probability of default of firm i, on z, the productivity of the firm and on $\widetilde{z}$ which is the productivity below which the firm defaults. Therefore, the risk adjusted interest rate is

$$R_{j,t} = \frac{R_f}{1 - \pi_{j,t} + \xi \int_{\underline{z}}^{\widetilde{z}} \min\left\{\frac{y_{j,t} - w_t l_{j,t} + (1-\delta)k_{j,t}}{B_{j,t}}, 1\right\} f(z)dz} \tag{7}$$

To provide some intuition to the expression above, assume that $\min\left\{\frac{y_{j,t}-w_t l_{j,t}+(1-\delta)k_{j,t}}{B_{j,t}}, 1\right\} = 1$, $\forall z$. In this case, condition 6 becomes

$$R_f = R_{j,t}\left(1-\pi_{j,t}\right) + R_{j,t}\pi_{j,t}\xi$$

implying that interest rate is

$$R_{j,t} = \frac{R_f}{1 - \pi_{j,t}\left(1-\xi\right)} \tag{8}$$

where it is clear that interest rate is positively related with the probability of default, and negatively related with the recovery rate.

For the labor market to clear it must be that the supply of labor of the representative consumer must equal the labor demand such that

$$\overline{L} = \int_0^1 l_{j,t} di$$

Furthermore, the savings accumulated by households and deposited with financial intermediaries will have a rate of return such that in equilibrium it this supply of funds equals to the demand of entrepreneurs. Therefore in the steady-state equilibrium

$$S_t = \int_0^1 \frac{B_{j,t}}{R_{j,t}} dj$$

Finally, the market for the consumption good clears when aggregate demand is equal to the aggregate supply

$$c + S_t + \int_0^1 (d_{j,t} + i_{j,t})\, di = \int_0^1 y_{j,t} di + S_{t-1}$$

Stationary Competitive Equilibrium - A stationary competitive equilibrium for this economy consists of: an invariant distribution of firms $G(K, B, z, \tau)$, policy functions of households c, i_c policy functions of entrepreneurs $d(K, B, z, \tau), i(K, B, z, \tau), B(K, B, z, \tau), l(K, B, z, \tau)$ loan contracts offered by creditors $R_{j,t}(K, B, z, \tau)$ and prices w, R_f such that:

1. Given the schedule of loan contracts offered, the policy and value functions of firms satisfy their optimization problem;
2. Loan contracts reflect the firm's default probabilities such that all financial intermediaries make zero profits in expectation on all contracts;
3. The distribution of firms is fixed and is consistent with the policy functions of firms and idiosyncratic shocks;
4. Labor, capital and goods markets clear.

3.2 Example - Simple productivity process

To provide some intuition on the basic mechanisms of the model, I use a parameterization that allows for analytical results. The main assumption in this example is that all uncertainty is resolved as soon as firms enter the market. This is equivalent to say that the productivity of a firm is constant throughout its life, and that it becomes known as soon as production occurs. Furthermore, and for analytical convenience, I assume that the productivity of the firm is drawn from an uniform distribution. Therefore,

$$\begin{aligned} z_{j,t} &= z_j \\ z &\sim U[\underline{z}, \overline{z}] \end{aligned}$$

The remaining parameters, described in the Table 6, are such that: i) Firms use capital as the only input, have constant returns to scale and an exogenous cap on size. ii) The rate of discount is the same for entrepreneurs and lenders. iii) There is complete depreciation of capital. iv) There is no punishment for a defaulter.

Table 6: Example

Description	**Parameters**
Technology	$\alpha = 1$
Cap on size	$k_t \leq \overline{k}$
Discount rate	$\beta_e = \beta_c$
Depreciation	$\delta = 1$
No exclusion	$p = 1$

As stated above, since all uncertainty is resolved after entry occurs, there are only two types of firms: *entrants* and *incumbents*. The difference between these firms, is in the uncertainty and therefore default-premium that entrant firms face. Conversely, and since all uncertainty is resolved after entry, incumbent firms face a risk–free interest rate.[35]

This difference in the cost of credit is reflected in differences in the size of firms. Entrant firms, who have no personal funds, have a size

$$k^e = \frac{B^e}{R^e} \leq \overline{k}$$

with $R^e = \frac{R_f}{1-\pi(1-\xi)}$ which is dependent on the quality of investor protection ξ and on the probability of default π.[36] Incumbent firms, who face no uncertainty and therefore no risk-premium, have their size capped by $\overline{k}$. Therefore

$$k^i = \overline{k}$$

3.2.1 Firm Value and Productivity Threshold

In order to pin down the magnitude of the selection mechanism we must obtain the entrant firm's value along with the productivity cutoff. As endogenous exit *only* occurs at the end of the first period,

[35]Incumbent firms, face a risk-free rate up to the indebtedness point where default has higher value than continuation.
[36]In this example I am assuming that in equilibrium $\theta_L > R^e$.

and given that z has an uniform distribution, the value of an entrant firm is

$$\begin{aligned} V^o &= \max_{\{B,\}} z^e \frac{B^e}{R^e} - B^e \left(1 - \pi^e \left(1 - \xi\right)\right) + \\ &\quad \beta \left(1 - \phi\right) \left(\pi^e V^o + \left(1 - \pi^e\right) \frac{\left(z^i - R_f\right) \overline{k}}{1 - \beta \left(1 - \phi\right)} \right) \\ \pi^e &= \frac{\widetilde{z} - \underline{z}}{\overline{z} - \underline{z}};\ z^e \equiv \frac{\underline{z} + \overline{z}}{2};\ \ z^i \equiv \frac{\widetilde{z} + \overline{z}}{2} \end{aligned} \tag{9}$$

where $\widetilde{z}$ represents the productivity threshold below which firms exit, π^e represents the probability of exit in the first period, and z^e and z^i represent the average productivity of the entrant and incumbent cohorts respectively. After receiving the productivity draw z the manager of an entrant firm compares the profits from continuing $\left(i.e.\text{ receive } \left(z_j - R_f\right) \overline{k} \text{ in perpetuity}\right)$ with the decision to default and start a new firm $\left(1 - \xi\right) B^e + \beta \left(1 - \phi\right) V^o$. Therefore, he decides to exit if the continuation value is lower than the exiting value. Consequently, the productivity at which the entrepreneur is indifferent between exiting or continuing is such that

$$B^e \left(1 - \xi\right) + \beta_e \left(1 - \phi\right) V^o = \frac{\left(\widetilde{z} - R_f\right) \overline{k}}{1 - \beta \left(1 - \phi\right)} \tag{10}$$

and from this condition we can derive the productivity cutoff

$$\widetilde{z} = \frac{1 - \beta \left(1 - \phi\right)}{\overline{k}} \left(V^o + \left(1 - \xi\right) B^e\right) + R_f \tag{11}$$

indicating that the productivity below which entrepreneurs decide to exit depends on both the value of starting a new firm and on the value of outstanding debt.

From this example we obtain a series of predictions on both the dynamics of the firm as well as on more aggregate outcomes such as income per capital and aggregate productivity. The first result concerns the value of opening a new firm:

Lemma 1 *The value of an entrant firm V^o is increasing in the recovery rate.*

Proof. See section 7.4 in the appendix. ■

The main intuition of this result is that in countries with worse investor protection the cost of debt is higher, lowering the value of opening a new firm. This result leads us to the main prediction of the paper, concerning the productivity threshold of the economy and consequently the magnitude of the selection effect:

Proposition 2 *The productivity threshold below which firms exit, is increasing in the recovery rate. Consequently, the difference in productivities between incumbent and entrant cohorts is increasing in the recovery rate.*

Proof. See section 7.4 in the appendix. ■

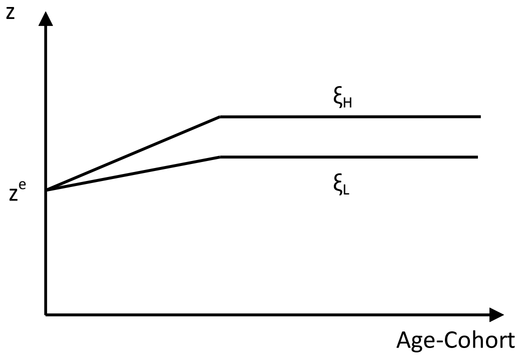

Figure 1: Firm Age and Productivity - Productivity per age-cohort increases with age, especially in countries with better investor protection ($i.e.$ higher ξ).

The main intuition of Proposition 2 is as follows: From Lemma 1 we know that $\frac{\partial V^o}{\partial \xi} > 0$, which means that the value of the outside option of creating a new firm is increasing in the quality of the recovery rate. This higher outside option implies that the minimum quality acceptable from a given project increases with investor protection $\frac{\partial \tilde{z}}{\partial \xi} > 0$. This result, graphed in Figure 1, shows that the average productivity per age-cohort increases with age, and that this increase is stronger in firms with better recovery rate.

3.2.2 Firm Growth and Leverage

The next result regards the size of entrant firms. From condition 8, and assuming everything else constant, we know that an increase in the recovery rate decreases the interest rate. Therefore in countries where the cost of credit is lower, entrant firms will demand more credit for a larger initial investment. This leads us to the following result:

Lemma 3 *Entrants are larger in countries with higher recovery rate*

Proof. See section 7.4 in the appendix. ■

In this simple example, all firm growth occurs from the first to the second period. As all uncertainty is resolved in the first period, and given the linearity of production, we have that the size of all incumbents is $\bar{k}$. This fact, in addition to Lemma 3 implies that:

Proposition 4 *Young firms grow faster in countries with lower recovery rate.*

Proof. Using lemma 3 we know that entrants' size is increasing in recovery rate, while the incumbents' size is independent of recovery rate. Hence the result. ■

This proposition, graphed in Figure 2, implies that firms grow faster in countries with worse recovery rates due to their smaller entry size.

