Early and Late Human Capital Investments, Borrowing Constraints, and the Family*

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Abstract

We develop a dynastic human capital investment framework to study the importance of potential market failures – family borrowing constraints and uninsured labor market risk – as well as the process of intergenerational ability transmission in determining human capital investments in children at different ages. We explore the extent to which policies targeted to different ages can address these market failures, potentially improving economic efficiency and equity. We show that dynamic complementarity in investment and the timing of borrowing constraints are critical for the qualitative nature of investment responses to income and policy changes. Based on these analytical results, we use data from the Children of the NLSY (CNLSY) to establish that borrowing constraints bind for at least some families with young and old children.

Calibrating our model to fit data from the CNLSY, we find a moderate degree of dynamic complementarity in investment and that 12% of young and 14% of old parents borrow up to their limits. While the effects of relaxing any borrowing limit at a single stage of development are modest, completely eliminating all lifecycle borrowing limits dramatically increases investments, earnings, and intergenerational mobility. Additionally, the impacts of policy or family income changes at college-going ages are substantially greater when anticipated earlier, allowing early investments to adjust. Finally, we show that shifting the emphasis of investment subsidies from college-going ages to earlier ages increases aggregate welfare and human capital.

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1 Introduction

The growing importance of parental income for child achievement and educational attainment (Belley and Lochner 2007, Duncan and Murnane 2011, Reardon 2011) raises serious questions about the capacity (or willingness) of disadvantaged families to make efficient investments in their children. In this paper, we investigate the importance of potential market failures – family borrowing constraints and uninsured labor market risk – as well as the process of intergenerational ability transmission in determining human capital investments in children at different ages. We also explore the extent to which policies targeted to different ages can address these market failures, potentially improving economic efficiency, equity, and intergenerational mobility.

Sizeable gaps in childhood investments and achievement by parental income are already evident at early ages and persist (Carneiro and Heckman 2002, Caucutt, Lochner and Park 2017). Kaushal, Magnuson, and Waldfogel (2011) find that families in the bottom family expenditure quintile spend 3% of their total expenditures on educational enrichment items, while families in the top quintile spend 9%. Parental time is also an important input for a young child's development that poor parents may be unable to afford (Del Boca, Flinn, and Wiswall 2014, Mullins 2016). For example, Guryan, Hurst and Kearney (2008) show that higher-educated parents spend more time on childcare than less educated parents, whether or not one controls for employment status.

There are many mechanisms consistent with these child investment and achievement gaps by family income. Caucutt, Lochner and Park (2017) use a lifecycle human capital investment framework to explore several common explanations, including intergenerational ability transmission, consumption value to investment, imperfect information and uncertainty, and borrowing constraints. They conclude that one cannot explain the high estimated marginal returns to early investments (especially among poor children) without information or credit market frictions.[1] Both labor market risk and credit constraints can explain the growing evidence that family income significantly raises investments in children and child achievement, while borrowing constraints are crucial to match evidence that the timing of parental income matters for

[1] Heckman, et al. (2010) estimate private internal rates of return to Perry Preschool of about 8%, while Barnett and Masse (2007) estimate benefit-cost ratios of Abecedarian to be around 2.5. For broader surveys, see Karoly et al. (1998), Blau and Currie (2006), Cunha, et al. (2006), Almond and Currie (2011), Heckman and Kautz (2014), Kautz et al. (2014).

child achievement and educational outcomes.[2] Studies of consumption behavior also estimate important distortions due to credit market frictions (including imperfect insurance), especially for younger households.[3] While Caucutt, Lochner, and Park (2017) show which mechanisms are needed to explain a broad set of stylized facts in the child development literature, they do not evaluate the relative importance of those mechanisms. This paper does.[4]

We develop a dynastic model of early and late human capital investments in children to study and quantify the importance of intergenerational ability transmission, labor market uncertainty, and borrowing constraints over the lifecycle and across generations.[5] Our analysis starts with the recognition that investment in human capital is a multi-stage process that begins early in life.[6] As a result, we model human capital investment as an intergenerational family problem.[7] Our model accounts for the fact that later investments build on earlier investments, that early childhood investments are made by young parents at the beginning of their careers, and that

[2]Several recent studies demonstrate that exogenous increases in family income lead to real increases in child investments and improvements in child development (Løken 2010, Duncan, Morris and Rodrigues 2011, Milligan and Stabile 2011, Dahl and Lochner 2012, Løken, Mogstad and Wiswall 2012, Jones, et al. 2015). Carniero and Ginja (2016) estimate that investments respond to permanent income shocks but not transitory shocks. Caucutt and Lochner (2006), Aakvik, Salvanes, and Vaage (2005), and results presented below demonstrate that family income received at early childhood ages has a greater impact on adolescent achievement and educational attainment when compared with income received at later ages. Carneiro, et al. (2015) estimate a U-shaped relationship between the effect of income on education and the age at which income was received. Carneiro and Heckman (2002) estimate no differences in the effects of income on college enrolment based on when income was earned.

[3]See, e.g., Meghir and Weber (1996), Alessie, Devereux, and Weber (1997), Gourinchas and Parker (2002), Attanasio, Goldberg and Kyriazidou (2008), and Stephens (2008).

[4]Cunha (2014) estimates the extent to which a similar set of factors explain racial differences in early investment using a static one-period early investment framework. His analysis emphasizes biases in beliefs rather than uninsurable risk as an important information friction, concluding that these biases may help explain some of the differences between black and white family investments in their children.

[5]Our quantitative framework also allows for unmeasured costs of late investment, which could reflect a 'psychic cost' or 'consumption value' of schooling; however, we do not consider individual heterogeneity in these costs as in, for example, Carneiro, Hansen, and Heckman (2003) and Cunha, Heckman and Navarro (2005).

[6]See, e.g., Becker (1975), Todd and Wolpin (2003, 2007), Restuccia and Urrutia (2004), Cunha, et al. (2006), Cunha and Heckman (2007, 2008), Cunha, Heckman and Schennach (2010), Cunha (2013), Del Boca, Flinn, and Wiswall (2014), Gayle, Golan and Soytas (2014), Attanasio, Meghir and Nix (2015), Attanasio, et al. (2015), Agostinelli and Wiswall (2016), Lee and Seshadri (2016), and Mullins (2016).

[7]See, e.g., Becker and Tomes (1979, 1986), Loury (1981), Glomm and Ravikumar (1992), Galor and Zeira (1993), Aiyagari, Greenwood and Seshadri (2002), Caucutt and Kumar (2003), Restuccia and Urrutia (2004), Cunha and Heckman (2007, 2008), Gayle, Golan and Soytas (2014), and Lee and Seshadri (2016).

desired borrowing may differ substantially over the lifecycle and across families.

In our framework, young parents make early investments in their children and provide them with consumption. These parents, who are subject to earnings shocks, make their own consumption choices and borrow or save to intertemporally allocate resources. Constraints on their borrowing may limit consumption and investments in young children. Older children make additional investments in themselves (e.g. college), using their own earnings, transfers from their parents, and student loans to cover schooling costs and consumption. Again, choices may be impacted by imperfect credit markets and labor market uncertainty. Older parents must decide how much to transfer to their college-age children and how much to borrow or save for their own current and future consumption. Once a child leaves the home to establish his own family, parents continue to work, save, and consume until retirement. This cycle repeats itself, as young adults grow into parenthood.

We posit that a child's ability depends on his parent's ability and human capital. This relationship, along with market frictions, accounts for the sizeable investment gaps by parental income produced by the model. We find that children with parents in the top income quartile receive about \$3,000/year more in early investments and nearly \$8,000/year more in late investments relative to those in the bottom income quartile. Conditioning on child ability indicates that 15-25% of these gaps is due to ability transmission, leaving the remainder to be driven by market frictions.

Consistent with the analysis of Cunha and Heckman (2007), we show that *dynamic complementarity* in investment – the complementarity between early and late investments in human capital – plays a central role in determining the impacts of family income, investment subsidies and borrowing constraints on investment over the lifecycle. When investments are sufficiently complementary, a policy that encourages investment at one stage of development will also tend to increase investment at other stages.

An important consequence of dynamic complementarity is that studying the impacts of a policy change exclusively in that period can be misleading. For example, a large literature considers the effects of college-age policies on schooling and labor market outcomes holding early investment and adolescent achievement levels fixed.[8] The degree of dynamic complementarity we

[8] See, e.g., structural analyses, including Cameron and Heckman (1998), Heckman, Lochner and Taber (1998), Keane and Wolpin (2001), Caucutt and Kumar (2003), Cameron-Taber-2004, Hanushek, Leung and Yilmaz (2003), Johnson (2013), Abbott, et al. (2016), Hai and Heckman (2017), and Navarro and Zhou (2017). Quasi-experimental studies studying changes or differences in tuition or aid levels on college-going also implicitly hold early investment

calibrate suggests that these policies not only affect college-going, but also earlier investments in children.[9] Our quantitative analysis highlights that ignoring these earlier investment responses can lead researchers to under-estimate the total wage impact of college-age investment subsidies by almost 60%. We also show that when parents of college-age children experience a large, unanticipated income windfall or loss, the impacts on child outcomes appear to be small, consistent with evidence from Bulman, et al. (2016) and Hilger (2016) on the impacts of lottery winnings and paternal job loss, respectively. However, we demonstrate that if the income transfer is anticipated and parents can adjust early investments accordingly, the effects are much larger. Long-run differences in family income are likely to produce much greater differences in child investments and labor market outcomes than is suggested by empirical analyses exploiting 'exogenous shocks' to family resources during adolescence.

The timing of borrowing constraints can interact with dynamic complementarity in investment in a way that masks the importance of credit market frictions when focusing on limits at only one stage of development at a time. Individuals would like to adjust both early and late investments together due to complementarity, but relaxing one constraint does not help with (and can even exacerbate) the distortions caused by constraints at other ages. In our calibrated model, we find that no college-age children borrow up to their limits, while 10-15% of young and old parents do. Of course, the decisions of many more families are distorted by the possibility of binding constraints due to uncertainty about future income. Still, our calibration implies no effect of expanding student loan opportunities for old children, while increasing borrowing limits on either young or old parents one at a time has only modest impacts on investment behavior.[10] It is tempting to conclude from this that borrowing constraints are unimportant. However, we find that eliminating all lifecycle borrowing constraints simultaneously would generate substantial increases in investments and earnings, while shrinking the intergenerational correlation in human capital by one-quarter.

Keane and Wolpin (2001) and Johnson (2013) both emphasize the importance of differences

behaviors fixed (e.g. Dynarski 2003, Van der Klaauw 2002, Kane 2007).

[9] Our calibrated measure of dynamic complementarity is consistent with indirect evidence discussed in Cunha, et al. (2006) and estimates by Cunha, Heckman and Schennach (2010) and Cunha (2013).

[10] Consistent with these results, other recent studies estimating structural lifecycle models of schooling and labor supply in the presence of borrowing constraints estimate small effects of expansions in student loans on college attendance (Keane and Wolpin 2001, Johnson 2013, Abbott, et al. 2016, Hai and Heckman 2017, Navarro and Zhou 2017).

in parental transfers by socioeconomic background in explaining differential schooling outcomes; however, parental transfers are exogenously determined and unaffected by policy and economic conditions in their models. By endogenizing parental transfers, we account for the fact that parents respond to different policies by adjusting transfers to their children. Furthermore, our dynastic approach to human capital investment enables us to study dynamic effects of lasting economic policies that are often ignored. We simulate the long-run effects of permanent policy changes in addition to the short-run effects typically measured in empirical studies. While short-run effects are based on the current distributions of wealth and human capital in the population, long-run effects take into account changes in these distributions over time.

This paper is most closely related to Cunha and Heckman (2007) and Cunha (2013). The former develops a similar framework to our baseline dynastic lifecycle investment model of Section 2 in which early and late investments in human capital are complementary and may be distorted by intergenerational and lifecycle borrowing constraints. Their analysis highlights the potential for investment policies at one stage of development to impact investments at other stages due to dynamic complementarity and discusses the impacts of borrowing constraints on the *ratio* of early to late investment. We analytically study the effects of borrowing constraints and income transfers at different stages of development on the *levels* of both early and late investment, as well as human capital. As we demonstrate, these effects are not always what one might expect. More income or expanded borrowing opportunities do not always lead to increases in investment. The qualitative movements depend on the extent of dynamic complementarity/substitutability of investment and the timing of when constraints bind. More importantly, we study these effects quantitatively, and establish the empirical importance of cross-period policy impacts. For example, the impacts of a subsidy to late investments on average post-school earnings are more than twice as large when early investments are allowed to respond.

Cunha (2013) extends the dynastic framework of Cunha and Heckman (2007) to incorporate annual investment periods up to age 20. Following the approach of Cunha, Heckman and Schennach (2010), he estimates the technology of skill production over the investment period, taking advantage of noisy measures of investments and cognitive skills in the CNLSY. Incorporating idiosyncratic labor market risk, Cunha (2013) focuses attention on the impacts of imperfect insurance markets on human capital investment decisions. While our economic framework is similar (with fewer periods of investment), we study a different set of issues, including the importance of intergenerational ability transmission and lifecycle borrowing constraints (as well as imperfect insurance and intergenerational constraints) for investment behavior and intergenerational mobility. As such, we incorporate a number of features absent in Cunha (2013): (i) systematic lifecycle earnings growth (with retirement), (ii) lifecycle borrowing constraints, (iii) a parental altruism parameter accounting for the fact that parents may care less about their children than themselves, and (iv) intergenerational persistence in innate learning ability.

Del Boca, Flinn, and Wiswall (2014) explore the role of investments in children over the lifecycle; however, their emphasis is primarily on the relative importance of different parental time inputs (e.g. mother vs. father time) within and across periods.[11] Given their many inputs, they assume a Cobb-Douglas technology across inputs within and across periods and abstract from borrowing and saving. While we do not explicitly model multiple investment inputs within each period, we show in the Online Appendix that our analysis is consistent with the optimal allocation of time and goods inputs within periods where we effectively study total investment expenditures across all inputs. Because our theoretical analysis highlights the importance of dynamic complementarity, we use the more general CES production function in our quantitative analysis.[12] Our results indicate that considerable intertemporal smoothing is available, even if borrowing constraints play a critical role.

This paper proceeds as follows. In Section 2, we develop a dynastic model of human capital investment in children with borrowing constraints. Allowing for two periods of investment, we analytically study the effects of changes in family income in both periods. Key results provide testable empirical predictions about the relative importance of early vs. late income for educational attainment, which we briefly examine using data from the Children of the National Longitudinal Survey of Youth (CNLSY). Our findings are broadly consistent with strong dynamic complementarity in investments and binding borrowing constraints for at least some families when children are both young and old. We also analyze the effects of relaxing borrowing constraints at different child ages, demonstrating the importance of dynamic complementarity and the timing

[11]Lee and Seshadri (2016) and Gayle, Golan and Soytas (2014) also study the role of both goods and time inputs, the latter focusing more on the importance of family structure and endogenous fertility choices as contributors to racial gaps in skill development.

[12]Cunha and Heckman (2007), Cunha, Heckman and Schennach (2010), Cunha (2013), and Lee and Seshadri (2016) also base their analyses on a similar CES production function. Unlike Cunha, Heckman and Schennach (2010), we do not distinguish between cognitive and non-cognitive skills. To the extent that these skills are combined to create a composite productivity (i.e. human capital) level used in the labor market, we effectively identify the technology mapping early and late investments (in cognitive or non-cognitive skills) into this productivity measure.

of constraints for the qualitative nature of investment responses.

In Section 3, we extend the model to incorporate a number of other features of the economic environment to facilitate a realistic quantitative analysis. Most notably, we include earnings uncertainty, a direct effect of parental human capital on intergenerational ability transmission, and government policies. We discuss identification and calibrate this model using data from the CNLSY on parental income and wealth levels, educational attainment by children and their parents, noisy measures of early investments in children, and the wage outcomes of children. We also explore a number of counterfactual exercises aimed at understanding the determinants of intergenerational mobility and responses to family income/wealth shocks.

In Section 4, we simulate the impacts of various policy changes including increases in borrowing limits, marginal investment subsidies, and publicly provided early investment. We consider the sensitivity of our quantitative results to alternative calibrations of our model in Section 5 and conclude in Section 6.

2 Dynastic Model with Early and Late Investments

In this section, we develop a dynastic lifecycle human capital framework to study analytically the behavior of human capital investment when borrowing constraints may limit the ability to smooth consumption over the lifecycle. The next section generalizes and extends this basic framework to facilitate an empirically based quantitative analysis.

We assume that people live through six periods in their lives: young and old childhood (periods 1 and 2), young and old parenthood (periods 3 and 4), post-parenthood (period 5), and retirement (period 6).[13] Human capital investment takes place in the first two periods (i.e. 'childhood'), followed by three periods of work and a period of retirement. Conceptually, investments may include various forms of goods inputs like computers and books, parental time

[13]We abstract from fertility choice and timing, which may also be affected by borrowing constraints. In response to husband job displacement (generating substantial earnings declines for at least 8 years), Lindo (2010) documents a small short-term increase in fertility followed by a decline over the next several years such that the total effect over 8 years is slightly negative. In a lifecycle model of fertility and wealth accumulation, Scholz and Seshadri (2009) conclude that poorer households are typically credit constrained longer than wealthier households, because poorer households have more children. As we show below (see Table 1), maternal age at child's birth and the number of siblings do not affect the linkages between parental income (at different child ages), maternal education, and child investments – the key relationships used to calibrate our quantitative model. See Gayle, Golan, and Soytas (2014) for a recent analysis of child investments with endogenous fertility decisions.

Diagram 1: Generations of a Dynasty

3. Young Parent	**4. Old Parent**	5. *Post−Parent*	6. *Retirement*		
1. Young Child	**2. Old Child**	**3. Young Parent**	**4. Old Parent**	5. *Post−Parent*	6. *Retirement*
		1. Young Child	**2. Old Child**	**3. Young Parent**	**4. Old Parent**
				1. Young Child	**2. Old Child**

in child development activities, formal schooling, and other time inputs by older children. Our analysis is agnostic about the form of investments, instead focusing on the intertemporal nature of skill production and investment choices throughout childhood.[14]

Parents consume, save, and make transfers to their children, who consume, invest in their own human capital, and save (during old childhood) for their future. Children then grow up to become parents themselves with the cycle repeating. Assuming parents are altruistic towards their children, valuing their lifetime utility makes the problem dynastic in the sense of Becker and Tomes (1986). The lifecycle of different generations in a dynasty is given by Diagram 1.

2.1 Technology for Human Capital Production and Earnings

Investments in young and old childhood are given by i_1 and i_2, respectively. These investments produce adult human capital:

$$h = \theta f(i_1, i_2). \tag{1}$$

The total factor productivity of investments, θ, reflects a child's ability to learn as well as a parent's ability to teach the child. Despite these different interpretations, we will typically refer to it as an individual's learning productivity or ability. This learning productivity may vary across dynasties at any point in time or within dynasties across generations, creating a potentially important source of inequality and social mobility (Becker and Tomes 1979, 1986, Cunha and Heckman 2007).[15] The human capital production function $f(\cdot,\cdot)$ is strictly increasing and strictly

[14]We show in the Online Appendix that what we refer to as 'investment' each period can be thought of as total investment expenditures in those periods given the optimal within-period allocation of expenditures across all inputs (e.g. parental time and goods inputs as in Del Boca, Flinn and Wiswall (2014) and Mullins (2016)).

[15]Variation in θ may also reflect local differences in school quality or input prices (see the Online Appendix).

concave in both of its arguments.[16] To guarantee appropriate second order conditions hold in the decision problems described below, we assume the following throughout our analysis:

Assumption 1. $f_{12}^2 < f_{11}f_{22}$, $f_{12} > max\left\{f_{22}\left(\frac{f_1}{f_2}\right), f_{11}\left(\frac{f_2}{f_1}\right)\right\}$.

The first condition limits the degree of complementarity in investments and ensures strict concavity of the production function. The second condition implies that the least costly way to produce additional human capital h is to increase both early and late investments. Most specifications for human capital production entail dynamic complementarity (i.e. $f_{12} \geq 0$), satisfying this condition.[17]

In our quantitative analysis below, we employ a CES human capital production function of the form

$$f(i_1, i_2) = (ai_1^b + (1-a)i_2^b)^{d/b}, \tag{2}$$

where $a \in (0,1)$, $b < 1$, and $d \in (0,1)$; however, our theoretical analysis does not rely on any particular functional form. (Assumption 1 holds for this production function.) We impose decreasing returns to scale (i.e. $d < 1$); otherwise, unconstrained individuals may want to invest an infinite amount.

