Ascending Auctions with Costly Monitoring*

Joachim R. Groeger
Carnegie Mellon University

Robert A. Miller
Carnegie Mellon University

September 22, 2012

Abstract

This paper analyzes discriminatory ascending auctions in which bidders have limited opportunities to place bids throughout the auction and rival bidder activity is not continuously monitored. We characterize the theoretical properties of a Perfect Bayesian Equilibrium in which bids are increasing in valuations, analyze identification and develop a flexible estimation approach. Then we apply the framework to a financial market where state governments purchase savings vehicles by an auction mechanism. The data cover the period before and after the onset of the recession in 2008. This allows us to study the effects of the crisis on this local market.

1 Introduction

This is a theoretical analysis and empirical application of discriminatory ascending auctions with costly monitoring. In our framework there is a fixed finite time horizon for conducting the auction, and bidders have limited opportunities to place and update their bids. Bids cannot be withdrawn or reduced, and the next opportunity to revise old bids or place new ones is a random event whose probability distribution is determined by the bidder's monitoring choices. We characterize symmetric pure strategy perfect equilibria, analyze identification for a cross section of auction records on bidding, and develop an estimator for such models that accommodates unobserved heterogeneity including different equilibria being selected. The model is applied to a monthly financial market where the treasurers of state governments purchase from local banks savings vehicles, certificates of deposit, for unallocated state funds via a procurement auction mechanism. We use our estimates to explain how local financial markets reacted to the credit crisis of 2008, by focusing on changes before and after that time, and analyze the performance of this auction mechanism relative to a sealed bid format in order to assess the role of monitoring costs.

The main features of the data are described in the Section 2. The mechanism for the allocation of state funds is an open cry multiunit auction. Some bidders submit bids above the reserve price at the beginning of the half hour auction period, identical units are sold at different prices, entry occurs after the auction commences, and nobody knows whether bidders with stale orders (that is past bids now out of the money) have dropped out of the auction or not. These stylized facts cannot be reconciled to a Japanese auction, where the field of bidders is announced before the auction begins, and bidders publically withdraw as the auctioneer increases the price until the supply of

*PRELIMINARY AND INCOMPLETE. PLEASE DO NOT QUOTE WITHOUT PERMISSION. We would like to thank Timothy Derdenger for pointing us to this data, John Carver and Jon Smith at Grant Street for providing it, plus Quang Vuong and Isabelle Perrigne for their comments. We have benefited from presenting this paper at Pennsylavania State and Princeton Universities.

units matches the demand by the bidders who are left. In our data some bidding, and even some winning bids, are made well before the end of the auction, giving other bidders plenty of time to respond. The data suggest that bidding activity is not focused at the end of the auction; for example bids above the reserve price are sometimes submitted close to the beginning of an auction. Sniping, engaging in last minute bidding in the final instants of the auction so that no other bidders have time to revise their own bids in response, is not a common occurrence. These facts are inconsistent with the prediction that all bidders snipe in an electronic ascending auction which ends at a fixed known time. In our data bids tend to increase with the lowest price that is in the money, evidence against using a sealed bid auction to model our data. Yet our data also exhibits bids that jump in a discrete manner, in contrast to an English auction, where the ask price increases incrementally.

Section 3 develops a theoretical model estimated in the latter parts of the paper to explain and accommodate the peculiar features of our data described above. We treat the auction as a dynamic game where players with private valuations have limited opportunities to place new bids, update stale bids by reacting to the current lowest price in the money, and decide how closely to monitor proceedings.[1] The player's monitoring choice determines the rate at which opportunities to bid arrive throughout the auction game; a higher arrival rate is more costly to implement than a lower rate, and it can be revised with each bid update. In this way we capture alternative uses of the player's time, such as monitoring other securities the player trades, seeking new clients, or conducting administrative duties for his or her employer. The players' equilibrium choices of monitoring endogenize the intensity of bidding activity throughout the auction. At the beginning of the game, bidding is relatively frequent: until a player has made his first bid, the cost of losing an opportunity to bid is zero from the auction, and we show that everyone who has has an opportunity to make an initial bid will take it; anticipating the value of the initial bid players choose a relatively frequent monitoring. Having placed an initial bid, a player might not increase it at every opportunity he has, depending on whether it is still in the money. For this reason players might economize on monitoring costs after making their first bid. The rush of bids at the end, sniping, is a well understood response by bidders trying to prevent being pushed out of the money from rival bids that arrive later but still in time for the auction close. These collective choices are exacerbated by rational herd behavior. Since players have rational beliefs about their rivals, they reduce their monitoring during the the middle phase of the auction because fewer rival bids arise, compounding the reduced frequency of bidding, but in the last minutes of the auction, anticipating that others will snipe, they increase their monitoring as a defensive strategy, which in turn increases bidding activity.

Our empirical approach and implementation does not impose any parametric structure on the distribution of the valuations, that is apart from the assumption that opportunities arise to monitor follow an endogenously determined Poission arrival rate. Our analysis of identification, discussed in Section 5, is based on the equilibrium bidding behavior of each player and the frequency of bidding throughout the auction. We assume the data comprises numerous auctions that each bid in the data can be matched to the player making it. We begin our structural analysis by partitioning all bids into isolated points versus bids clustered just in the money. Our model implies that bids made on isolated points satisfy a first order condition, allowing us to identify the player's valuation if the equilibrium monitoring rate he chooses is identified.

We show that the highest bid ever observed by a group of players in different auctions that are

[1] Theoretical models that analyse jump bidding are, amongst others, Hörner and Sahuguet (2007) and Avery (1998). Birulin and Izmalkov (2011) consider English auctions with re-entry. There are a number of theoretical papers that deal with ascending bidding models that are not strategically equivalent to a Japanese auction. For example, Avery (1998) considers jump bidding models. Daniel and Hirshleifer (1998) consider auctions with costly sequential bidding. The latter shows that in the absence of bidding costs an equilibrium can exist where the transaction price is not equal to the second highest valuation. Birulin and Izmalkov (2011) considers English auctions with re-entry.

observed to make the same isolated bid in reponse to the same history up until that point, is a consistent estimate of their common valuation. Then we note that as soon as a player falls out of the money he will take the first opportunity to bid if at some later point in the auction he is observed to bid. This establishes a simple set of sufficient conditions in which a bidder would bid if he or she could do so, thus identifying the monitoring technology selected as a function of the history of the auction (including the remaining amount of time to bid) and the previous bid which is a sufficient statistic for the player's valuation if the bid satisfied a first order condition. Having identified both the valuation and the choice of monitoring rate for the set of bids that satisfy a first order condition, we now apply the first order conditions for the monitoring choices to identify the marginal cost of increasing monitoring at the the player's equilibrium choice. Since there is a strictly positive probability that every player will bid first, and therefore satisfy a first order condition, and that every player will subsequently fall out of the money. Thus even though the first order condition does not necessarily hold at all points in the auction, it certainly holds at the beginning of the auction, at which point anyone with a valuation above the auctioneer's reservation price, would enter a bid. Together with our estimates of the monitoring technology that lead to the first bid we finally identify the distribution of valuations.

We develop a flexible estimation approach that avoids a number of assumptions on strategic behavior, aside from a monotonicity restriction on strategies. Estimation follows the spirit of indirect estimation strategies such as Hotz and Miller (1993), Guerre, Perrigne, and Vuong (2000) and Manski (1993). The literature on the structural estimation of auctions was initiated by the work of Donald and Paarsch (1993), Elyakime, Laffont, Loisel, and Vuong (1994) and Guerre, Perrigne, and Vuong (2000). Donald and Paarsch (1993) estimate auction models, where the underlying valuation distribution is parameterized and bidding strategies must be computed. Guerre et al. (2000) consider two step indirect non-parametric estimators that can avoid the computation of strategies and the specification of underlying valuation distributions. The estimation of dynamic auctions is considered in Jofre-Bonet and Pesendorfer (2003) and Groeger (2010). These papers consider repeated first price auctions in dynamic environments and extend Guerre et al. (2000). Jofre-Bonet and Pesendorfer (2003) consider the effect of capacity constraints on bidding. Groeger (2010) studies repeated first price auctions with endogenous participation and synergies in participation. Our paper differs from the aforementioned papers in that on the one hand we consider the auction as a dynamic game, but on the other hand we exclude dynamic linkages between auctions. The presence of multiple units and the open cry format makes this market differ from standard ascending auction models and existing multiunit auction mechanisms in the literature. The literature on the empirical analysis of ascending auctions has mainly made use of the Japanese button auction model to estimate underlying valuations. In this setting the transaction price is the second highest valuation. One exception is Haile and Tamer (2003) who attempt to construct an incomplete model of ascending auctions that allows for a number of different equilibrium outcomes. The authors are able to provide informative bounds on the valuation distribution.

Our estimates are presented in Section 6. Our estimates allow us to estimate the effects of the late 2000 recession. Moreover, given that this is a financial market, we would also like to understand how the financial crisis and recession in the late 2000s affected this market. Section 8 concludes.

2 Auctioning Certificates of Deposit

A Certificate of Deposit (CD) is similar to a standard savings account. The key difference is that a CD has a specific, fixed term, usually three, six or twelve months. At maturity the money may be withdrawn together with the accrued interest. Our application analyzes CD auctions in Texas. Prior to entering a CD auction a bank must undergo a pre-qualification process. During this process the level of collateral a bank holds is ascertained. Texas limits participation to local banks to ensure that tax money does not leave the state and can be used to inject liquidity in the local economy. Participating banks compete to sell these savings vehicles to the state treasurer through an on line auction. The money sold in these auctions from state funds have no immediate purpose.[2]

This section describes the auction mechanism in our application, summarizes the main features of the bidding, and thus demonstrates why and how existing auction models should be modified to accommodate the stylized facts that characterize this auction. The overview of the institutional setting provides some basic descriptive statistics of the auctions along with evidence that these auctions cannot be treated as Japanese or sealed bid auctions. We show that bidders in these auctions submit multiple bids at different points of the auction with some bidders submitting bids early in the game. We find that a number of winning bids are submitted early on in the auction and that sniping is not the only type of play observed. Bidding activity is focused at the beginning and the end of the auction, with less activity in the middle. The spread between interest rates in the money is significant. Behaviour in these auctions differs quite markedly from other online auction platforms such as eBay, where sniping is common.

2.1 Auction format

Auctions take place at the beginning of each month, usually the first week. The orders are cleared by ranking each submission solely by the level of the interest rate. The pot is then allocated to the highest interest rates until there is no money left. An order that is at the lowest interest in the money, might be broken up and partially filled. For example, if there is $500,000 to be allocated and the last order in the money is for $1 million, only $500,000 of that order will be filled and the other half of the order falls out the money. The key features of the auction are the following:

- Before the auction begins, a minimal bid is determined by the the state government announcing a reserve interest rate.
- Bidding takes place over 30 minutes.
- Banks can submit up to five separate bids at different interest rates for different dollar amounts.
- Banks can submit a minimum bid of $100,000 and a maximum of $7million in Texas, with minimum bid increment of $100,000. The maximum quantity of dollars a bank can bid on is determined by the collateral available to that bank.
- A bid can be updated as many times as the bank wants.
- At the time its bids, a bank observes whether its bid is in the money or not.
- When the auction ends, the funds are allocated to those banks whose bids are in the money at that time.
- Winning banks pay the interest rate they bid, while losing banks pay nothing. In other words, the auction is a discriminatory multiunit auction.

[2] There are a number of other states that use the same online auction platform, for example Idaho, Iowa, Louisiana, Massachusetts, New Hampshire, Ohio, Pennsylvania and South Carolina. We focus on Texas since these are the most active states

2.2 Auction size and Bidding Patterns

Our data set contains 140 CD auctions in Texas for the years 2006 - 2010. The majority of auctions involve six month CDs. There are five auctions in our data set that are for 12 month CDs. The amount to be auctioned is on average $76 million. The majority of auctions were for $80million. Eleven auctions were for $50 million. Various dimensions of the size of the auction are displayed in Table 1. There is a potential pool of 53 bidders, and an average of 25 bidders enter each auction, with an average order comprising 1.6 parcels per auction. The table below highlights that bidders do not alter the dollar amount they bid on throughout the auction. This can be seen by the row labelled, "Number of Quantity Changes", in the table. On average a bidder submits only a parcel of one size throughout the auction. On average, 76 percent of bidders end up winning, the total award from each auction being $75 million.

Table 1: Summary Statistics on Size of Auctions

	Mean	**Standard Deviation**	**Min**	**Max**
Number of Bidders	24.5000	6.4983	9.0000	41.0000
Number of Bids (for each bidder)	8.0062	13.3484	1.0000	139.0000
Prop. of Submitted Bids In The Money	0.7405	0.3238	0.0000	1.0000
Prop. of Submitted Bids Out of The Money	0.1624	0.2810	0.0000	1.0000
Prop. of Submitted Bids On The Money	0.0971	0.1995	0.0000	1.0000
Prop. of Submitted Bids with Quantity Changes	0.0029	0.0275	0.0000	0.5000
Number of Parcels	1.6007	1.0264	1.0000	5.0000
Winners (as fraction of bidders)	0.7673	0.1806	0.4137	1.0000
Annual Reserve Coupon Rate	2.1928	2.1072	0.0600	5.3000
Total Award Amount (millions)	75.7692	10.5090	50.0000	80.0000
Award Amount to Winning Bank (millions)	4.1068	2.7458	0.1000	7.0000

The table shows that on average 74 percent of the bids are in the money, in other words so high that at least one previous lower bid would win it the auction were to end immediately. Conversely 16 percent of the bids are out of the money; nothing is gained from making such a bid aside from gaining information about the lowest bid that would win if the auction were to end immediately. This third category of bids, accounting for the remaining 9.7 percent, are called bids on the money, and would be the only type of bid observed in an English auction. Figures 1 through 3 amplify the importance of these three bidding categories by providing some simple dynamics. Figure 1 plots the lowest bid in the money against each current bid. The first one shows that right throughout the auction, as measured by the distance from the reserve which increase with time, there are many bids in all three categories.

