Beyond Cobb-Douglas: Estimation of a CES Production Function with Factor Augmenting Technology*

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Abstract

Both the recent literature on production function identification and a considerable body of other empirical work assume a Cobb-Douglas production function. Under this assumption, all technical differences are Hicks neutral. I provide evidence from US manufacturing plants against Cobb-Douglas and present an alternative production function that better fits the data. A Cobb Douglas production function has two empirical implications that I show do not hold in the data: a constant cost share of capital and strong comovement in labor productivity and capital productivity (revenue per unit of capital). Within four digit industries, differences in cost shares of capital are persistent over time. Both the capital share and labor productivity increase with revenue, but capital productivity does not. A CES production function with labor augmenting differences and an elasticity of substitution between labor and capital less than one can account for these facts. To identify the labor capital elasticity, I use variation in wages across local labor markets. Since the capital cost to labor cost ratio falls with local area wages, I strongly reject Cobb-Douglas: capital and labor are complements. Many results in economic growth and macroeconomics depend both upon the bias of technical change and the value of the elasticity of substitution. Specifying the correct form of the production function is more generally important for empirical work, as I demonstrate by applying my methodology to address questions of misallocation of capital.

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1 Introduction

Plant level capital labor ratios are extremely dispersed, even within narrowly defined four digit SIC industries. I document this dispersion in Table 1. In the 1987 Census of Manufacturers for the US, the 75th percentile plant has two and a half times the capital per worker of the 25th percentile plant for the median industry. Looking further into the tails of the distribution, the 90th percentile plant has more than six times the capital per worker of the 10th percentile plant. This dispersion is not solely due to differences in wages or worker quality across plants. The factor cost ratio (the ratio of capital costs to labor costs) is also disperse, with the 75th percentile plant having capital costs relative to labor costs twice as large as the 25th percentile plant and the 90th percentile plant more than five times larger than the 10th percentile plant.

My basic assumption is that firms minimize costs given competitive factor markets. For some results, I also assume that firms maximize profits facing a downward sloping demand curve. Under these assumptions, a Cobb-Douglas production function, the most common choice of the empirical literature, implies that the factor cost ratio is constant. As I showed above, the ratio of capital costs to labor costs within industry is far from constant. This dispersion in capital shares is not just temporary, due perhaps to measurement error or adjustment costs. I find that the factor cost ratio and capital-labor ratio are both strongly autocorrelated over a span of 10 years, about as autocorrelated as conventionally measured TFP. I also show that the factor cost ratio is strongly correlated with firm output, with the largest firms within the same industry having a 45% higher factor cost ratio than the smallest plants in 1987 and more than 100% higher factor cost ratio in 2002. Labor productivity and capital productivity (output per unit of capital) should also move together if the production function is Cobb Douglas. But in the US Census data, the average revenue product of labor rises with revenue while the average revenue product of capital is flat beyond the smallest plants.

A production function with non neutral productivity can better explain these facts. Productivity is always Hicks neutral when the production function is Cobb Douglas; improvements in productivity do not affect the relative marginal products of capital and labor and so do not alter the relative allocations of the factors. Under a CES production function with an elasticity of substitution not equal to one, productivity that augments labor affects the ratio of the marginal products of the factors. Labor augmenting productivity is akin to having more effective labor for the same number of workers. If labor and capital are complements (so the elasticity of substitution is less than one), firms with more effective labor have a higher marginal product of capital relative to labor. Cost minimizing firms set relative marginal products equal to the relative factor prices they face, so firms with more labor augmenting productivity expand their capital labor ratios. Labor augmenting productivity is thus labor saving as well.

Profit maximizing firms facing a downward sloping elastic demand curve expand output when labor augmenting productivity rises. These firms also increase their capital per worker and capital share. Bigger firms then have larger capital shares and a higher average revenue product of labor but a lower average revenue product of capital. These patterns are what I see in the US Census data, though I find that the average revenue product of capital is constant rather than falling with size.

The response of firms to labor augmenting productivity depends upon the value of the long run elasticity of substitution. The elasticity of substitution measures how much firms change their capital intensity when factor prices change. When labor and capital are complements, firms increase their capital labor ratio less than an increase in wages, so the factor cost ratio falls. Under a Cobb-Douglas production function, the capital-labor ratio rises exactly in proportion to the rise in wages, so the factor cost ratio is constant. The slope of the relation between wages and the factor cost ratio then identifies the elasticity of substitution.

I find sharp rejections of the Cobb-Douglas specification when I use differences in wages across local areas for identification. I construct wages from both worker data and establishment data and estimate the elasticity using state, MSA, and county differences in wages, as well as county differences in wages within a state. The wages based on worker data are adjusted for differences in worker quality across areas. I first estimate the plant level elasticity using an entire manufacturing cross-section, assuming that the elasticity is the same across industries, but controlling for four digit SIC industry differences in the factor cost ratio. As local area wages rise, the factor cost ratio falls. For overall manufacturing, this decreasing relationship implies that the elasticity of substitution less than one. My preferred estimate using MSA level wage differences is .44.

I then estimate the elasticity of substitution separately for major industries at the 2 digit level within manufacturing. I can reject Cobb-Douglas for 17 out of 19 two digit industries using state level wages, 17 out of 19 using county level wages, and 15 out of 19 using within state county level wages. I also construct a sample of ten large four digit industries with considerable geographic variation. I reject Cobb-Douglas for 8 out of 10 of these industries with state wages and 10 out of 10 with county wages.

I also examine the elasticities of capital with skilled labor and unskilled labor separately, where I use production workers as a proxy for unskilled labor and nonproduction workers for skilled labor. The high school wage proxies the unskilled labor wage and the college wage the skilled labor wage. The elasticity of capital with unskilled labor is still lower than one for most of my specifications. Consistent with skill capital complementarity, the elasticity of capital with skilled labor is below .5 in all specifications and much lower than the elasticity with unskilled labor.

The production function is one of the basic building blocks of economic theory. Thus, my results on the production function have implications for a whole host of economic questions. In Industrial Organization, economists are investigating how productivity differences across firms are related to market structure, and more generally what causes differences in productivity across firms (Bartlesman and Doms (2000), Syverson (forthcoming)). Many macroeconomic models assume that productivity shocks cause business cycle fluctuations.

Productivity plays a central role in the recent trade literature as well. In these models, productivity is heterogeneous across firms and high productivity firms decide to enter into trade (Bernard et al. (2003), Melitz (2003)).

I can now look at the implications of both Hicks neutral productivity and labor augmenting productivity. Assuming cost minimization, I can identify a labor augmenting productivity from the plant's first order conditions for labor and capital. With data on revenue, I can identify Hicks neutral productivity together with price differences across plants. I find that labor augmenting productivity is correlated with both size and size growth, which holds up using employment or value added as a measure of size. My Hicks neutral measure is negatively correlated with both, although this result may be due to large and growing firms having low prices.

The long run elasticity of substitution is central to many questions of growth theory, including changes in income shares and relative convergence over time. The qualitative implications of many growth models depend on whether the elasticity of substitution is below or above 1, but the value of the elasticity is important for many questions as well. Since innovation and improvements in productivity drive economic growth, the bias of productivity affects how and why innovation occurs. Acemoglu (forthcoming) characterizes when technology improvements are labor saving, for example.

The type of technical differences we see has important implications for questions of misallocation as well. A recent literature studies whether developing countries are poor because resources are not allocated efficiently (Banerjee and Duflo (2005), Restuccia and Rogerson (2008)). Productive firms do not get enough capital while unproductive firms get too much capital, and firms may operate under output constraints or benefit from subsidies. These allocation frictions lower aggregate productivity. Hsieh and Klenow (2009) take this theory to the micro data and find that eliminating misallocation frictions would increase aggregate TFP by 40% in the US and more than 100% in China and India. In their model, firms with Cobb-Douglas production functions face capital and output wedges in a static environment.

Identifying allocation frictions in the data requires assumptions on the form of the production function and productivity. In a Cobb-Douglas world with misallocation frictions, a high capital share of cost implies a low capital wedge and a low labor share of revenue implies a high output wedge. Labor augmenting productivity would both increase the capital share of cost and decrease the labor share of revenue, as well as increase revenue. Thus, labor augmenting productivity would imply a set of testable implications for the misallocation wedges and revenue. Using data from Chile, I find that firms with low capital wedges have high output wedges and high revenue, as differences in labor augmenting productivity would predict. To avoid overestimating the level of misallocation of capital, we also need to consider differences in labor augmenting productivity.

Table 1: Dispersion in K/L and Factor Cost Ratio within 4 digit Industries for the 1987 Census of Manufacturers

		Median	25%	75%
Capital-Labor Ratio	75/25 Ratio	2.5	2.3	2.8
	90/10 Ratio	6.4	5.5	8.2
Factor Cost Ratio	75/25 Ratio	2.1	1.9	2.4
	90/10 Ratio	5.4	4.6	6.7

For each industry, I calculated the 75/25 ratio and 90/10 ratio for each variable. I have then reported the median, 25%, and 75% of these ratios across industries.

1.1 Literature Review

My work is related to a couple of different literatures. One literature tries to estimate the micro level production function and productivity. When estimating the micro production function, we first have to decide what functional form to assume. The Cobb-Douglas production function has been the most widely used production function in empirical work, but is extremely restrictive as it sets the elasticity of substitution between factors to one. The

CES production function due to Arrow et al. (1961) allows any constant elasticity of substitution. The translog production function (Christensen et al. (1973)) is even more general, as it does not impose that the elasticity of substitution is constant across plants or across multiple inputs. The translog is a second order approximation to any production function, as it contains all first and second order terms of the inputs.

The main stumbling block in production function estimation has been accounting for the endogeneity of productivity. Inputs are generally correlated with productivity because a firm takes into account its productivity when making input decisions. This correlation biases production function parameters estimated using OLS, as first pointed out by Marshak and Andrews (1944). To focus on endogeneity, economists have simplified the estimated form of the production function to Cobb-Douglas.

Olley and Pakes (1996) provide the main IO approach to endogeneity problems. Assuming that investment is a monotonic function of productivity and capital, they replace productivity with the inverted function of capital and investment. Since endogeneity problem is gone, they can estimate the labor coefficient and remove measurement errors in output. Assuming that productivity is first order Markov, a set of timing assumptions on when input decisions are made imply that observed variables or their lags are uncorrelated with the innovation in productivity. A GMM estimator from these moments identifies the capital coefficient.

Gandhi et al. (2009) are the only paper to apply the Olley Pakes type methodology to a wider array of production functions. They use revenue share equations from the first order conditions of the production function rather than a nonparametric input demand equation to separate measurement error from Hicks neutral productivity. They then estimate the production function parameters with similar GMM moments to Olley Pakes. The revenue share equations allow them to estimate more complicated production functions than Cobb Douglas, such as the CES or translog. By adding an input demand equation, they can handle imperfect competition as well. In their CES estimation case, they find an elasticity considerably above 1, very different from both my results and the rest of the literature.

However, they assume only Hicks neutral productivity, so labor augmenting productivity would become measurement error in their approaches as labor augmenting productivity enters the revenue share equations. Ignoring labor augmenting productivity could lead to severe biases in their estimation procedure.

A broader literature in macroeconomics and labor economics focuses on estimating the elasticity of substitution between labor and capital, using a variety of different techniques. Most of these estimate the macro elasticity of substitution. Since we can substitute factors both within firms and across firms at the macro level, the macro elasticity of substitution will be higher than the micro elasticity of substitution.

The early debate on the elasticity of substitution focused on differences between time series and cross section estimates of the elasticity. These early papers used relations between labor productivity and wages, as capital data was unavailable. The cross section estimates, based on 2 digit sector aggregates across states or countries and local area wage differences, found high elasticities, often above one. However, the cross-section estimates had severe biases due to differences in labor quality and industry composition across areas. The early time series estimates were significantly below one. However, after using improved time series data on labor, capital, output, and factor payments, Berndt (1976) concludes that one not reject that the aggregate production function was Cobb-Douglas.