Adult earnings depend on human capital acquired through childhood investments. Given our emphasis on childhood human capital investment (i.e. early childhood and schooling investments), we assume that earnings grow exogenously after childhood:

$$W_j(h) = w\Gamma_j h, \text{ for } j \in \{3,4,5\}, \tag{3}$$

where $w > 0$ reflects the wage per unit of skill.[18] Lifecycle growth in earnings implies $\Gamma_5 > \Gamma_4 > \Gamma_3$, where we normalize $\Gamma_3 = 1$. In Section 3, we introduce idiosyncratic period-specific shocks to adult earnings; however, we abstract from this uncertainty throughout this section to simplify the analysis.

[16]Specifically, we assume that $f_j(i_1, i_2) > 0$ and $f_{jj}(i_1, i_2) < 0$ for $j = 1, 2$, where the subscript j denotes the partial derivative with respect to its jth argument. We also assume standard Inada conditions to ensure interior solutions.

[17]For prior evidence on the extent of dynamic complementarity, see Cunha, et al. (2006), Cunha, Heckman and Schennach (2010), Cunha (2013), Attanasio, et al. (2015), Attanasio, Meghir and Nix (2015), and Agostinelli and Wiswall (2016). Del Boca, Flinn and Wiswall (2014) and Mullins (2016) assume a Cobb-Douglas specification, implying a modest degree of dynamic complementarity.

[18]See Gayle, Golan and Scholz (2014) and Lee and Seshadri (2016) for recent childhood investment models that incorporate adult skill accumulation through learning-by-doing or on-the-job investment, respectively.

Finally, we assume older children earn $W_2 \geq 0$, which is assumed to be independent of their ability and early investments. As discussed further below, W_2 is meant to reflect potential earnings over ages 16-23, in which case investments among old children include foregone earnings while in school.

2.2 Preferences, Constraints and Household Decisions

We assume time separable preferences for consumption, where the time discount rate $\beta \in (0,1)$ and utility function $u(c)$ is strictly increasing, strictly concave and satisfies standard Inada conditions. Let $\rho > 0$ indicate the degree of altruism across generations. To explore the impacts of exogenous income transfers to families on investments in children, we incorporate income transfers y_3 and y_4 to the parents of young and old children, respectively.

The gross rate of return on borrowing and saving is $R \geq 1$. Assets saved in period j are given by a_{j+1}, and total borrowing (negative a_{j+1}) may be limited by a restriction on debt carried over to the next period, L_j. During retirement, individuals consume their savings and do not work.

We assume that young children cannot borrow or save themselves (i.e. $a_2 = 0$), and that young parents make investment and consumption decisions for their young children. Although old children make investment decisions, we assume that it is their last period of financial interaction with their parents, so there is no scope for strategic behavior. Given any level of transfers from parents to children, both generations agree on how to allocate those resources to investment and consumption. Therefore, it is possible to write the entire family problem from the point of view of parents.[19]

To simplify the exposition, in this section, we assume dynasties are characterized by a single learning productivity θ' for all generations; however, we relax this assumption in our quantitative analysis below.[20] Letting prime superscripts denote the child's variables, the problem facing a young parent with a young child is described by the following value function:

$$V_3(a_3, h) = \max_{c_3, c_4, a_4, a_5, c'_1, c'_2, i'_1, i'_2, a'_3} \left\{ u(c_3) + \beta u(c_4) + \beta^2 V_5(a_5, h) + \rho \left[u(c'_1) + \beta u(c'_2) + \beta^2 V_3(a'_3, h') \right] \right\}$$

[19]See Brown, Scholz and Seshadri (2012) for an interesting analysis of tied and unrestricted transfers in a dynamic setting when children may wish to under-invest in their human capital knowing their parents will provide greater transfers later. The capacity for parents to make tied transfers (i.e. transfers linked directly to human capital investments) helps alleviate the potential for under-investment.

[20]It is straightforward to generalize the results of this section to account for stochastic θ' that follows a Markov process depending only on prior generations' θ values. As discussed in greater detail below (see Section 3), allowing the distribution of θ' to also depend on parental human capital alters the problem in more fundamental ways.

subject to

$$
\begin{aligned}
a_4 &= Ra_3 + W_3(h) + y_3 - c_3 - c_1' - i_1', \\
a_3' + a_5 &= Ra_4 + W_4(h) + y_4 + W_2 - c_4 - c_2' - i_2', \\
a_4 &\geq -L_3, && (4) \\
a_5 &\geq -L_4, && (5) \\
a_3' &\geq -L_2, && (6) \\
h' &= \theta' f(i_1', i_2'), && (7)
\end{aligned}
$$

$c_3 \geq 0$, $c_4 \geq 0$, $c_1' \geq 0$, $c_2' \geq 0$, $i_1' \geq 0$ and $i_2' \geq 0$. Because young children are not allowed to borrow on their own, the only constraint on borrowing during early childhood/parenthood is that imposed on young parents. The value function $V_3(a_3', h')$ in the maximization problem reflects the fact that children grow up to become parents themselves and face the same general decision problem, making the problem one of overlapping dynasties with parental altruism.

The problem facing a post-parent with no child at home is a standard lifecycle consumption/savings problem:

$$V_5(a_5, h) = \max_{a_6} \{u(Ra_5 + W_5(h) - a_6) + \beta u(Ra_6)\}. \tag{8}$$

2.3 Consumption and Investment Behavior

Consumption allocations when parents and children co-reside satisfy $u'(c_3) \geq \beta R u'(c_4)$, $u'(c_4) \geq \beta R u'(c_5)$, $u'(c_1') \geq \beta R u'(c_2')$, and $u'(c_2') \geq \beta R u'(c_3') = \beta \frac{\partial V_3(a_3', h')}{\partial a_3'}$, where an inequality is strict if and only if the borrowing constraint for that period binds.[21] That is, individuals efficiently smooth consumption across periods when borrowing constraints are non-binding, while consumption growth is relatively high whenever borrowing constraints bind. Optimality also implies that $u'(c_3) = \rho u'(c_1')$ and $u'(c_4) = \rho u'(c_2')$, so families efficiently smooth consumption across generations within periods.[22]

[21] Because individuals always wish to save for the retirement period, borrowing constraints are slack during post-parenthood. As such, parental consumption is fully smoothed once children leave the household, i.e. $u'(c_5) = \beta R u'(c_6)$.

[22] Our quantitative analysis below incorporates an additional restriction that parents must make non-negative transfers to their children each period, which can distort intra-temporal allocations between parents and children. In this case, $u'(c_4) > \rho u'(c_2')$ if and only if parental transfers are constrained.

First order conditions for investment imply:

$$u'(c_1') = \beta^2 \frac{\partial V_3(a_3', h')}{\partial h'} \theta' f_1(i_1', i_2') \tag{9}$$

$$u'(c_2') = \beta \frac{\partial V_3(a_3', h')}{\partial h'} \theta' f_2(i_1', i_2'). \tag{10}$$

Taking the ratio of these equations reveals that optimal investment equates the technical rate of substitution in the production of human capital with the marginal rate of substitution for consumption: $\frac{f_1(i_1', i_2')}{f_2(i_1', i_2')} = \frac{u'(c_1')}{\beta u'(c_2')} \geq R$.

As first noted by Becker (1967), unconstrained optimal investments for an individual of ability θ, denoted $i_1^u(\theta)$ and $i_2^u(\theta)$, equate the marginal returns on investment to the return on savings: $\theta\chi_3 f_1(i_1^u(\theta), i_2^u(\theta)) = R^2$ and $\theta\chi_3 f_2(i_1^u(\theta), i_2^u(\theta)) = R$, where $\chi_3 \equiv w(1+R^{-1}\Gamma_4+R^{-2}\Gamma_5)$ is the discounted present value of an additional unit of human capital for a young parent.[23] Unconstrained families make investment choices to maximize the discounted present value of lifetime earnings net of discounted investment costs, because they can freely borrow and save to allocate those resources across family members and over time. As a consequence, unconstrained investments are independent of preferences, initial wealth, parental earnings, and income transfers.

The separation between investment and consumption choices no longer exists when borrowing constraints restrict intertemporal allocations. As the next proposition demonstrates, binding constraints on a household typically lead to under-investment in the child's human capital. (See Appendix B for proofs of all propositions.)

Proposition 1. *Consider a child and his parent. (i) If and only if any borrowing constraint for the child binds (i.e. $a_3' = -L_2$, $a_4' = -L_3$, or $a_5' = -L_4$) or his young parent's borrowing constraint binds (i.e. $a_4 = -L_3$), then: optimal early investment in the child is strictly less than the unconstrained amount and adult human capital is strictly less than the unconstrained level. (ii) If any borrowing constraint for the child binds (i.e. $a_3' = -L_2$, $a_4' = -L_3$, or $a_5' = -L_4$) and either (a) $f_{12} > 0$ or (b) his young parent's borrowing constraint does not bind (i.e. $a_4 > -L_3$), then optimal late investment is strictly less than the unconstrained amount.*

A child that faces a binding borrowing constraint at any point, even later in life, under-invests

[23] If an individual is unconstrained during his adult life (periods 3-5), then he does not care in which form he holds his wealth: assets or human capital. He only cares about the combined value: $Ra_3 + \chi_3 h$. In this case, we can write $V_3(a_3', h') = v_3(Ra_3' + \chi_3 h')$. This implies that $\frac{\partial V_3(a_3',h')}{\partial a_3'} = Rv_3'$ and $\frac{\partial V_3(a_3',h')}{\partial h'} = \chi_3 v_3'$, so $\left(\frac{\partial V_3(a_3',h')}{\partial a_3'}\right)\frac{1}{R} = \left(\frac{\partial V_3(a_3',h')}{\partial h'}\right)\frac{1}{\chi_3}$. Combining this with $u'(c_1) = \beta R u'(c_2) = \beta^2 R \frac{\partial V_3(a_3',h')}{\partial a_3'}$ and Equations (9) and (10) yields the unconstrained conditions. Constraints on future generations have no bearing on these results.

in human capital during early childhood. When constraints bind, the returns to investment in the form of higher earnings come in periods of plenty (i.e. when consumption levels are relatively high) while costs must be paid when resources are scarce. This raises the marginal cost relative to the marginal benefit of early investment. A binding constraint on young parents discourages early child investment for the same reasons; however, the constraint on old parents does not, by itself, distort investment decisions for the child, because old children can borrow themselves (unless they are also constrained). When the constraint on old parents binds, parents will transfer less to their children, which distorts investments if and only if the children are also constrained at that time or later.

If investments are complementary over time, then there is also under-investment during old childhood if the child ever faces a binding constraint. By contrast, if investments are substitutable over time (i.e. $f_{12} \leq 0$), then binding constraints on young parents could shift investment from early to later stages of development. In this case, late investments in children could exceed the unconstrained optimal amount.

The complementarity/substitutability of investments across periods not only affects the impacts of borrowing constraints on investment, but it also affects investment responses to changes in parental income. If investments are substitutable, families can shift investment from constrained periods to unconstrained periods with little sacrifice in terms of human capital accumulation. Their ability to do this diminishes as investments become more complementary. Letting $HEC(i_1, i_2) \equiv \frac{f_{12}(i_1,i_1)f(i_1,i_1)}{f_1(i_1,i_1)f_2(i_1,i_1)}$ reflect Hicks' partial elasticity of complementarity between early and late investments, the following dynamic complementarity condition is important for a number of results below.[24]

Condition 1. $HEC(i_1', i_2') > -\frac{\left[\frac{\partial^2 V_3(-L_2,h')}{\partial h'^2}\right]h'}{\frac{\partial V_3(-L_2,h')}{\partial h'}}$.

This condition requires that early and late investments be sufficiently complementary relative to the amount of curvature in lifetime utility (as of young parenthood) with respect to acquired human capital. If credit constraints are non-binding for the child throughout his adult life, then the condition simplifies to

$$HEC(i_1', i_2') > \frac{\eta_{c_3',h'}}{IES(c_3')}, \tag{11}$$

where $IES(c) \equiv \frac{-u'(c)}{u''(c)c}$ is the consumption intertemporal elasticity of substitution and $\eta_{c3,h} \equiv$

[24]See Sato and Koizumi (1973) for a discussion of Hicks' partial elasticity of complementarity and its relationship to other elasticity of substitution measures.

$\left(\frac{\partial c_3}{\partial h}\right) / \left(\frac{c_3}{h}\right)$ is the elasticity of period 3 consumption with respect to human capital.[25] This inequality is more likely to hold as q-complementarity between early and late investment increases (as measured by Hicks' partial elasticity of complementarity) or as individuals become less concerned about maintaining smooth consumption profiles (as measured by the consumption intertemporal elasticity of substitution). Put another way, when individual preferences for smooth consumption are strong, Condition 1 requires strong dynamic complementarity between early and late investments.

As noted earlier, changes in parental income have no effect on investments for unconstrained families. This is not the case for families facing binding borrowing constraints. Among constrained families, changes in parental income at different stages of child development have complicated effects on investment choices depending on when income is received and the dynamic complementarity/substitutability of investments in the production of human capital. The following proposition characterizes the impacts of changing income transfers y_3 and y_4 for a single generation (i.e. parents of the child under consideration), leaving transfers to future generations unchanged. These results focus on the role of income transfers but would apply equally to exogenous differences in parental earnings (i.e. differences not directly related to the productivity of investments).

Proposition 2. *Consider a child-parent pair:*

I. If the parent is unconstrained when the child is young (i.e. $a_4 > -L_3$) but the borrowing constraint binds for the child when old (i.e. $a_3' = -L_2$), then:

(i) $\frac{\partial i_1'}{\partial y_3} = R\frac{\partial i_1'}{\partial y_4} = \frac{\partial i_1'}{\partial (R^{-1}y_4)} > 0$;

(ii) $\frac{\partial i_2'}{\partial y_3} = R\frac{\partial i_2'}{\partial y_4} = \frac{\partial i_2'}{\partial (R^{-1}y_4)} > 0$;

(iii) $\frac{\partial h'}{\partial y_3} = R\frac{\partial h'}{\partial y_4} = \frac{\partial h'}{\partial (R^{-1}y_4)} > 0$.

II. If the parent is borrowing constrained when the child is young (i.e. $a_4 = -L_3$) but the child is not constrained later in life, then:

(i) $\frac{\partial i_1'}{\partial y_3} > 0$ *and* $\frac{\partial i_1'}{\partial y_4} < 0$;

(ii) $\frac{\partial i_2'}{\partial y_3} > 0 \iff f_{12} > 0$; *and* $\frac{\partial i_2'}{\partial y_4} < 0 \iff f_{12} > 0$;

[25] The simplified condition of equation (11) is obtained by recognizing that when borrowing constraints (4) and (5) do not bind for the child when he grows up, it is straightforward to show that $\frac{\partial V_3(a_3,h)}{\partial h} = w\chi_3 u'(c_3)$ and $\frac{\partial^2 V_3(a_3,h)}{\partial h^2} = w\chi_3 u''(c_3(a_3,h))\frac{\partial c_3(a_3,h)}{\partial h}$. For the CES production function given in equation (2), Hicks' partial elasticity of complementarity between early and late investments is simply $\frac{d-b}{d}$. The condition cannot hold for $d \leq b$, but this only rules out very strong substitution between early and late investments such that $f_{12} \leq 0$.

(iii) $\frac{\partial h'}{\partial y_3} > 0$ *and* $\frac{\partial h'}{\partial y_4} < 0$.

III. If the parent is borrowing constrained when the child is young (i.e. $a_4 > -L_3$*) and the child is borrowing constrained when old (i.e.* $a_3' = -L_2$*), then:*

(i) $\frac{\partial i_1'}{\partial y_3} > 0$*; and* $\frac{\partial i_1'}{\partial y_4} > 0 \iff$ *Condition 1 holds;*

(ii) $\frac{\partial i_2'}{\partial y_3} > 0 \iff$ *Condition 1 holds; and* $\frac{\partial i_2'}{\partial y_4} > 0$*;*

(iii) $\frac{\partial h'}{\partial y_3} > 0$ *and* $\frac{\partial h'}{\partial y_4} > 0$.

We highlight two key implications of this proposition. First, if the parents of young children are unconstrained but the child is constrained during late childhood, then investments depend only on the discounted present value of family income transfers $y_3 + R^{-1}y_4$, not the timing of income (conditional on discounting y_4). Thus, the timing of parental income only affects child investments and human capital when borrowing constraints limit the choices of young parents.[26]

Second, when young parents are borrowing constrained, investment responses to changes in income depend on when those changes take place, the extent of dynamic complementarity, and whether later constraints (for the child) also bind. While constrained early investment is always increasing in early income, it is not always increasing in income at later ages. Because an increase in late income exacerbates the early borrowing constraint, early investment is unambiguously decreasing in y_4 when the child is unconstrained at later ages. Families would like to consume some of the increased late income in the earlier period; however, if the young parent is borrowing constrained, they can only do this by reducing early investment. When only the early (i.e. young parent) constraint binds, the impacts of income on late investment depend entirely on its effect on early investment and whether early investment raises ($f_{12} > 0$) or lowers ($f_{12} < 0$) the marginal return to late investment. Perhaps surprisingly, when $f_{12} > 0$ and only the early constraint binds, an increase in family income during late childhood *reduces* skill investments in both periods. By contrast, when constraints bind throughout childhood (for parents during early childhood and the child during late childhood), increases in income during either childhood period increase investment in both periods if and only if there is sufficient dynamic complementarity.[27]

The results in Proposition 2 can be explored empirically by estimating the effects of early and late family income on educational attainment (late investment, i_2', in the context of the

[26] Because children themselves can borrow at older ages, the borrowing constraint for old parents, by itself, has no bearing on the signs of the effects of income transfers on investments.

[27] A similar result can also be obtained when borrowing constraints bind during early childhood and when the child becomes an adult, even if the child is unconstrained during late childhood.

model) using the random sample of children from the CNLSY – the same data used in the quantitative analysis of our model below. Table 1 reports results from regressing educational attainment indicators on early and late family income, where income is measured in \$10,000 year 2008 dollars and is averaged over child ages 0-11 (early income) and 12-23 (late income) after discounting income each year back to the child's birth.[28] Estimates reported in Panel A control only for maternal education, while those in Panel B also control for other child and mother characteristics. Columns (1)-(4) report results for specifications that measure family income using total reported parental earnings, while columns (5)-(8) report results when using an adjusted 'full' earnings measure that adjusts for the possibility that some mothers may work part-time to spend more time investing in their children.[29] The estimated effects are quite similar across specifications and reveal that a \$10,000 increase in annual early income significantly reduces high school dropout (i.e. less than 12 years of schooling) rates by about 2.5 percentage points, while it increases college attendance (i.e. greater than 12 years of schooling) rates by as much as 4.6 percentage points. The same increase in late income has smaller (and statistically insignificant) effects on these education margins; however, the difference between the effects of early and late income are consistently significant across specifications only for college attendance. Income at both early and late ages raises college completion (i.e. 16 or more years of schooling) rates by 2-3 percentage points.[30]

Interpreting these results through the lens of Proposition 2 suggests that, for at least some

[28]A discount rate of 5% is used. The assumptions and age ranges used here are consistent with those used later in the calibration of our model.

[29]Panel B specifications include the average number of children in the household over child ages 0-6, measures of child's year of birth, race/ethnicity, and gender, as well as mother's characteristics including educational attainment, whether she was a teenager when the child was born, living in an intact family at age 14, foreign-born, and Armed Forces Qualifying Test scores. The adjusted 'full' earnings measure inflates earnings for mothers working less than 1,500 hours per year to its 1,500 hour equivalent. Because NLSY mothers were ages 14-22 in 1979, many of their children are still young. Thus, our sample sizes are smaller when looking at college attendance or completion at age 24 compared with measures of high school dropout as of age 21. We also lose some observations due to missing mother or child characteristics (Panel B) or missing measures of hours worked (columns 5-8). See Appendix A for additional details on the CNLSY data and our sample.

[30]Also using the CNLSY data, Carneiro and Heckman (2002) cannot reject that income has the same effects on college enrolment regardless of the age at which it was received. Our analysis benefits from a sample size that is roughly twice as large, allowing for greater precision. Furthermore, because Carneiro and Heckman (2002) are more concerned with the importance of borrowing constraints at college-going ages, they control for age 12 mathematics achievement levels, which might absorb much of the effect of early income.

families, borrowing constraints are binding at both early and late ages. Stronger estimated effects of early (relative to late) income on college attendance suggest that early constraints bind for at least some young parents. The fact that attendance is not decreasing in late income further suggests that later constraints also bind and that early and late investments are sufficiently complementary (part III of Proposition 2). Results for high school dropout are broadly consistent with these same conclusions. The finding that both early and late family income increase college completion by similar amounts is consistent with either binding early and late borrowing constraints coupled with sufficient dynamic complementarity (part III of Proposition 2) or constraints that bind only at later ages (part I of Proposition 2). Altogether, these empirical results demonstrate the practical value of Proposition 2 in helping to identify the importance of borrowing constraints at different stages of development as well as the extent of dynamic complementarity. A similar set of empirical relationships are, therefore, used below in the calibration of our quantitative model.

