BIOLOGY AND THE ARGUMENTS OF UTILITY[*]

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Abstract

Why did evolution not give us a utility function that is offspring alone? Why do we care intrinsically about food, for example? We answer these questions on the basis of informational asymmetries. On the one hand, there has been a long evolutionary history establishing that food is advantageous. On the other, individuals possess local information that is highly relevant—where food is located, for instance. If Nature shapes utility to embed the evolutionary knowlege that food is desirable and the individual maximizes expected utility conditional on local information, the optimal choice can be attained.

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1 Introduction

Despite a steady trickle of research on the issue over the last 20 years, it remains unconventional to consider the biological basis of utility or preferences.[1] However, this approach holds out the promise of generating a utility function with the key economic properties of being exogenous and invariant. At first blush, such a biologically derived utility would value commodities in accordance with their impact on fitness—we should value food, for example, in terms of its marginal contribution to biological success. However, on reflection, a serious conceptual problem arises—why have we been made to care about *anything other* than offspring?[2] Why, that is, if we are rational, are we not programmed to like *only* offspring and then to treat food, for example, as purely instrumental, as a means to an end? There seems little doubt that there are many biologically intermediate goods that are nevertheless valued intrinsically. In modern times, indeed, we readily sacrifice expected offspring to increase consumption of other commodities. The recent "demographic transition," during which incomes rose but fertility fell is dramatic *prime facie* evidence on this score.

We consider a solution to this conundrum in terms of informational asymmetries. On the one hand, there are relevant aspects of the environment that Nature "knows" better than the individual. That is, there is available a long evolutionary history during which food exhibited a clear effect in enhancing survival and hence fitness. For an individual to ascertain the effect of food would be expensive and pointless. On the other hand, there are aspects of the environment, perhaps local in time and space, with which the individual is more familiar—the likely location of food, for example. These aspects could have already arisen only rarely, or never, so that natural selection was not brought to bear.

We find an evolutionarily optimal scheme under which utility is first shaped by the information that Nature has. The individual then maximizes the expectation of this utility, conditioning on local information. The action chosen is then the best possible in the light of both sources of information. The marginal utility of an action does not then simply reflect the marginal contribution of the action to fitness, but also reflects the information of Nature. The more precise Nature's information relative to that of the individual, the larger the additional effect.

The focus here is on "primary" arguments rather than "secondary" arguments of utility.

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[1] For example, Robson (2001) argues that a utility function serves as a method for Nature to partially decentralize control, thus achieving a flexible response to novelty.

[2] The number of offspring is assumed to be appropriately adjusted for quality throughout the paper.

That is, we consider arguments that are desired as ends in themselves rather than as means to an end. There are many primary arguments, of course. For example, Linden's list includes food, prestige, temperature, a view, and sex (see Linden (2011)). Bentham lists 26 categories of "simple" pleasures and pains (Bentham (1789)). Perhaps the most salient example of a secondary argument is money.[3] This is fundamentally only a means to an end from the perspective of the individual.[4]

More specifically, the theoretical approach we take is an instance of the principal-agent problem. In this interpretation, the principal, Nature, maximizes expected fitness by choosing the utility function for the agent in the light of partial information Nature has. The agent, the individual, chooses an action to maximize expected utility, conditional on additional local information. This action, and the state, determine fitness. We show that Nature can design the utility function so that the individual chooses the best action conditional on all information. The principal-agent perspective is illuminating, but there no formal techniques or results that are available from that literature.[5] This is because it is not a form of the principal-agent problem that would be meaningful in conventional economics. Most conspicuously, the principal has the power here to shape the agent's preferences. Furthermore, the problem here is apparently trivial in that all actions are contractible. Perhaps most significantly, the principal has information that cannot be directly communicated, despite the parties having parallel interests in the information.

A paper that can be described in analogous terms is Samuelson & Swinkels (2006), who also suppose both Nature and the Agent possess relevant information. In an ideal case, the Agent would maximize expected fitness. The informational structure is quite different, however, and there is an emphasis on second-best solutions that lead to behavioral biases.

The remainder of the paper is as follows. Section 2 outlines the basic model and considers different mechanisms by which evolution could implement the optimal choice. We dismiss the possibility that individuals are born knowing Nature's information as unrealistic. Although it is not unrealistic that utility is contingent on the individual's information to some extent, it is unrealistic that this conditionning is complete and detailed. This leaves us with the

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[3] Not all of Bentham's categories seem clearly primary. For example, he nominates wealth as a simple pleasure, but then defends this choice in terms of what the money can buy.

[4] Since since money is a very familiar means, it induces a rather automatic response. It was once thought that the fMRI signature of money could not be distinguished from that of sex. However, Sescousse *et al* (2010) show that money and sex have subtly distinct fMRI signatures, reflecting the instrumental role of money.

[5] In spirit, ours is a model of delegation. See, for example, Holmstrom (1984), Aghion & Tirole (1997), Dessein (2002), Alonso & Matouschek (2008), and Armstrong & Vickers (2010).

mechanism that we study here where Nature shapes utility in the light of her information, and the individual maximizes expected utility conditional on his.

Section 3 makes some remarks about existence. Although existence is delicate in a continuous formulation, it is a straightforward question in any discrete approximation. We nevertheless stick with the continuous formulation, but place more structure on the problem. The main point of this structure is to obtain further results, but it has the side-effect of ensuring existence. Section 4 proves the main theoretical result of the paper—that optimal choice can be generated by a utility function that has a simple additive form.

Section 5 considers various examples. These are simple with quadratic fitness and normal distributions. They illustrate, for example, that the weight put on Nature's information depends on its precision relative to that of the individual. In a labor-leisure example, the perceived cost of work may vary with the task, although the actual fitness cost does not.

Section 6 shows how the present framework can be readily adapted to consider the evolution of interdependent preferences. We derive a "keeping up with the Jones's effect" in general. In a simple example, we show how greater precision for Nature's information may lead to a greater weight being placed on another individual's choice, which is of interest in the light of the psychological literature on "prepared learning."

2 Model

There are two players: a *principal* (mother nature) and an *agent* (the individual). The agent faces a one-shot opportunity to produce fitness $y \in \mathbb{R}$ (representing quality-adjusted offspring). Fitness is a function of the agent's action $x \in \mathbb{R}^N$ (representing an abstract decision) and an underlying state $\sigma \in \mathbb{R}$:

$$y = \varphi(x, \sigma).$$

The players' have only partial knowledge of the state. The principal and the agent observe, respectively, variables $s, t \in \mathbb{R}$, which are imperfectly correlated with the state. The distribution of σ conditional on (s, t) is given by the p.d.f. $f(\sigma \mid s, t)$, and the distribution of σ conditional on t is given by the p.d.f. $g(\sigma \mid t)$.

We interpret s and t as follows. On the one hand, s represents those aspects of the environment that remain constant over evolutionary time. For primates, for instance, UV light exposure aids the production of Vitamin D (which in turn is essential for health) and fatty food is a vital source of energy. On the other hand, t represents local information that becomes available once the agent is alive. For instance, a hunter observes the position and strength of his prey, as well as the current abundance of alternative sources of food. By observing these local conditions, in the language of Binmore (1994), the individual serves as mother nature's "eyes."

The model proceeds in three stages:

1. The principal observes s and selects a utility function U for the agent (the potential arguments of this function are specified below). The goal of the principal is to maximize the agent's expected fitness.

2. The agent learns his utility function U and selects an action x. The goal of the agent is to maximize his expected utility conditional on available information (also specified below).

3. The state σ is drawn and the payoffs of both players – fitness for the principal and utility for the agent – are realized.

We interpret this setting as a metaphor for the long-run outcome of an evolutionary process (which we do not model) in which the utility functions of humans are heritable and are the object of natural selection. Over time, through a trial-and-error process, those individuals endowed with utility functions that best promote their own fitness dominate the population. Rather than explicitly modelling such trail-and-error process, we grant mother nature the ability to directly "choose" a fitness-maximizing utility function for each human being. In this metaphor, mother nature has had time to learn the constant features of the evolutionary environment (represented by s). That is, evolution has had time to shape the utility function appropriately.

