eBay's Second-Chance Offer

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Abstract

We study the second-price offer feature of eBay auctions in the case where the seller has multiple units. The opportunity to make second-chance offers reduces seller revenue relative to sequential auctions for some regular environments. If the seller can commit to making fewer second-chance offers than she would in equilibrium, then the second-chance offer feature increases seller revenue. Uncertainty regarding the auction period hurts the seller. The seller is better off if she can credibly announce when she is offering her last unit.

VERY PRELIMINARY

1 Introduction

eBay has become one of the most important market places for retail goods worldwide. Yet important aspects of the strategic bidding incentives in eBay auctions remain unexplored. eBay's main sales mechanism is closely related to a second-price auction, which has been extensively studied. There are, however important difference to a standard second-price auction. In particular, the eBay auction allows the seller to sell multiple units of the same good.[1] While the seller is committed to sell the first unit to

[1] Another important difference is that eBay allows sequential bids. The sequentiality, however, does not play a crucial role in the private-value environments that we consider.

the highest bidder at essentially the second-highest bid, she also retains the option to make "second-chance offers". She may offer a second unit to the second-highest bidder at the second-highest bid, a third unit to the third-highest bidder at the third-highest bid, and so on.[2]

The opportunity to make second-chance offers raises a number of interesting questions. In particular, what is the impact of second-chance offers on the seller's revenue, the buyers' welfare, and on the aggregate welfare of all market participants? And: does the eBay mechanism have optimality properties as a multi-unit auction, parallel to the well-known optimality properties of the second-price auction mechanism in a single-unit world? Surprisingly, the existing literature has not addressed these questions. We can find only two papers which address the topic from a game-theoretic perspective, and both these works consider a generalized second-stage mechanism in which the seller is not restricted to charge the losing bidder's stage-one bid. Salmon and Wilson (2008) assume that the seller and losing bidder play an ultimatum game in stage two. They find that the only equilibria that exist are those in which stage-one bidders play mixed strategies. However, this result is of limited empirical relevance since it is driven by the assumption that there are two bidders and two items for sale. Joshi et al. (2005) allow the seller to freely price discriminate when making stage two offers. They assume the number of bidders is at least twice the number of items and propose an equilibrium in which bidders with sufficiently high value pool their bids at a market clearing price. The empirical relevance of these results is limited by the fact that they assume IID uniform values and a button auction in stage one with irreversible and publicly observed dropout bids.

Our goal is to stay as close as possible to eBay rules and environment. We consider a setting in which buyers have symmetric independent private values for a single unit.[3] The seller may have a second unit for sale, but this is unknown to the bidders. In a given auction, the seller may set a minimum bid, and decides about second-chance offers depending on the observed bids. We characterize a symmetric bidding equilibrium, taking into account the buyers' uncertainty about the seller's endowment (i.e., number of units available). The equilibrium bid function is strictly increasing in the buyer's willingness to pay (her "type"). We assume that the seller chooses an optimal threshold bid (i.e., minimal acceptable bid) for a second-chance offer, but that she cannot commit to a particular threshold by announcing it before the auction.

We begin with an observation that pertains to the case where the seller has a constant marginal cost function (i.e., each unit of the good has the same cost), that the buyers know the seller that has two units, and that the environment is regular in the sense of Myerson (1981). Here, the eBay auction with an optimal minimum bid maximizes the seller's expected profit among all (multi-unit) sales mechanisms. At the optimal minimum bid, a buyer's virtual valuation equals the seller's marginal cost. A second-chance offer is made whenever a second buyer has submitted an eligible bid, and is always accepted. While the seller-optimality of the resulting allocation is well-known (Maskin and Riley, 1989), we believe it has not been reported before in the context of

[2]eBay's description of the second-chance offer is available at http://pages.ebay.com/help/sell/second_chance_offer.html.

[3]The symmetry assumption fits well to the largely anonymous trading environment in which eBay auctions take place. Also, the assumption that each buyer demands at most one unit is reasonable in many contexts.

second-chance offers. The result provides an important benchmark. In particular, a different auction format where the seller is not bound to auction bids, but can freely choose second-chance offer prices (Joshi et al., 2005, Salmon and Wilson, 2008) will typically be harmful to the seller because of the resulting distortion of the allocation away from the optimal one.