Recently, however, a number of papers have challenged his result, as Berndt assumes that all technical change was Hicks neutral. Labor augmenting technical change causes estimates of the elasticity to be biased towards one. At the aggregate level, the capital share of cost is constant while the capital-labor ratio is rising, leading regressions to conclude that the production function is Cobb Douglas. However, if labor augmenting productivity increases at the same rate as the capital labor ratio, any elasticity is consistent with a stable aggregate capital share of cost. Antras (2004) and Klump et al. (2007) both control for labor augmenting technical change through time trends (implying exponential growth in labor augmenting productivity) or other parametric functional forms and estimate the elasticity to be .8 and .6 for the US, respectively. Depending upon the type of technical change and country studied, researchers have found the elasticity to be below, equal, or greater than one. León-Ledesma et al. (2010) show in Monte Carlo simulations that estimation approaches that use both the production function and its first order conditions jointly can identify the elasticity of substitution given biased technical change.

Another literature estimates the firm level elasticity of substitution using shocks to the rental rate of capital, such as changes in capital taxes or investment tax credits. Chirinko (2008) provides a recent survey of this literature. Studies using changes in the rental rate of capital have to face a couple of challenges. First, since adjustment costs mean that capital cannot adjust instantaneously, the short run elasticity of substitution can be much lower than the long run elasticity of substitution. Thus, controlling for adjustment costs through lag values of variables can have a large impact on the estimated elasticities. Second, some of the changes in the rental rate for capital are transitory, so firms may not respond much to them. Labor augmenting technical change can also bias the micro level estimates as well. As a result, estimates of the elasticity of substitution vary considerably, depending upon the panel data set and type of user cost variation used.

To account for some of the estimation problems, compares changes in the firm's average capital-output ratio over two long time intervals to changes in the average user cost, to avoid problems of transitory changes in the user cost or adjustment costs. Controlling for industry effects and biased technical change, they find estimates of .44, similar to my baseline estimates using MSA level wages. One advantage of my approach using differences in local area wages is that Chirinko et al. (2004)wage differences are fairly persistent over time, so I do not have to worry about adjustment costs. Since I am using a large crosssection, I can also look at estimates for particular years and industries.

2 Basic Theory

In this section I go over the basic theory of firm production under a CES production function with labor augmenting productivity. If you are comfortable with this theory, feel free to skip the section and move on to the empirical results.

I assume that the production function has a constant elasticity of substitution σ, which allows labor and capital to either be complements or substitutes depending upon the value of σ. Hicks (1932) shows that, for any constant returns production function:

$$\sigma = \frac{F_k F_l}{F_{kl} F} \tag{2.1}$$

The elasticity of substitution depends upon the curvature of the production function through the second derivative F_{kl}, as the firm's isoquants reveal. The isoquants are linear when the production function is linear, with an infinite elasticity of substitution. As the elasticity of substitution falls, the isoquants become more and more curved as labor and capital become more and more complementary. When the elasticity of substitution reaches zero, the isoquants take the L shape of the Leontief production function.

Productivity can enter into the production function in a number of different ways. If productivity is Hicks neutral, improvements in productivity affect labor and capital symmetrically. If productivity is labor augmenting (Harrod neutral), an increase in productivity is equivalent to having more labor. The CES Production Function with labor and capital and both Hicks neutral and labor augmenting technical differences is:

$$Y = A(\alpha K^{\frac{\sigma-1}{\sigma}} + (1-\alpha)(BL)^{\frac{\sigma-1}{\sigma}})^{\frac{\sigma}{\sigma-1}} \tag{2.2}$$

Here, A is Hicks neutral productivity and B is labor augmenting productivity. Y is the output of the firm, not revenue. σ is the elasticity of substitution: labor and capital are complements if $\sigma < 1$ and substitutes if $\sigma > 1$. The distribution parameter α governs how much capital contributes to output relative to labor. If the elasticity is not equal to 1, α cannot be identified separately from A and B. This production function is also constant returns to scale. [1]

The elasticity of substitution determines how the ratio of marginal products depends upon productivity. Taking derivatives of the CES production function, we have that:

$$\frac{MPK}{MPL} = (B)^{\frac{1-\sigma}{\sigma}} (\frac{K}{L})^{-\frac{1}{\sigma}} \frac{\alpha}{1-\alpha} \tag{2.3}$$

Since Hicks neutral productivity increases the marginal productivity of capital and labor by the same percentage, the ratio of marginal products is unaffected. Labor augmenting productivity does affect the ratio of marginal products, though its effects depend upon the elasticity of substitution. Keeping the capital labor ratio constant, the marginal product of capital rises relative to that of labor when labor augmenting productivity increases if the elasticity of substitution σ is less than 1. If the elasticity of substitution is greater than 1, increases in labor augmenting productivity decrease the marginal product of capital relative to labor. In the Cobb-Douglas case, where σ equals 1, increases in productivity do not affect the ratio of marginal products.

[1] One useful property of the CES production function is that it nests a number of famous simple cases. When the elasticity of substitution σ converges to 0, we have the Leontief production function, $Y = A\min(K/\alpha, BL/(1-\alpha))$. When the elasticity of substitution σ is 1, we have the Cobb-Douglas production function $Y = AK^{\alpha}(BL)^{1-\alpha}$. When the elasticity of substitution σ converges to infinity, we have the linear production function $Y = A(\alpha K + (1-\alpha)BL)$.

2.1 Cost Minimization

So far I have kept factor proportions constant. A cost minimizing firm sets marginal products equal to factor prices by adjusting the levels of its factors. Cost minimization implies:

$$\frac{r}{w} = \frac{MPK}{MPL} = (B)^{\frac{1-\sigma}{\sigma}} (\frac{K}{L})^{-\frac{1}{\sigma}} \frac{\alpha}{1-\alpha} \tag{2.4}$$

I invert the above equation to examine the capital labor ratio and factor cost ratio.

$$\frac{K}{L} = B^{1-\sigma} (\frac{r}{w})^{-\sigma} (\frac{\alpha}{1-\alpha})^{\sigma} \tag{2.5}$$

$$\frac{rK}{wL} = B^{1-\sigma} (\frac{r}{w})^{1-\sigma} (\frac{\alpha}{1-\alpha})^{\sigma} \tag{2.6}$$

First, the elasticity of substitution σ controls how the capital labor ratio and factor cost ratio react to changes in factor prices. [2] As wages increase, the capital labor ratio will rise unless the production function is Leontief. Wage increases will reduce the factor cost ratio when $\sigma < 1$. A Cobb-Douglas production function has a constant capital share of cost, as $\frac{rK}{wL} = \frac{\alpha}{1-\alpha}$.

I can also use equations 2.5 and 2.6 to characterize what happens to the capital labor ratio and factor cost ratio when the level of labor augmenting productivity changes. If $\sigma < 1$, firms respond to increases in labor augmenting productivity by raising their capital-labor ratio and factor cost ratio until marginal products are equal to factor prices. Thus, increases in labor augmenting productivity are also *labor saving* if $\sigma < 1$. If $\sigma > 1$, firms decrease

[2] Cost minimization thus implies the Robinson (1933) definition of the elasticity of substitution, $\sigma = -\frac{d\log(K/L)}{d\log(r/w)}$

their capital-labor ratio when labor augmenting productivity rises.

The intuition here is that increases in labor augmenting productivity B are equivalent to increases in labor for firms. If labor and capital are complements, firms then want to increase the amount of capital they hold, so the marginal product of capital rises until firms increase capital relative to labor. Since the elasticity of the capital-labor ratio to changes in labor augmenting productivity B is $1-\sigma$, the capital labor ratio increases less than proportionately with B unless the production function is Leontief.

In the Cobb-Douglas case we cannot separate Hicks neutral from factor augmenting productivity. The labor augmenting productivity B can be merged with the Hicks neutral productivity A to form a new Hicks neutral shifter $\tilde{A}$:

$$Y = AK^{\alpha}(BL)^{1-\alpha} = AB^{1-\alpha}K^{\alpha}L^{1-\alpha} = \tilde{A}K^{\alpha}L^{1-\alpha}$$

I summarize this section in the following proposition:

Proposition 1. *If the firm production function is CES with labor and capital as inputs, and firms cost minimize facing competitive factor markets, then when $\sigma < 1$:*

1. *The factor cost ratio $\frac{rK}{wL}$ will decrease with increases in w/r with an elasticity of $\sigma - 1$.*
2. *The capital-labor ratio $\frac{K}{L}$ will increase with increases in w/r with an elasticity of σ.*
3. *Both the factor cost ratio and capital-labor ratio increase with labor augmenting productivity B with an elasticity of $1 - \sigma$.*

If the firm production function is Cobb-Douglas with labor and capital as inputs, and firms cost minimize facing competitive factor markets, the factor cost ratio $\frac{rK}{wL}$ is constant.

2.2 Profit Maximization

So far I have not placed any assumptions on the level of output. I now introduce a demand side through an isoelastic demand function. Each firm produces a differentiated product and has a downward sloping demand curve for their product. The demand curve is:

$$Y = \frac{D^{\epsilon-1}}{P^{\epsilon}} \tag{2.7}$$

Here D is a demand shifter, as firms with higher D can sell more of the product at the same price. ϵ is the elasticity of demand for the firm's product. I assume that the elasticity of demand is greater than one, as is required in a model with price setting firms. I can rewrite expression 2.7 in terms of revenue, so firm revenue depends upon price in the following way:

$$PY = (D/P)^{\epsilon-1} \tag{2.8}$$

Because the demand function is isoelastic, the optimal price for the firm is a simple constant markup over marginal cost:

$$P = \frac{\epsilon}{\epsilon-1}C \tag{2.9}$$

where C is the marginal cost of the firm's product. Since the production function has constant returns to scale, the marginal cost of production does not depend upon the amount produced. Cost minimization implies that the firm's marginal cost is:[3]

[3] The marginal cost is the Lagrange multiplier on the production function in the cost minimization problem. To obtain the marginal cost, substitute in the first order conditions for labor and capital into the production function and then solve for the Lagrange multiplier.

$$C = \frac{1}{A}(\alpha^{\sigma} r^{1-\sigma} + (1-\alpha)^{\sigma}(\frac{w}{B})^{1-\sigma})^{\frac{1}{1-\sigma}} \tag{2.10}$$

Both A and B reduce the marginal cost of the firm. The capital distribution parameter α and labor augmenting productivity B govern how much labor contribute to the marginal cost relative to capital. I can then solve for the price substituting the marginal cost from equation 2.10 into the markup from equation 2.9. Then, equation 2.8 implies that the firm's revenue is:

$$PY = (AD)^{\epsilon-1}(\frac{\epsilon-1}{\epsilon})^{\epsilon-1}(\alpha^{\sigma} r^{1-\sigma} + (1-\alpha)^{\sigma}(\frac{w}{B})^{1-\sigma})^{-\frac{\epsilon-1}{1-\sigma}} \tag{2.11}$$

Both the Hicks neutral productivity A and demand shifter D increase revenue in the same way. With heterogeneous goods, A and D are isomorphic: one can not tell whether a given product is produced more efficiently with higher A or just has higher demand D.