From the principal's perspective, the ideal choice of x solves

$$\max_{x} \quad \mathbb{E}\left[y \mid s, t\right]. \tag{1}$$

For simplicity, we assume that, for each pair (s, t), this problem has a unique solution, denoted $x^*(s, t)$. If a function U implements $x^*(s, t)$ for all (s, t), we say it is optimal.

We consider four scenarios, which differ along two dimensions: (1) the information available to the agent; and (2) the potential arguments of U. For every scenario, we assume that the agent's information includes t, and that the potential arguments of U include the agent's action x and his realized fitness y.

The four scenarios are:

	U depends on t	U is independent of t
Agent knows s	**I**	**II**
Agent ignores s	**III**	**IV**

We are ultimately interested in the fourth scenario, as we believe it best describes the evolutionary path followed by humans. To motivate this scenario, we begin by deriving optimal utility functions for the first three scenarios.

Scenarios I and II: the agent is fully informed

When the agent is informed of both s and t, his objective is

$$\max_x \quad \mathbb{E}\left[U(\cdot) \mid s, t\right]. \tag{2}$$

A trivially optimal utility function is then

$$U(y) \equiv y,$$

which perfectly aligns the agent's objective (2) with the principal's objective (1).

Such resolution, however, does not appear to be a realistic description of humans:

First, our ancestors appear to have been ignorant of important pieces of fitness-relevant information. For instance, a human is not born with knowledge that the compatibility between her immune system and that of her mate is relevant to the fitness of their offspring, or with any direct information about such compatibility.

Second, our actual utility functions appear to have arguments other than fitness. For instance, mates with compatible immune systems appear to smell good (e.g. Wedekind et al., 1995), which amounts to this smell being an argument of utility. In the language of Barash (1979), a pleasant smell produces a "whisper within" that motivates us to select such a mate.

Scenarios I and III: utility is contingent on all information

When U is contingent on the agent's signal t (in addition to the principal's signal s and the agent's action x), a trivially optimal utility function is

$$U(x,s,t) = \begin{cases} 1 & \text{if } x = x^*(s,t), \\ 0 & \text{if } x \neq x^*(s,t). \end{cases}$$

This utility function effectively transforms the agent into an automaton hard-wired to select $x^*(s,t)$ upon observing t.

The actual utility function of a human does appear to respond, at least partially, to local information. For instance, when our peripheral temperature is above (respectively, below) homeostasis, a cold (respectively, hot) drink is pleasurable – see Kandel et al. (2000) for a description of the neurological basis of this phenomenon.

However, the above extreme is not realistic: unlike an automaton, humans constantly form statistical inferences and alter their choices according to those inferences. A speculative argument why we did not evolve to be automatons is that an intelligent animal that infers how to optimally tailor her choices to intricate features of the environment is likely to be more efficient than an animal hard-wired with a immense list of contingent actions.

Scenario IV: the agent is imperfectly informed and utility is "coarse"

This scenario recognizes two realistic features. First, humans ignore important fitness-related information (which we represent in the model by s). Second, we are not perfect automatons (which we represent, for simplicity, by assuming that U is fully independent of t).

The agent's problem becomes

$$\max_x \ \mathbb{E}\left[U(x,y,s) \mid t\right] \quad = \quad \max_x \quad \int U(x,y,s) g(\sigma \mid t) d\sigma,$$

with $y = \varphi(x,\sigma)$. Note that t affects the agent's decision exclusively though the conditional distribution of σ, and s serves exclusively as a parameter of the utility function.

In the remainder of the paper, we restrict attention to the present scenario. This represents a reasonable abstraction, in particular, from allowing utility to be contingent to a limited extent on the agent's information. That this scenario best captures reality may be a matter of historical accident, phylogenetic happenstance. But we will show that this mechanism is constrained optimal, for hunter-gatherers, so there would have been no selection pressure for modification.

3 Existence—Finite Case

In general, the basic theoretical question is whether there exists a function $\bar{U}$ that depends only on x, σ, and s and that satisfies the integral equation

$$\int \varphi(x,\sigma) f(\sigma|s,t) d\sigma = \int \bar{U}(x,\sigma,s) g(\sigma|t) d\sigma,$$

where the functions $\varphi(x,\sigma)$, $f(\sigma|s,t)$ and $g(\sigma|t)$ have been specified exogenously already. If $y = \varphi(x,\sigma)$ were strictly monotonic in σ, for each x, then the existence of a function $U(x,y,s)$ with the desired properties would be a consequence. However, the existence of a solution for $\bar{U}(x,\sigma,s)$ to such a "Fredholm equation of the first kind" (Hochstadt (1973)) is a delicate issue.

The choice of a continuous formulation over a discrete one is just a matter of convenience. Indeed, from a conceptual point of view, a discrete formulation seems unobjectionable. In such a formulation, existence can be readily addressed. Suppose, then that σ and t are restricted to $\{1, ..., S\}$, so, given s, the problem is to find $\bar{U}(x,\sigma,s)$ such that

$$\sum_{\sigma} \overbrace{\varphi(x,\sigma)}^{1\times S}\overbrace{f(\sigma|s,t)}^{S\times S} = \sum_{\sigma} \overbrace{\bar{U}(x,\sigma,s)}^{1\times S}\overbrace{g(\sigma|t)}^{S\times S}, \text{ for all } x.$$

This equation has a unique solution for the row vector $\overbrace{\bar{U}(x,\sigma,s)}^{1\times S}$ if and only if the matrix $\overbrace{g(\sigma|t)}^{S\times S}$ is non-singular, which is a condition that holds generically.[6] More generally, σ and t might be restricted to finite sets of different sizes, $\{1, ..., S\}$ and $\{1, ..., T\}$, say. Perhaps the plausible alternative case is where the number of signals is less than the number of states, so that $S > T$. If the matrix $\overbrace{g(\sigma|t)}^{S\times T}$ has full rank, T, then there is again no problem of

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[6] Generically in $\mathbb{R}^{S^2}$, the matrix $\overbrace{g(\sigma|t)}^{S\times S}$ has a nonzero determinant. Now normalize each column by dividing by the sum of the entries in that column to obtain $\overbrace{g(\sigma|t)}^{S\times S}$. This normalization does not affect the determinant.

existence; rather there is an issue of multiplicity—there are many solutions for $\overbrace{\bar{U}(x,\sigma,s)}^{1\times S}$.

The approach we take below renders this discussion moot. We maintain the continuous formulation, but we sidestep the most general issue of existence by imposing substantive additional structure. This structure is mainly intended to generate a tractable model, and to make available further results, but, as a side effect, it ensures existence.

4 The Main Result

A Monotone Environment

We revert to seeking an optimal utility function of the form $U(x,y,s)$. We first relax the requirement of matching the entire function $\int \varphi(x,\sigma)f(\sigma|s,t)d\sigma$, asking only that expected utility be maximized by $x^*(s,t)$. That is, we require only that, given s,

$$x^*(s,t) = \arg\max_x \int \varphi(x,\sigma)f(\sigma|s,t)d\sigma = \arg\max_x \int U(x,y,s)g(\sigma|t)d\sigma, \text{ for all } t$$

This relaxation of the restrictions on utility is helpful. It is reasonable as well, since there would have been be no biological selection that did more than this.

Assumption 1 *i) The pdf $f(\sigma|s,t)$ is continuously differentiable in (s,t) and strictly increasing, in the sense of first-order stochastic dominance, in s and in t.*

ii) The pdf $g(\sigma|t)$ is continuously differentiable, and strictly increasing, in the sense of first-order stochastic dominance, in t.[7]

iii) Fitness $\varphi(x,\sigma)$ is twice continuously differentiable and strictly concave in x.[8]

iv) Increasing the state σ increases the marginal product of each action so that $\varphi_{x_i\sigma}(x,\sigma) > 0$, for $i = 1,...,N$.

v) Actions are complements in that $\varphi_{x_ix_j}(x,\sigma) \geq 0$, for all $i,j = 1,...,N, i \neq j$.