We can also (theoretically) characterize the effects of borrowing constraints themselves on human capital investments. First, consider relaxing the constraint on older children (for a single generation).

Proposition 3. *Consider a child that is borrowing constrained during late childhood (i.e. $a_3' = -L_2$) but unconstrained later as an adult. Then, $\frac{\partial i_2'}{\partial L_2} > 0$ and $\frac{\partial h'}{\partial L_2} > 0$; if the parent is unconstrained when the child is young (i.e. $a_4 > -L_3$) or Condition 1 holds, then $\frac{\partial i_1'}{\partial L_2} > 0$.*

Relaxing the child's borrowing constraint during late childhood unambiguously increases late investment. If the parent's constraint is non-binding when the child is young or if early and late investments are sufficiently complementary, then any increase in late investment encourages additional early investment as well. For sufficiently strong intertemporal substitutability in investments, it is possible that early investment declines when later borrowing opportunities are expanded if parents are constrained when the child is young. In this case, investment may shift from early to late childhood. Still, children acquire more human capital.

Next, consider relaxing the borrowing constraint on the parents of young children.

Proposition 4. *Consider a child whose parent is constrained when the child is young (i.e. $a_4 = -L_3$). (i) If no other borrowing constraint binds for the child, then: $\frac{\partial i_1'}{\partial L_3} > 0$; $\frac{\partial i_2'}{\partial L_3} > 0 \iff f_{12} > 0$; and $\frac{\partial h'}{\partial L_3} > 0$. (ii) If the child is also borrowing constrained during late childhood (i.e. $a_3' = -L_2$) and Condition 1 does not hold, then: $\frac{\partial i_1'}{\partial L_3} > 0$ and $\frac{\partial i_2'}{\partial L_3} < 0$.*

When family choices are only limited by the young parent's borrowing constraint, relaxing it leads to an increase in early investment. This, in turn, encourages late investment if and only if the marginal productivity of late investment is increasing in early investment.

When both early (i.e. young parent's) and late (old child's) constraints bind, relaxing the early constraint shifts resources from late to early childhood. With sufficient dynamic complementary, early and late investments will move in the same direction. It is likely that investments will increase, but the increases will tend to be modest, because the intertemporal shift in resources raises the cost of investing late. If the production technology is such that small changes in early investment must be matched with large changes in late investment, it is possible that relaxing the early borrowing constraint (thereby tightening the late constraint) could cause families to *reduce* investment in both periods. By contrast, if investments are sufficiently substitutable over time (i.e. Condition 1 does not hold), then shifting resources from late to early childhood by relaxing the early constraint causes investment to shift from the late to the early period as well.

These results demonstrate that the effects of parental income and expanded borrowing opportunities depend on the extent of dynamic complementarity in investments as well as the timing of when borrowing constraints bind. These forces not only determine the magnitude of investment level responses (studied next), but also their signs.

3 An Empirically Based Quantitative Analysis

In this section, we generalize our framework to incorporate additional features of the family investment problem for a more realistic and empirically grounded quantitative analysis. After specifying this more general problem, we consider the effects of these additional features on investment behavior relative to the stylized problem of the previous section. We further discuss identification and calibration of this model using intergenerational data on investment behavior, savings, and wages/earnings. Using our calibrated model, we explore several counterfactual exercises to better understand the impacts of income and wealth shocks on investment and the determinants of intergenerational mobility.

3.1 A More General Quantitative Framework

We begin by describing several extensions to the family problem of the previous section before specifying the complete problem used in our quantitative analysis.

3.1.1 Investment Subsidies

Subsidies for education are a key feature of the market for human capital investment. We incorporate a lump sum amount of free/public investment, $p_j \geq 0$, in childhood periods $j = 1, 2$ that all children receive at no private cost to families, as well as additional, proportional subsidies $S_j(i_j)$ as functions of private investments i_j for $j = 1, 2$.

We abstract from taxation; however, investment choices are unaffected by a constant labor income tax rate τ if net investments are tax deductible and borrowing limits are reduced by the factor $1 - \tau$.[31] The former is consistent with investments in terms of foregone earnings and considerable tax breaks for direct educational expenditures, while the latter is conceptually consistent with the link between borrowing limits and future lifetime earnings discussed in Section 3.1.3. In this case, the solution to the household's problem is equivalent in terms of investment choices and human capital levels; however, consumption and asset allocations are reduced by the factor $1 - \tau$. (Income transfers y_3 and y_4 should also be read as net of taxes.)

3.1.2 Earnings Shocks

To account for unpredictable variation in earnings over the lifecycle, we introduce period j-specific earnings shocks ϵ_j, so adult earnings are given by

$$W_j(h, \epsilon_j) \quad = \quad w\Gamma_j(h + \epsilon_j), \quad \text{for } j \in \{3, 4, 5\}, \tag{12}$$

where we assume that income shocks are *iid* log normal, i.e. $\epsilon_j \sim logN(m, s^2)$ for $j = 3, 4, 5$.[32] This assumption implies that the minimum level of earnings in any adult period j is given by $w\Gamma_j h$. The parameters Γ_j continue to reflect lifecycle growth in expected earnings relative to young parenthood. Of course, individuals receiving a low earnings shock initially will have higher than average earnings growth, while the opposite is true for those with high initial earnings shocks.

[31] This requires that $u((1-\tau)c) = g(1-\tau)u(c)$ for some positive function $g(\cdot)$, which is satisfied for the CRRA utility function we use in our analysis.

[32] Abstracting from any intertemporal correlation in earnings shocks, which greatly reduces the computational burden, is unlikely to be very problematic given the length of our periods. Accounting for ex ante known unobserved heterogeneity in a similar framework, Cunha (2013) estimates an annual autocorrelation for earnings shocks of 0.791, which implies a correlation between shocks 12 years apart (the length of a period in our analysis) of only 0.06.

3.1.3 Human Capital-Specific Borrowing Constraints

We allow borrowing constraints to depend on the future human capital and earnings of an individual to account for the possibility that higher education increases borrowing opportunities. This is both theoretically and empirically attractive for reasons discussed in Lochner and Monge-Naranjo (2011).[33] Specifically, we assume that borrowing limits are a fraction $\gamma \in [0, 1]$ of the lowest possible discounted value of future earnings, so

$$L_j(h) = \gamma R^{-1} \chi_{j+1} h, \quad \text{for } j = 2, 3, 4,$$

where (analogous to χ_3) we define $\chi_4 \equiv w(\Gamma_4 + R^{-1}\Gamma_5)$ and $\chi_5 \equiv w\Gamma_5$ to reflect the discounted present value of human capital as of periods 4 and 5, respectively. One can think of γ as a measure of credit accessibility and contract enforceability. A value of γ near zero implies very little availability of credit, consistent with negligible contract enforcement, while γ near one means that individuals can borrow fully against guaranteed future earnings, consistent with full enforceability as in the models of Laitner (1992), Huggett (1993), and Aiyagari (1994).[34] While enforcement and γ could vary across stages of the lifecycle, we abstract from this possibility given data limitations.

As demonstrated by Lochner and Monge-Naranjo (2011), the fact that borrowing limits increase with human capital means that investment behavior tends to be less distorted than when borrowing limits are unrelated to future earnings. Furthermore, increases in γ expand credit more for individuals of high ability and those who have invested more in their human capital, because human capital is increasing in ability and investment.

3.1.4 Non-Negative Transfer Constraint

We assume that intergenerational borrowing constraints prevent parents from borrowing against their children's future income as first emphasized in Becker and Tomes (1979, 1986). Our problem for young parents implicitly imposes this by assuming that all child investment and consumption is paid for by parents; however, an additional restriction is needed to ensure that old parents transfer non-negative resources to their old children (the last period of their financial interaction).

[33] Lochner and Monge-Naranjo (2011) argue that more skilled individuals can commit to re-pay higher debts, explaining why private lenders offer them more credit. This is also broadly consistent with the federal student loan system, which directly links loan amounts to post-secondary enrollment and the level of schooling attended.

[34] See Hai and Heckman (2017) for an interesting generalization of the 'natural' borrowing limit in an educational choice model with endogenous lifecycle labor supply and minimum income support.

Specifically, we assume that

$$a'_3 \geq W_2 - c'_2 - i'_2 + S_2(i'_2), \tag{13}$$

which requires that old children do as well in the family as they would on their own. This intergenerational transfer constraint limits the extent of intergenerational consumption smoothing the family can achieve, with parents consuming too little relative to future generations when the constraint binds.[35] It may also distort investment decisions, because parents may withhold some productive investments in both periods if they cannot access the future returns. This situation is most likely to arise when the child is high ability and the parent is poor.

3.1.5 Intergenerational Transmission of Investment Productivity

Differences in learning productivity θ (see equation (1)) are a source of cross-sectional inequality and intergenerational mobility. To account for this heterogeneity, we assume a two-state process for ability with $\theta \in \{\theta_1, \theta_2\}$ where the probability of low vs. high ability ($\theta_1 < \theta_2$) depends on parental ability and human capital. Specifically, we assume that:

$$\Pi(\theta, h; \pi) \equiv Pr(\theta' = \theta_2 | \theta, h; \pi) = \frac{exp(\pi_0 + \pi_1\theta + \pi_2 h)}{1 + exp(\pi_0 + \pi_1\theta + \pi_2 h)}. \tag{14}$$

A positive (raw) intergenerational transmission of ability implies $\pi_1 > 0$. If parental human capital further improves the learning productivity of children (or makes parents better teachers), then we would also expect $\pi_2 > 0$.[36] This provides an additional incentive to invest in human capital, beyond that which maximizes one's own lifetime income, and reinforces intergenerational correlations in wages and educational attainment.

[35] Our restriction (made for computational tractability) that intergenerational transfers are zero after children grow up eliminates the possibility for smoothing across generations in response to idiosyncratic earnings shocks experienced in period 5 for parents and period 3 for their grown children. Because we force parents to make all 'bequests' before these earnings shocks are realized, parents and older children likely over-save somewhat in an effort to self-insure against these shocks when they might otherwise be able to rely on some access to family insurance.

[36] Estimates by Cunha and Heckman (2008), Cunha, Heckman and Schennach (2010), Cunha (2013), Attanasio, et al. (2015), and Attanasio, Meghir and Nix (2015) suggest that parental education or skill is a direct input into the production of child human capital. For computational reasons, we incorporate the effects of parental human capital on child productivity through the ability transmission process rather than introducing parental human capital directly into the human capital production function $f(\cdot, \cdot)$. Heterogeneity in θ may also reflect differences in very early or pre-natal investments or in local school quality across families, which we do not model explicitly but which might be related to parental human capital.

3.1.6 Unobserved Costs of Schooling

There are many difficult-to-measure schooling costs, including transportation costs and potentially higher costs of living associated with post-secondary schooling. There may also be 'psychic' costs (or benefits) of schooling as well (e.g. Carneiro, Hansen and Heckman 2003, Cunha, Heckman and Navarro 2005). For simplicity, we model all of these as unmeasured financial costs, $\zeta(i_2')$, where $\zeta(0) = 0$ and $\zeta'(i_2') > 0$. We assume these additional expenditures do not affect human capital levels but must be paid nonetheless. Thus, $\zeta(i_2')$ is subtracted from family resources in the budget constraint but does not appear as investment in the production function. Taking into account government subsidies and public investments, the total effective investment in old children is given by $\tilde{i}_2' \equiv p_2 + i_2'$, while total private family expenditures on late investment amount to $i_2' + \zeta(i_2') - S_2(i_2')$.

3.1.7 Decision Problem

With uncertainty in earnings, it is useful to break the decision problem into different life stages. The problem facing a young parent with a young child is given by:

$$V_3(a_3, h, \epsilon_3, \theta') = \max_{c_3, a_4, c_1', i_1'} \left\{ u(c_3) + \rho u(c_1') + \beta \mathbb{E}_{\epsilon_4} V_4(a_4, h, \epsilon_4, \tilde{i}_1', \theta') \right\}$$

subject to

$$\begin{aligned} a_4 &= Ra_3 + W_3(h, \epsilon_3) + y_3 - c_3 - i_1' + S_1(i_1') - c_1', \\ a_4 &\geq -L_3(h), \\ \tilde{i}_1' &= p_1 + i_1', \end{aligned}$$

$c_3 \geq 0$, $c_1' \geq 0$ and $i_1' \geq 0$. The expectation of V_4 is taken over the earnings shock of the old parent, ϵ_4. Because young children do not borrow or save on their own, the only constraint on borrowing during this period is that imposed on young parents.

The problem facing an old parent (with old child) is given by:

$$V_4(a_4, h, \epsilon_4, \tilde{i}_1', \theta') = \max_{c_4, a_5, c_2', i_2', a_3'} \left\{ u(c_4) + \beta \mathbb{E}_{\epsilon_5} V_5(a_5, h, \epsilon_5) + \rho \left[u(c_2') + \beta \mathbb{E}_{\epsilon_3', \theta''} \left(V_3(a_3', h', \epsilon_3', \theta'') | h', \theta' \right) \right] \right\}$$

subject to the intergenerational transfer constraint, equation (13),

$$\begin{aligned} a_3' + a_5 &= Ra_4 + W_4(h, \epsilon_4) + y_4 + W_2 - c_4 - c_2' - i_2' - \zeta(i_2') + S_2(i_2'), \\ a_3' &\geq -L_2(h'), \\ a_5 &\geq -L_4(h), \\ h' &= \theta' f(\tilde{i}_1', p_2 + i_2'), \end{aligned}$$

$c_4 \geq 0$, $c_2' \geq 0$ and $i_2' \geq 0$.[37] Both the old parent and the old child face constraints on their borrowing. The expectation of V_3 is taken over the earnings shock the old child receives as a young parent, ϵ_3', and over the ability level of the future grandchild, θ'', conditional on the ability of the child, θ', and the child's human capital, h'.

The problem facing a post-parent with no child at home is a standard lifecycle consumption/savings problem without any remaining uncertainty:

$$V_5(a_5, h, \epsilon_5) = \max_{a_6}\{u(Ra_5 + W_5(h, \epsilon_5) - a_6) + \beta u(Ra_6)\}.$$

The first order conditions for investment in this problem help illustrate the impact of the extensions we have made to the model of Section 2. To simplify notation, denote the expected difference in period 3 value functions for a young parent with a high vs. low ability child by $\Delta(a_3, h) \equiv \int [V_3(a_3, h, \epsilon_3, \theta_2) - V_3(a_3, h, \epsilon_3, \theta_1)]\, dF(\epsilon_3) > 0$. Let λ_3, λ_4, and λ_2' be Lagrange multipliers on the young parent, old parent and old child's borrowing constraints, respectively, and let ξ be the Lagrange multiplier on the old parent's non-negative transfer constraint. Note that adding a prime to any of these Lagrange multipliers indicates that it applies to the child's constraint. Optimal late investment i_2' then solves the following:

$$(\Psi_2 - \Upsilon_2 + \chi_3)\, \theta' \frac{\partial f}{\partial i_2'} = R\left[1 + \zeta'(i_2') - S_2'(i_2')\right], \tag{15}$$

where the two investment distortion wedge terms are defined (generally for periods $j = 1, 2$) as

$$\Upsilon_j \equiv \left(\frac{1-\gamma}{u'(c_j')}\right)\left(\rho^{-1}\lambda_2'\chi_3 + \beta\mathbb{E}_{\epsilon_3',\theta''}[\lambda_3'|h',\theta']\chi_4 + \beta^2\mathbb{E}_{\epsilon_3',\epsilon_4',\theta''}[\lambda_4'|h',\theta']\chi_5\right), \tag{16}$$

$$\Psi_j \equiv \beta R \frac{\partial \Pi(\theta', h')}{\partial h'} \frac{\Delta(a_3', h')}{u'(c_j')}, \tag{17}$$

[37] To simplify the problem computationally, we reduce the state space by introducing, $z_j = Ra_j + w\epsilon_j\Gamma_j$, which combines the asset state variable and the earnings shock into one continuous state variable. In this case, we have value functions $V_3(z_3, h, \theta')$ and $V_4(z_4, h, \tilde{i}_1)$ and substitute z_j where appropriate in the budget constraints.

and χ_k reflects the discounted present value of human capital as of period k as defined earlier.

The two wedges Υ_2 and Ψ_2 distort investment relative to the expected lifetime income maximizing amount. The following conditions eliminate these two distortions:

Condition 2. *No child borrowing constraint ever binds (i.e. $\lambda'_2 = \lambda'_3 = \lambda'_4 = 0$) for any state of the economy, and/or individuals can borrow up to their guaranteed lifetime income (i.e. $\gamma = 1$).*

Condition 3. *Parental human capital does not affect the distribution of child ability, so $\partial\Pi(a'_3, h')/\partial h' = 0$ for all (a'_3, h').*

Under Conditions 2 and 3, optimal i'_2 will be the (net) lifetime income maximizing amount, equating marginal returns in the labor market with marginal costs:

$$\chi_3 \theta' \frac{\partial f}{\partial i'_2} = R\left[1 + \zeta'(i'_2) - S'_2(i'_2)\right]. \tag{18}$$

Because earnings shocks are separable from human capital, they do not distort investment behavior in the absence of borrowing constraints. As expected, subsidies encourage investment, while additional unmeasured schooling expenditures discourage investment.

The two investment distortion wedges are relevant when Conditions 2 and/or 3 do not hold. Equation (16) shows that investment is discouraged by current and future constraints on borrowing. Due to future earnings uncertainty, investment will be distorted downward for everyone who might possibly end up being constrained at a later age, regardless of whether they ever actually borrow up to their limit. Turning to Equation (17), there will be more investment when $\partial\Pi/\partial h' > 0$. In this case, investment not only raises the individual's own income, but it also raises the expected income of future generations.

The Lagrange multipliers for the parental borrowing constraints and the non-negative transfer constraint do not directly appear in the condition for optimal late investment given by equation (15). However, these constraints will affect the marginal utility of consumption in late childhood, $u'(c'_2)$, which scales both wedges Υ_2 and Ψ_2. Parental borrowing constraints and the non-negative transfer constraint may also affect the extent to which children themselves are constrained (i.e. λ'_2, λ'_3, and λ'_4). Still, if the child's borrowing constraints are always non-binding and $\partial\Pi/\partial h' = 0$, then the extent to which the old parent's constraint or the non-negative transfer constraint bind is irrelevant for late investments.

The first order condition for early investment is more complicated due to uncertainty about late investment decisions:

$$\beta R\theta'\mathbb{E}_{\epsilon_4}\left[(\Psi_1-\Upsilon_1)\frac{\partial f}{\partial i_1'}\right]+\beta R\chi_3\theta'\text{Cov}\left(\frac{u'(c_2')}{u'(c_1')},\frac{\partial f}{\partial i_1'}\right)+\left(1-\frac{\lambda_3+\beta R\mathbb{E}_{\epsilon_4}[\xi]}{\rho u'(c_1')}\right)\chi_3\theta'\mathbb{E}_{\epsilon_4}\left(\frac{\partial f}{\partial i_1'}\right)=R^2[1-S_1'(i_1')],$$

where the covariance term comes from variation in ϵ_4 realizations for parents, which can impact c_2' and i_2' as discussed above.

There are now four distinct wedges that distort i_1' relative to the expected lifetime income maximizing amount. The first two are similar to the period 2 investment wedges, only expectations are now taken over the uncertainty about period 4 earnings shocks ϵ_4. This equation contains the expected values of Ψ_1 and Υ_1 multiplied by the marginal return on early investment, which depends on the (uncertain) level of i_2' to be chosen. As with late investment, a positive effect of parental human capital on expected child ability encourages early investment, while the possibility that borrowing constraints might bind for the child in the future discourages early investment. The third wedge (i.e. the covariance term) reflects the distortionary effects of parental income risk on early human capital investment. This term would be zero if $f_{12}=0$, because the marginal return to early investment would not depend on (uncertain) late investment choices.[38] More generally, both late consumption and investment are increasing in late earnings realizations (assuming constraints bind for some realizations of ϵ_4), so this covariance term has the opposite sign of f_{12}. Under dynamic complementarity ($f_{12}>0$), labor market risk has a discouraging effect on early investment, because the marginal productivity of early investment is high when late parental income and, consequently, late investment are high but the marginal value of consumption is low. The final wedge derives from distortions due to borrowing constraints on the young child's parents (i.e. λ_3) and the non-negative transfer constraint (i.e. ξ), both discouraging early investment.