Complications arise if the marginal cost function is increasing. In this case, the seller will make a second-chance offer only if the observed bid is higher than the marginal cost of the unit that is at stake. This obviously changes the buyers' incentives towards bidding more aggressively. In particular, a buyer type who obtains a second-chance offer with probability 0 will bid her value. This complicates the derivation of the seller's optimal minimum bid. Also, if the marginal cost function is increasing, then a seller-optimal sales mechanism would include the announcement of a minimum bid for selling the second unit (higher than the minimum bid for the first unit), another minimum bid for the third unit, and so on. This cannot be achieved via the eBay auction. Hence, the eBay auction will not be a seller-optimal sales mechanism anymore if the marginal cost function is increasing.

Another important direction of departure from the benchmark setting is to allow that the buyers may be uncertain about the seller's endowment. This uncertainty is important for the seller because the more likely buyers think it is that a second unit is available, the less they will bid in equilibrium.

The benchmark case refers to a static environment in which the seller keeps the units that she fails to sell through her auction. This fits the sale of perishable goods such as concert tickets. For the sale of durable goods, however, it makes more sense to consider a dynamic environment. We do so and make the simplifying assumption that the seller is much more patient than the buyers. If the seller fails to sell all her units, then she waits for some time, until a new set of buyers has arrived, and holds another auction. We acknowledge that in reality the future auction could contain some of the bidders from the original auction, but we do not address this complication. We contend that our approach is reasonable in a huge anonymous market place like eBay.

We study the dynamics in a two period model in which the seller initially has an endowment of one or two units, and in which she cannot use a minimum bid. We prohibit the seller from choosing a minimum bid in part because this is an empirically relevant case, but also because allowing for a minimum bid would generate an informed principal game à la Myerson (1983) in which the seller's choice of a minimum bid in the initial auction would signal information about her endowment.[4] We find that in regular environments, the seller would like to precommit to making fewer second-chance offers than she makes in equilibrium. In fact, for some well-known regular environments we find that, in the absence of commitment the seller would be better off if second-chance offers were excluded altogether. If the seller can precommit to making fewer second chance offers than she would in equilibrium, the seller is made better off by the second-chance offer feature.

[4]We do not have statistics on the prevalence of minimum bids. There is some evidence that high minimum bids discourage participation in auctions (see Bajari and Hortacsu, 2003) and this may influence their use by sellers. Also, eBay charges a small fee for placing a minimum bid which may discourage some sellers.

2 The static environment

Consider an environment with $n \geq 2$ buyers with symmetric independent private values, with distribution F with positive density f on $[0,1]$. Let X_i denote the random variable for buyer i's value.

The buyers participate in a variant of a second-price auction where, with some probability $\lambda > 0$, the seller has a second unit that she may offer at the second-highest bid to the second-highest bidder ("second-chance offer"). The seller can announce a minimum bid $r \geq 0$. The buyers expect the seller to make a second-chance offer if and only if the second-highest bid is not smaller than some number $b_0 \geq r$. Our analysis of the game will be based on the following equilibrium.

Proposition 1. *There exists a symmetric equilibrium. All types $x < r$ stay out of the auction. The equilibrium bid function $\beta : [r,1] \to [r,1]$ is strictly increasing. All types $x \in [r, b_0]$ bid their values $\beta(x) = x$, and, for all $x \in [b_0, 1]$,*

$$\begin{aligned} 0 &= (x-\beta(x))(n-1)F(x)^{n-3}f(x)(F(x)(1-\lambda)+\lambda(1-F(x))(n-2)) \\ &\quad -\lambda(n-1)(1-F(x))F(x)^{n-2}\beta'(x). \end{aligned} \tag{1}$$

All types $x \in (b_0, 1)$ submit bids below their values, $\beta(x) < x$.

Any second-chance offer is accepted.