To help explain why we observe all three categories of bidding throughout the auction, we separately investigated those bids on the money and those that are in it. Figure 2 shows that the preponderance of bids submitted on the money were preceded by a bid that was quite close.

Figure 3 shows that relatively few such bids are preceded by a submission that is out of the money by very much, and indeed the previous bids tend to be clustered around the on the money bid. Moreover many of the bids that are not within the cluster are submitted by banks who are simultaneously bidding on another parcel which is in the money. These bids are labelled as circles in Figure 3.

Figure 1: Submitted bid against lowest ITM bid

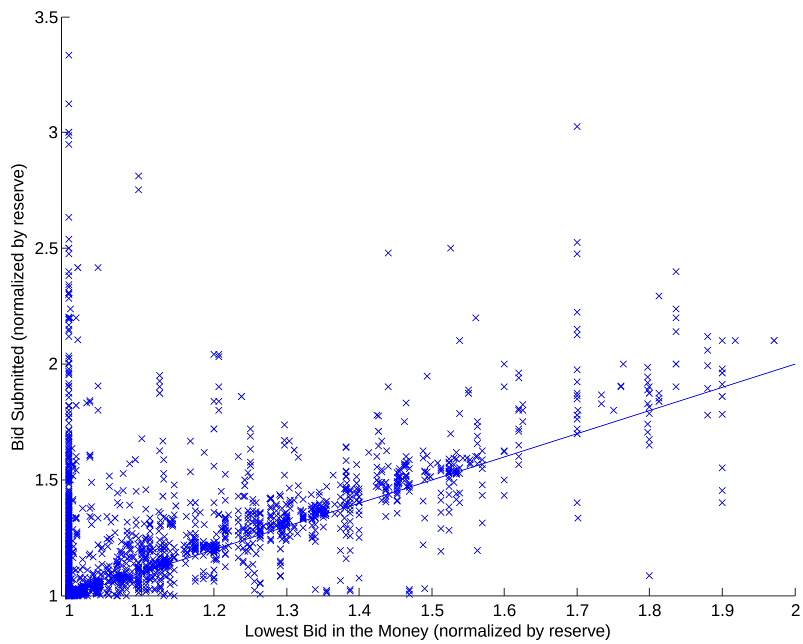


Figure 2: On the Money Bids versus Previously Submitted Bids

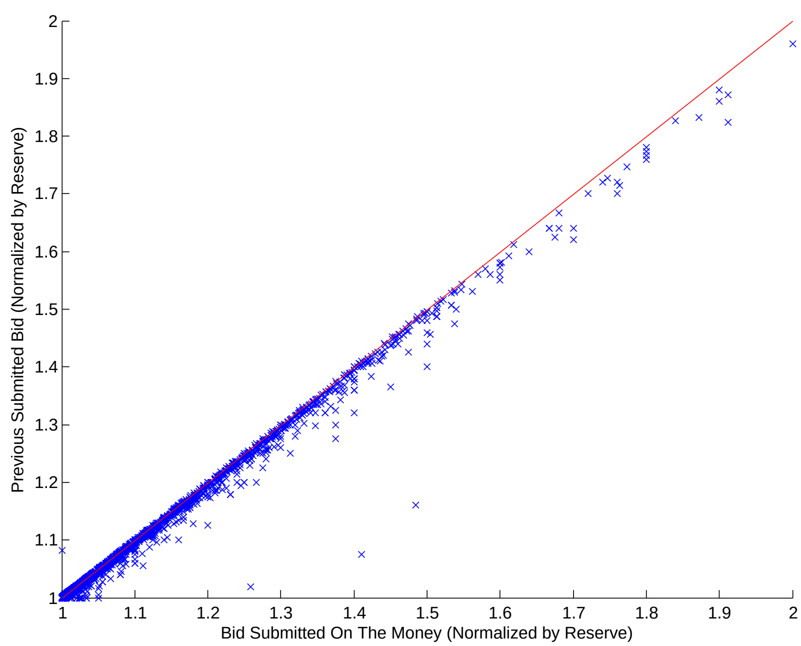
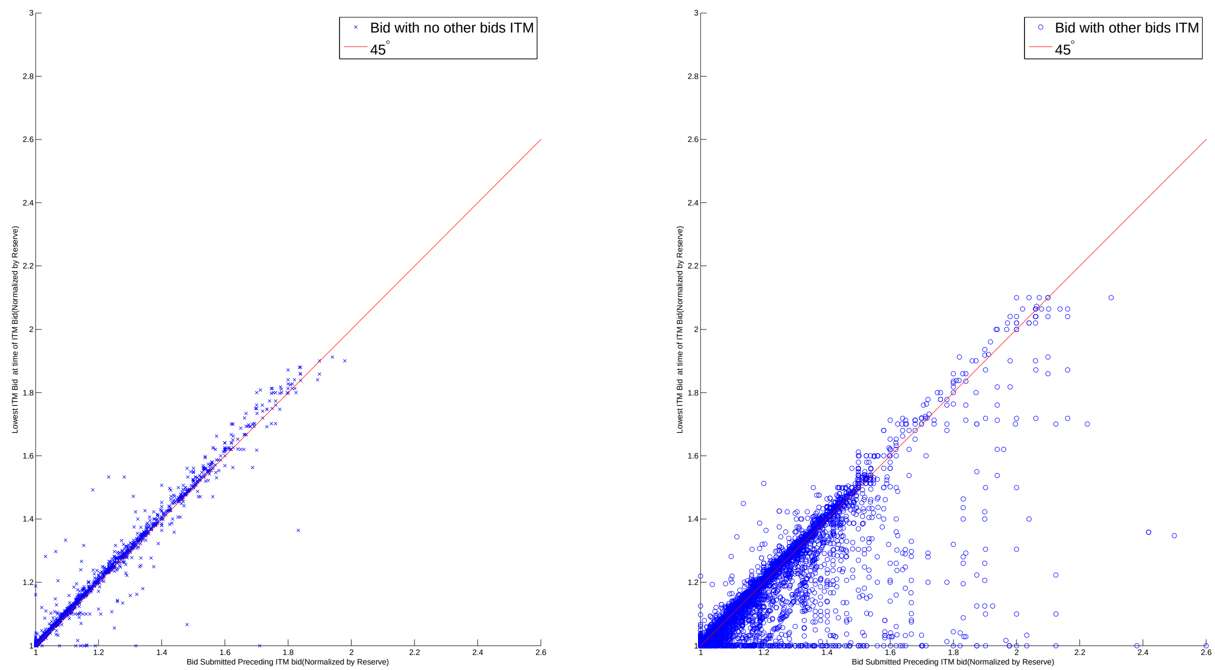


Figure 3: In the Money Bids versus Previously Submitted Bids



Taken together the figures illustrate a pronounced tendency for those bidders who ultimately place a bid that is on or in the money to first determine where the boundary is by creeping to it with incremental bids, and only then deciding how much to jump into the money, if at all. Given this tendency, and recognizing that making increasing bids are not costly, we treat out of the money bids as simply a procedure for determining the boundary, and assume that whenever a person bids in the money, she knows the current on the money bid.

2.3 Interest Rate Spread

The following descriptive statistics provide an insight into the amount of uncertainty in the bidding game. Low interest rate spreads would indicate that bidders have a clear picture of their competitor's behaviour when submitting bids.

We first consider the interest rate spread between the winning interest rates and highest losing interest rate (when all bids are in the money we use the lowest winning rate). We compute this by taking the difference between the highest winning interest rate and highest losing interest rate, normalized by the highest winning rate. This statistic provides a measure of the level of uncertainty bidders face during the course of the auction. Given the open nature of bidding, i.e. all bids are visible at all times, it is surprising to see that there is still substantial money left on the table. With an average of 21 percent difference between the highest and lowest winning interest rate. Table 2 summarises the results in terms of dollar amounts. We compute these statistics by taking the interest a bidder had to pay on their winning bid and comparing this to the interest they could have paid had they submitted the highest bid that was out of the money. We then average across all winning bidders and then across auctions. On average we find that the MLT is $1,107, as shown in Table 2.

The interest rate spread observed at the end of the auctions could not occur unless some bidders

Table 2: Summary Statistics on Money Left on the Table

	Pre and Post $	Pre $	Pos $	Pre and Post Rate
Mean	1790.85	1097.89	2322.52	0.21
Standard Deviation	6522.17	7664.25	5431.26	0.20
Min	0.00	0.00	0.00	0.01
Max	210000.00	210000.00	101617.50	0.91
Number of Observations	1921.00	834.00	1087.00	78.00

engage in jump bidding by submitting offers that exceed the level necessary to stay in the money. In Table 3 we summarise statistics on the amount of jump bidding. To be precise we compute the percentage difference between the currently submitted bid and the current lowest bid in the money for every bid submission. We then average across all bidders and all auctions.

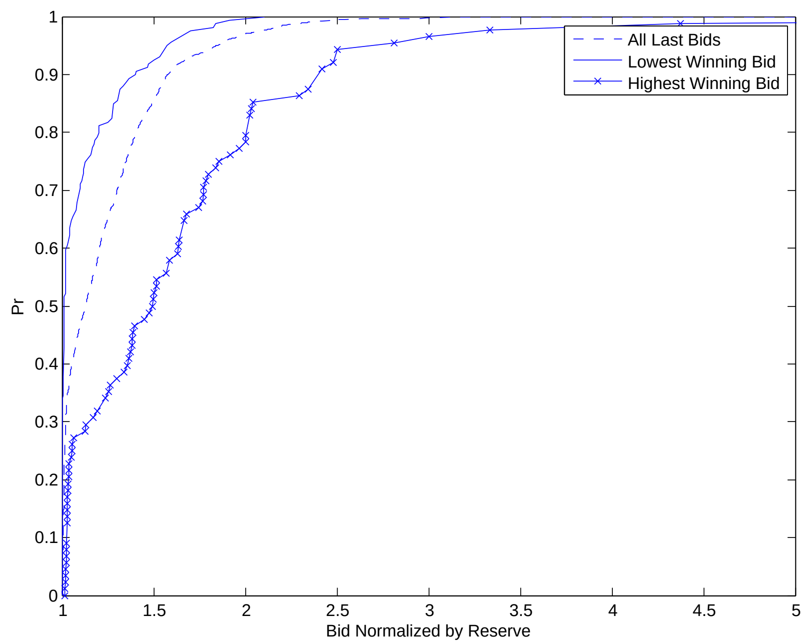
Table 3: Summary Statistics on Jump Bids

	$\frac{\textbf{Current Bid}}{\textbf{Lowest ITM Bid}} - 1$
Mean	0.0439
Standard Deviation	0.1987
Min	0.0000
Max	11.6957

Because bidders leave the money on the table, these auctions do not seem to belong to the class of auctions Haile and Tamer (2003)(HT) investigate. Their analysis derives bounds on the probability distribution of valuations under two plausible assumptions. First, a bidder never bids above his valuation. Second, rivals do not win auctions at prices against opponents who are willing to raise the price. Together these assumptions imply the valuations of all losing bidders are less than the lowest winning bid in private value auctions; the auction is an efficient allocation mechanism. Under these assumptions no bidder has no incentive to jump bid, because winning the auction is guaranteed by keeping abreast of the lowest bid in the money. Therefore the distribution of highest winning bids coincides with the distribution of the lowest winning bids in their framework. Moreover the distribution of all final bids (both winning and losing) are first order dominated by the distribution of (lowest) winning bids. In Figure 4 we plot the empirical analogues of these bidding distributions for our setting.

Neither prediction is validated by the auctions we investigate. Since individual rationality in a private value setting ensures their first assumption holds, their second assumption looks more suspect. If monitoring costs were important, the second assumption might not hold because bidders might economize on opportunities to bid by placing bids above the lowest amount necessary to stay in the money. Note too that if the second assumption is violated, the auction might be inefficient in the first best sense described above. Summarizing, there is substancial evidence against the view that the auctions we investigate resemble Japanese or English auctions.

Figure 4: Bid Distributions

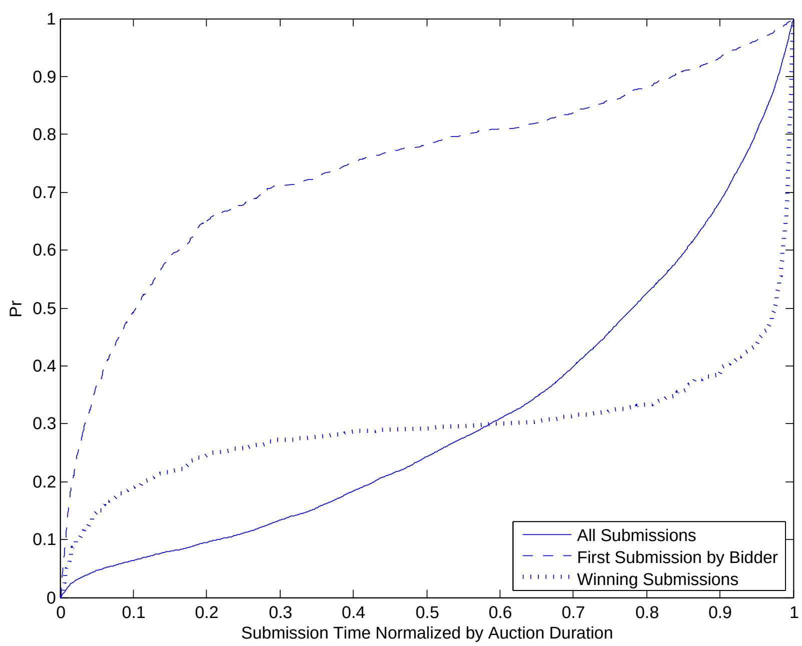


2.4 Submission Times

The data on submission times is useful for bidders observe can react therefore not simultaneous In Figure 5 we provide the empirical distribution, that is averaged across all auctions, of all submission times (for the entire thirty minute period): for all bids; for all winning bids; and all first bids. Most first bids are made before the five minute mark, and the convex shape of the distribution of all bids betrays less activity in the middle of the auction than at either end. The winning bid distribution climbs to the 20th percentile within 6 minutes of the auction beginning but flattens between 9th and of 24th minute.