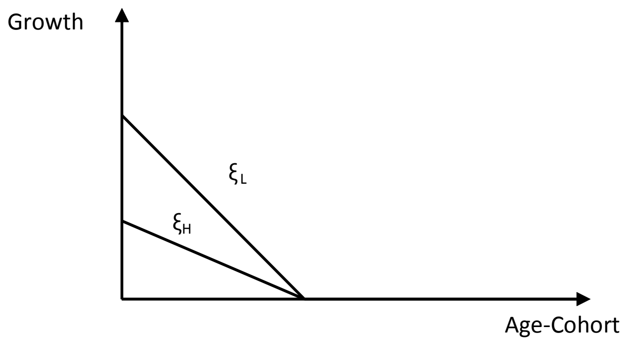

Figure 2: Firm Age and Growth

Additionally, the model provides predictions on the dynamics of firm leverage.

Proposition 5 *Firms in countries with better recovery rate have a faster deleverage*

Proof. See section 7.4 in the appendix. ■

This proposition is graphed in Figure 3. The intuition for the faster deleverage is simple. Firms start larger and with higher profits in countries with better recovery rate. Using the assumption that the discount rate is equal to the risk-free rate, this implies that they pay their outstanding debt and achieve their long-run leverage faster.

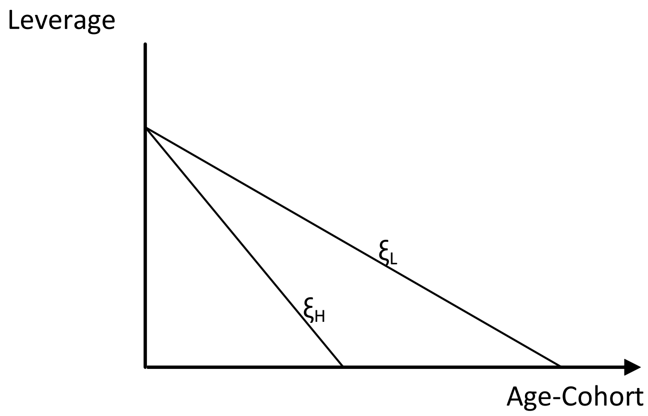

Figure 3: Firm Age and Leverage - Firms deleverage faster in countries with better investor protection.

3.2.3 Share of Entrant Firms

To determine the stationary equilibrium of the economy I must calculate the share of entrant firms. This share depends on the number of firms that exit the market endogenously as well as on those that and are born and die exogenously. As all uncertainty is resolved by the second period, endogenous exit will *only* occur in the first period. More specifically, firms exit if the productivity of the project z is below a certain threshold $\widetilde{z}$. In addition to the endogenous exit, firms die and exit exogenously with probability ϕ. Therefore, in equilibrium the number of entrants λ must be equal to the endogenous exit $\lambda\frac{\widetilde{z}-\underline{z}}{\overline{z}-\underline{z}}$ plus the exogenous one $(1-\lambda)\,\phi$, such that

$$\lambda = \lambda\frac{\widetilde{z}-\underline{z}}{\overline{z}-\underline{z}} + (1-\lambda)\,\phi$$

implying that the fraction of entrant firms is

$$\lambda = \frac{1}{1+\frac{1}{\phi}\frac{\overline{z}-\widetilde{z}}{\overline{z}-\underline{z}}} \tag{12}$$

which is positively dependent on both the birth rate ϕ and on the productivity cutoff $\widetilde{z}$. A direct consequence of the stricter selection is that there is a larger firm turnover in the form of higher entry and exit. This leads us to the following lemma

Lemma 6 *The share of entrant firms, on the total number of firms, is increasing in the recovery rate*

Proof. Directly from condition 12 and proposition 2. ■

This result, graphed in Figure 4, implies that the in countries with better investor protection the larger turnover leads to a larger share of young firms.

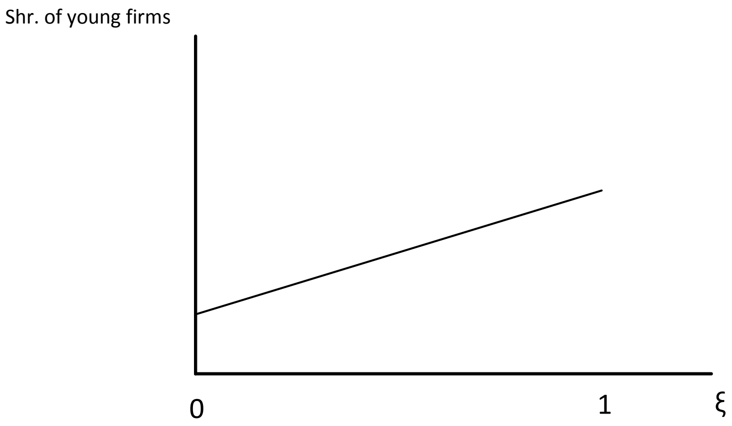

Figure 4: Investor protection and share of young firms.

3.2.4 Aggregate Production and Aggregate Productivity

The main interest of this paper is in analyzing the impact that the recovery rate has on aggregate production and productivity. Aggregate production Y is measured as the production of all firms. Formally,

$$\begin{aligned} Y &= \lambda y^e + (1-\lambda) y^i \qquad (13) \\ y^e &= \frac{\underline{z}+\overline{z}}{2}\frac{B^e}{R^e} \\ y^i &= \frac{\widetilde{z}+\overline{z}}{2}\overline{k} \end{aligned}$$

where y^e and y^i represent the average production of entrant and incumbent firms. From the definitions of λ and Y in 12 and 13 we obtain the following result.

Proposition 7 *Aggregate production is increasing in recovery rate.*

Proof. See section 7.4 in the appendix ■

The above proposition resolves the trade-off between the number of entrants, their relative size and aggregate production. On the one hand in countries with better recovery rates the number of, the relatively small, entrant firms is higher, but on the other hand entrants are also larger in those countries. Proposition 7 shows that this last effect is stronger and that aggregate production is increasing in recovery rate.

The model also allows us to make inferences regarding firm productivity. From it we can uncover the relation between recovery rate and the productivity of the *average* firm's productivity

$$z^{avg} = \lambda \frac{\underline{z} + \overline{z}}{2} + (1 - \lambda) \frac{\widetilde{z} + \overline{z}}{2}$$

Using the definitions of λ and $\widetilde{z}$ allows us to obtain the following relation

Lemma 8 *Countries with higher recovery rate have higher average firm productivity.*

Proof. See section 7.4 in the appendix ■

The aggregate productivity of an economy is measured as the ratio of total production to total inputs used. Formally,

$$\begin{aligned} z^{prod} &= \frac{Y}{K} \\ Y &= \lambda y^e + (1 - \lambda) y^i \\ K &= \lambda k^e + (1 - \lambda) k^i \end{aligned}$$

where Y is the overall production of the economy while K represent all the inputs used. Using the above lemma on average productivity we establish the following relation

Proposition 9 *Countries with better recovery rate have higher aggregate productivity.*

Proof. See section 7.4 in the appendix ■

In this section I have established the important results that income level and aggregate productivity are positively related with the quality of *recovery rate*.

3.3 Quantitative Results

In this section, I assess quantitatively how the quality of investor protection affects firm decisions, such as growth and leverage, as well as more aggregate outcomes such as income per worker and aggregate productivity.

3.3.1 Calibration

I calibrate the technological parameters so that the model economy matches key aspects of the United Kingdom's data, my benchmark economy. I specify values for the sixteen parameters: $\beta_c, \alpha, \gamma, \delta, p, \phi, \overline{L}, \beta_e$ and $prtr_j^{j'}, \rho_j, \sigma_j | j \in \{e, i, o\}$. From these parameters, the first four are set to standard values in the literature. The discount rate for consumers is set at 0.96, the depreciation rate is set at $\delta = 0.06$, the decreasing returns parameter $\alpha + \gamma = 0.8$, following Buera, Kaboski and Shin, and I choose $\frac{\alpha}{\alpha+\gamma} = 0.33$ to match the aggregate income share. All these benchmark parameters are shown in the first rows of Table 7.

Table 7: Benchmark Parameters

	Parameter	Value
Discount Rate - Consumers	β_c	0.96
Depreciation rate	δ	0.06
Technology	$\alpha + \gamma$	0.9
Capital share	$\frac{\alpha}{\alpha+\gamma}$	0.33
Probability of transition	$prtr_e^i, prtr_i^o$	0.25, 0.15
Persistence	ρ_e, ρ_i, ρ_o	0.77, 0.82, 0.87
Variance shocks	$\sigma_e^2, \sigma_i^2, \sigma_o^2$	0.12, 0.08, 0.05

As referred in Section 2, the uncertainty on the returns of the firm is decreasing with age. Therefore, and for this calibration, I assume that a firm can have three possible age-states, *entrant*, *incumbent* and *old*. The difference across these states is in the persistence and variance of the idiosyncratic component. All firms are born *entrants* and each period they have the probability $prtr_e^i$ of becoming *incumbents*. Similarly, *incumbent* firms have each period the probability $prtr_i^o$ of becoming *old*. I set the transition probabilities so that an *entrant* firm waits on average four years before becoming an *incumbent*, while an *incumbent* firm waits on average eight years before becoming *old*. Therefore, $prtr_e^i = 0.25$ and $prtr_i^o = 0.125$.

I assume that the log-productivity of firms, follows a first-order idiosyncratic process of the type

$$\begin{aligned} \ln z_{j,t} &= \rho_\tau \ln z_{j,t-1} + \varepsilon_{j,\tau,t} \\ \varepsilon_{j,\tau} &\sim N\left(0, \sigma_\tau^2\right) \end{aligned}$$

where the persistence parameter and the variance of the idiosyncratic shock are age-dependent. I estimate the parameters ρ_j and σ_j^2 for each of the three age-states using the data from UK firms for each set of age-cohorts. The results of the estimation are present in the last three rows of Table 7.

I am left then with four parameters that are more specific to my study: $\phi, \beta_e, \overline{L}, p$. I calibrate them so that my model can replicate four relevant moments of the UK data. The relative productivity of old firms to entrant firms (1.26), the leverage of old firms (0.62), the share of entrant firms (0.41) and the average number of workers per firm (21).[37]

Although in the model all parameters affect all the target moments in a non-linear fashion, some parameters have a stronger impact on certain moments. Therefore, the identification of these parameters is done according to the following logic: The share of entrant firms is positively impacted by the death rate, and therefore birth rate, of entrepreneurs. Hence, the entry/death rate parameter ϕ is identified by the share of entrant firms 0.41. The discount rate of entrepreneurs β_e affects the long-term leverage of firms. Therefore β_e is calibrated such that the mean long-term leverage in the model economy, matches its empirical counterpart 0.62. The number of employees per entrepreneur $\overline{L}$ is calibrated such that the number of workers per *active* entrepreneurs matches the value in the U.K. 21. Finally, the length of exclusion from credit markets p is pinned-down by the strength of the selection mechanism. The longer the waiting period, the lower the incentive to exit the market and therefore the weaker the selection mechanism. In the U.K. data, the cohort of firms with more than 12 years old is on average 25% more productive than the cohort of firms with less than 4-years. This relative productivity occurs in the model when $p = 0.5$.