Proof. Consider a buyer (say, buyer 1) of type $x \in [0,1]$ who believes that everybody else uses the strictly increasing and continuous bid function β with $\beta(r) = r$ and all types $< r$ staying out. Her expected payoff from bidding $b \in [r, \beta(1)]$ is

$$\begin{aligned} \Pi(b,x) &= E[\mathbf{1}_{\max_{i\neq 1}\beta(X_i)\leq b}(x - \max\{r, \max_{i\neq 1}\beta(X_i)\})] \\ &\quad +\lambda \mathbf{1}_{b\geq b_0}(x-b)(F^{1,n-1}(\beta^{-1}(b)) - F^{2,n-1}(\beta^{-1}(b))), \end{aligned}$$

where $F^{k,n-1}$ denotes the c.d.f. for the kth largest among $n-1$ values that are drawn i.i.d. according to F.

For all types $x \in [r, b_0]$ the expected payoff is maximized by value-bidding, for the same reason as in a standard second-price auction.

Consider then $x > b_0$. Any bid $b < b_0$ is suboptimal because $\Pi(b,x) < \Pi(b_0,x)$ for the same reason as in a standard second-price auction.

For any $b \in [b_0, \beta(1)]$, we can write the expected payoff as

$$\begin{aligned} \Pi(b,x) &= F^{1,n-1}(r)(x-r) + \int_r^{\beta^{-1}(b)} (x-\beta(y))\mathrm{d}F^{1,n-1}(y) \\ &\quad +\lambda(x-b)(F^{1,n-1}(\beta^{-1}(b)) - F^{2,n-1}(\beta^{-1}(b))) \\ &= F(r)^{n-1}(x-r) + \int_r^{\beta^{-1}(b)} (x-\beta(y))(n-1)F(y)^{n-2}f(y)\mathrm{d}y \\ &\quad +\lambda(x-b)(n-1)(1-F(\beta^{-1}(b)))F(\beta^{-1}(b))^{n-2}. \end{aligned}$$

The payoff change from a marginal bid increase is

$$\begin{aligned}\frac{\partial \Pi}{\partial b} &= (\beta^{-1})'(b)(x-b)(n-1)F(\beta^{-1}(b))^{n-2}f(\beta^{-1}(b))(1-\lambda)\\ &+\lambda(x-b)(n-1)(1-F(\beta^{-1}(b)))(n-2)F(\beta^{-1}(b))^{n-3}f(\beta^{-1}(b))(\beta^{-1})'(b)\\ &-\lambda(n-1)(1-F(\beta^{-1}(b)))F(\beta^{-1}(b))^{n-2}.\end{aligned}$$

Because this function is increasing in x, the same argument as for a standard first-price auction shows that Π is quasi-concave in b. Hence, to show the optimality of the bid $b=\beta(x)$, it is sufficient to verify the first-order condition

$$\begin{aligned}0 &= \left.\frac{\partial \Pi}{\partial b}\right|_{b=\beta(x)}\\ &= \frac{x-\beta(x)}{\beta'(x)}(n-1)F(x)^{n-3}f(x)(F(x)(1-\lambda)\\ &\quad +\lambda(1-F(x))(n-2))-\lambda(n-1)(1-F(x))F(x)^{n-2}.\end{aligned}$$

We have to solve the differential equation (1) for $x\in[b_0,1]$, with the boundary condition $\beta(b_0)=b_0$. Because the differential equation is linear in β and β', a unique solution exists.

We use the equation (1) in order to show that $\beta'(x)>0$ for all $x\in(b_0,1)$, implying that β is strictly increasing, thus justifying the use of the inverse above.

The differential equation (1) has the form $(x-\beta(x))h(x)=k(x)\beta'(x)$, where $h(x)>0$ and $k(x)>0$ for all $x\in[b_0,1)$.

Fix any $\overline{x}<1$. First we show that

$$\arg\min_{x\in[b_0,\overline{x}]} x-\beta(x) \quad = \quad \{b_0\}. \tag{2}$$

Suppose otherwise. Then there exists $y\in(b_0,\overline{x}]$ where $x-\beta(x)$ is minimized, implying $1-\beta'(y)\le 0$ by the standard first-order condition (we write "≤ 0" instead of "$=0$" to include the possibility of a minimum at the right boundary $\overline{x}$). Hence, $\beta'(y)>0$. Thus, using the differential equation, $(y-\beta(y))h(y)=k(y)\beta'(y)>0$, implying $y-\beta(y)>0$. Because y is a minimizer, we conclude that $b_0-\beta(b_0)\ge y-\beta(y)>0$, a contradiction.