The figure shows that most winning bids have already been submitted by the 25 minute mark. Noting that bidders see all previous submissions when they bid and only have to beat the lowest bid in the money to win, we infer that sniping does not predominate bidding. The fact that a winning bid is frequently submitted in the early stages of the auctions provides further dramatic evidence that some bidders do not win by incrementally increasing their bids as necessary but enter with a jump.

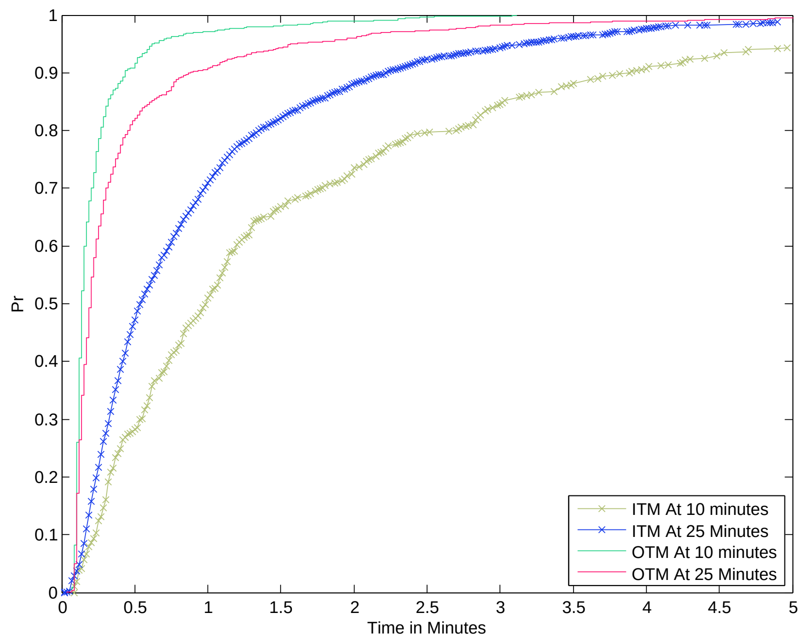
Figure 5: Empirical Distribution of Order Submission Times Normalized by Auction Duration



How quickly bidders respond to rivals by updating their previous bids reflects how intensively they monitor the auction. However the average duration time between submissions by the same bidder is a biased estimators of the expected length of durations between successive monitoring opportunities, because bidders do not necessarily update their bids at every opportunity. But if a bidder was pushed out of the money, had limited bidding opportunities to revise his previous bid, and a sufficiently high valuation, he might increase his offer at the first available opportunity. That motivates why we display duration times between a bidder being pushed out of the money to when an updated bid restores him into the money relative to all bidders who fall out of the money.

Figure 6 shows three empirical distributions of reaction times. For every auction at 10, 15 and 25 minute mark into the bidding, we select all bidders who fall out of the money at some point within the preceding five minute window. We then form percentiles measuring the proportion of those bidders who re-submit a new bid in the money within a given amount of time up to five minutes.

Figure 6: Empirical Distribution of Reaction Times



This measure of monitoring intensity does not account for bidders who withdraw from the auction because the lowest bids in the money have overtaken their gross value from winning since their previous bids were placed, a factor that shifts all three curves to the left. Nevertheless it is noteworthy that the distribution of reaction times at earlier points in the auction first order stochastically dominate the distribution of reaction times at later points in the auction, suggesting that bidders increase their monitoring as the auction proceeds. Reviewing the data on submission times, it is implausible to argue that the final bid distribution in these auctions matches those we would observe in a discriminatory first price sealed bid auction.

3 Model

The model we construct to rationalize the facts described above is a dynamic bidding game between bidders who have imperfect monitoring capabilities. Because the monitoring of bids by rival players demands attention, given limited powers of concentration, each player faces a trade off between devoting exclusive attention to a particular auction and reacting immediately to changes in the bidding history of that security, versus bidding or trading on a larger number of financial securities markets and consequently being less responsive to changes in bidding or trading patterns. In addition there may be other tasks the player undertakes for his employer to draw his attention away from the bidding updates placed by rival players. Monitoring opportunities are modeled as random events, and the shadow value of alternative activities as a cost of increasing the arrival rate of monitoring opportunities. Denoting by λ the parameter of the Poisson distribution for the next bidding opportunity, we let $g(\lambda)$ signify the cost of monitoring with intensity λ until that time. The arrival rate of the next bidding opportunity is reset every time the player bids.

At the beginning of the bidding game, the auctioneer announces the reserve coupon rate. Players then observe their private values and the number of units to be bid on. Then prior to the beginning of the auction they individually and privately determine at a cost their monitoring intensities, which stochastically determine when they have their first bidding opportunity. Once the auction starts, when a player receives his first bidding opportunity, he observes the entire path of play up to the current time period and submits a bid and determines the intensity of arrival of future monitoring opportunities. The bidder also has the option at any time to pass and not to increase their current bid. The number and identity of active bidders is never fully observed by the players.

- *Time* is continuous on the interval $[0, T]$.

- *Units to be awarded:* The maximal total amount to be awarded is denoted by Q.

- *Reserve Price:* A reservation price denoted by R is announced prior to bidding.

- *Number of Bidders:* The players in the game are a set of potential bidders denoted $\mathcal{I} \equiv \{1, ..., I\}$.

- *Value and Size of Bidding Parcels:* Player $i \in \mathcal{I}$ draws a single indivisible parcel of exogenous size, denoted by $q_i \in \mathbf{Q} \equiv [0, Q]$, along with a private value $v_i \in \mathbf{V} \equiv [R, \infty)$, independently and identically from commonly known distribution $F_{q,v}(.)$ prior to the beginning of the auction at time $t = 0$. In an appendix we extend our analysis by allowing players to make a fixed number of draws from $F_{q,v}(.)$ which they may make separate bids on.

- *Action Space:* Letting $s \in \{1, 2, \ldots, \rho_i\}$, at the random time $\tau_{is} \in (\tau_{i,s-1}, T]$ player $i \in \mathcal{I}$ submits or updates his bid, denoted by $b_{is} \in \mathbf{B}_s \equiv [b_{is-1}, \infty)$, and selects a monitoring intensity, denoted by $\lambda_{is} \in \Lambda \equiv [0, \infty)$, where λ_{is} is the poisson parameter which determines the distribution of the $(s+1)^{th}$ monitoring time.

- *Monitoring Costs:* The cost of choosing monitoring intensity λ_{is} at time τ_{is} is defined on Λ by a convex increasing positive real valued mapping $g(\cdot)$ with $g(0) = g'(0) = 0$.

- *Information:* The information set of player i at time t is his valuation and bid parcel size (v_i, q_i), his sequence of past bidding times $\{\tau_{i1}, \ldots, \tau_{i,s-1}\}$, his previous bids $\{b_{i1}, \ldots, b_{i,s-1}\}$, and the on-the-money at those times $\{r_{i1}, \ldots, r_{i,s-1}\}$. We consolidate the information acquired during the bidding process by denoting the information of player i at his s^{th} decision node as $h_{is} \equiv (v_i, q_i, h'_{is})$ where $h'_{is} \equiv \{\tau_{is'}, b_{is'}, r_{is'}\}_{s'=1}^{s-1} \in \mathbf{H}_s$.

- *Awarding and Payment Rules:* The auction is similar to a multiunit discriminatory auction. At time T the auction ends and up to Q units are allocated, those bidding the highest prices winning the auction. Winning bids will be called in-the-money (ITM) and losing will be called out-of-the-money (OTM). If bidders tie with the same price then earlier bids receive higher priority. A function denoted by $M(h_T, b_{i\tau})$ equals 1 if the bid is ITM and 0 otherwise.

- *Expected Payoffs* for bidder i are given by:

$$E\left[q_i (v_i - b_{i\tau}) M(h_T, b_{i\tau}) - \sum_{s=1}^{\rho_i} g(\lambda_{i\tau_s}) \,|q_i, v_i\right] \tag{1}$$

  where $b_{i\tau}$ is the last bid submitted by bidder i. The expectation is taken over the actual number of competitors, his future monitoring opportunities, the future path of play and the transaction price.

- *A Strategy* for player i at opportunity s consists of a bidding and monitoring strategy and is denoted by $\sigma_{is} = (b^{\sigma}_{is}, \lambda^{\sigma}_{is})$, where b^{σ}_{is} is a mapping defined by $b^{\sigma}_{is} : \mathbf{V} \times \ \mathbf{Q} \times \mathbf{H}_s \rightarrow \mathbf{B}_s$,and λ^{σ}_{is} is a mapping defined by $\lambda^{\sigma}_{is} : \mathbf{V} \times \ \mathbf{H}_s \rightarrow \Lambda$. We restrict attention to bidding strategies that are monotone increasing in valuations.

4 Equilibrium

The existence of a symmetric Perfect Bayesian Equilibrium with monotone weakly increasing bidding strategies can be established by extending the analyses of Reny (2011) and Athey (2001). This section describes its main features. Summarizing the analysis in this section, in equilibrium players with valuations that dominate bids ITM submit bids on projects at their first opportunity. Depending on his valuation and the existing offers ITM, a player might enter a bid at an isolated point or cluster on-the-money. In the first case the player's valuation is revealed to the remaining players throught the first order condition he solves. In the second case, the remaining player's update their posterior distribution of the bidder's valuation by truncating the unconditional distribution from which project valuations are drawn. A first order condition also characterizes the choice of monitoring.

Comparing the equilibrium properties with a sealed bid discriminatory auctions, one obvious difference is that in our framework players choose how often to monitor the book and update their bids. A second difference is reflected in the necessary first order conditions for bids satisfying necessary conditions for an interior solution, which must reflect the fact that their current bid may not be their last. Third, players in this auction might optimally locate on-the-money rather solve a first order condition. We show that players whose valuations differ may settle on the same price point, implying that they only partially reveal their valuations through their bidding.

4.1 Optimization problem in equilibrium

To reduce notational clutter we now drop i subscripts, for example setting $\tau_{is} = \tau_s$. We show the bidder optimally locates at a price point by adding his bid to other bids at that precise price, or by solving a first order condition. Let v denote the expected value of a unit of the object at the end of the auction.

Suppose that at the time the bidder has his j^{th} opportunity to bid, the bidding history is h_j. We denote his previous bid by b_{j-1} where $b_0 \equiv 0$ and constrain bids to be nondecreasing. We also denote by h_T the entire bidding history of the game and denote by $\tau \in [0, T]$ the last occasion the player bids, denoting his final, highest bid by b_τ. The indicator function $M\left(h_T, b_\tau\right)$ is defined by setting $M\left(h_T, b_\tau\right) = 1$ if the bidder is successful and earns $(v - b_\tau)$ and setting $M\left(h_T, b_\tau\right) = 0$ if his final (and highest) bid is too low to be in the money. At his j^{th} opportunity to bid, the player chooses $b_j \geq b_{j-1}$, his bid, and λ_j the intensity with which he monitors the auction book until he bids again (if ever), to sequentially maximize:

$$E\left[(v - b_\tau) M\left(h_T, b_\tau\right) - \sum_{k=j}^{\tau} g(\lambda_k) \middle| h_j \right] \tag{2}$$

where the expectation is taken over b_τ, his final bid, h_T, the entire bidding history, current plus expected future monitoring costs incurred.

With probability $\left[1 - e^{-\lambda_j(T-\tau_j)}\right]$ his bid at τ_j is his final opportunity, in which case b_j and b_τ coincide and the only monitoring cost incurred is $g(\lambda_j)$. Otherwise he will be able to raise his bid at least one more time and further adjustments in monitoring might also take place. To recognize these two distinct possibilities, we rewrite (2) as:

$$\begin{aligned} & e^{-\lambda_j(T-\tau_j)}(v-b_j)E_{t0}\left[M\left(h_T,b_j\right)\right]-g(\lambda_j) \\ & +\left[1-e^{-\lambda_j(T-\tau_j)}\right]E_{t1}\left[(v-b_\tau)M\left(h_T,b_\tau\right)-\sum_{k=j+1}^{\tau}g(\lambda_k)\right] \end{aligned} \quad (3)$$

where:

$$E_{t0}\left[\cdot\right]\equiv E\left[\cdot\left|h_j,b_j,\lambda_t \text{ and no further bidding opportunity}\right.\right]$$

and:

$$E_{t1}\left[\cdot\right]\equiv E\left[\cdot\left|h_j,b_j,\lambda_t \text{ with further bidding opportunity}\right.\right]$$

Let us now consider the initial choice over monitoring intensity. After the announcement of the reserve price and bidders observe their valuations, the bidder must determine the distribution of the first monitoring time. This decisions is taken prior to bidding starting. The programme the bidder solves is given by:

$$\max_{\lambda_0}\left[1-e^{-\lambda_0 T}\right]E\left[\left(v-b_\tau\right)M\left(h_T,b_\tau\right)-\sum_{k=1}^{\tau}g(\lambda_k)\middle|h_0\right]-g(\lambda_0)$$

where h_0 is the initial history.

4.2 First order conditions

One can show that if his valuation exceeds the maximum of the reservation price and lowest existing ITM bid, then a player will bid if he has not bid before or if his previous bids have been displaced. Intuitively consider what happens when a player receives an opportunity to bid at a point close sufficiently close to the end of the auction, in which there is little chance anybody will have a further opportunity to bid. If the player's previous bids are OTM then he will place an ITM bid if his valuation exceeds the least ITM bid. If the there are already OTM bids, this action will displace an existing ITM bid. By an induction a player bids whenever he has the opportunity and it OTM.

