Lifetime Labor Supply and Human Capital Investment

Rodolfo E. Manuelli* Ananth Seshadri† Yongseok Shin‡

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Abstract

We develop a model of retirement and human capital investment to study the effects of tax and retirement policies on the supply of effective labor over an individual's lifetime. Workers choose the supply of raw labor (career length) and also the human capital embodied in their labor. Our model explains a significant fraction of the schooling/retirement differences between Europe and the US. The model predicts that reforms of the European retirement benefit regimes modeled after the US system can deliver 15–35 percent gains in output per worker in the long run. Increased human capital investment in and out of school accounts for most of such output gains, with relatively small changes in career length. This result reconciles the view that taxes can have a large economic impact (Prescott, 2006) with the view that labor supply at the extensive margin may be inelastic owing to institutional rigidities (Ljungqvist and Sargent, 2010). We conclude that models that ignore human capital investment decisions will significantly underestimate the economic impact of tax and retirement policies.

*Washington University in St. Louis and Federal Reserve Bank of St. Louis
†University of Wisconsin–Madison
‡Washington University in St. Louis and Federal Reserve Bank of St. Louis

1 Introduction

Micro and macro estimates of how aggregate labor input responds to shocks and policies display significant differences. Micro estimates of this elasticity are close to zero, while macro studies obtain a much larger number. Recently, researchers have developed models that are consistent with both micro and macro estimates; for example, Imai and Keane (2004).

In this paper we take a different tack. We view the economically meaningful notion of labor input as having two components: a quantity component associated with career length, and a quality component that depends on education and other forms of investment in human capital. From this perspective, to assess the economic effects of policies and shocks, it is necessary to understand how they affect human capital accumulation decisions as well as participation and retirement choices. We discover that even the macro elasticity substantially underestimates the economic impact of tax and retirement policies.

We develop a hybrid life-cycle model that complements the standard continuous-time life-cycle model—which is essential to understand retirement decisions—with a dynastic preference structure that guarantees a tractable characterization of the long-run equilibrium. We assume that individuals choose years of schooling, human capital investment in school, and on-the-job training, as well as the supply of raw labor. In our model, leisure, which gives extra utility, is indivisible (i.e., labor supply decisions are at the extensive margin only). Retirement is defined as the endogenously-chosen back-loaded leisure consumption. Finally, we model retirement benefits as a non-linear function of lifetime income and retirement age that encapsulates the most salient features of real-world policies.

The key elements of our model are labor-leisure choices at the extensive margin, human capital investment in and out of school, and retirement policies that introduce rigidities in workers' labor supply decisions. The first two are what Keane and Rogerson (2010) consider to be important for useful models of labor supply, and the third is emphasized by Ljungqvist and Sargent (2011) in their review article. One important distinction between our model and previous work lies in the specification of human capital accumulation. We use the Ben-Porath (1967) technology, while the literature almost exclusively uses learning-by-doing (i.e., costless) human capital accumulation. This distinction allows us to complement the literature along two dimensions. First, unlike in learning-by-doing models, human capital investment is not hard-wired into decisions on raw labor supply in our model. Consequently, we can analyze the interaction between human capital investment and the supply of raw labor. Second, our model allows us to consider schooling decisions as well as on-the-job human capital investment within a single framework, while most existing work in the literature on lifetime labor supply abstracts from schooling decisions.

We use the model to explore, both theoretically and quantitatively, the impact of changes in taxes, retirement policies, and other shocks. A given change affects several margins simultaneously. For example, higher taxes discourage individuals from acquiring human capital in and out of school, and, at the same time, induce workers to retire earlier.

Theoretically, we first show that, in the long run, reductions in the degree of redistribution in taxes and transfers increase workers' retirement age. We then explore the effects of unanticipated shocks. First, we show that negative wealth shocks increase the retirement age. Second, we find that a permanent decrease in the wage rate induces older workers to retire earlier, but has an ambiguous impact on younger workers' retirement since it depends on the relative strength of income and substitution effects. In addition, for young workers, such a shock flattens their age-earnings profile: upon impact, their effective labor supply increases as they allocate more time to market work and less to human capital investment, but over time their human capital and hence effective labor supply is lower than what they would have been in the absence of the shock. Finally, we study the effect of an unanticipated drop in the stock of human capital (e.g., a loss associated with reallocation in the presence of firm- or sector-specific human capital). We show that older workers supply less effective labor and retire earlier. Again, this shock has an ambiguous impact on the retirement of younger workers.

We calibrate the model to data on output per worker, demographic variables, and tax and retirement policies for the US and several European countries. We obtain two main results. First, the impact of taxes and retirement policies on effective labor supply is substantial. For example, these policies explain a large fraction of the gap in retirement ages between European countries and the US, and almost all of their differences in schooling. Second, a policy reform that institutes the US-style retirement regime has large effects on output per worker in the European countries that we study, long-run increases of 15 to 35 percent. In all cases we find that the response of career length—the extensive margin emphasized by the more recent views—is relatively small: One reason is that changes in retirement age are often accompanied by changes in schooling in the same direction, thereby muting the response of career length. In our model, the large long-run economic impact materializes primarily through human capital investment decisions. In this sense, we reconcile the views of Prescott (2006), Ljungqvist and Sargent (2007), and Prescott, Rogerson, and Wallenius (2009), who emphasize the large impact of tax policies on aggregate output, with the seemingly opposing views of Ljungqvist and Sargent (2010), who emphasize the rigidities built into nonemployment benefits that result in the typical worker being at a corner and, hence, his choice of career length not responding significantly to policy changes.[1]

In another set of exercises, we use the model to evaluate the economic impact of certain policy changes in the US. We find that raising the normal retirement age from 65 to 67 increases output per worker by only three percent in the long run. On the other hand, we find that eliminating the social security benefits and taxes altogether has a sizeable impact: Output per worker increases by 23 percent in the long run, while career length increases by six percent.

Through our analysis, we find that the retirement and human capital investment decisions reinforce each other, and that the full impact of tax and retirement policies can be captured only in a model that allows individuals to adjust both the quantity and the quality of their labor. For example, most studies on the fiscal sustainability of retirement policies abstract from the

[1]Note that in a learning-by-doing model of human capital, human capital investment is hard-wired into labor supply decisions. If there is no change in labor supply, there cannot be any change in human capital either.

endogenous response of workers' human capital to policy changes (Gruber and Wise, 2007). In light of our results, we conjecture that these studies may grossly underestimate the full economic impact of the proposed reform of tax and retirement policies.

2 Model

We adapt the model of human capital accumulation laid out in Ben-Porath (1967) to allow for endogenous labor supply at the extensive margin. Our model incorporates a dynastic preference structure into the standard continuous-time life-cycle framework. Individuals choose years of schooling and human capital investment while in school. When in the labor force, they also decide how to allocate their time between market work and investment in their human capital. Leisure, which gives extra utility out of goods consumption, is indivisible, and retirement in the model is endogenous back-loading of leisure. Finally, we model retirement benefits as a non-linear function of lifetime income and retirement age.

Life Cycle The individual life span is deterministic and runs from age 0 to T. From age 0 to I, an individual is attached to his parent, who makes decisions for him. He then becomes independent at age I. At age B, he gives birth to e^f children, who remain attached to him until they turn age I. We assume $I \leq B$.

Choices From age $a = 6$ through T, an individual is endowed with one unit of time at any given instant. Time can be spent on leisure, $\ell(a)$, investment in human capital, $n(a)$, and market work, $1 - n(a) - \ell(a)$. Leisure is a binary variable: $\ell(a) \in \{0, 1\}$.[2] Both $n(a)$ and $1 - n(a) - \ell(a)$ must belong to the closed interval $[0, 1]$. The individual also chooses consumption, $c(a)$, and goods input to human capital production, $x(a)$.

In addition, he makes decisions for his children while they are attached to him. He chooses goods inputs for early childhood human capital production, x_E, when they turn 6. Between their age 6 and I, with subscript k denoting child-specific variables, he chooses their time allocation and goods inputs in human capital accumulation: $\{n_k(a), \ell_k(a), x_k(a)\}$. When children are between ages 0 and I, he chooses their consumption, $c_k(a)$. Finally, when they reach age I and become independent, he gives each child a bequest of q_k.

Preferences An independent adult orders allocations in terms of the utility derived from his own consumption and leisure, and also all his descendants' utility. The parent's intergenerational

[2] Since leisure is binary, measured hours of work, which equals one minus leisure in the post-schooling period, is also binary. The assumption of indivisible labor supply has a long tradition in both macro and microeconomics. Two examples are the lotteries of Rogerson (1988) and the retirement model of Rust and Phelan (1997). Fixed costs associated with working can justify this indivisibility, especially at retirement. French and Jones (2011) estimate the daily fixed time cost to be about 3.26 hours at age 60. Rogerson and Wallenius (2010) show that the fixed time cost needs to be more than the equivalent of a half-time job in order to create the downward discontinuity in hours at retirement, with an intertemporal elasticity of substitution of labor supply lower than or equal to 0.5.

altruism may be imperfect as in Barro and Becker (1989). His preferences are represented by

$$\int_I^T e^{-\rho(a-I)} U(c(a), \ell(a)) da + \sum_{t=1}^{\infty} e^{-(\alpha_0 - \alpha_1 f)t} \int_0^T e^{-\rho(a+Bt-I)} U(c_t(a), \ell_t(a)) da.$$

The first term is the utility from his own consumption and leisure, where ρ is the subjective discount rate. The second term is the utility he derives from his descendants, with t indexing generations. Note that α_0 and α_1 control the degree of intergenerational altruism. The case with $\alpha_0 = 0$ and $\alpha_1 = 1$ corresponds to the standard dynastic preferences, while positive α_0 and α_1 less than 1 imply imperfect altruism.

We use the following specification for the flow utility:

$$U(c, \ell) = \frac{(c(1+\zeta\ell))^{1-\theta}}{1-\theta},$$

with parameters ζ and θ satisfying:

$$\frac{(c(1+\zeta))^{1-\theta}}{1-\theta} > \frac{c^{1-\theta}}{1-\theta}.$$

This specification can accommodate both the view that consumption drops at retirement because of the substitutability between home and market goods (French, 2005; Laitner and Silverman, 2008) and the view that, in order to supply labor to the market, it is necessary to purchase some goods such as transportation and meals outside home (Aguiar and Hurst, 2009).[3]

Human Capital Production We adopt the Ben-Porath (1967) formulation of the human capital production technology, augmenting it with an early childhood period.

$$\dot{h}(a) = z_h (n(a)h(a))^{\gamma_1} x(a)^{\gamma_2} - \delta_h h(a), \qquad a \in [6, T] \tag{1}$$

$$h(6) = h_B x_E^{\nu} \tag{2}$$

Equation (1) is the standard human capital production developed by Ben-Porath, with $h(a)$ denoting the human capital stock at age a and δ_h the depreciation rate. The technology to produce a child's human capital at age 6, $h(6)$, is given by (2), where h_B is the stock of human capital at birth and x_E is goods input. This specification captures the idea that nutrition and health care are important determinants of early levels of human capital, and that these inputs are essentially market goods. We assume that γ_1, γ_2, and ν are all between 0 and 1.

[3]Consider a utility function $\hat{u}(c_l, c_w, \ell)$. In this formulation c_l denotes consumption not related to supplying labor, and c_w denotes consumption necessary to supply labor. Given that total consumption expenditures are given by $c_l + c_w = c$, the indirect utility function is $u(c, \ell) = \max_{c_l} \ \hat{u}(c_l, c - c_l, \ell)$. In particular, let

$$\hat{u}(c_l, c_w, \ell) = \left(\frac{c_l^{\phi(1-\ell)+\ell} c_w^{(1-\phi)(1-\ell)}}{\phi^{\phi}(1-\phi)^{1-\phi}} \right)^{1-\theta} \frac{1}{1-\theta}.$$

The indirect utility function $u(c, \ell)$ coincides with our specification with $1 + \zeta = (\phi^{\phi}(1-\phi)^{1-\phi})^{-1}$.