Table 8: Calibrated Parameters

Target Moments	Calibrated Parameter	UK Data	Model
Share of entrant firms	$\phi = 0.06$	0.41	0.40
Workers per entrepreneur	$\overline{L}= 18$	21	21
Long-run leverage	$\beta_e= 0.85$	0.60	0.54
Relative productivity $\frac{z^{old}}{z^{ent}}$	$p = 0.5$	1.25	1.26

3.3.2 Results

In this section I examine the results of the calibration. I compute and simulate an economy with 10,000 firms for 500 periods. The last period of the simulation provides me with the long-term cross-section

[37] I define the TFP levels as Solow residuals. The $\ln TFP$ of firm i in sector j at time t is

$$\ln z_{ijt} = \ln y_{ijt} - \alpha_j \ln k_{ijt} - \gamma_j \ln l_{ijt}$$

where y is the value added, k total assets, l number of employees. The elasticities α and λ are sector-j specific. As in the previous papers I set them equal to input cost shares at the sectoral level. For more details see Appendix XXX.

from which I obtain all the statistics. I divide the cross-section of firms between *entrant, incumbents* and *old*.[38]

Quality of Fit

Figure 5 compares the distribution of firms in the U.K. data and in the simulated model calibrated to certain moments of the U.K. data. The model is able to fit the tails of the empirical distribution as well as the slope. The model however cannot capture the largest firms in the distribution presumably due to the low returns to scale of the model.

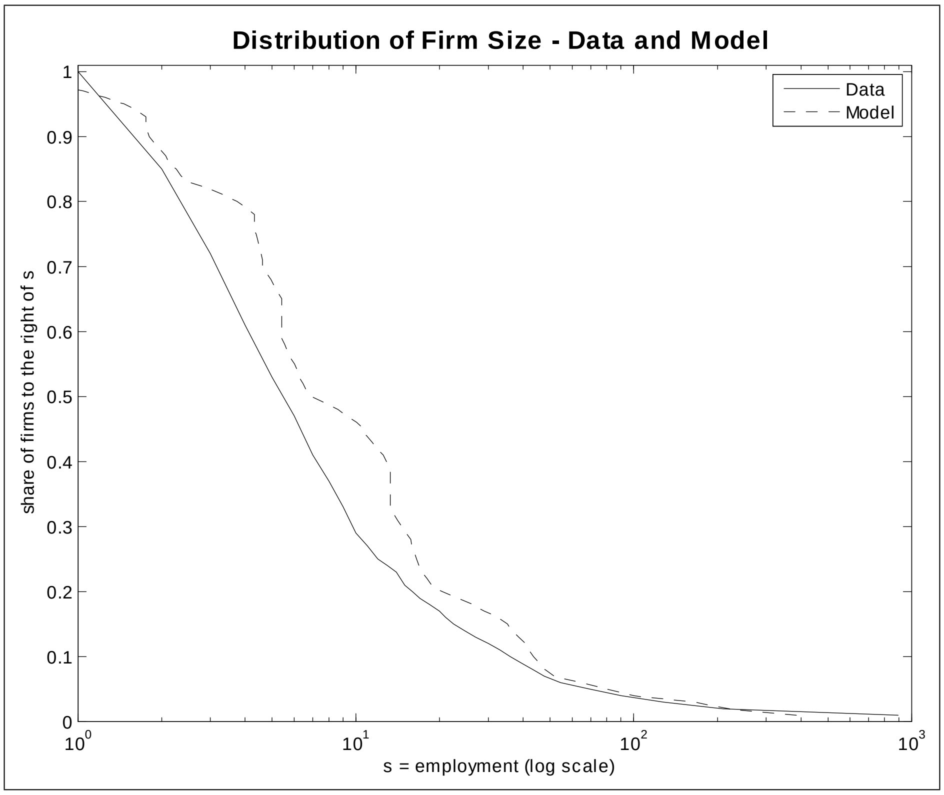

Figure 5: Distribution of firms per number of employees in the U.K. data and in the model.

In Table 9, I present additional results of the quality of fit of the model to several moments in the data. We observe that the model is able to replicate several moments of data that were not targets in the calibration. Moments such as relative productivity, leverage, growth and the share of entrant firms.[39] As predicted in the example, the mean productivity of cohorts increases with age. In fact,

[38]Firms start the project as entrants, and have a predetermined probability of transitating into incumbency. Once incumbents they have a certain probability of becoming old, which is an absorving state.

[39]I calculated the empirical counterpart of the revenue productivity as

$$TFP_i = \frac{Y_i}{K_i^{\alpha_j} L_i^{1-\alpha_j}}$$

old firms are on average 29 percent more productive than entrants are. Regarding growth, the model is also able to replicate the patterns present in the data. Namely, that younger firms grow faster than their older counterparts. More concretely, young firms grow on average 30 percent per year whereas older firms grow only at 8 percent per year. The model also presents some predictions on leverage. In the example we assumed that while entrant firms relied mostly on external financing to start a project, older firms were able to finance a larger part of their assets with internal generated funds. This is precisely what we see in the data and what is replicated in the quantitative model. Young firms present leverage ratios of 0.80 whereas older firms have leverage ratios of around 0.60.

Simulations

		U.K. $\xi = 0.85$	
		Data	Model
Productivity	entrants	1	1
	incumbents	1.20	1.08
	old	1.29	1.19
Growth	entrants	0.34	0.42
	incumbents	0.17	0.21
	old	0.08	0.10
Leverage	entrants	0.80	0.56
	incumbents	0.69	0.60
	old	0.60	0.51
Share of Firms	entrants	0.31	0.39
	incumbents	0.37	0.35
	old	0.32	0.25

Table 9: *Entrant* firms have less than 5 years, *incumbents* have between 5 and 11 years and *old* have more than 11 years

Cross-country variation

A main interest of this work is on the importance that recovery rate has on firm dynamics across countries. Therefore, it is of interest to compare simulations upon varying the recovery rates. In this section I check whether the various simulations for different levels of recovery rates replicate the empirical facts.

Productivity Across Age–Cohorts - For the first comparison, I contrast the data across age-cohorts

where Y_i represents the operating revenue of firm i, K_i the assets of the firm, and L_i the employees. The capital and labor shares are sector-j specific.

from the benchmark economy, the United Kingdom, with that of Russia. I chose Russia, as its recovery rate is in the median of the lowest quartile of my sample and given the good quality of its data. In Figure 6, I compare the empirical productivity per age-cohort with the simulated model. In the left panel, I present the empirical data, whereas in the right panel I compare the simulations of two economies mimicking the recovery rates of the U.K. and Russia, 0.85 and 0.25 respectively. As predicted by the example, older firms are relatively more productive than younger firms in countries with better investor protection. More specifically, in the U.K. 10 years-old firms are 25 percent more productive relative to 1 year-old firms. Conversely, in countries with worse investor protection such as Russia, the average productivity of older cohorts is roughly the same as that of their younger counterparts.

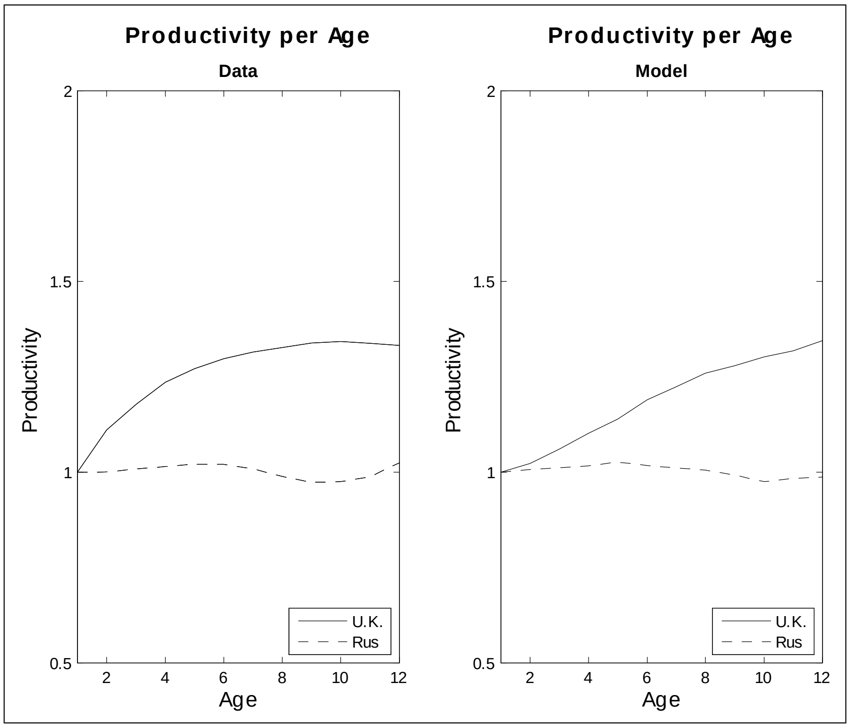

Figure 6: Productivity per Age - Both panels represent the mean productivity per age-cohort normalized by the productivity of the entrant cohort. Productivity is measured as $z_i = \frac{valadd_i}{k_i^{\alpha_j} l_i^{1-\alpha_j}}$. The right panel represents the results from the calibrated model. The only difference in the two simulations is in the recovery rate. For the U.K. simulation the recovery rate is 0.85 whereas the Russian recovery rate is 0.25.

Firm Growth Across Age-Cohorts - The model also has predictions regarding the relative growth of firms. In Figure 7, I present the growth rates per age-cohort. In the left-panel, and as in the previous Figure, I present the data for the U.K. and Russia. As predicted by the example, not only are growth rates in the country with worse investor protection but they also converge with age. In the right panel I present the results of the two simulations mimicking the recovery rate of the U.K. (0.85) and of Russia (0.25). The quantitative model is able to capture the differences in levels of growth across countries.