Lemma 1 *Under Assumption 1, $x_i^*(s,t)$ is differentiable and $\frac{\partial x_i^*(s,t)}{\partial s} > 0$, $\frac{\partial x_i^*(s,t)}{\partial t} > 0$, for all i. Furthermore, if $\hat{x}(t) = \arg\max_x \int \varphi(x,\sigma)g(\sigma|t)d\sigma$, it similarly follows that $\hat{x}_i(t)$ is differentiable and $\frac{\partial \hat{x}_i(t)}{\partial t} > 0$, for all i.*

Proof. *See the Appendix.* ■

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[7] Specifically, we require that $\int v(\sigma)\frac{\partial g(\sigma|t)}{\partial t}d\sigma > 0$ for all continuous and strictly increasing functions v; similarly for $f(\sigma|s,t)$.

[8] Specifically, we require that the matrix of second derivatives of $\varphi(\cdot,\sigma)$ is everywhere negative definite.

The Main Result

In order to set the stage for the main result, consider an arbitrary $x \in \mathbb{R}^N$. We associate to the component x_i the value of $t \in \mathbb{R}$ such that the ith component of $x^*(s,t)$ is x_i.

Definition 1 *Let* $t^i : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ *be such that* $x_i^*(s, t^i(x_i, s)) \equiv x_i$, *for all* i.

We can now demonstrate the main result concerning the existence of a utility function which is maximized by the optimal x.

Theorem 1 *Under Assumption 1, for all* s, t *the following utility function implements* $x^*(s,t)$—

$$U(x,y,s) = \varphi(x,\sigma) + \alpha(x,s) = y + \alpha(x,s)$$

where

$$\alpha(x,s) = \sum_i \alpha^i(x_i, s)$$

and

$$\alpha^i(x_i, s) = -\int_0^{x_i} \int \varphi_{x_i}(x^*(s, t^i(z,s)), \sigma) g(\sigma | t^i(z,s)) d\sigma dz.^9$$

Proof. See Appendix. ■

This is the simplest utility function that delivers $x^*(s,t)$, in that $\alpha(x,s)$ is deterministic (independent of σ), additively separable from y and across the x_i. Note also that such an additive term must depend on both x and s and thus cannot be further simplified.

Proof in One Dimension

When x is one-dimensional, so $N = 1$, the first-order condition becomes

$$\int \varphi_x(x,\sigma) g(\sigma|t) d\sigma - \int \varphi_x(x^*(s, t(x,s)), \sigma) g(\sigma | t(x,s)) d\sigma = 0.$$

Since $t(x,s)$ is the value of t that induces x as the solution to the constrained optimum, it follows that $x^*(s, t(x,s)) = x$. Hence if $x = x^*(s,t)$ then $t(x,s) = t$ and this first-order condition is satisfied. Further, if $x < x^*(s,t)$, then $t(x,s) < t$. Since $g(\sigma|t)$ is increasing in the sense of first-order stochastic dominance, it follows that $\int \varphi_x(x,\sigma) g(\sigma|t(x,s)) d\sigma > \int \varphi_x(x,\sigma) g(\sigma|t) d\sigma$, so that marginal expected utility is then positive. Similarly, if $x > x^*(s,t)$, then marginal expected utility is negative. Hence $x = x^*(s,t)$ is the global maximizer of expected utility.

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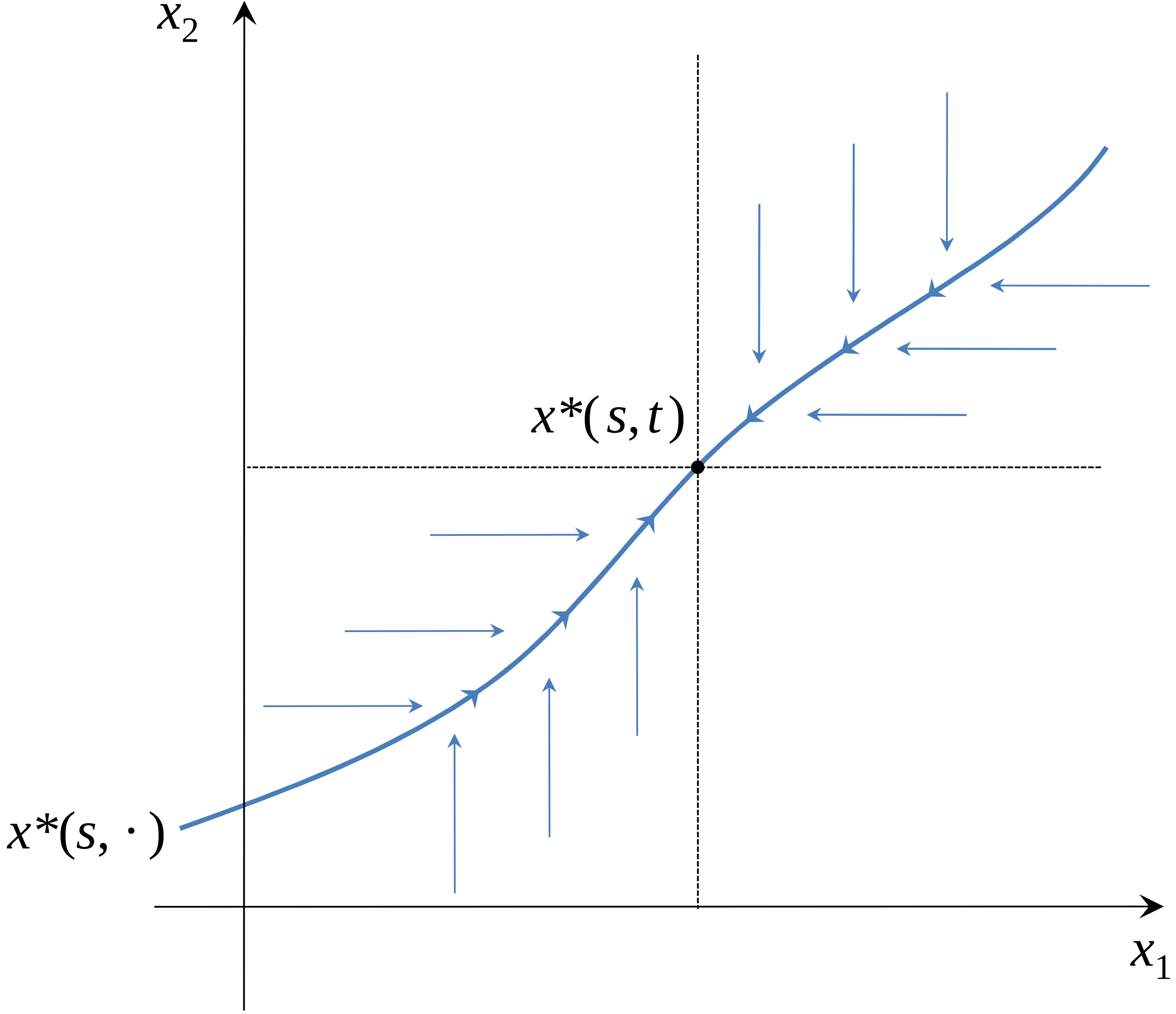
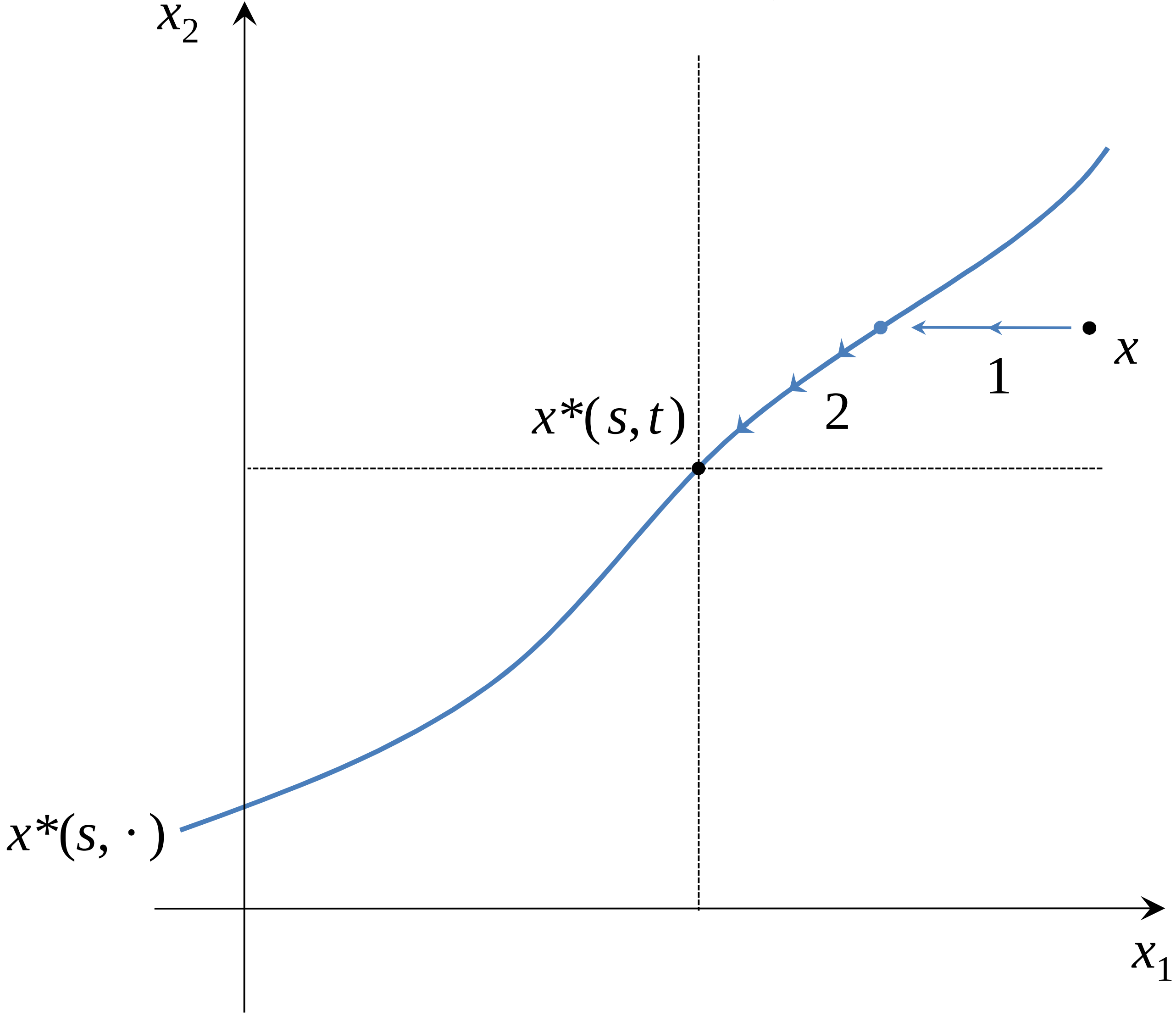
[9] The lower limit in the outer integral is taken to be 0 to ensure convergence.