Schooling and Retirement Human capital produced at age a can be used until age T, subject to depreciation, and human capital has no value once the individual reaches age T. He will choose to front-load his human capital production, and $n(a)$ weakly decreases with age. In fact, there can be an age interval $[6, 6+s]$ in which individuals are at a corner, using all his time allotment on human capital production: i.e., $n(a) = 1$. We interpret this interval as the schooling period, with s standing for years of schooling. Note that we use the same human capital production function (1) both in and out of school.

At the same time, individuals in our model will choose to back-load their leisure, with $\ell(a) = 1$ for $a \in (R, T]$ and $\ell(a) = 0$ for $a \in [6, R]$, with $6 < R \leq T$. We interpret R as the retirement age. As we show in the appendix, back-loaded leisure is the consequence of human capital depreciation and the equilibrium interest rate being greater than or equal to subjective discount rate.[4]

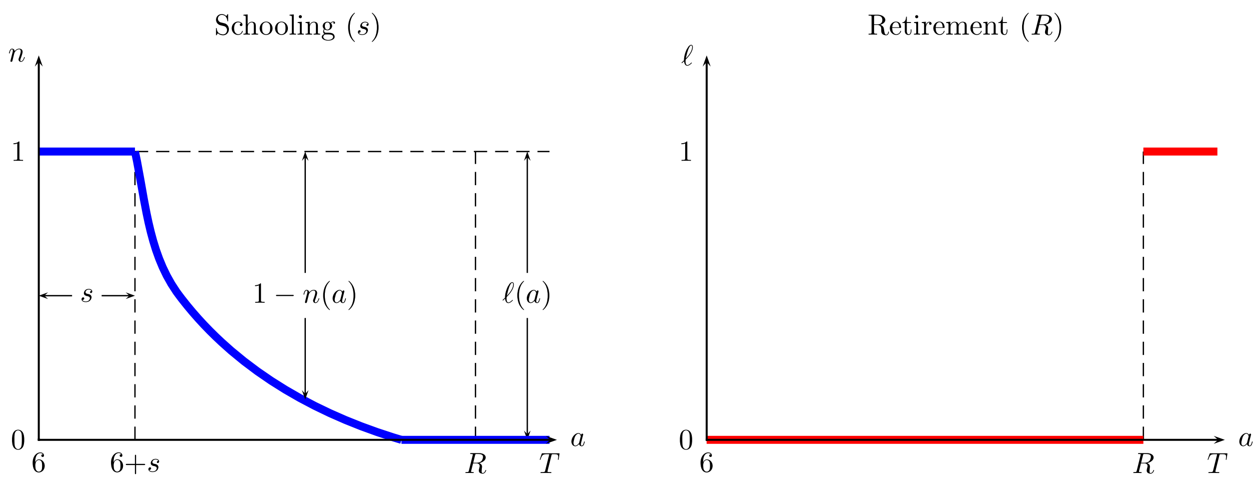

Figure 1: Schooling and Retirement Decisions

Figure 1 shows a typical lifecycle choices of $n(a)$ and $\ell(a)$. On the left panel, an individual uses all his time endowment for human capital production, i.e., $n(a) = 1$, from age 6 to $6 + s$. This is the schooling period, which lasts s years. While in the labor force, i.e., between age $6 + s$ and R, he supplies $1 - n(a)$ of his time to the labor market, and uses $n(a)$ of his time producing human capital. The latter can be labeled as on-the-job training. Whereas in our model $n(a)$ is the time spent on human capital production during the working period and not on market work, one can think of an alternative interpretation. Imagine an alternative environment where jobs are different in terms of learning contents/opportunities. In the labor market equilibrium of this alternative economy, jobs with more learning possibilities will pay lower rental rate of human capital, and a worker's choice of $n(a)$ is equivalent to a choice over different jobs. This is essentially the theory of occupation mobility in Rosen (1972).

The right panel shows the leisure choice through the lifecycle. Only after he reaches age R, which will be chosen endogenously, he will spend all his time enjoying leisure, i.e., $\ell(a) = 1$, until

[4] Age-dependent retirement benefits can also induce the back-loading of leisure, but they are not necessary.

the terminal period T. This is interpreted as retirement.

Effective Labor Supply We are primarily interested in the economic impact of taxes and retirement policies. As such, it is important that we make a distinction between pure quantity measures of raw labor supply (e.g., hours and years worked) and labor service measured in efficiency units. In our framework, the conventional measures of raw labor supply is career length or the number of years worked, since our leisure vs. labor supply decision is only at the extensive margin. The career length is $R - (6 + s)$, with the worker entering the labor market at the age of $6 + s$ and exiting to retire at the age of R.

In the model, labor service in terms of efficiency units is measured by the amount of human capital supplied for market work. To be more specific, we define $h^e(a) \equiv (1 - n(a) - \ell(a))h(a)$ as the effective labor supply. The amount of effective labor an individual supplies to the market through his lifetime depends on two things: his raw labor input, $1 - n(a) - \ell(a)$, which is 0 when he is school ($n(a) = 1$) and when he is in retirement ($\ell(a) = 1$); and also the amount of human capital, $h(a)$, he brings to the market per his raw labor input.

Taxes and Transfers The government taxes labor income at rate τ. In our formulation, $\tau \equiv \tau_I + \tau_S$, where τ_S is the social security tax rate. Capital income is taxed at rate τ_K.

With the income tax revenue, the government pays for retirement benefits, other transfers not tied to retirement (denoted with u), and government consumption. We denote with $b(a, R)$ the retirement benefit at age a for an individual whose chosen retirement age is R. Retirement benefits will depend on a retiree's past earnings, and we will fully specify them in Section 3.1.

Net Labor Income We describe the lifecycle profile of an individual's labor income net of taxes and goods input to human capital production. In particular, take an individual with $s \geq 0$ years of schooling and retirement age R. Let w be the rental rate of human capital and p be the tax-adjusted price of goods input to human capital production. The unit price of early-childhood goods input is p_E. His net (of taxes and goods input) income at age a is as follows.

$$y(a, R) = \begin{cases} 0 & 0 \leq a < 6 \\ -p_E x_E & a = 6 \\ -(1-\tau)px(a) & 6 \leq a \leq 6+s \\ (1-\tau)[wh(a)(1-n(a)) - px(a)] & 6+s < a \leq R \\ b(a, R) & R < a \leq T \end{cases}$$

The interpretation is straightforward. Until age 6, individuals are not endowed with time and there is no human capital production or market work. We summarize the expenditures on early childhood human capital investment as a lump-sum payment at age 6, $p_E x_E$. From age 6 to $6 + s$, net income is the negative of the market goods expenses for schooling, $-(1 - \tau)px(a)$, as the individual supplies no market labor. The active labor market period runs from age $6 + s$ to R. During this period, the net income is $(1 - \tau)[wh(a)(1 - n(a)) - px(a)]$. We allow for the possibility that a fraction ξ of goods input to human capital is tax-deductible. Thus, given the income tax rate τ, p is given by $p = (1-\tau\xi)/(1-\tau)$. Note that the tax treatment of goods input is assumed to be the same whether the individual is in school or working. Finally, $b(a,R)$ is the retirement benefit, which we assume is not taxed.

Individual's Problem We assume that an individual can freely borrow and lend at the after-tax real interest rate $r \equiv (1-\tau_K)\hat{r}$, although he is not allowed to die in debt. The budget constraint is given by

$$\int_I^T e^{-r(a-I)}c(a)da + e^f \int_0^I e^{-r(a+B-I)}c_k(a)da + e^f e^{-rB}q_k \\ \leq \int_I^T e^{-r(a-I)}y(a,R)da + e^f \int_6^I e^{-r(a+B-I)}y_k(a,R_k)da + q + u, \quad (3)$$

where q is the bequest that the individual starts his independent life with, and u is the non-retirement transfer from the government. The variables with subscript k are his children's. Note in particular that the parent not only pays for his children's consumption and goods input but also receives their labor income—if any—until they become independent.

Now, the maximized value of an independent individual at age I who starts with h units of human capital, bequest q, and non-retirement transfer from the government u is:

$$V(h,q,u) = \max \int_I^T e^{-\rho(a-I)}U(c(a),\ell(a))da + \\ e^{-\alpha_0+\alpha_1 f}\int_0^I e^{-\rho(a+B-I)}U(c_k(a),\ell_k(a))da + e^{-\alpha_0+\alpha_1 f-\rho B}V_k(h_k(I),q_k,u_k).$$

The maximization is over $\{c(a),x(a),\ell(a),n(a)\}_{a=I}^T$, $\{c_k(a)\}_{a=0}^I$, $\{x_k(a),\ell_k(a),n_k(a)\}_{a=6}^I$, x_E, and q_k, and is subject to the budget constraint (3) and the human capital production technology in (1) and (2).

3 Theoretical Analysis

We first show that the individual's problem can be transformed into an income maximization problem. We then analyze how retirement decisions are made. Finally, we characterize how an individual's human capital investment and retirement decisions respond to unexpected shocks and policy changes.

3.1 Solution of the Individual Problem

Given our preference specification and the assumption of perfect financial markets in particular, we can transform the individual's problem into a version of income maximization problems.

For a child, his parent makes decisions for him until he turns I. However, there is no discrepancy between the choices his parent makes and what he would have chosen himself, even though the intergenerational altruism is not perfect. To obtain this result with our dynastic preferences, bequests must not be restricted. A heuristic argument establishing this equivalence is as follows. Suppose that the parent does not choose investment in human capital to maximize the present value of his children's lifetime income. In this case, the parent could increase the utility of each child by adopting the income-maximizing human capital investment and adjusting the bequest to exactly offset the cost differential. The cost to the parent is the same, but his children are—and hence he is given his (imperfect) altruism—now made better off.

To solve the individual problem, one also needs to take into account the retirement benefits, which typically depend on the labor income history. Let $W(R)$ be the maximized present value of net income between age 0 and R, for a worker who retires at the age of R. That is,

$$W(R) = \max \int_0^R e^{-r(a-I)} y(a,R) da, \tag{4}$$

where the maximization is over $\{x(a), n(a)\}_{a=6}^{R}$ and x_E, subject to the human capital production in (1) and (2). This problem, for a given R, is solved in the appendix. We assume that the benefits are affine in $W(R)$, during the eligible retirement period, so that the individual's optimal time and goods allocation is the same whether he maximizes the present value of his net labor income between 0 and R or between 0 and T. The retirement benefits at age a are given by

$$b(a, W(R); R_n) = \begin{cases} 0 & a \le \max\{R, R_n\} \\ b_m + b_y W(R) & a > \max\{R, R_n\}. \end{cases}$$

Here, b_m is a constant and $b_y W(R)$ is the variable component of retirement benefits. In addition, R_n is the normal retirement age set by law. Those who retire before R_n will have to wait until they turn R_n to collect benefits. On the other hand, those who retire past R_n forfeit the benefits between R_n and their retirement without actuarial adjustment in future benefits.

Admittedly, our specification of retirement benefits is overly simplistic. However, it still captures the main features of actual programs. In particular, the assumption of the completely forgone benefits while working for those who work past the normal retirement age is an accurate description of the US social security system between 1950 and 1972. After 1972, a delayed retirement credit was reintroduced, but it is only with rules that recently came into effect that the compensation became close to actuarially fair (Schulz, 2001). This assumption is also consistent with the retirement benefit systems in many European countries (Gruber and Wise, 1999). In addition, the assumption that the benefits are a function of the present value of net earnings but not the entire earnings history is a reasonable approximation to real-world programs. For example, the US social security benefits are determined by the average of a worker's highest 35 years of earnings.

Let $y^*(a, R)$ be the net labor income at age a that constitutes the maximized present value of lifetime net labor income in (4). For later use, it is convenient to develop a notation for partial sums of net labor income. Let

$$W(a_1, a_2, R) = \int_{a_1}^{a_2} e^{-r(a-I)} y^*(a, R) da$$

be the properly discounted value of the net labor income between age a_1 and a_2. Obviously, $W(0, R, R) = W(R)$.

We proceed in two steps. The first is to simplify and solve the individual's problem for a given retirement age R. This way, we obtain an indirect value function in R. In the second step, we solve for the retirement age that maximizes the indirect value function.

