Catching Up and Falling Behind

Nancy L. Stokey

University of Chicago

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Abstract

This paper studies the interaction between technology, which flows in from abroad, and human capital, which is accumulated domestically, as the twin engines of growth in a developing economy. The model displays two types of long run behavior, depending on policies and initial conditions. One is sustained growth, where the economy keeps pace with the technology frontier. The other is stagnation, where the economy converges to a minimal technology level that is independent of the world frontier. Transitions to the balanced growth path display features seen in modern growth miracles: a high savings rate and rapid investment in education.

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This paper develops a model of growth that can accommodate the enormous differences in observed outcomes across countries and over time: periods of rapid growth as less developed countries catch up to the income levels of those at the frontier, long periods of sustained growth in developed countries, and substantial periods of decline in countries that at one time seemed to be catching up. The sources of growth are technology, which flows in from abroad, and human capital, which is accumulated domestically. The key feature of the model is the interaction between these two forces.

Human capital is modeled here as a private input into production, accumulated using the agent's own time (current human capital) and the local technology as inputs. Thus, improvements in the local technology affect human capital accumulation directly, by increasing the productivity of time spent in that activity. Improvement in the local technology also raise the local wage, increasing both the benefits and costs of time spent accumulating human capital. If the former outweigh the latter, improvements in the local technology spur human capital accumulation through this channel as well.

In the reverse direction, human capital affects the rate of technology inflow. There are many specific mechanisms that this channel might represent. For example, better educated entrepreneurs and managers are better able to identify new products and processes that are suitable for the local market. In addition, a better educated workforce makes a wider range of new products and processes viable for local production, an important consideration for both domestic entrepreneurs interested in producing locally and foreign multinationals seeking attractive destinations for direct investment. Because local human capital has this positive external effect, public subsidies to its accumulation are warranted, and here they can affect the qualitative behavior of the economy in the long run.

The framework here shares several features with the one in Parente and Prescott

(1994), including a world technology frontier that grows at a constant rate and "barriers" that impede the inflow of new technologies into particular countries. The local technology is modeled here as a pure public good, with the rate of technology inflow governed by three factors: (i) the domestic technology gap, relative to a world frontier, (ii) the domestic human capital stock, also relative to the world frontier technology, and (iii) the domestic "barrier." The growth rate of the local technology is an increasing function of the technology gap, reflecting the fact that a larger pool of untapped ideas offers more opportunities for the adopting country. As noted above, it is also increasing in local human capital, reflecting the role of education in enhancing the ability to absorb new ideas. Finally, the "barrier" reflects tariffs, internal taxes, capital controls, currency controls, or any other policy measures that retard the inflow of ideas and technologies.

For fixed parameter values, the model displays two types of long run behavior, depending on the policies in place and the initial conditions. If the barrier to technology inflows is low, the subsidy to human capital accumulation is high, and the initial levels for the local technology and local human capital are not too far below the frontier, the economy displays sustained growth in the long run. In this region of policy space, and for suitable initial conditions, higher barriers and lower subsidies imply slower convergence to the economy's balanced growth path (BGP) and wider long-run gaps between the local technology and the frontier. But inside this region of policy space, changes in the technology barrier or the subsidy to human capital accumulation do not affect the long-run growth rate. Policies that widen the steady-state technology gap also produce lower levels for capital stocks, output and consumption along the BGP, but the long-run growth rate is equal to growth rate of the frontier technology.

Thus, the model predicts that high- and middle-income countries can, over long periods, grow at the same rate as the world frontier. In these countries the gap between the local technology and the world frontier is constant in the long run, and not too large.

Alternatively, if the technology barrier is sufficiently high, the subsidy to human capital accumulation is sufficiently low, or some combination, balanced growth is not possible. Instead, the economy stagnates in the long run. An economy with policies in this region converges to a minimal technology level that is independent of the world frontier, and a human capital level that depends on the local technology and the local subsidy. In addition, even for policy parameters that permit balanced growth, for sufficiently low initial levels of technology and human capital, the economy converges to the stagnation steady state instead of the BGP.

Thus, low-income countries—those with large technology gaps—cannot display modest, sustained growth, as middle- and high-income countries can. They can adopt policies that trigger a transition to a BGP, or they can stagnate, falling ever farther behind the frontier. Moreover, economies that enjoyed technology inflows in the past can experience technological regress if they raise their barriers: local TFP and per capita income can actually decline during the transition to a stagnation steady state.

In the model here two policy parameters affect the economy's performance, the barrier to technology inflows and the subsidy to human capital accumulation. Although both can be used to speed up transitional growth, the simulations suggest that stimulating technology inflows is a more powerful tool. The reasons for this are twofold. First, human capital accumulation takes resources away from production, reducing output in the short run. In addition, for reasonable model parameters, human capital accumulation is necessarily slow. Thus, while subsides to its accumulation eventually lead to faster technology inflows and higher productivity, the process is prolonged. Policies that enhance technology inflows increase output immediately. In addition, they increase the returns to human (and physical) capital, thus stimulating further investment and growth in the long run.

The rest of the paper is organized as follows. Section 1 discusses evidence on the importance of technology inflows as a source of growth. It also documents the fact that many countries are not enjoying these inflows, instead falling ever further behind the world frontier. In section 2 the model is described, and in section 3 the BGPs and stagnation steady states are characterized in detail. In section 4 their stability is discussed and a method is described for analyzing transitional dynamics computationally. Sections 5 and 6 describe the baseline calibration and computational results. Section 7 provides sensitivity analysis for parameters about which there is little direct evidence, and section 8 concludes.

1. EVIDENCE ON THE SOURCES OF GROWTH

Five types of evidence point to the conclusion that differences in technology are critical for explaining differences in income levels over time and across economies. Collectively, they make a strong case that developed economies share a common, growing 'frontier' technology, and that less developed economies grow rapidly by tapping into that world technology.[1]

First, growth accounting exercises for individual developed countries, starting with those in Solow (1957) and Denison (1974), invariably attribute a large share of the increase in output per worker to an increase in total factor productivity (TFP). Although measured TFP in these exercises—the Solow residual—surely includes the influence of other (omitted) variables, the search for those missing factors has been extensive, covering a multitude of potential explanatory variables, many countries, many time periods, and many years of effort. It is difficult to avoid the conclusion that technical change is a major ingredient.

Second, development accounting exercises using cross-country data arrive at a

[1]See Prescott (1997) and Klenow and Rodriguez-Clare (2005) for further evidence supporting this conclusion.

similar conclusion, finding that differences in physical and human capital explain only a modest portion of the differences in income levels across countries. For example, Hall and Jones (1999) find that of the 35-fold difference in GDP per worker between the richest and poorest countries, inputs—physical and human capital per worker—account for 4.5-fold, while differences in TFP—the residual—accounts for 7.7-fold. Klenow and Rodriguez-Clare (1997b) arrive at a similar conclusion.

To be sure, the cross-country studies have a number of limitations. Data on hours are not available for many countries, so output is measured per worker rather than per manhour. No adjustment is made for potential differences between education attainment in the workforce and the population as a whole, which might be much greater in countries with lower average attainment. Human capital is measured imprecisely, consisting of average years of education in the population with at best a rough adjustment for educational quality.[2] Nor is any adjustment made for other aspects of human capital, such as health. And as in growth accounting exercises, the figure for TFP is a residual, so it is surely biased upward. Nevertheless, the role allocated to technology in development accounting exercises is large enough to absorb a substantial amount of downward revision and survive as a key determinant of cross-country differences.

A third piece of evidence is Baumol's (1986) study of the OECD countries. Although criticized on methodological grounds (see DeLong, 1988, and Baumol and Wolff, 1988), the data nevertheless convey an important message: the OECD countries (and a few more) seem to share common technologies. It is hard to explain in any other way the harmony over many decades in both their income levels and growth rates. Moreover, as Prescott (2002, 2004) and Ragan (2006) have shown, much of the

[2]Differences in educational quality probably have a modest impact, however. Hendricks (2002) reports that many studies find that immigrants' earnings are within 25% of earnings of native-born workers with the same age, sex, and educational attainment.

persistent differences in income levels can be explained by differences in fiscal policy that affect work incentives.

A fourth piece of evidence for the importance of technology comes from data on 'late bloomers.' As first noted by Gerschenkron (1962), economies that develop later have an advantage over the early starters exactly because they can adopt technologies, methods of organization, and so on developed by the leaders. Followers can learn from the successes of their predecessors and avoid their mistakes. Parente and Prescott's (1994, 2000) evidence on doubling times makes this point systematically. Figure 1 reproduces their scatter plot, updated to include data through 2006. Each point in this figure represents one of the 55 countries that had reached a per capita GDP of \$4000 by 2006. On the horizontal axis is the year that the country first reached \$2000, and on the vertical axis is the number of years required to first reach \$4000.

As Figure 1 shows, there is a strong downward trend: countries that arrived at the \$2000 figure later, doubled their incomes more quickly. The later developers seem to have enjoyed the advantage of fishing from a richer pool of ideas, ideas provided by advances in an ever-improving world technology.[3]

A fifth and final piece of evidence supporting the importance of technology is the occurrence, infrequently, of 'growth miracles.' The term is far from precise, and a stringent criterion should be used in classifying countries as such, since growth rates show little persistence from one decade to the next. Indeed, mean reversion in income levels following a financial crisis or similar event implies that an especially bad decade

[3]Figure 1 has a built-in bias, which should be noted. Among countries that have recently reached the \$2000 figure, many have not yet reached \$4000. In particular, the slow growers have not yet reached that goal. The dotted line indicates a region where, by construction, there cannot yet be any observations. Ignoring the pool of countries that in the future will occupy this space biases the impression in favor of the 'advantage of backwardness' hypothesis. An easy way to mitigate the bias is to truncate the last twenty years of data, which eliminates many of the observations for which the \$4000 goal lies in the future. The strong downward trend seems to survive this truncation.

in terms of growth rates is likely to be followed by a good one. But recovery from a disaster is not a miracle.

Nevertheless, over the period 1950-2006 twelve countries (i) enjoyed at least one 20-year episode where average per capita GDP growth exceeded 5%, and (ii) in 2006 had GDP per capita that was at least 45% of the U.S. This group has five members in Europe (Germany, Italy, Greece, Portugal, and Spain), five in east Asia (Japan, Taiwan, Hong Kong, Singapore, and Korea), and two others (Israel and Puerto Rico). The jury is still out on several others candidates: China, Thailand, Malaysia, and Botswana have met criterion (i) but not (yet) accomplished (ii).[4]

Although each of these five sources of evidence has individual weaknesses, taken together they make a strong case for the importance of international technology spillovers in keeping income levels loosely tied together in the developed countries, and occasionally allowing a less developed country to enjoy a growth spurt during which it catches up to the more developed group.

Not all countries succeed in tapping into the global technology pool, however. Figure 2 shows the world pattern of catching up and falling behind, with the U.S. taken as the benchmark for growth. It plots per capita GDP relative to the U.S. in 2000, against per capita GDP relative to the U.S. in 1960, for 104 countries. Countries that are above the 45^{o} line have gained ground over that 40-year period, and those below it have lost ground. It is striking how few have gained. The geographic pattern of gains and losses is striking as well. The countries that are catching up are almost exclusively European (plus Israel) and East Asian. With only a few exceptions, countries in Latin America, Africa, and South Asia have fallen behind.

Figure 3 shows 69 of the poorest countries, those in Asia and Africa, in more

[4] It is sobering to see how many countries enjoyed 20-year miracles, yet gave up all their (relative) gains or even lost ground over the longer period. Among countries in this groups are Bulgaria, Yugoslavia, Jamaica, North Korea, Iran, Gabon, Libya, and Swaziland.

detail. The six Asian miracles (Israel, Japan, Taiwan, Hong Kong, Singapore, and Korea) are omitted, since they significantly alter the scale.[5] Here the plot shows per capita GDP in 2000 against per capita GDP in 1960, both in levels. The rays from the origin correspond to various average growth rates. Keeping pace with the U.S. over this period requires a growth rate of 2%. The number of countries that have gained ground relative to the U.S. (points above the 2% growth line) is modest, while the number that have fallen behind (points below the line) is much larger. A shocking number, mostly in Africa, have suffered negative growth over the whole period.

The model developed below focuses on technology inflows as the only source of sustained growth. Other factors are neglected, although some are clearly important.

For example, it is well documented that in most developing economies, TFP in agriculture is substantially lower than it is in the non-agricultural sector.[6] Thus, an important component of growth in almost every fast-growing economy has been the shift of labor out of agriculture and into other occupations. The effect of this shift on aggregate TFP, which is significant, cannot be captured in a one-sector model.

More recently, detailed data for manufacturing has allowed similar estimates for the gains from re-allocation across firms within that single sector. Such misallocation can result from financial market frictions, from frictions that impede labor mobility, or from taxes (or other policies) that distort factor prices. For China, Hsieh and Klenow (2009) find that improvements in allocative efficiency contributed about 1/3 of the 6.2% TFP growth in manufacturing over the period 1993-2004.

Although this gain in Chinese manufacturing is substantial, it is a modest part of overall TFP growth in China—in all sectors—over that period. In addition, evidence

[5] Also excluded are the oil producers Bahrain, Oman, Saudia Arabia, and countries with population under 1.2 million in 2000, Bahrain, Cape Verde, Comoro Islands, Djibouti, Equatorial Guinea, Mauritius, Reunion, Seychelles Islands.

[6] See Caselli (2005) for recent evidence that TFP differences across countries are much greater in agriculture than they are in the non-agricultural sector.

from other countries gives reallocation a smaller role. Indeed, Hsieh and Klenow find that in India allocative efficiency declined over the same period, although per capita income grew. Similarly, Bartelsman, Haltiwanger, and Scarpetta (2008) find that in the two countries—Slovenia and Hungary—for which they have time series, most of the (substantial) TFP gains those countries enjoyed during the 1990s came from other sources: improvements in allocative efficiency played a minor role. And most importantly, TFP gains from resource reallocation are one-time gains, not a recipe for sustained growth.[7]

The model here is silent about the ultimate source of advances in the technology frontier, which are taken as exogenous. Thus, it is complementary to the large body of work looking at the incentives to invest in R&D, the role of learning by doing, and other factors that affect the pace of innovation at the frontier. Nor does it have anything to say about the societal factors that lead countries to develop institutions or adopt policies that stimulate growth, by stimulating technology inflows, encouraging factor accumulation, or any other means. Thus, it is also complementary to the work of Acemoglu, Johnson, and Simon (2001, 2002, 2005) and others, that looks at the country characteristics associated with economic success, without specifying the more proximate mechanism.

[7]In addition, it is not clear what the standard for allocative efficiency should be in a fast-growing economy. Restuccia and Rogerson (2008) develop a model with entry, exit, and fixed costs that produces a non-degenerate distribution of productivity across firms, even in steady state. Their model has the property that the stationary distribution across firms is sensitive to the fixed cost of staying in business and the distribution of productivity draws for potential entrants. There is no direct evidence for either of these important components, although they can be calibrated to any observed distribution. Thus, it is not clear if differences across countries reflect distortions that affect the allocation of factors, or if they represent differences in fundamentals, especially in the 'pool' of technologies that new entrants are drawing from. In particular, one might suppose that the distribution of productivities for new entrants would be quite different in a young, fast-growing economy like China and a mature, slow-growing economy like the U.S.

2. THE MODEL

The representative household has preferences over intertemporal consumption streams. It allocates its time between work and human capital accumulation, and it allocates its income between consumption and savings, taking as given the paths for wages and interest rates, as well as the subsidy to human capital accumulation. The representative firm operates a constant-returns-to-scale (CRS) technology, taking as given the path for local TFP. It hires capital and labor, paying them their marginal products. The government makes one-time choices about two policy parameters, the barrier to technology inflows and the subsidy to human capital accumulation. It finances the subsidy with a lump sum tax, maintaining budget balance at all times. Technology inflows depend on the average human capital level, as well as the barrier. In this section we will first describe the technology inflows in more detail, and then turn to the household's and firm's decisions.

A. The local technology

The model of technology is a variant of the diffusion model first put forward in Nelson and Phelps (1966) and subsequently developed elsewhere in many specific forms.[8] There is a frontier (world) technology $W(t)$ that grows at a constant, exogenously given rate,

$$\dot{W}(t) = gW(t), \tag{1}$$

where $g > 0$. In addition, each country i has a local technology $A_i(t)$. Growth in $A_i(t)$ is described by

$$\frac{\dot{A}_i}{A_i} = 0, \qquad \text{if} \quad A_i = A_i^{st} \quad \text{and} \quad \frac{\psi_0}{B_i}\frac{\overline{H}_i}{W}\left(1 - \frac{A_i^{st}}{W}\right) < \delta_A, \tag{2}$$

[8] See Benhabib and Spiegel (2005) for an excellent discussion of the long-run dynamics of various versions.

$$\frac{\dot{A}_i}{A_i} = \frac{\psi_0}{B_i}\frac{\overline{H}_i}{W}\left(1 - \frac{A_i}{W}\right) - \delta_A, \qquad \text{otherwise,}$$

where A_i^{st} is a lower bound on the local technology, B_i is a policy parameter, $\overline{H}_i$ is average human capital, and $\delta_A > 0$ is the depreciation rate for technology.

The technology floor A_i^{st} allows the economy to have a 'stagnation' steady state with a constant technology. Above that floor, technology growth is proportional to the ratio $\overline{H}_i/W$ of local human capital to the frontier technology, and to the gap $(1 - A_i/W)$ between the local technology and the frontier. The former measures the capacity of the economy to absorb technologies near the frontier, while the latter measures the pool of technologies that have not yet been adopted. A higher value for the frontier technology W thus has two effects. It widens the technology gap, which tends to speed up growth, but also reduces the absorption capacity, which tends to retard growth. The first effect dominates if the ratio A_i/W is high and the second if H_i/W is low, resulting in a logistic form.[9]

As in Parente and Prescott (1994), the barrier $B_i \geq 1$ can be interpreted as an amalgam of policies that impede access to or adoption of new ideas, or reduce the profitability of adoption. For example, it might represent impediments to international trade that reduce contact with new technologies, taxes on capital equipment that is needed to implement new technologies, poor infrastructure for electric power or transportation, or civil conflict that impedes the flow of people and ideas across border. Notice that because of depreciation, technological regress is possible. Thus, an increase in the barrier can lead to the abandonment of once-used technologies.

[9]Benhabib and Spiegel (2005, Table 2) find that cross-country evidence on the rate of TFP growth supports the logistic form: countries with very low TFP also have slower TFP growth. Their evidence also seems to support the inclusion of a depreciation term.