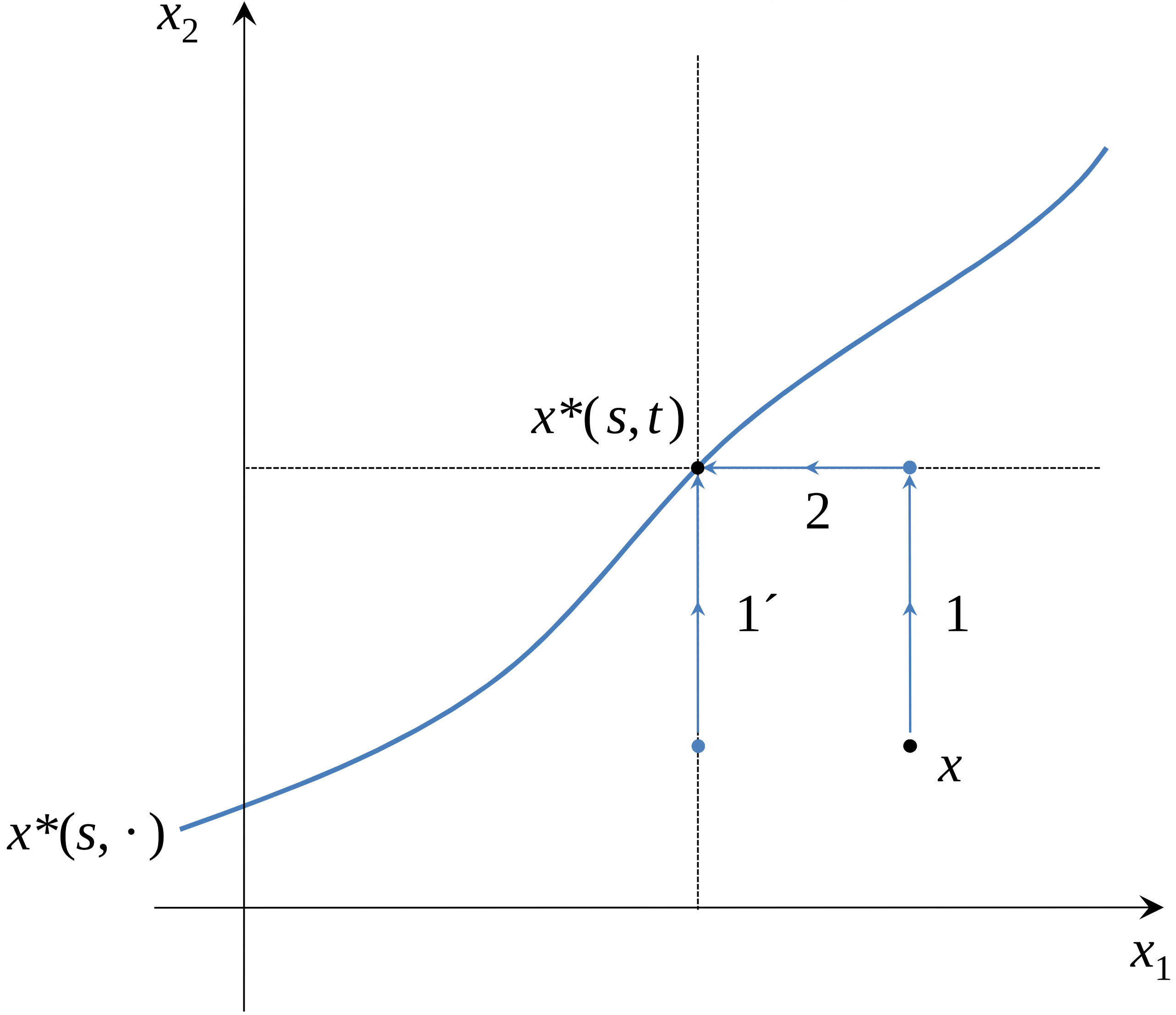
Directions of utility increase



Case 1: x and $x^*(s,t)$ ordered



Case 2: x and $x^*(s,t)$ unordered



Sketch of Proof in Two Dimensions

The case in which x is two-dimensional serves to demonstrate the intuitive idea of the general proof, although the general case is substantially more complex technically. With $N = 2$, the first-order conditions for maximizing expected utility are

$$\int \varphi_{x_i}(x, \sigma) g(\sigma|t) d\sigma = \int \varphi_{x_i}(x^*(s, t^i(x_i, s)), \sigma) g(\sigma|t^i(x_i, s)) d\sigma \text{ for } i = 1, 2.$$

As required, these first-order conditions are satisfied by $x = x^*(s, t)$ since this implies $t^i(x_i, s) = t$, for $i = 1, 2$.