From (2) and $b_0-\beta(b_0)=0$ it follows that $x-\beta(x)>0$ for all $x\in(b_0,1)$. Hence, $\beta'(x)>0$ by (1). *QED*

Because the bid function β is strictly increasing, we know the equilibrium allocation: one unit of the good is assigned to the buyer with the highest value, as along as this value is at least r; with probability λ, a second unit is assigned to the buyer with the second-highest value, provided that value is not lower than b_0.

Because the seller cannot commit to b_0, it is optimal for her to set b_0 equal to the maximum of r and her opportunity cost of selling the second unit.

Recall from Myerson (1981) that the environment is *regular* if the virtual valuation function $\psi(x)=x-(1-F(x))/f(x)$ is strictly increasing. The following result provides an important benchmark.

Corollary 1. *Suppose the seller has two units ($\lambda = 1$), constant marginal costs $c \in [0, 1]$ of selling each unit, and the environment is regular.*

Then the second-price auction with the minimum bid $r = \psi^{-1}(c)$ and the second-chance offer threshold $b_0 = r$ is an optimal mechanism for the seller.

To see this, recall that it is optimal for the seller to assign one unit to the buyer with the highest virtual valuation, and assign a second unit to the buyer with the second-highest virtual valuation, as long as these virtual valuations are not below c.

Elsewhere (Joshi et al., 2005, Salmon and Wilson, 2008) a different second-chance offer mechanism is analyzed, in which the seller is free to make any second-change offer, as a take-it-or-leave-it price, after seeing the second-highest bid. Under the assumptions of Corollary 1, this freedom to choose a price can never be advantageous for the seller. In fact, a buyer would get no rent from the trade of the second unit if her bid revealed her value; as a result, the buyer may randomize her bidding (cf. Salmon and Wilson, 2008), so that the resulting final allocation is distorted away from the seller-optimal allocation.

Corollary 1 does not apply to environments with increasing marginal costs. Here, the optimal allocation requires that a second-chance offer is made only if the second-highest virtual valuation is not below the cost of the second unit. Because the seller cannot commit to any particular threshold b_0 before the auction, the optimal allocation cannot generally be achieved by the eBay auction.[5]

We also observe that, the higher the probability buyers put on the event that the seller has two units, the less they will bid.

Corollary 2. *For all $x \in (b_0, 1)$, the equilibrium bid $\beta(x)$ is strictly decreasing in λ.*

Sketch of proof. To see this, consider $\lambda > 0$ and divide both sides of (1) by λ:

$$\begin{aligned}(n-1)(1-F(x))F(x)^{n-2}\beta'(x) \quad &= \\ &(x-\beta(x))(n-1)F(x)^{n-3}f(x)(F(x)(\frac{1}{\lambda}-1)+(1-F(x))(n-2))\end{aligned}$$

The right-hand side is decreasing in λ, and the left-hand-side is independent of λ. Hence, the larger λ, the smaller the slope prescribed by the differential equation, as was to be shown. *QED*

Corollary 2 shows that the seller wants the buyers to believe that she has only one unit. One way this may possibly be achieved is via reserve price signaling. However, the signaling raises serious coordination issues on the buyers' side. In fact, such issues may contribute to the evidently sparse use of minimum bids on eBay.

3 The dynamic environment

A disadvantage of the static model discussed so far is that the seller's cost of selling is exogenous. In a dynamic world, a major source of costs is the lack of opportunity

[5]Corollary 1 also does not apply if $\lambda < 1$. Here, the seller is privately informed about her endowment so that her optimal design problem becomes an informed-principal game; cf. Myerson, 1983.

to sell in the future under possibly more favorable market conditions. In this section, we focus on precisely these costs. In particular, costs become endogenous. Secondly, we endogenize the buyers' uncertainty about the seller's endowment. Assuming the seller set no minimum bid, our main question is whether the opportunity to make a second-chance offer increases the seller's total expected revenue.[6]

We consider a 2-period game where in a second period n new buyers arrive (each of them with an independent private value distributed according to F), and the first n buyers are not present anymore.