At that point he will also optimally adjust his monitoring of the auction book. Denote $g'\left(\lambda\right)\equiv\partial g\left(\lambda\right)/\partial\lambda$. By assumption $g'(0)=0$ and $g'(\infty)=\infty$. Consequently the optimal choice of λ is found by solving its associated first order condition given by:

$$\begin{aligned} g'(\lambda_j) \;=\; & \left[e^{-\lambda_j(T-\tau_j)}\right](T-\tau_j)\left\{E_{t1}\left[M\left(h_T,b_\tau\right)\right]-E_{t0}\left[M\left(h_T,b_j\right)\right]\right\}v \\ & -\left[e^{-\lambda_j(T-\tau_j)}\right](T-\tau_j)\left\{E_{t1}\left[b_\tau M\left(h_T,b_\tau\right)+\sum_{k=j+1}^{\tau}g(\lambda_k)\right]-b_jE_{t0}\left[M\left(h_T,b_j\right)\right]\right\} \end{aligned} \quad (4)$$

Intuitively, at τ_j the player sets $g'(\lambda_j)$ the marginal cost of more intense monitoring to offset the extra expected benefit, net of future monitoring costs, from raising his bid before the auction closes to increase the chance of being ITM.

If the player is the first bidder and bids above the reservation price, or more generally bids a price that has not been selected by anyone else yet, then the necessary first order condition is:

$$\begin{aligned} v\frac{\partial E[M(h_T,b_\tau|h_j,b_j)]}{\partial b} \;=\; & \frac{\partial E[b_\tau M(h_T,b_\tau)|h_j,b_j]}{\partial b}+e^{-\lambda_j(T-\tau_j)}E_{t0}(M(h_T,b_j)) \\ & +\left[1-e^{-\lambda_j(T-\tau_j)}\right]\frac{\partial}{\partial b}E_{t1}[\sum_{k=j+1}^{\tau}g(\lambda_k)\,|h_j,b] \end{aligned} \quad (5)$$

There are three differences between a sealed bid auction equilibrium choice and this ascending auction. First there are two extra terms. One reflects the probability that only with probability $e^{-\lambda_j(T-\tau_j)}$ will the bidder pay b_j as opposed to an updated bid price. The other arises because changes in the current bid affect the probability distribution of future rival bids, and hence the choice of monitoring later in the auction. Finally the derivative of the expected payment with respect to b_j is not necessarily evaluated b_j because it is not the amount necessarily paid in the event of being ITM.

The first order condition for the initial monitoring intensity λ_0 is given by:

$$Te^{-\lambda_0 T}E\left[(v-b_\tau)\,M\left(h_T,b_\tau\right)-\sum_{k=1}^{\tau}g(\lambda_k)\middle|h_0\right]=g'(\lambda_0) \tag{6}$$

5 Identification and Estimation

The primitives of the model are the reservation price, R, the number of players I, the number of units for sale, Q, the probability density function for the valuations of the players, denoted by $f\,(v)$, and the monitoring cost function, $g(\lambda)$. Given values for these primitives, an equilibrium for the model generates a joint probability distribution for the sequence of bids, b_t, the monitoring rates, λ_t, along with the identities of the bidders (the participating players). For each auction in the data set we observe the bid sequence, denoted by h_T, and the identities of the bidders, as well as the reservation price R, and the number of units for sale, Q. However we do not observe the λ_t choice sequence or I, the number of players. However it is straightforward to show that with strictly positive probability, all the players bid. Consequently I is identified, and is consistently estimated by the maximum number of players observed bidding in one of the sampled auctions. This only leaves $f\,(v)$ and $g(\lambda)$ to identify. We follow a standard assumption in the literature on structural estimation that the data for all the auctions are generated by same equilibrium, that they have the same monitoring cost structure, and all valuations are drawn from the same probability distribution.

For the most part our estimators are constructed from sample analogs to the moment conditions and nonlinear regression functions that establish identification. Estimation is conducted in steps. First we estimate the monitoring choices of players who make more than two bids. Then we estimate the monitoring cost function from choices made by players who are observed to make three bids. This allows us to estimate the valuations associated with bids on isolated points. From that we can infer and estimate the first waiting time distribution as a function of estimated valuations, along with the potential number of bidders in the auction, and the probability distribution of the valuations.

5.1 Forming auction histories

Players see boundary bids every time they bid, and also keep track of their own past bids still in the money. A sufficient statistic for their choices is the unobserved state of the auction, their individual valuation, the incomplete history of boundary prices they saw, and the time left in the auction. Histories for each auction are formed from the sequence of bids that are in or on the money along with their associated bidding times. Denote a generic auction history by the sequence of coordinate pairs $\{b_\tau,t_\tau\}_{\tau=1}^{T}$ where b_τ registers the τ^{th} bid in the auction, and t_τ gives the time it occurs. We also denote the weakly increasing sequence of boundary bids constructed from the bidding history by $\{\beta_\tau\}_{\tau=1}^{T}$.

A stepwise procedure identifies the monitoring cost function, $g(\lambda)$, and the probability density function for the valuations of the players, $f\,(v)$. We first identify a subset of the isolated bids in each auction. Given one such isolated bid, say b_s and its bidding history h_s, we then show that the valuation giving rise to it, say v_i, is also identified, and consistently estimated by the highest bid of anyone who bid b_s with bidding history h_s. The associated monitoring choice λ_s is also identified. Intuitively, a strictly positive proportion of isolated bids fall OTM and are later updated; since players with sufficiently high valuations revise their bids at the first opportunity after their bids fall OTM, the duration times to their next bid identify λ_s. From the first order conditions for monitoring, knowledge of (v_i, b_s, λ_s) for the isolated bids can be used to identify the cost function for monitoring, $g(\lambda)$. Although the first order condition includes the expected sum of all future monitoring costs, we exploit recursive methods to difference out the role of monitoring costs that occur beyond one future opportunity. Since the subset of isolated bids includes all first bids by definition, and since every player with a valuation above R bids if the book is empty in the equilibrium we investigate, the selected distribution of participating players is therefore identified from the valuations of first bidders. Finally because the selection rule is determined by the monitoring rate, I, the number of potential bidders, is also identified.

5.2 Valuations

Assuming bids are strictly monotone in valuations, it now follows that for a given history all identical isolated bids (across different auctions) were induced by the same valuation. Therefore the highest bid ever observed amongst this subset of players lower bounds their common valuation. Since the lowest winning bid may be arbitrarily close to any valuation, we can show that the highest winning bid observed amongst the subset converges from below to the common valuation.

Let $b_{it} \equiv b(h_t, v_i)$ denote an equilibrium bid by player i at t when the partial auction history is h_t and his valuation is v_i. Suppose that $b_{it} = b_{it}^{(int)}$, which is the bid he would make if he solved the first order condition. We let $h_s \succeq (h_t, b_{it})$ mean that for any time $s \in (t, T]$, the partial history h_s succeeds h_t or is a continuation of h_t. Then:

$$v_i = \sup_{h_s \succeq (h_t, b_{it})} \{b(h_s, v_i) | b_{it}\} \tag{7}$$

Intuitively, given the opportunity, a player bids up to his valuation in an IPV ascending auction and quits if the (lowest ITM) bid rises above it. So to prove this result we need to show that with positive probability there are histories $h_s \succeq (h_t, b_{it})$ such that $b(h_s, v_i)$ is arbitrarily close to v_i, say because with positive probability there is a cluster point at v_i^-.

In estimation we make use of the fact that bidders with the same history will have the same valuation. We can then follow these bidders then face different futures. Some future auction states will be very competitive. As a result we would expect markups to be more compressed in the more competitive auctions. Essentially, we find bids with similar histories and collect the transaction prices. We then take the maximum bid submitted. This then provides a lower bound on valuations for these selected bids.

This approach relies on finding close bids. We use a number of different metrics. We first present results based on the euclidian distance between history vectors.

5.3 Monitoring choices of isolated bids

Now consider a player with valuation v_i who bids for the m^{th} time at $r \equiv \tau_i^{(m)}$. Suppose her bid at $\tau_i^{(m)}$ falls OTM at some later point during the auction denoted by $\rho_i^{(m)} > \tau^{(m)}$. Then suppose she subsequently raises her bid at $\tau_i^{(m+1)} > \rho_i^{(m)}$. The latter bid shows her valuation can justify ITM bids at $\rho_i^{(m)}$. Thus in equilibrium she bids at her first opportunity after falling OTM. Comparing her with those who make the same bid given the same history, and therefore, by strict monotonicity of the bid function, have the same valuation, we conclude that the monitoring rate is identified from the truncated distribution for all isolated bids that fall OTM and are subsequently revised upwards, since they have the same chance that their next opportunity to bid will occur if they fall OTM. That is:

$$\lambda_i^{(m)} \equiv \lambda\left(h_r, v_i\right) = \left\{E\left[\tau_i^{(m+1)} - \rho_i^{(m)} \left| h_r, v_i\right.\right]\right\}^{-1}$$

We can extend this concept by exploiting the fact that the valuations of isolated bids are (point) identified. Once a bid falls OTM, the player takes his first opportunity to rebid so long as his valuation is ITM, because choosing the same monitoring rate and updating his bid to ITM dominates passing. (The per time cost is the same, as are future bidding opporunities, but he gains from bidding if no further bidding opportunities arrive and his revised bid remains ITM.) This extends the period of time from when he falls OTM to the point where the lowest ITM bid crosses his valuation. Moreover since the player would choose not to bid after that point in the auction, we obtain the censored distribution of first passage times to a revised bid.

Even more information in the data generating process is available to increase time frame for estimating the monitoring rate. Consider the duration from when the original bid is made until it falls out of the money or there is a rebid, whichever comes first. Comparing the history of auction for this bid with all other identical histories, we check for the times in other such auctions when a rebid is ever made. In this way we identify an indicator function that defines when the bidder would rebid if he could. The union of all such durations then provide the basis for estimating the equilibrium monitoring rate for all isolated bids. Using this method has the advantage of generating the longer durations and greater number of updates, but its drawback is that we must compare histories that remain comparable after the first bid is made, straining the data along a different dimension.

Our data for the estimation of monitoring rates is constructed as follows. Our model suggests that a bidder who valuation allows the bidder to be in the money will want to submit a bid as soon as being pushed out of the money. We measure the seconds between being pushed OTM and re-submitting a new bid that is ITM as non-monitored time, in other words this measures the duration between monitoring opportunities. These times must therefore be the realization of a monitoring rate distribution set at the time of the previous bid. Denote ρ_i^k as the kth observation of the time at which bidder i is pushed OTM and τ_i^k the associated time bidder i is back ITM. We also have the history $h_{\rho_i^k}$ associated with τ_i^k as constructed above. As a result we know that the duration $\tau_i^k - \rho_i^k$ is the realization of duration from an exponential distribution whose parameter is

$$\lambda\left(b_{\rho i^k}, h_{\tau_i^k}\right)$$

If we had multiple realizations of these durations for the same $b_{\rho_i^k}, h_{\tau_i^k}$ we could estimate λ by averaging across all durations.

Our estimators for the valuations also provide an estimate of the maximum amount of time available to rebid. Denote by T_{ki} the time elapse between ρ_i^k and the time when the lowest bid ITM exceeds the valuation of bidder i, or T, which ever comes first. Conditioning on history and bidding, the log likelihood of the censored sample is:

$$\sum_i \left(1 - d_i\right) \ln\left(\lambda e^{-\lambda \rho_i^k}\right) + d_i \ln e^{-\lambda T_{ki}}$$

where d_i is an indicator for whether i rebids or not. Rearranging the first order condition yields an estimator of

$$\frac{1}{\lambda} = \frac{\sum_i \left(1 - d_i\right) \rho_i^k}{\sum_i \left(1 - d_i\right)} + \frac{\sum_i d_i T_{ki}}{\sum_i \left(1 - d_i\right)}$$

Again we use nonparametric analogues to smooth over nonidentical but close histories to handle the conditioning.

5.4 Valuation distribution

The truncated distribution of first bidders is identified since they are certainly isolated points and therefore solve a first order condition if they bid above R. From the timing of first bids for those players with identical valuations we identify the initial choice of the monitoring rate as a function of their valuation, denoted by $\lambda_0(v)$. Since the selection rule and the truncation distribution of valuations for first bidders is identified, and the truncated distribution covers the support of the underlying distribution of valuations, it follows that the latter is identified too.

More specifically, let T_i denote the random time that player i has his first opportunity to bid when his initial monitoring rate is $\lambda_0(v_i)$. Then:

$$\ln T_i = -\ln \lambda_0(v_i) + \eta_i$$

where η_i has a Type 1 extreme value distribution. Suppose v_i is drawn from a finite set $\{v_1, \ldots, v_I\}$ Suppose momentarily that the support of parent population of the truncated population, ranked from the lowest to the highest, is $\{v_1, \ldots, v_I\}$ with probabilities $\{q_1, \ldots, q_I\}$, and that in the truncated population the probabilities are $\{p_1, \ldots, p_I\}$. Then:

$$p_i = \frac{q_i \lambda_0(v_i)}{\sum_{j=1}^{I} q_j \lambda_0(v_j)}$$

That is:

$$\frac{p_i}{\lambda_0(v_i)} = \frac{q_i}{\sum_{j=1}^{I} q_j \lambda_0(v_j)}$$

But:

$$\sum_{i=1}^{I} \frac{p_i}{\lambda_0(v_i)} = \sum_{i=1}^{I} \frac{q_i}{\sum_{j=1}^{I} q_j \lambda_0(v_j)} = \frac{1}{\sum_{j=1}^{I} q_j \lambda_0(v_j)}$$

implying:

$$q_i = \left(\frac{p_i}{\lambda_0(v_i)}\right) \Big/ \sum_{i=1}^{I} \frac{p_i}{\lambda_0(v_i)}$$

and more generally for any v_k:

$$\Pr\{v \le v_k\} = \left\{\sum_{i=1}^{k} \left(\frac{p_i}{\lambda_0(v_i)}\right) \Big/ \sum_{i=1}^{I} \frac{p_i}{\lambda_0(v_i)}\right\}$$

or taking limits we have:

$$\Pr\{v \le v_k\} = \frac{E\left[I\{v \le v_k\} / \lambda_0(v)\right]}{E\left[1 / \lambda_0(v)\right]}$$

where the expectation is taken over the truncated distribution.