Indeed, it follows that $x = x^*(s, t)$ is the unique global maximum. To see why, consider any $x \neq x^*(s, t)$. Figure 1 describes the directions in which expected utility unambiguously increases. These directions can be established by signing the corresponding derivatives. These directions lie in the NE quadrant and the SW quadrant relative to $x^*(s, t)$. There are two representative cases to consider. Case i) $x \geq x^*(s, t)$. In this case, Figure 2 sketches how it is possible to move from x to $x^*(s, t)$ in a fashion that increases expected utility. That is, first reduce the coordinate that is too large relative to being on the $x^*(s, \cdot)$ curve. Then move along this curve $x^*(s, \cdot)$ to $x^*(s, t)$. (The case in the SW quadrant where $x \leq x^*(s, t)$ is analogous.) Case ii) $x_1 \geq x_1^*(s, t)$ and $x_2 < x_2^*(s, t)$. Refer to Figure 3. Consider a path from x to $x^*(s, t)$ that first increases x_2 to $x_2^*(s, t)$, as in Step 1 in Figure 3, and then reduces x_1 to $x_1^*(s, t)$, as in Step 2. Step 2 is a limiting case from Figure 1 where expected utility must increase, but Step 1 is apparently ambiguous. Consider, however, Step 1', where x_2 increases to $x_2^*(s, t)$, with $x_1 = x_1^*(s, t)$. Expected utility must increase in Step 1' since it is again a limiting case from Figure 1. The assumption that $\varphi_{x_1 x_2}(x, \sigma) \geq 0$ implies that expected utility must increase by at least as much in Step 1 as it does in Step 1', so it must increase in the two-step process—first Step 1 and then Step 2. The case in which x lies in the NW quadrant is analogous, so the sketch of the proof is complete.

Observations

Note that this particular decomposition of utility generates a particular tradeoff between y and x, so the individual would sacrifice expected offspring for, say, more food. This decomposition into y and x is not unique, however, since y is itself a function of x as $y = \varphi(x, \sigma)$. Utility could then be formulated as a function of x and σ alone. However, utility cannot be simply y.

Claim 1 *Generically, the utility function* $U = y = \varphi(x, \sigma)$ *is not optimal.*

Proof. Suppose otherwise, so that $x^*(s,\cdot) = \hat{x}(\cdot)$. It follows that

$$\int \varphi_{x_i}(\hat{x}(t),\sigma) f(\sigma|s,t) d\sigma = 0$$

This equality can hold for at most one value of s since

$$\frac{\partial}{\partial s} \int \varphi_{x_i}(\hat{x}(t),\sigma) f(\sigma|s,t) d\sigma = \int \varphi_{x_i}(\hat{x}(t),\sigma) \frac{\partial f(\sigma|s,t)}{\partial s} d\sigma > 0.$$

■

Remark 1 *A measure of the contribution of Nature to the Individual's decision is*

$$\int \varphi_{x_i}(x^*(s,t),\sigma) g(\sigma|t) d\sigma.$$

This measure is expressed purely in terms of the fitness function and so is independent of the utility representation. It is a measure of how much the optimal choice of $x^*(s,t)$ involves "bending" the Individual's preferences away from expected fitness, generating then derivatives of expected offspring that differ from zero.

5 Examples

Consider first the case of $N = 1$ and a simple quadratic fitness function, so that $\varphi(x,\sigma) = -(x-\sigma)^2$. Suppose further that the state has a diffuse prior, so that $\sigma \sim N(\mu_0, \infty)$ loosely speaking. Suppose that the variables s and t are distributed as $s \sim N(\sigma, v_s)$ and $t \sim N(\sigma, v_t)$, where s, and t are independent.

It follows that the mean of the posterior distribution after observing t alone is $\mu' = t$.[10] The mean after observing both s and t is

$$\mu'' = \lambda s + (1-\lambda)t \text{ where } \lambda = \frac{\frac{1}{v_s}}{\frac{1}{v_s} + \frac{1}{v_t}} \in (0,1),$$

so that λ is the precision of Nature's information relative to that of the Individual. With a quadratic fitness, the optimal x is, in general, the mean of the distribution of σ. Hence, it follows that $x^*(s,t) = \lambda s + (1-\lambda)t$. Using the general formula $\alpha(x,s) = -\int^x \int \varphi_x(z,\sigma) g(\sigma|t(z,s)) d\sigma dz$

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[10] Assuming that the variance of the prior distribution of σ is infinite implies that the mean of this prior is irrelevant.

where $\lambda s + (1-\lambda)t(z,s) = z$, so $t(z,s) = \frac{z-\lambda s}{1-\lambda}$, we have

$$U(x,y,s) = y - \frac{\lambda}{1-\lambda}(x-s)^2.$$

The weight placed on the term $-(x-s)^2$ reflects how the individual's choices are "bent" away from those that maximize expected fitness. If, for example, $v_s = \infty$ or $v_t = 0$, so the individual's signal is infinitely more precise than is Nature's, then $u(x,y,s) = y$ and no weight is put on $-(x-s)^2$ at all. In this limiting case, the individual maximizes expected fitness, but only because Nature's information is unimportant. At the other extreme, if $v_s = 0$ or $v_t = \infty$, so Nature's signal is infinitely more precise than is the individual's, zero weight is given to y and full weight is given to $-(x-s)^2$. In this limiting case, the Individual makes the choice that is optimal purely in the light of Nature's information, that is, $x = s$.

In this example,

$$\int \varphi_{x_i}(x^*(s,t),\sigma)g(\sigma|t)d\sigma = \lambda(t-s).$$

Hence, relative to maximizing expected fitness conditional only on t, the Individual is urged to choose a larger x when $t < s$ and to choose a smaller x when $t > s$.

Recall that $\hat{x}(t)$ is the fitness-maximizing choice given only the signal t—

$$\hat{x}(t) = \arg\max_x \int \varphi(x,\sigma)g(\sigma|t)d\sigma,$$

which is increasing in t. Then a general measure of the autonomy of the Individual is

$$\frac{\frac{\partial x^*(s,t)}{\partial t}}{\frac{d\hat{x}(t)}{dt}} = a(s,t)$$

which derives from the fitness function alone and is therefore independent of the representation of utility.

In this example, $\hat{x}(t) = t$, so

$$a(s,t) = 1-\lambda$$

and the Individual's autonomy is the precision of his signal relative to that of Nature.

An Example with Two Dimensional State and Information

To expand on the point that the bending of the individual's utility depends on the relative precision of Nature's information, consider an example where the state has two components, each of which has a diffuse prior, so that $\sigma_i \sim N(\mu_0^i, \infty)$, $i = 1, 2$.[11] Nature's information is given by $s_i \sim N(\sigma_i, v_s^i)$, for $i = 1, 2$. Similarly, the Individual's information is $t_i \sim N(\sigma, v_t^i)$, $i = 1, 2$. The random variables σ_i, s_i and t_i, $i = 1, 2$, are all independent.

Further, the Individual's action is $x = (x_1, x_2)$ and fitness is

$$y = \varphi(x, \sigma) = -(x_1 - \sigma_1)^2 - (x_2 - \sigma_2)^2.$$

This quadratic function is maximized by choosing each x_i as the mean of the distribution of σ_i, $i = 1, 2$. It follows that

$$x_i^*(s, t) = \bar{\sigma}_i = \lambda_i s_i + (1 - \lambda_i) t_i \text{ where } \lambda_i = \frac{\frac{1}{v_s^i}}{\frac{1}{v_s^i} + \frac{1}{v_t^i}}, \; i = 1, 2,$$

so λ_i is the precision of Nature's signal relative to the Individual's, for each component $i = 1, 2$.

It is then straightfoward to show that an optimal utility function is

$$u(x, y, s) = y - \frac{\lambda_1}{1 - \lambda_1}(x_1 - s_1)^2 - \frac{\lambda_2}{1 - \lambda_2}(x_2 - s_2)^2$$

This example demonstrates that the Individual's two choices are *independently* "bent" away from those that maximize expected fitness, according to the relative precision of the corresponding signals.

An Anecdote

To illustrate the implications of the relative precision of the information held by Nature and that available to the individual, consider the situation faced by an individual hiking across Baffin Island. There are two main dangers faced on such a trek—polar bears and rivers. Of these, bears are more psychologically salient. A hiker inevitably worries more about encountering a bear than encountering a river. However, although rivers are less dangerous per encounter, there are many more encounters with them and the aggregate danger posed by rivers exceeds that posed by polar bears. One needs to take river crossings seriously.