If the seller still has one unit of the good after period 1 is over, then she will sell this unit in period 2 in a second-price auction.

Suppose first that each buyer knows the number of the period in which she is playing. A buyer of type x who is playing in period 1 obtains a unit of the good with probability

$$q^1(x) \quad = \quad F(x)^{n-1} + \lambda \mathbf{1}_{x \geq b_0}(n-1)F^{n-2}(x)(1-F(x)).$$

If she uses the equilibrium strategy of type $\hat{x}$, then she obtains the payoff $q^1(\hat{x})x - t^1(\hat{x})$, where $t^1(\hat{x})$ denotes the equilibrium (expected) payment of type $\hat{x}$. By incentive compatibility, her payoff is maximized at $\hat{x} = x$. Hence, the envelope theorem in integral form (Milgrom and Segal, 2002) implies that the equilibrium payoff of type x is

$$U^1(x) \quad = \quad \int_0^x q^1(y)\mathrm{d}y.$$

Hence, the ex-ante expectation of a period-1 buyer's payoff is

$$\begin{aligned}
\int_0^1 U^1(x)f(x)\mathrm{d}x \quad &= \quad \int_0^1 \int_0^x F(y)^{n-1}\mathrm{d}y f(x)\mathrm{d}x \\
&\quad\; + \lambda \int_{b_0}^1 \int_{b_0}^x (n-1)F(y)^{n-2}(1-F(y))\mathrm{d}y f(x)\mathrm{d}x \\
&= \quad \int_0^1 (1-F(y))F(y)^{n-1}\mathrm{d}y + \lambda \int_{b_0}^1 (n-1)F(y)^{n-2}(1-F(y))^2\mathrm{d}y,
\end{aligned}$$

where we have exchanged the order of integration in both double integrals to obtain the last equality. Using the notation

$$f^{(1)}(x) = nF(x)^{n-1}f(x), \quad f^{(2)}(x) = n(n-1)F(x)^{n-2}(1-F(x))f(x)$$

for the densities of the highest and second-highest order statistics of n i.i.d. random variables, we can write the ex-ante expectation of the period-1 buyers' aggregate payoffs as

$$n \int_0^1 U^1(x)f(x)\mathrm{d}x \quad = \quad \int_0^1 \frac{1-F(y)}{f(y)} f^{(1)}(y)\mathrm{d}y + \lambda \int_{b_0}^1 \frac{1-F(y)}{f(y)} f^{(2)}(y)\mathrm{d}y,$$

[6] Joshi et al. (2005) also compare a second-chance offer mechanism to consecutive auctions, albeit in a different model where the seller is free to choose any second-chance offer price.

Similarly, the ex-ante expectation of the period-2 buyers' aggregate payoffs is

$$n \int_0^1 U^2(x) f(x) \mathrm{d}x \quad = \quad \lambda P(b_0) \int_0^1 \frac{1-F(y)}{f(y)} f^{(1)}(y) \mathrm{d}y,$$

where $P(b_0) = \int_0^{b_0} f^{(2)}(y)\mathrm{d}y$ denotes the probability that the second-highest value among period-1 buyers is too low for a second-chance offer.

The expected total welfare created by the 2-period game is

$$W(b_0) \quad = \quad (1 + \lambda P(b_0)) \int_0^1 y f^{(1)}(y) \mathrm{d}y + \lambda \int_{b_0}^1 y f^{(2)}(y) \mathrm{d}y.$$

Subtracting from this the buyers' aggregate payoffs, we obtain obtain the seller's expected revenue

$$R(b_0) \quad = \quad (1 + \lambda P(b_0)) \int_0^1 \psi(y) f^{(1)}(y) \mathrm{d}y + \lambda \int_{b_0}^1 \psi(y) f^{(2)}(y) \mathrm{d}y, \tag{3}$$

where we use the virtual valuation function

$$\psi(y) \quad = \quad y - \frac{1-F(y)}{f(y)}.$$

Also observe that, at the beginning of period 2, if the seller has 1 unit to sell, then her expected revenue is

$$R^2 \quad = \quad \int_0^1 \psi(y) f^{(1)}(y) \mathrm{d}y.$$

In equilibrium, the seller will offer the second unit in period 1 if and only if the second-highest value is not below R^2, that is, her equilibrium b_0 equals

$$b_0^* \quad = \quad R^2.$$

Recall that F is called *regular* (Myerson, 1981) if the function $\psi(\cdot)$ is strictly increasing.