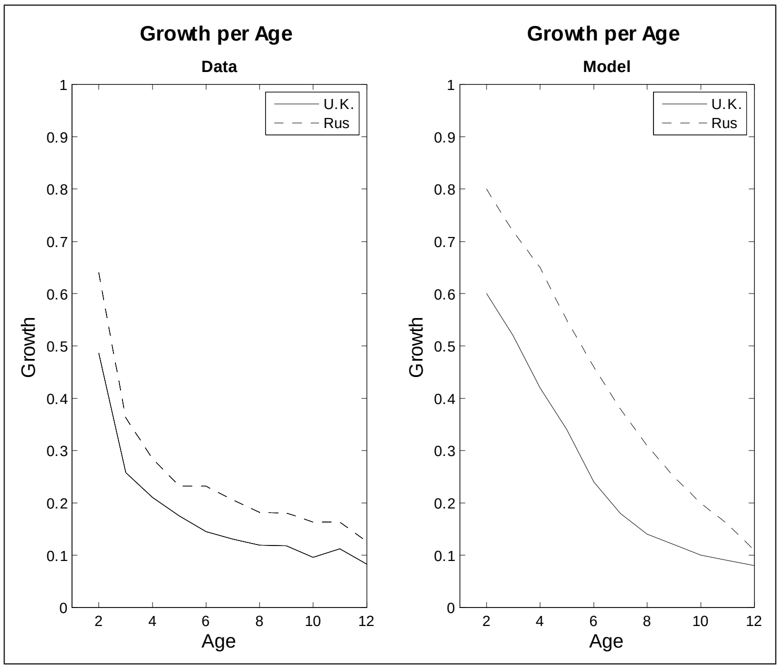

Figure 7: Growth per age-cohort - In the left panel I present the average real growth (CPI-deflated) of sales across age-cohorts for the U.K. and the Russian sample. In the right panel I present the simulations of two economies with the same calibration apart from the recovery rate. The U.K. simulation has a recovery rate 0.85 whereas the Russian simulation presents a recovery rate of 0.25.

Firm Leverage Across Age-Cohorts - Finally, in the Figure 8 I analyze the impact of different recovery rates on leverage. As assumed in the example, entrant firms are highly dependent on external financing. This is verified in the left panel. In this panel, I present the average leverage per age-cohort for the U.K. and for Russia.[40] In both countries external financing is around 0.80 for entrant firms, and it reaches a long-term value of around 0.60. In the right panel, I present the simulations for the average leverage per age-cohorts. The model presents similar results relative to the data.

Another prediction of the model is that, due to the larger firm turnover, the share of young firms should be increasing in recovery rate. To verify whether that is true both in the data and in the quantitative simulations, I graph the share of firms with less than 4 years in all the countries in the sample. Results are present in Figure 9 and indicate that this mechanism does indeed hold empirically and

[40]Leverage is defined as the ratio of total liabilities to total assets.

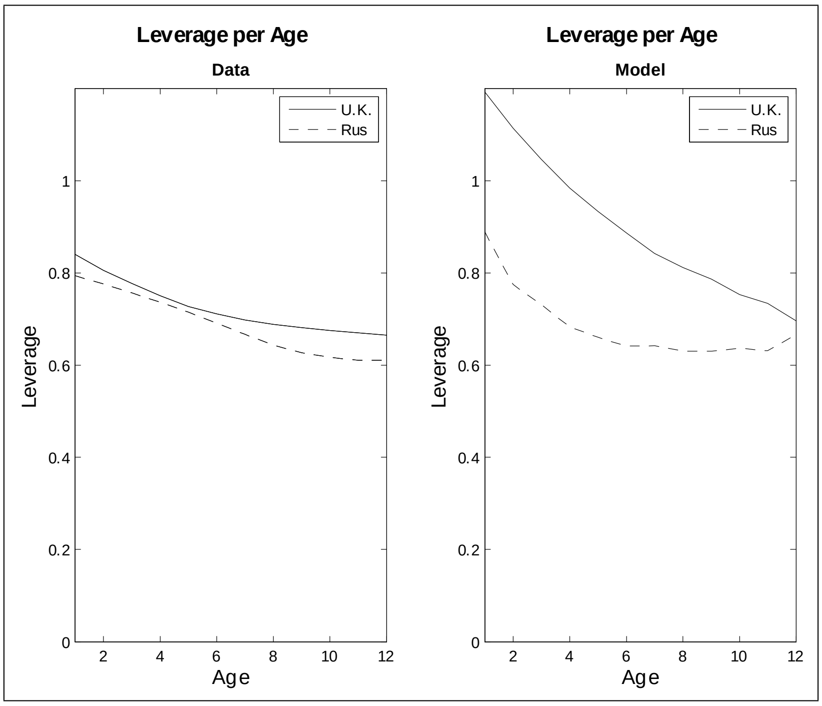

Figure 8: Leverage per age-cohort - In the left panel, I present the data on the average leverage per age-cohort for the U.K. and for Russia. In the right panel, I present the results for the simulations of two economies with recovery rates mimicking those of the U.K. and Russia.

quantitatively. For example and economy with a recovery rate in the top quartile has a share of young firms representing around 25 percent of all firms in the economy. This number is reduced to 12 percent when we analyze the bottom quartile of recovery rate.

The main interest of this research is in the analysis of the impact of the recovery rate on aggregate outcomes. In Figure 10, I present both the empirical and the simulated effect of recovery rate on income level and aggregate TFP. Both income and TFP are relative to the U.K. In the model, the variation in recovery rate explains around 50 percent of the cross-country differences in income. Although this difference does not completely explain the large income discrepancy between the U.K. and the poorest countries in the sample, its magnitude is considerable since I am only varying the recovery rate. In the right-hand panel, I present the relation between recovery rate and aggregate productivity for data and for the simulated effect. As in the data, differences in productivity in the quantitative model explain around 80 percent of the variation in income.

Policy - Optimal Punishment on Default

In this section I analyze the optimal punishment, measured as the periods of exclusion from credit markets after default, that maximizes aggregate income.

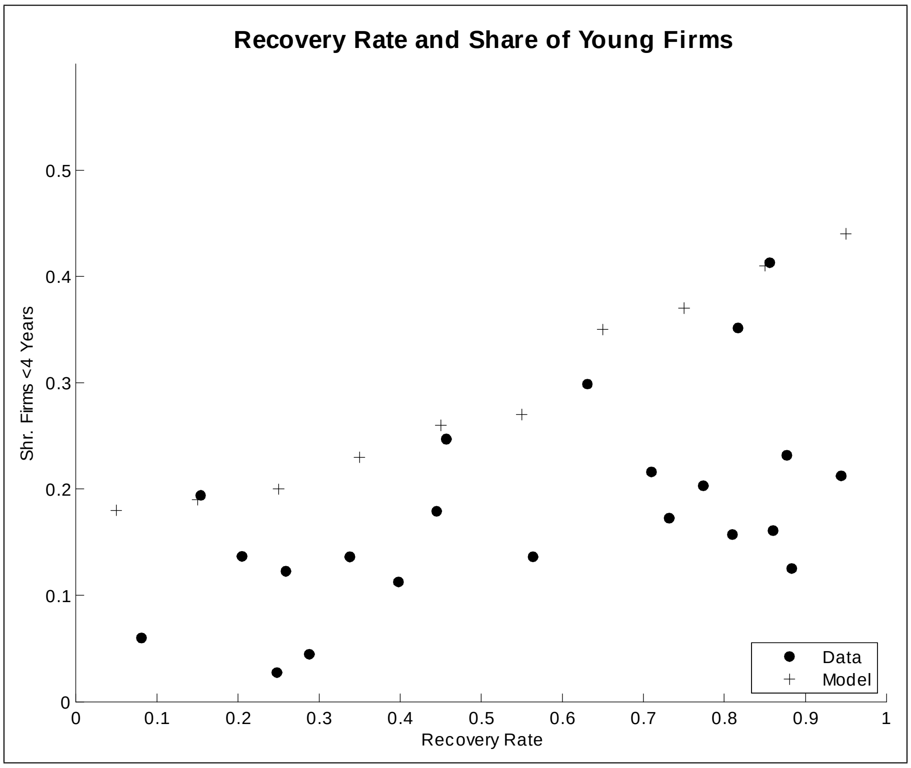

Figure 9: This figure relates, at the country level, the share of firms with less than 4 years relative to all the firms in the economy with the level of recovery rate.

For each level of recovery rate ξ I calculate the optimal p^* that maximizes production. Results are in Figure 11. An increase in the length of exclusion from credit markets has two, opposing, effects. On the one hand, it reduces the incentives to default, which reduces the default-premium demanded and consequently the interest rates. On the other hand, by increasing the negative impact of a default it makes entrepreneurs more conservative in their growth policies. The model simulations, suggest that the first effect is more important than the second effect in countries with lower recovery rate, where the default premia are highest. Conversely, in countries with higher recovery rate, and given that default premia are relatively low, it is more efficient to have relatively lower punishments on default.

4 Testing Model Predictions

In this section, I test empirically the main predictions obtained in section 3.2.