If, in addition to Conditions 2 and 3, the child's parent is also unconstrained when the child is young and the family is not transfer constrained for any value of ϵ_4, then $\lambda_3=E[\xi]=0$, and early investment i_1' will be the lifetime income maximizing amount, satisfying

$$\chi_3\theta'\frac{\partial f}{\partial i_1'}=R^2\left[1-S_1'(i_1')\right]. \tag{19}$$

[38] If $f_{12}=0$, then the first order condition for early investment simplifies to

$$\left(\beta RE_{\epsilon_4}[\Psi_1-\Upsilon_1]-\chi_3\left(\frac{\lambda_3+\beta RE[\xi]}{\rho u'(c_1')}\right)+\chi_3\right)\theta'\frac{\partial f}{\partial i_1'}=R^2[1-S_1'(i_1')].$$

Aside from the subsidy, this first order condition is equivalent to that determining unconstrained investment in the problem of Section 2.[39]

This rich framework alters investment behavior in five main ways compared to the stylized model of Section 2: (i) a positive effect of parental human capital on the expected productivity of child investment (i.e. $\partial\Pi/\partial h' > 0$) provides an additional incentive for investment, (ii) the presence of labor market uncertainty means that future borrowing constraints discourage investment even for children that do not hit up against those future limits; (iii) non-negative intergenerational transfer constraints discourage investments by limiting the capacity for some parents to reap the rewards from investments in their children; (iv) government subsidies encourage investment; and (v) the positive dependence of borrowing limits on human capital produces a credit-expansion benefit of investment relative to consumption, encouraging the former.

3.2 Discussion of Identification

In this subsection, we briefly discuss identification of parameters of the human capital production technology, ability distribution, earnings growth, and the distribution of earnings shocks from lifecycle data. (A more detailed discussion is provided in Appendix C.) We then discuss the use of additional data on wealth and intergenerational data on investments and earnings to identify parameters related to borrowing constraints, parental altruism, the intergenerational transmission of ability, and unmeasured late investment expenditures. Throughout this discussion, we assume that public investment amounts (p_1 and p_2) and subsidy functions ($S_1(\cdot)$ and $S_2(\cdot)$) are known and that late investment levels i_2 and i_2' are observed for parents and children. Subsection 3.3.1 discusses how we obtain these values/functions from our data and how we map annual lifecycle data into the six life stages of our model.

First, data on growth rates in average earnings across lifecycle periods can be used to identify Γ_4 and Γ_5. We can then identify $Var(\epsilon_3) = Var(W_3) - \Gamma_4^{-1}Cov(W_3, W_4)$ with panel data on earnings over the first two periods of adulthood.

Next, consider identification of the human capital production technology (a, b, d), two ability levels (θ_1, θ_2), and the mean of earnings shocks $E(\epsilon_3)$. (For this discussion, we drop prime superscripts on variables where the analysis focuses on a single generation.) Given our assumptions,

[39] There is no uncertainty about i_2' when Conditions 2 and 3 hold (see equation (18)), so (separable) earnings shocks only distort investments through their interactions with borrowing constraints, the non-negative transfer constraint, and the intergenerational transmission of human capital (i.e. $\partial\Pi/\partial h' \neq 0$).

period 3 human capital for individual n is given by:

$$h_n = \theta_n f(i_{1n}, i_{2n}) = \theta_n \left(a(p_1 + i_{1n})^b + (1-a)(p_2 + i_{2n})^b \right)^{d/b}. \tag{20}$$

While we assume that late investments i_2 are observed, early investments i_1 are not. Instead, J noisy measures of early investment are available for each individual n. De-meaning these measures to obtain Z_{nj}, we have

$$Z_{nj} = \alpha_j \Phi_n + v_{nj}, \quad j = 1, ..., J, \tag{21}$$

where we normalize $\alpha_1 = 1$, $E[\Phi_n] = 0$, and v_{nj} are independent across individuals and measures. We also assume that the v_{nj} measurement errors are independent of all other choice and outcome variables (e.g. i_{1n}, i_{2n}, W_{3n}).

From data on $(Z_{n1}, Z_{n2}, .., Z_{nJ}, i_{2n}, W_{3n})$ for $J \geq 3$ early investment measures, we can identify the joint distribution of (Φ_n, i_{2n}, W_{3n}), then proceed as though we observe this distribution directly. (See Cunha, Heckman and Schennach (2010) for a similar line of argument.) It is important to recognize, however, that the factor Φ_n has no meaningful location or scale. To map these factors to early investments, we assume that $\Phi_n = \phi(i_{1n})$ where $\phi(\cdot)$ is a known function up to a few unknown parameters and $\phi'(\cdot) > 0$. Thus, higher factor scores reflect higher investment, and we can substitute $i_{1n} = \phi^{-1}(\Phi_n)$ into the production function given by equation (20). Appendix C provides greater details and shows how one can use the joint distribution (Φ_n, i_{2n}, W_{3n}) to identify the human capital production parameters (a, b, d), learning ability levels (θ_1, θ_2), parameters defining $\phi(\cdot)$, and $E(\epsilon_3)$.[40] Knowledge of $E(\epsilon_3)$ and $Var(\epsilon_3)$ identifies parameters (m, s) of the log normal distribution for earnings shocks. The cross-sectional distribution of ability can also be identified, but it is more difficult to identify the intergenerational ability transition matrix $\Pi(h, \theta_3; \pi)$ without direct observations on the ability of children and parents.

The remaining parameters to be identified include those determining the intergenerational transmission of ability π, the extent of parental altruism ρ, the severity of borrowing constraints

[40] This analysis builds on the approaches of Cunha, Heckman and Schennach (2010) and Agostinelli and Wiswall (2016), accounting for a discrete number of unobserved ability θ types. Because we rely on a single measure of post-investment earnings W_{3n}, we cannot directly apply the results (for unobserved time-invariant heterogeneity) of Cunha, Heckman and Schennach (2010). Nor can we use parental income as an instrument for early investments or skill levels (as in their approach for time-varying unobserved skills), because parental income is likely to be correlated with unobserved parental and, therefore, child ability. Agostinelli and Wiswall (2016) do not consider unobserved heterogeneity in ability.

γ, and the unmeasured late investment expenditure function $\zeta(\cdot)$. Because borrowing levels are non-increasing in the borrowing limit, we exploit data on the fraction of older parents with negative debt to help identify γ. We also exploit data on child investments and wages conditional on parental income and educational attainment to help identify γ, π, ρ, and $\zeta(\cdot)$.

As shown in Proposition 1, the effects of early and late parental income on child investment are important sources of identification regarding the extent of dynamic complementarity (determined largely by b) and the extent to which borrowing constraints bind, determined largely by γ. More generally, the first order conditions for early and late investments under uncertainty (see subsection 3.1.7) show that γ and ρ both play important (though distinct) roles when borrowing constraints and the non-negative transfer constraint bind or may bind in the future.

The intergenerational correlation in ability also affects intergenerational investment and wage relationships. To better understand this, first consider the case without constraints on borrowing or parental transfers and when the distribution of child ability θ' does not depend on parental human capital (i.e. under Conditions 2 and 3). In this case, child investments (i'_1, i'_2) and wages W'_3 will be independent of parental earnings (and parental investment choices i_1 and i_2) conditional on the child's ability θ'. Intergenerational wage and investment relationships would depend entirely on the effects of parental ability on child ability, providing a valuable source of identification for $Pr(\theta' = \theta_2|\theta)$.

To see this clearly, assume Conditions 2 and 3 are satisfied and that $S_j(\cdot)$ and $\zeta(\cdot)$ are linear functions. Ignoring corner solutions where investments are zero, it is straightforward to show that effective early and late investments ($\tilde{i}_1$ and $\tilde{i}_2$) are both proportional to $\theta^{1/(1-d)}$. This should be true for both parents and children, implying that $Corr(\ln(\tilde{i}_1), \ln(\tilde{i}'_1)) = Corr(\ln(\tilde{i}_2), \ln(\tilde{i}'_2)) = Corr(\ln(\theta), \ln(\theta'))$, so the intergenerational correlation of ability could be directly identified from intergenerational correlations in the log of effective investments.[41] More generally, knowledge of (i_j, i'_j) and all other parameters (identified above) would directly identify all (θ, θ') pairs and, therefore, their joint distribution $Pr(\theta', \theta)$. In this case, a child's investment should be independent of parental earnings conditional on parental investment.[42]

[41] For $S_j(i_j) = s_j i_j$ and $\zeta(i_2) = \bar{\zeta} i_2$, optimal investments are $\tilde{i}_1 = \kappa_1 \theta^{1/(1-d)}$ and $\tilde{i}_2 = \kappa_2 \theta^{1/(1-d)}$ where, $\kappa_1 \equiv \left[\frac{da\chi_3}{R^2(1-s_1)}\right]^{1/(1-d)} \left[a + (1-a)\kappa_0^b\right]^{\frac{d-b}{b(1-d)}}$ and $\kappa_2 \equiv \kappa_0 \kappa_1$ for $\kappa_0 \equiv \left[R\left(\frac{1-a}{a}\right)\left(\frac{1-s_1}{1+\bar{\zeta}-s_2}\right)\right]^{1/(1-b)}$. One can further show that $h = \kappa_3 \theta^{1/(1-d)}$ where $\kappa_3 \equiv \left[a\kappa_1^b + (1-a)\kappa_2^b\right]^{d/b}$.

[42] While the absence of constraints simplifies identification of the intergenerational correlation of ability, it complicates identification of other production function parameters. Borrowing constraints lead to variation in early and late investments conditional on ability, which is critical for identifying parameters of the human capital

If this independence does not hold, then parental human capital may directly affect the distribution of child ability, as would be the case if $\pi_2 \neq 0$ in our assumed ability transition function, $\Pi(h, \theta; \pi)$. In this case, investment decisions would now depend on parental human capital as well as parental ability, which need not be perfectly correlated anymore. For $\pi_2 > 0$, child schooling will tend to be positively correlated with parental earnings conditional on parental schooling, even in the absence of any binding borrowing or non-negative transfer constraints. Borrowing constraints and the non-negative parental transfer constraint may also distort investment (downward), creating a direct causal link between parental income and child investments, conditional on parental investments. These distortions are particularly strong when parents have high levels of education (and, therefore, ability) but low earnings realizations. Therefore, intergenerational investment relationships at the top vs. bottom of the parental income distribution are informative about π, γ and ρ.

Finally, the distribution of i_2' can be used to identify the unmeasured late investment expenditure function $\zeta(\cdot)$. This is most easily seen under Conditions 2 and 3 (ruling out borrowing constraints and an effect of parental human capital on child ability), because the parameters defining $\zeta(\cdot)$ could then be directly identified from the cross-sectional distribution of i_2' and equation (18) given knowledge of the production function, lifecycle earnings growth rates, and distribution of ability. More generally, $\zeta(\cdot)$ would need to be identified in conjunction with (π, γ, ρ) using the distribution of i_2' along with the wealth and intergenerational investment and wage/income moments already discussed.

3.3 Calibration

For our quantitative analysis, we rely primarily on data from the National Longitudinal Survey of Youth 1979 Cohort (NLSY79) and CNLSY to calibrate our model to the U.S. economy. All earnings are in 2008 dollars (deflated by the CPI-U). We normalize $w = 1$, so human capital is measured in 2008 dollars per year. In mapping model periods to the data, we assume that the six periods are 12 years each, corresponding to ages 0-11, 12-23, 24-35, 36-47, 48-59, and 60-71.

We assume a CES human capital production function, as in equation (2), and define preferences for consumption each period as

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma}, \qquad \sigma \geq 0,$$

production process. Observable variation in the price of investments across families could serve a similar purpose as in Attanasio, et al. (2015) and Attanasio, Meghir and Nix (2015).

so individuals have a constant intertemporal elasticity of substitution. We assume $\sigma = 2$, which implies an intertemporal elasticity of substitution for consumption of 0.5, consistent with estimates in the literature (Browning, Hansen and Heckman 1999). An annual interest rate of $r = 0.05$ is assumed throughout, so $R = (1 + r)^{12} = 1.7959$. We assume $\beta = R^{-1}$, so individuals desire constant lifecycle consumption profiles.

We consider four values of late investment, i_2, corresponding to different observed schooling levels: high school dropouts (less than 12 years of completed schooling), high school graduates (exactly 12 years of completed schooling), some college (13-15 years of completed schooling), and college graduates (16 or more years of completed schooling).[43] We assume unobserved late investment costs $\zeta(i_2)$ are related to time spent in school, recognizing that there may be different costs associated with years in high school vs. college. Specifically, we assume that $\zeta(i_2)$ equals zero for high school dropouts, $2\zeta_1$ for high school graduates, $2\zeta_1 + 2\zeta_2$ for those with some college, and $2\zeta_1 + 5\zeta_2$ for college graduates. For computational purposes, we also assume a finite grid for early investments i_1, which together with finite grids for i_2 and θ, produces a finite grid for human capital h. The grid for i_1, values for i_2 associated with different schooling levels, and calibration of (ζ_1, ζ_2) are discussed in greater detail below.

Along with using data to guide our choice for the investment grids, the following parameters must be determined empirically: potential earnings in school (W_2), post-school income shock distributions (m, s), lifecycle earnings growth rates (Γ_4, Γ_5), the human capital production function, (a, b, d), the Markov process for ability $(\theta_1, \theta_2, \pi_0, \pi_1, \pi_2)$, parental altruism towards children (ρ), debt constraints (γ), and unobserved late investment cost parameters (ζ_1, ζ_2). We first discuss a few parameters that are chosen to directly match data without having to simulate the model and then outline the calibration process for all remaining parameters.

3.3.1 Second Period Earnings, Investment Costs, and Investment Subsidies

We directly estimate potential earnings for ages 12-23, W_2, using the CNLSY. We also estimate foregone earnings from these data, which are combined with direct educational expenditures by schooling level (from the Digest of Education Statistics 2008) to determine publicly provided investments p_1 and p_2, late investment expenditure amounts i_2, and late subsidy functions $S_2(i_2)$.

Using the random sample of the CNLSY, we estimate the discounted present value of average

[43] Following much of the literature on schooling choice, we abstract from quality differences in post-secondary institutions. To the extent that these differences are important, they are likely to be captured in the distribution of θ.

earnings for high school dropouts over ages 16-23.[44] Dividing the average annual discounted income over this period by 12 yields an annualized potential income measure of $W_2 = 11,187$. This also reflects the total amount of foregone earnings for individuals in our highest schooling category: college completion. Foregone earnings for 'high school graduates' (those with 'some college') are given by the discounted present value of earnings for dropouts over ages 16-18 (16-20), dividing by 12 to annualize the amounts. We assume no foregone earnings for high school dropouts, because individuals cannot typically work before age 16.

We distinguish between total measured investment expenditures and the amount privately paid by individuals themselves, because education is heavily subsidized in the U.S. Total investment expenditures include foregone earnings and total public and private education expenditures. Consider first the investments made by old children ages 12-23. To calculate expenditures associated with grades 6-12, we use average expenditure per pupil for all public elementary and secondary schools. For the schooling category 'some college', we add two years of current-fund expenditures per student at all post-secondary institutions to the costs of high school. For 'college graduates', we add five years of current-fund expenditures per student at four-year post-secondary institutions to the costs of high school.[45] Combining foregone earnings with direct expenditures and dividing by 12 to annualize the amounts, we obtain total measured investments $\tilde{i}_2$ of \$3,563, \$5,912, \$13,369, and \$29,805 for high school dropouts, high school graduates, some college, and college graduates, respectively.

Foregone earnings are borne by individuals, but we assume that primary and secondary schooling is otherwise publicly provided at no private cost. Because dropping out of high school entails no foregone earnings or other private costs, we set $p_2 = 3,563$. This amount is subtracted from total observed investment expenditures to obtain private measured (pre-subsidy) investment expenditures i_2 of \$0, \$2,260, \$9,374, and \$25,082 for high school dropouts, high school graduates, some college, and college graduates, respectively. High school graduates only pay foregone earnings (roughly two-fifths of their total investment), while college students pay both foregone earnings and a share of direct costs, which are heavily subsidized. Dividing revenue from tuition and

[44] A discount rate of $r = 0.05$ was used to discount earnings to age 18.

[45] All schooling expenditure figures are taken from the *Digest of Education Statistics* (2008) and are adjusted to year 2008 dollars using the CPI-U. Primary and secondary expenditures (\$8,552 per year) are based on averages over the 1990-91 to 1994-95 period (Table 181). Post-secondary expenditures are based on all degree-granting institutions in 1995-96 (Table 360). Annual expenditures per student are \$25,902 at two-year institutions and \$32,712 at four-year institutions.

fees by total revenue for all degree-granting post-secondary institutions in 1995-96 suggests that student tuition payments account for only 28% of college revenues. We assume the remaining 72% of direct college expenditures reflect additional subsidies (beyond p_2 free public investments) and apply that to the tuition component of i_2. This yields $S_2(i_2)$ values of \$0, \$1,425, \$4,537, and \$11,251 for high school dropouts, high school graduates, some college, and college graduates, respectively.

Because there are no forgone earnings for young children, we take the annualized value of \$3,563 as the minimum period one investment.[46] Assuming this level of investment is completely subsidized for young children, we set $p_1 = 3,563$ and consider a 12-point grid for i_1 ranging from zero to \$12,000.[47] We set $S_1(i_1) = 0$ for all i_1, because private investments by parents in their young children are not typically subsidized in the U.S.

3.3.2 Earnings Growth Rates

We set $\Gamma_4 = E[W_4(h, \epsilon_4)]/E[W_3(h, \epsilon_3)] = 1.4778$ based on growth in average earnings levels between ages 24-35 and 36-47 for men in the NLSY79. Given Γ_4, we use growth in average earnings for men ages 36-47 and 48-59 in the 2006 March Current Population Survey to obtain $\Gamma_5 = \Gamma_4 \times E[W_5(h, \epsilon_5)]/E[W_4(h_4, \epsilon_4)] = 1.5919$.[48]

3.3.3 Calibrating other Parameters Using Simulated Method of Moments

The remaining parameters are calibrated by simulating the model and comparing the resulting allocations with those observed in the data.[49] In particular, we determine parameters of the

[46] This corresponds to the sum of average annual expenditures per pupil of \$8,552 for grades 1–5 divided by 12 (to annualize the amount).

[47] For the calibration, we used equally spaced points of \$1,000 from \$0 to \$10,000 and an additional point at \$12,000. The highest early investment chosen by anyone is \$8,000 in our calibration. For policy/counterfactual simulations that lead to higher levels of investment, we add additional grid points above \$12,000 in increments of \$2,000 as needed.

[48] In both cases, we use data for men deflated to year 2008 dollars and discount within period earnings to ages 30, 42, and 54 using a 5% interest rate. We drop observations for respondents with annual earnings less than \$200 or greater than \$275,000 or those with less than 9 years of completed schooling.

[49] We could, in principle, estimate the technology of skill production and unmeasured schooling cost parameters in a first step using only moments for post-school earnings conditional on early childhood investment measures and educational attainment (see Section 3.2). However, we estimate all remaining unknown parameters simultaneously, because the relationship between parental income and child investments and wages is also informative about some of these parameters, especially b determining the extent of dynamic complementarity.

earnings shock distribution (m, s), the human capital production function (a, b, d), parental altruism towards their children (ρ), the ability distribution and its intergenerational transmission $(\theta_1, \theta_2, \pi_0, \pi_1, \pi_2)$, debt constraint parameter (γ), and unmeasured cost parameters (ζ_1, ζ_2) using a simulated method of moments procedure to best fit moments based on data from the CNLSY. This step entails fully solving the dynastic fixed point problem of Section 3.1.7 in steady state, simulating a number of conditional moment conditions, and comparing those moments with their empirical counterparts.

Our calibration approach is equivalent to the nested fixed point approach of many recent dynamic structural estimation analyses in the literature on schooling choice and lifecycle earnings (e.g. Keane and Wolpin 2001, Johnson 2013, Hai and Heckman 2017, Navarro and Zhou 2017); however, we do not calculate standard errors, because our objective function is not differentiable and has many local minima. The non-differentiability rules out standard asymptotic formulas, while the combination of a non-smooth function and local minima make bootstrapping methods computationally prohibitive. Instead, we conduct a comprehensive sensitivity analysis, calibrating our model under different parameter restrictions to see how that affects our estimates and policy simulations. This analysis is summarized in Section 5 and detailed in the Online Appendix.

We fit moments related to (i) the education distribution, (ii) the distribution of annual earnings for men ages 24-35 by educational attainment and the covariance in earnings between ages 24-35 and 36-47, (iii) measures of early child investments conditional on early and late parental income and maternal schooling, (iv) child schooling attainment levels conditional on early and late parental income and maternal schooling, (v) child wages at ages 24-35 conditional on their own educational attainment, maternal schooling, and early parental income levels, and (vi) the fraction of families with older children that have zero or negative net worth.[50] In calibrating the model, all moments are weighted by the inverse of their sample variances. Here, we briefly discuss these moments, summarize the extent to which the model replicates them, and describe a few

[50]Our baseline calibration uses reported total parental earnings (mother's plus father's earnings) as the conditioning measures of family income in moment sets (iii)-(v); however, in Section 5, we also calibrate the model using an adjusted 'full' family income measure that adjusts for the possibility that mothers may work part-time in order to spend time investing in their children. We use family income based on age of the child (not mother), in an effort to account for differential fertility timing across families. Thus, we use family income levels and growth rates over the lifecycle of the child to determine whether constraints are binding over the child's lifetime. Notably, the regression analysis reported in Table 1 produces very similar effects of family income on child educational attainment whether or not we control for whether the mother was a teenager when the child was born and the number of children in the household once we condition on maternal education.

other important features of the calibrated baseline steady state. Appendix C provides further details.