B. Households and firms

Next consider the decisions of households and firms. Households accumulate physical capital, which they rent to firms, and in addition each household is endowed with one unit of time (a flow), which it allocates between human capital accumulation and goods production. Investment in human capital uses the household's own time and human capital as inputs, as well as the local technology. The economy-wide average human capital may also play a role. In particular,

$$\dot{H}_i(t) = \phi_0 \left[v_i(t)H_i(t)\right]^{\eta} A_i(t)^{\zeta}\overline{H}_i(t)^{1-\eta-\zeta} - \delta_H H_i(t), \tag{3}$$

where v_i is time allocated to human capital accumulation, $\delta_H > 0$ is the depreciation rate for human capital, and $\eta, \zeta > 0,\ \ \eta+\zeta \leq 1$. This technology has constant returns to scale (CRS) jointly in the stocks $\left(A_i, H_i, \overline{H}_i\right)$, which permits sustained growth. The restriction $\zeta > 0$ rules out sustained growth in the absence of technology diffusion, and the restriction $\eta < 1$ implies an Inada condition as $v_i \to 0$, so the time allocated to human capital accumulation is always strictly positive. Average human capital $\overline{H}_i$ plays a role if $\eta + \zeta < 1$, but this channel may be absent.

The technology for producing effective labor has a similar form. Specifically, the household's effective labor supply is

$$L_i(t) = \left[1 - v_i(t)\right] H_i(t)^{\omega} A_i(t)^{\beta}\overline{H}_i(t)^{1-\beta-\omega}, \tag{4}$$

where $(1 - v_i)$ is time allocated to goods production, and $\omega, \beta > 0,\ \omega + \beta \leq 1$. Here the time input enters linearly, but an Inada condition on utility insures that time allocated to production is always strictly positive. Here, too, average human capital $\overline{H}_i$ may or may not play a role.

The technology for goods production is Cobb-Douglas, with physical capital K_i and effective labor L_i as inputs,

$$Y_i = K_i^{\alpha} L_i^{1-\alpha},$$

so factor returns are

$$\begin{aligned} R_i(t) &= \alpha \left(\frac{K_i}{L_i}\right)^{\alpha-1}, \\ \hat{w}_i(t) &\equiv (1-\alpha)\left(\frac{K_i}{L_i}\right)^{\alpha}, \end{aligned} \tag{5}$$

where $\hat{w}$ is the return to a unit of effective labor. Hence the wage for a worker with human capital H is

$$w_i(H_i,t) = \hat{w}_i H_i^{\omega} A_i^{\beta} \overline{H}_i^{1-\beta-\omega}. \tag{6}$$

Including a subsidy to human capital accumulation allows the model to be used to study the effect of policies that promote formal schooling, vocational training, on-the-job training programs, and so on. To limit the policy to one parameter, the subsidy is assumed to have the following form. Time spent investing in human capital is subsidized at the rate $\sigma_i w_i(\overline{H}_i, t)$, where $\sigma_i \in [0,1)$ is the policy variable and $\overline{H}_i$ is average human capital in the economy. Thus, the subsidy is a fraction of the current average wage, a form that permits balanced growth. The subsidy is financed with a lump sum tax τ_i, and the government's budget is balanced at all dates, so

$$\tau_i(t) = \overline{v}_i \sigma_i w_i(\overline{H}_i, t), \qquad t \geq 0, \tag{7}$$

where $\overline{v}_i$ is the average time allocated to human capital accumulation.

Thus, the budget constraint for the household is

$$\dot{K}_i(t) = (1-v_i)\, w_i(H_i,t) + v_i \sigma_i w_i(\overline{H}_i, t) + (R_i - \delta_K)\, K_i - C_i - \tau_i, \tag{8}$$

where v_i is its own time allocation and $\delta_K > 0$ is the depreciation rate for physical capital. Households have constant elasticity preferences, with parameters $\rho, \theta > 0$. Hence the household's problem is

$$\max_{\{v_i(t), C_i(t)\}_{t=0}^{\infty}} \int_0^{\infty} e^{-\rho t} \frac{C_i^{1-\theta}(t)}{1-\theta} \qquad \text{s.t. (3) and (8)}, \tag{9}$$

given σ_i, H_{i0}, K_{i0} and $\left\{A_i(t), \overline{H}_i(t), R_i(t), w_i(\cdot, t), \tau_i(t), \quad t \geq 0\right\}$.

C. Competitive equilibrium

A competitive equilibrium requires utility maximization by households, profit maximization by firms, and budget balance for the government. In addition, the law of motion for the local technology must hold.

Definition: Given the policy parameters B_i and σ_i and initial values for the state variables W_0, A_{i0}, H_{i0}, and K_{i0}, a *competitive equilibrium* consists of $\{A_i(t), H_i(t), \overline{H}_i(t), K_i(t), v_i(t), C_i(t), R_i(t), w_i(\cdot, t), \tau_i(t), \quad t \geq 0\}$ with the property that:

i. $\{v_i, C_i, H_i, K_i\}$ solves (9), given σ_i, H_{i0}, K_{i0}, and $\{A_i, \overline{H}_i, R_i, w_i, \tau_i\}$;

ii. $\{R_i, w_i\}$ satisfies (5) and (6), and $\{\tau_i\}$ satisfies (7);

iii. $\{A_i\}$ satisfies (2) and $\{\overline{H}_i(t) = H_i(t), \quad t \geq 0\}$.

The system of equations characterizing the equilibrium is in the Appendix.

Two types of behavior are possible in the long run. If the barrier B_i is low enough and/or the subsidy σ_i is high enough, balanced growth is possible. Specifically, if the policy parameters (B_i, σ_i) lie inside a certain set, the economy has two balanced growth paths (BGPs), on which the time allocation v_i^{bg} is constant and the variables (A_i, H_i, K_i, C_i) all grow at the rate g. On BGPs, the ratios A_i/W, H_i/W, and so on depend on B_i and σ_i.

Balanced growth is not the only possibility, however. For any policy parameters (B_i, σ_i) the economy has a unique stagnation SS with no growth. In a SS, the local technology is A_i^{st}, the time allocation v_i^{st} is constant, and the variables $(H_i^{st}, K_i^{st}, C_i^{st})$ are also constant. In a SS, the levels $(H_i^{st}, K_i^{st}, C_i^{st})$ depend on the subsidy σ_i, but not on the barrier B_i. The next section describes the BGPs and stagnation SS in more detail. Before proceeding, however, it is useful to see how growth accounting and development accounting work in this setup.

D. Growth and development accounting

Consider a world with many economies, $i = 1, 2, ..., I$. In each economy i, output per capita at any date t is

$$Y_i(t) = K_i(t)^\alpha \left\{ [1 - v_i(t)] \, A_i(t)^\beta H_i(t)^{1-\beta} \right\}^{1-\alpha}.$$

Assume that at each date, any individual is engaged in only one activity, and hours per worker are the same over time and across countries. Then all differences in v—in the allocation of time between production and human capital accumulation—take the form of differences in labor force participation. Let

$$\hat{Y}_i(t) \equiv \frac{Y_i(t)}{1 - v_i(t)} \qquad \text{and} \qquad \hat{K}_i(t) \equiv \frac{K_i(t)}{1 - v_i(t)}$$

denote output and capital per worker, and note that human capital H_i is already so measured. Then output per worker can be written three ways,

$$\begin{aligned} \hat{Y}_i(t) &= \hat{K}_i(t)^\alpha \left[H_i(t)^{1-\beta} A_i(t)^\beta \right]^{1-\alpha}, \\ \hat{Y}_i(t) &= \left(\frac{\hat{K}_i(t)}{\hat{Y}_i(t)} \right)^{\alpha/(1-\alpha)} H_i(t)^{1-\beta} A_i(t)^\beta, \\ \hat{Y}_i(t) &= \left(\frac{\hat{K}_i(t)}{\hat{Y}_i(t)} \right)^{\alpha/\beta(1-\alpha)} \left(\frac{H_i(t)}{\hat{Y}_i(t)} \right)^{(1-\beta)/\beta} A_i(t). \end{aligned} \tag{10}$$

Suppose that α and β are known, and that H_i is observable, as well as $\hat{Y}_i$ and $\hat{K}_i$. The first equation in (10) is the standard basis for a growth accounting exercise, as in Solow (1957); the second is the version used for the development accounting exercises in Hall and Jones (1999), Hendricks (2002), and elsewhere; and the third is a variation suitable for the model here. In each case the technology level A_i is treated as a residual.

First consider a single economy. A growth accounting exercise based on the first line in (10) in general attributes growth in output per worker to growth in all three inputs, $\hat{K}_i, H_i$ and A_i, and along a BGP—where all four variables grow at a common, constant rate—the shares are α, $(1-\beta)(1-\alpha)$, and $\beta(1-\alpha)$.

An accounting exercise based on the second line attributes some growth to physical capital only if the ratio $\hat{K}_i/\hat{Y}_i$ is growing. The rationale for using the ratio is that growth in H_i or A_i induces growth in $\hat{K}_i$, by raising its return. Here the accounting exercise attributes to growth in capital only increases in excess of those prompted by growth in effective labor. Along a BGP $\hat{K}_i/\hat{Y}_i$ is constant, and the exercise attributes all growth to H_i and A_i, with shares $(1-\beta)$ and β.

The third line applies the same logic to human capital, since growth in A_i induces growth in H_i as well as K_i. Since $H_i/\hat{Y}_i$ is also constant along a BGP, here the accounting exercise attributes all growth on a BGP to the residual A_i.

The same logic applies in a development accounting exercise involving many countries. Differences across countries in capital taxes, public support to education, and other policies lead to differences in the ratios $\hat{K}_i/\hat{Y}_i$ and $H_i/\hat{Y}_i$, so a development accounting exercise using any of the three versions attributes some differences in labor productivity to differences in physical and human capital. But the second line attributes to physical capital—and the third to both types of capital—only differences in excess of those induced by changes in the supplies of the complementary factor(s), H_i and A_i in the second line and A_i in the third.

3. BALANCED GROWTH PATHS AND STEADY STATES

In this section the BGPs for growing economies and the SS for an economy that stagnates are characterized. It is convenient to analyze BGPs in terms of variables that are normalized by the world technology, A_i/W, H_i/W, and so on, while it is convenient to analyze the SS in terms of the levels A_i, H_i, and so on. It will be important to keep this in mind in the next section, where transitions are discussed.

A. BGPs for economies that grow

For convenience drop the country subscript. It is easy to show that along a BGP the variables A, H, K, L, and C grow at the same rate g as the frontier technology, the time allocation $v > 0$ is constant, the factor returns R and $\hat{w}$ are constant, the average wage per manhour $w(\overline{H})$ grows at the rate g, and the costate variables Λ_H and Λ_K for the household's problem grow at the rate $-\theta g$.

Therefore, to study growing economies it is convenient to define the normalized variables

$$\begin{aligned} a(t) &\equiv \frac{A(t)}{W(t)}, \qquad h(t) \equiv \frac{H(t)}{W(t)}, \qquad \lambda_h(t) \equiv \frac{\Lambda_H(t)}{W^{-\theta}(t)} \\ c(t) &\equiv \frac{C(t)}{W(t)}, \qquad k(t) \equiv \frac{K(t)}{W(t)}, \qquad \lambda_k(t) \equiv \frac{\Lambda_K(t)}{W^{-\theta}(t)}, \quad \text{all } t. \end{aligned} \tag{11}$$

Using the fact that in equilibrium $\overline{h} = h$, the equilibrium conditions can then be written as

$$\begin{aligned} \lambda_h \phi_0 \eta v^{\eta-1} &= (1-\sigma)\,\lambda_k \hat{w} \left(\frac{a}{h}\right)^{\beta-\zeta}, \\ c^{-\theta} &= \lambda_k, \\ \frac{\dot{\lambda}_h}{\lambda_h} &= \rho + \theta g + \delta_H - \phi_0 \eta v^{\eta} \left(\frac{a}{h}\right)^{\zeta} \left(1 + \frac{\omega}{1-\sigma}\frac{1-v}{v}\right), \\ \frac{\dot{\lambda}_k}{\lambda_k} &= \rho + \theta g + \delta_K - R, \\ \frac{\dot{h}}{h} &= \phi_0 v^{\eta} \left(\frac{a}{h}\right)^{\zeta} - \delta_H - g, \\ \frac{\dot{k}}{k} &= \kappa^{\alpha-1} - \frac{c}{k} - \delta_K - g, \\ \frac{\dot{a}}{a} &= \frac{\psi_0}{B} h\,(1-a) - \delta_A - g, \end{aligned} \tag{12}$$

where

$$\begin{aligned} \kappa &\equiv K/L = k/\,(1-v)\,a^{\beta} h^{1-\beta}, \\ R &= \alpha \kappa^{\alpha-1}, \qquad \hat{w} = (1-\alpha)\,\kappa^{\alpha}. \end{aligned} \tag{13}$$

The transversality conditions hold if and only if $\rho > (1-\theta)\, g$, which insures that the discounted value of lifetime utility is finite.

Let a^{bg}, h^{bg}, and so on denote the constant values for the normalized variables along the BGP. As usual, the interest rate is

$$r^{bg} \equiv R^{bg} - \delta_K = \rho + \theta g,$$

so the transversality condition implies that the interest rate exceeds the growth rate, $r^{bg} > g$. Also as usual, the input ratio κ^{bg} is determined by the rate of return condition

$$\alpha \left(\kappa^{bg}\right)^{\alpha-1} = R^{bg} = r^{bg} + \delta_K.$$

The return to effective labor $\hat{w}^{bg}$ then depends on κ^{bg}. Thus, all economies on BGPs have the same growth rate g and interest rate r^{bg}, which do not depend on the policy parameters (B, σ).

The time allocated to human capital accumulation on a BGP is determined by combining the third and fifth equations in (12) to get

$$v^{bg} = \left[1 + \frac{1-\sigma}{\omega\eta}\left(\frac{r^{bg}+\delta_H}{g+\delta_H} - \eta\right)\right]^{-1}. \tag{14}$$

Since $r^{bg} > g$ and $\eta < 1$, the second term in brackets is positive and $v^{bg} \in (0,1)$. Notice that v^{bg} is increasing in the subsidy σ, with $v^{bg} \to 1$ as $\sigma \to 1$. It is also increasing in ω and η, the elasticities for human capital in the two technologies. A higher value for ω increases the sensitivity of the wage rate $w(H)$ to private human capital, increasing the incentives to invest, while a higher value for η reduces the force of diminishing returns in time allocated to human capital accumulation. Finally, v^{bg} is increasing δ_H and g, since along a BGP investment must offset depreciation and H must keep pace with A.

The ratio of technology to human capital is then determined by the fifth equation in (12),

$$z^{bg} \equiv \frac{a^{bg}}{h^{bg}} = \left(\frac{g+\delta_H}{\phi_0 \left(v^{bg}\right)^{\eta}}\right)^{1/\zeta}. \tag{15}$$

Hence this ratio is decreasing in the subsidy σ. Note that v^{bg} and z^{bg} do not depend on the barrier B.

The last equation in (12) then implies that a BGP level for the relative technology a^{bg}, if any exists, satisfies the quadratic

$$a^{bg}\left(1-a^{bg}\right)=(g+\delta_A)\,\frac{B}{\psi_0}z^{bg}, \tag{16}$$

and the following result is immediate.

PROPOSITION 1: If

$$(g+\delta_A)\,\frac{4z^{bg}}{\psi_0}<\frac{1}{B}, \tag{17}$$

(16) has two solutions. These solutions are symmetric around the value 1/2, and there is a BGP corresponding to each. If (17) holds with equality, (16) has one solution, and the unique BGP has $a^{bg}=1/2$. If the inequality in (17) is reversed, no BGP exists.

Thus, a necessary and sufficient for the existence of BGPs is that $1/B$, which measures "openness," exceed a threshold that depends—through z^{bg}—on σ. The threshold for $1/B$ increases as σ falls, as shown in Figure 4a, creating two regions in the space of policy parameters. Above the threshold—for high values of $1/B$ and σ—the economy has two BGPs, and below the threshold there is no BGP.

If (17) holds, call the solutions a_L^{bg} and a_H^{bg}, with

$$0<a_L^{bg}<\frac{1}{2}<a_H^{bg}.$$

Figure 4b displays the solutions to (16) as functions of $1/B$, for three values of the subsidy σ and fixed values for the model parameters. For fixed σ, a small decrease in $1/B$ moves both solutions toward the value 1/2. For fixed B, a small decrease in σ has the same effect. For $1/B$ or σ or both sufficiently small, the inequality in (17) fails and no BGP exists.

Notice that the higher solution a_H^{bg} has the expected comparative statics—it increases with openness $1/B$ and with the subsidy σ—while the lower solution a_L^{bg} has the opposite pattern. As we will see below, a_H^{bg} is stable and a_L^{bg} is not. Therefore, since $a_H^{bg} \in [1/2,\ 1]$, this model produce BGP productivity (and income) ratios of no more than two across growing economies. Poor economies cannot grow along BGPs, in parallel with richer ones.[10]

In addition, since v^{bg} and z^{bg} do not vary with B, and only the higher solutions to (16) are stable, looking across economies with similar education policies σ, and all on BGPs, those with higher barriers B lag farther behind the frontier, but in all other respects are similar. Stated a little differently, an economy with a higher barrier looks like its neighbor with a lower one, but with a time lag.

B. Steady states for economies that stagnate

While BGPs exist only for policy parameters (σ, B) where (17) holds, every economy has a stagnation SS. At this SS, the technology level, capital stocks, factor returns, consumption, and time allocation are constant, as are the costates for the household's problem. Let A^{st}, H^{st}, K^{st}, and so on denote these levels.

The SS can be calculated in the same way as the BGPs. Here, since consumption is constant, the interest rate is equal to the rate of time preference,

$$r^{st} = R^{st} - \delta_K = \rho.$$

The input ratio κ^{st} and the return to effective labor $\hat{w}^{st}$ are then determined as before. The steady state time allocation and input ratio, the analogs to (14) and (15), are

$$v^{st} \quad = \quad \left[1 + \frac{1-\sigma}{\eta\omega}\left(\frac{r^{st}+\delta_H}{\delta_H} - \eta\right)\right]^{-1}, \tag{18}$$

[10]Other factors, like taxes on labor income (which reduce labor supply) can also increase the spread in incomes across growing economies. See Prescott (2002, 2004) and Ragan (2006) for models of this type.

$$z^{st} \equiv \frac{A^{st}}{H^{st}} = \left(\frac{\delta_H}{\phi_0 \left(v^{st} \right)^{\eta}} \right)^{1/\zeta}, \tag{19}$$

so they are like those on a BGP except that the interest rate is r^{st} and there is no growth.