In terms of the currrent model, it seems reasonable, on the one hand, that the information

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[11] This example goes beyond the general framework in allowing the state space to be two-dimensional. It sidesteps some of the complications that arise in general by the assumption of independence of the distributions and the assumption that fitness is additively separable.

held by Nature concerning bears is rather precise relative to any information available to the individual. Indeed, polar bears belong to the category of large carnivores with sharp yellow fangs and claws that one is instinctively afraid of. There are not many sufficiently good reasons to voluntarily approach a polar bear. On the other hand, rivers can be useful. Thus the information held by the agent may well be decisive. There are indeed many sufficiently good reasons to cross a river, and the overall evolutionary strategy is to rely to a much larger extent on the agent's assessment of the local circumstances.

An Example of Labor-Leisure Choice

As a specific economic application, consider the following example of labor-leisure choice. Suppose that fitness is given by $y = \varphi(x, \sigma) = \sigma x - c(x)$, where x is effort, σx is earnings and $c(x)$ is the cost of effort. Suppose $E[\sigma|s,t] = \lambda s + (1-\lambda)t$ and $E[\sigma|t] = t$ as derived before. The first-order condition characterizing $x^*(s,t)$ is

$$\lambda s + (1-\lambda)t = c'(x^*(s,t)).$$

An optimal utility is then

$$u(y, x, s) = y - \frac{\lambda}{1-\lambda}[c(x) - sx] = \sigma x - \frac{c(x)}{1-\lambda} + \frac{\lambda s x}{1-\lambda}.$$

It is readily verified that the first-order condition for $x^*(s,t)$ is satisfied by maximizing expected utility conditional on t alone.

In this case, the Individual perceives a cost of effort that varies as λ varies with the task, even though the actual fitness cost of effort is invariant. It seems psychologically plausible, for example, that the perceived effort involved in walking 10km is less when there is a magnificient view than when walking to work.

6 Application to a Concern with Status—A Sketch

Finally, consider how the present model yields a framework for considering the evolution of interdependent preferences.[12] Suppose that individuals obtain independent signals concern-

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[12] Samuelson (2004) is a key antecedent in the literature on relative consumption. He also supposes that the observable but inherently irrelevant consumption of others conveys information about an unobservable but relevant state. In contrast to the model here, Samuelson adopts a different informational structure. In particular, there is no counterpart to the information of Nature. In addition, he focusses on second-best resolutions since the individual misperceives the precision of his own signal relative to those of others.

ing a shared local environment. This implies that the signal of another individual is useful to any given individual. Such signals cannot be observed directly, but must be inferred from observed choices.

For simplicity, consider two individuals who choose sequentially, with Individual 1 choosing in the light of his signal alone, and Individual 2 choosing in the light of his signal and Individual 1's choice. Given the appropriate utility function, Individual 1 effectively maximizes $\int \varphi(x_1,\sigma)f(\sigma|s,t_1)d\sigma$ by choosing $x_1^*(s,t_1)$. This can be achieved with the utility

$$\begin{aligned} U_1(x_1,y_1,s) &= y_1+\alpha(x_1,s)=\varphi(x_1,\sigma)+\alpha(x_1,s) \\ \text{where } \alpha(x_1,s) &= -\int_0^{x_1}\int \varphi_x(z,\sigma)g(\sigma|t^1(z,s))d\sigma dz, \text{ for all } t, \text{ given } s, \end{aligned}$$

exactly as in Theorem 1. Under Assumption 1, it follows that $\frac{\partial x_1^*(s,t_1)}{\partial t_1}>0$.

Consider now Individual 2's ideal choice, $x_2^*(s,t_1,t_2)$, given both t_1 and t_2. This is the choice that maximizes $\int \varphi(x_2,\sigma)h(\sigma|s,t_1,t_2)d\sigma$. In this expression, h represents the pdf for σ conditional on s, t_1, and t_2. Assume that increases in any of s, t_1 and t_2 increase the distribution for σ in the sense of first-order stochastic dominance. The problem facing Individual 2 remains analogous to that described in detail in Section 4, with s and t_1 togther playing the role that was played by s alone and t_2 playing the role of t.

Under Assumption 1, it follows that $\frac{\partial x_2^*(s,t_1,t_2)}{\partial t_1}>0$ and $\frac{\partial x_2^*(s,t_1,t_2)}{\partial t_2}>0$. Furthermore, extending Theorem 1 to this context, it follows that there exists a utility function whose expectation is uniquely maximized by $x_2^*(s,t_1,t_2)$ of the form

$$\bar{U}_2(x_2,y_2,s,t_1)=y_2+\bar{\beta}(x_2,s,t_1)=\varphi(x_2,\sigma)+\bar{\beta}(x_2,s,t_1),$$

where $\bar{\alpha}(x_2,s,t_1)=-\int^{x_2}\int \varphi_x(z,\sigma)g(\sigma|t^2(z,s,t_1))d\sigma dz$, for all t, given s, and $t^2(z,s,t_1)$ is such that $x_2^*(s,t_1,t^2(z,s,t_1))=z$. In terms of x_1, which Individual 2 observes directly, this expression becomes

$$U_2(x_2,y_2,s,x_1)=y_2+\beta(x_2,s,x_1)=\varphi(x_2,\sigma)+\beta(x_2,s,x_1),$$

where $U_2(x_2,y_2,s,x_1)=\bar{U}_2(x_2,y_2,s,{x_1^*}^{-1}(x_1,s))$ and $\beta(x_2,s,x_1)=\bar{\beta}(x_2,s,{x_1^*}^{-1}(x_1,s))$, where ${x_1^*}^{-1}(\cdot,s)$ is the inverse function of $x_1^*(s,\cdot)$.

The following result captures a "keeping up with the Jones's" effect.

Proposition 1 *Under Assumption 1,*

$$\frac{\partial^2 \beta(x_2, s, x_1)}{\partial x_2 \partial x_1} > 0$$

so an increase in the action taken by Individual 1 spurs an increase in the optimal action taken by Individual 2.

Proof. Under Assumption 1,

$$\frac{\partial \beta(x_2, s, x_1)}{\partial x_2} = -\int \varphi_x(x_2, \sigma) g(\sigma | t^2(x_2, s, \, x_1^{*-1}(x_1, s)) d\sigma$$

which is an increasing function of x_1 since $t^2(x_2, s, t_1)$ is decreasing in t_1, $x_1^{*-1}(x_1, s)$ is increasing in x_1, and t^2 increases g in the sense of first-order stochastic dominance. ■

An Example

Suppose fitness for each individual is of the form $\varphi(x, \sigma) = -(x - \sigma)^2$. Suppose that $\sigma \sim N(\mu_0, \infty)$, $s \sim N(\sigma, v_s)$, and $t_i \sim N(\sigma, v_t)$, $i = 1, 2$, where s, t_1 and t_2 are independent. The optimal choice for 1 is then $x_1 = \lambda_1 s + (1 - \lambda_1) t_1$, where $\lambda_1 = \frac{\frac{1}{v_s}}{\frac{1}{v_s} + \frac{1}{v_t}} \in (0, 1)$. This choice can be implemented by the utility function

$$U_1(x_1, y_1, s) = y_1 - \frac{\lambda_1}{1 - \lambda_1}(x - s)^2,$$

where $y_1 = -(x_1 - \sigma)^2$.