Table 2 shows the distribution of educational attainment for our NLSY calibration sample along with the calibrated steady state distribution produced by our model. Roughly 80% of youth in our sample attained at least a high school degree, while slightly more than 40% went on to attend some college or more. Only about 20% completed at least four years of college. The model matches educational attainment levels in the data quite well.

Average earnings for young parents, W_3, is \$41,650 in the baseline economy, while the standard deviation is \$23,108. Given $\Gamma_4 = 1.4778$, average earnings grow to roughly \$60,000 for older parents (W_4). These are quite close to the empirical counterparts for men ages 24-35 and 36-47 in the NLSY79.[51] As shown in Appendix C, average earnings for young parents conditional on educational attainment match the data quite well, ranging from \$29,500 for high school dropouts to \$59,700 for college graduates. While the model closely matches the overall variance in earnings for young men in the NLSY79, it under-states the increase in variance with educational attainment and the covariance between W_3 and W_4. The latter is not particularly surprising given it receives very little weight in the calibration (the variance of this moment is large in the NLSY79) and the fact that we discretize investment choices and human capital in our model, which limits the extent of variation that can be explained.

We use the CNLSY to calculate average early investment factor scores, $\hat{\Phi}_n$, and the distribution of educational attainment by maternal education, early family income, and late family income. Factor scores are estimated for children ages 6-7 using data on eight early investment measures, such as the number of books in the home, whether the child receives special lessons, or whether the mother regularly reads to the child.[52] We condition on three categories of early and late family income: bottom quartile, second quartile, and top half of the age-specific family income distributions. We fit these moments assuming the function mapping early investment amounts to early factor scores, $\phi(\cdot)$, is quadratic.[53]

Tables 3 and 4 report average early investment factor scores and educational attainment by parental education and by parental income when the child is young and old. First, consider the

[51] Average earnings for men ages 24-35 in the NLSY79 is \$41,650 with a standard deviation of \$23,415. Average earnings for men ages 36-47 is \$61,490 with a standard deviation of \$41,416.

[52] See Appendix C for a detailed description of the factor analysis, factor score estimation, and the full set of moments and weights used in estimation, along with the calibrated model counterparts.

[53] For $\phi(i_1) = \phi_0 + \phi_1 i_1 + \phi_2 i_1^2$, our calibration yields $\phi_0 = -1.07$, $\phi_1 = 0.00085$, and $\phi_2 = 0.0000001$.

relationship between investments and parental education shown in Table 3. The model produces the sharp increases in early investment (as measured by estimated factor scores) and educational attainment by parental schooling observed in the data. For example, in both the data and model, high school graduation rates are about 30 percentage points higher for the children of college graduates relative to high school dropouts. College attendance and graduation rates are even more strongly increasing in parental education. Table 4 shows that early investment and educational attainment also increase with both early and late family income. Conditioning on both income measures simultaneously, we observe that investments are increasing in both early and late income (throughout both income distributions); however, early income differences appear to be more important with the model and data in general agreement.

We use the CNLSY to calculate average earnings for children over ages 24-35, W_3', conditional on the child's and mother's educational attainment and early family income.[54] Conditional on the youth's own educational attainment, early family income and parental education can affect the child's earnings through early investment choices and the child's ability. Consistent with the discussion above for male earnings in the NLSY79 (the parent's generation), we observe that children's earnings are strongly increasing in their own education. This is true in the CNLSY and the model even when we condition on maternal education and early family income. The model is also consistent with the data in that parental education is largely unrelated with child earnings conditional on the child's education and early family income; however, the model produces too little variation in child earnings with early family income when conditioning on both child and maternal education.

Finally, we match the fraction of parents in the CNLSY who reported zero or negative net worth when their child was ages 17-19. Our model suggests that 22% of old parents have zero or negative wealth (i.e. $a_4 \leq 0$) compared to 17% in the data.

Table 5 reports the calibrated parameter values for our model. The model implies more weight on early relative to late investments in the production of human capital, with $a = 0.58$. A value of $b = 0.26$ suggests that early and late investments are slightly less complementary than Cobb-Douglas, and there is modest diminishing returns to investment in that $d = 0.82$.

[54] We use weekly earnings for our measure of W_3' (due to data availability and the desire to best capture differences in human capital), while we use the distribution of annual earnings for men in helping identify earnings growth and the distribution of shocks (as described above). Because the units for these are quite different, we scale weekly earnings for each individual by average earnings for our sample of youth, calibrating to fit these ratios. See Appendix C for additional details.

In a similar framework (with more investment periods), Cunha (2013) estimates that past skills and current investments (analogous to early and late investments in our framework) are slightly more complementary than Cobb-Douglas in the production of new skills, with a similar degree of decreasing returns to scale.[55]

Values for θ_1 and θ_2 suggest that high ability individuals are roughly 2.5 times as productive as their low ability counterparts. Calibrated values of (π_0, π_1, π_2) imply that 70% of all individuals are of high ability with a strong intergenerational correlation in θ. The positive intergenerational correlation in θ of 0.31 reflects two distinct forces. First, $\pi_1 > 0$ implies that average child ability is directly increasing in parental ability. Second, $\pi_2 > 0$ means that the child's expected ability is also increasing in parental human capital, which is generally increasing in parental ability. The probability that a high ability parent has a high ability child ranges from 76-83% depending on parental human capital. Low ability parents have much less variation in human capital levels and a roughly 50% chance of having a high ability child. This lower probability mainly reflects the direct role of ability in the transmission process, but the low level of parental human capital among low ability parents is also partly responsible. The modest direct effects of parental human capital on child development are broadly consistent with several recent estimates of similar production technologies (Cunha, Heckman and Schennach 2010, Cunha 2013, Attanasio, Meghir and Nix 2015, Attanasio, et al. 2015). It is more difficult to find estimates of the raw intergenerational transmission of ability, because most measures of 'ability' reflect not only raw innate ability but also any investments made up until the measurement period. One recent study for Sweden (Grönqvist, Öckert, and Vlachos, forthcoming) addresses important concerns about measurement error and estimates that age 18 father-son intergenerational correlations for cognitive and non-cognitive abilities range from 0.41 to 0.48. They further estimate intergenera-

[55]Several important specification differences make it difficult to compare our parameter values with the estimates produced in other recent studies (Cunha, Heckman and Schennach 2010, del Boca, Flinn and Wiswall 2014, Attanasio, Meghir and Nix 2015, Attanasio, et al. 2015, Agostinelli and Wiswall 2016): First, these studies generally consider frameworks with shorter 1-5 year periods (compared to our 12-year periods) and typically end at earlier ages. Second, these studies do not typically examine human capital or wages as the output produced by child investments (e.g. outcomes are sometimes in normalized test score units or anchored to years of completed schooling) and investments are not typically monetized. Third, these studies often examine the simultaneous development of multiple skills (e.g. cognitive and non-cognitive or health) or consider multiple types of investment (e.g. time and goods) each period. Finally, several studies abstract from important features of our technology (e.g. imposing constant returns to scale or a Cobb-Douglas specification, abstracting from unobserved heterogeneity in ability). Only Agostinelli and Wiswall (2016) finds evidence against dynamic complementarity.

tional ability correlations of 0.12-0.13 using a sample of fathers and adopted sons, which suggests a non-trivial role for nurture (i.e. investments in our context). Given this, it is not surprising that their estimated intergenerational correlation for biological fathers and sons lies between our estimated intergenerational correlation in innate ability θ (0.31) and in acquired skill h (0.5).

The calibrated value of $\rho = 0.86$ implies that considerable value is placed on children and grandchildren. The calibrated value for γ implies that individuals can only borrow up to 22% of their minimal discounted lifetime earnings at any age with the implied limits increasing in human capital.[56] Thus, credit limits are far more stringent than the 'natural limit' of Laitner (1992), Huggett (1993), and Aiyagari (1994).

Finally, the calibrated value $\zeta_1 = 47$ implies negligible unobserved costs of high school, while $\zeta_2 = 761$ implies moderate unobserved costs associated with college attendance. The higher costs of college are not surprising, given the additional travel and living expenses often associated with attending college.

3.4 Additional Features of the Baseline Steady State

Table 6 shows how average early and late private investment amounts vary with parental education in our baseline steady state. On average, parents annually spend \$1,888 investing in their young children and \$5,629 investing in their older children (including unobserved expenditures $\zeta(i_2)$ less subsidies $S_2(i_2)$ as reported in the final column). Comparing columns 2 and 3 shows how late subsidy amounts differ, on average, by education, while comparing columns 3 and 4 shows how average unmeasured investment expenses $\zeta(i_2)$ differ. Based on columns 1 and 4, total (net-of-subsidy) private investment expenditures in young (old) children are roughly 6.0 (4.6) times as great for the children of college graduates compared to high school dropouts. These ratios are in line with that of Kaushal, Magnuson, and Waldfogel (2011), who find that parents with a college degree spend 5.7 times more on their children than parents without a high school degree.[57]

Table 7 reports the fraction of young and old parents that are borrowing up to their limits, along with the fraction of old parents that are transfer constrained (i.e. transferring zero to their

[56] This implies $L_2(h)$ limits of \$1,109-10,246; $L_3(h)$ limits of \$1,132-10,457; and $L_4(h)$ limits of \$762-7,041. Because a period represents 12 years, these amounts should be multiplied by 12 years in thinking about their implications for actual borrowing observed in the U.S. economy. (That is, an extra \$1,000 of assets in our model reflects \$1,000 of additional spending per year for 12 years.)

[57] See Table 3 of the online appendix from Kaushal, Magnuson, and Waldfogel (2011). The ratio of 5.7 is based on expenditure amounts that exclude enrichment spending allocated to parents.

children). Our calibrated steady state suggests that 12% of all young parents and 14% of all old parents are borrowing as much as they can. The share of young parents borrowing up to their limit is greater among those who only finished high school (20%) or who dropped out (13%) relative to those who attended (6%) or completed (1%) college. The overall share of old parents borrowing up to their limit is similar to the share of young parents, with the highest rates of constrained old parents among high school graduates and those who attended some college (both 17%). Constraints among more educated families are more likely to be binding at the later age when they are typically financing high levels of late investment in their children. For example, Table 6 shows that college graduate parents, on average, spend nearly $8,000 more on late than early investments. The differences between late and early investment expenditures are much smaller among less-educated parents (whose children are of lower average ability), so constraints tend to be more binding for them at early ages due to consumption-smoothing motives. We find no evidence that older children are borrowing up to their limits; although, their investments may still be distorted at this age due to potentially binding future constraints during early and late adulthood. More generally, many families may be affected by the presence of borrowing limits even if they never actually hit up against them.

Beginning with Becker and Tomes (1979, 1986), much of the literature on human capital investment in dynastic intergenerational frameworks has emphasized the role of 'intergenerational constraints' – the non-negative transfer constraint in our model. Our calibrated economy suggests that this constraint is not particularly salient with only 1% of high school dropout parents choosing not to make any transfers to their old children. The least-educated parents are affected, because they tend to have low income relative to what their children can expect. While all high school dropouts are of low ability, nearly half of them will have a high ability child. Some of these parents would like to take resources from their older children but are prevented from doing so by the transfer constraint. As discussed below, this 'intergenerational constraint' would become more salient if lifecycle constraints were eliminated; however, it would still directly impact very few families.

3.5 Income/Wealth Effects on Investment

Two recent studies estimate the effects of exogenous family income/wealth shocks in the form of lottery winnings (Bulman, et al. 2016) or paternal job loss (Hilger 2016) on the college attendance rates and earnings of children finishing high school at the time. The findings in both studies imply that \$100,000-150,000 in additional wealth would increase college attendance rates by 1-4%. These modest effects lead them to conclude that borrowing constraints are relatively unimportant for college attendance.

We explore this type of financial windfall in our model, both as an external validity check on our calibration and to gain a deeper understanding of the economic forces at play. Standing in for a big lottery win, row 1 of Table 8 simulates the average impacts of a one-time \$10,000 unanticipated transfer to old parents on their children's human capital investment and post-school earnings. Because each period in our model reflects 12 years, this \$10,000 transfer is analogous to a \$120,000 increase in parental wealth (or 12 years of \$10,000 more in income each year). Our model suggests that this large windfall would produce only a 3% increase in college attendance rates, consistent with the quasi-experimental findings of Hilger (2016) and Bulman, et al. (2016). A comparison with row 2 of Table 8 shows that late investment responses are low (with no change in college completion rates), because families are unable to optimally adjust early investments when the shock is unanticipated. If the same late transfer is anticipated by parents when their children are still young, college attendance rates increase 7.2%. Average early investment increases by 8% and late investments increase by more than four times as much as when the transfer is unanticipated (6.2% vs. 1.4%). With adjustments in both early and late investment, the \$10,000 transfer to the parents of old children would increase their children's post-school earnings by 1.3%, more than 6 times as much as when the transfer is unanticipated and early investments are held fixed. These results highlight the implications of dynamic complementarity for investment responses to changes in family income/wealth. They also suggest that quasi-experimental evidence from unanticipated income/wealth shocks for parents of high school-aged children under-estimates the importance of long-run predictable differences in family income/wealth for child development.

We can learn about the role of borrowing constraints on young parents by simulating an equivalent transfer (in discounted value) given to young parents rather than old parents. The effects of this are reported in the final row of Table 8. Consistent with Proposition 2 and the fact that early borrowing constraints bind for some families (see Table 7), we observe stronger investment responses to the transfer to young parents (compare rows 2 and 3); however, the differences are modest relative to the differences between unanticipated and anticipated transfers to old parents (rows 1 vs. 2).

3.6 Decomposing Investment Gaps: Ability and Market Frictions

Heterogeneity in ability and market frictions (i.e. borrowing constraints, the non-negative transfer constraint, imperfect insurance against earnings risk) generate the sizeable differences in early and late educational investments by parental background in our framework. Table 9 explores the relative importance of these forces, beginning with the 'raw' or unconditional gaps in investment between children from the highest and lowest parental income quartiles.[58] As reported in row 1, children from high income families invest roughly \$3,000 more at early ages and nearly \$8,000 more at later ages compared to children from low income families. The college attendance gap by income is 38 percentage points. With no market frictions, these investment differences would be driven entirely by differences in intergenerational transmission of ability θ in our model.

To quantify the importance of market frictions, we begin by exploring the extent to which gaps in investment by family income remain after conditioning on ability. Row 2 of Table 9 conditions only on parental ability, which is informative about the correlation between parental ability and income. This reduces investment gaps by as much as 10%, with more modest effects on early investment. Row 3 conditions on the child's own ability, fully accounting for any differences in the productivity of investments across children. In this case, differences in investment are explained entirely by market frictions. These results suggest that 15-27% of the raw gaps in investment by family income are due to differences in ability; the remaining 73-85% is due to various market frictions.

To isolate the effects of lifecycle borrowing constraints (from other market frictions), we can relax all lifecycle borrowing constraints to their 'natural limits' (Laitner 1992, Huggett 1993, and Aiyagari 1994) by setting $\gamma = 1$. In this case, reported in the final two rows of Table 9, families are only constrained by the requirement that they must repay their loans under all circumstances, in which case they would never wish to borrow more than these implied limits (given standard Inada conditions are satisfied for $u(c)$).[59] While this effectively eliminates distortions related to lifecycle borrowing constraints, the fraction of parents constrained from making negative transfers rises from less than 1% to 5%. These effects are concentrated among lower ability parents with high ability children, because the latter have better lifetime opportunities than the former. Despite the increased salience of the non-negative transfer constraint, investments increase substantially for

[58] The average differences between the highest and lowest quartiles in parental income are about \$54,000 among young parents and \$79,000 among old parents.

[59] For computational purposes, we set $\gamma = 0.99$ so that consumption is always strictly positive.

most children – average early and late investment amounts more than double. As Table 9 shows, eliminating borrowing constraints increases unconditional early and late average investment gaps by 16% and 6%, respectively; although, it reduces college-going differences by 14%.

So, why do the gaps widen, when market frictions decline? While not reported in the table, the intergenerational correlation of ability increases slightly to 0.33, because parental ability and human capital become more strongly correlated and both raise expected child ability. The final row in the table reveals that the larger investment gaps by parental income largely reflect a more efficient allocation of investment by ability and the strong correlation between parental ability and income. Compared to the unconditional investment gaps by family income in our baseline economy (row 1), average early investment gaps are substantially reduced when borrowing constraints are eliminated and we condition on ability. This is particularly true for average late investments and college attendance gaps, which are reduced by half and two-thirds, respectively. The reduction in early investment gaps is much more modest at 19%. Alternatively, comparing the economy with (row 3) and without (row 5) lifecycle borrowing constraints, we observe substantial reductions in investment gaps by parental income conditional on ability. The remaining gaps can be attributed to the distortions caused by the non-negative transfer constraint and imperfect insurance against labor market risk.

In Table 10, we further examine the role of intergenerational ability transmission. We begin by studying the importance of parental human capital as a direct determinant of child ability by shutting down its effect on the intergenerational transmission of ability. Specifically, we set $\pi_2 = 0$ in equation (14), adjusting π_0 and π_1 to hold constant $Pr(\theta'|\theta)$. In doing so, we maintain the same intergenerational correlation of ability, but eliminate the force that directly links parental investments in their own human capital to their child's ability. Simulating this counterfactual economy reveals that eliminating the direct effects of parental human capital on children would lead to an increase in both early and late investment for the children of high school dropouts (whose children are now more able, on average) but would reduce investments among the children born to more educated parents (whose children are now less able, on average). Comparing columns 1 and 2 of Table 10, we observe that the early investment gap between the children of college graduates and high school dropouts would decline by nearly \$400 and the late investment gap would decline by more than \$1,300. The intergenerational correlation for late investments, acquired human capital, and the present value of (adult) lifetime earnings would also fall.[60] These

[60] Adult lifetime earnings are given by $W_3 + R^{-1}W_4 + R^{-2}W_5$.

findings suggest a modest role for direct effects of parental human capital on child ability (or the production of child skills more generally) in the determination of intergenerational mobility.

In column 3 of Table 10, we fully eliminate the ability correlation between parent and child. Not surprisingly, investment gaps by parental education shrink dramatically. In this case, the children of high school dropouts are just as likely to be of high ability as the children of college graduates. Investment gaps by parental education fall by about one-third relative to baseline differences, but remain sizeable, reflecting the influence of market frictions (i.e. borrowing constraints, uninsured risk, and non-negative transfers) on intergenerational mobility. Even with no intergenerational correlation in raw ability, the intergenerational correlations in late investments (0.29), human capital (0.28), and lifetime earnings (0.19) would remain sizeable.

The final column of Table 10 shows how much stronger intergenerational correlations in investments are if ability is perfectly correlated. In this case, all high school dropouts are of low ability and always have low ability children, while all college graduates are of high ability and always have high ability children. Investments in the children of high school dropouts fall dramatically; investments in the children of college graduates increase. Investment gaps by parental education increase by roughly half relative to the baseline. The intergenerational correlation in late investments increases to 0.85, with market frictions continuing to generate a limited amount of intergenerational mobility. These simulations imply that the intergenerational correlation of ability is a significant determinant of intergenerational investment and earnings relationships; however, market frictions also play an important role.

4 Policy Analysis

This section analyzes three separate policy interventions. First, we consider different loan policies to evaluate the importance of borrowing constraints at different stages of child development. Here, we show that eliminating all lifecycle constraints has a much greater impact than relaxing constraints in any single period. Second, we study fiscally equivalent early and late investment subsidy policies. The stronger investment response to early subsidies highlights the interaction between dynamic complementarity and early borrowing constraints. We also show that the investment response to late subsidies is much stronger when early investments are allowed to adjust than when they are held fixed, underscoring the economic importance of dynamic complementarity and endogenous early investment behavior. We compute the optimal ratio of early to late subsidies and show that shifting resources from late to early childhood increases aggregate welfare.

Third, we consider the effects of a fiscally equivalent increase in the level of early public investment. This exercise underscores the extent to which different types of human capital investment policies impact different ends of the education distribution.