Firms with higher labor augmenting productivity B also have higher revenue. Since the cost minimizing conditions mean that higher B firms have a factor cost ratio $\frac{rK}{wL}$, firms with higher revenue should have a higher factor cost ratio. Since revenue increases proportionately with cost through the constant markup, the average revenue products of capital and labor depend upon B:

$$\frac{PY}{L} = \frac{\epsilon}{\epsilon-1} w \frac{\alpha^{\sigma} r^{1-\sigma} + (1-\alpha)^{\sigma}(\frac{w}{B})^{1-\sigma}}{(1-\alpha)^{\sigma}(\frac{w}{B})^{1-\sigma}} \tag{2.12}$$

$$\frac{PY}{K} = \frac{\epsilon}{\epsilon-1} r \frac{\alpha^{\sigma} r^{1-\sigma} + (1-\alpha)^{\sigma}(\frac{w}{B})^{1-\sigma}}{(1-\alpha)^{\sigma} r^{1-\sigma}} \tag{2.13}$$

Hicks neutral productivity does not affect the average revenue products. Improvements in Hicks neutral productivity induce the firm to produce more until the marginal return of factors meets factor prices. This increase in production pushes the firm down its demand curve until the price falls and average revenue products remain constant. Labor augmenting productivity, by contrast, shifts the average revenue products of labor and capital in opposite directions. A firm with high labor augmenting productivity B increases its capital labor ratio, depressing its average revenue product of capital and pushing up its average revenue product of labor. I summarize this section with the following proposition:

Proposition 2. *If the firm production function is constant returns to scale and CES with labor and capital as inputs, and firms profit maximize facing competitive factor markets and an isoelastic demand function, then when $\sigma < 1$:*

1. *Revenue will increase with labor augmenting productivity B.*
2. *The average revenue product of labor $\frac{PY}{L}$ increases with labor augmenting productivity B and the average revenue product of capital $\frac{PY}{K}$ decreases with B.*

3 Data

In this study, I primarily use US data on manufacturing plants from the Census of Manufacturers and Annual Survey of Manufacturers (ASM). The Census of Manufacturers is a census of all manufacturing plants taken every five years. For some small plants with less than 5 workers, called Administrative Record plants, the Census only records payroll and employment gathered from IRS data. Since capital is an important variable in my study, I drop these plants as is common in the literature.

The primary data constraint for this study is data on capital. Before 1987 the Census did not ask questions on capital stocks for non ASM plants. Thus, I use the 1987, 1997, and 2002 Census of Manufacturers. The Annual Survey of Manufacturers tracks about 50,000 plants over five year periods and is more heavily weighted towards big plants. I use the manufacturing Censuses for my main results, but I do look at the ASM only plants for some robustness checks.

For the manufacturing Census samples, I drop all plants that enter in the given Census year. Entering plants could have high levels of inputs (for example capital) but no output if the plant entered late in the year. In this case, the plant would look extremely unproductive only because the true levels of inputs are measured incorrectly. Similarly, if the entering plant has bought its capital but not yet employed workers, the capital-labor ratio may look high relative to other plants in the industry.

I also clean the data for outliers. First, I drop all observations where data on a number of variables is either missing, zero, or negative: these include the average product of capital, average product of labor, capital share of cost, capital-labor ratio, and wage (measured as payroll over employment). I also drop outlier observations in the bottom .5% and top 99.5% tails of these variables relative to their industry, which amounts to about 4% of the dataset for each Census. This data cleaning prevents huge outliers due to mismeasurement from affecting the results, but the main results are similar when these outliers are included.

For the 2002 Census of Manufacturers, data on the value of non monetary compensation given to employees, such as health care or retirement benefits, are available for most plants. I use this benefits data to better measure payments for labor. I also then drop data from plants whose benefits data is imputed.

I measure capital by the end year book value of capital, deflated using a current cost to historic cost deflator. The 1987 Census has book values for equipment capital and structures capital separately, so I construct capital stocks for each and then combine them. For the general Census of Manufacturers I cannot use perpetual inventory methods, because investment is not recorded in non-Census years.

To measure the capital share of cost, I also need measures of factor prices. I use unpublished 2 digit BLS rental rates calculated by standard formulas converting capital prices to rental rates. For wage payments I use the total wage bill of the plant, except in 2002 when

I add benefits payments as well.

I measure age for plants using the Longitudinal Business Database, which records the first year and last year of each plant. However, the LBD began in 1975 so any plants existing before 1975 are given a first year of 1975. To better measure age for older plants, I record the first year of the plant as 1972 if it existed in the 1972 Census of Manufacturers.

My empirical strategy in this paper is to look within cross-sections, using the panel nature of the data only to calculate correlations or growth rates over time. In all of my results, I control for 4 digit industry so I use within industry variation. My measures of the factor cost ratio assume that the rental rate of capital is constant between plants within the same industry. I also assume that the wage faced by the plant can vary across local areas, but within the same local area differences in wages between plants are due to skill differences.

4 Elasticity of Substitution

4.1 Estimates

The first order conditions for capital and labor of the CES production function under cost minimization imply that:

$$\log(rk/wl) = -(1-\sigma)\log(w/r) + (1-\sigma)\log B + \sigma \log \frac{\alpha}{1-\alpha} \tag{4.1}$$

If the production function is Cobb-Douglas, plants adjust their capital-labor ratio proportionately to increases in wages, so the factor cost ratio remains constant. If the elasticity of substitution is less than one, plants do not increase their capital intensity enough to compensate for the rise in wages and so the factor cost ratio falls. Thus, the slope of the relationship between the wage and the factor cost ratio depends upon whether the elasticity of substitution is greater than or less than one.

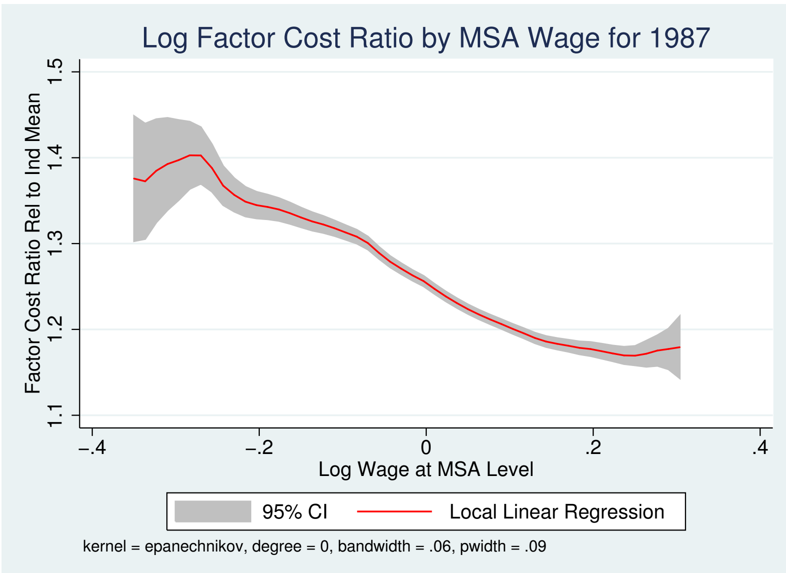
I compute the factor cost ratio as before, from data on capital and the wage bill at the plant level. I then use local area wage variation to estimate the elasticity of substitution. I am assuming that the local area wage is the wage that the plant faces when it chooses its levels of factors, and that the local area wage is orthogonal to the plant's rental rate for capital and its level of labor augmenting productivity. Under these conditions, local area wage variation should provide a consistent estimate of the elasticity of substitution. Since I have plant level data, I am estimating the elasticity of substitution at the plant level, which may differ from the industry or aggregate elasticity of substitution.

I calculate measures of local area wages from a worker based dataset and an employer based dataset. The first source of wage variation is from the Census 5% sample of Americans, where I match the 1990 Census to the 1987 Census of Manufacturers and the 2000 Census to the 1997 Census of Manufacturers. I calculate the individual's wage as wage and salary income divided by the number of hours worked times weeks worked, for men working in the private sector with ages between 25 and 55. This wage is thus an individual hourly wage. I then construct the average log wage for each state and each MSA, after controlling for industry, occupation, a quartic in experience, race, and education. These wage measures are thus adjusted for differences in worker quality across areas which would otherwise act as measurement error. I have experimented with using median log wages or average log wages for manufacturing workers only in unpublished results and get similar estimates to those below.

The second source of wage variation is on the firm side, using the Longitudinal Business Database (LBD). The LBD contains employment and payroll data for every establishment in the US (so around 7 million establishments). I define the wage as payroll divided by employment, so this wage is the average yearly wage for the establishment. I then construct average log wages for each state, MSA, and county in the United States. Since the LBD is yearly, I match the Manufacturing Censuses to wages from the appropriate year of the LBD.

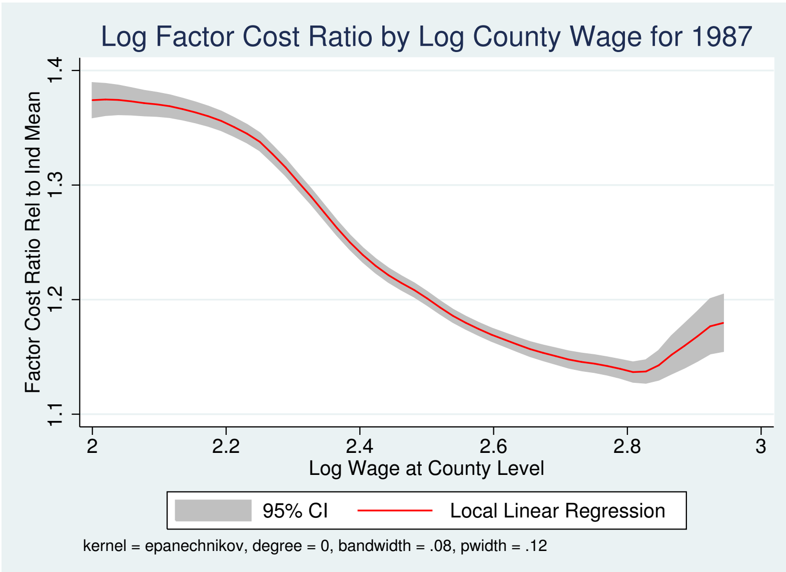
Figures 4.1 and 4.2 nonparametrically plot the industry demeaned factor cost ratio against the state level wage and the county level wage. In both plots, the factor cost ratio is strongly decreasing in the local area wage. The relationship does flatten out for very low wages and very high wages, especially in the county level wage graph. However, there are not many plants in areas with these extreme wages, so the confidence bands are quite wide.

Figure 4.1: Factor Cost Ratio by MSA Level Wage for 1987



The X axis is the average log wage at the MSA level after controlling for education, experience, race, industry, and occupation, where the wage is calculated as the wage and salary income over total number of hours worked for a worker in the Census 5% sample data. The Y axis is the log factor cost share after taking out industry averages.

Figure 4.2: Factor Cost Ratio by County Level Wage for 1987



The X axis is the average log wage at the county level, where the wage is computed as payroll/number of employees at the establishment level for establishments in the Longitudinal Business Database. The Y axis is the log factor cost share after taking out industry averages.

Table 2 displays estimates of the elasticity of substitution for all of manufacturing. Each column of the table provides estimates of the elasticity of substitution from a different source of wage variation. In these regressions, I assume that the elasticity of substitution is the same for every 4 digit industry, but the distribution parameter α and average level of labor augmenting productivity B can vary across industries through industry fixed effects. I cluster standard errors at the 2 digit SIC- local area level where the local area is based on the source of the wage variation, so the state-level regressions have standard errors clustered at the 2 digit SIC- state level, the MSA level regressions have standard errors clustered at the 2 digit SIC- MSA level, etc. This level of clustering adjusts the standard errors for correlated shocks to the factor cost ratio in local areas for plants in the same broad industry.

Table 2: Elasticities of Substitution between Labor and Capital for All Manufacturing

	State Level	MSA Level	State Level	County Level	County Level, Within State
CMF 1987	.37 *(.03)*	.44 *(.03)*	.53 *(.02)*	.60 *(.02)*	.65 *(.02)*
CMF 1997	.17 *(.05)*	.42 *(.02)*	.47 *(.03)*	.60 *(.01)*	.67 *(.01)*
Source of Wage Data	Census 5% individual samples		Longitudinal Business Database		
State Dummies	No	No	No	No	Yes
N	~180,000	~125,000	~180,000	~180,000	~180,000

Note: All regressions include industry dummies and have standard errors clustered at the 2 digit industry-area level (so for state-level regressions, 2 digit sic-state, etc.) Wages used are the average log wage for the geographic area, where the wage is computed as payroll/number of employees at the establishment level for the LBD wages and the wage is wage and salary income over total number of hours worked for the Census 5% sample data. The average log wages using worker data are adjusted for differences in education, experience, race, occupation, and industry.