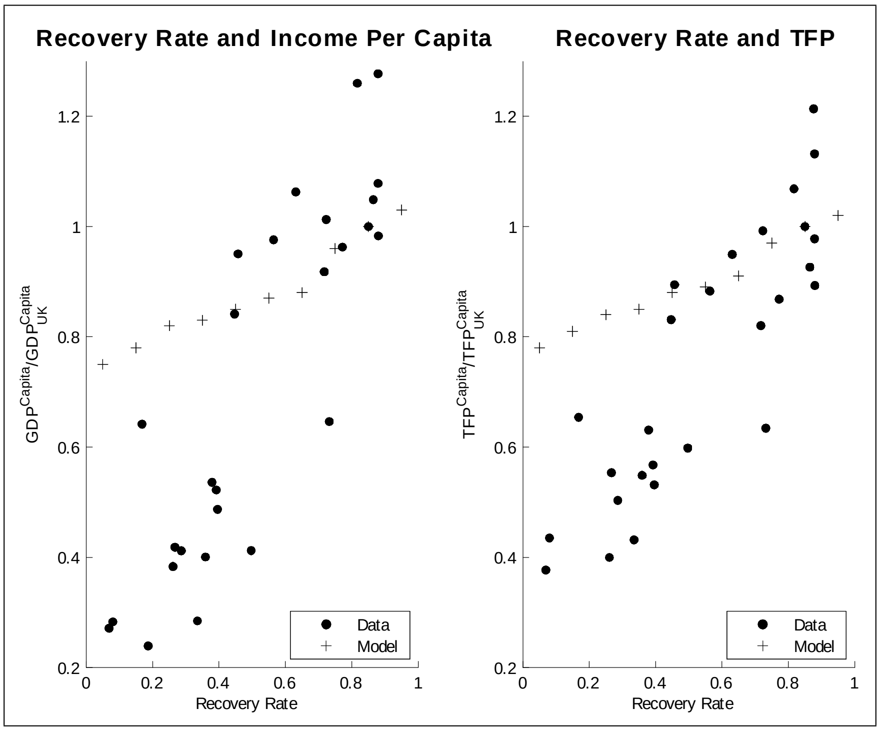

Figure 10: This graph relates the recovery rate to the income per capita and aggregate productivity relative to the UK in 2005. The circular points represent the observations for each country, whereas the crosses represent the simulations of the model for a given recovery rate.

4.1 Firm Selection

The main prediction of the model (Proposition 2) posits that selection is stronger in countries with better investor protection.[41] This implies that firm productivity should not only increase across age-cohorts, but it should increase more in countries with better investor protection.

Using the definition of productivity derived in Section 2 I test the first prediction by estimating

$$\ln TFP_{j,t} = \alpha_0 + \alpha_1 \ln (age_{j,t}) + \alpha_2 \ln (age_{j,t}) \ln (RR_c) + cty \times sect \times year + \varepsilon_j$$

where the dependent variable is the log of the firm productivity and the regressors are the *log of firm age*, the interaction of the *log firm age* with *log recovery rate* along with country, sectoral and year fixed-effects. The *age* and and the *recovery rate* regressors are transformed into logs given the non-linearities predicted by the model. The country fixed-effects control for country characteristics such as institutional quality, accounting procedures and political system. Industry effects are at the NACE

[41]Selection on productivity refers to the process in which entrepreneurs with low productivity firms exit the market. This process increases the average productivity of the remaining incumbents.

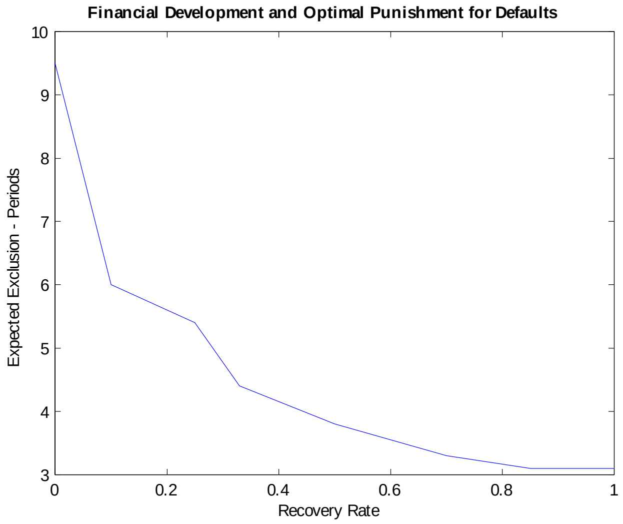

Figure 11: This graph represents the average length of exclusion from credit markets after default, that maximizes aggregate income.

2-digit level and control for industry characteristics such as factor intensity, competition structure and external financial dependence. Finally, year fixed-effects controls for business cycle fluctuations. Furthermore, the interaction between the fixed-effects allows us to capture all the idiosyncratic features of each sector for each year in a particular country. Finally, the standard errors are clustered at the country level for all regressions.

I present the results of the estimation in the second column of Table 10. As predicted by the model, both the age and the interaction coefficients are positive and significant at the 5% level. This implies that older firms are relatively more productive than younger firms with this difference increasing in the country's recovery rate. The interpretation of the interaction coefficient is analogous to that of a second derivative. To give a sense of its magnitude I compare the difference in the median productivity of 1-year-old and 5-year-old firms in Russia and the United Kingdom where the recovery rates are 0.25 and 0.85 respectively.[42] The difference in the median productivity of firms in the 5-year cohort, relative to the 1-year cohort, is 3 percentage points higher in the United Kingdom than in Russia. This difference understates, potentially by a great margin, the overall effect that selection has on aggregate productivity since the distribution of TFP is highly skewed.

An important caveat, regarding the sample, concerns the fact that roughly half of the countries in the sample were under the Communist regime up until 1991. Therefore it is possible, if not probable,

[42] As referred above, I chose the U.K. and Russia for comparison purposes, given the quality of their data and given that their recovery rates are in the median of the top and bottom quartiles respectively.

Table 10: Recovery Rate and Firm Selection

	Sales		Val. Added	Sales/Empl
	All Sample	<15 Yrs.	All Sample	All Sample
$\log(Age_{j,t})$	0.39	0.32	0.41	0.14
	(0.07)**	(0.08)*	(0.04)**	(0.07)**
$\log(Age_{j,t}) \times \log(RR_c)$	0.03	0.02	0.06	0.07
	(0.01)**	(0.01)**	(0.03)**	(0.01)**
Adjusted R-squared	0.32	0.33	0.35	0.39
Number of observations	5,522,718	4,274,790	3,376,835	5,522,718

The dependent variable is firm productivity, which is calculated under three definitions: *Sales Value Added* and *Sales/Employees*. Regressions have country×sector×year fixed-effects. Standard errors are in parenthesis and clustered at country level. ** denotes significant at 5%.

that the older firms in these countries have different productivities and firm dynamics simply due to this legacy. To control for this possibility, I conduct a robustness check for all predictions by restricting the maximum firm age in the sample to 15 years. In the third column of Table 10, I present the results for a censored sample of firms with less than 15 years. This does not alter neither the sign of nor the magnitude of the previous results

Another potential concern regards the way productivity is measured. In this test I measure productivity using the value added definition. More concretely, for this test I define productivity as

$$TFP_{j,t}^{VA} = \frac{Sales_{j,t} - Materials_{j,t}}{k_{j,t}^{\alpha_s} l_{j,t}^{1-\alpha_s}}$$

where $Materials$ represents the cost of materials used in productions. In this robustness check, I run the regression

$$\ln TFP_{j,t}^{VA} = \alpha_0 + \alpha_1 \ln(age_{j,t}) + \alpha_2 \ln(age_{j,t}) \ln(RR_c) + cty \times sect \times year + \varepsilon_{j,t}$$

The results are present in the third column of Table 10 and under this definition of productivity, the magnitude of the interaction coefficient is twice as large than for the benchmark definition of productivity.

An additional concern regards the fact that assets are measured using their accounting value which may be at historical values not reflecting their economic value. This may imply that the productivity for different cohort of assets may be measured differently due to this accounting fact. I attempt to control for this problem, by estimating the above equation using sales per employee as the measure of productivity. The coefficients are present in the last column of Table 10 and confirm the previous results.

Further Tests

In this section I perform additional checks on the robustness of the results. In the first test I split the sample in half with countries with recovery rate above and below the median. Then I run the regression 14 twice. Once for firms in countries with a recovery rate above median recovery rate, and once for firms in countries with a recovery rate below the median.

$$\log TFP_{j,t} = \theta_1 \left(Age_{j,t} < 4\right) + ... + \theta_5 \left(Age_{j,t} > 18\right) + cty \times sect \times year + \varepsilon_{j,t} \tag{14}$$

The results of the estimation are present in Table 11.

Table 11: Age-effect on productivity

Age	High RR	Low RR
$[4, 7]$	0.05	0.01
	(0.001)**	(0.003)**
$[8, 11]$	0.05	-0.07
	(0.001)**	(0.002)**
$[12, 18]$	0.06	-0.07
	(0.001)**	(0.003)**
> 18	0.09	-0.37
	(0.001)**	(0.006)**
Adjusted R-squared	0.06	0.14
Number of observations	4,331,012	1,194,408

The dependent variable is firm productivity. Countries are divided into two samples: high and low recovery rate. All regressions have country×sector×year fixed-effects. Std errors are in parenthesis and clustered at country level. ** denotes significant at the 1% level.

These results indicate that the selection effect is clearly positive for countries with better recovery rate since their θ-coefficients are increasing with age-cohort. Conversely, the average productivity across age-cohorts is *decreasing* with age in countries with lowest recovery rate. This test confirms the predictions of the model that older firms are relatively more productive in countries with better investor protection.

4.2 Growth

Another important result of the model (Proposition 4) is that firms, and in particular young firms, grow faster in countries with low recovery rate. To test the validity of the prediction I estimate

$$salgrt_{j,t} = \alpha_0 + \alpha_1 \ln(RR_c) + \alpha_2 \ln(age_{j,t}) + \alpha_3 \ln(age_{j,t}) \,.\, \ln(RR_c) + sect \times year + \varepsilon_{j,t}$$

where the dependent variable $salgrt_i$ is the growth of sales and the regressors are log of the recovery rate, the log of the firm age, the interaction between the log of age and the log of recovery rate, and country×sector×year fixed-effects. The second column of Table 12 presents the results of the regression. The coefficients are significant and of the expected sign. Sales growth of firms decreases with age, and this growth rate decreases faster with age in countries with worse recovery rates. The coefficient of the interaction term can be interpreted as a second derivative. The difference in growth rates between a 1-year old firm and a 5-year-old firm is 4.5 percentage points higher in Russia than in United Kingdom. This difference is significant since average sales yearly growth in Russia are 35 percent.

Table 12: Recovery Rate and Sales Growth

	All Sample	<15 Yrs.
$\log(Age_{j,t})$	-0.073	-0.0138
	(0.01)**	(0.01)**
$\log(Age_{j,t}) \times \log(RR_c)$	0.022	0.017
	(0.009)**	(0.009)*
Adjusted R-squared	0.03	0.03
Number of observations	4,699,844	3,426,290

The dependent variable is real sales growth. Regressions have sector×year fixed-effects. Standard errors are in parentheses and clustered at country level. ** denotes significant at the 5% level.