Notice that for two economies with the same education subsidy σ, more time is allocated to human capital accumulation in the growing economy, $v^{bg} > v^{st}$, if and only if

$$\frac{r^{bg} + \delta_H}{g + \delta_H} < \frac{r^{st} + \delta_H}{\delta_H},$$

or

$$(\theta - 1)\, \delta_H < \rho.$$

If θ is sufficiently large, the low willingness to substitute intertemporally discourages investment in the growing economy. For $\theta = 2$ and $\rho = \delta_H$, as will be assumed in the simulations here, the steady state time allocations are the same, $v^{bg} = v^{st}$. In this case the growing economy has a higher ratio of technology to human capital,

$$\frac{z^{bg}}{z^{st}} = \left(\frac{g + \delta_H}{\delta_H} \right)^{1/\zeta} > 1.$$

4. TRANSITIONAL DYNAMICS

With three state variables, A, H, and K, and two costates, Λ_h and Λ_k (or their normalized counterparts), the transitional dynamics are complicated. The more interesting interactions involve technology and human capital, with physical capital playing a less important role. Thus, for simplicity we will drop physical capital to study transitions. To this end, set $\alpha = 0$ and drop the equations for $\dot{K}$ and $\dot{\Lambda}_k$. Then $\hat{w} = 1$ and $R = 0$, and $C^{-\theta}$ takes the place of Λ_k.

Recall from Figure 4a that if the policy $(1/B, \sigma)$ lies above the threshold, condition (17) holds, and the economy has two BGPs and one SS. As we will see below, in this case the SS and one of the BGPs are stable, while the other BGP, which separates them, is unstable. In this case the economy converges to the stable BGP if the initial conditions (a_0, h_0) lie above a certain threshold in the state space, and converges to the SS if the initial conditions lie below the threshold. If the policy $(1/B, \sigma)$ lies below the threshold in Figure 4a, the economy has no BGP, and it converges to the SS for all initial conditions.

A. Catching up: transitions to a BGP

Suppose that $(1/B, \sigma)$ lies above the threshold in Figure 4a, and consider the local stability of the two BGPs. Since in equilibrium $\overline{h} = h$, and in the absence of physical capital all output is consumed, consumption is

$$c = (1 - v)\, a^{\beta} h^{1-\beta}.$$

Using $c^{-\theta}$ for λ_k, the first equation in (12) implies that the time allocation on either BGP satisfies

$$v^{\eta-1} (1 - v)^{\theta} = \frac{1 - \sigma}{\eta \phi_0} a^{\Delta} h^{-(\Delta+\theta)} \lambda_h^{-1}, \tag{20}$$

where $\Delta \equiv \beta (1 - \theta) - \zeta$. The transitional dynamics are then described by the three equations in (12), for $\dot{\lambda}_h/\lambda_h$, $\dot{h}/h$, and $\dot{a}/a$.

Recall that the normalized variables are constant along either BGP. This system of three equations can be linearized around the each of the steady states for the normalized variables, and the characteristic roots determine the local stability of each. In all of the simulations reported here, all of the roots are real, exactly two roots are negative at a_H^{bg}, and exactly one root is negative root at a_L^{bg}. Moreover, extensive computations suggest that this configuration for the roots holds for all reasonable parameter values.[11]

[11] The Appendix contains a further analysis of the characteristic equation, and provides an example with complex roots.

With this pattern for the roots, the higher steady state, a_H^{bg}, is locally stable: for any pair of initial conditions (a_0, h_0) in the neighborhood of $\left(a_H^{bg}, h_H^{bg}\right)$, there exists a unique initial condition λ_{h0} for the costate with the property that the system converges asymptotically. The lower steady state, a_L^{bg}, is unstable in the sense that there is only a one-dimensional manifold of initial conditions (a_0, h_0) in the neighborhood of $\left(a_L^{bg}, h_L^{bg}\right)$ for which the system converges. This manifold is the boundary of the basin of attraction for $\left(a_H^{bg}, h_H^{bg}\right)$. Above it the economy converges to the stable BGP, and below it the economy converges to the stagnation SS.

The boundary is computed by perturbing around the point $\left(a_L^{bg}, h_L^{bg}\right)$, using as the perturbation the eigenvector associated with the single negative root, and running the ODEs backward. The eigenvector is multiplied by $\pm\varepsilon$, for small $\varepsilon > 0$, producing the two arms of the boundary in (a, h)-space. The region above this boundary is the basin of attraction for $\left(a_H^{bg}, h_H^{bg}\right)$, and the region below it is the basin of attraction for the stagnation SS. Notice that this boundary involves the normalized variables, so it depends on W, as well as (A, H).

Transitional dynamics to the stable BGP can be computed by perturbing the system away from the point $\left(a_H^{bg}, h_H^{bg}\right)$ and running the ODEs backward. Any linear combination of the eigenvectors associated with the two negative roots, with small weights, can be used as an initial perturbation, giving a two-dimensional set of allowable perturbations.

b. Falling behind: transitions to the SS

Our interest here is in transitions to the SS from above. A transition of this sort would be observed in a country that had enjoyed growth above the stagnation level, by reducing its barrier or increasing subsidy to human capital accumulation, and then reversed those policies. This is not merely a theoretical possibility. As shown in Figure 3, a number of African countries had negative growth in per capita GDP over the four decade period 1960-2000, and even more had shorter episodes—one or two decades—of negative growth.

By the same line of argument as above, the time allocation during the transition to the SS satisfies the analog to (20),

$$v^{\eta-1}\left(1-v\right)^{\theta}=\frac{1-\sigma}{\eta\phi_0}A^{\Delta}H^{-(\Delta+\theta)}\Lambda_H^{-1}, \tag{21}$$

where Δ is as before, and the law of motion for the costate is

$$\frac{\dot{\Lambda}_H}{\Lambda_H}=\rho+\delta_H-\phi_0\eta v^{\eta}\left(\frac{A}{H}\right)^{\zeta}\left(1+\frac{\omega}{1-\sigma}\frac{1-v}{v}\right). \tag{22}$$

The transitional dynamics are described by these two equations and the laws of motion for W, A, H in (1)-(3).

Linearizing around the stagnation steady state is problematic, for two reasons. First, the law of motion for A in (2) has a kink at A^{st}. Only the region of the state space where $A \geq A^{st}$ is of interest, but the kink nevertheless comes into play if A reaches A^{st} before H reaches H^{st}. In addition, since the frontier technology W appears in (2), the system is not autonomous: (1) is needed.

A computational method that deals with both issues is the following two-step procedure. For the first step, let $A_0 = A^{st}$ and conjecture that $\dot{A} = 0$. Then the transition is described by the equations for $\dot{H}$, $\dot{\Lambda}_H$, which do not involve W. Linearize (3) and (22) around (H^{st}, Λ^{st}), using (21) for v. As shown in the Appendix, the resulting pair of equations has roots that are real and of opposite sign. Hence for any H_0 sufficiently close to H^{st}, there exists a unique Λ_{H0} near Λ_H^{st} with the property that for the initial conditions (H_0, Λ_{H0}) and $A_0 = A^{st}$, the linearized system converges to the steady state. This solution constitutes a competitive equilibrium provided that $\dot{A} = 0$ when (2) is used. That will be the case provided that H_0/W_0 is small enough. Then, since (3), (21), and (22) are continuous in their arguments, and for $A \geq A^{st}$ (2) is also continuous, the argument extends to the case where $A_0/A^{st} = 1 + \varepsilon$, for $\varepsilon > 0$ sufficiently small.

For the second step, choose any pair (H, Λ_H) generated by the first step, a (large) value W, and a small time interval Δt. As shown in the Appendix, it is possible to choose $A = (1+\varepsilon) A^{st}$, with $\varepsilon > 0$, that has the following property: for the initial condition (W, A, H, Λ_H), the ODEs in (2) implies that A converges to A^{st} in Δt units of time. Thereafter (H, Λ_H) adjust as described in the first step.

To calculate longer transitional paths, run the system of ODEs in (1) - (3) and (22) backward from the 'initial' condition (W, A, H, Λ_H). Notice that transitions to the steady state involve the world technology W directly. Thus, viewing the transition paths going forward, the value W_0 for the world technology is required, as well as (A_0, H_0).

An economy converging to that SS stagnates in the long run, but it may grow—slowly—for a while. Its rate of technology adoption must eventually fall short of g, however, and then decline further. In the long run depreciation more than offsets new inflows, and the technology declines to its stagnation level.

5. CALIBRATION

The model parameters are the long run growth rate g; the preference parameters (ρ, θ); and the technology parameters $(\eta, \zeta, \phi_0, \delta_H)$ for human capital accumulation, (β, ω) for effective labor, and (ψ_0, δ_A) for technology diffusion. Baseline values for these parameters are described below. Experiments with alternative values are also conducted, to assess the sensitivity of the results.

The growth rate is set at $g = 0.019$, which is the rate of growth of per capita GDP in the U.S. over the period 1870-2003.

The preference parameters are set at $\rho = 0.03$ and $\theta = 2$, which are within the range that is standard in the macro literature.

The depreciation rate for human capital is set at $\delta_H = 0.03$, which is close to the value ($\delta_H = 0.037$) estimated in Heckman (1976), and the same rate is used for technology, $\delta_A = 0.03$. For these parameters the time allocation is the same along the BGP and in the stagnation steady state, $v^{bg} = v^{st}$, for economies with the same subsidy σ.

For human capital accumulation, the elasticity η with respect to own human capital input vH_i is set at $\eta = 0.50$, which is close to the value ($\eta = 0.52$) estimated in Heckman (1976). The division of the remaining weight $1 - \eta$ between A_i and $\overline{H}_i$ affects the speed of convergence, with more weight on A_i producing faster transitions. The baseline simulations use $\zeta = 0.50$, so $1 - \eta - \zeta = 0$, and average human capital plays no role. The effect of a positive elasticity with respect to $\overline{H}_i$ is studied as part of the sensitivity analysis.

For goods production, the large wage increases enjoyed by migrants who move from poor countries to rich ones suggests that $1-\omega$, the weight on the technology and average human capital in the country of employment, is large. Here, as above, the division of $1 - \omega$ between A_i and $\overline{H}_i$ affects the speed of convergence. The baseline simulations use $\omega = 0.50$ and $\beta = 0.50$, so $1 - \omega - \beta = 0$, and average human capital plays no role. The effect of a positive elasticity with respect to $\overline{H}_i$ is studied as part of the sensitivity analysis.

The constants ϕ_0, ψ_0 involve units for A_i and H_i, so one can be fixed arbitrarily. Here ϕ_0 is chosen as follows. Define a "frontier" economy as one with no barrier, $B = 1$, and with an education subsidy $\sigma = \sigma_F$ set so that the steady state time allocation v_F^{bg} coincides with the one that solves a social planner's (welfare maximization) problem. The parameter ϕ_0 is chosen so that $a_F^{bg}/h_F^{bg} = z_F^{bg} = 1$ in the frontier economy. From (15), this requires

$$\phi_0 = (g + \delta_H) \left(v_F^{bg} \right)^{-\eta}. \tag{23}$$

The level parameter ψ_0 affects the speed of convergence, and it also governs the maximal difference in income levels along BGPs. In particular, recall from (16) that the relative technology on a stable BGP is bounded below by $a_{\text{crit}} = 1/2$. For an upper bound we can use $a_F < 1$, the relative technology for a "frontier" economy. Then the maximum income ratio for economies on BGPs is $a_F/a_{\text{crit}} = 2a_F$. Hence a_F must be close to unity to get BGP incomes that vary by a factor close to two. The baseline simulations here use $a_F = 0.96$.

To summarize, the benchmark parameters are

$$\begin{array}{llll} g = 0.019, & \theta = 2.0, & \rho = 0.03, & \\ \eta = 0.50, & \beta = 0.50, & \delta_H = 0.03, & \\ \omega = 0.50, & \zeta = 0.50 & \delta_A = 0.03, & a_F = 0.96, \end{array}$$

which implies

$$\phi_0 = 0.12676, \qquad \psi_0 = 1.276.$$

The steady state subsidy and time allocation in a frontier economy are $\sigma_F = 0.051282$ and $v_F = 0.14943$.

6. BASELINE SIMULATIONS

In the first group of simulations the baseline model parameters are used and the subsidy to human capital is fixed at its level in the frontier economy, $\sigma_F = 0.051$.

A. Basins of attraction, phase diagrams

Figures 5 and 6 show some effects of varying the barrier B. Figure 5a shows the (normalized) BGPs, $s_{Ji} = \left(a_{Ji}^{bg}, h_{Ji}^{bg}\right)$, $J = H, L$, $i = 1, 2, 3$, and basins of attraction for three barriers, $B_1 < B_2 < B_3$. Since the subsidy σ is the same for all three economies, the input ratios a_{Ji}^{bg}/h_{Ji}^{bg} on the BGP are also the same, and the normalized steady states lie on a ray from the origin. The normalization for ϕ_0 used here implies that ray is the 45° line. Since $\sigma = \sigma_F$ and $B_1 = 1$, the point s_{H1} represents the frontier economy, with $a_{H1}^{bg} = a_F$. For $B > B_{crit} \approx 6.51$ the system does not have a

BGP.

For each B_i, the threshold is displayed that separates the state space into two regions. For initial conditions above the threshold, an economy with barrier B_i converges to the (stable) BGP described by s_{Hi}, and for initial conditions below the threshold, it converges to its stagnation SS. For initial conditions exactly on the threshold it converges to the (unstable) BGP described by s_{Li}.

As in Figure 4, increasing B shifts the threshold upward, $s_{L1} < s_{L2} < s_{L3}$, shrinking the set of initial conditions that produce balanced growth. Increasing B also shifts the normalized levels for the stable BGP downward, $s_{H1} > s_{H2} > s_{H3}$. Thus, comparing across economies on BGPs for different barriers, those with higher barriers lag farther behind the frontier economy.

The barrier B is also important for the speed of convergence to the stable BGP. Figure 5b displays the values for the two negative roots, which govern that speed, as functions of $1/B$. Call the roots "fast" and "slow," with $-R_f > -R_s > 0$. Greater openness—a higher value for $1/B$—increases both roots, speeding up transitions. The fast root is approximately linear in $1/B$, implying that a low barrier will be needed to construct a growth miracle. The slow root is an extremely concave function of $1/B$, converging to zero as $1/B$ declines to the threshold where a BGP ceases to exist.

The two panels of Figure 6 show the phase diagrams for $B = 1$ and $B = 4$. In each case the solid lines are transition paths to the stable BGP, the broken lines are the thresholds from Figure 5, and the dashed lines are the eigenvectors, E_f and E_s, associated with the negative roots. (The latter are linear in the log space.) The adjustment paths in panel a, for $B = 1$, are very flat, indicating that with a low barrier the technology level a adjusts much more rapidly than human capital h. The paths in panel b, for $B = 4$, show much more adjustment in human capital for a given adjustment in the technology. This pattern is consistent with Figure 5b. Roughly speaking, the fast root seems to govern the adjustment in a and the slow root the adjustment of h, and increasing the barrier has a much larger impact on the former than on the latter.

B. Transition paths: fast growth and lost decades

Figures 7 and 8 display transitions for changes in the barrier B that take the economy from one BGP to another. Thus, the barriers before and after the transition are both low enough to be consistent with balanced growth.

Figure 7 displays the transition for an economy that reduces its barrier from $B_3 = 6$ to $B_1 = 1$. Thus, the initial and final conditions for this economy are s_{H3} and s_{H1} in Figure 5. The ratio $z^{bg} = a^{bg}/h^{bg}$ is the same on both BGPs, and both state variables start at about 67% of their final levels. In each panel of Figure 7 both the exact path (solid) and the linear approximation (dashed) are displayed.

Figure 7a shows the transition path in $a, h-$space. Over the first decade of the transition, the normalized technology a grows rapidly while the normalized human capital stock h grows very slowly. Over the next four decades a is approximately constant, since the local technology has caught up with the world level, and human capital h continues its steady but slow growth. The linear approximation is close to the exact path over the whole transition, although it overstates the speed of adjustment during the early years.

Figures 7b - 7d display time plots for the first 50 years after the policy change. Figure 7b shows the time v allocated to human capital accumulation. Interestingly, it falls rather sharply immediately after the policy change. Two factors are at work. First, consumption smoothing provides a direct incentive to shift the time allocation toward goods production. In addition, since technology is an important input into human capital accumulation, there is an incentive to delay the investment of time in that activity until after the complementary input has increased.

Figure 7c shows the local technology a, human capital h, and output y relative to the frontier economy. As the phase diagram showed, the technology adjusts rapidly toward the BGP, while human capital adjusts more slowly. The shift in the time allocation just after the policy change allows consumption to adjust rather rapidly, jumping immediately from 67% to about 70% of the value in the frontier economy.

Figure 7d shows the growth rate of output, which jumps from 1.9% to about 8.3% immediately after the policy change, and then falls gradually back to the steady state level.

Figure 8 displays the transition for an economy that increases its barrier from $B_1 = 1$ to $B_2 = 4$. The result is a "lost decade." Figure 8a shows the transition in the state space. As before, the technology adjusts more rapidly than human capital. Here the adjustments are slower, however, and the two stocks adjust at more similar rates. Panel b shows the time allocation, which here is the mirror image of the previous one. The logic is the same as before. Since human capital is the only vehicle for smoothing consumption, and since the technology for human capital accumulation uses technology as an input, the household has two reasons for shifting its time allocation toward accumulation. Figure 8c shows the transition paths for a, h and y, relative to their new BGP levels. In the long run all three variables decline by about 18%, with the technology adjusting most rapidly. Panel d displays the growth rate of output, which jumps from 1.9% down to 1.0%, and then gradually returns to its old level. The lost decade is never recovered, however: output on the new BGP is about 18% below what it would have been on the old one.

C. Transitions: miracles and disasters

Although the economy in Figure 7 enjoys substantial growth, it is not a miracle by the criterion in section 1. A miracle requires an average growth rate of at least 5% over 20 years, so income must increase by at least a factor of $e^{(.05)(20)} = 2.57$ over that period. But the initial condition for an economy on a BGP is at least half of income in the frontier economy, so its income cannot double. Thus, growth miracles must have initial conditions below those possible on any BGP. What sort of initial condition for a miracle does the data suggest?

The growth miracles described in section 1 can be grouped into four categories according to GDP per capita relative to the U.S. at the beginning of their period of rapid growth: (a) Germany and Italy, (b) Greece, Portugal, Spain, Puerto Rico, Japan, Hong Kong, Singapore, Israel; (c) S. Korea and Taiwan; and (d) China. Table 1 shows, for each group, their initial GDP per capita relative to the U.S. ($\hat{y}_0 = y_0/y_{0,US}$), their GDP per capita relative to the U.S. at date T ($\hat{y}_T = y_T/y_{T,US}$), their average growth rate over that period (g), the number of years in their period of rapid growth (T), and the initial and final dates for that period..

		$\hat{y}_0$	$\hat{y}_T$	$g(\%)$	T	date 0	date T
(a)	Ger., Italy	0.39	0.68	5.3	20	1950	1970
(b)	8 countries	0.23	0.53	5.4	30	1950*	1980
(c)	S. Kor., Taiwan	0.10	0.59	6.2	43, 50	1963, 1950	2006, 2000
(d)	China	0.065	0.195	7.1	24	1982	2006

*1960 for Singapore

Table 1

The miracles in Germany and Italy are probably explained in large part as recovery in the physical capital stock after the destruction in World War II, a quite different mechanism from the one proposed here. Of the others, initial per capita incomes were between 6.5% and 23% of the U.S. level when their periods of rapid growth began.