Lemma 1. *If F is regular, then the function $R(\cdot)$ is strictly quasi-concave and its unique maximizer b_0^{**} satisfies $R^2 = \psi(b_0^{**})$.*

Proof. Compute

$$\frac{\mathrm{d}R}{\mathrm{d}b_0} \quad = \quad \lambda f^{(2)}(b_0) \int_0^1 \psi(y) f^{(1)}(y) \mathrm{d}y - \lambda \psi(b_0) f^{(2)}(b_0).$$

Hence, $\frac{\mathrm{d}R}{\mathrm{d}b_0} \gtrless 0$ if and only if $R^2 \gtrless \psi(b_0)$. *End.*

We call the unique maximizer of $R(\cdot)$ the seller's optimal precommitment threshold for second-chance offers. In regular environments, the seller would like to precommit to some second-chance offers, but not as many as she makes in equilibrium.

Lemma 2. *If F is regular, then the seller's optimal precommitment threshold for second-chance offers, b_0^{**}, is strictly between b_0^* and 1.*

Proof. Observe that $R^2 = b_0^* > \psi(b_0^*)$. Hence, $\frac{\mathrm{d}R}{\mathrm{d}b_0}(b_0^*) > 0$, implying that $b_0^{**} > b_0^*$.

On the other hand $R^2 < 1 = \psi(1)$. Hence, $\frac{\mathrm{d}R}{\mathrm{d}b_0}(1) < 0$, implying that $b_0^{**} < 1$. *End.*

Example 1. *Suppose that F is the uniform distribution. In equilibrium, $b_0 = b_0^* = (n-1)/(n+1)$. The seller's optimal precommitment threshold is $b_0^{**} = n/(n+1)$.*

Remark 2. *By very similar methods, it can be established that the welfare maximizing threshold for second-chance offers satisfies $b_0^W = \int_0^1 y f^{(1)}(y)dy$. Hence, if $\psi(\cdot)$ is strictly concave, then $\psi(b_0^W) > \int_0^1 \psi(y) f^{(1)}(y)dy = R^2$, implying $b_0^W > b_0^{**}$ for regular F. In these cases, according to the seller's optimal precommitment (and, hence, in equilibrium) inefficiently many second-chance offers are made. In the opposite cases where $\psi(\cdot)$ is strictly convex, inefficiently few second-chance offers are made according to the seller's optimal precommitment. If $\psi(\cdot)$ is linear (e.g., when F is uniform), then the precommitment level is efficient.*

Another interesting question is whether excluding second-chance offers increases seller revenue or improves efficiency. Without second-chance offers, the seller sells one unit in each period and obtains the revenue $(1+\lambda)R^2$. From (3), the expected revenue for the seller when second chance offers are allowed is

$$
\begin{aligned}
R(b_0) &= (1+\lambda P(b_0)) \int_0^1 \psi(y) f^{(1)}(y)\mathrm{d}y + \lambda \int_{b_0}^1 \psi(y) f^{(2)}(y)\mathrm{d}y \\
&= R^2 + \lambda P(b_0) R^2 + \lambda \int_{b_0}^1 \psi(y) f^{(2)}(y)\mathrm{d}y \\
&= R^2 + \lambda P(b_0) R^2 + \lambda \int_{b_0}^1 \left(y - \frac{1-F(y)}{f(y)} \right) f^{(2)}(y)\mathrm{d}y \\
&= R^2 + \lambda P(b_0) R^2 + \lambda \int_{b_0}^1 y f^{(2)}(y)\mathrm{d}y - \lambda \int_{b_0}^1 \frac{1-F(y)}{f(y)} f^{(2)}(y)\mathrm{d}y \\
&= R^2 + \lambda R^2 + \lambda P(b_0) R^2 - \lambda \int_0^{b_0} y f^{(2)}(y)\mathrm{d}y - \lambda \int_{b_0}^1 \frac{1-F(y)}{f(y)} f^{(2)}(y)\mathrm{d}y,
\end{aligned}
$$