In the third column of Table 12, I present the results of the same estimation, but restricting the sample to firms with less than 15 years. The results still hold and remain economically significant.

Other explanations

Arellano, Bai and Zheng (2010) document that smaller firms grow faster than older ones especially in countries with better financial development.[43] They argue that the size of firms acts as a

[43] Arellano *et al* use two indicators to proxy for *financial development*. The ratio of private credit to GDP and the percentage of population covered by national credit bureaus.

collateral on debt and that in countries with worse financial development firms grow faster to decrease financing costs. To account for this explanation, and given the high correlation between age and size, I estimate the same regression including $log\left(AsstShr_{i,c}\right)$ as a measure of size.[44] The results are present in the third column of Table 12 and the coefficients of interest remain with the same signal and magnitude.

4.3 Leverage

The model offers some prediction on dynamics of firm financing. More concretely, Proposition 5 in the model predicts that the rate at which firms deleverage, is increasing in the country's recovery rate. To test this hypothesis I run

$$lev_{j,t} = \alpha_0 + \alpha_1 \ln\left(age_{j,t}\right) + \alpha_2 \ln\left(age_{j,t}\right) . \ln\left(RR_c\right) + cty \times sect \times year + \varepsilon_j$$

with leverage as the dependent variable, and where the regressors are firm age and the interaction of age with the log of the recovery rate, along with country and sector fixed-effects.[45] The second column of Table 13 reports the estimated coefficients. As predicted by the model, the coefficient on firm age is negative indicating that firms deleverage with age, while the coefficient of the interaction term is positive implying that the speed of deleverage is increasing in recovery rate.

Table 13: Recovery Rate and Leverage

	All Sample	<15 Years
log(Age)	-0.13	-0.112
	(0.005)**	(0.005)**
log(RR) $\times$ log(Age)	-0.034	-0.027
	(0.001)**	(0.001)**
Adjusted R-squared	0.09	0.07
Number of observations	9,976,994	8,384,639

The dependent variable is leverage. Leverage is defined as the ratio of total liabilities to total assets. All regressions have country$\times$sector$\times$year fixed-effects. Standard errors are parentheses and are robust to heteroskedasticity. ** denotes significant at the 5% level.

Other Explanations

Arellano *et al* document that size has important effect on firm dynamics in that higher colateralization In the second column I report that the results of the estimation are robust to the truncation of

[44] $\log\left(AsstShr_{i,c}\right)$ is the log share of firm i assets to total assets in country c.

[45] Leverage is measured as the accounting ratio between total liabilities and total assets.

the sample to firms with less than 10 years of age.

4.4 Share of Entrant Firms

In Lemma 6 of the model we have that, due to larger firm turnover, the share of young firms is higher in countries with better recovery rate.[46] To test whether it is true that countries with better recovery rate present a higher share of young firms I run

$$shrent_c = \alpha_0 + \alpha_1 \log RR_c + \varepsilon_c$$

where $shrent$ represents the fraction of firms with less than 4 years and RR_c the recovery rate in country c. I present the results in Figure 12. The coefficient of interest α_1 is positive but it is not significantly different from zero.

Before verifying whether this prediction holds in the data we must note that this particular testing is highly sensitive to sample selection, as it is plausible that certain groups of firms are relatively underrepresented. Therefore, I analyze the coverage and comparability of the Amadeus dataset across countries. In this exercise, I compare the coverage of the Amadeus dataset with the data present in the European Commission report. The latter report contains information on the distribution of the universe of firms per number of employees. I follow Arellano *et al*(2010) in comparing the fraction of firms, for each employment category, in the Amadeus dataset with that in the universe from the report.[47] Furthermore, and given that my sample is from 2002 to 2005, it is likely that Eastern European economies are still in transition from the end of the Communist regime. This presents the possibility that Eastern European countries have a different distribution of older firms simply due to this ongoing transition. I attempt to control for this possibility by restricting the sample to countries that have at least 1 percent of firms with more than 15 years old.[48]

In Figure 12, I graph the results, along with the fitted regression.

[46]Before verifying this prediction a disclaimer is in order. This particular testing is highly sensitive to sample selection, as it is plausible that younger firms are relatively more underrepresented. Therefore I consider the numbers of this test an indication of the results, rather than the documentation of a fact.

[47]From the European Comission Report, we have that for all the countries in the sample the share of large firms (i.e. >250 employees) is on average 6.9%. In this exercise, I discarded countries whose share of larger firms was larger than 25%. This leads me to discard the Dutch data, as its share of large firms represents 31.4 percent of the as the share of small firms. If I altered the cutoff to 10% Poland and Ukraine would also be excluded.

[48]Using this criterium I exclude the Czech Republic, Estonia, Hungary, Latvia, Lithuania and Romania.

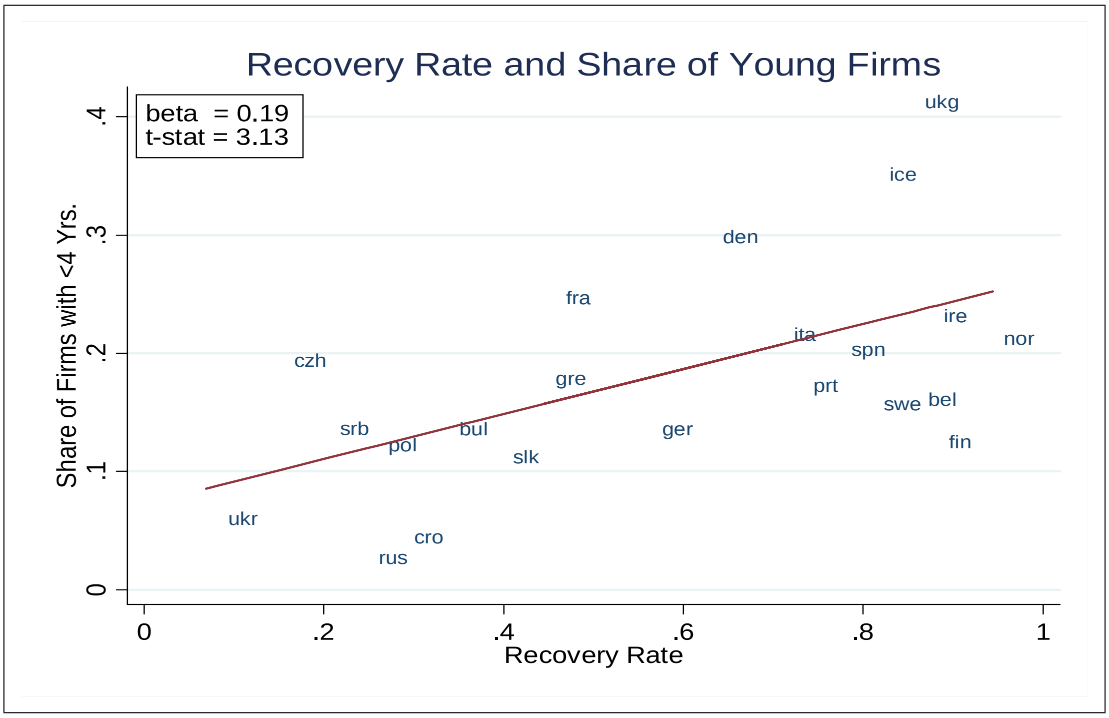

Figure 12: The graph represents the 2005 observations relating recovery rate with the share of firms with less than 4 years-old. The straight line represents the linear fit.

4.5 Income Level

The model has a prediction on income which states that the income level is increasing in recovery rate. To attest the veracity of this prediction I run

$$\log Y_c = \alpha_0 + \alpha_1 \ln RR_c + \varepsilon_c$$

where the dependent variable is the log of income per capita in PPP while the regressor in the log of the recovery rate. The data for this regression was taken from the Penn World Tables 6.3 and I used the data for 2004. The data from the recovery rate was from 2004 and it was taken from Doing Business.

The results of this regression are in the first column of Table 14.

Table 14: Recovery Rate and Income Level

	All Sample
log(RR)	0.631
	(0.0814)**
Number of observations	132
Adjusted R-squared	0.31

Standard errors in parentheses. * denotes significant at 5%; ** significant at 1

4.6 Aggregate Productivity

Finally, the model predicts that aggregate productivity is increasing with recovery rate. To test this hypothesis I run

$$\log TFP_c = \alpha_0 + \alpha_1 \ln RR_c + \varepsilon_c$$

where the dependent variable is the log of TFP while the regressor in the log of the recovery rate.[49] The results of this regression are in the first column of Table 15.

Table 15: Recovery Rate and Income Level

	All Sample
log(RR)	0.3683
	(0.0475)**
Number of observations	132
Adjusted R-squared	0.31

Standard errors in parentheses. * denotes significant at 5%; ** significant at 1

5 Conclusion

In this paper I proposed a theory of aggregate productivity based on firm selection. Improvements in investor protection reduce disproportionally the risk-premium of entrant firms augmenting the value of starting a project anew. This increase in the outside option of an incumbent raises the exit rate of lower quality firms improving overall productivity. A testable implication of this mechanism

[49] I calculate TFP as

$$TFP_c = \frac{GDP_c}{K_c^{0.33} L_c^{0.67}}$$

where GDP is the gross production at international prices, K_c is the total capital in the economy, and L_c is the total population. The intial capital for each country was calculated using the perpetual inventory method.

is that older firms should be relatively more productive in countries with better investor protection, where this entry/exit is stronger. I present empirical evidence that this effect is significantly present in the data. Quantitative exercises indicate that this selection mechanism accounts for around 25 percent of the cross-country patterns in aggregate productivity. Additionally, the model provides a series testable predictions that present indirect evidence on the empirical plausibility of this selection mechanism. Using a comprehensive cross-country panel database of firms, I show that in countries with better investor protection there is a higher share of young firms, firms start larger, grow more slowly and deleverage more quickly. Finally, I show that after default, the length of exclusion from credit markets that maximizes aggregate income is decreasing in the quality of investor protection.

6 References

Aghion, P. and P. Bolton, (1997) - "A Theory of Trickle-Down Growth and Development" *Review of Economic Studies*, 64, 151-172.