Figures 9 and 10 show transitions for economies whose initial incomes are 20% and 10%, respectively, of the level in the frontier economy. The initial conditions for the technology levels and human capital stocks are chosen to be consistent with the description of a stagnation SS. (The human capital subsidy is assumed to be σ_F throughout, but using $\sigma = 0$ in the initial condition does not change the results much.)

The patterns in Figure 9 are similar to those in Figure 7, but much more pronounced. The technology grows very rapidly, reaching 90% of the BGP level after 20 years. The initial swing in the time allocation is very pronounced here, and (relative) human capital declines over the first decade, recovering to the (relative) level it had at the start after about 15 years. Output growth surges in the first years after the policy change, and remains high for twenty years.

The transition in Figure 10 is like the one in Figure 9, but from an even lower starting point. Technology inflows are quite rapid for 50 years, and output growth remains above the 5% level for almost 30 years. Interestingly, the growth rate is not monotonic: after its initial jump it declines slightly and then recovers.

Figure 11 shows a "growth disaster." The initial values A, H, and Y, for this economy are above their stagnation levels, and the Figure 11 shows the transition back to the economy's stagnation SS. In this economy, the model produces wild swings in the time allocation, as households try to save for their (dim) future in the only way they can, by accumulating human capital. Thus, in Figure 11 the time allocation is fixed at its SS level.

D. Transitions: policies and starting points

[Note: Figures 12 and 13 use a different set of model parameters.]

Figure 12 displays the transition paths for two different policies, for an economy with fixed initial conditions. Policy 1 (solid line) involves a low barrier to technology adoption, $B_1 = 1.3$, but a low subsidy to education, $\sigma_1 = 0.087$. Policy 2 (broken line) involves a higher barrier and a higher subsidy, $B_2 = 3.0$ and $\sigma_2 = 0.35$. These policies are constructed so that consumption—and hence welfare—on the BGP is the same under both.

As Figure 12a shows, the economy enjoys a much more rapid transition under Policy 1: technology increases very quickly. Figure 12b shows the time allocations. Under Policy 2 the time allocation is very close to its BGP level throughout the (very long) transition. Under Policy 1, the incentive to smooth consumption leads to a sharp reduction in time devoted to human capital accumulation, in order to increase production. The result is a high growth rate for output (consumption) in the first decade of the transition. Under policy 2 consumption growth adjusts very gradually to the new steady state level.

Figures 12d – 12f show the transition paths for technology, human capital, and consumption. The two policies lead to different BGP levels for a and h, but the same level for c. Under Policy 1, human capital is lower over the whole transition, but technology and consumption are higher. Thus, welfare is higher under Policy 1.

Figure 13 shows the transitions for two economies with different initial conditions. Economy D (the solid line) has as its initial condition the (normalized) steady state for an economy with $B = 3.5$ and $\sigma = .035$. Economy C (the dashed line) has a higher initial stock of human capital but a lower technology level. This point could be interpreted as the BGP for a regime with an even higher initial barrier than economy D, but also has a higher education subsidy. The initial condition for Economy C is chosen so that it has the same productive capacity as Economy D, if all time is allocated to goods production. Both economies are assumed to adopt the policies of the frontier economy.

As Figure 13a shows, Economy C enjoys a much more rapid transition. It makes a more dramatic shift in its time allocation, away from human capital accumulation and toward production, as shown in Figure 13b, and hence it enjoys a larger immediate jump in consumption growth, as shown in Figure 13c. As shown in Figure 13d, Economy C's technology soon overtakes Economy D's, and its human capital and consumption exceed D's over the entire transition path.

The experiment in Figure 13 shows that poor but well educated countries grow rapidly when technology inflows accelerate. It does not imply that policies to promote education are the most valuable ones, however.

7. SENSITIVITY ANALYSIS

The baseline experiments give elasticities of one half with respect to own human capital in both technologies, $\omega = \eta = 0.5$, give the rest of the weight to the local technology, $\beta = \zeta = 0.50$, and allow no role for average human capital $\overline{H}$ in either technology. The latter fact implies that the subsidy σ_F in the frontier economy is quite small.

This section looks at the effects of allowing a positive role for $\overline{H}$. In particular, ω and η are kept at their baseline values, and β and ζ are reduced. Since the frontier economy sets its subsidy σ_F at the level that is efficient (welfare-maximizing) on the BGP, the values for σ_F are much higher in this experiment. For these model parameters, it is interesting to rerun the policy experiment in Figure 12, reducing the barrier and increasing the subsidy.

[To be completed.]

8. CONCLUSIONS

The model developed here studies the interaction between technology, which flows in from abroad, and human capital, which is accumulated domestically, as an explanation of cross-country differences in income levels and growth rates. As argued in section 1, several different types of evidence, from many countries and over long time periods, point to the importance of technology inflows. But technology, by itself, does not seem to be a sufficient explanation. In the model here local human capital is critical in letting countries effectively exploit technologies imported from abroad and in allowing that inflow to continue.

The model developed here has a number of empirical implications. First, as shown in Proposition 1, economies with sufficiently unfavorable policies—a combination of high barriers to technology inflows and low subsidies to human capital accumulation—have no BGP. Thus, the model implies that only middle and upper income economies should grow like the technology frontier over long periods. Low income economies can grow faster (if they have reduced their barriers) or slower (if they have raised them) or stagnate. The empirical evidence is at least consistent with this prediction. Higher income countries grow at very similar rates over long periods, while low income countries show more heterogeneity (in cross section) and variability (over time).

Second, unfavorable policies reduce the level of income and consumption on the BGP, and also shrink the set of initial conditions for which an economy converges to that path. Thus, the model predicts that middle-income countries that grow with the frontier should have displayed slower growth during their transitional phases.

Third, growth miracles feature a modest period of very rapid TFP growth, accompanied by rapid growth in income and consumption. This initial phase is followed by a (longer) period during which human (and physical) capital are accumulated. Income growth declines toward the rate of frontier growth as the income level approaches the frontier level. Thus, the model has predictions for some features of the transition paths of growth 'miracles.'

Fourth, low income countries with higher human capital are better candidates to become growth miracles, since TFP and income grow more rapidly in such economies.

Finally, the model suggests that policies stimulating technology transfer are more effective in accelerating growth than policies stimulating human capital accumulation. Investment in human—and physical—capital responds to returns, and those returns are high when technology is growing rapidly. Thus, the empirical association between high investment rates and rapid growth is not causal: technology drives both.

APPENDIX

A. Equilibrium conditions

The Hamiltonian for the household's problem in (9) is

$$\begin{aligned}\mathfrak{H} &= \frac{C^{1-\theta}}{1-\theta} + \Lambda_H \left[\phi_0 \left(vH\right)^{\eta} A^{\zeta} \overline{H}^{1-\eta-\zeta} - \delta_H H\right] \\ &\quad + \Lambda_K \left[\left(1-v\right) \hat{w} A^{\beta} H^{\omega} \overline{H}^{1-\beta-\omega} + v\sigma\hat{w} A^{\beta} \overline{H}^{1-\beta} + \left(R-\delta_K\right) K - C - \tau\right],\end{aligned}$$

where to simplify the notation the subscript i's have been dropped. Taking the first order conditions for a maximum, using the equilibrium conditions $\overline{H} = H$ and $\tau = v\sigma\hat{w}A^{\beta}H^{1-\beta}$, and simplifying gives

$$\begin{aligned}\Lambda_H \phi_0 \eta v^{\eta-1} &= \Lambda_K \left(1-\sigma\right) \hat{w} \left(\frac{A}{H}\right)^{\beta-\zeta}, \qquad (24)\\ C^{-\theta} &= \Lambda_K, \\ \frac{\dot{\Lambda}_H}{\Lambda_H} &= \rho + \delta_H - \phi_0 \eta v^{\eta} \left(\frac{A}{H}\right)^{\zeta} \left(1 + \frac{\omega}{1-\sigma}\frac{1-v}{v}\right), \\ \frac{\dot{\Lambda}_K}{\Lambda_K} &= \rho + \delta_K - R, \\ \frac{\dot{H}}{H} &= \phi_0 v^{\eta} \left(\frac{A}{H}\right)^{\zeta} - \delta_H, \\ \frac{\dot{K}}{K} &= \left(\frac{K}{L}\right)^{\alpha-1} - \frac{C}{K} - \delta_K,\end{aligned}$$

where $L = \left(1-v\right) A^{\beta} H^{1-\beta}$, and $\dot{A}/A$ is in (2). The transversality conditions are

$$\lim_{t\to\infty} e^{-\rho t}\Lambda_K(t)K(t) = 0 \qquad \text{and} \qquad \lim_{t\to\infty} e^{-\rho t}\Lambda_H(t)H(t) = 0. \qquad (25)$$

Equations (1), (2), (24) and (25) characterize the competitive equilibrium, given the policy parameters (B, σ) and initial values for the state variables (W, A, H, K).

The law of motion for A requires H/W and A/W to be constant along a BGP, so A and H must also grow at the rate g. Since the production functions for effective labor in (4) and for output have constant returns to scale, the factor inputs L and K also grow at the rate g, as do output Y and consumption C. Hence the factor returns R and $\hat{w}$ are constant on a BGP, and the costate variable Λ_K grows at the rate $-\theta g$. The costate Λ_H grows at the same rate as Λ_K.

The normalized conditions in (12) follow directly from (2) and (24).

B. Linear approximations and stability

Define the constants

$$\chi \equiv \eta \frac{\omega}{1-\sigma}, \qquad \Pi_H \equiv \frac{r^{bg}+\delta_H}{g+\delta_H}, \qquad \Delta \equiv \beta(1-\theta)-\zeta,$$

$$\begin{aligned}
\Gamma_2 &\equiv \eta - \frac{\chi}{\eta v^{bg} + \chi\left(1-v^{bg}\right)} < \eta, \\
&= \eta - \frac{\chi / v^{g}}{\eta + \chi\left(1/v^{bg}-1\right)} \\
&= \eta - \frac{\chi + \Pi_H - \eta}{\Pi_H}, \\
\frac{1}{\Gamma_3} &\equiv \eta - 1 - \frac{\theta}{1/v^{bg}-1} \\
&= \eta - 1 - \frac{\theta\chi}{\Pi_H - \eta} < 0,
\end{aligned}$$

and recall that

$$\frac{1}{v^{bg}} - 1 = \frac{1}{\chi}\left(\Pi_H - \eta\right) > 0.$$

In addition define the log deviations

$$x_1 \equiv \ln\left(a/a^{bg}\right), \qquad x_2 \equiv \ln\left(h/h^{bg}\right), \qquad x_3 \equiv \ln\left(\lambda_h/\lambda_h^{bg}\right),$$

Take a first-order approximation to (20) to get

$$\frac{v-v^{bg}}{v^{bg}} = \Gamma_3\left[\Delta x_1 - (\Delta+\theta)\,x_2 - x_3\right].$$

Then linearize the laws of motion for a, h, and λ_h in (12) to find that

$$
\begin{aligned}
\dot{x}_1 &\approx (g+\delta_A)\left[-\frac{a_J^{bg}}{1-a_J^{bg}}x_1 + x_2\right], \\
\dot{x}_2 &\approx (g+\delta_H)\left[\zeta(x_1-x_2)+\eta\frac{v-v^{bg}}{v^{bg}}\right] \\
&= (g+\delta_H)\left\{\zeta(x_1-x_2)+\eta\Gamma_3\left[\Delta(x_1-x_2)-\theta x_2 - x_3\right]\right\}, \\
\dot{x}_3 &\approx -\left(r^{bg}+\delta_H\right)\left[\zeta(x_1-x_2)+\Gamma_2\frac{v-v^{bg}}{v^{bg}}\right] \\
&= -\left(r^{bg}+\delta_H\right)\left\{\zeta(x_1-x_2)+\Gamma_2\Gamma_3\left[\Delta(x_1-x_2)-\theta x_2 - x_3\right]\right\}.
\end{aligned}
$$

Hence

$$
\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{pmatrix} \approx \begin{pmatrix} -q_{1J} & q_2 & 0 \\ q_3 & -q_3-\theta q_4 & -q_4 \\ -q_6 & q_6+\theta q_5 & q_5 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix},
$$

where

$$
\begin{aligned}
q_{1J} &= (g+\delta_A)\, a_J^{bg}/\left(1-a_J^{bg}\right), \\
q_2 &= g+\delta_A, \\
q_3 &= (g+\delta_H)(\zeta+\eta\Gamma_3\Delta), \\
q_4 &= (g+\delta_H)\,\eta\Gamma_3, \\
q_5 &= \left(r^{bg}+\delta_H\right)\Gamma_2\Gamma_3, \\
q_6 &= \left(r^{bg}+\delta_H\right)(\zeta+\Gamma_2\Gamma_3\Delta).
\end{aligned}
$$

The local stability of each steady state depends on the roots of the associated characteristic equation,

$$
0 = \det\begin{pmatrix} -R-q_{1J} & q_2 & 0 \\ q_3 & -R-(q_3+\theta q_4) & -q_4 \\ -q_6 & q_6+\theta q_5 & -R+q_5 \end{pmatrix}
$$

$$
= \det \begin{pmatrix} -\left(R+q_{1J}\right) & q_2 & 0 \\ q_3 & -\left(R+q_3\right) & -q_4 \\ -q_6 & q_6+\theta R & -R+q_5 \end{pmatrix}
$$

$$
\begin{aligned}
&= -\left(R+q_{1J}\right)\left[\left(R+q_3\right)\left(R-q_5\right)+q_4\left(q_6+\theta R\right)\right] \\
&\quad -q_2\left(-Rq_3+q_3q_5-q_4q_6\right) \\
&= -\left(R+q_{1J}\right)\left[R^2-Rm_1-m_2\right]+q_2\left(Rq_3-m_2\right) \\
&= -R^3+\left(m_1-q_{1J}\right)R^2+\left(m_2+m_1q_{1J}+q_2q_3\right)R+m_2\left(q_{1J}-q_2\right),
\end{aligned}
$$

where

$$
\begin{aligned}
m_1 &\equiv q_5-q_3-\theta q_4, \\
m_2 &\equiv q_3q_5-q_4q_6 \\
&= \left(g+\delta_H\right)\left(r^{bg}+\delta_H\right)\Gamma_3\zeta\left(\Gamma_2-\eta\right)>0,
\end{aligned}
$$

and the last line uses the fact that $\Gamma_3<0$ and $\Gamma_2-\eta<0$.

Write the cubic as

$$
0=\Psi_J(R)\equiv -R^3+A_{2J}R^2+A_{1J}R+A_{0J}, \qquad J=H,L,
$$

where

$$
\begin{aligned}
A_{2J} &\equiv m_1-q_{1J}, \\
A_{1J} &\equiv m_2+m_1q_{1J}+q_2q_3, \\
A_{0J} &\equiv m_2\left(q_{1J}-q_2\right), \qquad\qquad J=H,L.
\end{aligned}
$$

Since $a_H^{bg}>1/2>a_L^{bg}$, it follows that $q_{1H}>q_2>q_{1L}$. Hence

$$
\Psi_H(0)=A_{0H}>0, \qquad \Psi_L(0)=A_{0L}<0,
$$

so Ψ_H has at least one positive real root, and Ψ_L has at least one negative real root.

The other roots of Ψ_H are real and both are negative if and only if Ψ_H has real inflection points, the smaller one is negative—call it $I_H < 0$, and $\Psi_H(I_H) < 0$. Similarly, the other roots of Ψ_L are real and both are positive, if and only if Ψ_L has real inflection points, the larger one is positive—call it $I_L > 0$, and $\Psi_L(I_L) > 0$. The inflection points are solutions of the quadratic $\Psi'_J(I) = 0$, so they are real if and only if $\Psi'_J(0) = A_{1J} > 0$. Write this condition as

$$0 < m_2 + (m_1 + q_3) q_2 + m_1 (q_{1J} - q_2) . \tag{26}$$

To construct examples where Ψ_J has complex roots, consider cases where the inequality in (26) fails. As noted above, $m_2 > 0$. The term $(q_{1J} - q_2)$ is small in absolute value if a_j^{bg} is close to $1/2$. As shown in Figure 3, this occurs if B is close to the highest value for which a BGP exists. For the middle term note that $q_2 > 0$ and

$$\begin{aligned}
m_1 + q_3 &= q_5 - \theta q_4 \\
&= (g + \delta_H) \Gamma_3 (\Pi_H \Gamma_2 - \theta \eta) \\
&= -(g + \delta_H) \Gamma_3 [\theta \eta - \Pi_H \eta + \chi + \Pi_H - \eta] \\
&= -(g + \delta_H) \Gamma_3 \left[\eta \left(\theta - 1 + \frac{\omega}{1 - \sigma} \right) + \Pi_H (1 - \eta) \right] .
\end{aligned}$$

Since $\Gamma_3 < 0$,this term has the sign of the expression in square brackets. The terms $\Pi_H (1 - \eta)$ is positive, but for η close to one it is small. The term $\omega / (1 - \sigma)$ is also positive, but for ω close to zero it is also small. The term $\theta - 1$ is negative if θ is close to zero. Thus, complex roots can occur if B is large, η is close to unity, and ω, θ are close to zero.

For example, for with the preference and technology parameters

$$\theta = 0.10, \quad \eta = 0.95, \quad \zeta = 0.05, \quad \omega = 0.05, \quad \beta = 0.45,$$

and the others at the baseline values, and the policy parameters

$$\sigma = 0.90907, \quad B = 1.6616,$$

the steady states are $a_H^{bg} = 0.68383$ and $a_L^{bg} = 0.31617$, and the system has complex roots—with negative real parts—at both steady states. For nearby policy parameters, the complex roots can have positive real parts at the low steady state, and the roots at the high steady state can be real while those at the low steady state remain complex. Notice that ω close to zero implies that an individual's wage depends very little on his own human capital, while $1 - \omega - \beta = 0.50$ implies that average human capital is quite important for productivity. Hence the optimal steady state subsidy is quite high—here it is $\sigma_F = 0.91594$, and the example uses a subsidy almost that high. In addition notice that θ close to zero implies a very high elasticity of intertemporal substitution. For both reasons, these parameter values are rather implausible.

Check sum of roots for $\sigma = 0,\ x_1 \equiv 0$.—

For the case where $x_1 = \dot{x}_1 \equiv 0$ the system is

$$\begin{pmatrix} \dot{x}_2 \\ \dot{x}_3 \end{pmatrix} \approx \begin{pmatrix} -q_3 - \theta q_4 & -q_4 \\ q_6 + \theta q_5 & q_5 \end{pmatrix} \begin{pmatrix} x_2 \\ x_3 \end{pmatrix},$$

so the roots satisfy

$$\begin{aligned} 0 &= (R + q_3 + \theta q_4)(R - q_5) + q_4 (q_6 + \theta q_5) \\ &= R^2 - m_1 R - m_2. \end{aligned}$$

Since $m_2 > 0$, the roots are real and of opposite sign.