Obviously, the retirement decision affects the human capital investment decision. For example, it can be easily shown that an increase in R—holding other factors constant—results in higher levels of schooling and human capital. The intuition is simple: As the time horizon over which human capital can be utilized lengthens, the returns to human capital investment become higher and individuals respond by investing more in human capital.

For a given retirement age R, which we will solve for in the second step, we proceed to study optimal consumption decisions. Optimal consumption must satisfy:

$$c(a) = c(I)e^{(r-\rho)(a-I)/\theta}, \ a \in [I, R], \tag{5}$$

$$c(a) = c(I)e^{(r-\rho)(a-I)/\theta}(1+\zeta)^{1/\theta-1}, \ a \in (R, T], \tag{6}$$

where $c(I)$ is the consumption at age I. To make consumption drop at retirement, $\lim_{a\downarrow R} c(R) > c(a)$, as is observed in the data, we assume $\theta > 1$. The attached children's consumption must satisfy:

$$c_k(a) = c(I)e^{\frac{(r-\rho)(a+B-I)-\alpha_0-(1-\alpha_1)f}{\theta}}, \ a \in [0, I). \tag{7}$$

We further simplify the problem by looking at the sum of the utility that directly accrues to the parent between his age I and T and the utility derived from his attached children. Simple calculations using the consumption Euler equations (5), (6), and (7) turn this sum into the right-hand side of (8).

$$\int_I^T e^{-\rho(a-I)}U(c(a), \ell(a))da + e^{-\alpha_0+\alpha_1 f}\int_0^I e^{-\rho(a+B-I)}U(c_k(a), \ell_k(a))da = \frac{c(I)^{1-\theta}}{1-\theta}G(I, R) \tag{8}$$

Here, we have introduced the following set of new notations:

$$G(I, R) \equiv e^{\upsilon(r)I}\left[(\Delta(R) - \Delta(I)) + (1+\zeta)^{\frac{1-\theta}{\theta}}(\Delta(T) - \Delta(R))\right] + e^{\upsilon(r)I}e^{f-rB}e^{-\mu/\theta}\Delta(I) \tag{9}$$

$$\Delta(x) \equiv \int_0^x e^{-\upsilon(r)a}da = \frac{1-e^{-\upsilon(r)x}}{\upsilon(r)} \tag{10}$$

$$\upsilon(r) \equiv \frac{\rho-(1-\theta)r}{\theta} > 0 \tag{11}$$

$$\mu \equiv \alpha_0 + (1-\alpha_1)f + \rho B - rB \tag{12}$$

The μ term captures the difference between the current interest rate r and the effective discount rate for the utility of different generations $(\alpha_0 + (1-\alpha_1)f)/B + \rho$.

The consumption expenditures for the parent, inclusive of the consumption of the attached children, similarly simplify into the right-hand side using (5), (6), and (7):

$$\int_I^T e^{-r(a-I)}c(a)da + e^f \int_0^I e^{-r(a+B-I)}c_k(a)da = c(I)G(I,R).$$

With these notations, the maximized utility of a parent of generation t at age I is:

$$V(h_t(I), q_t, u_t; R_t) = \max_{c_t(I), h_{t+1}(I), q_{t+1}} \frac{c_t(I)^{1-\theta}}{1-\theta} G(I, R_t) + e^{f-rB-\mu} V(h_{t+1}(I), q_{t+1}, u_{t+1}; R_{t+1}),$$

subject to the budget constraint:

$$c_t(I)G(I,R_t) + e^{f-rB}q_{t+1} \leq W(I,T,R_t) + e^{f-rB}W(0,I,R_{t+1}) + q_t + u_t. \tag{13}$$

We now discuss the second step: the determinants of the retirement age R. The individual's problem can be re-written into a sequence problem, again with t indexing generations. The objective function is:

$$\sum_{t=0}^{\infty} e^{(f-rB-\mu)t} \frac{c_t(I)^{1-\theta}}{1-\theta} G(I, R_t)$$

and the budget constraint is obtained by forward iteration:

$$\sum_{t=0}^{\infty} e^{(f-rB)t} c_t(I)G(I,R_t) \leq$$
$$\sum_{t=0}^{\infty} e^{(f-rB)t} \Big[W(I,T,R_t) + e^{f-rB}W(0,I,R_{t+1})\Big] + \sum_{t=0}^{\infty} e^{(f-rB)t}u_t + q_0. \tag{14}$$

Assuming that $W(R)$ is differentiable with respect to R, although it is not at R_n because of the policy-induced discontinuity in retirement benefits, the first-order conditions are:

$$e^{-\mu t}c_t(I)^{-\theta} = \Phi,$$
$$e^{-\mu t}\frac{c_t(I)^{1-\theta}}{1-\theta}G_R(I,R_t) - \Phi c_t(I)G_R(I,R_t) = -\Phi W_R(R_t),$$

where Φ is the Lagrange multiplier on (14), and G_R and W_R denote the partial derivatives with respect to R. These conditions further simplify into

$$W_R(R_t) = \frac{\theta}{\theta - 1} G_R(I, R_t)c_t(I). \tag{15}$$

The second-order conditions require that the left-hand side intersects the right-hand side from above.

Since the retirement benefits are discontinuous at R_n, there are two cases to consider. For $R \geq R_n$, with the notation $D(R) \equiv \left(e^{-rR} - e^{-rT}\right)/r$,

$$W_R(R_t) = e^{-r(R_t - I)}\left[(1 + b_y D(R_t))(1-\tau)wh(R_t) - (b_m + b_y W(R_t))\right], \tag{16}$$

while for $R < R_n$ we obtain

$$W_R(R_t) = e^{-r(R_t - I)}(1 + b_y D(R_n))(1 - \tau)wh(R_t). \tag{17}$$

Since $G_R(I, R) > 0$, condition (15) implies that the marginal benefit of working exceeds the marginal monetary cost at the equilibrium retirement age—if they were equal, the condition would be $W_R(R_t) = 0$. The reason is obvious. The opportunity cost of work is the additional utility associated with leisure, which is captured by the right-hand side. In other words, condition (15) equates the marginal benefit of additional work (i.e., additional net income) with the forgone utility from leisure converted into units of income. Note that, if $R_t > R_n$, the forgone retirement benefits are subtracted from the marginal benefit of additional work, as shown in (16).

3.2 Properties of the Solution

In this section we characterize some properties of the solution to the individual problem. In particular, we describe the response of effective labor supply to unanticipated changes in the economic environment. To derive results in this section, we assume that the rental rate of human capital (w) and the interest rate (r) are exogenously given.

We restrict our attention to the implications of the model when $\mu = 0$, which corresponds to the steady state. To see this, consider the case in which the interest rate exceeds the generational discount rate, i.e., $\mu < 0$. The definition of μ is in equation (12). One consequence is that consumption increases from generation to generation, shifting the right-hand side of (15) upward over t. The second-order condition implies that the retirement age, R_t, must decrease over generations indexed by t.[5] Intuitively, this is driven by income effects: Earlier retirement allows individuals to enjoy more leisure. The case with $\mu > 0$ can be worked out in a straightforward manner.

In the steady state ($\mu = 0$), from the budget constraint (13), we obtain

$$c(I)G(I, R) = W(I, T, R) + e^{f - rB}W(0, I, R) + (1 - e^{f - rB})q + u. \tag{18}$$

Now, the relevant first-order condition for the choice of retirement age is, except at the policy-induced kink:

$$\frac{W_R(R)}{W(I, T, R) + e^{f - rB}W(0, I, R) + (1 - e^{f - rB})q + u} = \frac{\theta}{\theta - 1}\frac{G_R(I, R)}{G(I, R)}, \tag{19}$$

where we have replaced $c(I)$ using (18).

3.2.1 Long-Run Responses

In this section, we consider the effects of unanticipated, permanent changes in tax and retirement policies. These are steady-state comparisons. The short-run transition effects are explored in Section 3.2.2.

[5] This model predicts that there is a downward trend in retirement age when interest rates exceed their long-run values, which happens during the transition to the steady state.

Effect of Changes in Retirement Benefits The above first-order condition can be used to study how the level and the redistributive nature of the retirement system affect the retirement age. Consider an increase in the generosity of the retirement benefits as given by an increase in the constant portion b_m, with all else held constant. Through a standard income effect, this policy change will lower the equilibrium retirement age. To see this, note that the positive income effect associated with an increase in b_m raises the denominator of the left-hand side of (19). Moreover, depending on whether the worker retires after R_n or before, $W_R(R)$ either decreases or stays the same—see equations (16) and (17). Thus, the left-hand side of equation (19) goes down while the right-hand side remains the same, resulting in a decrease in the retirement age. Recall that the left-hand side intersects the right-hand side from above, according to the second-order condition.

To isolate the impact of the redistributive nature of the retirement benefits, we consider a joint change in b_m and b_y that keeps the benefit level constant at the initial retirement age R^*. More specifically, let the benefit level be $\bar{b} = b_m + b_y W(R^*)$. The new b'_m and b'_y satisfy at the initial retirement age R^*:

$$\bar{b} = b'_m + b'_y W(R^*).$$

It follows that an increase in b_y accompanied by a decrease in b_m to maintain $\bar{b}$ shifts the numerator of the left-hand side of equation (19) upward. Its denominator will change, but it will be the same as before at R^* by construction. Hence we know for sure that the left-hand side is now higher than before at least at R^*. The right-hand side is unchanged. As a result, the retirement age rises.

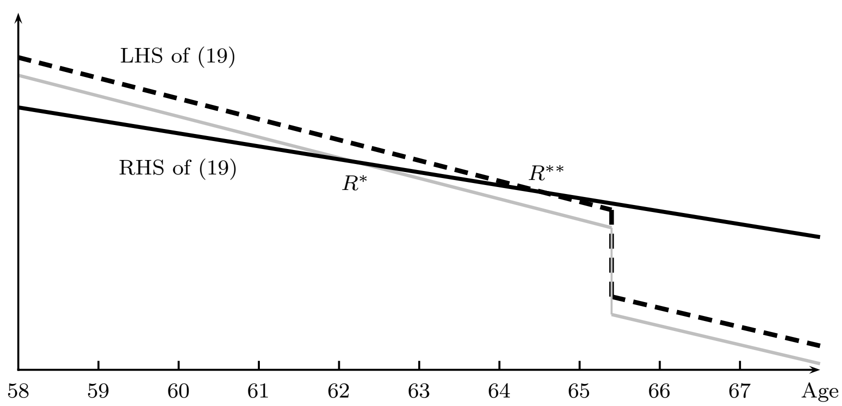

Figure 2: Retirement Decision Responding to Changes in b_m and b_y

Figure 2 depicts this exercise. Initially, the left-hand side (solid, gray line) of condition (19) and the right-hand side (solid, black line) cross at R^*, which is the optimal retirement age. Given the changes in b_m and b_y, the left-hand side moves up (dashed line), while the right-hand side remains the same. The new optimal retirement age is at the new intersection, denoted by R^{**}.

The intuition is straightforward. Returns to work increase as the system reduces the pure tax-and-redistribute component of the retirement benefits. Now that the worker will retire later, he will increase his human capital investment. He will go to school for longer, and will invest more in his human capital on the job, ultimately increasing his effective labor supply. We conclude that it is not only the level of benefits but also their redistributive nature that influences retirement and human capital investment decisions. This will play a role in our analysis of the differences between Europe and the US.

Effect of Changes in Tax Rates To better understand how the model works, and to build an intuition that is relevant to the US-Europe comparison, we consider the following change in taxes and transfers: We increase the tax rate τ and then commensurately increase the non-retirement transfer u such that the denominator of the left-hand side of (19) evaluated at R^* is left unchanged. That is, we control for the income effect channel at R^*.

Unlike the changes in retirement benefits we considered above, this tax-transfer change will affect the individual human capital investment even when the retirement age is held constant. Of course, this tax-transfer change will have an impact on the retirement age itself, and the response of the retirement age will generate further responses in terms of human capital investment.