5.5 Monitoring costs

Since the optimal choices of λ_r at $r \equiv \tau^{(m)}$ and λ_s at $s \equiv \tau^{(m+1)}$ satisfy the monitoring FOC we can first difference to obtain:

$$\begin{aligned}
&\frac{g'(\lambda_s)}{(T-s)}e^{\lambda_s(T-s)} - \frac{g'(\lambda_r)}{(T-r)}e^{\lambda_r(T-r)} \\
= &\left\{ \begin{array}{c} E_{r1}\left[bM\left(h_T,b\right)\right] - b_t E_{r0}\left[M\left(h_T,b_t\right)\right] \\ -E_{s1}\left[bM\left(h_T,b\right)\right] + b_s E_{s0}\left[M\left(h_T,b_s\right)\right] \end{array} \right\} \\
&-v\left\{ \begin{array}{c} E_{r1}\left[M\left(h_T,b\right)\right] - E_{r0}\left[M\left(h_T,b_t\right)\right] \\ -E_{s1}\left[M\left(h_T,b\right)\right] + E_{s0}\left[M\left(h_T,b_t\right)\right] \end{array} \right\} \\
&E_{r1}\left[\sum\nolimits_{k=m+1}^{\tau} g(\lambda^{(k)})\right] - E_{s1}\left[\sum\nolimits_{k=m+2}^{\tau} g(\lambda^{(k)})\right]
\end{aligned}$$

But:

$$E_{r1}\left[\sum\nolimits_{k=m+1}^{\tau} g(\lambda^{(k)})\right] - E_{s1}\left[\sum\nolimits_{k=m+2}^{\tau} g(\lambda^{(k)})\right] = g(\lambda_s) + \epsilon_s$$

where ϵ_s is orthogonal to (h_r, v, λ_s). Consolodating we have:

$$vy_{1s} - y_{0s} = g(\lambda_s) - g'(\lambda_s)A_{2s} + g'(\lambda_r)A_{1r} + \epsilon_s \tag{8}$$

where:

$$\begin{aligned}
y_{0s} &= E_{r1}\left[bM\left(h_T,b\right)\right] - b_t E_{r0}\left[M\left(h_T,b_t\right)\right] \\
&\quad -E_{s1}\left[bM\left(h_T,b\right)\right] + b_s E_{s0}\left[M\left(h_T,b_s\right)\right] \\
y_{1s} &= E_{r1}\left[M\left(h_T,b\right)\right] - E_{r0}\left[M\left(h_T,b_t\right)\right] \\
&\quad -E_{s1}\left[M\left(h_T,b\right)\right] + E_{s0}\left[M\left(h_T,b_s\right)\right] \\
A_{1r} &= \frac{e^{\lambda_r(T-r)}}{(T-r)} \\
A_{2s} &= \frac{e^{\lambda_s(T-s)}}{(T-s)}
\end{aligned}$$

In this way we establish that $g(\lambda)$ is identified within the class of analytic functions.

More generally $g(\lambda)$ can be modelled as depending on the amount of time left in the auction. For example suppose that monitoring costs are paid per unit time until the next monitoring opportunity, or the auction ends, whichever comes first. We write:

$$\begin{aligned}
g\left(\lambda, s\right) &\equiv h\left(\lambda\right)\int_0^{T-s} t\lambda \exp\left(-\lambda t\right) dt \\
&= h\left(\lambda\right)\left\{-\left[t\exp\left(-\lambda t\right)\right]_0^{T-s} + \int_0^{T-s}\exp\left(-\lambda t\right) dt\right\} \\
&= h\left(\lambda\right)\left\{\frac{1}{\lambda} - \left(T - s + \frac{1}{\lambda}\right)\exp\left[-\lambda\left(s-T\right)\right]\right\} = h(\lambda)A_3
\end{aligned}$$

Given that we now have estimated λ and valuations, we can now use:

$$vy_{1s} - y_{0s} = g(\lambda_s) - g'(\lambda_s)A_{2s} + g'(\lambda_r)A_{1r} + \epsilon_s$$

To estimate the parameters of the monitoring cost function as a simple regression instrumenting for λ_s using previous λs. We then apply a two stage least squares estimator. To be more precise we assume that:

$$h(\lambda) = h(\lambda;\theta) = \sum_{p=1}^{P}\theta_p\lambda^p$$

It follows then that: We can then re-write:

$$vy_{1s} - y_{0s} = \sum_{p=1}^{P} \theta_p \left[\lambda_s^p \left(A_{3s} - A_{4s}A_{2s} - p\lambda_s^{-1} A_{3s}A_{2s} \right) + p\lambda_r^{p-1} A_{1r}A_{4r} + \lambda_r^p A_{1r}A_{3r} \right] + \epsilon_s \quad (9)$$

where A_4 is the derivative of A_3.

Substituting out our estimates for v and λ we can estimate θ in the above directly.

The nonparametric estimator for $E(M(b_{\tau_j}, h_{j\tau_j})|h_{js}, b_{j\tau_j})$ is the Nadaraya-Watson estimator. See Pagan and Ullah (1999) for a discussion of the properties of these estimators. Similar, estimators have been used by Hotz et al. (1994) for optimal choice probabilities. The estimator is defined as:

$$\widehat{M}(x) = \frac{\sum_{l=1}^{L} \sum_{j=1}^{N_1} K_X \left((x - x_{jt}^l)/\xi_x \right) M(h_T, b_{jt}^l)}{\sum_{l=1}^{L=1} \sum_{j=1}^{N_1} K_X \left((x - x_{jt}^l)/\xi_x \right)}$$

where $x = (b, h, R, q)$ and K_X is a multi dimensional kernel and ξ is the bandwidth.We use a product kernel for the conditioning variables. We estimate $E[b_{j\tau} M(h_T, b_{j\tau})|h_{\tau_j}, b_{\tau_j}]$ by:

$$\widehat{Mb}(x) = \frac{\sum_{l=1}^{L} \sum_{j=1}^{N_1} K_X \left((x - x_{tj}^l)/\xi_x \right) b_{jt}^l M(h_T, b_{jt}^l)}{\sum_{l=1}^{L=1} \sum_{j=1}^{N_1} K_X \left((x - x_{jt}^l)/\xi_x \right)}$$

As noted before we require the derivatives of the above objects. To estimate these derivatives we simply differentiate the above estimators with respect to the current bid (which is a conditioning variable in the conditional expectation). This approach is described in Härdle (1992) and Pagan and Ullah (1999). Details can be found in the appendix. Bandwidths are selected using a rule of thumb as discussed in Bowman and Azzalini (1997).

5.6 Inference (incomplete)

To construct standard errors we make use of subsampling techniques.

6 Structural Estimates and Tests

Our structural estimates are organized around three issues. The first two pertain to model specification. We compare the results of our model with bounds obtained by directly focusing on the final bid of each auction participant, and with results that would be generated by two alternative auction formats. Then we report our findings on the role of unobserved heterogeneity and the scope for multiple equilibrium. For each issue we show how the distribution of valuations and the monitoring costs changed with the onset of the financial crisis, in this way providing several measures of its impact on this market.

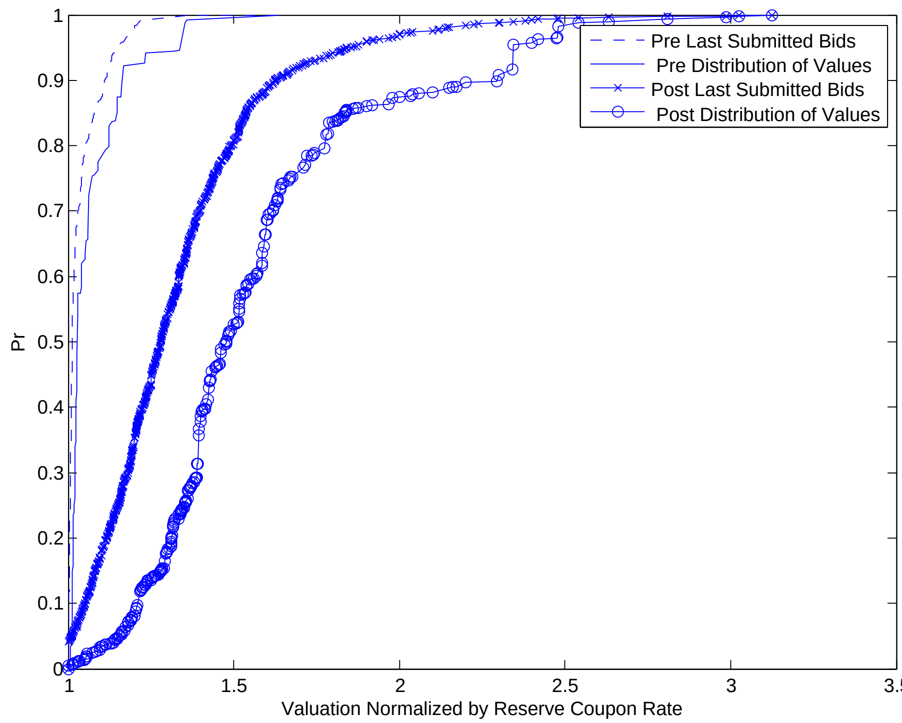
6.1 The Empirical Importance of the Auction Format

Does modeling the equilibrium for a detailed description of the institutional detail have a noticeable effect on the results that can be obtained without modeling the equilibrium? Are the empirical results from our auction format robust to misspecification? To answer these questions we compare the results of our model with bounds obtained by directly focusing on the final bid of each auction participant, and with results that would be generated by two alternative auction formats, a first price sealed bid (FPSB) auction, and a sequential auction where bidders know the order of their last bid as well as the reservation price at that time.

6.1.1 Valuations and final bids

How close are the bounds we would obtain from not putting any structure on the institutional detail surrounding the bidding. Figure 8 compares the distribution of final bids with the distribution of valuations estimated from our model for interior bids for data prior to 2008 and post 2008. These distributions are based on the sample of observed isolated bids. Note that we can combine these

Figure 7: Valuation Distributions Using Close Bids, normalized by reserve coupon rate



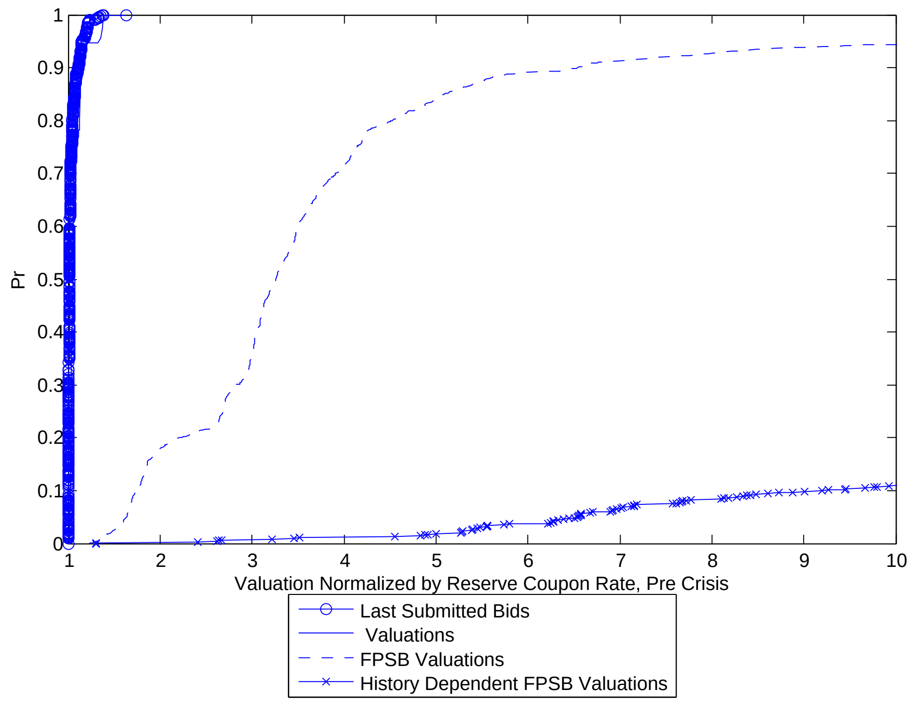
estimates with the valuations of players who cluster by noting that when a player clusters he has a lower valuation than the player who begins the cluster, and a higher valuation than the next lowest bid. A tighter bound can be found by exploiting the indifference condition discussed in the equilibrium section.

The lower bounds, given by the final bids, key finding is that applying the model significantly shifts the distribution function to the left.

6.1.2 Valuations implied by a first price sealed bid auction

We now the question of whether our estimates of the valuation distribution are robust to the form of the auction, by comparing our estimates with alternative auction models that might be applied to this market. Suppose that the end of the auction resembles a first price sealed bid (FPSB) mechanism and that all previous bidding activity is non-informative. This is similar to the setting of Bajari and Hortacsu (2003). With this framework it is possible to simply extend the approach of Guerre et al. (2000) and estimate private values. Details of the estimation are provided in the appendix. Estimation uses a first order condition to infer the private valuations from observed bids. This framework assumes that bidders do not observe the bids of other players.

Figure 8: Valuations compared to FPSB estimates, normalized by reserve coupon rate



The distribution function estimated this way first order stochastically dominates the estimated distribution from our specification in quite dramatic fashion. Valuations are implausibly estimated to be orders of magnitude higher than the bids even though there are on average moe than twenty bidders in each auction. Intuitively, the derivative of the probability of winning with repsect to the current bid is lower than it might be if the player knew he only had only one opportunity to bid. Since there may be an opportunity to bid later on, lowballing can be corrected at some future time before the auction ends. The low derivative is complemented by at least one additional term that separates the bid from its valuation in the first order condition; thus not all the difference between the quotient of the winning probability and its derivative can be attributed to the difference between the bid and the valuation.