The elasticity of substitution is .37 in the state-level wage regressions for 1987 and .17 in the state-level wage regressions for 1997 when I can control for worker quality differences in the wages. Controlling for differences in worker quality matters as these estimates are lower than those from the LBD that are not adjusted for differences in quality. When I use more disaggregated wages at the MSA and county level, estimates are slightly higher. At the MSA level, I estimate that the elasticity is .44 for 1987 and .42 for 1997. Since the MSA level regressions drop all plants not located in an MSA, these estimates are based only on plants in major metropolitan areas. Using wages at the county level, I can incorporate plants outside major metro areas and still have a more disaggregated measure of the wage, though these wages will not be adjusted for quality differences. County level wage regressions imply that the elasticity of substitution is .6. In all of these cases, I can easily reject that the production function is Cobb-Douglas.

I also run county-level wage regressions with state level fixed effects, so that all of the wage variation is within state. The previous regressions could have problems if state-level regulations affect both wages and the factor cost ratio.[4]

Using within state county level wage variation, I find that the elasticity of substitution is .65 for 1987 and .67 for 1997. These estimates are slightly higher than those without state fixed effects, but I can still easily reject an elasticity of 1.

So far, I have assumed that the elasticity of substitution is constant across all industries. In Table 3, I show estimates of the elasticity of substitution using state, county, and within state county level wage variation for each two digit SIC industry for 1987. I exclude the tobacco industry because it is much smaller than the other two digit industries. Two digit SIC industries are major broad industry groupings within manufacturing. For example, Textiles or Primary Metals are two digit SIC industries, while Carpets and Rugs (SIC 2273) and Steel Blast Furnaces (SIC 3312) are four digit SIC industries within these broader two digit SIC industries.

[4] For example, right to work laws make it more difficult for firms to unionize, which could both lower wages and make it easier for firms to automate and change their factor cost ratio. Holmes (1998) shows that plants do indeed respond to right to work laws, as industrial activity is higher than average in right to work states adjacent to non right to work states.

Table 3: Elasticities of Substitution between Labor and Capital for 2 digit SIC Industries

SIC Two Digit Industry:	Level of Wage Variation				N
	State Level, 1987	MSA Level, 1987	County Level, 1987	County Level, Within State, 1987	
20: Food Products	.4 *(.12)*	.54 *(.12)*	.66 *(.04)*	.64 *(.05)*	∼10,000
22: Textiles	-.08 *(.16)*	.47 *(.18)*	.55 *(.09)*	.87 *(.09)*	∼3,500
23: Apparel	.50 *(.12)*	.75 *(.17)*	.92 *(.06)*	1.17 *(.06)*	∼12,000
24: Lumber and Wood	.35 *(.15)*	.19 *(.13)*	.39 *(.05)*	.41 *(.05)*	∼15,000
25: Furniture	.22 *(.17)*	.33 *(.18)*	.38 *(.05)*	.43 *(.07)*	∼6,000
26: Paper	-.20 *(.16)*	.08 *(.22)*	.35 *(.06)*	.39 *(.08)*	∼4,000
27: Printing and Publishing	.57 *(.05)*	.52 *(.06)*	.67 *(.03)*	.68 *(.03)*	∼26,000
28: Chemicals	.19 *(.17)*	.23 *(.16)*	.40 *(.09)*	.38 *(.09)*	∼6,500
29: Petroleum Refining	.38 *(.27)*	.56 *(.23)*	.70 *(.12)*	.86 *(.14)*	∼1,500
Source of Wage Data	Census 5% individual samples		Longitudinal Business Database		
State Dummies	No	No	No	Yes	

SIC Two Digit Industry:	Level of Wage Variation				N
	State Level, 1987	MSA Level, 1987	County Level, 1987	County Level, Within State, 1987	
30: Rubber	.14 *(.19)*	.56 *(.14)*	.46 *(.05)*	.54 *(.05)*	~8,500
31: Leather	.63 *(.32)*	.50 *(.33)*	.77 *(.11)*	.86 *(.18)*	~1,000
32: Stone, Clay, Glass, Concrete	.19 *(.14)*	.34 *(.12)*	.62 *(.04)*	.75 *(.05)*	~9,000
33: Primary Metal	.19 *(.13)*	.28 *(.19)*	.60 *(.06)*	.69 *(.08)*	~4,000
34: Fabricated Metal	.20 *(.11)*	.23 *(.10)*	.47 *(.04)*	.52 *(.04)*	~20,000
35: Machinery	.47 *(.08)*	.52 *(.08)*	.65 *(.02)*	.68 *(.03)*	~25,000
36: Electrical Machinery	.23 *(.14)*	.32 *(.13)*	.53 *(.07)*	.64 *(.06)*	~8,000
37: Transportation Equip	.49 *(.20)*	.65 *(.17)*	.64 *(.06)*	.77 *(.07)*	~5,000
38: Instruments	.65 *(.12)*	.64 *(.11)*	.61 *(.06)*	.55 *(.09)*	~4,500
39: Misc	.37 *(.17)*	.46 *(.17)*	.51 *(.04)*	.51 *(.06)*	~6,500
Source of Wage Data	Census 5% individual samples		Longitudinal Business Database		
State Dummies	No	No	No	Yes	

Note: All regressions include industry dummies and have standard errors clustered at the 2 digit industry-area level (so for state-level regressions, 2 digit sic-state, etc.) Wages used are the average log wage for the geographic area, where the wage is computed as payroll/number of employees at the establishment level for the LBD wages and the wage is wage and salary income over total number of hours worked for the Census 5% sample data. The average log wages using worker data are adjusted for differences in education, experience, race, occupation, and industry.

The elasticity of substitution does vary considerably between 2 digit industries, but most of these differences are not statistically significant. For an example of statistically significant differences, the lumber and wood industry has an elasticity of about .4 which is significantly different from the leather industry with an elasticity of .77. While there are some differences in estimates between different types of wage variation within the same industry, only for Textiles and Apparel are these estimates significantly different from each other.

I can reject that the elasticity of substitution is one for 18 of 19 industries using state level wages, 16 of 19 industries using MSA level wages, 17 of 19 industries using county level wages, and 15 of 19 industries using within state county level wages. In only one case, for the Apparel industry with within state county level wage variation, do I find a point estimate of the elasticity that is above one. Thus, disaggregating to the SIC 2 digit level does not alter my main conclusion that the Cobb-Douglas specification can be rejected.

4.2 Potential Caveats

4.2.1 Local Area Agglomerations

Many industries exhibit agglomeration effects, which lead firms in those industries to concentrate in particular geographic areas. For example, much of the US furniture industry is located in North Carolina, the computer industry in Silicon Valley, and the auto industry in Detroit. If the plants inside the agglomeration area were no different than the plants outside, such local area industry concentrations would mean less wage variation to identify the elasticity of substitution and so higher standard errors on estimates.

To bias the regression estimates, plants in the agglomeration area must have a different production function than those outside and wages would have to be different in agglomeration areas than non agglomeration areas. For the furniture industry, North Carolina plants would need to pay higher wages than the rest of the US and have a production function with a lower capital share (or vice versa). Holmes and Stevens (2010) argue that some industries are characterized by a large mass (bottom 80%) of small producers producing for specialized local demand, and a few (top 20%) of producers producing on a mass scale for the entire market. If the production functions for local demand and global demand were different, agglomeration areas where the big producers operate could have systematic differences in their factor cost ratio from the rest of the US.

To check whether these forces are driving my results, I look at 10 four digit SIC industries with substantial geographic variation. I select all the industries that are located in at least 300 MSAs or at least 250 MSAs and 48 states, dropping industries that have less than 1,000 plants or are miscellaneous industries (plants that the SIC code did not classify anywhere else are sometimes put into miscellaneous industries). Newspaper Publishing, Commercial Lithographic Printing, and Ready Mixed Concrete are among the biggest of these industries. Ready Mixed Concrete is perhaps the best test case; since ready mixed concrete cannot be shipped very far, every location that has construction activity must have concrete plants. For concrete, there are no agglomerations and no differences between local and non-local producers. Table 4 displays the estimates of the elasticity of substitution using state and county level wages for these industries. I can reject Cobb-Douglas for eight out of ten industries with state level wages and ten out of ten industries with county level wages. For ready mixed concrete, I find an elasticity of substitution of .36 in the state-level regressions and .8 in the county level regressions.

Table 4: Elasticities of Substitution between Labor and Capital for Geographically Varying Industries

SIC Four Digit Industry	Level of Wage Variation		N
	State Level, 1987	County Level, 1987	
2711: Newspaper Publishing	.37 *(.13)*	.75 *(.07)*	~4,000
2752: Commercial Printing, Lithographic	.74 *(.07)*	.71 *(.03)*	~12,000
3272: Concrete Products, Except Block and Brick	.39 *(.20)*	.80 *(.07)*	~2,000
3273: Ready Mixed Concrete	.16 *(.19)*	.80 *(.07)*	~4,000
3441: Fabricated Structural Metal	.25 *(.22)*	.48 *(.11)*	~1,500
3444: Sheet Metal Work	.40 *(.15)*	.45 *(.09)*	~3,000
2051: Bread and other Bakery Products, except Crackers	.63 *(.28)*	.63 *(.12)*	~1,000
2421: Sawmills and Planing Mills	.89 *(.32)*	.75 *(.08)*	~3,000
2431: Millwork	-.17 *(.22)*	0 *(.09)*	~1,500
2434: Wood Kitchen Cabinets	.08 *(.23)*	.23 *(.11)*	~2,000
Source of Wage Data	Census 5% individual samples	Longitudinal Business Database	
State Dummies	No	No	

Note: All regressions include industry dummies and have standard errors clustered at the 2 digit industry-area level (so for state-level regressions, 2 digit sic-state, etc.) Wages used are the average log wage for the geographic area, where the wage is computed as payroll/number of employees at the establishment level for the LBD wages and the wage is wage and salary income over total number of hours worked for the Census 5% sample data. The average log wages using worker data are adjusted for differences in education, experience, race, occupation, and industry.

To improve on this analysis, I plan to estimate the elasticity of substitution separately for the industries that Holmes and Stevens identify as not having differences between local demand plants and mass market plants. I will also estimate the elasticity of substitution separately for industries with low agglomerations using an agglomeration index incorporating differences in output across areas, as in Ellison and Glaeser (1997), and not just dispersion in location of plants.

4.2.2 Different Types of Workers

So far in this paper, I have combined all labor into one plant level aggregate. This aggregation implies that different types of workers are perfect substitutes for each other and complementary with capital with the same elasticity of substitution. Capital could be more complementary with some kinds of workers than others.

The simplest disaggregation of labor is into skilled workers and unskilled workers. In fact, one major agenda of the recent labor literature has been investigating skill biased technical change, where productivity increases are biased towards skilled workers. Also, many papers assume that capital and skilled labor are complementary. In the Manufacturing Censuses, I do have information on the number of production workers and non production workers and their wages. I use production workers are a proxy for unskilled workers and nonproduction workers for skilled workers, as is done by a number of papers in the literature including Kahn and Lim (1998) and Blum (2010).

I then estimate the elasticity of substitution between capital and skilled labor, capital and unskilled labor, and skilled and unskilled labor. The regression equations are similar to that for capital and labor, except that I use the high school average log wage from the Census 5% files for a measure of the unskilled wage and the college average log wage for a measure of the skilled wage. Table 5 shows the results using state and MSA level wage variation. I find that the elasticity between capital and skilled workers is always significantly lower than that between capital and unskilled workers, consistent with capital-skill complementarity. The elasticity of substitution between capital and skilled labor is very low, at .19 for 1987 and .01 for 1997 using state wages. Estimates at the MSA level are higher, at .33 for 1987 and .43 for 1997, but are still fairly low.