First, we will go through the effect of this increase in tax rate τ for a fixed retirement age. One immediate implication is that the goods input to human capital production is now more expensive relative to the after-tax return to human capital. This is true as long as the goods input is not fully deductible, i.e., $\xi < 1$.[6] As a result, there will be less investment in human capital. For one, the individual will choose less schooling, and he will use less goods input for human capital production while in school. Likewise, he will reduce his investment in human capital on the job. In summary, a higher tax rate, unless goods input to human capital production is fully tax-deductible, will reduce the amount of human capital a worker acquires and, in turn, the supply of effective labor, even when the retirement age is held constant.

Next, we explore the effect of this tax-transfer change on the retirement age. We revisit the first-order condition (19). Note that the right-hand side does not change. The numerator of the left-hand side goes down, because (i) the tax rate is now higher and (ii) for a given R, the worker now accumulates less human capital for the reason explained above. Thus, if we compensate the worker with a higher transfer u such that the denominator of the left-hand side of (19) evaluated at R^* is left unchanged, then we see that the retirement age goes down unambiguously, together with the left-hand side of (19).[7]

Now that the worker will retire earlier, he will further reduce his human capital investment. This is an important interaction between retirement—i.e., a decision on the career length—and the human capital investment decision. Note that the decrease in the pure quantity measure of raw labor supply (career length) underestimates the decrease in labor services adjusted for human capital (effective labor). In our quantitative analysis in Section 4, we find that the magnitude of this downward bias can be substantial.

[6] If $\xi = 1$, then we have $p = 1$, which means the relative price is not affected by tax rates.

[7] The higher u for this purpose must compensate not only for the higher taxes but also for the lower income due to lower levels of human capital.

3.2.2 Short-Run Responses

We now analyze the short-run response of a worker's effective labor supply to three different kinds of unanticipated shocks: a one-time drop in the value of accumulated non-human wealth, a one-time drop in the stock of human capital, and a permanent decrease in the rental rate of human capital.

For convenience, we assume that such shocks hit a worker of age $a' > I + B$—that is, his children have already left the household. He will not adjust the bequest in response to shocks, and his responses capture standard life-cycle effects.[8] In addition, for simplicity, we consider the case in which the worker chooses $R^* < R_n$ in the absence of shocks, and the new retirement age remains in this region.[9]

Shock to Non-Human Wealth The relevant version of (19) at age a' for a worker who plans to retire at age $R^* < R_n$ is,

$$\frac{e^{-rI}W_R(a',T,R^*)}{(e^{-rI}+D(R_n)b_y)W(a',T,R^*)+D(R_n)(b_m+b_yW(0,a',R^*))+e^{-ra'}A(a')} = \frac{\theta}{\theta-1}\frac{G_R(a',R^*)}{G(a',R^*)}, \quad (20)$$

where $A(a')$ is the value of financial wealth accumulated up until age a'. As before, the second-order condition requires that the left-hand side of (20) crosses the right-hand side from above.

Now, consider the effect of an unanticipated (one-time) drop in the value of financial wealth $A(a')$ the worker has accumulated so far. This shock decreases the denominator of the left-hand side of (20), but $W_R(a',T,R)$ is not affected. The left-hand side shifts up and the right-hand side stays the same, increasing the retirement age—i.e., $\partial R/\partial A < 0$. Note that this shock only has a negative income effect, and leaves other things—e.g., returns to work—unchanged. The worker chooses to consume less leisure, which implies a delayed retirement.

Shock to Human Capital Next, we consider the effect of an unanticipated (one-time) partial destruction of human capital at age a'. This could be interpreted as, for example, an unanticipated reallocation of workers across sectors or firms when human capital is at least partly sector- or firm-specific.

The effect of this one-time shock on effective labor supply diminishes over time. To be precise, for a fixed retirement age R, the effect of an exogenous change in $h(a')$, is given by, from equations (32) and (33) in the appendix

$$\frac{\partial h^e(a)}{\partial h(a')} = e^{-\delta_h(a-a')}, \quad a' \le a < R,$$

where $h^e(a) \equiv h(a)(1-n(a))$. That is, the younger the worker is when he is hit by this negative shock, the smaller is the long-run impact on effective labor supply, for a given retirement age.

[8]The results are qualitatively similar for ages under $I+B$, but the expressions are algebraically more cumbersome and the income effects must include the adjustment of bequests.

[9]If this is not the case, again, the qualitative results remain similar but the algebra needs to be adjusted.

The intuition is clear: a one-time shock to the stock of human capital does not alter the returns to human capital (i.e., the rental rate of human capital w) in the future. Since younger workers have a longer horizon of human capital utilization, they accumulate more human capital than older workers .

Unlike in the previous case with the shock to non-human wealth, there is a substitution effect in addition to a negative income effect. While the returns to human capital are not affected, the shock reduces the returns to the worker's time, as one unit of his time translates now into less human capital supplied to the market or used for human capital production. The income and substitution effects push the retirement age in opposite directions. A sufficient condition for the substitution effect to dominate, and hence, for the worker to retire earlier than originally planned is

$$\frac{r+\delta_h}{e^{(r+\delta_h)(R^*-a')}-1} \geq \frac{\theta}{\theta-1}\frac{G_R(a',R^*)}{G(a',R^*)}.$$

It is easy to check that this condition is more likely to be satisfied for older workers (i.e., a' closer to R^*). Intuitively, for an older worker, only a small fraction of his working life is in the post-shock period, and hence the labor income generated after the shock accounts for only a small fraction of his lifetime income. Thus, the income effect is much smaller relative to his younger counterpart's.

Permanent Wage Shock Finally, we ask what is the effect of an unanticipated permanent decrease in the rental rate of human capital w for a worker who is a' years old. From equation (33) in the appendix, for a fixed retirement age R,

$$\frac{\partial h^e(a)}{\partial w} = \frac{\gamma_2}{1-\gamma}\frac{C_h}{w}\left(\frac{w}{p}\right)^{\frac{\gamma_2}{1-\gamma_1-\gamma_2}}\pi(a',a),\ \ a > a',$$

where $h^e(a)$ is the supply of effective labor and

$$\begin{aligned}
\pi(a',a) &\equiv (r+\delta_h)\int_{a'}^{a} e^{-\delta_h(a-u)}m(u)^{\frac{\gamma_1+\gamma_2}{1-\gamma_1-\gamma_2}}du - \gamma_1 m(a)^{\frac{1}{1-\gamma_1-\gamma_2}},\\
m(a) &\equiv 1-e^{-(r+\delta_h)(R-a)},\\
C_h &\equiv \left(\frac{z_h}{r+\delta_h}\gamma_1^{\gamma_1}\gamma_2^{\gamma_2}\right)^{\frac{1}{1-\gamma_1-\gamma_2}}.
\end{aligned}$$

The sign of the impact on effective labor depends on the sign of $\pi(a',a)$. Let us first fix R and $a' < R$. Simple calculations show that for each a' there exists $\varphi(a') \in (a',R)$ such that

$$\pi(a',a)\begin{cases} <0 & \text{if } a' \leq a < \varphi(a') \\ =0 & \text{if } a = \varphi(a') \\ >0 & \text{if } \varphi(a') < a \leq R \end{cases}$$

This result shows that, immediately following a permanent decrease in the rental rate of human capital, effective labor supply increases. The size of the increase depends on the worker's age: Young workers increase their effective labor supply more than old workers. Over time, the effect on the supply of effective labor of a change in the wage rate reverses itself. For a given age a' at the time of the shock, there exists another age, $\varphi(a') > a'$, such that the post-shock effective labor supply at that age coincides with what the effective labor supply at that age would have been in the absence of the shock. Eventually, for $a > \varphi(a')$, the post-shock effective supply of labor, $h^e(a)$, drops below what it would have been without the shock.

The intuition is as follows. When faced with a permanent decrease in the return to human capital, workers choose to reduce investment in human capital. In the short run, this results in more human capital supplied to the market as he changes his allocation of time away from human capital production—i.e., a decrease in $n(a)$. Eventually, the lower levels of investment in human capital is such that the stock of human capital drops below what it would have been without the shock, bringing down effective labor supply. The qualitative response of earnings mirrors that of effective labor. For a fixed retirement age, the adjustment in human capital investment implies that a drop in the human capital rental rate flattens the age-earnings profile.

What is the impact on the retirement age? In general, it is not possible to determine the effect on retirement because both the income and the substitution effects are at play. A lower w reduces the marginal benefit of working, $W_R(a', T, R)$, but the negative income effect reduces the denominator in the left-hand side of (20), pushing the retirement age in opposite directions. It is possible to show that for a' close to R—i.e., for older workers—the magnitude of the income effect is arbitrarily small, again because the post-shock working period is only a small fraction of their overall working life. Thus, for older workers, the model predicts that a decrease in w results in earlier retirement. The effect on the retirement plan of younger workers is ambiguous.

To summarize, we find that the impact of unanticipated shocks on retirement depends on the standard tension between income and substitution effects as well as on the specifics of the determinants of the age-earnings profile. Unlike in most other models of lifetime labor supply, the age-earnings profile in our model is endogenously determined and hence is not independent of the nature of shocks.

4 Quantitative Analysis

In Section 4.1, we close the model by specifying the goods production side of the economy and the government budget constraint. For our quantitative analysis, the preference and technology parameters are calibrated to the relevant data for the US (Section 4.2).

Section 4.3 begins by asking whether differences in tax and retirement policies across countries can explain their differences in schooling and retirement behavior. Next, we consider three sets of counterfactual exercises. First, we impose the retirement policies of the US onto selected European countries. Second, we study the impact on the US economy of various policy and demographic changes. Finally, we ask whether our model can explain the labor market outcomes in the US circa 1900. In all the exercises, we focus on the impact of taxes and retirement policies on schooling, retirement, and overall effective labor supply along the lifecycle.

4.1 Equilibrium

To define the steady-state equilibrium, we develop notations for the age distribution in the population. Let $\eta = f/B$ be the population growth rate. With the population size at any give point in time normalized to one, the density for age a is

$$\phi(a) = \frac{\eta e^{-\eta a}}{1 - e^{-\eta T}}.$$

We assume that the aggregate goods production function is Cobb-Douglas in physical capital and effective labor, with total factor productivity z. Let κ be the ratio of physical capital to effective labor, and δ_k be the depreciation rate of physical capital. Using this notation, output per worker is

$$y = zF(\kappa, 1)\frac{\int_{6+s}^{R} h^e(a)\phi(a)da}{\int_{6+s}^{R} \phi(a)da}, \tag{21}$$

and output per capita is

$$\bar{y} = zF(\kappa, 1)\int_{6+s}^{R} h^e(a)\phi(a)da. \tag{22}$$

The steady-state equilibrium requires that the goods market clear:

$$\int_0^T [c(a) + x(a)]\,\phi(a)da + \phi(6)x_E + g = [zF(\kappa, 1) - (\delta_k + \eta)\kappa]\int_{6+s}^{R} h^e(a)\phi(a)da, \tag{23}$$

where g is per-capita government consumption.

We assume competitive factor markets, and hence firms equate the marginal product to factor prices:

$$r = (1 - \tau_K)(zF_K(\kappa, 1) - \delta_k), \tag{24}$$

$$w = zF_H(\kappa, 1), \tag{25}$$

where F_K and F_H respectively denote the marginal product of physical and human capital.

We assume that the retirement benefits are fully financed by the social security taxes on labor income at all times. That is, the revenues collected from labor income at rate τ_S are equal to retirement benefit payments:

$$\int_{R_n}^{T} [b_m + b_y W(R)]\,\phi(a)da = \tau_S \int_6^R [wh^e(a) - \xi x(a)]\,\phi(a)da,$$

where we take into account the partial tax deductibility of goods input to human capital production. Furthermore, we assume that the government runs a balanced budget in terms of non-retirement revenues and expenditures:

$$g + \phi(I)u = \tau_I \int_6^R [wh^e(a) - \xi x(a)]\,\phi(a)da + \tau_K(zF_K(\kappa, 1) - \delta_k)\kappa \int_{6+s}^{R} h^e(a)\phi(a)da,$$

where the left-hand side is the non-retirement expenditures and the right-hand side is the sum of non-retirement labor income tax revenue and capital income tax revenue.