4.1 Increasing Borrowing Limits

Given the complementarity we find between early and late investments and the fact that borrowing constraints bind for many parents in our baseline steady state, relaxing borrowing constraints should lead to increases in investment during both early and late childhood (see Section 2). To investigate this quantitatively, we simulate the 'short-run' and 'long-run' responses to a permanent \$2,500 increase in borrowing limits for all young parents and then again for all old parents (leaving all other borrowing limits unchanged in both cases). In the case of short-run responses, we consider the effects on children that are young when the expanded loan policy is implemented, so both early and late investment choices can respond. Long-run responses are based on behavior in the new steady state relative to the baseline economy, reflecting changes in asset and human capital distributions that take place across generations.

We start by permanently increasing borrowing limits for young parents. The effects of this on early and late investments in children and on their average post-school earnings are reported in Table 11. Focusing first on short-run impacts, we see that relaxing borrowing constraints on young parents would lead to modest increases in investment. The increase in early investment would be greatest among children of high school graduates, while the children of high school dropouts and those with some college would also experience above average increases. This is not surprising given the shares of young parents constrained by education level reported in Table 7. Due to dynamic complementarity, the increases in early investment are met with increases in late investment, especially in college attendance. The average wages of young adults increase by 0.4% in the short-run.

The long-run changes (also in Table 11) incorporate the fact that some young parents borrow more and accumulate more debt as old parents, causing them to transfer less to their children. Despite the fact that constrained persons with any given level of assets and human capital are likely to invest more in their children, asset distributions shift leftward over the long-run such that the fractions of young and old parents that are borrowing constrained change very little. This decline in asset levels leads to lower overall investment levels and negligible long-run effects on average wages.

These results suggest that relaxing borrowing constraints can be a double-edged sword in terms of human capital investment. In the short-run, investment and debt increase among constrained families leading to reductions in intergenerational transfers. Unconstrained parents are also likely to reduce transfers to their children, even though they do not benefit directly from increased loan limits. To the extent that their descendants may benefit from higher loan limits, these parents will attempt to capture some of the 'family' gains by transferring less to their children. While these responses are good in terms of 'family' or 'dynastic' welfare, they saddle future generations with more debt and can lead to long-run reductions in human capital investment. These results underscore the potential conflict between short-run effects on current generations and long-run effects on future generations. They also highlight the fact that some policies may have important indirect effects on asset accumulation if future generations are affected: a policy may cause current generations to respond even if they themselves are not directly affected by the policy.

Relaxing constraints on older parents has greater impacts on investments in children, largely because the families that are most constrained at this stage (especially higher educated parents) have a greater propensity to spend additional resources on child investment rather than consumption. See Table 12. In the short-run, early investment increases by 10.9%, on average, while the college attendance rate increases by 5%. Average earnings rise by 1.8%. The sizeable increase in earnings helps offset the consequences of greater borrowing on intergenerational transfers, so short- and long-run impacts are similar.

Because old children are not borrowing constrained in our baseline steady state, relaxing their borrowing limits has no effect on investment behavior.[61] Yet, this does not mean that investment decisions for old children are at unconstrained optimal levels (even conditional on early investment choices), because many of these children face binding constraints as young and old parents. Still, allowing them to borrow more as old children does nothing to alleviate these future constraints.

So far, these results suggest a modest role for credit market limits. We now show that this is not the case. Instead of increasing borrowing limits one period at a time, we simultaneously relax all lifecycle borrowing constraints to their 'natural limits' (Laitner 1992, Huggett 1993, Aiyagari 1994) by setting $\gamma = 1$. Individuals never want to borrow more than these limits allow. As shown in Table 13, fully relaxing borrowing constraints leads to sizeable increases in human capital investments that are an order of magnitude larger than those observed for $2,500 increases in

[61] Keane and Wolpin (2001), Johnson (2013), Abbott, et al. (2016), and Hai and Heckman (2016) also find small effects of expanding loan limits at college-going ages, although, for varied reasons.

borrowing limits for young or old parents alone.[62] Early investments increase more than late investments, and investments increase the most among the children of high school graduates and dropouts. The intergenerational correlation of h falls from .5 to .4 as a result. These investment responses raise the average earnings of young adults by 11.7% in the short-run and 17.7% in the long-run.

Table 13 suggests that borrowing limits considerably discourage family investments, despite the fact that less than 15% of parents are actually at their borrowing limits in the baseline economy. Uncertainty is one important aspect of this result, because most, if not all, families face the potential of binding constraints in the future. This can discourage investment even if families never actually end up borrowing to their limits. The dynamics of the problem are also important, because relaxing borrowing constraints in one period can have limited effects on investment and borrowing if future borrowing constraints are likely to bind. Indeed, the L_3 borrowing limits cannot be relaxed much more than $2,500 for low human capital individuals if L_4 is not also relaxed.[63] However, relaxing all borrowing constraints together (i.e. raising γ to 1 from 0.22), allows us to more than quadruple borrowing opportunities at each stage of life.

4.2 Subsidizing Investments

We next study the consequences of increasing subsidy rates for early and late human capital investments. This analysis highlights the implications of dynamic complementarity in investments and borrowing constraints when considering policies targeted to different stages of development.

In this exercise, we consider constant changes in marginal subsidy rates. In particular, let $S_1(i_1) = s_1 i_1$ where our baseline calibration assumes $s_1 = 0$, and late investments are subsidized according to $\tilde{S}_2(i_2) = S_2(i_2) + s_2 i_2$ where the baseline subsidy function $S_2(i_2)$ is described earlier Subsection 3.3.1. We begin by separately increasing s_1 and s_2 so that total expenditures on all education subsidies (i.e. $S_1(i_1) + \tilde{S}_2(i_2)$) increase by the same amount, making the policies comparable.[64]

[62] As in Table 9, we set $\gamma = 0.99$ for computational purposes to ensure that consumption is always strictly positive.

[63] More specifically, $L_3(h)$ cannot be raised by more than $2,515 (to $3,647) for the lowest h value given that the borrowing limit must ensure that (i) all debts must always be repaid, and (ii) future debts can never exceed future borrowing limits. If $L_4(h)$ is also set to the natural limit, then $L_3(h)$ can be raised to $5,156 for the lowest h value.

[64] These results abstract from distortions that might be generated from taxation in order to raise revenue to cover the costs. As discussed in subsection 3.1.1, there are no investment distortions of flat taxes (rate τ) on earnings

Table 14 reports the short- and long-run effects of additional subsidies for early and late investments. Because they are so similar, we discuss only the short-run results. The first row reports the effects of subsidizing early human capital investment at a rate of 10%. The per student total cost of this policy is roughly \$1,420, with roughly three-quarters of this coming from the increased costs associated with subsidies for late investments. Not surprisingly, there is a large increase in early investment (64%). Because investments are complementary, this policy also increases late investment by roughly 23%. Most of the changes in the education distribution come from increases at the upper end with a 43% increase in the college graduation rate. Average post-school wages increase 6.5%.

We next consider the effects of increasing the marginal subsidy rate to late investments by $s_2 = 0.026$ (also costing \$1,420 per student). We begin by discussing the effects of this policy when parents are aware of the higher subsidy rate when their children are young (row two of Table 14). Thus, both early and late investments may respond. Although this policy costs the same as a 10% subsidy to early investment, it has weaker effects on human capital accumulation. Early investments increase by only 13%, compared with 64% for the early investment subsidy. Perhaps more surprisingly, the increase in average late investment is only slightly greater than that generated by the early investment subsidy. While late subsidies have weaker impacts on college completion compared to early subsidies, they appear to increase high school graduation rates more. These investment responses imply a 3.6% increase in average entry wage rates, much less than the response to an early investment subsidy.

These results underscore the interaction between credit constraints and dynamic complementarity. While unconstrained families increase both early and late investments in response to an (anticipated) increase in s_2, constrained young parents are limited in how much they can increase investments in their young children. Complementarity implies that if children do not receive adequate early investments, it may not be worth it for parents to make later investments, even if they are heavily subsidized. By contrast, early investment subsidies enable families to increase investments in their young children without having to sacrifice current consumption or borrow more. Those early investments can then be matched with later investments.

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if investments are tax deductible and borrowing constraints are proportional to $1 - \tau$. Imposing non-deductible flat taxes on earnings to cover the costs of new subsidies has only modest effects on the simulated policy impacts. For example, Table 14 indicates that the average long-run increase in W_3 associated with the fiscally equivalent early and late investment subsidies we consider would be 8.4% and 4.4%, respectively. Imposing non-deductible flat taxes to cover these costs, the comparable long-run increases in average W_3 would be 7.8% and 3.6%.

Row three of Table 14 reports the effects of an increase in s_2 that is announced after early investments have already been made. This measures the (very) short-run effects for families with older children when the policy is first announced and introduced. Here, we see more modest effects on late investment and human capital accumulation, because early investment is held fixed. Overall, average late investment increases about 15.4%, a little more than half the effect observed when early investment is also able to adjust. This, coupled with no change in early investment, produces a much smaller short-run increase in wages (1.6% vs. 3.6% when early investment adjusts). Increases in high school completion and college attendance (i.e. some college or more) rates are very similar whether or not early investments are able to adjust. (Notably, simulated effects on college attendance rates are consistent with most estimates of the impacts of tuition and financial aid on college attendance in the U.S.)[65] By contrast, effects on college completion are less than half when early investment cannot respond (15% compared to 39%). Substantial early investments are needed to make a college degree worthwhile.

These results demonstrate the importance of accounting for the interaction between early and late investments when considering education policies. Holding constant adolescent skill levels when analyzing policies that affect high school or college attendance decisions is not innocuous. Failing to account for adjustments in early investment not only neglects those responses, but it also leads one to underestimate the policy's full impact on late investments. Together, these imply substantial underestimation of policy effects on human capital and wages (except, of course, for those families with older children at the time of the policy change). Our results suggest that failure to account for early investment responses would cause researchers to underestimate the full impact of post-secondary subsidies on earnings by almost 60%.[66]

Given the differential effects of early and late investment subsidies, we consider whether it is efficient to increase s_1 while reducing s_2 to hold government investment expenditures constant.[67]

[65] Our s_2 increase of 0.026 is roughly equivalent to a $1,200 reduction in annual tuition for the first two years of college. Our simulations suggest that this increases college attendance by 6.8 percentage points in the short-run. Kane (2006) and Deming and Dynarski (2009) provide recent surveys of the related empirical literature, concluding that a $1,000 reduction in tuition leads to a 3-6 percentage point increase in college attendance.

[66] These concerns not only apply to structural models of schooling decisions, but they also apply to more standard regression or quasi-experimental estimates of the effect of tuition or financial aid changes on college attendance. These strategies may identify the very short-run effects on older cohorts of college-age children when the policy is implemented, but they are unlikely to identify the medium- or long-term effects on younger or future cohorts.

[67] We consider policy changes that hold constant the discounted present value of all investment expenditures from the present and forever after (i.e. including the transition period).

We find that increasing early investment subsidies to $s_1 = 0.43$, offset by a reduction in late investment subsidies ($s_2 = -0.36$), maximizes the expected value function for the current generation of young parents, $E[V_3(a_3, h, \theta')]$. This policy increases welfare by an amount equivalent to increasing consumption in every period (for every generation) by 0.33%. Not surprisingly, the increase in early investment subsidies coupled with a reduction in late subsidies leads to a shift in investment to the earlier period. Table 15 shows that average early investment increases by more than 300% while average late investment declines by nearly half (with a dramatic drop in college graduation rates). Average earnings during early adulthood increase 11%. The impacts of this policy change are not the same for all children. Shifting subsidies towards early investment exacerbates differences in early investment by parental education, while reducing differences in late investments. The wage gains most benefit the children of highly educated parents, with the difference between wages of children of college graduates and high school dropouts increasing by 17%. Comparing the two steady states, the intergenerational correlation in discounted adult lifetime earnings rises from 0.29 to 0.34, reflecting a sizeable reduction in intergenerational mobility.

4.3 Public Provision of Early Investment

Lastly, we consider the effects of increasing the amount of publicly provided (lump-sum) early investment, p_1. Conceptually, changes in p_1 and s_1 are quite different. While an increase in the marginal subsidy rate lowers the price of and encourages private investment for all families, this is not the case for an expansion of public lump-sum investments. Among families already investing heavily in their young children, an increase in p_1 largely crowds out private investment activity. It is equivalent to an income transfer for those initially investing more than the increase in p_1. By contrast, there is little scope for crowd-out among children who initially receive very little or no early private investments. Their total early investments increase one-for-one with increases in public investments.

We consider an increase in p_1 of \$880, equivalent in cost to the early and late subsidies studied earlier. On average, this increase crowds out \$344 of early private investment, or 39% of the added public investment. In the long-run, high school completion rates increase by 16%, and the fraction that attends some college (or more) increases by 20%. Because the policy mainly increases total early investment for those who invest very little to begin with, it has a small effect on college completion rates, 5%. Average wages increase by 2.8%, roughly one-third of the response to an increase in early subsidy rates.

It is noteworthy that increasing early public investments (p_1) and early subsidies (s_1) affect educational outcomes at opposite ends of the distribution. A modest increase in p_1 does not raise early investments enough to make college completion worthwhile for those who were investing little to begin with. By contrast, an increase in s_1 encourages those who were already making investments to invest more, pushing many of them across the college completion threshold. Yet, modest early investment subsidies are ineffective at raising high school completion rates, because most dropouts appear to be at a 'corner' solution during early childhood, wishing to invest less than is already publicly provided for free. Of course, these are precisely the children whose early investments increase one-for-one with increases in p_1.

5 Sensitivity Analysis

We conduct a comprehensive sensitivity analysis for our calibration, counterfactual, and policy simulations. We summarize our main findings after re-calibrating the model imposing three different values of b, which vary the extent of dynamic complementarity. We also re-calibrate our model using a 'full' family income measure that adjusts for the possibility that mothers may work part-time in order to spend time investing in their children. Detailed results are provided in the Online Appendix, where we also include similar sensitivity analyses to other parameter restrictions on the borrowing constraint parameter ($\gamma = 0.5$), effect of parental human capital on the probability of the child's ability ($\pi_2 = 0$), and the unmeasured cost of high school ($\zeta_1 = 0$).

Given the importance of dynamic complementarity as defined by b, we explore the sensitivity of our analysis to other plausible values (0.5, 0, and -0.5) consistent with the range implied by previous estimates. Recall that our baseline estimate is $b = 0.26$, so the case of $b = 0.5$ implies more substitutability, while the other two cases imply stronger complementarity with $b = 0$ reflecting a Cobb-Douglas technology. The estimates for most other parameters are largely unaffected by the assumed value of b, and the model fit is only slightly worse for the two closest cases to the baseline. The fit for stronger complementarity ($b = -0.5$) is notably worse, especially for the schooling distribution and moments related to the conditional wage distributions. Early investment levels vary somewhat across specifications (ranging from an average of \$1,296 for the Cobb-Douglas case to \$3,389 for $b = -0.5$); however, the levels of late investment (targeted by the calibration) are quite similar for all cases. The ratios of investment for children of college graduates relative to high school dropouts are quite similar to the baseline case, ranging from

4.8-6.6 for the early period and 3.9-4.6 for the later period. The implied shares of families up against their borrowing or transfer constraints (as well as the patterns of constraints by parental education) are also comparable to the baseline calibration. These results suggest that key features of the baseline economy we study are robust to other, reasonable, values of b.

We also consider our counterfactual and policy simulations under these different parameter sets. As one would expect, the late investment and wage effects of an unanticipated income/wealth transfer to parents with old children are closer to the effects of an anticipated transfer when investments are more substitutable ($b = 0.5$), while the opposite is true when dynamic complementarity is strong ($b = -0.5$). In the latter case, the effects of an unanticipated transfer on investments are negligible, while the effects of an anticipated transfer are similar to those of our baseline case. Our counterfactual simulations aimed at studying intergenerational mobility suggest a comparable role for the intergenerational transmission of ability and a similar or stronger role role for lifecycle borrowing constraints as compared to the baseline. Results are also similar across specifications for our policy simulations that relax borrowing constraints or subsidize investments: The effects of relaxing borrowing constraints at only one stage of development are modest, while the impacts of fully eliminating constraints are substantial. Subsidies for early investments always have greater effects on wages than late investment subsidies. Announcing late subsidies at early ages has greater effects on wages than announcing them late; however, the difference is modest for strong intertemporal substitutability ($b = 0.5$) and much greater for strong complementarity ($b = -0.5$). The main conclusions from our baseline calibration are largely unchanged for other reasonable degrees of dynamic complementarity.

Our baseline calibration, as well as the restricted versions just discussed, uses reported total parental earnings (mother's plus father's earnings) as the conditioning measures of family income in moment sets (iii)-(v); however, we have also calibrated the model using a 'full' family income measure that adjusts for the possibility that mothers may work part-time in order to spend time investing in their children. This 'full' income measure inflates earnings for mothers working less than 1,500 hours per year to its 1,500 hour equivalent by multiplying mother's earnings by 1,500 and dividing by reported annual hours. This implicitly assumes that mothers working fewer than 1,500 hours/year spend the balance of that time investing in their children. Because this measure does not move many families into different quartiles of the family income distribution, the moments are highly correlated with those used in our baseline analysis. Not surprisingly, this calibration produces fairly similar parameter estimates, with slightly higher θ, b, and ζ_2

values and a lower γ, and fits the adjusted data equally well. Early investment amounts are also comparable to those of our baseline, as are the fractions of constrained young parents. About 5 percent more old parents are constrained, with the largest discrepancy among the children of college graduates. This is consistent with the higher unmeasured costs of college and lower estimated γ. As in the baseline analysis, the intergenerational transfer constraint is empirically irrelevant. The slightly higher estimated $b = 0.32$ implies that anticipated vs. unanticipated income/wealth transfers and subsidies for late investment have more similar effects than in the baseline analysis. Still, we find that anticipated transfers would have more than twice the effect on post-school earnings compared to unanticipated transfers. The impacts of changes in early vs. late borrowing constraints and investment subsidies are comparable to those found for the baseline calibration. Our analysis of the role of intergenerational ability transmission and market frictions as determinants of intergenerational mobility also yields quite similar results to those of our baseline calibration. If anything, we find that borrowing constraints may play an even stronger role in explaining late investment gaps by parental income.

6 Conclusions

Our theoretical analysis of borrowing constraints and multi-period human capital investment establishes the complexity of the interaction between dynamic complementarity and constraints, especially when one constraint is relaxed in isolation. Borrowing constraints do not necessarily imply that investments will increase with income transfers or expansions in borrowing opportunities, especially in other periods. Relaxing one constraint can make others more binding, causing investment to decline rather than increase. When investments are sufficiently complementary over time, they will move together so policies that encourage investment in one period will tend to raise investments in other periods as well. These findings highlight the value of empirically grounded quantitative work.

We use a simulated method of moments strategy to calibrate our dynastic model of multi-period human capital investment to a wealth of intergenerational data on earnings, schooling, early investment measures, and family assets from the CNLSY. The estimated parameters for our CES human capital production function, especially the extent of dynamic complementarity between early and late investment, are broadly consistent with previous estimates in related frameworks. We also obtain new estimates of the intergenerational transmission of innate (unobserved) learning abilities, the degree of parental altruism, and a measure of credit market frictions. These may all be of independent interest for future researchers working with similar intergenerational models of human capital investment.

Our quantitative analysis demonstrates the importance of credit market frictions, more so than might be expected given the methodologies most researchers use to measure them. Despite the fact that relaxing any one period's borrowing constraint in isolation has very modest effects (consistent with much of the literature), we find that eliminating lifecycle constraints altogether has substantial impacts on investment and intergenerational mobility due to both dynamic complementarity and uncertainty. Families want to adjust investments in all periods together but may find this difficult when only a single period's constraint is relaxed. Furthermore, the investment decisions of many more individuals are distorted by the potential for binding future constraints than ever end up borrowing to their limits. We show that unanticipated changes in income for parents of college-age children (e.g. due to lottery winnings or job loss) have modest effects on their college-going behavior and future wages. However, if parents anticipate the future income change when their children are young, the impacts on college attendance are more than twice as large. Impacts on post-school earnings are more than six times as large due to the combined effects of higher early and late investments. This suggests that quasi-experimental estimates of wealth/income effects on educational attainment using 'exogenous' wealth/income shocks to the families of adolescent children substantially under-estimate the impacts of long-run differences in family income. As noted by Cunha and Heckman (2007), the impacts of family income differences on higher education decisions begin with investment choices made long before children reach high school.

While we identify strong distortions caused by lifecycle borrowing constraints, we find that very few families are constrained by parents' inability to 'take' from their children. Even if lifecycle borrowing constraints were completely eliminated, our simulations suggest that only about 5% of all parents would like to saddle their children with debt in order to improve their own lot.