where the last equality follows since $\int_{b_0}^1 y f^{(2)}(y)\mathrm{d}y = R^2 - \int_0^{b_0} y f^{(2)}(y)\mathrm{d}y$. Hence, the seller is better off when second chance offers are allowed provided

$$
\lambda P(b_0) R^2 - \lambda \int_0^{b_0} y f^{(2)}(y)\mathrm{d}y - \lambda \int_{b_0}^1 \frac{1-F(y)}{f(y)} f^{(2)}(y)\mathrm{d}y > 0. \tag{4}
$$

Note that if $\lambda = 0$ the second chance offer is irrelevant, and hence revenue with and without the second chance offer is the same. Moreover, for positive λ, the sign of (4) does not depend on the magnitude of λ and hence the revenue ranking does not depend on the (positive) likelihood of the the seller having the second good.

Starting from (4), add and subtract the expression

$$\int_{b_0}^{1} y f^{(2)}(y) dy.$$

Dropping λ this yields the expression

$$\begin{aligned} P(b_0)R^2 - \int_0^1 y f^{(2)}(y)\mathrm{d}y + \int_{b_0}^1 y - \frac{1-F(y)}{f(y)} f^{(2)}(y)\mathrm{d}y \\ = -R^2(1-P(b_0)) + \int_{b_0}^1 \psi(y) f^{(2)}(y)\mathrm{d}y. \end{aligned} \tag{5}$$

Hence, the seller is better off when second chance offers are allowed if and only if the expression in (5) is positive. Rearranging (5) we see that second chance offers increase seller revenue if and only if

$$\int_{b_0}^1 \psi(y) \frac{f^{(2)}(y)}{1-P(b_0)} \mathrm{d}y > R^2. \tag{6}$$

It is relatively easy to check condition (6) for given value distributions. For instance, if F is uniform or a power distribution, then the seller is worse off when second-chance offers are allowed. Both of these examples are regular distributions. We do not know whether regularity is a sufficient condition for the seller to be worse off when second-chance offers are allowed. Below we provide an example which shows that the seller may be better off with second chance offers in non-regular environments.

Example 3. *If F puts almost all weight on the points 0 and 1, then the seller is better off when second-chance offers are allowed.*

Proof. Since $\psi(1) = 1$,

$$\int_{b_0^*}^1 \psi(y) \frac{f^{(2)}(y)}{1-P(b_0)} \mathrm{d}y \approx \int_{b_0^*}^1 \frac{f^{(2)}(y)}{1-P(b_0)} \mathrm{d}y = \frac{1-P(b_0^*)}{1-P(b_0^*)} = 1.$$

Likewise,

$$R^2 = \int_0^1 y f^{(2)}(y)\mathrm{d}y \approx \int_{b_0^*}^1 f^{(2)}(y)\mathrm{d}y = 1 - P(b_0^*).$$

Hence, (6) is satisfied. *End.*

We conclude this section with a proposition which shows that the harm done to sellers by the introduction of second chance offers arises for a somewhat subtle reason. It is not that second chance offers are inherently bad in some regular environments. Rather, it is the seller's inability to precommit to a threshold that leads to the reduction in revenue.

Proposition 2. *If F is regular, then a seller who can precommit to a threshold for second-chance offers is better off when second-chance offers are allowed.*

Proof. From Lemma 1 we know that a seller who can precommit to a threshold for second-chance offers chooses the threshold b_0^{**} such that $R^2 = \psi(b_0^{**})$ and therefore, by regularity, $\psi(y) > R^2$ for $y \in (b_0^{**}, 1]$. Hence,

$$\int_{b_0^{**}}^{1} \psi(y) \frac{f^{(2)}(y)}{1 - P(b_0^{**})} \mathrm{d}y > R^2 \int_{b_0^{**}}^{1} \frac{f^{(2)}(y)}{1 - P(b_0^{**})} \mathrm{d}y = R^2,$$

which satisfies (6). *End.*

3.1 Uncertainty over the auction period

Another model variant worthy of investigation seems to be a model where each buyer is uncertain whether she is playing in period 1 or in period 2. Formally speaking, half of the buyers are each assigned the label "period 1" and each buyer from the other half is assigned the label "period 2". Each of the $\binom{2n}{n}$ possible labelings has the same probability, and no buyer observes the labeling.