The definition of a steady-state competitive equilibrium is standard, and is omitted here.

4.2 Calibration

Certain parameters are standard in the macro literature. The interest rate is set at $r = 0.04$ and the depreciation rate is set at $\delta_k = 0.09$. The income share of (physical) capital is set at 0.33. The parameters governing the altruism function, $\alpha_0 = 0.24$ and $\alpha_1 = 0.35$, are taken from Manuelli and Seshadri (2009) who use observations on intergenerational transfers and the endogenously chosen fertility rate to calibrate these parameters. In the steady state, these parameters together with the discount rate ($\rho = 0.027$), pins down the real interest rate ($r = 0.04$).

We use age-wage profiles to pin down the parameters that govern human capital production. Little information is available on the fraction of job training expenditures that are not reflected in wages, but there are many reasons why earnings ought not to be equated with $wh(1-n)-x$. First, some training is off the job and directly paid for by workers. Second, firms typically obtain a tax break on the expenditures incurred on training. Consequently, the government (and indirectly, the individual through higher taxes) pays for the training and this component is not reflected in wages. Third, some of the training may be firm-specific, in which case the employer is likely to bear the cost of the training, since the employer benefits more than the individual does through the incidence of such training. Finally, there is probably some smoothing of wage receipts in the data and consequently, the individual's marginal productivity profile could be steeper than his wage profile. For all these reasons, we equate measured earnings with $wh(1-n)-0.5x$.

Our theory implies that it is only the ratio $h_B^{1-\gamma}/(z_h^{1-\upsilon}w^{\gamma_2-\upsilon(1-\gamma_1)})$ that matters for the moments of interest. Consequently, we can choose w (which is determined by z, the productivity in the goods production function) and h_B arbitrarily, and then calibrate z_h to match a desired moment. The calibrated value of z_h is common to all countries. Thus, the model does not assume any cross-country differences in an individual's ability to learn. This leaves us with seven parameters, δ_h, z_h, γ_1, γ_2, ν, ζ, and θ.

Theory implies that, for a given retirement age, the human capital allocations that result from the solution to the parent's problem coincide with the allocations that result from the simpler income maximization problem studied by Ben-Porath (1967). Since the moments are block-separable, we proceed in two steps. For a given retirement age, we calibrate the parameters δ_h, z_h, γ_1, γ_2, and ν so as to match the following five moments.[10]

1. The ratio of wage rate at age 64 to wage rate at age 55: 0.75 (French, 2005).
2. The ratio of earnings at age 50 to earnings at age 25: 2.5 (authors' calculations using NLSY 79).
3. Years of schooling for males in the US as of 2005: 12.65 (Barro and Lee, 2010).
4. Schooling expenditures (primary and secondary) relative to GDP: 4.2% (UNESCO Institute for Statistics).

[10] Essentially, we use the properties of the age-wage profiles to identify the parameters of the production function of human capital. This, of course, follows a standard tradition in labor economics.

5. Pre-primary expenditures relative to GDP: 1.0% (UNESCO Institute for Statistics).[11]

Finally, we choose θ to match the average retirement age of 64.6, and ζ to generate a 15-percent drop in consumption upon retirement.

The resulting parameter values are as follows.

δ_h	z_h	γ_1	γ_2	ν	θ	ζ
0.024	0.14	0.68	0.21	0.13	1.14	1.89

Table 1: Calibration

The calibrated returns to scale in the human capital production ($\gamma_1 + \gamma_2 = 0.89$) is within the range of estimates in the literature, which runs from 0.6 to close to 1.

4.3 Results

With the preference and technology parameters calibrated to the US data, we vary the tax and retirement policies to mimic the real-world policies in selected other countries (Section 4.3.1). This exercise gives a sense of how much of the cross-country differences in labor market outcomes can be explained in our model.

We then consider three counterfactual exercises. In Section 4.3.2, we impose the US retirement policies on other countries, and ask how their labor market outcomes will change in the long run. In Section 4.3.3, we consider the long-run impact of demographic and retirement policy changes in the US economy. Finally, we ask whether our model is compatible with the observed trends in schooling and retirement in the US. In particular, we compare a version of our model for the US in 1900 with the data from that period.

4.3.1 Schooling and Retirement: Cross-Country Differences

To assess the usefulness of our model for understanding labor supply and human capital investment decisions, we ask how much of the cross-country differences in schooling and retirement can be explained in our framework. In particular, we will hold constant all the preference and technology parameters (with one exception) at the US values chosen in Section 4.2, and vary taxes and retirement policies across countries, guided by the data from the OECD.

Motivated by the large literature that emphasizes the US-Europe difference, we first select certain European countries in the OECD.[12] We also include Mexico, to see how well our model performs for countries at a lower stage of economic development.

[11] The UNESCO number includes only purchased inputs. We assume that home inputs are roughly of equal importance and hence arrive at the 1% figure.

[12] We report the results for Denmark, France, and Spain. Denmark and France are representative of many other European countries. Spain is included as a sort of an outlier because our model prediction on retirement, using the tax and retirement systems summarized by the OECD, misses the data. We explain this discrepancy below. The results for Germany are close to those for Denmark and France, and are not reported here.

Data	USA	Denmark	France	Spain	Mexico
R_n	65.4	65	60	65	65
$\bar{\rho}_{0.5}$	0.50	1.24	0.62	0.81	0.55
$\bar{\rho}_{1.5}$	0.34	0.67	0.48	0.81	0.35
τ	0.301	0.412	0.493	0.378	0.151
G/Y	0.09	0.17	0.16	0.12	0.12
Output per worker	1.00	0.81	0.80	0.72	0.34

Table 2: Tax and Retirement Policies

All the data in Table 2 are for 2005. The statutory full retirement age (R_n) and pension replacement rates ($\bar{\rho}_{0.5}$ and $\bar{\rho}_{1.5}$) are from OECD (2011).[13] The total labor tax rates (τ) are from OECD (2010). Finally, government consumption relative to output (G/Y) and output per worker are from Penn World Table Version 6.3.

For each of the countries, we use the preference and technology parameters of Section 4.2. We plug in R_n, G/Y, and τ directly. We then adjust b_m and b_y to match $\bar{\rho}_{0.5}$ and $\bar{\rho}_{1.5}$ in the data. The replacement rate $\bar{\rho}_{0.5}$ is for a retiree who used to make half the average earnings of the economy, and $\bar{\rho}_{1.5}$ is for one who used to make one and a half times the average earnings. The fact that $\bar{\rho}_{0.5}$ is greater than $\bar{\rho}_{1.5}$ implies a positive b_m, the redistributive component in the retirement benefits. The only non-policy parameter that we vary across countries is z, the TFP term in the goods production function as in (22). We choose this country-specific z to match the output per worker relative to the US.

From the computed steady-state equilibrium of each country, we report in Table 3 the years of schooling and retirement age.

	USA	Denmark	France	Spain	Mexico
Years of schooling (Data)	12.65	10.52	10.77	10.70	8.64
Years of schooling (Model)	12.65	11.23	10.45	11.01	8.39
Retirement age (Data)	64.6	63.3	58.7	61.4	73.0
Retirement age (Model)	64.6	63.5	60.0	65.0	65.0

Table 3: Schooling and Retirement (Data and Model)

The years of schooling data are the average for men aged 30–64 as of 2005 from Barro and Lee (2010). The retirement age data are the average effective retirement age for men constructed in OECD (2011).

We first note that the tax and retirement policies, coupled with TFP differences, can explain very well the cross-country variation in years of schooling.

Why does the model predict less schooling for the other countries? For a fixed retirement age, individuals will choose less schooling in response to lower after-tax rental rate of human capital.

[13] The US is in the process of gradually raising its full retirement age from 65 to 67, with the transition to be completed when those born in 1960 turn 62 in 2022. This age was 65.4 as of 2005.

This is because a lower after-tax rental rate implies relatively more expensive goods input to human capital production, which results in less human capital investment overall. The after-tax rental rate of human capital is lower in the other countries, because of higher taxes (in the case of Denmark, France, and Spain) and of lower TFP (in the case of Denmark, Spain, and Mexico).

This effect also interacts with the retirement age. In Denmark and France, earlier retirement implies a shorter horizon of human capital utilization, which also discourages human capital investment and schooling.

The model performance is mixed when it comes to retirement decisions. For Denmark and France, the picture is clear. These countries have higher labor income taxes than the US, and also more generous transfers in terms of retirement benefits (b_m and b_y) and non-retirement rebates (u). In light of the first-order condition (19), these policies decrease the numerator of the left-hand side and increase its denominator, with the overall impact of lowering the retirement age. In the case of France, the model retirement age of 60 is actually the corner (R_n) set by the retirement benefit program: If a French worker works past 60, the forfeiture of his retirement benefits acts as an effective tax rate of 50 percent on his labor income. In Section 4.3.2, we look at this case in more detail.

The model fails to explain the retirement decision of the Spaniards: In the data, the retirement age is 61.4, well below the US number, but the model predicts 65.[14] We now explain why.

In our model, the full retirement age R_n causes a kink in the value of retirement, because the worker's forfeiture of retirement benefits from R_n on imposes an effective tax on continued work. We set $R_n = 65$ for Spain, as in OECD (2011), and this happens to be the retirement age in our model. However, for many countries, a kink is posed not only by the full retirement age R_n, but also by the early retirement age. The early retirement age in Spain is 61, and it is estimated that the effective tax rate for the average Spanish worker working past 61 is about 35 percent (Duval, 2004). While this is much smaller than the effective tax rate at the full retirement age—which is as high as 90 percent—it is sizable enough to induce retirement for the majority of Spanish workers in reality. Indeed, when we modify our retirement benefit function in Spain to incorporate this early retirement age of 61, our model predicts that the average Spanish worker retires at this very kink. However, to maintain consistency across our calibration and counterfactual analyses, we choose to leave the early retirement age for Spain out of the model.[15]

More interesting, even though the model underpredicts the retirement age in Mexico, it shows that there is no inconsistency between fewer years of schooling and a higher retirement age. Comparing Mexico and the US, Mexican workers have less schooling (by 4.26 years), but retires 0.4

[14]The model retirement age of 65 for Spain is also the corner set by the retirement benefit program. The representative Spanish worker in the model would work for longer, were it not for this policy-induced kink.

[15]Even if we were to model the early retirement age for all countries in our analysis, only our results on Spain will change. For Denmark and France, the early retirement age coincides with the full retirement age, although in France those who are disabled and long-term unemployed can apply for early retirement at the age of 58. For the US, the early retirement age is 62, but there is an actuarially-fair adjustments for those who claim benefits before the full retirement age. That is, there is no kink induced by the US retirement policy at the age of 62.

years later in the model.[16] To understand this, the important difference for this phenomenon is the non-retirement transfer u. While the total tax wedge τ is twice as high in the US, government consumption relative to GDP is higher in Mexico. At the same time, the social security programs are very similar in the two countries. Overall, the lump-sum redistribution (u) is much smaller in Mexico. This explains why Mexican workers may retire later in the model. Although the substitution effect from the lower after-tax wage (owing to lower z) pushes retirement in the opposite direction, the income effect from the smaller transfer is found to prevail. We will see this very intuition at work again in Section 4.3.4, when we consider the US circa 1900.

In summary, we find that our model can explain much of the cross-country differences in schooling and retirement with their differences in tax and retirement policies. The shortcomings of our model regarding retirement in Spain and Mexico call for a richer model that can better capture the relevant institutional details, which is left for future research.