For $\zeta = 1 - \eta,\ \omega = 1 - \beta,\ \sigma = 0$, the household's optimization problem is undistorted, so the sum of the roots, $R_1 + R_2 = m_1$, should equal the discount rate. For the normalization here, the condition is

$$m_1 = r^{bg} - g.$$

Note that for these parameter values,

$$\chi = \eta (1 - \beta), \qquad \Delta = \beta (1 - \theta) - (1 - \eta).$$

Use the expression above for m_1 to write the required condition as

$$\left(r^{bg}+\delta_H\right)\Gamma_2\Gamma_3-(g+\delta_H)\left[1-\eta+\eta\Gamma_3\left(\Delta+\theta\right)\right]=r^{bg}-g,$$

or

$$\Pi_H\Gamma_2\Gamma_3-\left[1-\eta+\eta\Gamma_3\left(\Delta+\theta\right)\right]=\Pi_H-1,$$

or

$$\begin{aligned}\Pi_H\Gamma_2-\eta\left(\Delta+\theta\right) &= \left(\Pi_H-\eta\right)\frac{1}{\Gamma_3}\\ &= \left(\Pi_H-\eta\right)\left(\eta-1\right)-\theta\eta\left(1-\beta\right).\end{aligned}$$

or

$$\begin{aligned}\Pi_H\left(\Gamma_2+1-\eta\right) &= \eta\left(\Delta+\theta\right)+\eta\left(1-\eta\right)-\theta\eta\left(1-\beta\right)\\ &= \eta\beta.\end{aligned}$$

Since

$$\Gamma_2-\eta+1=\frac{\eta-\chi}{\Pi_H}=\frac{\beta\eta}{\Pi_H},$$

the required condition holds.

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Figure 1: doubling times for per capita GDP, 55 countries