In this model, the equilibrium bid function is derived as before, but λ must be replaced by the probability λ' (computed via Bayes rule from a buyer's point of view) that a second unit is available. Hence,

$$\lambda' \quad = \quad \frac{\lambda}{1 + P_0(b_0)\lambda}.$$

The seller's equilibrium threshold in this model, $b_0^{*'}$, satisfies

$$b_0^{*'} \quad < \quad b_0^{*}. \tag{7}$$

The reason is that now in period 2 all buyer types $x \in (b_0^{*'}, 1)$ bid below their values (whereas in the previous model all types bid their values), so that the seller's period-2 revenue—the expectation of the second-highest bid—is smaller than before.

However, for any given b_0, the seller's revenue is $R(b_0)$ just as in the previous model. This is because, for any of the $2n$ buyers, the probability that type x obtains a unit of the good is

$$q(x) \quad = \quad (1 + \lambda P(b_0))F(x)^{n-1} + \lambda \mathbf{1}_{x \geq b_0}(n-1)F^{n-2}(x)(1 - F(x)),$$

which is exactly the sum of a period-1 buyer's probability and a period-2 buyer's probability in the previous model. In other words, the envelope formula implies that the aggregate buyers' payoffs in the new model is exactly the same as in the old model. So is the aggregate welfare, implying that the seller's revenue is identical in both models for a given b_0.

Observe that in all cases with a regular F, Lemma 2 together with (7) implies that $b_0^{*'} < b_0^{*} < b_0^{**}$. Hence, $R(b_0^{*'}) < R(b_0^{*})$ by Lemma 1. Thus, we have established the following.

Proposition 3. *Suppose that F is regular. Then the seller is worse off in the model in which each buyer is uncertain about the period in which she is playing than in the model in which each buyer knows the period.*

4 Conclusion

Our investigation into eBay's second chance offer shows that the benefit to sellers depends crucially on the strategic response of bidders. If bidders are unaware that the second chance offer option is available to the seller or if they do not adjust their bidding, then second chance offers allow the seller to obtain all of the surplus from selling a second unit to a losing bidder. However, bidders who rationally respond to the potential for a second chance offer lower their bids (i.e., they do not value bid). This, of course, makes the second-chance offer option less attractive to sellers. We compare the equilibrium revenue obtained by the seller with an without the existence of second-chance offers, under the assumption that without second chance offers a seller with two units sells the second unit in a later auction to a new group of bidders. We find that the opportunity to make second-chance offers is revenue decreasing in examples of standard regular environments, however we do not know if this is true for all regular environments. We provide an example of a non-regular environment in which the second-chance offer opportunity increases seller revenue.

The reduction in seller revenue caused by the second-chance offer opportunity results, because eBay's second-chance offer rules give too much flexibility to the seller. If sellers could commit to use second-chance offers less frequently they they would in equilibrium, then second-chance offers are revenue increasing in all regular environments. In fact, we show that welfare maximizing threshold for second chance offers typically requires commitment by the seller; see Remark 2.

Our analysis includes environments where bidders are uncertain about the number of items the seller possesses and where bidders are uncertain about the auction period (i.e., whether the seller has offered the same item for sale before). We find that uncertainty regarding the auction period hurts the seller. The seller would be better off if she could credibly announce when she is offering her last unit.

Our analysis assumes the seller has at most two items. However, under eBay rules, second-chance offers can be extended for multiple additional items owned by the seller – one at the highest losing bid, the next at the second-highest losing bid and so on. We intend to extend our analysis to the case where the seller may have more than two units. In addition, we do not allow the seller to set a minimum bid in the dynamic version of our model. Exploration of minimum bids in the dynamic model is complicated because it sets up a signaling problem between the seller and the bidders. This is analogous to the case where the seller possesses private information about the good's quality which she may signal via her minimum bid requirement; see Cai et al. (2007). We will address this issue in future research.

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