Albuquerque, Rui and H. Hopenhayn, (2004) - "Optimal Lending Contracts and Firm Dynamics" *Review of Economic Studies*, 71: 285:315.

Amaral, P., and E. Quintin, (2010) - "Limited Enforcement, Financial Intermediation and Economic Development: A Quantitative Assessment", *International Economic Review,* 51: 785-811.

Arellano, C., Y. Bai, and Jing Zhang, (2010) - "Firm Dynamics and Financial Development" *NBER Working Paper.*

Buera, F., J. Kaboski, Y. Shin, (2009) - "Finance and Development: a tale of two sectors" *Northwestern University and University of Wisconsin, unpublished manuscript.*

Castro, R., G. Clementi, G. MacDonald, (2009) - "Legal Institutions, Sectoral Heterogeneity and Economic Development" *Review of Economic Studies*, 76. 2: 529-561.

Clementi, G., H. Hopenhayn (2006) "A Theory of Financing Constraints and Firm Dynamics" *Quarterly Journal of Economics* 121: 229-265.

Cooley, T. R. Marimon, and V. Quadrini, (2004) - "The Aggregate Consequences of Limited Enforceability", *Journal of Political Economy* 112 817-847.

Djankov, S., O. Hart, C. McLiesh, A. Shleifer, (2006) - "Debt Enforcement around the World" *Journal of Political Economy* 116, 6: 1105-1149.

Erosa, A., and A. Hidalgo-Cabrillana, (2010) - "On Capital Market Imperfections as a Source of Low TFP and Economic Rents" *International Economic Review*, 49, 2: 437-473.

Evans, D., (1987) - "The Relationship Between Firm Growth, Size, and Age: Estimates for 100 Manufacturing Industries", *The Journal of Industrial Economics*, 35: 566-581.

Greenwood, J., J. Sanchez, and C. Wang, (2010) - "Quantifying the Impact of Financial Development on Economic Development" *NBER Working No. 15893.*

Hall, Robert and Charles Jones (1999): "Why Do Some Countries Produce So Much More Output per Worker than Others?" *Quarterly Journal of Economics*, 114, 83-116.

Hopenhayn H. (1992) "Entry, Exit, and Firm Dynamics in Long Run Equilibrium: *Econometrica* 60: 1127-1150.

Jeong, H., R. Townsend (2007): "Sources of TFP Growth: Occupational Choice and Financial Deepening" *Economic Theory*, 32, 197-221.

Jovanovic, B. (1982) - "Selection and Evolution of Industry" *Econometrica,* 50, 649-670.

King, R.G., Levine, R., (1993) - "Finance, entrepreneurship and growth: theory and evidence". *Journal of Monetary Economics* 32(4), 513-542.

King, R., R. Levine (1993): "Finance, Entrepreneurship and Growth: Schumpeter Might Be Right" *Quarterly Journal of Economics*, 108, 717-737.

La Porta, Rafael, Florencio Lopez-de-Silanes, Andrei Shleifer, and Robert Vishny (1997) "Legal Determinants of External Finance" *Journal of Finance*, 52(3), 1131-1150.

La Porta, Rafael, Florencio Lopez-de-Silanes, Andrei Shleifer, and Robert Vishny (1998) "Law and Finance" *Journal of Political Economy*, 106(6), 1113-1155.

La Porta, Rafael, Florencio Lopez-de-Silanes, Andrei Shleifer, and Robert Vishny (2000) "Investor Protection and Corporate Governance" *Journal of Financial Economics*, 52(3), 1131-1150.

Levine, Ross, "Finance and Growth: Theory and Evidence" (2005) In: Philippe Aghion and Steven N. Durlauf (eds) *Handbook of Economic Growth*, Volume 1A. Amsterdam: Elsevier B.V.: 856-934.

Luttmer, E. (2007) "Selection, Growth, and the Size Distribution of Firms" *Quarterly Journal of Economics* 122: 1103-1144.

Musto, D., (1999) "The Reacquisition of Credit Following Chapter 7 Personal Bankruptcy" *Working Paper, Wharton School.*

Piketty, T. (1997): "The Dynamics of the Wealth Distribution and the Interest rate with Credit Rationing" *Review of Economic Studies*, 64, 173-189.

Quadrini, Vincenzo: Entrepreneurship in macroeconomics. Annals of Finance 5:295-311 (2009)

Rajan, R. and L. Zingales (1998): "Financial Dependence and Growth" *American Economic Review*, 88, 559-586.

Rossi-Hansberg, E. and M. Wright (2007): "Establishment Size Dynamics in the Aggregate Economy" *American Economic Review,* 97, 1639-1666.

Syverson, C. (2004): "Market Structure and Productivity: A Concrete Example" *Journal of Political Economy*, 112, 1181-1222.

7 Appendix

7.1 Data Cleaning Methodology

In this section I describe in detail de procedure for the cleaning of the data. First, I require firms to have some basic accounting information over the years. Therefore, I drop firms that do not provide positive assets, sales, labor payments and liabilities, or that do not provide information on the age of the firm.[50] Next, I delete from the sample, firms that report only consolidated statements, to avoid double counting firms and subsidiaries or operations abroad. I also exclude certain industries. I drop several primary sectors where the activity is very country-specific. These sectors include agriculture (NACE code 1), forestry (NACE code 2), fishing (NACE code 5), and mining (NACE codes 10-14). I also drop the financial services industries (NACE code 65-66) since the financial information for these firms is not-comparable to those of non-financial firms). Finally, I drop the public sector, education, health and social sector, and activities of organizations that cannot be classified (NACE code 75-99).

7.2 Variables

I construct all firm-level variables as follows. *Productivity* has three different definitions. $TFP_i^S = \frac{Sales_i}{k_i^{\alpha_j} l_i^{\gamma_j}}$, $TFP_i^{VA} = \frac{Sales_i - Materials_i}{k_i^{\alpha_j} l_i^{\gamma_j}}$, $TFP_i^E = \frac{Sales_i}{l_i}$ where capital k_i is measured as the total assets of firm i, labor l_i is the number of employees while m are the materials. Following Syverson (2004), I use the input cost shares of the median firm, across each sector, as the measure of input elasticities.[51] This procedure implicitly assumes that all plants in each industry and across all countries operate with the same technology and have constant returns to scale.[52,53] *Leverage* is defined as the ratio of total liabilities to total assets. *Sales* and *Asset* growth are at the firm-level and deflated using CPI.

[50] Less than 2 percent of the firms do not provide information regarding their age.

[51] A sector is defined as the two-digit NACE code.

[52] If the scale elasticity were different from one, each of the input elasticities should be multiplied by the scale elasticity.

[53] Capital expenditires are constructed using the total assets multiplied by a capital rental rate of 0.12. This rental rate was the average rental rate per sector reported by the U.S. Bureau of Labor and Statistics. I performed robustness checks using a rental rate of 0.05 and 0.20.

Regarding the information on growth, leverage and productivity, I drop the values on the top and bottom percentile to eliminate possible outliers.

7.3 Comparability of Country Samples

This section analyzes the coverage of the Amadeus dataset across countries. The European Commission Report (ECR) contains information on the universe of firms per business sector for the majority of countries in the sample. The ECR reports the percentage of firms that have 1-49 employees, 50-249 employees and more than 250 employees. I compare the fraction of firms in each employment category with the ones present in Amadeus. Unfortunately, and as reported in Arellano, Bai, and Zhang, employment information is not reported for all firms in Amadeus. On average, around 70% of the firms in the cleaned sample report the number of employees, with the incidence of missing information being larger for small firms. To deal with this lack of employment information, I follow Arellano *et al*, and impute employment measures for firms that do not report employment in Amadeus. I run regressions country by country of log(employment) on log(assets) and log(sales).[54]

7.4 Analytical Model - Proofs

Proof. (Lemma 1) - Note that condition 9 can be written as

$$V^o = \frac{\frac{z^e - R_f}{R_f} B^e \left(1 - \pi^e \left(1 - \xi\right)\right) + \beta \left(1 - \phi\right) \left(1 - \pi^e\right) \frac{\left(z^i - R_f\right)\overline{k}}{1 - \beta(1-\phi)}}{1 - \pi\beta\left(1 - \phi\right)}$$

Assume two economies $\{L, H\}$ with different recovery rates $\xi_L < \xi_H$. Assume also that while firms in country L chose their optimal debt B_L^*, firms in country H choose their debt such that $\pi_L = \pi_H$. In this case the only varying part in V^o is $B^e\left(1 - \pi^e\left(1 - \xi\right)\right)$. We know that for $\pi_L = \pi_H$ to occur it must be $\widetilde{z}_L = \widetilde{z}_H$. Given the indifference condition 11, if $\widetilde{z}_L = \widetilde{z}_H$ then $V_L^o + B_L\left(1 - \xi_L\right) = V_H^o + B_H\left(1 - \xi_H\right)$.

If $V_H^o < V_L^o$ then it must be that $B_H > B_L$. But in this case $V_H^o - V_L^o = \frac{\frac{z^e - R_f}{R_f}}{1 - \pi\beta(1-\phi)}$ $(B_H(1 - \pi\left(1 - \xi_H\right)) - B_L(1 - \pi\left(1 - \xi_L\right))) > 0$ which is a contradiction. Hence $V_H^o > V_L^o$. ■

Proof. (Proposition 2) - We want to prove that $\widetilde{z}_L < \widetilde{z}_H$ and that $\underline{z} < \widetilde{z}_j$.

[54] As referred above, I substitute operating revenue for sales for the Danish and Norwegian firms.