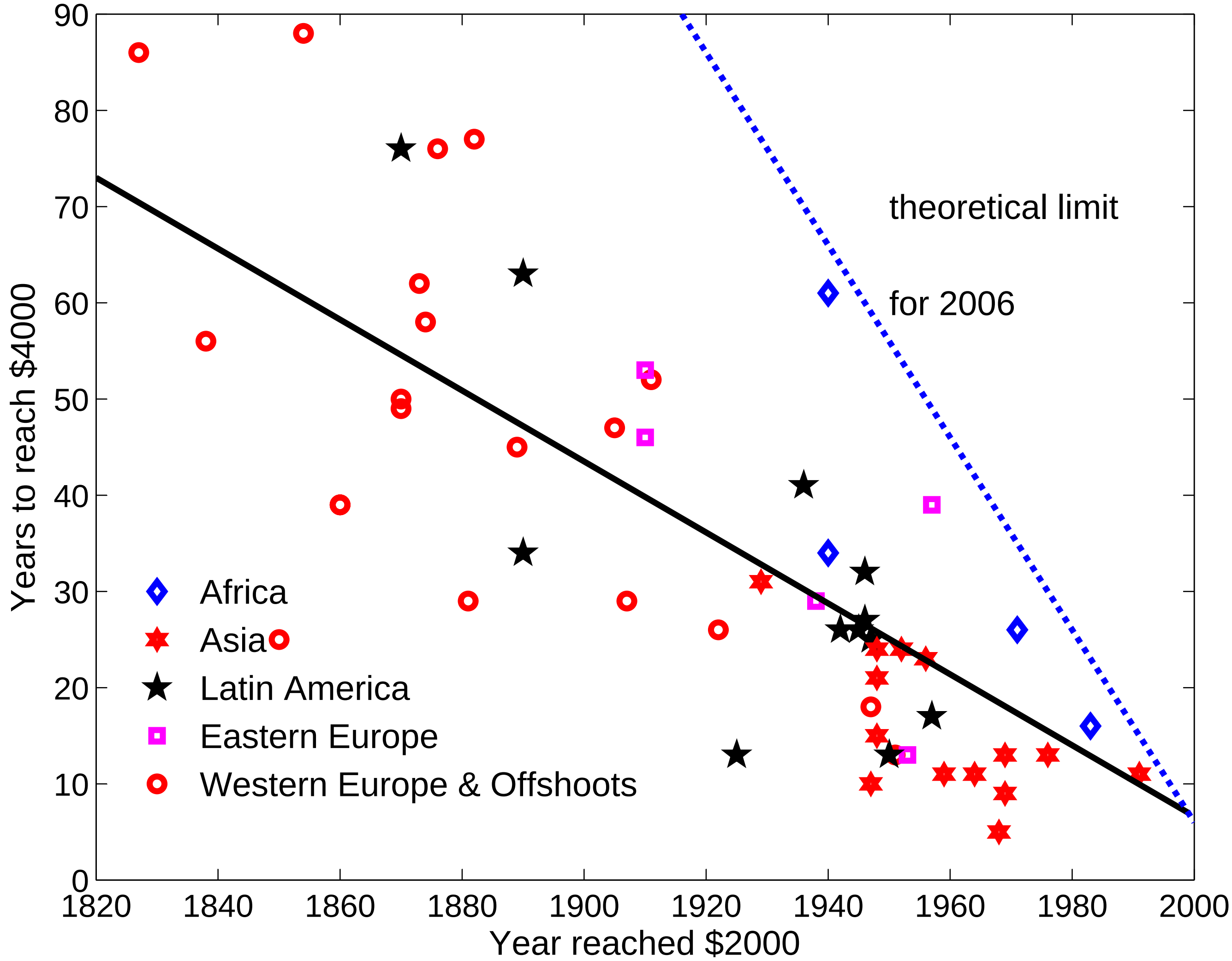


Figure 2: catching up and falling behind, 1960-2000

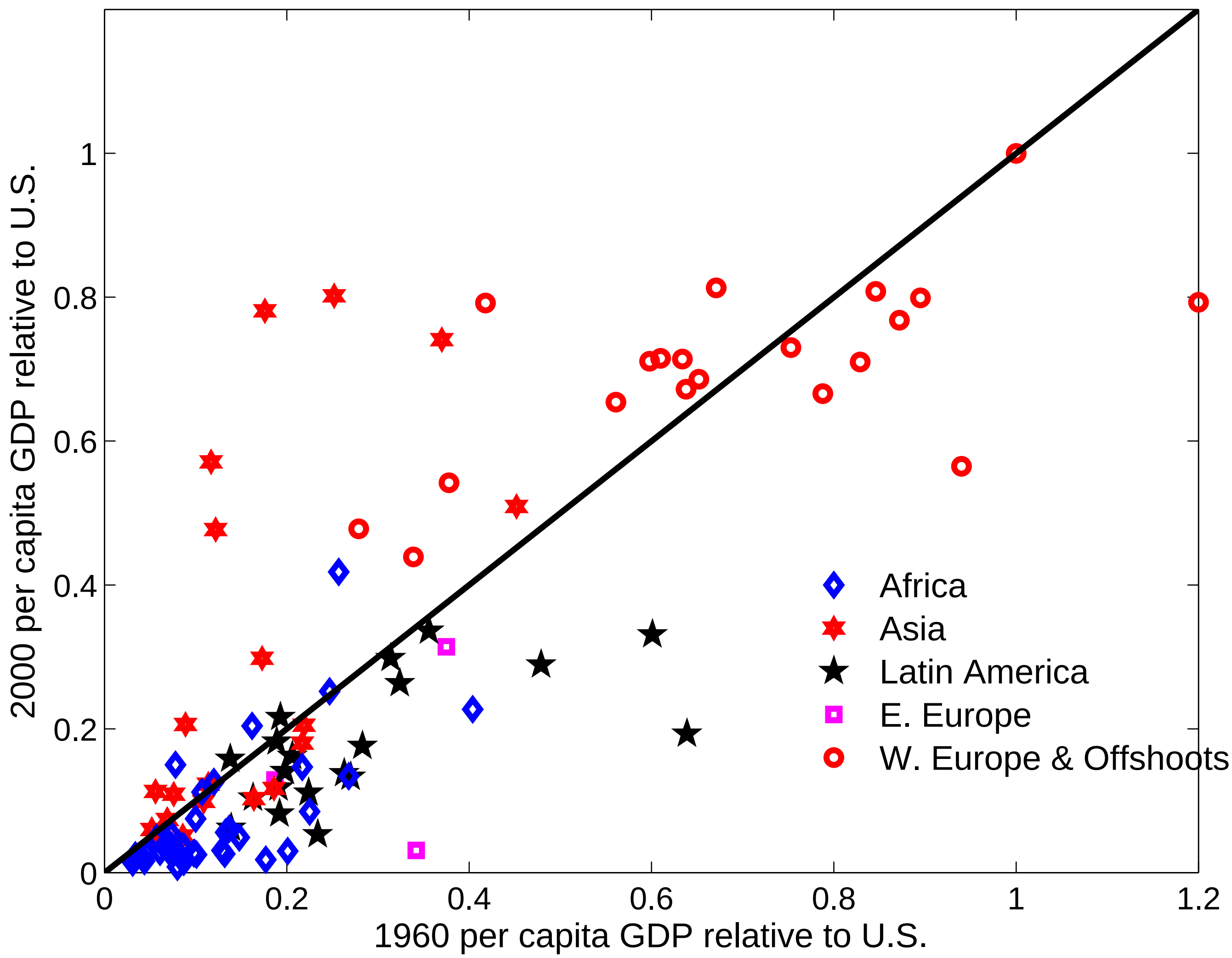


Figure 3: Asia and Africa, excluding 6 successes

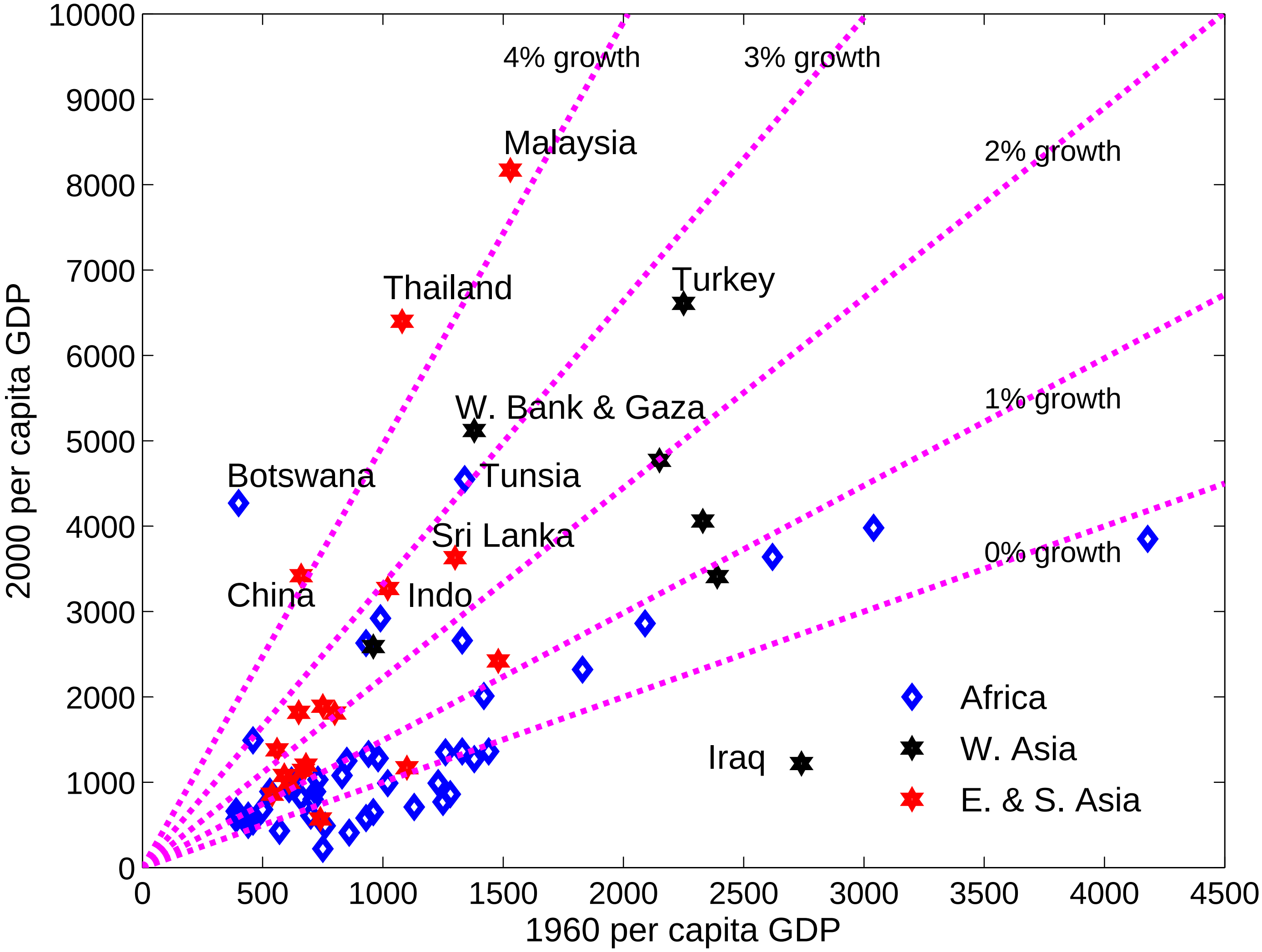


Figure 4A: policies for balanced growth

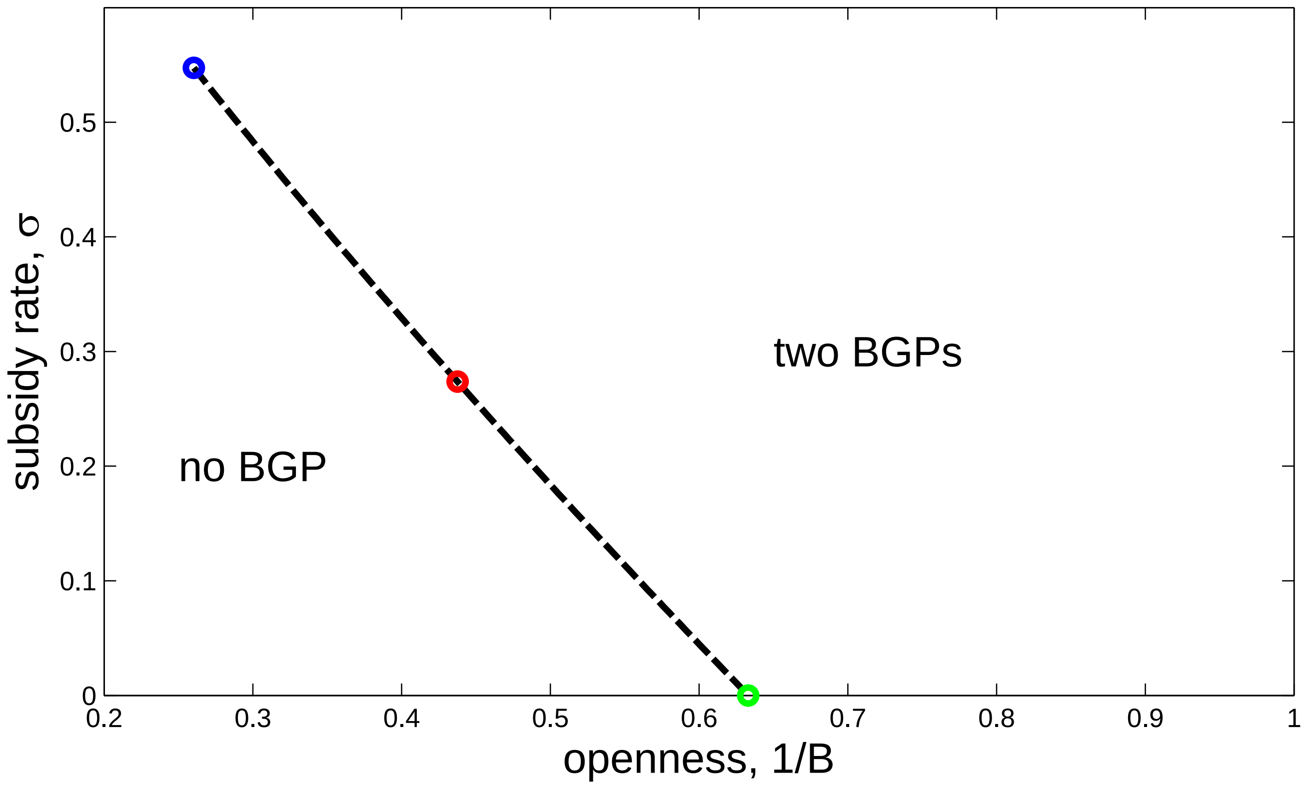


Figure 4B: relative technology on BGP, $0 = \sigma_0 < \sigma_1 < \sigma_2$

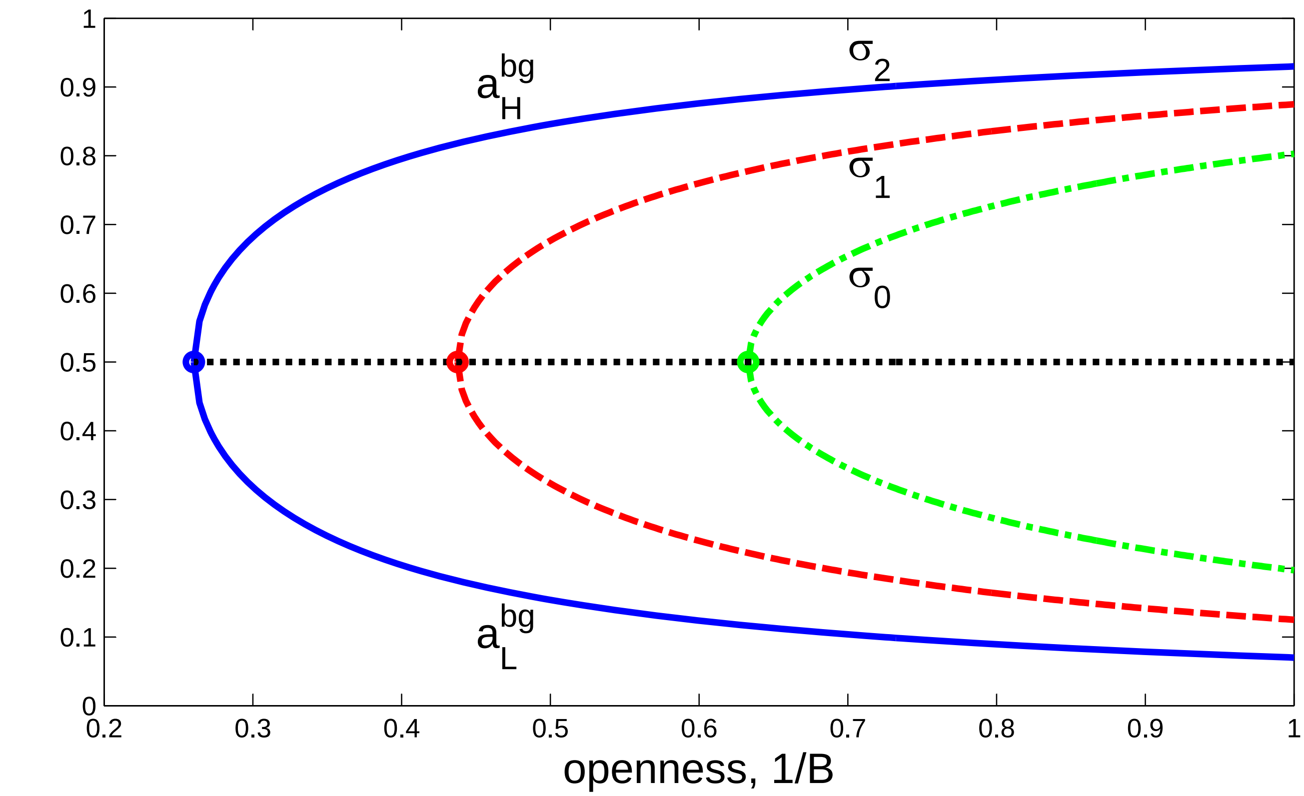


Figure 5a: basins of attraction for BGPs

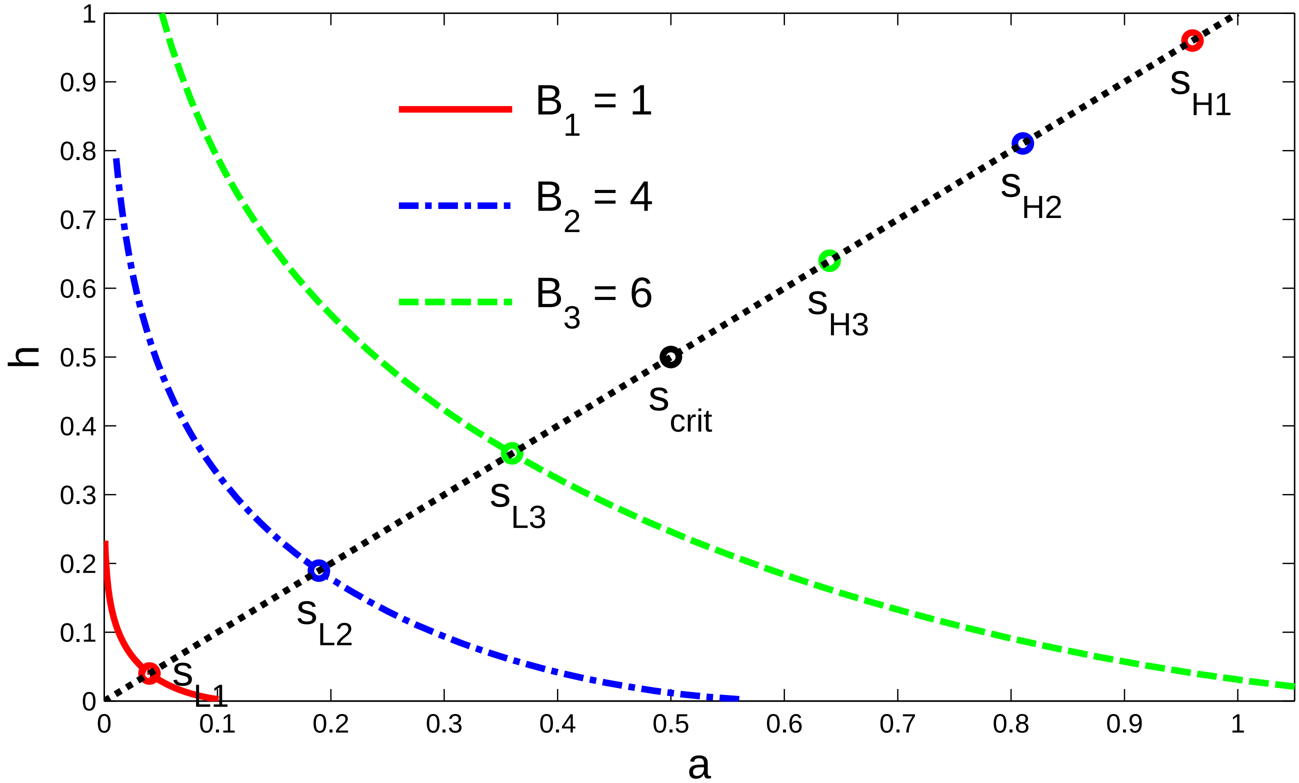


Figure 5b: "slow" and "fast" roots

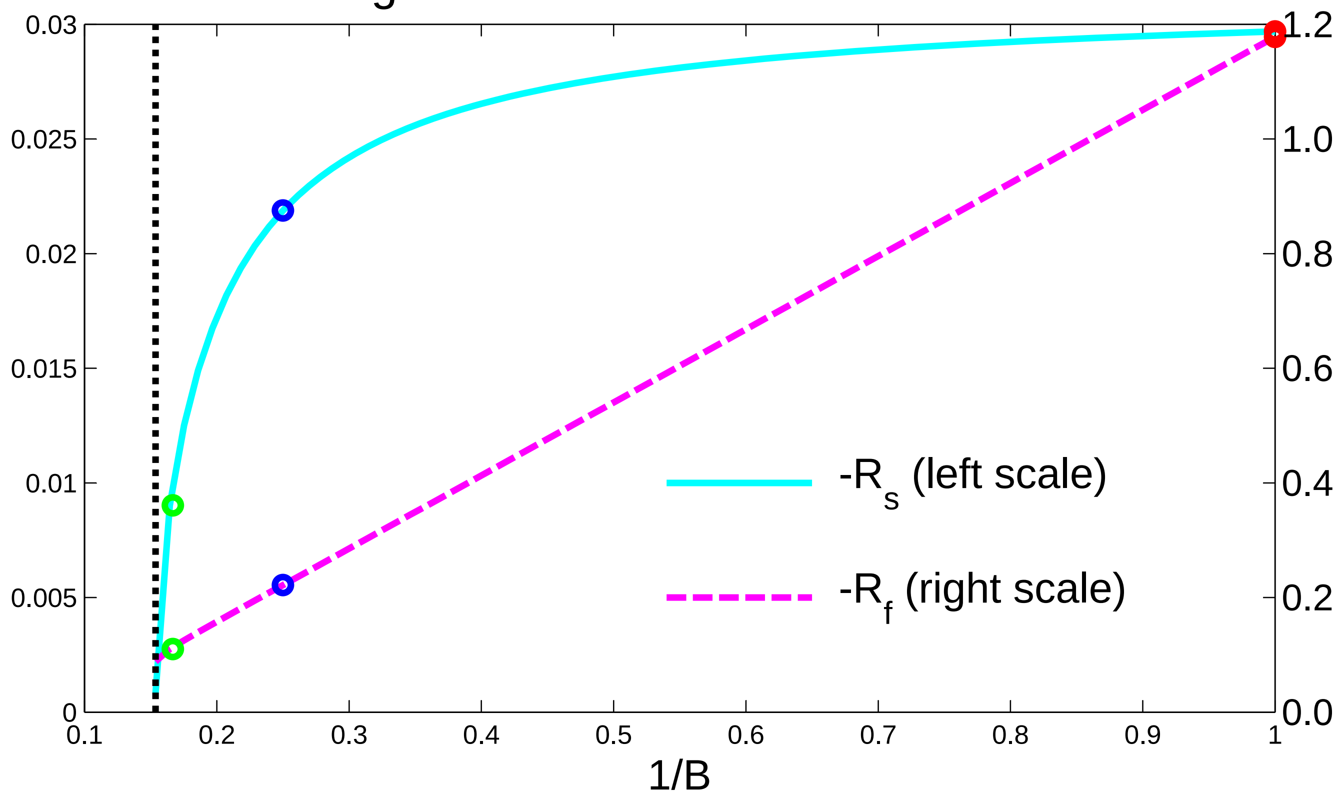


Figure 6a: phase diagram, B = B_1

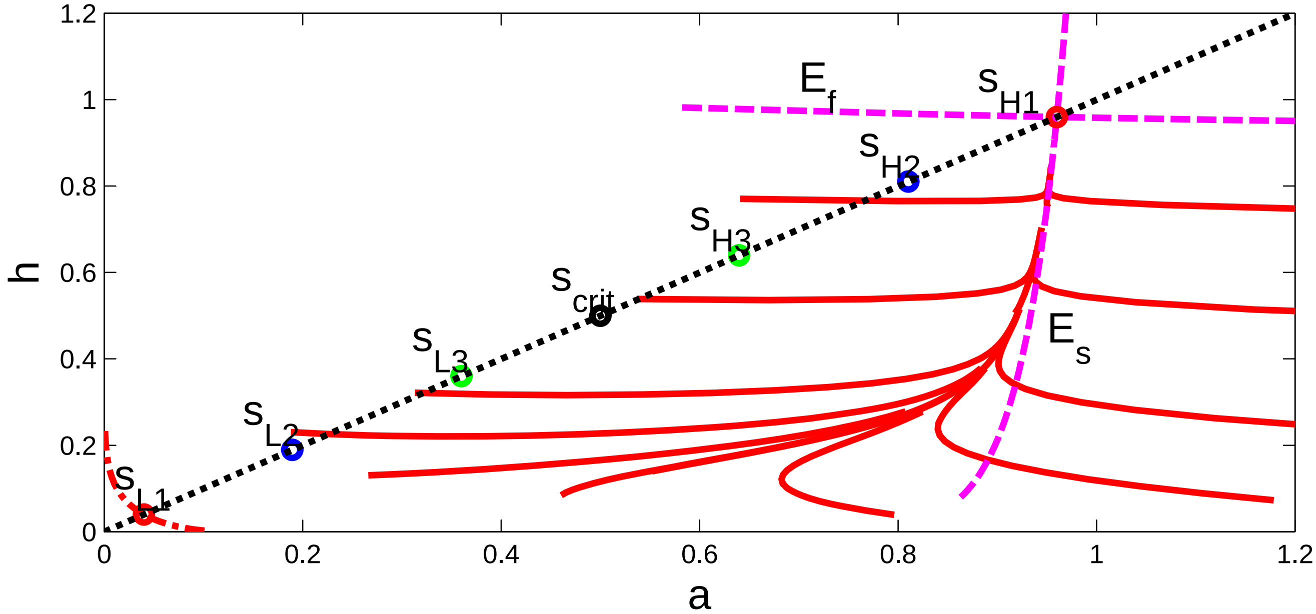
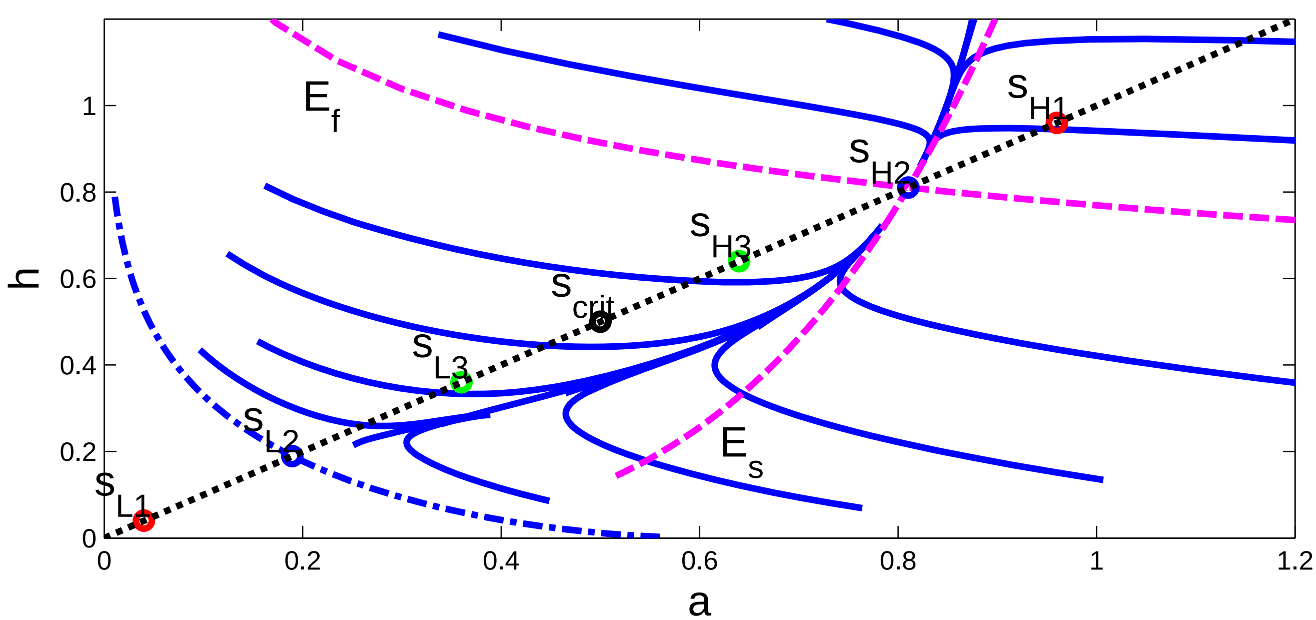


Figure 6b: phase diagram, B = B_2



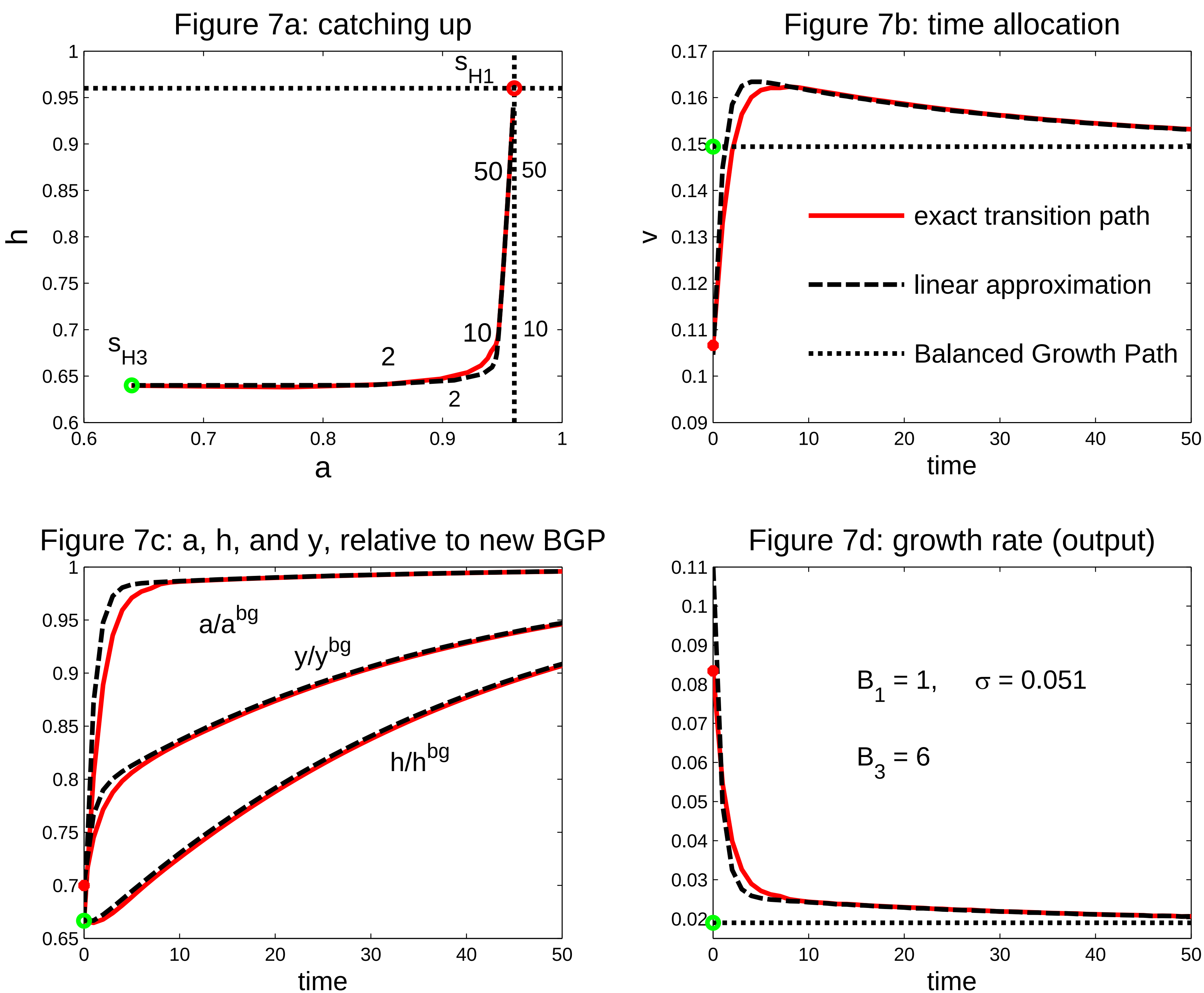


Figure 7a: catching up
h
a
s_H1
s_H3
50
50
10
10
2
2
Figure 7b: time allocation
v
time
exact transition path
linear approximation
Balanced Growth Path
Figure 7c: a, h, and y, relative to new BGP
a/a^bg
y/y^bg
h/h^bg
time
Figure 7d: growth rate (output)
B_1 = 1, σ = 0.051
B_3 = 6
time