The same incentives for intertemporal investment co-movements created by dynamic complementarity also have implications for human capital investment policy. Given the extent of dynamic complementarity we estimate (and estimated by others), policies enacted to encourage investment at one stage of development also encourage investment at other stages. Ignoring the early investment response to an increase in college subsidies under-estimates the impact on future wages by 60%. Thus, the long-run effects of many tuition subsidy policies are likely to be more than double what traditional empirical (structural or quasi-experimental) estimates suggest.

Still, we find that aggregate earnings and welfare levels increase by shifting marginal subsidies from the late to early investment stage. A fiscally equivalent increase in lump-sum public early investments produces smaller average benefits but has larger impacts on the bottom of the ability and education distribution.

Finally, we use our calibrated model to study the implications of intergenerational ability transmission and market frictions (i.e. lifecycle borrowing constraints, non-negative parental transfers, and uninsured risk) for intergenerational mobility. In our baseline calibration of the current economy, we find that differences in ability at birth can explain about one-quarter of the college attendance gap between high and low income families. Eliminating all lifecycle borrowing constraints further wipes out more than half of the remaining gap. We also consider moving from the current economy to one in which there are no intergenerational linkages in the transmission of learning abilities. The intergenerational correlation in late investments falls by almost half while the intergenerational correlation in lifetime earnings falls by one-third. While these changes are transformative, there is still considerable persistence in outcomes across generations. These exercises highlight the importance of both ability transmission and market frictions for intergenerational mobility.

Many simplifying assumptions have been made in order to make our intergenerational problem tractable. Future work should incorporate a richer structure that allows for fertility choices, marriage/divorce behavior, and labor supply decisions. More and shorter time periods would certainly enrich the nature of human capital production and allow for a more detailed analysis of other important lifecycle issues. While improvements along these lines should add credibility to any policy analysis, we have purposely focused on general lessons that should carry over to and guide future work in this area.

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Appendix A Data from the Children of the National Longitudinal Survey of Youth

We use data from the CNLSY, which follows the children born to all women in the NLSY79. The mothers in our sample are original NLSY79 respondents ages 14-22 in 1979 when the survey began. Our sample includes data collected up to 2010.

The data contains measures of family income every year from 1979 to 1994 and biennially thereafter. Our analysis uses the sum of reported earnings for the father and mother as the main measure of family income; however, we also consider an adjusted measure of earned 'full' income in Table 1 and (as part of our sensitivity analysis) in Section 5. This measure uses reported hours worked by mothers to adjust their earnings to a 1500 hour (30 hours per week) annual equivalent. Specifically, for all mothers working less than 1500 hours, we multiply reported earnings by 1500 and divide by reported hours. We then add this to father's earnings to get our measure of earned 'full' income. All income measures are deflated to 2008 values using the CPI-U.[68]

We discount combined family earnings back to age zero of the child using a 5% annual interest rate. Our measure of 'early' income averages family earnings over child ages zero to eleven, while our measure of 'late' income averages earnings over ages 12-23. These assumptions and age groups are used throughout.

We categorize individuals (mothers and children) with less than 12 years of completed schooling as high school dropouts, 12 years of schooling as high school graduates, 13-15 years of schooling as some college, and 16 or more years of schooling as college graduates. In Table 1, we refer to those with 13 or more years of completed schooling as having attended college. For children, if educational attainment is unavailable at age 21 (24), we use reported education at ages 22-24 (25-27). For mothers, we use educational attainment as of age 28 (or ages 29 and 30 if missing at earlier ages).

The CNLSY contains many potential measures of early investments. We use 8 measures from children ages 6-7 in calculating our early investment factor scores: (i) 10+ books in home, (ii) musical instrument in home, (iii) child taken to music/theater performance at least once in past year, (iv) child taken to a museum at least once in past year, (v) child gets special lessons or does extracurricular activities, (vi) family gets a daily newspaper, (vii) family encourages hobbies, and (viii) mother reads to the child 3+ times/week.

The CNLSY contains measures of many child and mother characteristics that may affect

[68] We impute missing earnings separately for mothers and fathers using individual-specific regressions of log earnings on an intercept, age and age-squared whenever at least 8 positive values are available and respondents are age 22 or older. Less than 10% of our final family earnings measures are imputed. Combined family earnings values of greater than $500,000 and less than $500 are set to missing.

educational attainment. In Panel B of Table 1, we include many of these variables as controls as described in the table notes.

Most of our analysis uses respondents from the random sample of the NLSY79 (or their children in the CNLSY). The only exceptions to this are our three sets of (early and late) investment and child wage moments conditional on parental income (i.e. moment sets (iii)-(v) as described in Section 3.3 and Appendix C). Because sample sizes for several conditioning sets are quite small using the random sample alone, we also include the black and hispanic oversamples as well. (Note that we use income distributions in the random sample in assigning families to their respective income quartiles.) This approach implicitly assumes that expected investments and early post-school wages are independent of race conditional on parental income and education (as well as own education in the case of wages).

Appendix B Theoretical Results

Propositions 1-4 extend the lifecycle analysis of Caucutt, Lochner and Park (2017), henceforth CLP, to the dynastic framework of Section 2. This appendix begins by showing how the dynastic problem of Section 2 can be mapped directly into the lifecycle problem of early and late human capital investment in CLP. While this is not necessary for proving Proposition 1, it helps link the two frameworks and allows us to apply key results in CLP to prove Propositions 2-4.

B.1 Mapping Dynastic Problem into Lifecycle Problem of CLP

In mapping the dynastic problem of Section 2 to the lifecycle problem studied in CLP, it is useful to define the following functions:

$$
\begin{aligned}
U_1(X) &= \max_{c_3} u(c_3) + \rho u(X - c_3) \\
U_2(X) &= \max_{c_4, c_5, c_6} u(c_4) + \beta u(c_5) + \beta^2 u(c_6) + \rho u(X + R^{-1} W_5(h) - c_4 - R^{-1} c_5 - R^{-2} c_6) \\
&\qquad\qquad s.t. \quad c_5 + R^{-1} c_6 - W_5(h) \geq -R L_4.
\end{aligned}
$$

The first function reflects total family utility when the child is young (with consumption allocated optimally across parent and child), while the second function reflects the sum of utility for the old child and the discounted remaining lifetime utility for the old parent when consumption is optimally allocated. The latter takes into account that the parent will earn $W_5(h)$ in post-parenthood and that borrowing for the old parent cannot exceed L_4. Importantly, both $U_1(\cdot)$ and $U_2(\cdot)$ are strictly increasing and strictly concave, because $u'(c) > 0$ and $u''(c) < 0$.

With these two functions, we can re-write the dynastic problem in Section 2.2 in terms of aggregated consumption amounts (C_1', C_2'), investments (i_1', i_2'), and assets (a_3', a_4):

$$V_3(a_3, h) = \max_{C_1', C_2', i_1', i_2', a_3', a_4} U_1(C_1') + \beta U_2(C_2') + \beta^2 \rho V_3(a_3', h')$$

subject to the human capital production function (equation (7))

$$\begin{aligned} C_1' &= Ra_3 + W_3(h) + y_3 - a_4 - i_1' && (22) \\ C_2' &= Ra_4 + W_4(h) + y_4 + W_2 - a_3' - i_2', && (23) \\ a_4 &\geq -L_3, \\ a_3' &\geq -L_2. \end{aligned}$$

This problem is nearly identical to the lifecycle problem studied in CLP. The most important difference is that $U_1(\cdot) \neq U_2(\cdot)$ here, whereas these 'utility' functions are the same in CLP. Fortunately, none of the results in CLP related to Propositions 2-4 of this paper require that $U_1(\cdot) = U_2(\cdot)$. Instead, all related proofs only require that both functions be strictly increasing and strictly concave (as is the case). A second distinction between this problem and that of CLP is that the borrowing constraint during early childhood applies to parents here rather than the child himself as in CLP. The implications of this constraint are exactly the same, however. A final difference is that the 'continuation value', $V_3(a_3', h')$, is the dynastic value function for the child here, while it more simply reflects the remaining lifetime continuation value for the individual in the lifecycle problem of CLP. Again, this distinction is irrelevant for Propositions 2-4 provided $V_3(a_3', h')$ is strictly increasing and strictly concave in each argument, which is proven in the Online Appendix.

Finally, we note that the old parent's constraint alone has no effect on child investment behavior if no other constraint binds for the parent-child pair (constraints may bind for future generations). When other constraints bind, the constraint on old parents may affect investment allocations, but it does not affect the sign of any investment responses to marginal changes in income transfers or other borrowing limits for the parent-child pair. As such, the old parent's borrowing constraint has no bearing on the results characterized in Propositions 2-4.

B.2 Proofs for Propositions 1-4

Proofs for all four propositions draw on those for analogous results of CLP, extending them from a lifecycle to a dynastic setting. In all cases, the borrowing constraint on young parents in the dynastic model plays the role of the borrowing constraint during early childhood in the lifecycle model of CLP. Proofs for Propositions 2-4 rely on the mapping between the dynastic and lifecycle frameworks established in B.1, where the proofs need to be trivially modified (not shown) to account for $U_1(\cdot) \neq U_2(\cdot)$ (with both strictly increasing and strictly concave functions).

Importantly, Propositions 2-4 apply only to changes in transfers or borrowing limits for a single generation and, therefore, do not affect the continuation value functions for the children when they grow up. Furthermore, they do not rely on any assumptions regarding borrowing constraints for future generations of the dynasty.

Proposition 1

Using the Envelope Theorem to substitute in for the marginal value of human capital, it is straightforward to show that equation (9) can be written as:

$$u'(c_1') = \beta^2\theta' f_1(i_1', i_2')w\left[\sum_{j=3}^{T}\beta^{j-3}\Gamma_j u'(c_j')\right] \leq \beta^2\theta' f_1(i_1', i_2')w\left[\sum_{j=5}^{T}\beta^{j-3}\Gamma_j(\beta R)^{1-j}u'(c_1')\right],$$

where the inequality follows from $u'(c_j') \geq \beta R u'(c_{j+1}')$ for all $j = 1, ..., 4$. This implies that $\theta\chi_3 f_1(i_1, i_2) \geq R^2$, with strict inequality if and only if any borrowing constraint for the child or for his young parent binds. Similarly, one can show that $\theta\chi_3 f_2(i_1, i_2) \geq R$, with strict inequality if and only if any borrowing constraint for the child binds from old childhood onwards.[69] As demonstrated in CLP, these two investment first order conditions, combined with Assumption 1, imply the results of Proposition 1. See Proposition 8 and its proof in CLP for details.

Proposition 2

The mapping from our dynastic framework to the lifecycle framework of CLP in B.1 allows us to apply the results of Proposition 9 in CLP where the constraint during early childhood refers to the constraint on young parents in our dynastic setting. We note that part II (young parent is borrowing constrained but the child is not at older ages) makes no assumptions about borrowing constraints faced by future generations. As long as the child is unconstrained in adulthood, $V_3(a_3', h')$ can be written as a strictly concave function of total physical and human wealth, $Ra_3' + \chi_3 h'$, as assumed in the proof of part II.

Proposition 3

Proposition 3 is analogous to Proposition 10 in CLP. Here, we also impose that the child is unconstrained during adulthood, so $V_3(a_3', h')$ can be written as a strictly concave function of $Ra_3' + \chi_3 h'$. This ensures that $\frac{\partial^2 V_3}{\partial a_3' \partial h'} < 0$ as required by the proof. While this condition seems likely to hold more generally (even when children are constrained during adulthood), we have not shown this.

[69] Notice, the old parent's borrowing constraint, by itself, does not imply that $u'(c_2') > \beta R u'(c_3')$; if this is the only binding constraint over the child's life, investment will be at the unconstrained optimal amount as determined by $\theta\chi_3 f_1(i_1, i_2) = R^2$ and $\theta\chi_3 f_2(i_1, i_2) = R$.

Proposition 4

Proposition 4 is analogous to Proposition 11 in CLP. In part (i), the statement that no other borrowing constraint binds for the child again allows for the possibility that borrowing constraints can bind for future generations of the dynasty.

Appendix C Details on Identification and Calibration

This appendix provides details on identification of key model parameters, the use of factor analysis in estimating early investments, and the calibration procedure.

C.1 Details on Identification

Here, we provide a detailed discussion of identification of the human capital production technology, ability distribution, earnings growth, and the distribution of earnings shocks using lifecycle data on investments and earnings for a single generation. We assume throughout that public investment amounts (p_1 and p_2) and subsidy functions ($S_1(\cdot)$ and $S_2(\cdot)$) are known and that late investment levels i_2 are perfectly observed.

Given our assumptions, adult human capital for individual n, h_n, is given by equation (20) in the text. Earnings for individual n in period j are given by $W_{jn} = w\Gamma_j[h_n + \epsilon_{3n}]$, where we normalize $w = \Gamma_3 = 1$, and $\epsilon_{jn} \sim logN(m, s^2)$ are iid over time and across individuals.

First, we identify $\Gamma_4 = E[W_{4n}]/E[W_{3n}]$ and $\Gamma_5 = \Gamma_4 E[W_{5n}]/E[W_{4n}]$ from growth in average earnings over the lifecycle. We then identify $\sigma_\epsilon^2 = Var(W_{3n}) - \Gamma_4^{-1} Cov(W_{3n}, W_{4n})$ with panel data on earnings over the first two periods of adulthood.

Next, consider identification of the production technology and mean of the earnings shock. While we assume that late investment is directly observed, we only observe J (de-meaned) noisy measures of early investment: $Z_{nj} = \alpha_j \Phi_n + v_{nj}$ for $j = 1, ..., J$, where we normalize $\alpha_1 = 1$, $E[\Phi_n] = 0$, and v_{nj} are independent across individuals and measures (i.e. $v_{nj} \perp\!\!\!\perp v_{n'j}$ and $v_{nj} \perp\!\!\!\perp v_{nj'}$ for all $n \neq n'$, $j \neq j'$). We also assume that the v_{nj} measurement errors are independent of all other choice and outcome variables (e.g. i_{1n}, i_{2n}, W_{3n}). In the language of factor analysis, Φ_n reflects the unobserved factor generating correlation across measures, α_j reflects each factor loading, and v_{nj} are the uniquenesses.

Because the factors Φ_n have no meaningful location or scale, we assume that $\Phi_n = \phi(i_{1n})$ maps actual early investments to factor scores, where the function $\phi(\cdot)$ has K_ϕ unknown parameters. We assume that $\phi'(i_1) > 0$ so that higher factor scores reflect higher investment, and we can write $i_{1n} = \phi^{-1}(\Phi_n)$.

From data on $(Z_{n1}, Z_{n2}, .., Z_{nJ}, i_{2n}, W_{3n})$ for $J \geq 3$ early investment measures, the conditional density function $G_{\Phi|i_2,W_3}(\Phi_n|i_{2n}, W_{3n})$ and density for measurement errors, $F_{v_j}(\cdot)$, can be identified using standard results in factor analysis conditioning on (i_{2n}, W_{3n}). From this conditional density, we can form the joint distribution of (Φ_n, i_{2n}, W_{3n}), $G_{\Phi,i_2,W_3}(\cdot,\cdot,\cdot)$, now proceeding as though we observe this distribution directly. See Cunha, Heckman and Schennach (2010) for a similar line of argument.

While the model has implications for investment behavior (by parental income and education) that can be useful in identifying parameters of the human production function (e.g. the complementarity parameter b as discussed in Proposition 2), we focus here on identification based only on the conditional density $G_{W_3|\Phi,i_2}(\cdot|\cdot,\cdot)$. This density provides the information needed to identify all model parameters related to human capital production and the mean of earnings shocks. To see this, notice that any conditional earnings moment of order-l can be written as

$$\begin{aligned}
E(W_{3n}^l|\Phi_n = \bar{\Phi}, i_{2n} = \bar{i}_2) &= E(\theta_n^l|\Phi_n = \bar{\Phi}, i_{2n} = \bar{i}_2)\left[f\left(\phi^{-1}(\Phi), i_2\right)\right]^l + \mu_\epsilon^l \\
&= \left[\theta_1^l + P_2\left(\bar{\Phi}, \bar{i}_2\right)\left(\theta_2^l - \theta_1^l\right)\right]\left[a\left(p_1 + \phi^{-1}(\bar{\Phi})\right)^b + (1-a)(p_2 + \bar{i}_2)^b\right]^{\frac{ld}{b}} + \mu_\epsilon^l,
\end{aligned}$$

where $\mu_\epsilon^l \equiv E(\epsilon_3^l)$ and $P_2(\bar{\Phi}, \bar{i}_2) \equiv Pr(\theta_n = \theta_2|\Phi_n = \bar{\Phi}, i_{2n} = \bar{i}_2)$ is the conditional probability that an individual is of high ability given their observed early investment factor score and late investment. If we treat $P_2(\bar{\Phi}, \bar{i}_2)$ as unknown, this equation contains $7 + K_\phi$ unknowns (p_1, p_2, $\bar{\Phi}$, and $\bar{i}_2$ are known) for any given moment order l. Because, we can write μ_ϵ^l for all $l > 1$ as known functions of $(\mu_\epsilon^1, \sigma_\epsilon^2)$ (given log normality of ϵ_3) and σ_ϵ^2 is already known, no new unknowns are introduced if we consider additional higher order moments for any known investment pair $(\bar{\Phi}, \bar{i}_2)$. Consequently, we always have only $7 + K_\phi$ unknowns regardless of the number of moments we simultaneously consider.

Importantly, the distribution of ϵ_3 does not depend on (Φ, i_2), so considering the set of all order moments for an additional investment pair (e.g. $(\bar{\Phi}', \bar{i}_2')$) adds only one new unknown parameter (i.e. $P_2(\bar{\Phi}', \bar{i}_2')$). Therefore, the first L order moments for any M pairs of observed (Φ, i_2) contains a total of $6 + K_\phi + M$ parameters to be identified and a total of $L \times M$ equations. Using the first L order moments for each pair (Φ, i_2) requires $M \geq (6 + K_\phi)/(L-1)$ pairs to identify the unknown abilities (θ_1, θ_2), production technology parameters (a, b, d), K_ϕ parameters determining $\phi(\cdot)$, μ_ϵ^1, and probabilities $P_2(\bar{\Phi}, \bar{i}_2), P_2(\bar{\Phi}', \bar{i}_2')$, etc.[70] For example, with $\phi(\cdot)$ a linear function ($K_\phi = 2$), using only first- and second-order moments requires 8 pairs of (Φ, i_2), while using first- through third-order moments requires 4 pairs.[71]

In addition to conditional expectations, the minimum of W_3 conditional on (Φ, i_2) also provides valuable information about abilities and the human capital production function given log

[70]Knowledge of μ_ϵ^1 and σ_ϵ^2 together directly identifies (m, s) of the log normal distribution for earnings shocks.

[71]One can use any order moment from all other pairs of observed (Φ, i_2) to identify all remaining $P_2(\Phi, i_2)$, then average over all values to obtain the unconditional probability of a high type, $Pr(\theta_n = \theta_2) = \int P_2(\Phi, i_2) dG_{\Phi,i_2}(\Phi, i_2)$.

normality of ϵ_3. Notice that

$$\min\{W_3|\Phi = \bar{\Phi}, i_2 = \bar{i}_2\} = \theta_1 \left[a(p_1 + \phi^{-1}(\bar{\Phi}))^b + (1-a)(p_2 + \bar{i}_2)^b\right]^{d/b} \quad \text{for } P_2(\bar{\Phi}, \bar{i}_2) < 1. \quad (24)$$

If a subset of observed investment pairs (Φ, i_2) is known to contain some low ability individuals (e.g. very low investment outcomes), then we could use the lowest earnings levels for those investment pairs to help identify θ_1, (a, b, d), and $\phi(\cdot)$ ($4 + K_\phi$ parameters). Fortunately, it is possible to test whether this is the case, because

$$Var(W_{3n}|\Phi_n = \bar{\Phi}, i_{2n} = \bar{i}_2) = P_2(\bar{\Phi}, \bar{i}_2)[1 - P_2(\bar{\Phi}, \bar{i}_2)](\theta_2 - \theta_1)^2 \left[\tilde{f}(\bar{\Phi}, \bar{i}_2)\right]^2 + \sigma_\epsilon^2,$$

which equals σ_ϵ^2 (already known from above) if and only if $P_2(\bar{\Phi}, \bar{i}_2) \in \{0, 1\}$. Thus, moments based on equation (24) can be used for all (Φ, i_2) satisfying $Var(W_3|\Phi, i_2) > \sigma_\epsilon^2$. They might also be used for lower investment pairs not satisfying this inequality, because these should only be observed for low ability, θ_1, types. For very high investment pairs satisfying $Var(W_3|\Phi, i_2) = \sigma_\epsilon^2$, we might reasonably assume that only high ability types are observed, enabling an analogous approach (using the conditional minimum earnings levels) to help identify θ_2 and the skill production parameters.