Consider now Individual 2. The optimal choice for 2 in the light of s, t_1 and t_2 is $x_2 = \lambda_2 s + (1 - \lambda_2)\frac{t_1 + t_2}{2}$, where $\lambda_2 = \frac{\frac{1}{v_s}}{\frac{1}{v_s} + \frac{2}{v_t}} \in (0, 1)$. This choice of x_2 can implemented by the utility function

$$U_2 = y_2 - \frac{1 + \lambda_2}{1 - \lambda_2}(x_2 - x_1)^2.$$

The term additional to y_2 penalizes any choice of x_2 that diverges from x_1. In terms of x_1, which is all that 2 observes, we have that $x_2 = (1 - \lambda_2)\frac{t_2}{2} + \frac{1 - \lambda_2}{2(1 - \lambda_1)} x_1$. It follows that an increase in x_1 spurs a positive but less-than-matching increase in x_2.[13]

There are some relevant results from the psychology literature on prepared learning. Monkeys do not exhibit an inborn fear of snakes or of flowers (less surprisingly). However, they readily learn to be afraid of snakes if they observe another monkey acting fearfully in

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[13] Note that $\frac{1 - \lambda_2}{2(1 - \lambda_1)} = \frac{\frac{1}{v_s} + \frac{1}{v_t}}{\frac{1}{v_s} + \frac{2}{v_t}} \in (0, 1)$.

the presence of a snake. It is much more difficult to teach them similarly to be afraid of flowers (Cook and Mineka (1989), for example).

The example sheds light on these phenomena. Suppose the relative precision of Nature's information increases. It follows that Individual 2 is *more* responsive to Individual 1's choice.[14] That is, individuals are *more* influenced by peer choices concerning snakes, where Nature's information is precise, than they are by those concerning flowers, where it is less so.

7 Conclusions

The motivating question that we began with was: Given we are rational, why is utility not simply offspring? We formulated a principal-agent model in which both Nature and the individual possessed information relevant to the agent's choices. One abstract option would be for Nature to explicitly and directly communicate her information to the agent, who could then choose optimally by maximizing expected fitness in the light of all the information. This option, however, is not realistic. Another abstract option would be for Nature to design a utility function that is contingent on all possible information that the individual may acquire. This option too can generate the optimal choice. Such an individual would function as an automaton, with no interest in statistical inference, since choice has already encoded all statistical inference. Although there is evidence that utility itself may adapt to the environment to some extent, it is unrealistic that this effect is, or could ever be, as complete as would be required to generate fully optimal choice.

Finally, we consider the option that Nature shapes the utility function in the light of the information Nature has, in a non-contingent fashion. The individual then maximizes the expectation of this utility conditional on the additional information that she receives. This option is the most realistic, and we show that it too would have generated optimal choice in hunter-gatherer societies. That this is the method actually employed, despite the existence of more direct abstract ways of achieving the same end, may then have been harmless phylogenetic happenstance.

We considered why utility is a "whispering within" urging individuals to take actions that reflect the evolutionary wisdom of a multitude of ancestors, in addition to accounting for local on-the-spot information. We derived the optimal way of combining the two sets

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[14] That is, $\frac{\frac{1}{v_s}+\frac{1}{v_t}}{\frac{1}{v_s}+\frac{2}{v_t}}$ is decreasing in $\frac{1}{v_s}/\frac{1}{v_t}$.

of information, showing that the bending of the utility function away from fitness reflects the weight that should be put on Nature's information. This weight (the loudness of the whisper) derives from the precision of Nature's information relative to that of the individual.

In vastly changed modern conditions—not the least of which involves contraception—the central mechanism may no longer be evolutionarily optimal. Nevertheless, the legacy of this mechanism is plausible—namely utility functions that convey evolutionary information, not the explicit transmission of the raw information, and not utility functions that destroy any interest in statistical inference.

8 Appendix—Proofs

8.1 Proof of Lemma 1.

The function $x^*(s,t)$ is characterized by the first-order conditions

$$\int \varphi_{x_i}(x^*(s,t),\sigma) f(\sigma|s,t)d\sigma = 0, \text{ for } i = 1, ..., N.$$

Hence

$$\sum_j A_{ij} \frac{\partial x_j^*(s,t)}{\partial t} = b_i \text{ for } i = 1, ..., N,$$

where

$$A_{ij} = \int \varphi_{x_i x_j}(x^*(s,t),\sigma) f(\sigma|s,t)d\sigma \text{ and } b_i = -\int \varphi_{x_i}(x^*(s,t),\sigma) \frac{\partial f(\sigma|s,t)}{\partial t} d\sigma < 0.$$

The $n \times n$ matrix A is symmetric, negative definite, and has non-negative off-diagonal elements. Hence $-A$ is a Stieltjes matrix, which must have a symmetric and non-negative inverse (see Varga (1962, p. 85)). Hence A^{-1} must be a symmetric and non-positive matrix. Since

$$\begin{bmatrix} \frac{\partial x_1^*(s,t)}{\partial t} \\ ... \\ \frac{\partial x_N^*(s,t)}{\partial t} \end{bmatrix} = A^{-1}b,$$

it follows that $\frac{\partial x_j^*(s,t)}{\partial t} \geq 0$, for $j = 1, ..., N$. Further, since A^{-1} is non-singular, it cannot have any row be entirely zero, and it must indeed be that $\frac{\partial x_j^*(s,t)}{\partial t} > 0$, for $j = 1, ..., N$.

The proof that $\frac{\partial x_j^*(s,t)}{\partial s} > 0$, for $j = 1, ..., N$ is analogous.

8.2 Proof of Theorem 1

Select an arbitrary s. To simplify notation, we drop the dependence of $x^*(\cdot)$ and $\alpha(\cdot)$ on s. Define, for all x and t,

$$V(x;t) = \mathbb{E}\left[\varphi(x,\sigma) \mid t\right] + \alpha(x),$$

where the expectation is taken over σ using the pdf g.

We wish to show that $V(x^*(t), t) > V(x, t)$ for all t and all $x \neq x^*(t)$.

Remark 2 *Properties of $V(x;t)$. For all i and all t:*

1. $\frac{\partial}{\partial x_i} V(x_i, x_{-i}; t)$ *is weakly increasing in x_{-i} for all x_i.*
2. $\frac{\partial}{\partial x_i} V(x;t)$ *is strictly increasing in t for all x.*
3. $\frac{\partial}{\partial x_i} V(x^*(t);t) = 0$.

Proof. From the definitions of $V(\cdot)$ and $\alpha(\cdot)$ we obtain

$$\frac{\partial}{\partial x_i} V(x;t) = \mathbb{E}\left[\varphi_{x_i}(x_i, x_{-i}, \sigma) \mid t\right] - \mathbb{E}\left[\varphi_{x_i}(x^*(t^i(x_i)), \sigma) \mid t^i(x_i)\right]. \tag{3}$$

For property 1, note that the first term on the R.H.S. of (3) is weakly increasing in x_{-i} (since, by assumption, $\frac{\partial^2}{\partial x_i \partial x_j}\varphi(x,\sigma) \geq 0$ for all x, σ and all $i \neq j$), and the second term is independent of x_{-i}.

For property 2, note that the first term on the R.H.S. of (3) is increasing in t (since, by assumption, $\frac{\partial^2}{\partial x_i \partial \sigma}\varphi(x,\sigma) > 0$ for all x, σ and all i, and the pdf g is increasing in t in first-order stochastic dominance), and the second term is independent of t.

For property 3, note that $t^i(x_i^*(t)) = t$ (by definition) and therefore

$$\mathbb{E}\left[\varphi_{x_i}(x^*(t), \sigma) \mid t\right] = \mathbb{E}\left[\varphi_{x_i}(x^*(t^i(x_i)), \sigma) \mid t^i(x_i)\right].$$

■

Now select an arbitrary t and an arbitrary $x \neq x^*(t)$. Let $\tau_i = t^i(x_i)$ for all i. Assume, WLOG, that $\tau_1 \leq \tau_2 \leq \ldots \leq \tau_N$. Also, select two numbers τ_0 and τ_{N+1} such that $\tau_0 \leq \min\{\tau_1, t\}$ and $\tau_{N+1} \geq \max\{\tau_N, t\}$.