Figure 8a: A lost decade

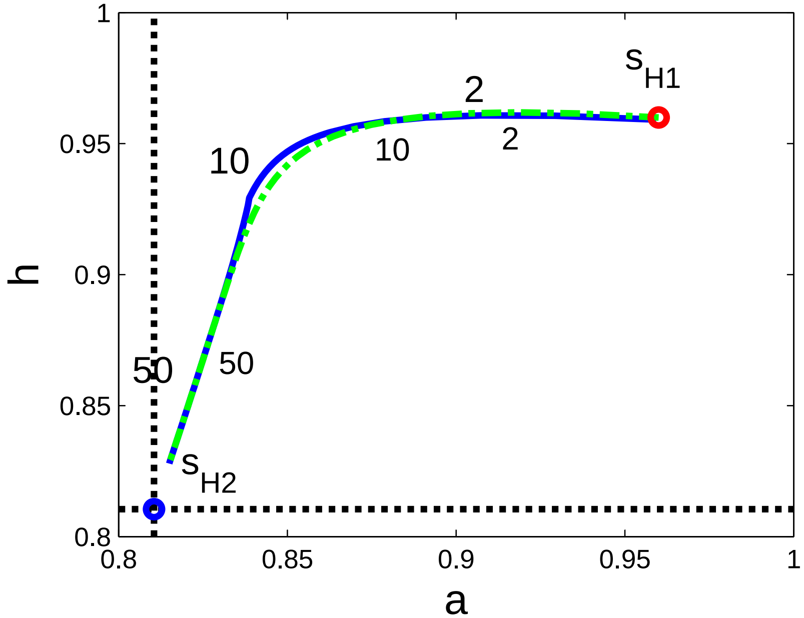


Figure 8b: time allocation

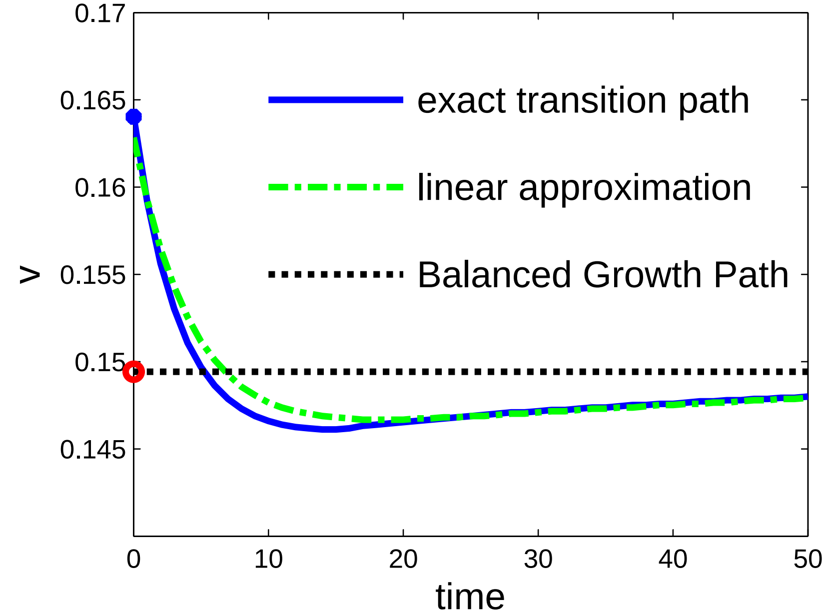


Figure 8c: a, h, and y, relative to new BGP

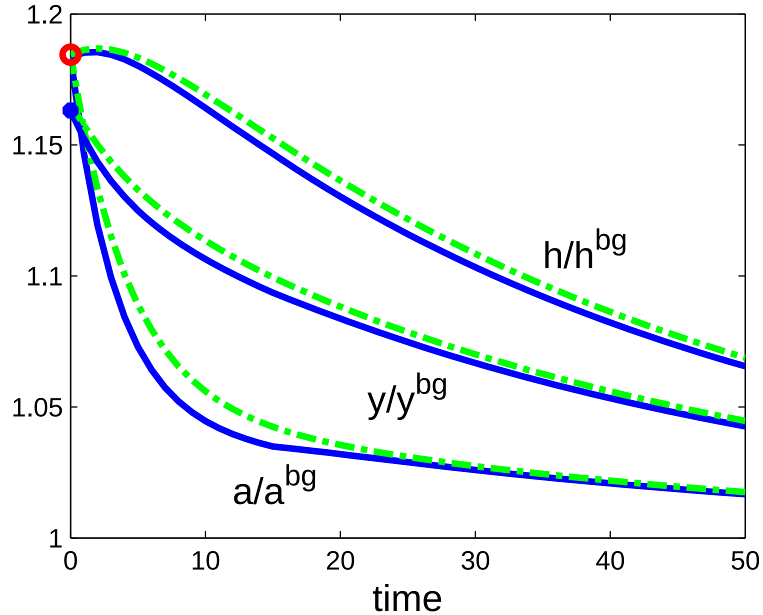
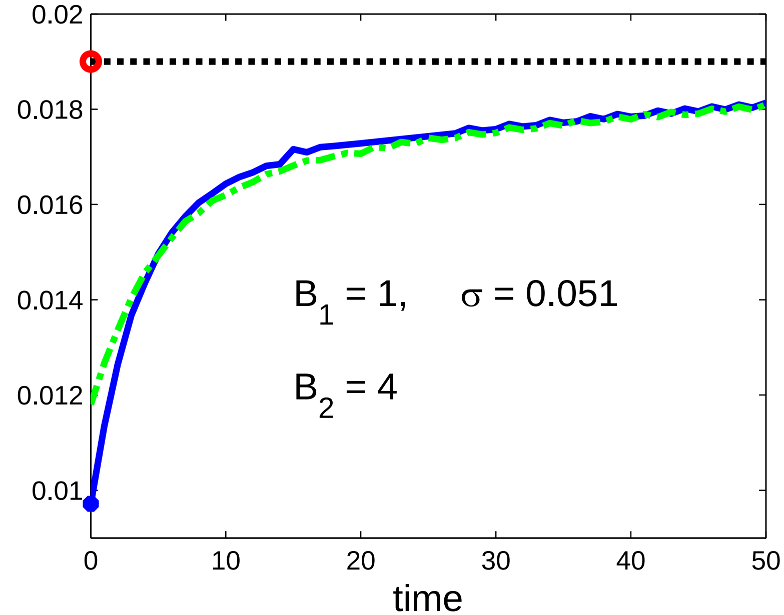


Figure 8d: growth rate (output)



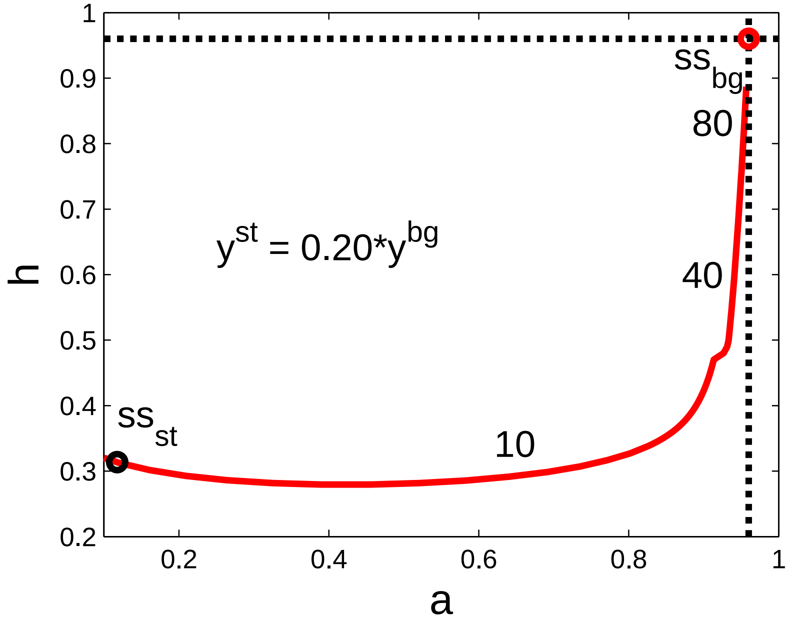


Figure 9a: a growth miracle

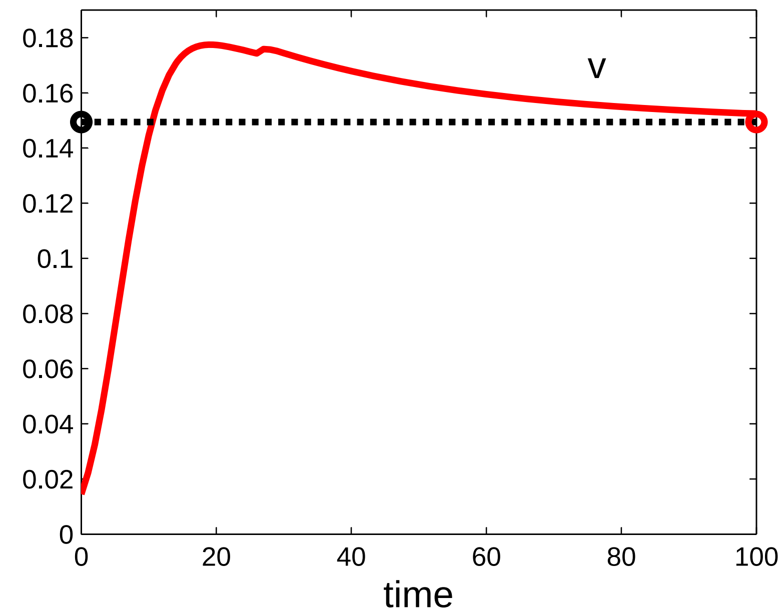


Figure 9b: time allocation

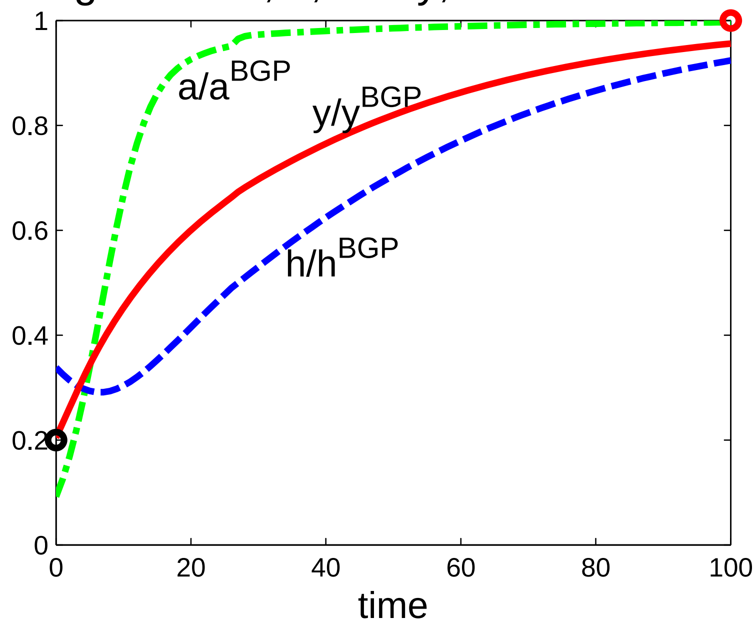


Figure 9c: a, h, and y, relative to BGP

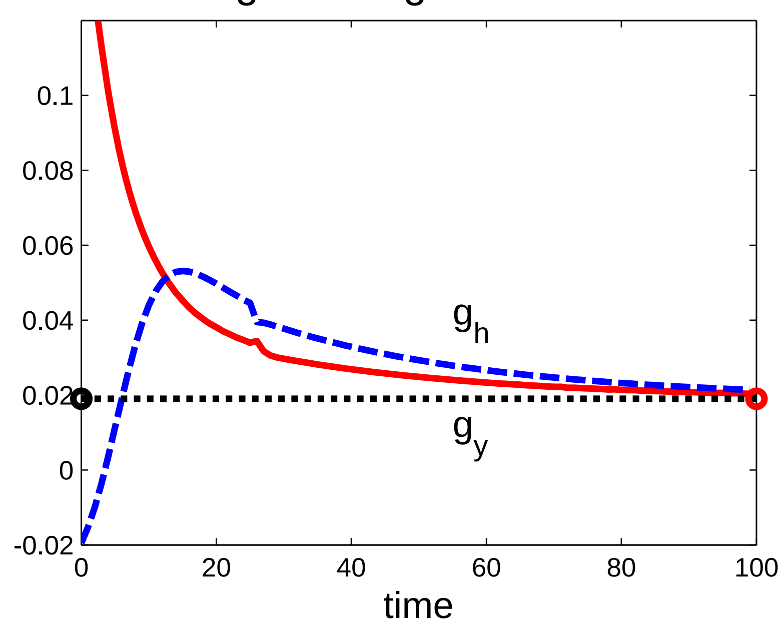


Figure 9d: growth rates

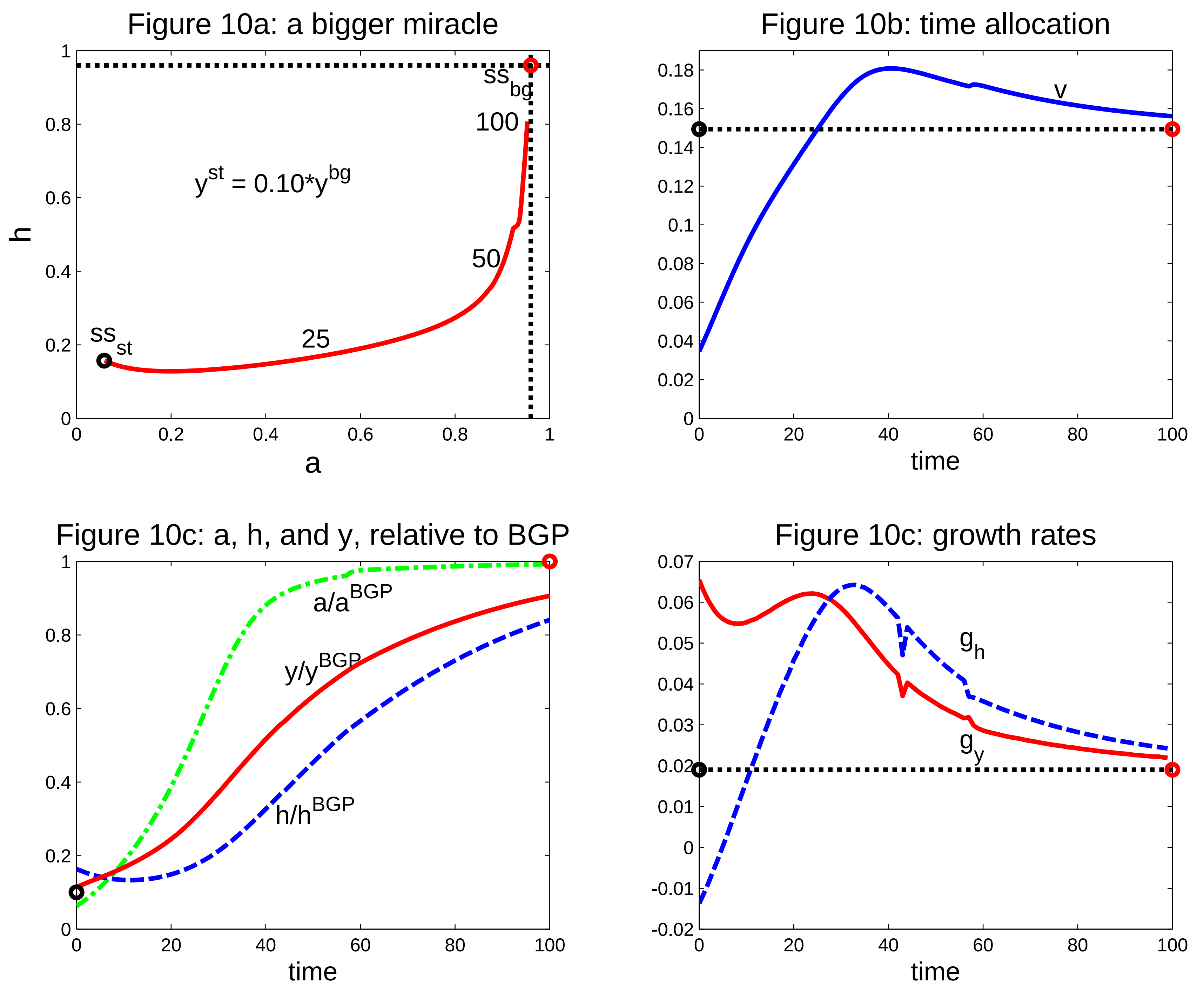


Figure 10a: a bigger miracle
h
a
ss_bg
100
50
25
ss_st
y^st = 0.10*y^bg
Figure 10b: time allocation
v
time
Figure 10c: a, h, and y, relative to BGP
a/a^BGP
y/y^BGP
h/h^BGP
time
Figure 10c: growth rates
g_h
g_y
time

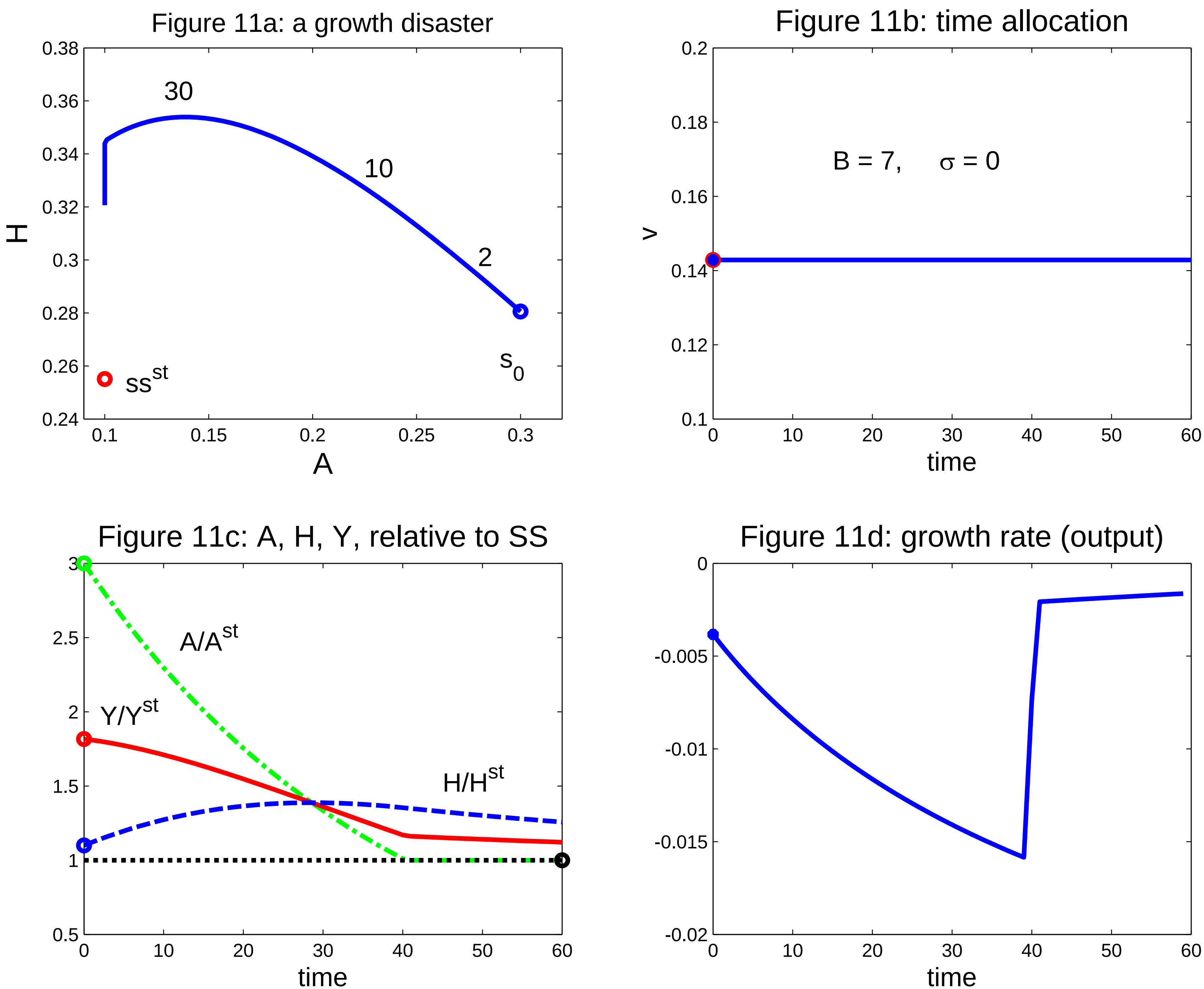


Figure 11a: a growth disaster
H
A
30
10
2
s_0
ss^{st}
0.38
0.36
0.34
0.32
0.3
0.28
0.26
0.24
0.1
0.15
0.2
0.25
0.3
Figure 11b: time allocation
B = 7, σ = 0
v
time
0.2
0.18
0.16
0.14
0.12
0.1
0
10
20
30
40
50
60
Figure 11c: A, H, Y, relative to SS
A/A^{st}
Y/Y^{st}
H/H^{st}
time
3
2.5
2
1.5
1
0.5
0
10
20
30
40
50
60
Figure 11d: growth rate (output)
time
0
-0.005
-0.01
-0.015
-0.02
0
10
20
30
40
50
60