Before moving on to counterfactual experiments, we briefly discuss our model implications on another dimension: age-earnings profile. With the tax and retirement policies in the European countries we consider, there is less schooling and less human capital accumulation both in and out of school. In our framework, this implies a flatter age-earnings profile. The earnings between age 25 and 50 in the US from our model grow faster than in Spain by 37 percent and than in France by 90 percent. The age-earnings profiles in our model seem to be consistent with the different slopes found in the US vs. European countries. While longitudinal data that span such a long period are not easily accessible for most countries, we found estimates for Spain (based on Spanish Social Security records) and Italy (based on the Bank of Italy Survey on Household Income and Wealth), as well as for the US (based on Social Security records). For the particular cohorts followed in these three countries, the average earnings in the US grow faster than in Spain by 83 percent and than in Italy by 20 percent.[17]

4.3.2 Imposing the US Retirement Policy

In this section, we evaluate the long-run effects of imposing the retirement benefit system of the US on each of the other four countries. We hold all other parameters constant, and re-calibrate R_n, b_m, b_y, τ_S, and τ. We plug in $R_n = 65.4$, which is the US full retirement age, and choose b_m and b_y for each country to match the US replacement rates ($\bar{\rho}_{0.5} = 0.5033$ and $\bar{\rho}_{1.5} = 0.3411$). We maintain the assumption that the retirement benefit system pays for itself with τ_S. Thus, we obtain a new τ_S for each country. We hold τ_I constant, which means that $\tau \equiv \tau_S + \tau_I$ will have to change by as much as does τ_S for each country. The first row of Table 4 shows this adjustment.

[16] The Mexican retirement age of 65 is again the corner set by the retirement benefit program. Although we assume that all Mexican workers are subject to the tax and retirement policies, we are keenly aware of the limited coverage of official pensions in Mexico, partly because of the vast informal sector. If we were to take into account this limited coverage and remove the $R_n = 65$ corner, the Mexican workers in our model will work for longer and our model will more closely match the retirement age in the data, 73.

[17] The age-earnings profile constructed from cross-sections also show that the US has a steeper slope than other European countries. See for example OECD (2006). However, we do not use this evidence, because the cross-section estimates confound age and cohort effects.

We compute the resulting new steady states, and report in Table 4 the impact of this policy change on years of schooling (s), retirement age (R), career lengths ($R - s - 6$), and output per worker. We also compute the average human capital supplied to the labor market by workers, divided by years of schooling: $\bar{h}^e/s$. This is a measure of human capital that is not fully captured by years of schooling. This measure and output per worker are normalized by their respective levels in the US. The change is indicated by the arrows from the initial steady states to the new steady states. We note that the initial steady state values are from the model, as they do not perfectly fit the data—see Table 3.

	Denmark	France	Spain	Mexico
τ_S	$0.24 \to 0.11$	$0.23 \to 0.11$	$0.23 \to 0.11$	$0.08 \to 0.07$
Schooling	$11.23 \to 12.68$	$10.45 \to 12.81$	$11.01 \to 12.08$	$8.39 \to 8.49$
Retirement age	$63.5 \to 65.4$	$60.0 \to 65.4$	$65.0 \to 65.4$	$65.0 \to 65.4$
Career length	$46.3 \to 46.7$	$43.5 \to 46.6$	$48.0 \to 47.3$	$50.6 \to 50.9$
Output per worker	$0.81 \to 0.97$	$0.80 \to 1.08$	$0.72 \to 0.83$	$0.34 \to 0.34$
$\bar{h}^e/s$	$0.93 \to 1.00$	$0.91 \to 1.00$	$0.93 \to 0.97$	$0.88 \to 0.88$

Table 4: Long-Run Impact of US-Style Reforms

In the three European countries with generous retirement benefits and high taxes, the US-style reform brings down τ_S and hence τ by more than ten percentage points. The social security tax rates fall for two reasons. First, the new replacement rates are lower, and the benefits can be financed with lower taxes. Second, workers are now eligible for benefits at an older age, especially so in France, and the beneficiaries make up for a smaller fraction of the population.

This is a significant cut in the total labor wedge (τ), which increases schooling by 1 (Spain) to 2.5 (France) years, as explained in Section 3.2.1. In relative terms, years of schooling increase by 10 percent (Spain) to 23 percent.

In addition, workers now retire later. In all three countries, the new retirement age is 65.4, the corner in the US retirement benefit program. In the pre-reform steady states, Danish workers were not at their corner, but their French and Spanish counterparts were. The retirement age rises by 2 full years in Denmark, and only by a half-year in Spain. The most dramatic change, again, is in France. With the French workers previously at the $R_n = 60$ corner, they not only face a much lower tax but also a new corner that is much farther out at $R_n = 65.4$. They now retire 5.4 years later. Such later retirement reinforces the incentive to acquire human capital driven by lower taxes, since it implies a longer horizon of human capital utilization.

One important distinction that we make in our framework is the one between pure quantity measures of raw labor supply (i.e., career length) and labor service measured in efficiency units (i.e., effective labor). We now ask how workers respond along these two margins in response to the reforms. It is clear from Table 4 that the model predicts only small changes in career length. The number of years worked rises by a half-half year in Denmark and three years in France, and actually declines by more than a half-year in Spain. This is because the rise in retirement age is accompanied by the increase in years of schooling—recall that career length is $R - (6 + s)$. These are small relative changes (Spain's -1.5 percent to France's +6.9 percent) in career length.

The big economic impact comes from the changes in human capital or the quality of labor. Not only do workers now have more schooling, they also invest more in their human capital both in and out of school. As a result, the average human capital supplied to the labor market by workers (not divided by years of schooling) rises by 15 percent (Spain) to 35 percent (France). The effective labor supplied per worker per year of schooling ($\bar{h}^e/s$) also rises by 4 percent (Spain) to 10 percent (France). Therefore, even if one proxies human capital with years of schooling, the impact of the reform on human capital will still be underestimated by a substantial margin—by about one-third, to be more precise.

All the increased human capital accumulation translates into the substantial increase in output per worker in the long run. With our goods production function, because the reforms do not alter the ratio of physical capital to human capital (κ), the steady-state output per worker increases one-to-one with the effective labor supplied per worker, as in equation (21), by 15 percent (Spain) to 35 percent (France).[18] To put this number in perspective, note that the convergence to a steady state in this model takes approximately 40 years owing to the demographic structure. The impact of the social security reform in Denmark and France is tantamount to extra 0.45 and 0.75 percentage point growth per year, respectively, over 40 years.

For a decomposition of the economic impact via the quantity and qualtiy of labor channels, we consider the changes in output per capita, which is the product of output per worker and the labor foce participation rate—equations (21) and (22). For the three countries, output per capita increase by 13 percent (Spain) to 39 percent (France). For France, 35 percentage points out of the 39 come from the increase in human capital, and the labor force participaion channel accounts for the small remainder. For the 13 percentage points in Spain, 15 come from the human capital channel, while the labor force participation channel negatively contributes to output per capita: In our exercise for Spain, years of schooling rises by more than the retirement age, reducing the career length and hence the labor force participation rate. Therefore, if one focuses only on the pure quantity measures of raw labor supply (i.e., labor force participation rate in this case), the economic impact of the reforms will be grossly underestimated—or, what is worse, even assigned the wrong sign.

In summary, we find that the adoption of a US-style retirement system in the three European countries will significantly raise their output, and most of the effect materializes through the human capital channel, which can be only partly captured by the increase in years of schooling. By contrast, a switch to the US-style retirement system has hardly any effect for Mexico, since its retirement benefit systems are largely similar to the US system to begin with.

Before we move on to the next exercise, we consider the French case in more detail. Adopting

[18] The model predicts that the new steady-state output per worker in France is even higher than the US level (normalized to one). This is driven by our country-specific TFP estimate. Since France has much higher tax rates than the US to begin with, the model dictates that its TFP be higher in order to match the observed levels of output per worker. When the taxes are reduced, its output per worker happens to exceed the US level.

the US retirement benefit program for France involves two important changes. One is the reduction in replacement rates, and the other is the increase in full retirement age R_n, from 60 to 65.4. To get a sense of the relative importance of these two elements, we work out an intermediate reform of raising R_n to 65.4 with the replacement rates left unchanged.[19] In the new steady state associated with this intermediate reform, retirement age is 64.0 (from 60.0), years of schooling is 12.18 (from 10.45), and output per worker relative to the US is 0.99 (from 0.80). The average effective labor per year of schooling ($\bar{h}^e/s$) is 0.97 (from 0.91). We conclude that the increase in full retirement age from 60 to 65.4 alone accounts for about two-thirds of the economic impact of the full reform in France. Again, the vast majority of the economic impact operates through the human capital channel both in and out of school.

4.3.3 Changes in Demographics and Tax/Retirement Policies

In this section we consider four experiments on the benchmark US economy. While these experiments do have policy implications (e.g., raising the full retirement age to 67, which is being gradually rolled out in the US), our main purpose is to further clarify the inner workings of the model by considering one change at a time.

As for demographic changes, we consider the long-run impact of an increase in the life span (T) from 78 to 80. Holding all other parameters constant, we re-calibrate b_m, b_y, and τ_S to maintain the replacement rates and to balance the social security budget. (Again, τ will change one-to-one with τ_S.) The results from the new steady state are reported in the second column of Table 5.

The two-year increase in life span has small effects on labor market outcomes. All else equal, a longer life span calls for a later retirement, as the worker needs to generate more income to pay for the additional consumption later in life. This would by itself result in more schooling, as the horizon for human capital utilization is now longer. However, in equilibrium, taxes must rise (τ_S and hence τ by 1.7 percentage point) to support the longer-living retirees. Through the channel explained in Section 3.2.1, this discourages human capital acquisition. The effect of the higher taxes on retirement age can be ambiguous because of the opposing income and substitution effects. In the end, schooling decreases slightly (by less than a month), but retirement age rises (by four months), lengthening the career by almost a half-year. However, the smaller investment in human capital results in lower output per worker, although the magnitude is very small.

In the third column, starting from the benchmark US economy, we raise the full retirement age from 65.4 to 67, while maintaining the replacement rates. In the benchmark, workers in the US retire at age 64.6, and are not at the $R_n = 65.4$ corner. Thus, there is no direct impact from pushing back this policy-induced corner. Still, now the retirement benefits are paid for those 67 and older, and the social security tax can be lowered, along with the total tax wedge τ. This policy implies a 1.66 percentage point drop in the labor tax rate. In the new steady state, workers retire later and acquire more schooling, with the career length increasing by slightly less than one percent.

[19] This requires a new calibration of R_n, τ_S, τ, b_m, and b_y, except that now we pick b_m and b_y to maintain the previous replacement rates ($\bar{\rho}_{0.5} = 0.6175$ and $\bar{\rho}_{1.5} = 0.4846$) in the face of the changes in R_n, τ_S, and τ.

	Benchmark	$T = 80$	$R_n = 67$	$\rho_{0.5} = \rho_{1.5}$	No Social Security
τ_S	0.1116	0.1286	0.0950	0.1116	0
Schooling	12.65	12.59	12.90	12.81	14.19
Retirement age	64.6	64.9	65.3	65.4	68.9
Career length	45.95	46.33	46.36	46.59	48.75
Output per worker	1.00	0.99	1.03	1.02	1.23
$\bar{h}^e/s$	1.00	0.99	1.01	1.01	1.09

Table 5: Long-Run Impact of Changes to the US

The 3.3-percent increase in output per worker reflects the increased human capital investment in and out of school: While years of schooling rises by 2 percent, the effective labor supply per year of schooling increases by one percent. The intuition is the exact reverse of the tax hike in Section 3.2.1.

The effect of this policy change is small, especially when compared with the impact of the similar reform for France that we considered at the end of Section 4.3.2. There are two reasons. First, the French (intermediate) reform pushed R_n from 60 to 65.4, while the experiment for the US is only from 65.4 to 67. Second, more important, French workers in our model were at the $R_n = 60$ corner prior to the reform. Therefore, the reform had a direct impact on retirement age and human capital investment, in addition to the indirect effect through the new, lower tax rates.

This policy change in the US will also affect the age-earnings profile. With more human capital investment early in the career, age-earnings profile will get steeper. Indeed, we find in the model that earnings between age 25 and 50 now grow 9 percent faster than in the benchmark. There is empirical evidence that corroborates this finding. Neumark and Stock (1999) exploit cross-state variation in the implementation of anti-age discrimination legislations in the US, and report that the age-earnings profile of young workers steepened with the elimination of mandatory retirement.