Table 5: Partial Elasticities of Substitution between Capital and Skilled Labor, Capital and Unskilled Labor, and Skilled and Unskilled Labor, for all Manufacturing

Elasticity	Level of Wage Variation			
	State Level, 1987	MSA Level, 1987	State Level, 1997	MSA Level, 1997
Capital and Unskilled Labor	.61 *(.03)*	.71 *(.03)*	.54 *(.04)*	.71 *(.03)*
Capital and Skilled Labor	.19 *(.04)*	.33 *(.03)*	.01 *(.06)*	.43 *(.04)*
Skilled and Unskilled Labor	3.20 *(.17)*	2.89 *(.14)*	2.43 *(.11)*	2.08 *(.09)*
State Dummies	No	No	No	No
N	∼180,000	∼110,000	∼180,000	∼130,000

Note: I run regressions with the high school wage (wage of high school completers from the Census 5% samples) to compute the unskilled labor-capital elasticity, regressions with the college wage (wage of college completers from the Census 5% samples) to calculate the skilled labor-capital elasticity, and regressions with the relative wage of high school completers to college completers to calculate the skilled labor-unskilled labor elasticity. The average log wages are adjusted for differneces in experience, race, occupation, and industry.

The elasticity of substitution between capital and unskilled labor is higher, at .61 for 1987 and .54 for 1997 using state level wages. I can still reject that the elasticity of substitution between capital and unskilled labor is 1 for all four estimates. The estimates of the elasticity of substitution between skilled and unskilled labor indicate that skilled and unskilled labor are substitutes, with elasticities substantially above one.

The basic CES production function cannot rationalize these estimates, as the CES form implies that the partial elasticities between all factors would be equal. The nested CES production function of Sato (1967) also fails here, as under the nested CES two of the three elasticities would have to be equal. The translog production function and a whole range of other possibilities considered in Fuss and McFadden, eds (1978) are all candidate production functions to explain these estimates. However, exploring these is beyond the scope of this paper.

4.2.3 Different Production Functions than CES

So far, I have assumed that the elasticity of substitution between labor and capital is constant. If a more broad production function characterizes an industry, the elasticity of substitution will not necessarily be constant. The translog production function, for example, does allow the elasticity of substitution to vary across plants. One way to test this is to run a regression of the factor cost ratio against the local area wage including polynomial terms in the local area wage as well. If the elasticity of substitution is constant, the relationship between the factor cost ratio and local area wage should be linear. In this case, one should not be able to reject that the coefficient on all of the non linear terms are jointly zero.

I include quadratic, cubic, and quartic local area wage terms in the regressions using all of manufacturing and conduct joint tests that the coefficients on all of these terms are equal to zero. In all cases, I reject the null hypothesis, raising the possibility that a more general production function than the CES characterizes the data. I plan to examine how large these deviations from linearity are, especially outside the tails of the local area wage distribution.

Another way to explain the above facts is that plants within the same industry have different Cobb-Douglas production functions. Differences in Cobb-Douglas production functions would lead plants to set different factor cost ratios. Since high wage areas have lower factor cost ratios, the Cobb-Douglas capital coefficient would have to be lower in high wage areas.

In models where plants can choose either their production technology or their location, plants have higher Cobb-Douglas capital coefficients in high wage areas. If a firm can choose between two different Cobb-Douglas production technologies, the relative cost of these technologies depends on factor prices. As wages increase, the more capital intensive Cobb-Douglas technology is favored because its relative cost falls. Firms could also choose where to locate. High capital intensive technology firms should be more likely to locate in high wage areas, as their costs rise less with high wages. Both of these scenarios predict that high capital share technologies would be observed in high wage areas, the opposite of what I find.

4.2.4 Endogeneity Concerns

My identification strategy relies upon differences in wages across local areas in the US. One potential concern is that the local area wage is endogenous to factors that affect firm factor cost ratios. To explore this, I first examine what causes differences in wages across local areas. If there are no frictions preventing people from moving around the country, people move seeking higher wages until the real wage that the worker faces is the same across locations. The wage important for my study is the wage that the plant pays its workers. In areas with a high cost of living (because of housing prices, for example), wages faced by the plant will be high even if real wages for workers are the same across areas. Thus, cost of living differences can lead to local area wage differences.

Migration costs are another reason for local area wage dispersion. Increases in labor demand will increase wages in the short run when labor supply is relatively fixed. Labor demand is affected by many factors, including the number of firms in the area and demand shocks to local industries. Local demand shocks will not affect the plant's optimal ratio of factor costs, however, as changes in demand do not affect the cost minimization conditions. Hicks neutral productivity improvements will also increase labor demand but not affect the plant's factor cost ratio. Only improvements in labor augmenting productivity B could both change the wage, by increasing labor demand, and change the plant's factor cost ratios. Even in this narrow case, manufacturing may not affect local area wages very much. Manufacturing is a small percentage of total employment, at 17.4% of total employment in 1987 and 14.5% of total employment in 1997.

Small local industries such as ready mixed concrete should have little effect on local area labor demand. To recall previous results in Table 4, I find an elasticity of substitution less than one for these industries as well. I also restrict the sample to only counties where the share of manufacturing employment is below the national median. Using only these counties, I find similar estimates in Table 6 of the elasticity of substitution.

Table 6: Elasticities of Substitution between Labor and Capital for All Manufacturing in Low Manufacturing Employment Counties

	State Level	State Level	County Level	County Level, Within State
CMF 1987	.49 *(.04)*	.60 *(.02)*	.60 *(.02)*	.66 *(.02)*
CMF 1997	.38 *(.06)*	.61 *(.02)*	.61 *(.01)*	.68 *(.01)*
Source of Wage Data	Census 5% individual samples	Longitudinal Business Database		
State Dummies	No	No	No	Yes
N	~80,000	~80,000	~80,000	~80,000

Note: All regressions include industry dummies and have standard errors clustered at the 2 digit industry-area level (so for state-level regressions, 2 digit sic-state, etc.) Wages used are the average log wage for the geographic area, where the wage is computed as payroll/number of employees at the establishment level for the LBD wages . For the Census 5% sample data, I calculate the residual wage after controlling for composition differences, where the wage is wage and salary income over total number of hours worked. I report results only for plants in counties where the manufacturing percentage of employment is below the national median for that year. The average log wages using worker data are adjusted for differences in education, experience, race, occupation, and industry.

I also examine instruments for the local area wage. One potential instrument exploits changes in labor demand, measured as changes in the national level of employment shares for 2 digit industries interacted with local area initial shares of these industries. With this instrument, I can look at labor demand changes excluding changes in manufacturing. In Table 7, I find extremely low estimates of the elasticity using these instruments (and even negative for 1997), but because standard errors are high I can only really say that the factor cost ratio falls with increases in wages. However, labor demand changes to the wage are temporary and firms may substitute much less when wage changes are temporary. I thus instrument using the 10 year previous wage in the area, which isolates the permanent component in wage differences across areas. Here I get similar estimates of the elasticity of substitution to the OLS estimates.

Table 7: Elasticities of Substitution between Labor and Capital for All Manufacturing using Instruments for Wages

	MSA Level	MSA Level	MSA Level
CMF 1987	.01 *(.43)*	.03 *(.44)*	.54 *(.03)*
CMF 1997	-.55 *(.60)*	-.60 *(.60)*	.32 *(.03)*
Source of Wage Data		Census 5% Samples	
Instrument	Labor Demand using All Industries	Labor Demand Using Non Manufacturing Industries	10 year lag wage
N	~120,000	~120,000	~120,000

Note: All regressions include industry dummies and have standard errors clustered at the 2 digit industry-area level (so for state-level regressions, 2 digit sic-state, etc.) Wages used are the average log wage for the geographic area.. For the Census 5% sample data, I calculate the residual wage after controlling for composition differences, where the wage is wage and salary income over total number of hours worked. The average log wages using worker data are adjusted for differences in education, experience, race, occupation, and industry.Labor demand instruments are based on the interaction between 10 year lag industry composition of employment in the MSA and nationwide changes in labor demand at the industry level. 10 year lag wage instrument is just the 10 year lagged wage.

Another concern is that the rental rate of capital also varies with the local area wage. For equipment capital, a national market for equipment should mean that the rental price is the same across local areas. For structures capital, however, local area wages and prices of materials can affect the price of capital and so the rental market. I thus look at the elasticity of substitution between equipment capital and labor in Table 8. Using only equipment capital, I find similar results on the elasticity of substitution.

Table 8: Elasticities of Substitution between Labor and Equipment Capital for All Manufacturing

	State Level	MSA Level	State Level	County Level	County Level, Within State
CMF 1987	.40 *(.03)*	.45 *(.03)*	.56 *(.02)*	.62 *(.02)*	.67 *(.02)*
Source of Wage Data	Census 5% individual samples		Longitudinal Business Database		
State Dummies	No	No	No	No	Yes
N	∼180,000	∼125,000	∼180,000	∼180,000	∼180,000

Note: All regressions include industry dummies and have standard errors clustered at the 2 digit industry-area level (so for state-level regressions, 2 digit sic-state, etc.) Wages used are the average log wage for the geographic area, where the wage is computed as payroll/number of employees at the establishment level for the LBD wages and the wage is wage and salary income over total number of hours worked for the Census 5% sample data. The average log wages using worker data are adjusted for differences in education, experience, race, occupation, and industry.

Another potential concern is that the plant's choice of its level of labor augmenting productivity B depends on the local wage. If plants adjust their level of labor augmenting technology because of the local wage, B will be related to the wage. When wages are high, labor augmenting technology that saves on labor is more valuable. Such wage based technology adoption would cause high wage areas to have high levels of labor augmenting technology and high capital shares. Thus, the relationship between the local area wage and labor augmenting technology would bias the estimate of the elasticity of substitution towards one.

5 Factor Bias of Productivity

5.1 Stylized Facts

Since I have found that the elasticity of substitution is less than one, I now look at stylized facts on the capital share of cost that could be generated by labor augmenting productivity.

5.1.1 Persistence over time

The capital-labor ratio and factor cost ratio are both persistent over time. If labor augmenting productivity is persistent, the factor cost ratio should be as well. On the other hand, measurement error or adjustment costs should not lead to persistent differences in the factor cost ratio and capital labor ratio. Measurement error should only cause temporary dispersion in capital shares unless measurement error in capital is serially correlated over a ten year span. Firms that face adjustment costs will not always match the static first order conditions of the simple theory above. Instead, they will only adjust capital infrequently when their capital stock gets too far away from the optimal capital stock. However, over a period of ten years the firm should have readjusted its capital stock, so adjustment costs would not predict long run persistence in the factor cost ratio or capital-labor ratio.

First, I regress each variable in 1997 against its value for the same plant in 1987, controlling for industry dummies. I compare these values with the autocorrelation values for conventionally measured TFP, measured by log value added subtracting log capital and log labor (total number of employees), both weighted using the industry level cost shares of the input. I use TFP as a comparison since TFP is well known to be autocorrelated over time.

Table 9: Persistence in Factor Cost Ratio and Capital-Labor Ratio between CMF87 and CMF97

	Ten Year	Implied One Year	Ten Year	Implied One Year
Log(Capital-Labor Ratio)	.42 *(.004)*	.92 *(.001)*	.48 *(.003)*	.93 *(.001)*
Log(Factor Cost Ratio)	.32 *(.004)*	.89 *(.001)*	.37 *(.003)*	.91 *(.001)*
TFP	.27 *(.003)*	.88 *(.001)*	.39 *(.003)*	.91 *(.001)*
Weights	No	No	Value Added	Value Added

All regressions contain 4 digit SIC industry dummies. TFP is measured by log(Value Added) minus log capital and log labor (no of employees) weighted by 4 digit industry level cost shares. The implied one year coefficient is the ten year coefficient to the 1/10 power.

Table 9 contains the estimates.