6.1.3 Valuations implied by a sequential auction

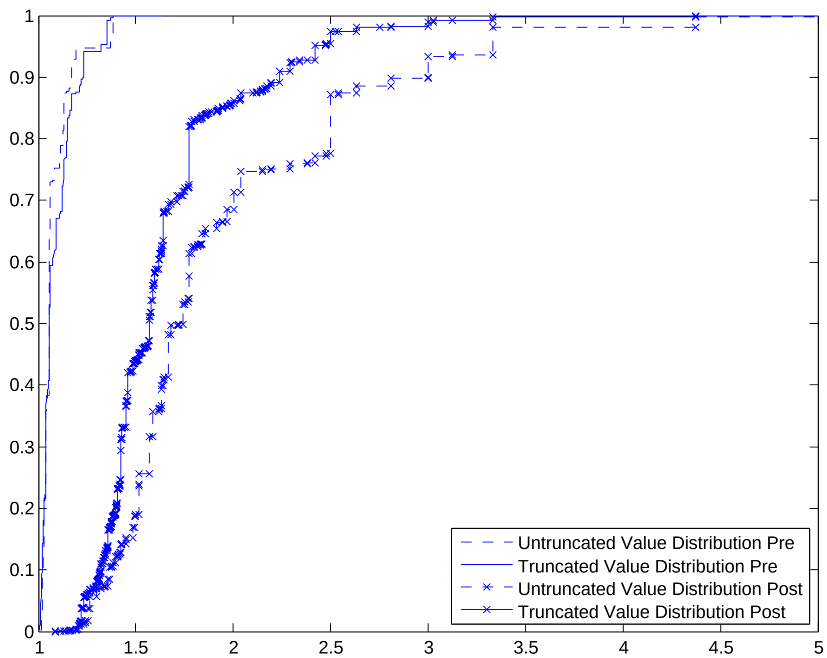
However, applying GPV approach so directly is misleading in our context. We can modify the above framework to capture the possibility that bidder can observe each other's moves. This model is similar in spirit to Daniel and Hirshleifer (1998). We modify the previous framework for an exogenous ordering of bidders who sequentially enter the auction. An equilibrium would involve similar behaviour as described in our main model. In particular, we can be sure that the first bidder will solve a first order condition for an optimal bid. The subsequent bidders, having observed previous bids, decide whether to cluster with an order already in the book (in other words submit an order slightly above an existing one) or to submit a bid that solves a first order condition.

As before, we will only be able to estimate valuations for isolated bids. We apply the same classification algorithm described before to identify isolated bids. We then use those isolated bids and the associated first order conditions to infer private valuations. The key difference to the first exercise is that bidders can condition their bidding behaviour on orders already in the book.

6.1.4 Parent Valuation Distribution

Figure 10 depicts our estimates of the parent distribution from which the pariticipating players are drawn through their first bidding opportunity. The most noticeable features are that bids are made on projects that, from a distributional perspective, first order dominate the untruncated valuations.

Figure 9: Untruncated versus Truncated Valuation Distribution Pre and Post



6.1.5 Monitoring Costs

We first present estimation of the valuations. We sometimes find a number of negative interest rates which we find implausible. We drop these estimates from the discussion. Then we present our estimates of the costs of monitoring, found by computing the expected benefits of increasing monitoring at any given point in the auction.

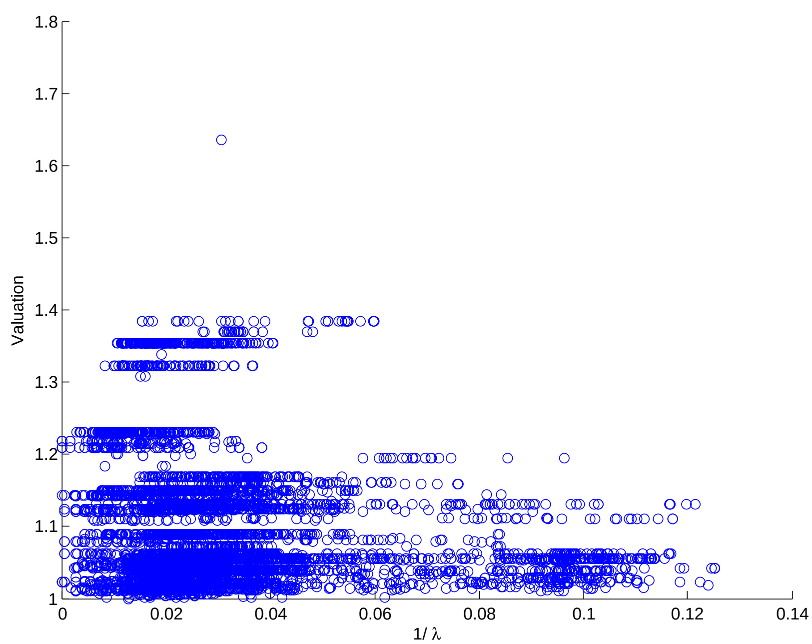
Table 4: Monitoring Cost Estimates

		Cost Function Parameters (std. error)	
		Poly of degree 2	Poly of degree 3
Pre 2008	θ_0	0.02 (0.00)	48.42 (1.25)
	θ_1 coeff. on λ	8.98 (0.42)	3.74 (0.47)
	θ_2 coeff. on $\lambda.^2$	0.00 (0.00)	0.03 (0.00)
	θ_3 coeff. on $\lambda.^3$		-0.00 (0.00)
Post 2008	θ_0	-0.17 (0.01)	52.37 (0.93)
	θ_1 coeff. on λ	79.76 (0.53)	134.58 (0.86)
	θ_2 coeff. on $\lambda.^2$	-0.00 (0.00)	0.04 (0.00)
	θ_3 coeff. on $\lambda.^3$		-0.00 (0.00)

Table 5: Monitoring Costs over Realized Profits $\frac{\sum_{t=1}^{\tau} g(\lambda_t)}{(v-b)M(b,h_T)}$

		Mean	**Standard Deviation**	**Min**	**Max**
Pre 2008	Poly of Degree 2	0.03	0.19	0.00	4.87
	Poly of Degree 3	0.02	0.13	0.00	3.27
Post 2008	Poly of Degree 2	0.14	1.07	0.00	36.28
	Poly of Degree 3	0.23	1.83	0.00	62.38

Figure 10: Valuations versus Estimated $\frac{1}{\lambda}$



7 Efficiency and Welfare

An important feature of this auction is the monitoring costs bidders pay to stay informed. It is reflected in the raw data by the jump bidding and the spread of winning bids; our structural estimates of monitoring costs are statistically significant and quantitatively important. Consequently the highest valuation players might not have an opportunity to bid, and amongst bidders the highest valuation players might not win. For these reasons the conditions of the revenue equivalence theorem are violated, and there is no reason to suppose the auction optimally allocates resources. Treating the monitoring costs as a given part of the economic environment, this section compares the auction with a social planning allocation.

7.1 Evaluating the Auction Format

In principle there are three differences social costs between the solution to the social planning problem and conducting the online ascending auction. First is the initial level of monitoring that occurs before the first bid in the auction. Second, there is no further monitoring after the first bid in the socially optimal mechanism, because the valuations of all the players who pass through the first stage are instantaneously revealed (as in a sealed bid auction), which contrasts with the online auction, where players continue to monitor after placing their first bid. Third in the online auction, successful bids do not necessarily correspond to those with the highest valuations. We quantified the differences between these three components.

The sum of realized profits for auction l is:

$$\sum_{i}^{I^l}[v_i^l - b_i^l]M(b_i^l)q_i^l$$

We then compute what the potential profits could have been with knowledge on individual valuations. We first determine which bidders should be in the money, rather than those who were found to be ITM by the mechanism. Let this updated ITM indicator be denoted by M^*. Then the sum of potential profits is given by:

$$\sum_{i}^{I^l}[v_i^l - b_i^l]M^*(b_i^l)q_i^l$$

The results are summarized in Table 6.

7.2 Potential Gains from Trade

A hypothetical social planner would solve a search and sorting problem to determine how the objects should be allocated. In the search phase, the planner identifies projects and their values by using the exponential monitoring technology. Then in the sorting phase the planner would allocate the fixed aqmount of funds to the projects of highest value. Suppose there are I projects of which J are identified by the planner, and a maximum of K can be funded. The independently and identically distributed probability density function for the projects is denoted by $f_\lambda(v)$, the subscript λ indicating the effect of monitoring on the projects tagged for possible funding. We denote by $\lambda(v)$ a monitoring function the planner might adopt, interpreting it as the monitoring intensity undertaken by the project manager to seek to contact the planner as instructed.

7.2.1 Monitoring costs

In our setup, there is no reason to pay monitoring costs after a project has been submitted or after the first phase has ended. Thus $g[\lambda^o(v)]$ is the expected monitoring cost of a project with value v,

Table 6: Private and Social Surplus

		Mean	Standard Deviation	Min	Max
Pre 2008	Fraction of OTM bidders who should be ITM	0.78	0.28	0.00	1.00
	$\sum_{i=1}^{I^l} v_i^l M(b_i^l)$	3879765.57	593490.85	2429404.00	4323249.00
	$\sum_{i=1}^{I^l} v_i^l M^*(v_i^l)$	3902577.93	588256.22	2435750.00	4323249.00
	$\frac{\sum_{i=1}^{I^l} v_i^l M(b_i^l)}{\sum_{i=1}^{I^l} v_i^l M^*(v_i^l)}$	0.99	0.01	0.93	1.00
	$\sum_{i=1}^{I^l} b_i^l M(b_i^l)$	3766933.11	578832.43	2391591.00	4269099.00
	$\sum_{i=1}^{I^l} b^{*l} M^*(v_i^l)$	3821871.43	571952.24	2381500.00	4244000.00
	$\frac{\sum_{i=1}^{I^l} b_i^l M(b_i^l)}{\sum_{i=1}^{I^l} b^{*l} M^*(v_i^l)}$	0.99	0.03	0.90	1.05
Post 2008	Fraction of OTM bidders who should be ITM	0.72	0.40	0.00	1.00
	$\sum_{i=1}^{I^l} v_i^l M(b_i^l)$	921950.19	912430.50	81315.00	3389840.00
	$\sum_{i=1}^{I^l} v_i^l M^*(v_i^l)$	944056.91	928547.94	81315.00	3414240.00
	$\frac{\sum_{i=1}^{I^l} v_i^l M(b_i^l)}{\sum_{i=1}^{I^l} v_i^l M^*(v_i^l)}$	0.98	0.03	0.88	1.00
	$\sum_{i=1}^{I^l} b_i^l M(b_i^l)$	760673.91	844874.28	57999.00	3346376.00
	$\sum_{i=1}^{I^l} b^{*l} M^*(v_i^l)$	838595.40	893196.08	37200.00	3360000.00
	$\frac{\sum_{i=1}^{I^l} b_i^l M(b_i^l)}{\sum_{i=1}^{I^l} b^{*l} M^*(v_i^l)}$	0.95	0.20	0.52	1.56

and:

$$I \int g\left[\lambda^o(v)\right] f_\lambda(v)\, dv$$

is the total monitoring cost

7.2.2 Benefits from optimal sorting

If $J \leq K$ then all the identified projects are funded, and the gross benefits are Jm_λ, where m_λ is the mean of the density for flagged projects when the monitoring technology $\lambda(v)$ is adopted. Alternatively $J > K$ and the planner selects the K most valuable projects for funding. Denoting by $m_{\lambda j:J}$ the mean value of the j^{th} least valuable project, it follows that the gross expected benefits in this case are:

$$\sum_{j=J-K+1}^{J} m_{\lambda j:J} = \sum_{j=1}^{J} m_{\lambda j:J} - \sum_{j=1}^{J-K} m_{\lambda j:J} = Jm_\lambda - \sum_{j=1}^{J-K} m_{\lambda j:J}$$

Denoting by $p_{\lambda J}$ the probability that J projects are identified to the social planner for potential funding, it now follows that the gross expected benefits to the social planner from selecting $\lambda(v)$ as a monitoring function are:

$$Jm_\lambda - \sum_{J=K+1}^{I} \sum_{j=1}^{J-K} p_{\lambda J} m_{\lambda j:J}$$

To solve the social planner's problem we must relate the set of potential projects to the set of identified projects through the monitoring technology. As before the unconditional probability density function characerizing valuations is denoted by $f(v)$. This density relates to $f_\lambda(v)$, the density from which flagged projects are drawn through a monitoring function $\lambda(v)$ via the mapping:

$$f_\lambda(v) = f(v)\lambda(v)/E[\lambda(v)] \tag{10}$$

where the expectation is taken with respect to $f(v)$. The mean of this distribution is then:

$$m_\lambda \equiv \int v f_\lambda(v)\, dv \tag{11}$$

while the mean of its j^{th} order statistic (out of J) is:

$$m_{\lambda j:J} = \frac{J!}{j!\,(J-j)!}\int v F_\lambda^{j-1}(v)\left[1-F_\lambda(v)\right]^{J-j} f_\lambda(v)\, dv \tag{12}$$

Similarly the probability that J projects are flagged for possible funding can be expressed as:

$$p_{\lambda J} = \frac{I!}{J!\,(I-J)!}\left[\int \exp(-\lambda(v)T) f(v)\, dv\right]^J \left[1-\int \exp(-\lambda(v)T) f(v)\, dv\right]^{I-J} \tag{13}$$

For simplicity we assume that

$$\lambda(v) = \sum_{p=1}^{P}\gamma_p\lambda^p \tag{14}$$

and that the social planner picks values of γ_p to maximize the expected social surplus.

We first present results on the quadratic information cost structure and a cubic $\lambda(v)$ function. The γ parameters are shown in Table 7. These parameters can be used to generate the truncated valuation distribution and can be compared to the selection the current auction mechanism induces. These are shown in Figure 11 and Figure 12

Summaries on auction outcomes across all simulations output can be found in Table **??**. To compute summary statistics we first average outcomes over all simulations and then across auctions.