In the fourth column, we remove the redistributive component of the social security by setting $b_m = 0$. We compute a new value of b_y that satisfies the social security budget constraint with the same $\tau_S = 0.1116$ as before. Hence, all tax rates remain the same. This is very close to the b_m-and-b_y exercise in Section 3.2.1, and here the obvious effect is the rise in retirement age. In turn, the schooling period lengthens, as the horizon of human capital utilization gets longer. The career length increases by 1.4 percent, and output per worker also rises by 2 percent driven by the higher level of human capital.

Finally, we eliminate the social security system. In the model, this is done by setting b_m, b_y, and τ_S all equal to zero. The total labor tax wedge τ decreases by 0.1116 from 0.3010. In addition, R_n is now irrelevant. This experiment can be thought of an extreme version of the second experiment above (third column), and hence the same logic—which has been discussed in detail in Section 3.2.1—applies. We obtain sizable long-run effects from this experiment. With much lower labor income taxes, workers acquire more schooling in and out of school: Years of schooling increases by 12 percent, and the effective labor supply per year of schooling rises by 9 percent. All in all, with the higher level of human capital, output per worker rises by 23 percent. They also retire later, and their career length increases, but only by 6 percent.

In summary, we again find that the reduction of distortions from taxes and retirement benefits can have significant economic impact. Most of the effect materializes through an increase in human capital acquisition both in and out of school. As a result, a pure quantity measure of raw labor supply will underestimate the whole economic impact of policy changes.

4.3.4 The US in 1900

As a final exercise, we ask whether our model can explain the trends in schooling and retirement over time for the US. In 1900, more than 60 percent of men aged 65 or older were in the labor force, while the figure in 2005 is 20 percent, with an average retirement age of 64.6.[20] At the same time, the average schooling among adult males in the US in 1900 was 5 years, while it has grown to 12.65 years by 2005. To summarize, over the past century, workers acquired more and more human capital (as proxied by schooling), but their career length shortened significantly.[21]

We cast our model in the year 1900 as follows. First, the life span was shorter in 1900. We use the life expectancy at age six, which implies a life span of $T = 65$ years.[22] The population growth rate was $\eta = 0.02$ around 1900. In addition, the government had a very small footprint in the economy. Consistent with the data, we set $\tau = \tau_I = \tau_S = 0$, $g = u = 0$, and $b_m = b_y = 0$. That is, there is virtually no tax or government expenditure, let alone the social security system. We then choose z in 1900 to match the output per worker relative to the 2005 level, which is 0.19.

With these changes, we compute the steady state of our economy in 1900, and report the predicted years of schooling and retirement age in Table 6 alongside the data. Given that the average worker circa 1900 did not retire at all, we use $T = 65$ as the retirement age in the data.

	Benchmark	1900 (Model)	1900 (Data)
Schooling	12.65	5.46	5
Retirement age	64.6	61.3	65 (T)
Career length	45.95	49.84	54

Table 6: The US circa 1900 (Data and Model)

There are several forces at play. First, the shorter life span, holding other things constant, would imply earlier retirement and, in turn, less schooling. The lower TFP (z), which we choose to match the lower output per worker in 1900, has both income and substitution effects on retirement, pushing it in opposite directions—income effect toward later retirement and substitution effect

[20]According to Carter and Sutch (1996), only about a fifth of men who reached 55 eventually retired. For many others, old age was a prolonged spell of unemployment, with the unemployment rate for those 60 or older estimated to exceed 30 percent.

[21]We use the estimates for schooling and output per worker in 1900 from Turner et al. (2007).

[22]The number is from Bell and Miller (2005). The life expectancy at birth was merely 52 years. This large discrepancy is explained by the high infant mortality rate in 1900. For our model, we find the expectancy at age six to be more relevant.

toward earlier retirement. Also, for any given retirement age, the lower z raises the price of goods input for human capital relative to the returns to human capital, reducing schooling and human capital investment. (This effect dominates the opposing effect from lower taxes.) Finally, the absence of retirement benefits and non-retirement transfers has a large negative income effect that pushes for later retirement.

The model predicts that the average worker in 1900 retires at 61.3 and attains 5.46 years of schooling, which imply a career length of 49.84 years. Remarkably, the model closey matches the average years of schooling in the data. Although the model falls short in terms of the retirement age, the average worker in 1900 is predicted to spend only 5.7 percent of his lifetime on leisure, while the figure in 2005 is 17 percent. To summarize, the model can explain all the changes in schooling, while it explains exactly half of the reduction in career length over the last century.

We draw the conclusion that, to understand long-run trends in human capital investment and retirement, one must consider the changes in all relevant factors jointly, including, but not limited to, demographic variables, technology, and tax-transfer policies.

5 Conclusion

In this paper we present a model in which labor input possesses two components: a quantity component associated with career length, and a quality component that depends on education and other forms of investment in human capital. We use the model to understand variations in retirement ages across countries and use the model for counterfactual policy simulations. Our paper makes several contributions. First, we solve a model of endogenous retirement and human capital investment, and show that our model can successfully explain the variation in retirement ages and schooling for a set of countries. Second, we find that the impact of taxes and retirement policies on effective labor supply and output per worker can be substantial. Third, while effective labor supply is highly elastic to changes in policies, in all our experiments, we find that the response of career length is relatively small. Hence, we see our analysis as reconciling the view that taxes can have a large economic impact (Prescott, 2006; Ljungqvist and Sargent, 2007; Prescott et al., 2009) with the view that labor supply at the extensive margin may be inelastic owing to institutional rigidities (Ljungqvist and Sargent, 2010).

Appendix

We first explain how the individual problem can be solved for a given retirement age R. The retirement decision has been studied in Section 3.1. We solve the individual problem backwards. That is, we first solve a worker's problem after he leaves school and chooses $n(a) < 1$, and then solve his problem when he is in school—i.e., $n(a) = 1$. We then show his early childhood human capital investment decision, which in turn pins down his years of schooling s.

Post-Schooling Problem We first present the value function and the optimal decisions rules of the Ben-Porath human capital accumulation problem in the post-schooling period. By this we mean that the fraction of the time allocated to work $(1-n)$ and to human capital accumulation (n) is not constrained to be between zero and one.

Recall that he representative worker maximizes the present value of his net labor income. Define the function $V(h,a)$ as

$$V(h(a),a) = \max \int_a^R e^{-r(t-a)}(1-\tau)[wh(t)(1-n(t)) - px(a)]da \tag{26}$$

subject to

$$\dot{h}(t) = z_h(n(t)h(t))^{\gamma_1}x(t)^{\gamma_2} - \delta_h h(t), \quad t \in [a,R). \tag{27}$$

Proposition 1 *The function $V(h,a)$ is given by*

$$V(h,a) = (1-\tau)w\left\{\frac{m(a)}{r+\delta_h}h + \frac{1-\gamma}{\gamma_1}C^{\frac{1}{1-\gamma}}\int_a^R e^{-r(t-a)}m(t)^{\frac{1}{1-\gamma}}dt\right\} \tag{28}$$

where

$$C = \frac{z_h\gamma_1}{r+\delta_h}\left(\frac{\gamma_2}{\gamma_1}\frac{w}{p}\right)^{\gamma_2}$$

and

$$m(a) = 1 - e^{-(r+\delta_h)(R-a)}.$$

Proof The Hamilton-Jacobi-Bellman equation corresponding to the problem is

$$rF(h,a) = \max_{n,x}\ (1-\tau)[wh(1-n) - px] + F_h(h,a)[z_h(nh)^{\gamma_1}x^{\gamma_2} - \delta_h h] + F_a(h,a) \tag{29}$$

with boundary condition $F(h,R) = 0$. A direct calculation shows that (28) satisfies (29) and the boundary condition. ■

The optimal time allocation, goods input, and the resulting human capital are given by

$$x(a) = \left(\frac{\gamma_2}{\gamma_1}\frac{w}{p}\right)C^{\frac{1}{1-\gamma}}m(a)^{\frac{1}{1-\gamma}}, \tag{30}$$

$$n(a)h(a) = C^{\frac{1}{1-\gamma}}m(a)^{\frac{1}{1-\gamma}} \tag{31}$$

$$h(a') = e^{-\delta_h(a'-a)}h(a) + \frac{r+\delta_h}{\gamma_1}C^{\frac{1}{1-\gamma}}\int_a^{a'} e^{-\delta_h(a'-t)}m(t)^{\frac{\gamma}{1-\gamma}}dt, \quad a' \geq a. \tag{32}$$

The effective labor supply is given by

$$h^e(a) = h(a) - C^{\frac{1}{1-\gamma}} m(a)^{\frac{1}{1-\gamma}}, \quad a \in [6+s, R] \tag{33}$$

For later use, we write down the Hamiltonian associated with this problem.

$$G(n,h,x,q) = (1-\tau)[w(1-n)h - px] + q[z_h(nh)^{\gamma_1} x^{\gamma_2} - \delta_h h],$$

where q is the costate variable. A standard argument, by either direct computation or the observation that $q(a) = \frac{\partial V(h,a)}{\partial h}$, shows

$$q(a) = (1-\tau)w\frac{m(a)}{r+\delta_h}. \tag{34}$$

Schooling Period We now describe the necessary conditions that the maximization problem must satisfy during the schooling period—i.e., when $n(a) = 1$. It is convenient at this point to treat the initial stock of human capital at age 6 (h_E) as given, even though this will be endogenously determined below.

To analyze this problem using Pontryagin's maximum principle, define the Hamiltonian as

$$Q(x,h,q) = -px + q(z_h h^{\gamma_1} x^{\gamma_2} - \delta_h h).$$

Then, the standard first order condition for the optimal choice of x is

$$x(t) = \left[\frac{\gamma_2 z_h}{p} q(t) h(t)^{\gamma_1}\right]^{\frac{1}{1-\gamma_2}}. \tag{35}$$

The Euler equation is simply

$$\frac{\dot{q}(t)}{q(t)} = (r+\delta_h) - [z_h \gamma_1 h(t)^{\gamma_1 - 1} x(t)^{\gamma_2}].$$

We combine it with the law of motion for human capital to obtain

$$\frac{\dot{q}(t)}{q(t)} + \gamma_1 \frac{\dot{h}(t)}{h(t)} = r + (1-\gamma_1)\delta_h. \tag{36}$$

Define, $v(t) \equiv q(t)h(t)^{\gamma_1}$, then the solution to (36) is $v(t) = v(6)e^{(r+(1-\gamma_1)\delta_h)(t-6)}$, or

$$q(t)h(t)^{\gamma_1} = q_E h_E^{\gamma_1} e^{(r+(1-\gamma_1)\delta_h)(t-6)}, \tag{37}$$

where q_E is the costate variable at age 6.