The capital labor ratio is extremely autocorrelated over time, with a 10 year coefficient of .42 implying a one year auto-correlation of .92. The factor cost ratio is somewhat less autocorrelated, with a 10 year coefficient of .32 implying a one year autocorrelation of .89. Small differences in the one year auto correlation rates can lead to big differences in the 10 year rates. TFP is about as autocorrelated as the factor cost ratio.

I also run weighted regressions with value added weights, which measure the autocorrelation of the biggest plants in the industry. In the weighted regressions, both TFP, the factor cost ratio, and the capital-labor ratio become even more autocorrelated, with ten year rates of .91, .91, and .93.

Another way to look at persistence is to compute transition tables. Within industry year, I assign plants to quartiles based on their factor cost ratio or capital-labor ratio. I then examine how much movement there is between quartiles over ten years. If the variables are not persistent, a plant in 1987 should be equally likely to be in either of the four quartiles

Table 10: Transition Table between Quartiles of Capital-Labor Ratio from CMF87 to CMF97

Quartile, CMF 87	Quartile, CMF 97			
	Q1	Q2	Q3	Q4
Q1	**36.3%**	29.1%	21.2%	13.4%
Q2	21.26%	**29.1%**	29%	20.7%
Q3	14.7%	22.4%	**31.4%**	31.5%
Q4	9.5%	13.7%	24.6%	**52.2%**

Here Quartile 1 have the smallest 25% of plants by within industry capital-labor ratio and Quartile 4 the largest 25% of plants by within industry capital-labor ratio.

Table 11: Transition Table between Quartiles of Factor Cost Ratio from CMF87 to CMF97

Quartile, CMF 87	Quartile, CMF 97			
	Q1	Q2	Q3	Q4
Q1	**39.3%**	27%	19.4%	14.3%
Q2	22.5%	**30.3%**	23.9%	23.4%
Q3	18.9%	26.1%	**26.8%**	28.2%
Q4	12%	19.3%	21.6%	**47.2%**

Here Quartile 1 have the smallest 25% of plants by within 4 digit industry capital share and Quartile 4 the largest 25% of plants by within 4 digit industry capital share.

in 1997, regardless of what its initial quartile was. Tables 10 and 11 contain these transition tables.

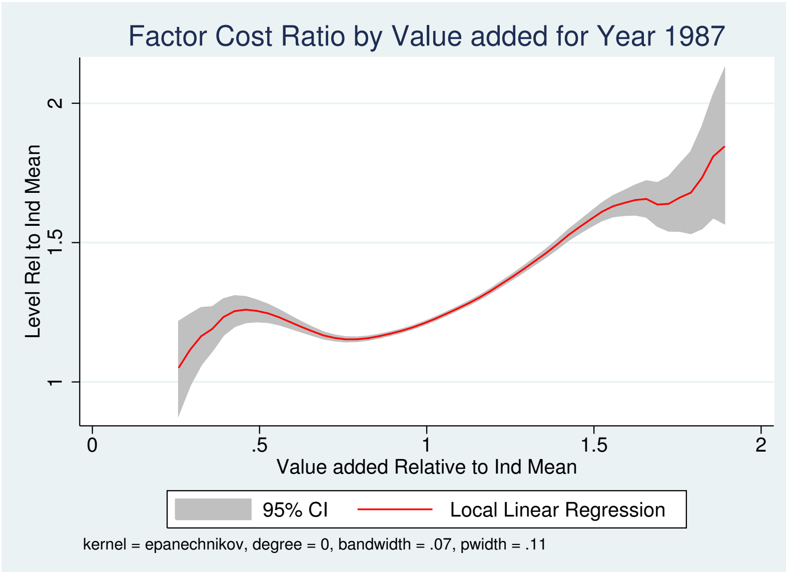
For the capital-labor ratio, more than 50% of the largest quartile plants in 1987 are in the largest quartile of plants in 1997. For the factor cost ratio, 47.2% of the largest quartile plants in 1987 are in the largest quartile of plants in 1997. Results are similar but smaller for the smallest quartile plants, with 37% and 39% for the capital-labor ratio and factor cost ratio. Thus, the capital-labor ratio and factor cost ratio have a high degree of persistence over time.

5.1.2 Correlation with Value added

I next look at how the factor cost ratio moves with output. For output, I use real double-deflated value added. [5]For each plant, I calculate its output percentile relative to the rest of the industry I then calculate each firm's log factor cost ratio taking out SIC 4 digit industry dummies (or 6 digit NAICS dummies, for the 2002 Census). Using local polynomial regression, I regress the demeaned plant log factor cost ratio on log output. I use local polynomial regressions to avoid placing functional form assumptions upon the relationship between the variables. Figure 5.1 and Figure 5.2 show the local polynomial graphs for 1987 and 2002. In both cases, the largest plants of the industry have a much higher factor cost ratio than the smallest plants- for 1987 about 45% higher and for 2002 about 150% higher. This increase is not just the biggest plants having a larger factor cost ratio than the smallest plants, however. Even among the largest 20% of plants in a given industry we see a substantial increase in capital costs relative to labor costs for larger plants.

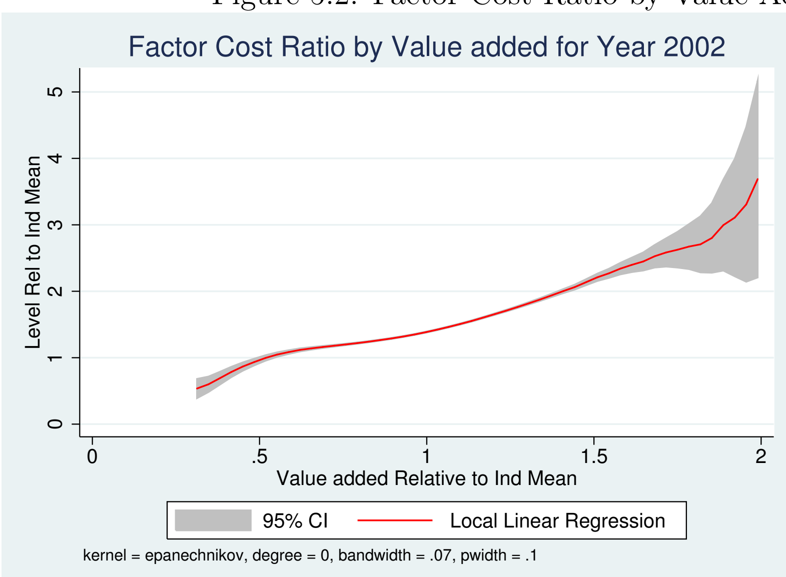
[5] As Basu and Fernald (1997) point out, value added is the correct measure of output if firms are perfectly competitive or materials are Leontief with labor and capital. I am assuming the elasticity of substitution between materials and labor and capital together is zero, so value added works as a measure of output. However, I do find the same patterns using total sales as the size measure instead of value added.

Figure 5.1: Factor Cost Ratio by Value Added for Year 1987



The X axis is a plant's value added relative to its industry mean. The Y axis is the factor cost ratio relative to its industry mean. The graph was generated using local polynomial regression.

Figure 5.2: Factor Cost Ratio by Value Added for Year 2002

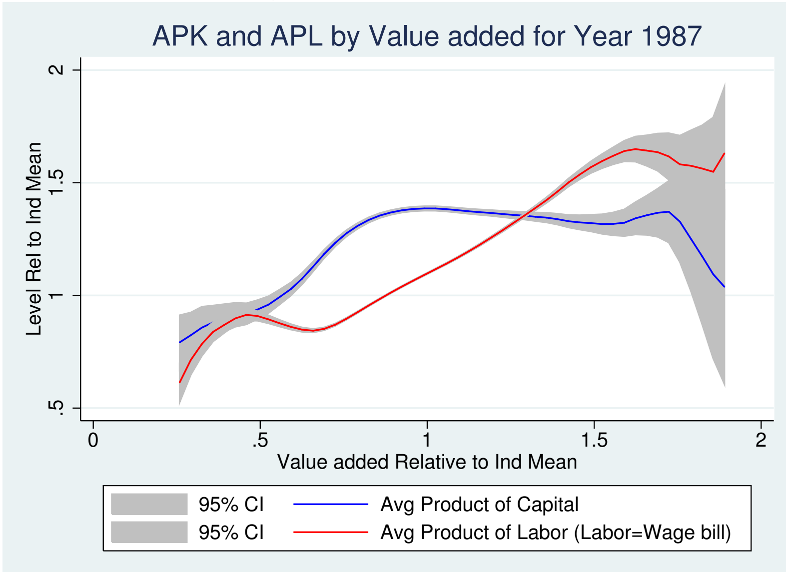


The X axis is a plant's value added relative to its industry mean. The Y axis is the factor cost ratio relative to its industry mean. The graph was generated using local polynomial regression.

For 1987 the factor cost ratio dips slightly for the smallest plants, which is not the case in 2002. In 1997 this dip is substantially bigger. The rise in the factor cost ratio after the smallest 20% of plants is always present, however. I can only measure the amount of capital owned by firms, not the amount of capital used by the firms. If a few firms shut down or produce much less than usual, they are not using much of their capital stock. These firms will have very low output but a high factor cost ratio relative to the industry. This example illustrates a general problem: capital utilization rates are not observed. Differences in utilization lower the true factor cost ratio for low output firms and raise the true factor cost ratio for high output firms, and so should only bolster my findings of rising capital costs relative to labor costs with output.

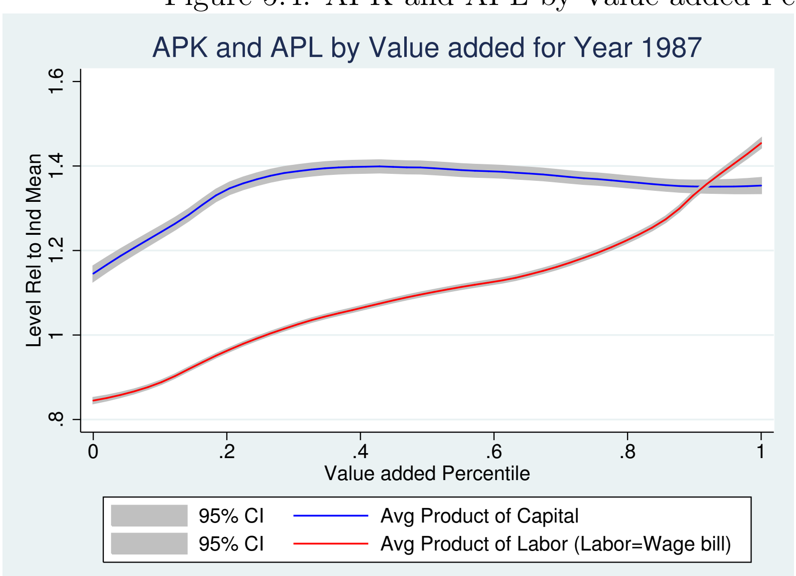
I also look at the correlation of value added with the average revenue product of capital and average revenue product of labor. I run similar local polynomial regressions to those for the capital share using the 1987 Census. I measure labor as the wage bill- labor productivity rises even faster if I measure labor by the number of employees. I look at output as value added relative to the industry mean, in Figure 5.3, and as the value added percentile relative to the industry in Figure 5.4. The average revenue product of capital is increasing, by about 40%, but only for the smallest 20% of plants. For the rest of the plants the average revenue product of capital is constant. The average revenue product of labor increases by about 100% using the wage bill as the measure of labor. Thus the average product of labor is increasing when the average product of capital is constant or slightly declining for the upper 80% of plants. These relationships look similar in 1997. In 2002, the average product of capital is actually falling for the largest plants.

Figure 5.3: APK and APL by Value added for Year 1987



The X axis is a plant's percentile of value added relative to its industry. The Y axis is the log average product of capital or average product of labor after taking out industry averages. Here labor is defined as the plant's wage bill. The graph was generated using local polynomial regression.