Define

$$\begin{aligned} M_+ &= \{i : x_i \geq x_i^*(t)\}, \\ M_- &= \{i : x_i < x_i^*(t)\}. \end{aligned}$$

Note that

$$V(x \vee x^*(t); t) =$$
$$V(x; t) + \sum_{n \in M_-} \int_{\tau_n}^{\min\{\tau_{n+1}, t\}} \frac{d}{d\tau} V(x^*_{i \leq n}(\tau), x_{i>n}; t) d\tau,$$

and

$$V(x \vee x^*(t); t) =$$
$$V(x^*(t); t) + \sum_{n \in M_+} \int_{\max\{\tau_{n-1}, t\}}^{\tau_n} \frac{d}{d\tau} V(x^*_{i \in M_-}(t), x_{i \in M_+^n}, x^*_{i \geq n}(\tau); t) d\tau,$$

where M_+^n is defined as the set $\{i \in M_+ : i < n\}$.

It follows that

$$\begin{aligned} V(x^*(t); t) - V(x; t) = & \sum_{n \in M_-} \int_{\tau_n}^{\min\{\tau_{n+1}, t\}} \frac{d}{d\tau} V(x^*_{i \leq n}(\tau), x_{i>n}; t) d\tau \\ & - \sum_{n \in M_+} \int_{\max\{\tau_{n-1}, t\}}^{\tau_n} \frac{d}{d\tau} V(x^*_{i \in M_-}(t), x_{i \in M_+^n}, x^*_{i \geq n}(\tau); t) d\tau. \end{aligned} \tag{4}$$

We begin by showing that $V(x^*(t); t) \geq V(x; t)$, for which we proceed in two steps.

Step 1. We show that all terms in the first sum of (4) are nonnegative. Fix $n \in M_-$. For all $\tau \in (\tau_n, \min\{\tau_{n+1}, t\})$ (a possibly empty interval) we have

$$\begin{aligned} & \frac{d}{d\tau} V(x^*_{i \leq n}(\tau), x_{i>n}; t) = \\ & \sum_{j \leq n} \frac{\partial}{\partial x_j} V(x^*_{i \leq n}(\tau), x_{i>n}; t) \cdot \frac{d}{d\tau} x^*_j(\tau) \\ & \geq \sum_{j \leq n} \frac{\partial}{\partial x_j} V(x^*_{i \leq n}(\tau), x_{i>n}(\min\{\tau_{n+1}, t\}); t) \cdot \frac{d}{d\tau} x^*_j(\tau) \\ & \geq \sum_{j \leq n} \frac{\partial}{\partial x_j} V(x^*_{i \leq n}(\tau), x_{i>n}(\min\{\tau_{n+1}, t\}); \min\{\tau_{n+1}, t\}) \cdot \frac{d}{d\tau} x^*_j(\tau) > 0. \end{aligned} \tag{5}$$

(Recall that $\frac{d}{d\tau} x^*_j(\tau) > 0$ for all j.)

The first weak inequality in (5) follows from property 1 of the remark: $x_i \geq x_i(\min\{\tau_{n+1}, t\})$

for all $i > n$ implies

$$\frac{\partial}{\partial x_j} V(x^*_{i \leq n}(\tau), x_{i>n}; t) \geq \\ \frac{\partial}{\partial x_j} V(x^*_{i \leq n}(\tau), x_{i>n}(\min\{\tau_{n+1}, t\}); t) \text{ for all } j \leq n.$$

The second weak inequality in (5) follows from property 2 of the remark: $t \geq \min\{\tau_{n+1}, t\}$ implies

$$\frac{\partial}{\partial x_j} V(x^*_{i \leq n}(\tau), x_{i>n}(\min\{\tau_{n+1}, t\}); t) \geq \\ \frac{\partial}{\partial x_j} V(x^*_{i \leq n}(\tau), x_{i>n}(\min\{\tau_{n+1}, t\}); \min\{\tau_{n+1}, t\}) \text{ for all } j \leq n.$$

Finally, the strict inequality in (5) follows from combining all three properties of the remark: $\tau < \min\{\tau_{n+1}, t\}$ implies $x^*_i(\tau) < x^*_i(\min\{\tau_{n+1}, t\})$ for all i and therefore

$$\frac{\partial}{\partial x_j} V(x^*_{i \leq n}(\tau), x_{i>n}(\min\{\tau_{n+1}, t\}); \min\{\tau_{n+1}, t\}) \geq \\ \frac{\partial}{\partial x_j} V(x^*(\tau); \min\{\tau_{n+1}, t\}) > \frac{\partial}{\partial x_j} V(x^*(\tau); \tau) = 0 \text{ for all } j \leq n.$$

Step 2. We show that all terms in the second sum of (4) are nonpositive. Fix $n \in M_+$. Note that for all $\tau \in (\max\{\tau_{n-1}, t\}, \tau_n)$ (a possibly empty interval) we have

$$\begin{aligned} &\frac{d}{d\tau} V(x^*_{i \in M_-}(t), x_{i \in M^n_+}, x^*_{i \geq n}(\tau); t) = \\ &\sum_{j \geq n} \frac{\partial}{\partial x_j} V(x^*_{i \in M_-}(t), x_{i \in M^n_+}, x^*_{i \geq n}(\tau); t) \cdot \frac{d}{d\tau} x^*_j(\tau) \\ &\leq \sum_{j \geq n} \frac{\partial}{\partial x_j} V(x^*_{i<n}(\max\{\tau_{n-1}, t\}), x^*_{i \geq n}(\tau); t) \cdot \frac{d}{d\tau} x^*_j(\tau) \\ &\leq \sum_{j \geq n} \frac{\partial}{\partial x_j} V(x^*_{i<n}(\max\{\tau_{n-1}, t\}), x^*_{i \geq n}(\tau); \max\{\tau_{n-1}, t\}) \cdot \frac{d}{d\tau} x^*_j(\tau) < 0. \end{aligned} \tag{6}$$

The first weak inequality in (6) follows from property 1 of the remark: $\left(x^*_{i \in M_-}(t), x_{i \in M^n_+}\right) \leq$

$x^*_{i<n}(\max\{\tau_{n-1}, t\})$ implies

$$\begin{aligned} \frac{\partial}{\partial x_j} V(x^*_{i\in M_-}(t), x_{i\in M_+^n}, x^*_{i\geq n}(\tau); t) &\leq \\ \frac{\partial}{\partial x_j} V(x^*_{i<n}(\max\{\tau_{n-1}, t\}), x^*_{i\geq n}(\tau); t) \text{ for all } j &\geq n. \end{aligned}$$

The second weak inequality in (6) follows from property 2 of the remark: $t \leq \max\{\tau_{n-1}, t\}$ implies

$$\frac{\partial}{\partial x_j} V(x^*_{i<n}(\max\{\tau_{n-1}, t\}), x^*_{i\geq n}(\tau); t) \leq$$
$$\frac{\partial}{\partial x_j} V(x^*_{i<n}(\max\{\tau_{n-1}, t\}), x^*_{i\geq n}(\tau); \max\{\tau_{n-1}, t\}) \text{ for all } j \geq n.$$

Finally, the strict inequality in (6) follows from combining all three properties of the remark: $\tau > \max\{\tau_{n-1}, t\}$ implies $x^*_i(\tau) > x^*_i(\max\{\tau_{n-1}, t\})$ for all i and therefore

$$\frac{\partial}{\partial x_j} V(x^*_{i<n}(\max\{\tau_{n-1}, t\}), x^*_{i\geq n}(\tau); \max\{\tau_{n-1}, t\}) \leq$$
$$\frac{\partial}{\partial x_j} V(x^*(\tau); \max\{\tau_{n-1}, t\}) < \frac{\partial}{\partial x_j} V(x^*(\tau); \tau) = 0 \text{ for all } j \geq n.$$

We now show that $V(x^*(t); t) > V(x; t)$. Since $x \neq x^*(t)$ there exists either an $n \in M_-$ such that the interval $(\tau_n, \min\{\tau_{n+1}, t\})$ is nonempty, or an $n \in M_+$ such that the interval $(\max\{\tau_{n-1}, t\}, \tau_n)$ is nonempty (or both). In the former case, it follows from step 1 above that at least one of the integrals in the first sum of (4) is positive. In the latter case, it follows from step 2 above that at least one of the integrals in the second sum of (4) is negative.

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