Table 7: Social Planner Solution

	Pre 2008	**Post 2008**
γ_1	-76.0917	0.1060
γ_2	990.3128	-8.1834
γ_3	15821.6614	105.4699

Figure 11: Pre 2008: Comparison of Truncated Distributions from Social Planning Problem (Quadratic Monitoring Costs)

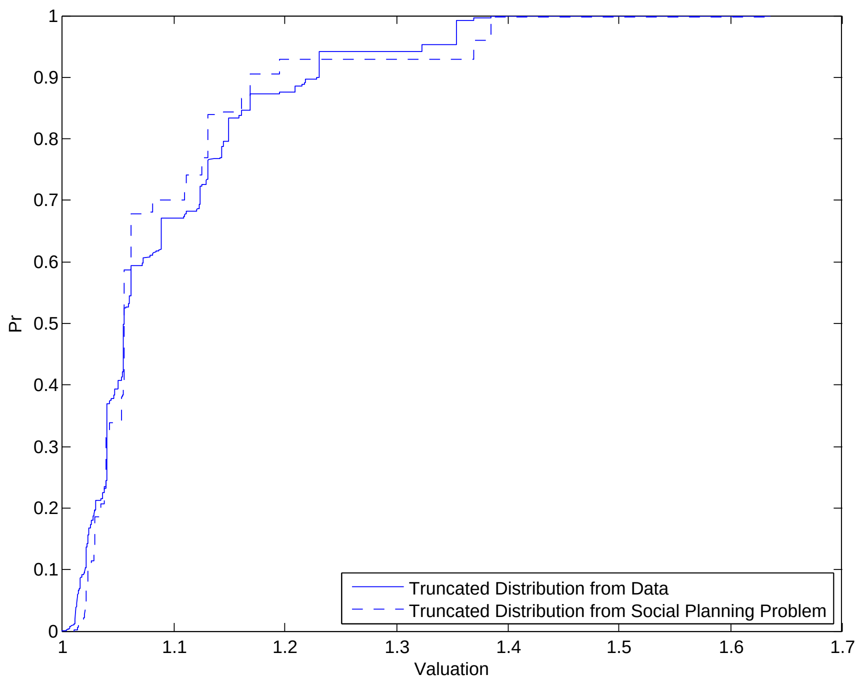


Figure 12: Post 2008: Comparison of Truncated Distributions from Social Planning Problem (Quadratic Monitoring Costs)

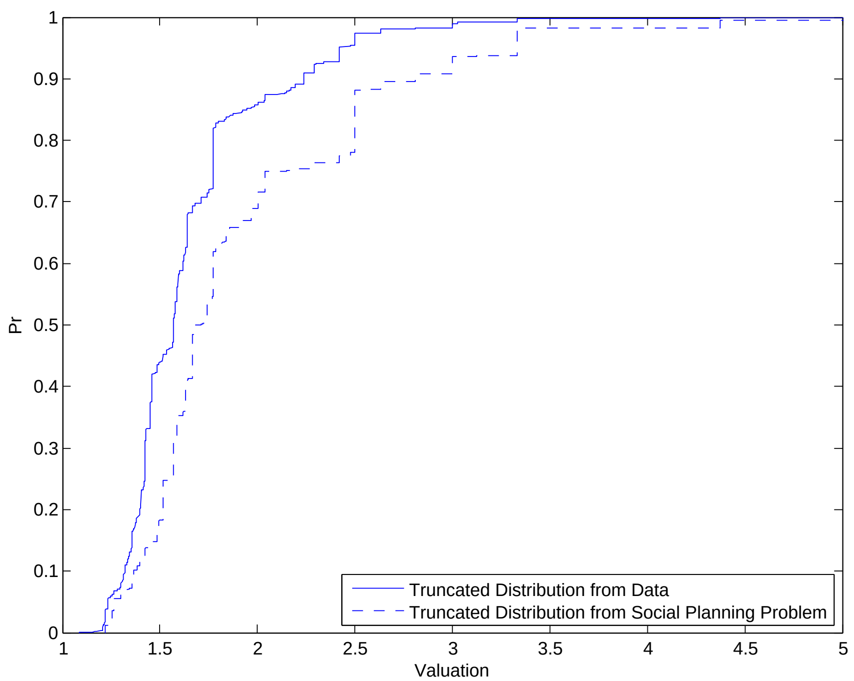


Table 8: Summary Statistics on Social Planner Simulation

		Mean	**Standard Deviation**	**Min**	**Max**
Pre 2008	Number of Bidders	47.8559	11.7195	18.1236	69.0202
	Avg. Monitoring Cost Across Bidders	160.5146	76.6329	23.8342	327.7212
	$\sum_{i=1}^{I} v_i M_{sp}(v_i)$	6955082.1417	1316351.9872	3764680.7801	9887093.0367
	$\frac{\sum_{i=1}^{I} v_i M^*(v_i)}{\sum_{i=1}^{I} v_i M_{sp}(v_i)}$	0.5909	0.1582	0.2933	0.9730
Post 2008	Number of Bidders	18.1071	28.9460	0.0094	71.7114
	Avg. Monitoring Cost Across Bidders	0.1234	0.2612	0.0000	1.1789
	$\sum_{i=1}^{I} v_i M_{sp}(v_i)$	1882073.5362	3027077.1545	96.2925	9246678.9770
	$\frac{\sum_{i=1}^{I} v_i M^*(v_i)}{\sum_{i=1}^{I} v_i M_{sp}(v_i)}$	0.3774	0.0936	0.1675	0.5012

8 Conclusion

This paper constructs and estimates a multiunit ascending auction with costly monitoring. Our modeling approach, estimation strategy and empirical findings establish that there is a role of monitoring costs. Paradoxically the value of a trader's time should be reflected by his inattention to detail. We would expect large successful traders to lose more money in any one market than their less successful rivals. Our estimation approach avoids computation of equilibrium strategies and could potentially be adapted to other environments with even more complex strategy spaces. We then apply the model to CD auctions held by State Treasurers to place unallocated government funds in savings vehicles. Our data allows us to trace the effects of the 2008 recession. We find that the distribution of interest rate valuations changes substantially after the crisis. In particular, the variance of valuations increases substantially. This indicates that bidders were affected very differently from each other by the recession.This paper contributes to understanding dynamic auction mechanisms that do not conform to the Japanese auction idealization.

Finally our model and approach sheds some light on daily trading in limit order markets. Viewed as an electronic limit order market, bidders compete by submitting limit buy orders before a seller enters at a preordained moment and places market orders at . Both types of trading mechanisms The general features of the data, in which order placement activity concentrated at the open and close of trading. Both exhibit isolated limit orders as well as price points, prices where several orders are inverted by placement time. Our equilibrium analysis draws upon and extends Hollifield, Miller, and Sandas (2004). As in our auctions activity is greatest at the beginning and end of the trading period. Our model explains why when there is mutual verification of the monitoring technology, traders tend to place orders at the beginning of the period to deter entry and at the end to prevent responses. At the trading level our model explains why such clumping occurs.

A Appendix

In this appendix we collect some results that are used in the text. We describe the procedure used for estimating the valuations under GPV.

A.1 Estimating valuations under the assumption of a sealed bid auction

We now describe our application of Guerre et al. (2000) to this data. The setup of the theoretical model is the same as the main paper. However, in this setting we assume that the bidding game has two stages. In the first stage bids are not necessarily binding or informative. In the second stage, the auctioneer announces that all bids are final. We assume that bidders know the number of bidders in the auction, I_A. Bidders then follow strategies that are equivalent to those in a FPSB auction. To be precise bidder i maximizes the following objective:

$$\max_{b_i}(v_i - b_i)E[M(b_i, h_T, I_A)]$$

where as before $M(.)$ is an indicator that is equal to 1 when bid b_i is in the money. h_T is the vector of all last bids which is unknown at the time of bidding. An optimal bid solves a first order condition which can be re-arranged to yield an expression for private information as follows:

$$v_i = b_i + \frac{E[M(b_i, h_T, I_A)]}{\partial E[M(b_i, h_T, I_A)]/\partial b_i}$$

The right hand side is completely observed. We estimate the regression and derivative of the regression function non-parametrically, as in the main text. We then use observed bids to estimate valuations.

The modified GPV approach outlined in the text assumes an exogenous ordering of bidders in the second stage. In this setting some bidders will be solving first order conditions and other will cluster with the lowest in the money bids . Bidder i's objective at subgame with history h_t is to maximize:

$$\max_{b_i}(v_i - b_i)E[M(b_i, h_T)|h_t] \tag{1}$$

For some valuations bidders will solve a first order condition and for others bidders will cluster with the lowest in the money orders.

A.2 Bidding on Multiple Parcels

First we describe the setup, then the equilibrium conditions and finally we show how our estimation technique extends to mutiple parcels.

A.2.1 Multi-tranche setup

The model we construct to rationalize the facts described above is a dynamic bidding game between bidders who have imperfect monitoring capabilities. Because the monitoring of bids by rival players demands attention, given limited powers of concentration, each player faces a trade off between devoting exclusive attention to a particular auction and reacting immediately to changes in the bidding history of that security, versus bidding or trading on a larger number of financial securities markets and consequently being less responsive to changes in bidding or trading patterns. In addition there may be other tasks the player undertakes for his employer to draw his attention away from the bidding updates placed by rival players. Monitoring opportunities are modeled as random events, and the shadow value of alternative activities as a cost of increasing the arrival rate of monitoring opportunities. Denoting by λ the parameter of the Poisson distribution for the next bidding opportunity, we let $g(\lambda)$ signify the cost of monitoring with intensity λ until that time. The arrival rate of the next bidding opportunity is reset every time the player bids.

At the beginning of the bidding game, the auctioneer announces the reserve coupon rate. Bidders then observe their private values and the number of units to be bid on. A Bidder then determines the monitoring intensity prior to the beginning of the auction. When a bidder receives their first monitoring opportunity, a bidder observes the entire path of play up to the current time period and submits a bid and determines the intensity of arrival of future monitoring opportunities. The bidder also has the option at any time to pass and not to increase their current bid. The set of active bidders is never fully observed by the players.

-

- *Time* is continuous on the interval $[0, T]$.

- *Units to be awarded:* The total amount to be awarded is denoted by S and is common knowledge to all bidders. Each bidder can win a maximum of $\overline{\Delta} < S$ units. $\overline{\Delta}$ is determined by the auctioneer prior to bidding and announced to all bidders. Bidders can submit bids for fewer than $\overline{\Delta}$ units. The minimum unit size is denoted by Δ and is also determined by the auctioneer prior to bidding.

- *Reserve Coupon Rate:* The seller publicly announces a reservation coupon rate R prior to bidding.

- *Number of Bidders:* The set of potential bidders is denoted $\mathcal{I} \equiv \{1, ..., I\}$ and is fixed over time. Bidder i does not observe the complete number of active bidders, $I_A \leq I$, until the end of bidding.

- *Bidder Demand:* Bidder i has an exogenously given quantity demand for funds. The quantity associated with bidder i will be denoted by $q_i \in \mathbf{Q} \equiv \{0, \Delta, 2\Delta, ...\overline{\Delta}\}$. The vector of quantities for all bidders will be denoted $\mathbf{q} = (q_1, ...q_I)$.

- *Private Values:* Bidder i draws a private value $v_i \in \mathbf{V} \equiv [R, \infty)$ independently and identically from commonly known distribution $F_v(.)$ at time $t = 0$ prior to bidding.

- *Action Space:* Bidder i's action space is given by the interest rate bid and choice of monitoring intensity. Bidder i submits interest rate bids at time t which will be denoted by $b_{it} \in \mathbf{B}_t \equiv [b_{it-1}, \infty)$ at every bidding opportunity. For each $s \in \{1, 2, \ldots\}$ he chooses a sequence of bids $\{b_{i1}, b_{i2}, \ldots\}$. To reduce notational clutter and without creating ambiguity we denote the last bid of the i^{th} player by $b_{i\mathcal{T}} \equiv b_{i\mathcal{T}_i}$ which in the single unit case specializes to $b_{i\mathcal{T}}$. Bidder i's choice of monitoring intensity will be denoted by $\lambda_i \in \Lambda$. Recall that λ is the poisson parameter of the distribution of the next monitoring time. The choice of λ is private information whereas bids are observed by all active players.

- *Monitoring Costs:* Bidder i faces a cost associated with every choice of monitoring intensity $\lambda_{i\tau_s}$ at time τ_s. We denote the cost function associated with a specific choice of intensity by $g(\lambda_{i\tau_s})$. $g(.)$ is common knowledge to all bidders.

- *Monitoring Opportunities:* Bidder i receives a sequence of random monitoring times which is private information. Future monitoring times are unknown to bidder i. The first monitoring time for bidder i, denoted $\tau_{i1} \in [0, T]$, is drawn prior to the beginning of the auction. Subsequent monitoring times, $\{\tau_{i2}, \tau_{i3}, \ldots\}$ are drawn sequentially after the most recently used monitoring opportunity. Each monitoring opportunity is defined as $\tau_{im} \in (\tau_{im-1}, T]$ for $m = 2, 3, \ldots$. The value of τ_{i2} is unknown at τ_{i1}. We also let τ_i denote the random time that maximizes τ_{is} on $s \in \{1, 2, \ldots\}$ subject to the constraint $\tau_{is} \leq T$. In other words, τ_i is bidder i's last time to bid.

- *Histories:* The public history at time t is denoted by h_t and contains all bid rate and quantity decision for all bidders up to time t as well as time itself. The set of all possible histories at time t is denoted $\mathbf{H}_t$. h_T denotes the final status of the auction. The information set of player i at time t is $(h_t, v_i) \in \mathbf{H}_t \times \ \mathbf{V}$.

- *Awarding and Payment Rules:* The auction is similar to a multiunit discriminatory auction. At time T the auction ends and the S units are allocated. Quantity is allocated by market clearing. Funds are allocated starting with the highest interest rate until all funds are allocated.Winning bids will be called in-the-money (ITM) and losing bids will be called out-of-the-money (OTM). If bidders tie with the same interest rate then later bids receive higher priority. A function denoted by $M\left(h_T, b_{i\tau}\right)$ equals 1 if the bid is ITM and 0 otherwise.