Figure 5.4: APK and APL by Value added Percentile for Year 1987



The X axis is a plant's percentile of value added relative to its industry. The Y axis is the log average product of capital or average product of labor after taking out industry averages. Here labor is defined as the plant's wage bill. The graph was generated using local polynomial regression.

Table 12: Correlations with Size for K/L Ratio, Capital Share, APK, APL

	1987		1997	
Log(Capital-Labor Ratio)	.15 *(.002)*	.18 *(.005)*	.12 *(.002)*	.17 *(.015)*
Log(Factor Cost Ratio)	.06 *(.001)*	.09 *(.005)*	.02 *(.001)*	.10 *(.013)*
Log(APK)	.07 *(.002)*	.03 *(.006)*	.10 *(.001)*	.07 *(.02)*
Log(APL)	.13 *(.001)*	.14 *(.001)*	.12 *(.004)*	.17 *(.009)*
Weights	No	Value Added	No	Value Added

All of these coefficients are from regressions with the LHS variable as the dependent variable and log of value added as the independent variable. Controls include dummy variables for age and state, single establishment status and 4 digit SIC industry. I use robust standard errors.

Table 12 contains regressions where each cell represents a different regression with log capital share, the log capital-labor ratio, the log average product of capital and the log average product of labor as the dependent variables and log value added as the independent variable, as well as controls for state, age, industry, and single establishment status, using the 1987 and 1997 Censuses. The basic results of the graphs remain, though a linear relation is not the best functional form for the relationship. A linear regression finds a positive linear relationship between the average product of capital and value added, even though all of the increase was only for the smallest 20% of plants. Consistent with this, the regressions weighted with value added have a lower coefficient on the average product of capital than the unweighted coefficients. Still, the average product of labor is increasing much faster than the average product of capital in these regressions.

I also examine whether the increasing relationship between capital share and value added holds for single establishment plants and multiestablishment plants separately. If multiestablishment plants are larger and have employees that contribute to production at the plant but are in other establishments, such as senior managers, the capital share of big firms could be overstated. To check for this, I regress the log capital share and the log capital labor ratio against log value added and controls, plus interaction terms between single establishment status and log value added. Table 13 contains these results. I find the capital-labor ratio and capital share to be higher for large plants for both single-establishment and multiestablishment plants, with coefficients of similar order of magnitude for both sets of plants. For example, in 1987 the capital share increases by 4% with value added for single establishment plants and 5% for multiestablishment plants in the unweighted regressions, or 6% and 7% in the weighted regressions. The only case where the single establishment and multi establishment plants move differently is for the 1997 unweighted regressions. In the 1997 unweighted regressions, the capital share increases by 5% with value added for multiestablishment plants but falls by 1% for single establishment plants. In the weighted regressions, though, I find that the capital share increases by 8% for single establishment plants and 6.5% for multi-establishment plants. I believe that the differences between the weighted and unweighted regressions are due to low capital utilization for the smallest single establishment plants in 1997.

So far I have used book values of capital for the capital stock. For the ASM plants it is possible to calculate capital using perpetual inventory methods. The advantage of perpetual inventory methods is that the vintage of each part of the capital stock is known, so I can depreciate each vintage by its age and deflate each vintage by its investment year's investment deflator. The disadvantage of the base perpetual inventory methods is that they do not take into account retirements of the capital stock. Plants retire their capital stock at a rate of about 4% a year, which is concentrated in a few plants retiring a lot of capital stock. Since firms retiring capital deduct the retirement values from their book value, the book value measures incorporate the depreciation from retirements. Data on retirements of capital stock are available for all years up to 1987, after which retirements are recorded only in Census years. Table 14 shows the correlations between value added and factor cost

Table 13: Robustness Checks: Single vs. MultiUnit Establishments

	1987		1997	
Single Unit:				
Log(Capital-Labor Ratio)	.16 *(.002)*	.16 *(.02)*	.11 *(.012)*	.19 *(.015)*
Log(Factor Cost Ratio)	.05 *(.002)*	.08 *(.017)*	-.01 *(.001)*	.10 *(.015)*
Multi Unit:				
Log(Capital-Labor Ratio)	.13 *(.002)*	.18 *(.006)*	.15 *(.002)*	.165 *(.016)*
Log(Factor Cost Ratio)	.06 *(.002)*	.10 *(.005)*	.08 *(.002)*	.10 *(.014)*
Weights	No	Value Added	No	Value Added

All of these coefficients are from regressions with the LHS variable as the dependent variable and log of value added as the independent variable. Controls include dummy variables for age and state, single establishment status and 4 digit SIC industry. The Single Unit and Multi Unit coefficients come from the same regression- I put an interaction of log(va) with multiunit status. I use robust standard errors.

Table 14: Capital Robustness Checks: Correlations with Size for K/L Ratio and Factor Cost Ratio, for ASM plants only

	1987	
Book Value Deflated:		
Log(Capital-Labor Ratio)	.17 *(.007)*	.19 *(.006)*
Log(Factor Cost Ratio)	.06 *(.007)*	.10 *(.006)*
Perpetual Inventory with Retirements:		
Log(Capital-Labor Ratio)	.14 *(.008)*	.17 *(.006)*
Log(Factor Cost Ratio)	.03 *(.007)*	.08 *(.006)*
Weights	ASM Weight	Value Added * ASM Weight

All of these coefficients are from regressions with the LHS variable as the dependent variable and log of value added as the independent variable. Controls include dummy variables for age and state, single establishment status and 4 digit SIC industry. I use robust standard errors. I construct the perpetual inventory stock using data on book values of capital, investment, and retirements.

ratio using the perpetual inventory measure of capital. Here I still find the same patterns, though the correlation between the factor cost ratio and value added is slightly lower using perpetual inventory capital in the non value added weighted case.

Another potential reason for my findings is differences in rental rates of capital across plants, as large plants may face low rental rates of capital. I attempt to control for rental rate differences by also controlling for firm size for multiunit plants, as the rental rate should depend upon the firm and not the plant. I control for firm size using total firm employment as a measure of size either by including deciles of the firm size distribution for manufacturing plants, or a quartic in log firm employment. As Table 15 shows, I still find that the factor cost ratio is correlated with plant value added, even after controlling for firm size. I have also varied firm size measures using payroll instead of employment and found similar results.

Table 15: Robustness Checks: Controls for Firm Size

	1987		1997	
Firm Size=Quartic in Log Firm Employment				
Log(Factor Cost Ratio)	.04 *(.002)*	.07 *(.007)*	.06 *(.003)*	.08 *(.014)*
Firm Size= Deciles of Log Firm Employment				
Log(Factor Cost Ratio)	.04 *(.002)*	.07 *(.007)*	.06 *(.003)*	.08 *(.015)*
Weights	No	Value Added	No	Value Added

All of these coefficients are from regressions with the LHS variable as the dependent variable and log of value added as the independent variable. Controls include dummy variables for age, state and 4 digit SIC industry. These regressions only include multiunit firms.

5.2 Structural Estimation of Productivity

The stylized facts in the previous section imply that plants may differ on their levels of labor augmenting productivity. Since I have estimated the elasticity of substitution, I can now examine the productivity of each plant. I do not want to hard wire that productivity is completely Hicks neutral or completely labor augmenting. Thus, I estimate both a Hicks neutral productivity parameter A and a labor augmenting productivity parameter B. Here, I assume that the quantity production function is a CES production function with constant returns to scale. I subsume α into labor augmenting productivity B, which is WLOG since σ is less than one. Manipulating equation 2.4, I have that:

$$\log B = \log(K/L) + \frac{\sigma}{1-\sigma}\log(\frac{rK}{wL}) \quad (5.1)$$

Thus, given the elasticity of substitution I can calculate the labor augmenting productivity of the plant straight from its factor allocations and payments, without using any data on output! I provide two estimates of B, the first where labor is measured as the number of employees and the second where labor is measured as the wage bill. The wage bill can account for higher quality labor but could also be influenced by higher local area wages or other factors. If labor is measured as the wage bill, my measure of labor augmenting productivity is effectively the factor cost ratio multiplied by $\frac{1}{1-\sigma}$, which increases the factor cost ratio since σ is less than one. I set σ to .6, which is the value of the elasticity of substitution I estimate in the county level wage regressions for all of manufacturing.

To estimate the Hicks neutral productivity A, I take the equation for the average product of capital and impose cost minimization conditions on the factors. The Hicks neutral productivity A is then:

$$\log A = \log(Y/K) - \frac{\sigma}{1-\sigma}\log(\frac{rK}{rK+wL}) \tag{5.2}$$

Since I do not have data on quantity, only revenue, one measure of Hicks neutral productivity that I can calculate includes both Hicks neutral productivity from the quantity production function and differences in prices between plants in the industry:

$$\log A + \log P = \log(PY/K) - \frac{\sigma}{1-\sigma}\log(\frac{rK}{rK+wL}) \tag{5.3}$$

My approach to estimation of productivity is similar to the cost share approach for the Cobb-Douglas, but generalized to the CES production function and allowing for both Hicks neutral and labor augmenting productivity. So far, I have not used the information in the level of output to improve the estimates of B, but increases in B should increase output as well as the capital share. Thus, it might be possible to provide better estimates of the labor augmenting productivity using this information. I can also use the isoelastic demand curve to solve for a combination of Hicks neutral productivity and the demand shifter:

$$\log A + \log D = \frac{\epsilon}{\epsilon - 1}\log(PY/K) - \frac{\sigma}{1-\sigma}\log(\frac{rK}{rK + wL}) - \frac{1}{\epsilon - 1}\log K \qquad (5.4)$$

Here, I can also measure Hicks neutral productivity plus demand shocks given assumptions on the elasticity of demand: here I assume that the elastic I find that my estimates of A and B using equations 5.1 and 5.3 are highly negatively correlated. First, errors in capital will tend to move A and B in opposite directions as higher capital stocks will increase B and decrease A mechanically, leading to substantial negative correlation. Second, most models with decreasing demand where firms can decide their price imply that high productivity firms have low prices. Foster et al. (2008) find that physical TFP and the plant price are strongly negatively correlated. Since my measures of A include differences in prices, plants that have low prices due to high B will have a negative correlation between my measured A and measured B.

I then examine some of the standard relationships between TFP and plant level variables found in the literature, checking how A and B vary with these variables. Table 16 examines how A, B, and TFP vary with the size of the plant. I construct TFP as before, as log value added minus an industry cost share weighted amounts of capital and labor. I use two measures of size, employment and value added. In the table, each cell is a separate regression with a log productivity measure as the dependent variable and a log size measure as the independent variable, along with industry dummies as controls. I find that TFP is positively correlated with revenue but not with employment. The labor augmenting productivity B is positively correlated with both employment and value added, while my measure of A is negatively correlated with both. These findings are consistent with simple models with decreasing demand, in which large productive plants have low prices and so low measured A.

I also look at 10 year size growth in Table 17, again measuring size both as employment

Table 16: Correlations between Productivity and Size

	Log(VA)		Log Employment	
Log(A)	-.05 *(.003)*	-.11 *(.002)*	-.08 *(.003)*	-.16 *(.002)*
Log(B) *(Labor=No of employees)*	.28 *(.003)*	.33 *(.003)*	.11 *(.004)*	.22 *(.003)*
Log(B) *(Labor= Wage Bill)*	.19 *(.003)*	.25 *(.003)*	.09 *(.004)*	.16 *(.003)*
TFP	.17 *(.001)*	.14 *(.001)*	.01 *(.001)*	.02 *(.0009)*
Weights	No	Value Added	No	Value Added

All regressions contain 4 digit SIC industry dummies. TFP is measured by log(Value Added) minus log capital and log labor (no of employees) weighted by 4 digit industry level cost shares. Log(VA) or Log(Employment) are the independent variables.

and value added. Conventional TFP is positively correlated with employment growth but negatively correlated with value added growth. The labor augmenting productivity B is positively correlated with both, while my measure of A is negatively correlated with both.