Coverage and Cross-Country Comparability

	Amadeus Dataset			EC Data		
	Small	Medium	Large	Small	Medium	Large
Country	1-49	50-250	>250	1-49	50-250	>250
Fin	0.957	0.032	0.953	0.985	0.012	0.003
Ire	0.929	0.070	0.000			
Ned	0.610	0.309	0.072	0.981	0.016	0.003
Nor	0.975	0.020	0.004			
Bel	0.971	0.023	0.006		0.009	
UK	0.907	0.071	0.020	0.978	0.018	0.004
Ice	0.987	0.011	0.002			
Spn	0.962	0.031	0.006	0.991	0.008	0.001
Prt	0.929	0.061	0.010			
Swe	0.966	0.027	0.006	0.990	0.008	0.002
Ita	0.944	0.047	0.008	0.994	0.005	0.001
Den	0.950	0.041	0.008			
Ger	0.691	0.220	0.088	0.972	0.023	0.005
Lit	0.711	0.225	0.039	0.952	0.043	0.005
Fra	0.966	0.027	0.006	0.987	0.010	0.003
Gre	0.894	0.078	0.014			
Slk	0.665	0.254	0.081	0.931	0.055	0.014
Est	0.968	0.027	0.004	0.966	0.030	0.004
Hun	0.982	0.015	0.002			0.001
Lat	0.731	0.211	0.053	0.970	0.027	0.003
Bul	0.683	0.217	0.094	0.982	0.016	0.002
Cro	0.905	0.072	0.021			
Pol	0.536	0.322	0.109	0.989	0.009	0.002
Rus	0.806	0.139	0.053			
Srb	0.856	0.099	0.043			
Czh	0.863	0.109	0.028	0.991	0.008	0.001
Ukr	0.325	0.414	0.256			
Rom	0.972	0.022	0.005	0.971	0.023	0.006

Table 16: The Amadeus dataset has the information used in the empirical exercises. The European Commission data, is obtained from the National Registrars and includes all formal firms within an economy

Assume that $\widetilde{z}_L > \widetilde{z}_H$. From the indifference condition

$$\widetilde{z} = \frac{1-\beta\,(1-\phi)}{\overline{k}}\,(V^o + (1-\xi)\,B^e) + R_f \tag{15}$$

and given that $V^o_H > V^o_L$ it must be that $B^e_L > B^e_H$. In the optimum, B^e is such that $\frac{\partial V^o}{\partial B} = 0$.

Maximizing V^o with respect to B gives us

$$\frac{\partial V^o}{\partial B} = \frac{z^e - R_f}{R_f}\left(1 - \pi^e (1-\xi) - \frac{\partial \pi}{\partial B}(1-\xi) B\right) + \beta(1-\phi)\left(\frac{\partial \pi^e}{\partial B}V^o - \left(\frac{\partial z}{\partial B}\frac{1-\pi}{2}\right)Y\right)$$

where $Y \equiv \frac{\overline{k}}{1-\beta(1-\phi)}$. In the optimum $\frac{\partial V^o}{\partial B} = 0$ and the condition becomes

$$B_j^e = \frac{Y}{(1-\xi)\left(1+\frac{1-\phi}{z^e R_f}\right)}\left((\overline{z} - \underline{z})\left(\frac{1}{1-\xi_j} - \pi_j\right) + (1-\phi) R_f\right) \tag{16}$$

Given the assumption that $\widetilde{z}_L > \widetilde{z}_H$, which is equivalent to $\pi_L > \pi_H$, we have that $B_H^e > B_L^e$ which is a contradiction. Hence $\widetilde{z}_L < \widetilde{z}_H$. ■

Proof. (Proposition 3) - In this prove we want to show that $k_L^e < k_H^e$. This proof will be done using contradiction arguments. As entrepreneurs start a project with no capital we have $k_j^e = \frac{B_j^e}{R_j^e} = \frac{B_j^e(1-\pi_c(1-\xi_c))}{R_f}$. We know that the expected value of an entrant firm is

$$V_j^o = \frac{z^e - R_f}{R_f}k_j^e + \beta(1-\phi)\left(\pi_c V_j^o + (1-\pi_c)\frac{z_L^i - R_f}{1-\beta(1-\phi)}\overline{k}\right)$$

and this can be rewritten as

$$V_j^o = P_j^e + \beta(1-\phi)\left(\pi_c V_j^o + (1-\pi_c)\left(z_L^i - R_f\right)Y\right)$$

where $P_j^e \equiv \frac{z^e - R_f}{R_f}k_j^e$ and $Y \equiv \frac{\overline{k}}{1-\beta(1-\phi)}$. Note that condition $k_L^e < k_H^e$ is equivalent to $P_L^e < P_H^e$. From the indifference condition 11 we know that $V^o < (\widetilde{z} - R_f)Y$. Since $\pi_L < \pi_H$ this implies that for condition $V_L^o < V_H^o$ to hold, it must be that $P_L^e < P_H^e$ which implies that $k_L^e < k_H^e$. ■

Proof. (Proposition 5) - Leverage is defined as $Lev_t = \frac{B_t}{k_t}$. Since firms deleverage as fast as possible then the leverage of a firm with τ years is

$$Lev^\tau = Max\left\{0, \frac{B_{\tau-1} - (z_i - R_f)k^\tau}{k^\tau}\right\}$$

which indicates that firm pays its outstanding debt $B_{\tau-1}$ with the profits it generates $(z_i - R_f)k^\tau$. As k^e is increasing in ξ we have that $\frac{\partial^2 Lev}{\partial \tau \partial \xi} < 0$ which implies that firms deleverage faster with higher *recovery rate*. ■

Proof. (Proposition 7) - Income level is

$$
\begin{aligned}
Y &= \lambda y^e + (1-\lambda)\, y^i \\
&= y^i + \lambda y^e + \lambda y^i
\end{aligned}
$$

The first two terms are clearly increasing in $\widetilde{z}$ and therefore in ξ[55]. The last term can be written as

$$
\lambda y^i = \frac{\frac{\widetilde{z}+\overline{z}}{2}\overline{k}}{1+\frac{1}{\phi}\left(1-\frac{\widetilde{z}}{\overline{z}}\right)}
$$

Differentiating it with respect to $\widetilde{z}$ gives us

$$
\frac{\partial \lambda y^i}{\partial \widetilde{z}} = \frac{1+\frac{1}{\phi}+\frac{\overline{k}}{2\phi\overline{z}}}{\left(1+\frac{1}{\phi}\left(1-\frac{\widetilde{z}}{\overline{z}}\right)\right)^2} > 0
$$

Since $\frac{\partial \widetilde{z}}{\partial \xi} > 0$ we have the result $\frac{\partial Y}{\partial \xi} > 0$. ■

Proof. (Lemma 8) - Average productivity is measured as

$$
\begin{aligned}
z^{avg} &= \lambda \frac{\underline{z}+\overline{z}}{2} + (1-\lambda)\frac{\widetilde{z}+\overline{z}}{2} \\
&= \frac{\overline{z}}{2} + \frac{\lambda \underline{z} + (1-\lambda)\,\widetilde{z}}{2}
\end{aligned}
$$

focusing on the variable part and using the definition of λ we have[56]

$$
\widetilde{z} - \lambda\left(\widetilde{z}-\underline{z}\right) = \widetilde{z} - \frac{\widetilde{z}-\underline{z}}{1+\frac{\overline{z}-\widetilde{z}}{\phi(\overline{z}-\underline{z})}}
$$

In this last expression only $\widetilde{z}$ depends on ξ. Differentiating it with respect to $\widetilde{z}$ gives us

$$
\frac{\partial\left(\widetilde{z}-\lambda\left(\widetilde{z}-\underline{z}\right)\right)}{\partial \widetilde{z}} = 1 - \frac{1+\frac{\overline{z}-\widetilde{z}}{\phi(\overline{z}-\underline{z})}+\frac{\widetilde{z}-\underline{z}}{\phi(\overline{z}-\underline{z})}}{\left(1+\frac{\overline{z}-\widetilde{z}}{\phi(\overline{z}-\underline{z})}\right)^2} = 1 - \frac{1+\frac{1}{\phi}}{\left(1+\frac{\overline{z}-\widetilde{z}}{\phi(\overline{z}-\underline{z})}\right)^2}
$$

[55] We have shown above that both $\widetilde{\theta}$ and k^E are increasing in ξ. Therefore both $y^I = \frac{\widetilde{\theta}+\theta_H}{2}\overline{k}$ and $\lambda y^E = \frac{\frac{\widetilde{\theta}+\theta_H}{2}k^E}{1+\frac{1}{\phi}\left(1-\frac{\widetilde{\theta}}{\theta_H}\right)}$ are also increasing in ξ.

[56] Recall that $\lambda = \frac{1}{1+\frac{1}{\phi}\frac{\overline{\varepsilon}-\widetilde{\varepsilon}}{\overline{\varepsilon}-\underline{\varepsilon}}}$.

if we minimize the above expression by assuming that $\widetilde{z} = 0$ we have that

$$\frac{\partial\left(\widetilde{z}-\lambda\left(\widetilde{z}-\underline{z}\right)\right)}{\partial\widetilde{z}} = 1 - \frac{1+\frac{1}{\phi}}{\left(1+\frac{1}{\phi\left(1-\frac{\underline{z}}{\overline{z}}\right)}\right)^2}$$

For $\frac{\partial(\widetilde{z}-\lambda(\widetilde{z}-\underline{z}))}{\partial\widetilde{z}} > 0$ we should have that

$$\begin{aligned}\left(1+\frac{1}{\phi\left(1-\frac{\underline{z}}{\overline{z}}\right)}\right)^2 &> 1+\frac{1}{\phi} \Rightarrow \\ \frac{2}{1-\frac{\underline{z}}{\overline{z}}} + \frac{1}{\phi\left(1-\frac{\underline{z}}{\overline{z}}\right)^2} &> 1\end{aligned}$$

which holds. Therefore, and given that $\frac{\partial(\widetilde{z}-\lambda(\widetilde{z}-\underline{z}))}{\partial\widetilde{z}} > 0$ and $\frac{\partial\widetilde{z}}{\partial\xi} > 0$ we have $\frac{\partial z^{avg}}{\partial\xi} > 0$. ■

Proof. (Proposition 9) - Overall productivity is measured as

$$z^{prod} = \frac{Y}{K} = \frac{\lambda\frac{\underline{z}+\overline{z}}{2}k^e + (1-\lambda)\frac{\widetilde{z}+\overline{z}}{2}k^i}{\lambda k^e + (1-\lambda)k^i}$$

which, given $\underline{z} < \widetilde{z}$, implies that the larger k^e the lower z^{prod}. k^e is capped by $\overline{k}$. if $k^e = \overline{k}$ then $z^{prod} = z^{avg}$. From Proposition 2 we know that $\frac{\partial z^{avg}}{\partial\xi} > 0$. Therefore we have the result $\frac{\partial z^{prod}}{\partial\xi} > 0$ ■