Figure 12a: Transitions for two policies

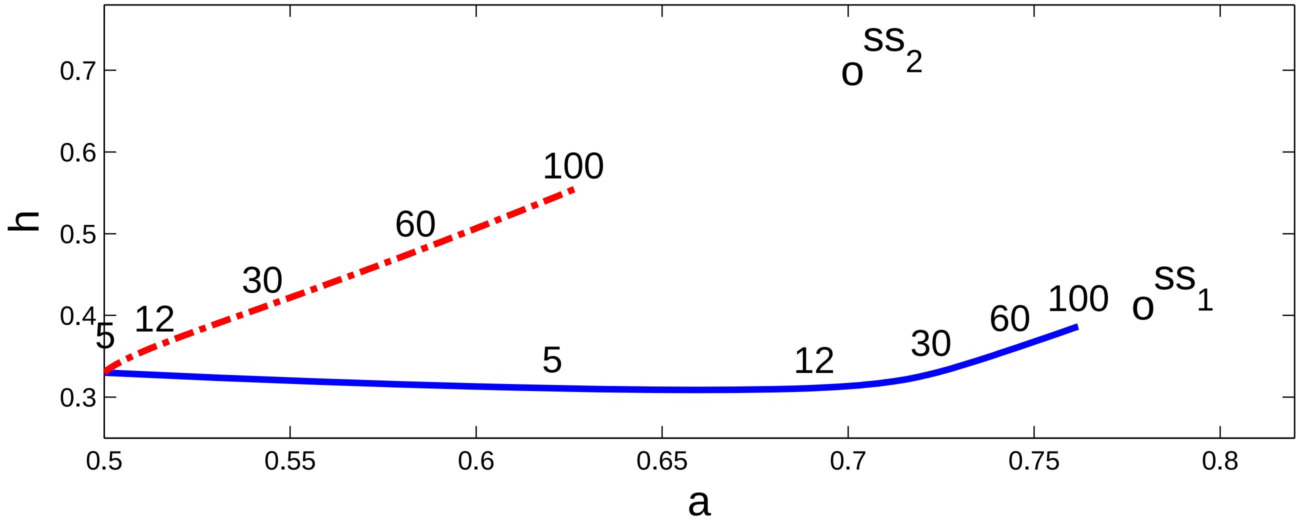


Figure 12b: time allocations

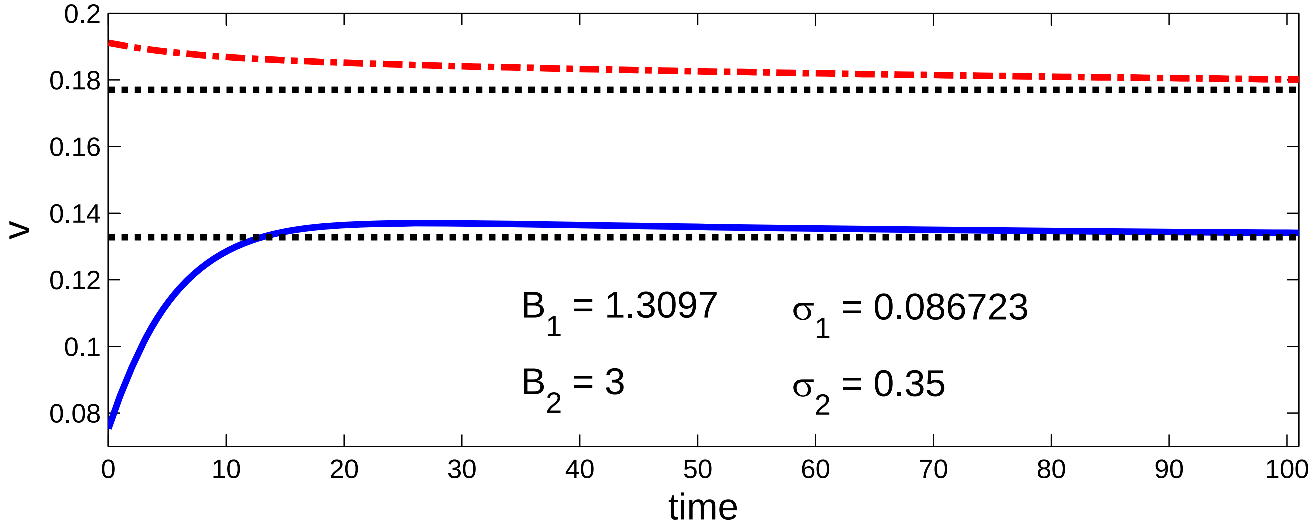


Figure 12c: growth rates (output)

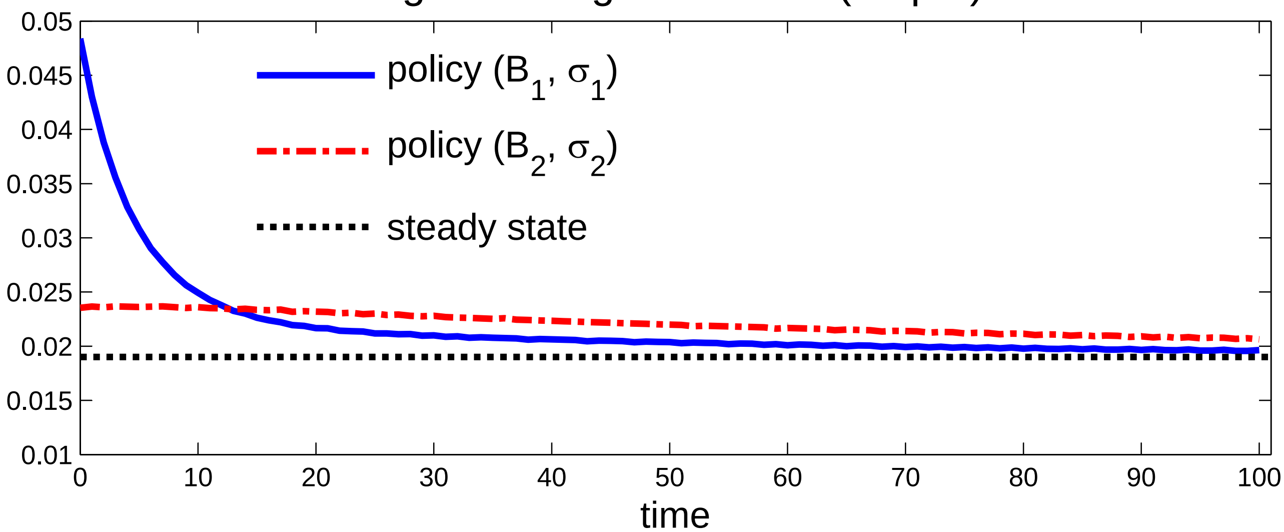


Figure 12d: technology levels, a

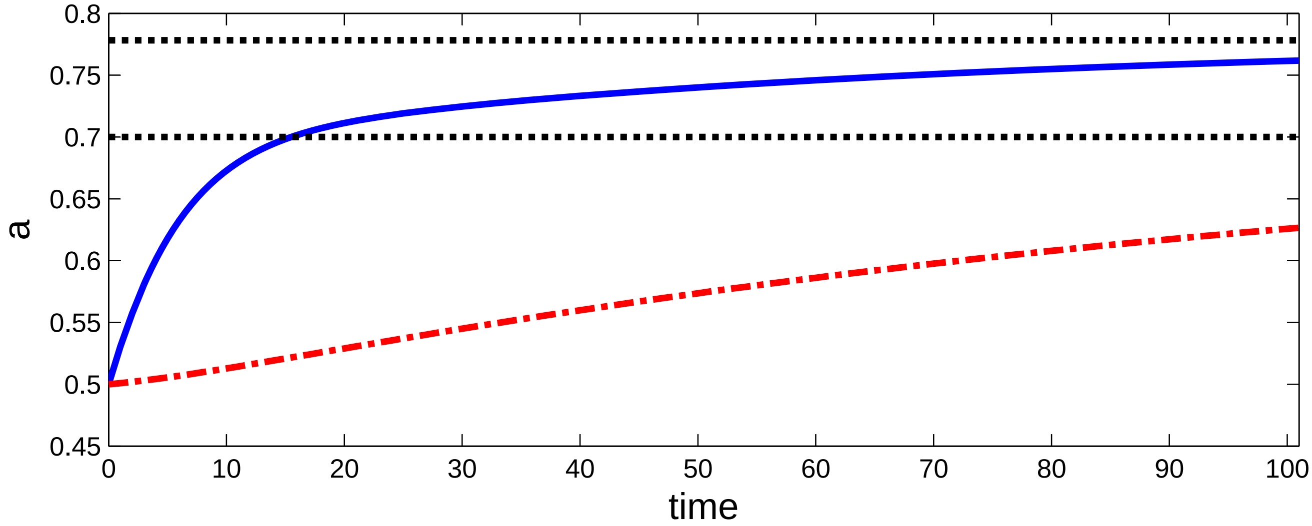


Figure 12e: human capital, h

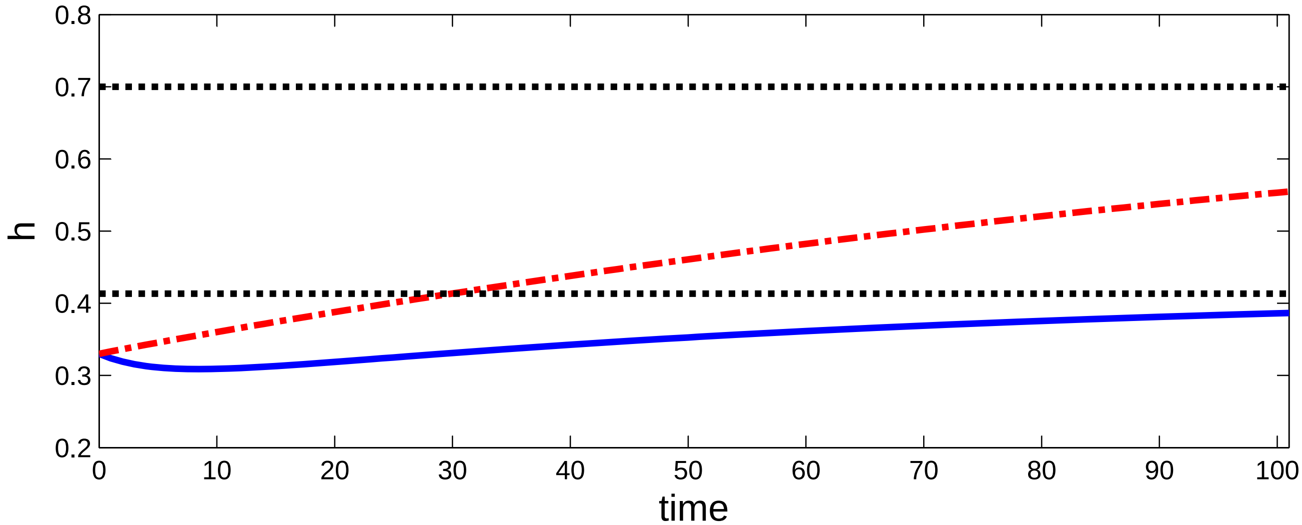


Figure 12f: consumption, c

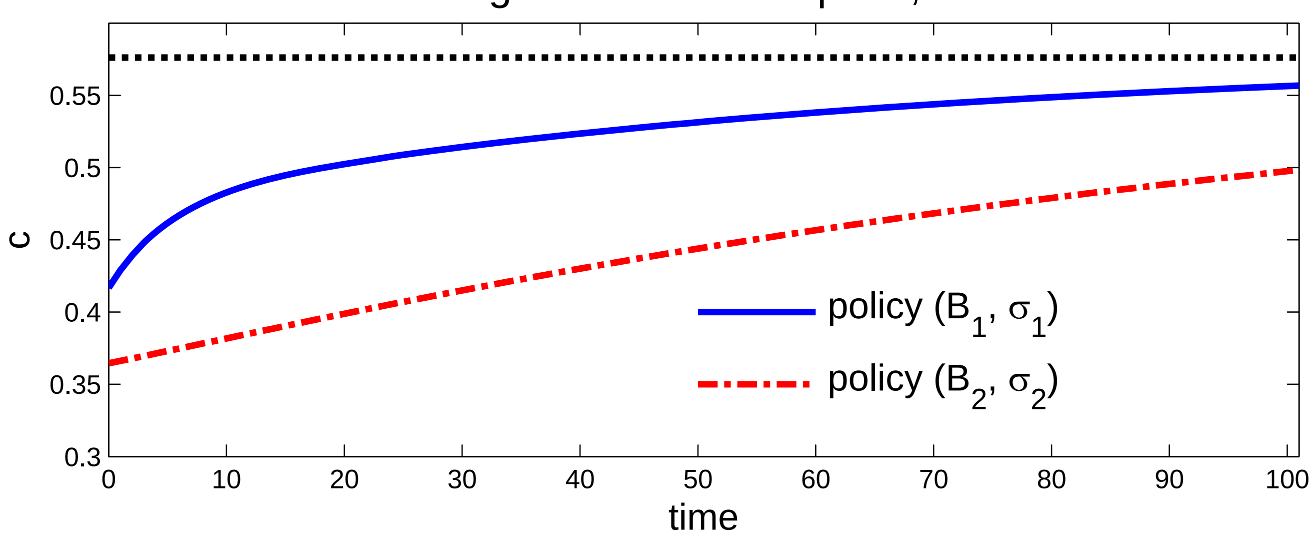


Figure 13a: Two transitions

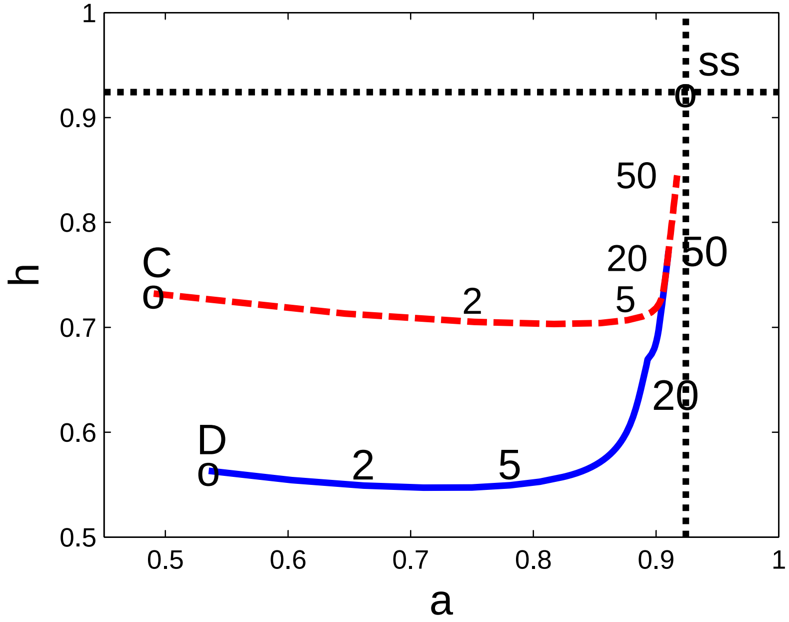


Figure 13b: time allocation

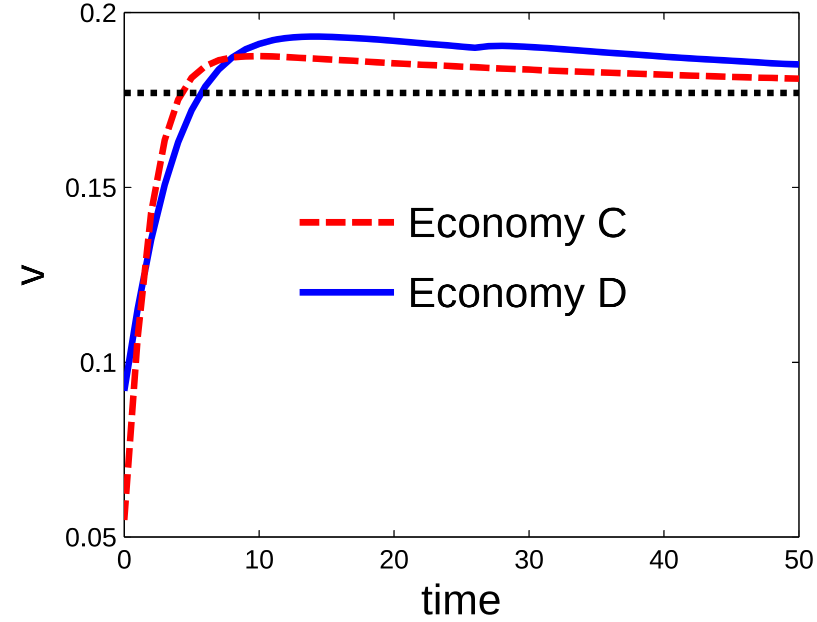


Figure 13d: a, h, c relative to BGP

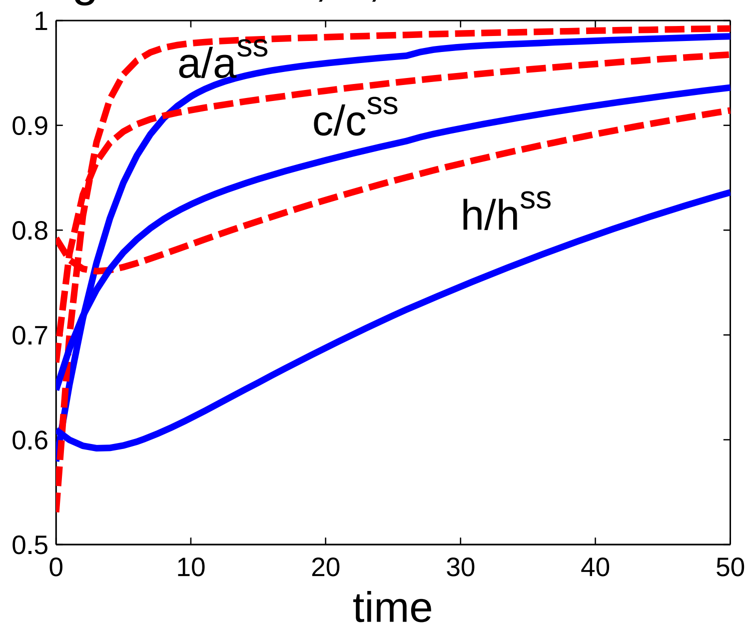


Figure 13c: growth rate (output)