- *Expected Payoffs* for bidder i are given by:

$$E_0\left[\left(v_i - b_{i\tau}\right) M\left(h_T, b_{i\tau}\right) - \sum_j g(\lambda_{i\tau_s}) \middle| R, q_i\right] \tag{2}$$

  where $b_{i\tau}$ is the last bid submitted by bidder i and R and the expectation is taken with respect to information at time 0. Expectations are taken over the actual number of competitors, the future monitoring opportunities, the future path of play and the transaction price.

- *A Strategy* for player i at opportunity s consists of a bidding and monitoring strategy and is denoted by $\sigma_{is} = (b^\sigma_{is}, \lambda^\sigma_{is})$. b^σ_{is} is a mapping defined by $b^\sigma_{is} : \mathbf{V} \times \ \mathbf{H}_s \to \mathbf{B}_s$ We restrict attention to bidding strategies that are monotone increasing in valuations. λ^σ_{is} is a mapping defined by $\lambda^\sigma_{is} : \mathbf{V} \times \ \mathbf{H}_s \to \Lambda$.

In the next section, we analyse the equilibria with single tranche and multi tranche demand. The single tranche case is equivalent to a multi unit auction with unit demand and the multi tranche case is similar to a multi unit demand.

A.2.2 First Order Conditions for Multi-Parcel Setting (incomplete)

We now detail the bidding decision in the multi tranche setting. Assume that a bidder updates all bids simultaneously. In the multi-tranche setting recall that a bidder i is allocated K_i tranches each with an associated private value. The expected payoff is given by:

$$\sum_{k=1}^{K_i} E_0\left[\left(v^k_i - b^k_{i\tau}\right) q^k_{i\tau} M\left(h_T, b^k_{i\tau}; \mathbf{b}^{-k}_{i\tau}, \mathbf{q}_i\right) \middle| R, \mathbf{q}_i\right]$$

where $\mathbf{b}_\tau^{-k}$ is the vector of bids on all tranches excluding tranche k at time τ. Let us focus on a subgame with history $h_{i\tau_s}$. The bidder's programme is given:

$$\max_{\{b_i^k,\lambda\}_{k=1}^{K_i}} \sum_{k=1}^{K_i} E[v_i^k q_{i\tau}^k M(h_T, b_{i\tau}; \mathbf{b}_{i\tau}^{-k})|h_{i\tau_s}, \mathbf{b}_{i\tau_s}, \mathbf{q}_i] \tag{3}$$
$$-\left(1 - e^{-\lambda(T-\tau)}\right) E[b_{i\tau}^k q_{i\tau}^k M(h_T, b_{i\tau}; \mathbf{b}_{i\tau}^{-k})|h_{i\tau_s}, \mathbf{b}_{i\tau_s}, \mathbf{q}_i]$$
$$-e^{-\lambda(T-\tau)} b_i^k E[q_{i\tau}^k M(h_T, b_{i\tau}; \mathbf{b}_{i\tau}^{-k})|h_{i\tau_s}, \mathbf{b}_{i\tau_s}, \mathbf{q}_i]$$
$$-g(\lambda) - E[\sum_{j=1}^{\tau} g(\lambda_j)|h_{i\tau_s}, \mathbf{b}_{i\tau_s}, \mathbf{q}_i]$$

The first order condition for parcel k is given by:

$$v_i^k \frac{\partial E\left[M\left(h_T, b_{i\tau}^k; \mathbf{b}_{i\tau}^{-k}\right)|h_{i\tau_s}, \mathbf{b}_{i\tau_s}, \mathbf{q}_i\right]}{\partial b_{i\tau_s}^k} = \frac{\partial E\left[b_{i\tau}^k M\left(h_T, b_{i\tau}^k; \mathbf{b}_{i\tau}^{-k}\mathbf{q}_i\right)|h_{i\tau_s}, \mathbf{b}_{i\tau_s}, \mathbf{q}_i\right]}{\partial b_{i\tau_s}^k} \tag{4}$$
$$-\sum_{m\neq k}^{K_i-1} v_i^m \frac{\partial E\left[M\left(h_T, b_{i\tau}^m; \mathbf{b}_{i\tau}^{-m}\right)|h_{i\tau_s}, \mathbf{b}_{i\tau_s}\mathbf{q}_i\right]}{\partial b_{i\tau_s}^k}$$
$$+\sum_{m\neq k}^{K_i-1} \frac{\partial E\left[b_{i\tau}^m M\left(h_T, b_{i\tau}^m; \mathbf{b}_{i\tau}^{-m}\right)|h_{i\tau_s}, \mathbf{b}_{i\tau_s}, \mathbf{q}_i\right]}{\partial b_{i\tau_s}^k}$$
$$-\left(e^{-\lambda(T-\tau_s)}\right) E(M(h_T, b_i^k; \mathbf{b}_{i\tau}^{-k}))$$
$$-\left(1 - e^{-\lambda(T-\tau_s)}\right) \frac{\partial}{\partial b_i^k} E[\sum_{j=1}^{\tau} g(\lambda_j)|h_{i\tau_s}, \mathbf{b}_{i\tau_s}, \mathbf{q}_i]$$

The first order condition highlights that a bidder will take into account how bid b_i^k on tranche k will affect the expected payoffs of the other $K_i - 1$ tranches. These are captured by the second and third line of Equation (4). Essentially, a bidder is concerned with a potential "ratchet effect": a higher bid on tranche k will induce other bidders to react at a later subgame which in turn might affect the expected

A.2.3 Multi Parcel Estimation (incomplete)

Let us return to the first order condition in Equation (4) derived in the previous section and define the following objects:

$$\mathbf{M} = \operatorname{diag}\left[\frac{\partial E\left[M\left(h_T, b_{i\tau}^k; \mathbf{b}_{i\tau}^{-k}\right)|h_{i\tau_s}, \mathbf{b}_{i\tau_s}, \mathbf{q}_i\ \right]}{\partial b_{i\tau_s}^k}\right]_{k=1}^{K_i}$$

is a $K_i \times K_i$ diagonal matrix and let

$$\mathbf{M}_b = \left[\frac{\partial E\left[b_{i\tau}^k M\left(h_T, b_{i\tau}^k; \mathbf{b}_{i\tau}^{-k}\right)|h_{i\tau_s}, \mathbf{b}_{i\tau_s}, \mathbf{q}_i\ \right]}{\partial b_{i\tau_s}^k}\right]_{k=1}^{K_i}$$

be a $K_i \times 1$ vector. Let $\mathbf{P}$ be a $K_i \times K_i$ matrix with diagonal entries equal to zero and entry in row k column m

$$\frac{\partial E\left[M\left(h_T, b_{i\tau}^m; \mathbf{b}_{i\tau}^{-m}\right)|h_{i\tau_s}, \mathbf{b}_{i\tau_s}, \mathbf{q}_i\ \right]}{\partial b_{i\tau_s}^k}$$

Let $\mathbf{P}_b$ be a $K_i \times K_i$ matrix with zero diagonal entries and entry in row k column m:

$$\frac{\partial E\left[b_{i\tau}^m M\left(h_T, b_{i\tau}^m; \mathbf{b}_{i\tau}^{-m}\right)|h_{i\tau_s}, \mathbf{b}_{i\tau_s}, \mathbf{q}_i\right]}{\partial b_{i\tau_s}^k}$$

Let $\mathbf{v}_i = [v_i^k]_{k=1}^{K_i}$ be a $K_i \times 1$ vector. Then we can write the set of first order conditions as

$$\mathbf{M}\mathbf{v}_i = \mathbf{M}_b + \mathbf{P}_b\iota - \mathbf{P}\mathbf{v}_i \tag{5}$$

where ι is a vector of ones. This can be solved for $\mathbf{v}_i$ as

$$\mathbf{v}_i = [\mathbf{M} + \mathbf{P}]^{-1}(\mathbf{M}_b + \mathbf{P}_b\iota) \tag{6}$$

Alternatively, the above can also be written as:

$$\mathbf{v}_i = [\mathbf{I} + \mathbf{M}^{-1}\mathbf{P}]^{-1}\mathbf{M}^{-1}(\mathbf{M}_b + \mathbf{P}_b\iota) \tag{7}$$

Invertibility requires that the matrix $\mathbf{M}+\mathbf{P}$ or $\mathbf{I}+\mathbf{M}^{-1}\mathbf{P}$ be diagonally dominant. This is equivalent to

$$\left|\frac{\partial E\left[M\left(h_T, b_{i\tau}^k; \mathbf{b}_{i\tau}^{-k}\right)|h_{i\tau_s}, \mathbf{b}_{i\tau_s}, \mathbf{q}_i\right]}{\partial b_{i\tau_s}^k}\right| > \sum_{m \neq k}^{K_i-1}\left|\frac{\partial E\left[M\left(h_T, b_{i\tau}^m; \mathbf{b}_{i\tau}^{-m}\right)|h_{i\tau_s}, \mathbf{b}_{i\tau_s}\mathbf{q}_i\right]}{\partial b_{i\tau_s}^k}\right|, \forall k \tag{8}$$

or

$$\left|1 + \left(\frac{\partial E\left[M\left(h_T, b_{i\tau}^k; \mathbf{b}_{i\tau}^{-k}\right)|h_{i\tau_s}, \mathbf{b}_{i\tau_s}, \mathbf{q}_i\right]}{\partial b_{i\tau_s}^k}\right)^{-1}\right| > \sum_{m \neq k}^{K_i-1}\left|\frac{\partial E\left[M\left(h_T, b_{i\tau}^m; \mathbf{b}_{i\tau}^{-m}\right)|h_{i\tau_s}, \mathbf{b}_{i\tau_s}\mathbf{q}_i\right]}{\partial b_{i\tau_s}^k}\right|, \forall k \tag{9}$$

In other words, the magnitude of the effect of a bid on its own winning probability is greater than the sum of the absolute value of the effects on other tranches.

The components of the matrices, $\mathbf{P}, \mathbf{P}_b, \mathbf{M}, \mathbf{M}_b$ are essentially derivatives of conditional expectations which, as in the single tranche case, can be estimated directly from the data. In the multi-tranche case, we first estimate these matrices and then make use of the matrix equation in equation (7) to estimate the vector of valuations for bidder i.

Assume that K_i is unknown and only the first valuation is known, i.e. v_i^1 is the first valuation. 5.6% of first bids are not equal to K_i or equal to just one bid. I am going to focus on the situation where bidders only learn 1 valuation first.

A.3 Estimators

In this section we present the non-parametric estimators we use. We make use of three estimators: conditional regression estimators, conditional density estimators and conditional distribution estimators. We use Nadaraya Watson mean estimators for the expectation of some variable y conditional on conditional on history h and current bid b:

$$\hat{m}(h, b) = \frac{\sum_{l=1}^{L}\sum_{j=1}^{N_1}\sum_{k=1}^{K_j} K_X\left((x - x_{tj}^l)/\xi_x\right) y_j^l}{\sum_{l=1}^{L=1}\sum_{j=1}^{N_1} K_X\left((x - x_{jt}^l)/\xi_x\right)}$$

where K_X is a multivariate kernel and x is the history vector.

References

S. Athey. Single crossing properties and the existence of pure strategy equilibria in games of incomplete information. *Econometrica*, 69(4):861–889, 2001.

C. Avery. Strategic jump bidding in english auctions. *Review of Economic Studies*, 1998.

P. Bajari and A. Hortacsu. The winner's curse, reserve prices, and endogenous entry: Empirical insights from ebay auctions. *RAND Journal of Economics*, 34(2):329–55, 2003.

O. Birulin and S. Izmalkov. On efficiency of the english auction. *Journal of Economic Theory*, 2011.

A. W. Bowman and A. Azzalini. *Applied Smoothing Techniques for Data Analysis: The Kernel Approach with S-Plus Illustrations (Oxford Statistical Science Series)*. Oxford University Press, USA, 1997.

K.D. Daniel and D.A. Hirshleifer. A theory of costly sequential bidding. Technical report, University of Michigan Business School Working Paper No. 98028, 1998.

S. G. Donald and H. J. Paarsch. Piecewise pseudo-maximum likelihood estimation in empirical models of auctions. *International Economic Review*, 34:121–148, 1993.

B. Elyakime, J.J. Laffont, P. Loisel, and Q. Vuong. First-price sealed-bid auctions with secret reservation prices. *Annales d'Economie et de Statistique*, 1994.

J.R. Groeger. A study of participation in dynamic auctions. *Carnegie Mellon University Working Paper*, 2010.

E. Guerre, I. Perrigne, and Q. Vuong. Optimal nonparametric estimation of first price auctions. *Econometrica*, 68:525–574, 2000.

P. A. Haile and E. Tamer. Inference with an incomplete model of english auctions. *Journal of Political Economy*, 111(1):1–51, 2003.

W. Härdle. *Applied Nonparametric Regression*. Cambridge Books. Cambridge University Press, 1992.

B. Hollifield, R. A. Miller, and P. Sandas. Empirical analysis of limit order markets. *Review of Economic Studies*, 2004.

J. Hörner and N. Sahuguet. Costly signalling in auctions. *The Review of Economic Studies*, 2007.

V. J. Hotz and R. A. Miller. Conditional choice probabilities and the estimation of dynamic models. *Review of Economic Studies*, 60:497–529, 1993.

V J. Hotz, R. A. Miller, S. Sanders, and J. Smith. A simulation estimator for dynamic models of discrete choice. *Review of Economic Studies*, 61(2):265–89, April 1994.

M. Jofre-Bonet and M. Pesendorfer. Estimation of a dynamic auction game. *Econometrica*, 71: 1443–1489, 2003.

C.F. Manski. Dynamic choice in social settings : Learning from the experiences of others. *Journal of Econometrics*, 58(1-2):121 – 136, 1993.

A. Pagan and A. Ullah. *Nonparametric econometrics*. Cambridge University Press, Cambridge; New York, 1999.

P. J. Reny. On the existence of monotone pure-strategy equilibria in bayesian games. *Econometrica*, 79:499–553, 2011.