Substituting equation (35) into the law of motion for human capital (27), with $n(t) = 1$, we obtain

$$\dot{h}(t) = z_h h(t)^{\gamma_1} \left[\frac{\gamma_2 z_h}{p} q(t) h(t)^{\gamma_1}\right]^{\frac{\gamma_2}{1-\gamma_2}} - \delta_h h(t), \quad t \in [6, 6+s).$$

Next, we substitute in equation (37) to get

$$\dot{h}(t) = z_h \left(\frac{\gamma_2 z_h}{p} q_E h_E^{\gamma_1} \right)^{\frac{\gamma_2}{1-\gamma_2}} e^{\frac{\gamma_2}{1-\gamma_2}(r+(1-\gamma_1)\delta_h)(t-6)} h(t)^{\gamma_1} - \delta_h h(t). \tag{38}$$

Equation (38) is a non-homogeneous non-linear differential equation, with initial condition $h(6) = h_E$. The solution is as follows.

$$h(t) = h_E e^{-\delta_h (t-6)} \left[1 + \frac{(1-\gamma_1)(1-\gamma_2)}{\gamma_2 r + (1-\gamma_1)\delta_h} z_h^{\frac{1}{1-\gamma_2}} \left(\frac{\gamma_2 q_E}{p} \right)^{\frac{\gamma_2}{1-\gamma_2}} h_E^{\frac{\gamma-1}{1-\gamma_2}} \left(e^{\frac{\gamma_2 r + (1-\gamma_1)\delta_h}{1-\gamma_2}(t-6)} - 1 \right) \right]^{\frac{1}{1-\gamma_1}} \tag{39}$$

The solution to the problem also requires deriving the time at which it is optimal for the individual to switch from school, $n(a) = 1$, to work, $n(a) < 1$. From (39) evaluated at $t = 6 + s$ and (31) with $n(6+s) = 1$, we derive the following equation.

$$h_E e^{-\delta_h s} \left[1 + \frac{(1-\gamma_1)(1-\gamma_2)}{\gamma_2 r + (1-\gamma_1)\delta_h} z_h^{\frac{1}{1-\gamma_2}} \left(\frac{\gamma_2 q_E}{p} \right)^{\frac{\gamma_2}{1-\gamma_2}} h_E^{\frac{\gamma-1}{1-\gamma_2}} \left(e^{\frac{\gamma_2 r + (1-\gamma_1)\delta_h}{1-\gamma_2} s} - 1 \right) \right]^{\frac{1}{1-\gamma_1}}$$
$$= C(w/p)^{1/(1-\gamma)} m(6+s)^{1/(1-\gamma)} \tag{40}$$

As the costate variable also needs to be continuous at $t = 6+s$, from (34), (37), and (39), we derive another equation in q_E and s for a given h_E.

$$q_E \left[C(w/p)^{1/(1-\gamma)} m(6+s)^{1/(1-\gamma)} \right]^{-\gamma_1} h_E^{\gamma_1} e^{(r+(1-\gamma_1)\delta_h)s} = (1-\tau) w \frac{m(6+s)}{r+\delta_h} \tag{41}$$

The solution to equations (40) and (41) gives the pair (q_E, s).

Early Childhood Investment So far, we took the stock of human capital at age 6, h_E, as given. Finally, we endogenize it here. Let $F(h, 6)$ be the value of the problem at age six. In other words,

$$F(h,6) = \max_{x(t), h(6+s)} \; - \int_6^{6+s} e^{-r(t-6)} p x(t) dt + e^{-rs} V(h(6+s), 6+s),$$

subject to the human capital production function in school. Then, with p_E denoting the price of goods input to early childhood human capital production, the optimal level of investment is the solution to

$$\max_{x_E, h} \; F(h,6) - p_E x_E$$

subject to the early childhood human capital production (2)

$$h = h_E = h_B x_E^{\nu}.$$

The first order condition of this problem is $F_h(h_E, 6)\nu h_B x_E^{\nu-1} = p_E$. The costate variable, q_E, satisfies $q_E = F_h(h_E, 6)$. The relevant first order condition is

$$h_E = h_B^{\frac{1}{1-\nu}} \left(\frac{\nu q_E}{p_E} \right)^{\frac{\nu}{1-\nu}}. \tag{42}$$

Adding equation (42) to the first order conditions (40) and (41), we obtain the optimal age-six stock of human capital and years of schooling.

Distribution of Leisure over Lifetime We give conditions under which individuals decide to continuously participate in market work—that is, once they leave school they do not take a "vacation" until retirement. In order to simplify the algebra, we restrict the exposition to the case in which the worker's children have already left. The result with attached children is essentially identical owing to the envelope theorem, except for the adjustments for the consumption of the attached children.

To be precise, we show that it decrease utility to stop working for a period of time of arbitrarily small length. Consider a worker who decides to stop working for a period of length ε and then return to the labor market. His continuation utilty, for a retirement age R that is to be newly chosen, is

$$\frac{c(a)^{1-\theta}}{1-\theta}\widetilde{G}(a,\varepsilon,R).$$

The budget constraint is

$$c(a)\widetilde{G}(a,\varepsilon,R) \leq \widetilde{W}(h,a,\varepsilon,R) + A(a),$$

where $A(a)$ is the worker's asset holding, and

$$\begin{aligned}
\widetilde{G}(a,\varepsilon,R) &= \left(1-(1+\zeta)^{\frac{1-\theta}{\theta}}\right)\left(\frac{e^{-\upsilon(r)\varepsilon}-e^{-\upsilon(r)(R-a)}}{\upsilon(r)}\right)+(1+\zeta)^{\frac{1-\theta}{\theta}}\left(\frac{1-e^{-\upsilon(r)(T-a)}}{\upsilon(r)}\right),\\
\widetilde{W}(h,a,\varepsilon,R) &= \widetilde{V}(h,a,\varepsilon,R)\left(1+b_y\frac{e^{-r(R-I)}-e^{-r(T-I)}}{r}\right)\\
&\qquad\qquad\qquad\qquad+\left(\frac{e^{-r(R-a)}-e^{-r(T-a)}}{r}\right)\left[b_m+b_yW(0,a,R^*)\right].
\end{aligned}$$

Here, $\widetilde{V}(h,a,\varepsilon,R)$ is the present value of net labor income between age a and R of a worker who takes ε time off for leisure. It sastisfies

$$\widetilde{V}(h,a,\varepsilon,R) = e^{-r\varepsilon}V(he^{-\delta_h\varepsilon},a+\varepsilon;R),$$

where V is from equation (28) and R^* is the optimal choice of retirement when $\varepsilon=0$.

The relevant first-order conditions for an interior solution (which we assume) are

$$\begin{aligned}
c(a)^{-\theta} &= \lambda(a),\\
c(a)\frac{\theta}{\theta-1}\widetilde{G}_R(a,\varepsilon,R) &= \widetilde{W}_R(h,a,\varepsilon,R),
\end{aligned}$$

where $\lambda(a)$ is the Lagrange multiplier on the budget constraint. A standard application of the envelope theorem shows that taking time off reduces utility if and only if

$$\widetilde{W}_\varepsilon(h,a,\varepsilon,R) \leq \frac{\widetilde{G}_\varepsilon(a,\varepsilon,R)}{\widetilde{G}_R(a,\varepsilon,R)}\widetilde{W}_R(h,a,\varepsilon,R).$$

This is equivalent to, when evaluated at $\varepsilon = 0$,

$$\widetilde{W}_\varepsilon(h, a, \varepsilon, R) \leq -e^{\upsilon(r)(R-a)}\widetilde{W}_R(h, a, \varepsilon, R).$$

Computing the derivatives, we turn this condition into the following.

$$\begin{aligned}&\left(1 + b_y \frac{e^{-r(R-I)} - e^{-r(T-I)}}{r}\right)(1-\tau)w\left[\left(\frac{m'(a)}{r+\delta_h}\left(1 - e^{\upsilon(r)(R-a)}\right) - m(a)\right)h(a)\right.\\&\left.-C_h^{\frac{1}{1-\gamma}}\frac{1-\gamma}{\gamma_1}\left(m(a)^{\frac{1}{1-\gamma}} + e^{\upsilon(r)(R-a)}\int_a^R e^{-r(t-a)}\frac{1}{1-\gamma}m(t)^{\frac{\gamma}{1-\gamma}}m'(t)dt\right)\right]\\&\qquad\qquad\qquad\qquad\qquad\qquad\leq e^{(\upsilon(r)-r)(R-a)}\left(b_m + b_y W(R^*)\right)\end{aligned}$$

Thus, a stronger condition is that the left-hand side of the above condition is less than or equal to zero. Applying integration by parts and the bound from equation (31), $h(a) \geq C^{\frac{1}{1-\gamma}} m(a)^{\frac{1}{1-\gamma}}$, a sufficient condition is

$$\begin{aligned}&m(a)^{\frac{1}{1-\gamma}}\left[\gamma_1\left(e^{-\upsilon(r)(R-a)} - e^{-(r+\delta_h)(R-a)}\right) - (1-\gamma)\left(1 - e^{-\upsilon(r)(R-a)}\right)\right]\\&\qquad\qquad\qquad\qquad\qquad\qquad + r\int_a^R e^{-r(t-a)}m(t)^{\frac{1}{1-\gamma}}dt \geq 0,\end{aligned}$$

which is satisfied for all a and R with our parameter values.

References

Aguiar, M. and E. Hurst (2009): "Deconstructing Lifecycle Expenditure," Manuscript.

Barro, R. J. and G. S. Becker (1989): "Fertility Choice in a Model of Economic Growth," *Econometrica*, 57, 481–501.

Barro, R. J. and J.-W. Lee (2010): "A New Data Set of Educational Attainment in the World, 1950–2010," Working Paper 15902, National Bureau of Economic Research.

Bell, F. C. and M. L. Miller (2005): "Life Tables for theUnited States Social Security Area 1900–2100," Actuarial Publications 120, Social Security Administration.

Ben-Porath, Y. (1967): "The Production of Human Capital and the Life Cycle of Earnings," *Journal of Political Economy*, 75, 352–365.

Carter, S. B. and R. Sutch (1996): "Myth of the Industrial Scrap Heap: A Revisionist View of Turn-of-the-Century American Retirement," *The Journal of Economic History*, 56, 5–38.

Duval, R. (2004): "Retirement Behaviour in OECD Countries," *OECD Economic Studies*, 2003, 7–50.

French, E. (2005): "The Effects of Health, Wealth, and Wages on Labour Supply and Retirement Behaviour," *The Review of Economic Studies*, 72, 395–427.

French, E. and J. B. Jones (2011): "The Effects of Health Insurance and Self-Insurance on Retirement Behavior," *Econometrica*, 79, 693–732.

Gruber, J. and D. A. Wise (1999): *Social Security and Retirement Around the World*, Chicago: University of Chicago Press.

Imai, S. and M. P. Keane (2004): "Intertemporal Labor Supply and Human Capital Accumulation," *International Economic Review*, 45, 601–641.

Keane, M. and R. Rogerson (2010): "Reconciling Micro and Macro Labor Supply Elasticities," Manuscript.

Laitner, J. and D. Silverman (2008): "Consumption, Retirement and Social Security: Evaluating the Efficiency of Reform that Encourages Longer Careers," Manuscript.

Ljungqvist, L. and T. J. Sargent (2007): "Do Taxes Explain European Employment? Indivisible Labor, Human Capital, Lotteries, and Savings," in *NBER Macroeconomics Annual 2006*, ed. by D. Acemoglu, K. Rogoff, and M. Woodford, Cambridge, MA: MIT Press, vol. 21, 181–246.

——— (2010): "Career Length: Effects of Curvature of Earnings Profiles, Earnings Shocks, Taxes, and Social Security," Manuscript.

——— (2011): "A Labor Supply Elasticity Accord?" *American Economic Review*, 101, 487–491.

Manuelli, R. E. and A. Seshadri (2009): "Explaining International Fertility Differences," *The Quarterly Journal of Economics*, 124, 771 –807.

Neumark, D. and W. A. Stock (1999): "Age Discrimination Laws and Labor Market Efficiency," *Journal of Political Economy*, 107, 1081–1125.

OECD (2006): *Live Longer, Work Longer*, Paris: OECD Publishing.

——— (2010): *Taxing Wages 2010*, OECD Publishing.

——— (2011): *Pensions at a Glance 2011*, OECD Publishing.

Prescott, E. C. (2006): "Nobel Lecture: The Transformation of Macroeconomic Policy and Research," *Journal of Political Economy*, 114, 203–235.

Prescott, E. C., R. Rogerson, and J. Wallenius (2009): "Lifetime aggregate labor supply with endogenous workweek length," *Review of Economic Dynamics*, 12, 23–36.

Rogerson, R. (1988): "Indivisible labor, lotteries and equilibrium," *Journal of Monetary Economics*, 21, 3–16.

Rogerson, R. and J. Wallenius (2010): "Fixed Costs, Retirement and the Elasticity of Labor Supply," Manuscript.

Rosen, S. (1972): "Learning and Experience in the Labor Market," *The Journal of Human Resources*, 7, 326–342.

Rust, J. and C. Phelan (1997): "How Social Security and Medicare Affect Retirement Behavior In a World of Incomplete Markets," *Econometrica*, 65, 781–831.

Schulz, J. H. (2001): *The Economics of Aging*, Westport, Conn: Auburn House, 7th ed.

Turner, C., R. Tamura, S. E. Mulholland, and S. Baier (2007): "Education and income of the states of the United States: 18402000," *Journal of Economic Growth*, 12, 101–158.