These variables are also related to entry and survival. I define a plant as an entrant if it entered in the previous two years and a plant as surviving if it is still in business after the next two years. TFP is positively correlated with survival and negatively correlated with entry. I find that entering firms and surviving firms both have high A and low B in Table 18. This result would imply that exiting firms have high levels of labor augmenting productivity, which is strange as almost every model and previous data study find that firms that exit are less productive. However, my estimates of A and B rely upon firms cost minimizing. If exiting firms begin to reduce levels of inputs before exit, they would stop investment in capital, fire workers, and lower wages. If they cannot sell their existing capital, however, they may look capital intensive with a higher capital-labor ratio and higher capital share,

Table 17: Correlations between Productivity and Size Growth

	Ten Year VA Growth		Ten Year Employment Growth	
Log(A)	-.16	-.16	-.08	-.12
	(.006)	*(.005)*	*(.008)*	*(.008)*
Log(B) *(Labor=No of employees)*	-.002	.06	.20	.29
	(.007)	*(.006)*	*(.009)*	*(.009)*
Log(B) *(Labor= Wage Bill)*	.02	.08	.16	.30
	(.007)	*(.006)*	*(.009)*	*(.009)*
TFP	-.16	-.13	.08	.06
	(.002)	*(.002)*	*(.002)*	*(.003)*
Weights	No	Value Added	No	Value Added

All regressions contain 4 digit SIC industry dummies. TFP is measured by log(Value Added) minus log capital and log labor (no of employees) weighted by 4 digit industry level cost shares.

and so have a high B for spurious reasons. In a sense, utilized capital is likely to be low but measured capital is high and so measured labor augmenting productivity B is high.

I also examine the autocorrelation of productivity, as TFP is known to be highly correlated. Table 19 contains estimates of the 10 year autocorrelation of A and B, as well as TFP, between the 1997 and 1987 Manufacturing Censuses. All three measures are fairly highly autocorrelated over time.

6 Application to Misallocation

A proposed explanation for the vast differences in TFP between rich and poor countries is that resources are not allocated well in poor countries. In this view, some highly productive firms in a poor country do not have enough capital, while other less productive firms have too much capital. This low allocative efficiency can then cause countries to have low aggregate

Table 18: Correlations between Productivity and Entry and Survival

	Entrant		Survival	
Log(A)	.03 *(.012)*	.06 *(.02)*	.16 *(.013)*	.09 *(.02)*
Log(B) *(Labor=No of employees)*	-.08 *(.01)*	-.20 *(.02)*	-.06 *(.015)*	.06 *(.02)*
Log(B) *(Labor=Wage Bill)*	.001 *(.01)*	-.12 *(.02)*	-.17 *(.015)*	-.03 *(.02)*
TFP	-.03 *(.004)*	-.11 *(.006)*	.10 *(.004)*	.12 *(.006)*
Weights	No	Value Added	No	Value Added

All regressions contain 4 digit SIC industry dummies. TFP is measured by log(Value Added) minus log capital and log labor (no of employees) weighted by 4 digit industry level cost shares.

Table 19: Autocorrelation of Productivity

	Ten Year	Implied One Year	Ten Year	Implied One Year
Log(A)	.30 *(.004)*	.89	.33 *(.004)*	.90
Log(B) *(Labor=No of employees)*	.37 *(.004)*	.91	.47 *(.004)*	.93
Log(B) *(Labor= Wage Bill)*	.35 *(.004)*	.90	.43 *(.004)*	.92
TFP	.29 *(.004)*	.88	.43 *(.004)*	.92
Weights	No	No	Value Added	Value Added

All regressions contain 4 digit SIC industry dummies. TFP is measured by log(Value Added) minus log capital and log labor (no of employees) weighted by 4 digit industry level cost shares. The implied one year coefficient is the ten year coefficient to the 1/10 power.

TFP. For the misallocation channel to be important, misallocation must generate large losses in aggregate TFP. Hsieh and Klenow (2009) solve for aggregate TFP in a setup where profit maximizing plants with Cobb-Douglas production functions face output and capital wedges. These wedges are meant to generalize many different reasons for misallocation. The dispersion in output and capital wedges lowers aggregate TFP, which in a frictionless world would only depend on the productivity of all of the plants. Applying their theory to plant level manufacturing data, they find that eliminating misallocation frictions can increase aggregate TFP by 40% in the US and more than 100% in China and India.

Midrigan and Xu (2009) explore a dynamic model with adjustment costs of capital and financial frictions and try to match the model to Korean plant data. They find that adjustment costs of capital and financial frictions lead to observed variation in the time series marginal product of capital but not the large cross-section differences in the marginal product of capital. Moll (2010) constructs a highly tractable dynamic model with financial frictions in which the autocorrelation of productivity determines whether there is misallocation. If firms are always highly productive they can easily self finance to obtain capital.

All of these models assume that the production function is Cobb-Douglas and so all differences in productivity are Hicks neutral. However, differences in labor augmenting productivity can cause dispersion in the capital cost share and so look like misallocation. Take the Hsieh and Klenow model. In their model, each plant faces downward sloping demand and has a constant returns to scale Cobb-Douglas quantity production function. All plants also face exogenous capital taxes, τ_k, and output taxes, τ_y. Thus each plant faces the following maximization problem:

$$\pi = (1 - \tau_y)PY - wL - (1 + \tau_k)rK \tag{6.1}$$

$$Y = AK^{\alpha}L^{1-\alpha} \tag{6.2}$$

Firms that face high capital taxes optimally choose low capital shares of cost, as the capital tax discourages them from purchasing capital. Firms with higher capital taxes also have lower revenue, as the capital tax implies a higher marginal cost and so higher prices.

Firms that face high output taxes also have less revenue as the output tax discourages them from producing more. These firms will also have a lower labor share of revenue, as their output restrictions mean higher prices and so higher revenue per unit produced. Hsieh and Klenow identify the frictions in the micro data as follows:

$$1 + \tau_k = \frac{wL}{rK}\frac{\alpha}{1-\alpha} \tag{6.3}$$

$$1 - \tau_y = \frac{wL}{PY}\frac{\sigma}{\sigma - 1}\frac{1}{1-\alpha} \tag{6.4}$$

Capital taxes are proportional to the labor cost to capital cost ratio, while output taxes are proportional to the inverse of the labor share of revenue.

They then solve for aggregate TFP and examine how aggregate TFP changes as the level of misallocation frictions change.

In a setup with labor augmenting productivity where capital and labor are complements, increases in labor augmenting productivity will increase the capital share of cost. If firms face a similar demand function as in Hsieh and Klenow, high labor augmenting productivity firms B will also have high revenue and a low labor share of revenue, as I derived earlier. Thus, labor augmenting productivity would imply the following correlations for the measured misallocation taxes and revenue:

$$Corr(1 + \tau_k, 1 - \tau_y) > 0 \tag{6.5}$$

$$Corr(1 + \tau_k, PY) < 0 \tag{6.6}$$

$$Corr(1 - \tau_y, PY) < 0 \tag{6.7}$$

Under the misallocation setup, there is no reason for output taxes and capital taxes to

Table 20: Correlations of Misallocation Frictions and Revenue for US Plant Level Data

	1987	1987	1997	1997
$Corr(1+\tau_k, 1-\tau_y)$	.29	.33	.20	.23
$Corr(1+\tau_k, VA)$	-.13	-.23	-.10	-.27
$Corr(1-\tau_y, VA)$	-.33	-.26	-.33	-.49
Weights	No	Value Added	No	Value Added

All of these correlations are within 4 digit SIC Industry.

have any correlation, as they are just exogenous frictions hitting firms. Firms facing capital taxes would have low revenue, as labor augmenting productivity would predict, but firms facing high output taxes would have lower revenue. I can then test in the micro data whether these restrictions hold. Table 20 displays the estimates, where all correlations are within 4 digit SIC industry. I do indeed find that firms with low capital taxes also have high output taxes, firms with low capital taxes have high output, and firms with high output taxes have high revenue. Thus, labor augmenting productivity B may be able to explain patterns in the data for which misallocation theories would require two frictions. I am currently working on a more quantitative assessment on what kinds of apparent misallocation TFP losses labor augmenting productivity can generate.

So far, I have shown that labor augmenting productivity can generate micro data patterns that appear to be misallocation. Thus, labor augmenting productivity could explain why Hsieh and Klenow find increases in aggregate TFP of around 40% for the US just from removing allocation frictions. But a key result in Hsieh and Klenow is that China and India have much higher TFP gains from reallocating efficiently. Misallocation is certainly a reasonable explanation for the cross country differences that Hsieh and Klenow find. But another explanation is that labor augmenting productivity is much more disperse in China and India than in the US.

A simple model based on Acemoglu and Zilibotti (2001) can perhaps explain why India and China would have higher dispersion in labor augmenting productivity than the US. Acemoglu and Zilibotti develop a model where productivity innovations are generated in the North and respond to Northern factor prices. In the North, wages are high while in the South wages are low, so Northern innovation is primarily labor saving. High wages in the North force all firms in the North to adopt the new labor saving technologies. Since wages in the South are low, firms in the South do not always adopt the latest labor saving technologies, leading to big gaps in TFP between the North and the South. If the best practice firms in a country like India adopt the newest labor saving technologies, but most firms do not, labor augmenting productivity will be more disperse in the South than in the North. This model could thus explain why India and China would have a higher dispersion of labor augmenting productivity than the US.

7 Conclusion

If plants in an industry have a Cobb Douglas production function, the capital share of cost should be constant across plants. I find that neither of these implications hold in micro data from plants in the US Manufacturing Censuses. The capital share of cost varies considerably within four digit industries. This dispersion is not just measurement error or temporary deviations from the optimal capital share. Capital shares are highly correlated across time within the same plant, and are increase with the plant's revenue. The Cobb Douglas specification also implies that the average revenue product of capital and average revenue product of labor should move together. Instead, I show that the average revenue product of labor increases much more with revenue than the average revenue product of capital.

A CES production function with labor augmenting productivity can better explain these data facts. If the elasticity of substitution between labor and capital is less than one, labor augmenting productivity is labor saving as plants with higher labor augmenting productivity increase their capital labor ratios. Firms with higher labor augmenting productivity will also have higher capital shares. Given downward sloping demand, labor augmenting productivity improvements will increase a firm's revenue and average revenue product of labor, but not its average product of capital. This process induces a positive correlation between revenue and both the capital share and the average revenue product of labor, which I find in the data.

I then identify the labor capital elasticity of substitution using local labor market wage variation. Areas with higher wages have lower factor cost ratios, just as an elasticity less than one would predict. For manufacturing as a whole, I estimate the elasticity of substitution to be between .45 to .65, depending on the level of wage variation and year. Estimating SIC 2 industries separately, I can reject the Cobb-Douglas specification for 17 out of 19 industries with state level wages and 15 out 19 industries with within state county level wages. I also examine a set of 4 digit industries with wide geographic variation. I can reject Cobb Douglas for 8 out of 10 industries using state level wages and 10 out of 10 industries using county level wages. Separating workers into unskilled workers and skilled workers, I find workers for skill-capital complementarity as well, as the elasticity between capital and skilled labor is lower than that between capital and unskilled labor.

Using cost minimization conditions and my estimates of the elasticity of substitution, I can identify measures of both Hicks neutral productivity and labor augmenting productivity, though the Hicks neutral measure includes differences in prices among firms. My measure of labor augmenting productivity is positively correlated with both size and size growth measures. I also apply my methodology to questions of misallocation to show why measuring the type of production technology is important. The misallocation setup of Hsieh and Klenow (2009) assumes that plants face a set of capital and output wedges that lower aggregate TFP. Labor augmenting productivity can cause dispersion in the micro data that looks like misallocation of factors. A simple theory with labor augmenting productivity implies that firms with measured output constraints should also have low capital wedges, and that firms with high revenue have high output wedges and low capital wedges. I find that measured misallocation frictions do have the predicted correlations using Chilean plant level data.

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