Finally, identification of $\phi(\cdot)$ together with identification of $F_{v_j}(\cdot)$ (discussed earlier) implies that the conditional density $G_{i_1|i_2,W_3}(\cdot|\cdot,\cdot)$ and, therefore, the joint density, $G_{i_1,i_2,W_3}(\cdot,\cdot,\cdot)$, is identified given independence of the measurement errors v_{nj} with each other and with (i_1, i_2, W_3).

C.2 Factor Analysis using Early Investment Measures

We do not observe early investments in our data, but instead observe J noisy measures of i_1 for each individual. We now show how we form conditional moments based on these noisy measures that are compared with conditional expectations of i_1 produced by simulating our model.

We first de-mean all measures of investment to obtain Z_{nj}. Based on measurement equation (21), we use standard techniques for linear factor models to estimate α_j and $\sigma_j^2 = Var(v_{nj})$ for all $j = 1, ..., J$ (normalizing $\alpha_1 = 1$). We then use the Thomson (1935) method to estimate factor scores $\hat{\Phi}_n$ for each individual such that $\hat{\Phi}_n = \sum_{j=1}^{J} w_j Z_{nj} = \Phi_n + \sum_{j=1}^{J} w_j v_{nj}$ and $\sum_{j=1}^{J} w_j = 1$.

Because Φ_n has no meaningful location or scale, we assume that $\Phi_n = \phi(i_{1n})$, where $\phi'(i_1) > 0$ (over the domain of i_1) so that higher factor scores reflect higher investment. Notice that $E[v_{nj}|X_n] = 0$ implies that $E[\Phi_n|X_n = x] = E[\phi(i_{1n})|X_n = x]$, where X_n reflects conditioning variables (parental education and income in our analysis). A first order Taylor approximation of the unknown function $\phi(i_1)$ around $E(i_1|X)$ yields $\phi(i_1) \approx \phi(E[i_1|X]) + \phi'(E[i_1|X])(i_1 - E[i_1|X])$. Assuming $\phi(i_1) = \phi_0 + \phi_1 i_1 + \phi_2 i_2^2$, our approximation yields the following moment conditions:

$$E[\hat{\Phi}_n|X_n = x] - \left\{\phi_0 + \phi_1 E[i_{1n}|X_n = x] + \phi_2 (E[i_{1n}|X_n = x])^2\right\} = 0 \quad (25)$$

used in calibration. In practice, we use a (weighted) regression of $E[\hat{\Phi}_n|X_n = x]$ (from data) on a constant, $E[i_{1n}|X_n = x]$, and $E[i_{1n}|X_n = x]^2$ (from the simulated model) where different values of x reflect different levels of maternal education, early and late family income.[72] With more than three different X_n types (we use 31 conditioning groups), these moments provide additional restrictions that aid in identification of structural parameters in our model. To see this, note that monotonicity of $\phi(\cdot)$ means that the ranking of $E[i_{1,n}|X_n = x]$ by x produced by the model should be the same as the ranking of $E[\hat{\Phi}_n^j|X_n = x]$ by x. Restricting $\phi(\cdot)$ to be a quadratic function further imposes conditions on relative differences in expected investments by x given relative differences in the factor scores by x. We calibrate (ϕ_0, ϕ_1, ϕ_2) along with all other structural parameters.

C.3 Calibration using Simulated Method of Moments

We calibrate parameters of the earnings shock distribution (m, s), the human capital production function (a, b, c), unobserved late investment costs (ζ_1, ζ_2), parental altruism towards children (ρ), the ability distribution and its intergenerational transmission $(\theta_1, \theta_2, \pi_0, \pi_1, \pi_2)$, and the debt constraint parameter γ by simulating the model in steady state to best fit a number of moments in the NLSY79 and CNLSY data. In particular, we fit moments related to (i) the education distribution, (ii) the distribution of annual earnings for men ages 24-35 by educational attainment, (iii) measures of early child investments conditional on early and late parental income and maternal schooling, (iv) child schooling attainment levels conditional on early and late parental income and maternal schooling, (v) child wages at ages 24-35 conditional on their own educational attainment, maternal schooling, and early parental income levels, and (vi) the fraction of families with older children that have zero or negative net worth.

As discussed in the paper, when classifying individuals by education (either mother or child), we categorize them by highest grade completed (completing less than 12 years of school, 12 years of school, 13-15 years, or 16 or more years).

We minimize the weighted sum of squared errors between the simulated model moments and the corresponding sample means in the data, where the weights are the inverse of the sample variance for each sample mean. In simulating the moments with our model, we solve for the steady state given any candidate set of parameter values, then compute the desired moments for comparison with the data. We briefly discuss each of the six sets of moments we fit.

First, we fit the model's steady state education probabilities (corresponding to values of i_2 in the model) using the random sample of all mothers in the NLSY79 (sample size of 2,478). Because

[72] This is equivalent to minimizing the (weighted) sum of squared errors for these moments, consistent with our strategy for all other moments as discussed below.

the education probabilities must sum to one across all four education groups we consider, we only use the proportion of high school dropouts, some college, and college dropouts (leaving out high school graduates) with weights of 16817.81, 14078.27, and 15792.09, respectively. Table 2 in the paper reports these moments in the data and our calibrated steady state. The mean weighted squared error (MWSE) for this subset of moments is 0.00013.[73]

Second, we fit key features on the male earnings distribution using data from the random sample of men in the NLSY79. Specifically, we (a) fit the model's steady state earnings distribution (mean and variance) conditional on educational attainment (i.e. $E(W_3|i_2)$ and $Var(W_3|i_2)$) for men ages 24-35, and (b) the covariance in male earnings between ages 24-35 and 36-47 (i.e. $Cov(W_3, W_4)$). In computing W_{3n} (W_{4n}) for each individual, we first discount all earnings over ages 24-35 (36-47) to age 30 (42) using a discount rate of $r = 0.05$. We then calculate the average annual discounted earnings (in $10,000s) over the available years for each person. Our total sample of men used in computing moments with only W_{3n} is 2,969, while our sample of men used in computing $Cov(W_3, W_4)$ is 2,372. Table C1 reports the conditional means and variances for W_3 and $Cov(W_3, W_4)$ along with their corresponding weights. The MWSE for this subset of moments is 0.65.

Third, we use data on all children ages 6-7 in the CNLSY to fit early investment factor scores conditional on maternal education (reflecting i_2) and early and late family income (W_3 and W_4, respectively). Our conditioning on family income is based on whether parental income (maternal plus paternal earnings) is in quartile 1, quartile 2, or above the median.[74] We use the following 8 early investment measures, Z_{nj}, from the CNLSY: (i) 10+ books in home, (ii) musical instrument in home, (iii) child taken to music/theater performance at least once in past year, (iv) child taken to a museum at least once in past year, (v) child gets special lessons or does extracurricular activities, (vi) family gets a daily newspaper, (vii) family encourages hobbies, and (viii) mother reads to the child 3+ times/week.

Using all available children ages 6-7 born to the random sample of mothers, we use principal factor analysis and the Thomson (1935) regression method to compute predicted factor scores, $\hat{\Phi}_n$ for each individual in the CNLSY sample (including over-samples).[75] For interpretability, we

[73]The MWSE for a subset of moments is calculated as the sum of weighted squared errors for those moments divided by the sum of the weights for the same moments.

[74]In calculating (period-specific) empirical income cutoffs for the first quartile and median, we use the distribution of average family income over maternal ages 24-35 and 36-47 (discounted at annual rate $r = 0.05$ to ages 30 and 42) based on all mothers in the random sample of the NLSY79. We use family income averaged over child ages 0-11 and 12-23 for the CNLSY to categorize children by parental income in periods 3 and 4.

[75]In practice, we obtain estimated factor scores that are very strongly correlated using either the Thomson (1935) or Bartlett (1937) estimators (i.e. correlation greater than 0.95). Scoring coefficients using the regression

re-scale these factor scores by subtracting off the mean and dividing by the standard deviation of scores based on the random sample of children. Thus, factor scores are in standard deviation units. Altogether, we calculate factor scores for 4,511 children. Table C2 reports estimated factor loadings α_j, uniqueness variances σ_j^2, and the factor scoring coefficients/weights w_j (scaled to sum to one). Table C3 reports the conditional moments $E[\hat{\Phi}_n|X_n = x]$ in the data and as predicted from the model, along with the weights used for each moment.[76] The MWSE for this subset of moments is 0.048.

Fourth, we use data on all CNLSY children's educational attainment conditional on maternal education and early and late family income, where the conditioning groups are the same as those used in the early investment factor score moments just discussed. Our moments include conditional probabilities of high school dropout, some college, and college graduate.[77] To determine child education probabilities, we use highest grade completed at age 21 to assign high school dropout status, and age 24 to assign college attendance and completion status. Table C4 reports sample sizes, probabilities, and weights from the CNLSY data and the simulated education probabilities obtained from our baseline calibration. The MWSE for this subset of moments is 0.0046.

Fifth, we fit period 3 average wages of all CNLSY children conditional on their own education, parental education, and parental income when they were young. We classify parental income and education as above and use average (discounted) weekly wages over ages 24-35 for all children in the CNLSY.[78] Because we consider weekly wages for children (rather than annual income) to better reflect human capital levels at younger ages, we re-scale average wage measures by dividing by the average wage for the full random sample. We perform the same re-scaling with the model counterpart, using $W_3'/E(W_3')$. Table C5 reports re-scaled average weekly wages, sample sizes, and weights from the CNLSY along with the simulated re-scaled period 3 earnings from our baseline calibration. The MWSE for this subset of moments is 0.028.

Finally, we fit the fraction of older parents with zero or negative net wealth. In particular, we match the fraction of parents in the CNLSY (based on the random sample) who reported zero or negative net worth when the child was ages 17-19. When more than one observation are available over these ages, we use the average value (with each observation discounted to child age

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method do not necessarily sum to one across all measures, so we re-scale them to sum to one, creating w_j.

[76]We estimate $\phi_0 = -1.07$, $\phi_1 = 0.00085$, and $\phi_2 = 0.0000001$.

[77]We do not include moments for the probability a child is a high school graduate, because this is simply one minus the sum of the other three probabilities we consider.

[78]We drop observations with weekly wages less than \$40 or greater than \$2,500. To calculate more precise average wage measures for high school dropouts and graduates, we also include weekly wage measures at ages 22-23. All wage measures are discounted to age 30 using $r = 0.05$ before taking individual averages.

18). Based on the sample of 3,056 families, this share is 16.7%, while our baseline calibration yields a 22% share of old parents with zero or negative wealth (i.e. $a_4 \leq 0$). This yields a squared error for this moment of 0.034.[79]

[79]The weight placed on this moment for calibration is 22,012.41.

Table 1: Random Sample: Effects of Early and Late Income (in $10,000s PDV as of birth year) on Child Educational Attainment

	Earned Income				Earned “Full” Income			
	Sample Size (1)	Early Income (2)	Later Income (3)	Equal Effects (p-value) (4)	Sample Size (5)	Early Income (6)	Later Income (7)	Equal Effects (p-value) (8)
A. Controls Only for Maternal Education								
HS Dropout	2,273	**-0.026**	-0.009	0.063	1,894	**-0.023**	-0.011	0.205
(Ages 21-24)		(0.004)	(0.006)			(0.005)	(0.006)	
Attended Any College	1,586	**0.046**	0.015	0.037	1,336	**0.045**	0.014	0.044
(Ages 24-27)		(0.007)	(0.009)			(0.008)	(0.010)	
Graduated College	1,586	**0.028**	**0.024**	0.765	1,336	**0.033**	**0.025**	0.500
(Ages 24-27)		(0.006)	(0.007)			(0.059)	(0.075)	
B. Control for Maternal Education and Child/Family Background								
HS Dropout	2,190	**-0.023**	-0.010	0.173	1,835	**-0.022**	-0.012	0.295
(Ages 21-24)		(0.005)	(0.006)			(0.005)	(0.006)	
Attended Any College	1,524	**0.040**	0.011	0.051	1,291	**0.042**	0.008	0.035
(Ages 24-27)		(0.008)	(0.009)			(0.008)	(0.010)	
Graduated College	1,524	**0.025**	**0.021**	0.778	1,291	**0.032**	**0.021**	0.361
(Ages 24-27)		(0.006)	(0.007)			(0.006)	(0.008)	

Notes: Coefficient estimates in bold typeface are statistically significant at 0.05 level. Estimates reported in Panel A control only for maternal education, while those in Panel B also control for important child characteristics (year of birth, race/ethnicity, gender), mother characteristics (educational attainment, whether she was a teenager when the child was born, living in an intact family at age 14, foreign-born, and Armed Forces Qualifying Test scores), and the average number of children in the household over child ages 0-6. Specifications in columns (1)-(4) use total reported parental earnings to measure family income, while those in columns (5)-(8) use an adjusted ‘full’ earnings measure that inflates earnings for mothers working less than 1,500 per year to its 1,500 hour equivalent. Early income reflects average discounted family income over child ages 0-11; late income reflects average discounted income over ages 12-23. A discount rate of 5% is used to discount income to age 0.

Table 2: Calibrated Education Distribution

Education	NLSY Data	Model
HS Graduate or More	.82	.83
Some College or More	.42	.44
College Graduate	.19	.21

Table 3: Average Early Investment Factor Scores and Educational Attainment by Parental Education (Baseline)

	Model				NLSY Data			
Parental Education	Early Invest. Score	HS Grad. or More	Some College or More	College Grad.	Early Invest. Score	HS Grad. or More	Some College or More	College Grad.
HS Dropout	-0.49	0.64	0.20	0.08	-0.92	0.60	0.24	0.05
HS Graduate	-0.40	0.81	0.27	0.08	-0.33	0.77	0.44	0.14
Some College	0.11	0.90	0.57	0.15	0.02	0.84	0.52	0.20
College Graduate	0.67	0.94	0.82	0.63	0.57	0.93	0.80	0.46

Table 4: Average Early Investment Factor Scores and Educational Attainment by Parental Income (Baseline)

		Model				NLSY Data			
Early Income Quartile	Late Income Quartile	Early Invest. Score	HS Grad. or More	Some College or More	College Grad.	Early Invest. Score	HS Grad. or More	Some College or More	College Grad.
1	Any	-0.56	0.73	0.18	0.06	-0.71	0.64	0.30	0.07
2	Any	-0.43	0.81	0.28	0.07	-0.30	0.79	0.43	0.16
3 or 4	Any	0.36	0.89	0.65	0.37	0.28	0.89	0.64	0.29
Any	1	-0.36	0.71	0.24	0.09	-0.69	0.65	0.31	0.07
Any	2	-0.24	0.81	0.35	0.11	-0.32	0.77	0.41	0.14
Any	3 or 4	0.16	0.90	0.59	0.33	0.27	0.87	0.60	0.28
1	1	-0.52	0.66	0.18	0.07	-0.76	0.62	0.29	0.06
2	1	-0.46	0.72	0.20	0.07	-0.46	0.72	0.35	0.10
3 or 4	1	0.00	0.78	0.38	0.15	-0.06	0.90	0.54	0.08
1	2	-0.60	0.73	0.17	0.06	-0.56	0.68	0.31	0.08
2	2	-0.44	0.80	0.26	0.06	-0.35	0.80	0.44	0.17
3 or 4	2	0.16	0.86	0.55	0.17	-0.04	0.84	0.54	0.17
1	3 or 4	-0.59	0.82	0.20	0.06	-0.41	0.69	0.36	0.14
2	3 or 4	-0.39	0.88	0.35	0.07	-0.10	0.81	0.48	0.18
3 or 4	3 or 4	0.49	0.92	0.75	0.47	0.37	0.90	0.68	0.34

Table 5: Calibrated Parameter Values

Parameter	Value
a	0.58
b	0.26
d	0.82
θ_1	4.85
θ_2	12.03
π_0	-0.88
π_1	0.15
π_2	0.000019
ζ_1	47.49
ζ_2	760.73
m	9.90
s	0.71
ρ	0.86
γ	0.22

Table 6: Average Baseline Investment Amounts by Parental Education

Parental Education	i_1	i_2	$i_2 - S_2(i_2)$	$i_2 + \zeta(i_2) - S_2(i_2)$
All Levels	1,888	8,744	4,757	5,629
HS Dropout	770	4,351	2,262	2,671
HS Graduate	907	5,217	2,691	3,212
Some College	1,857	8,739	4,713	5,716
College Graduate	4,600	18,687	10,563	12,304

Table 7: Fraction Borrowing and Transfer Constrained

	Fraction of Young Parents Constrained	Fraction of Old Parents Constrained	Fraction of Parents Transfer Constrained
All Levels	0.12	0.14	0.00
HS Dropout	0.13	0.06	0.01
HS Graduate	0.20	0.17	0.00
Some College	0.06	0.17	0.00
College Graduate	0.01	0.14	0.00

Table 8: Short-Run Effects (% Change) of One-Time Income/Wealth Transfers

Transfer Policy	Avg. i_1	Avg. i_2	HS+	Some Coll+	Coll Grad	Avg. W_3
\$10,000 unanticipated transfer to old parents	0.0	1.4	1.2	3.0	0.0	0.2
\$10,000 anticipated transfer to old parents	8.0	6.2	0.5	7.2	8.5	1.3
\$10,000/R transfer to young parents	9.0	7.0	0.9	8.0	9.6	1.4

Table 9: Decomposition of Investment Gaps between Parental Income Quartiles 1 and 4

	Investment Gaps			% Change Relative to Baseline		
	Avg. i_1	Avg. i_2	Some Coll+	Avg. i_1	Avg. i_2	Some Coll+
Baseline:						
Unconditional	3,057	7,743	.38			
Conditional on parent ability	2,940	6,938	.34	-3.8	-10.4	-9.9
Conditional on child ability	2,615	5,924	.28	-14.5	-23.5	-26.6
Relax all borrowing limits:						
Unconditional	3,555	8,174	.33	16.3	5.6	-14.1
Conditional on child ability	2,480	3,757	.12	-18.9	-51.5	-67.6

Notes: Income quartiles are based on young parent earnings for analysis of early investments ('Avg. i_1') and for old parent earnings for analysis of late investments ('Avg. i_2' and 'Some Coll +'). For cases under 'Relax all borrowing limits', we set $\gamma = 0.99$ and solve for the corresponding steady state.

Table 10: Intergenerational Ability and Investment Transmission

	Baseline	No effect of parental h on child θ'	No correlation between parent and child θ'	Perfect correlation between parent and child θ'
Avg. i_1 gap by parental education (college grad. - HS dropout parents)	3,829	3,468	2,525	5,385
Avg. i_2 gap by parental education (college grad. - HS dropout parents)	14,336	13,080	9,092	21,680
Intergen. corr. in θ	0.31	0.31	0.00	1.00
Intergen. corr. in i_2	0.52	0.46	0.29	0.85
Intergen. corr. in h	0.50	0.46	0.28	0.87
Intergen. corr. in lifetime earnings	0.29	0.26	0.19	0.44

Notes: All results are based on steady state simulations. For 'No effect of parental h on child θ', we set $\pi_2 = 0$ and adjust π_0 and π_1 to keep $Pr(\theta'|\theta)$ fixed at the baseline steady state probabilities. 'No correlation between parent and child θ'' sets $Pr(\theta'|\theta) = Pr(\theta')$ based on the unconditional cross-sectional probability in the baseline steady state. 'Perfect correlation between parent and child θ'' sets $\theta' = \theta$ and sets the fraction of each ability type equal to the unconditional cross-sectional probability in the baseline steady state. Lifetime earnings reflect the discounted present value of adult earnings: $W_3 + R^{-1}W_4 + R^{-2}W_5$.

Table 11: Effects of Increasing Young Parent's Borrowing Limit by $2,500

Parental Education	Short-Run Effects (% Change) Avg. i_1	Avg. i_2	HS+	Some Coll+	Avg. W_3	Long-Run Effects (% Change) Avg. i_1	Avg. i_2	HS+	Some Coll+	Avg. W_3
All Levels	2.6	1.9	0.5	4.8	0.4	-0.6	-0.6	0.1	1.5	-0.1
HS Dropout	3.2	3.2	3.9	5.4	0.3	1.4	1.7	3.1	2.3	0.1
HS Graduate	5.8	3.2	0.1	8.2	0.5	-0.8	-1.1	-0.6	0.5	-0.1
Some College	4.7	3.7	-0.4	7.9	0.8	-0.8	-0.5	-0.8	3.1	-0.1
College Graduate	0.5	0.1	0.0	0.4	0.1	0.5	0.1	0.0	0.4	0.1

Table 12: Effects of Increasing Old Parent's Borrowing Limit by $2,500

Parental Education	Short-Run Effects (% Change) Avg. i_1	Avg. i_2	HS+	Some Coll+	Avg. W_3	Long-Run Effects (% Change) Avg. i_1	Avg. i_2	HS+	Some Coll+	Avg. W_3
All Levels	10.9	9.6	2.2	5.0	1.8	11.0	9.8	2.5	5.3	1.8
HS Dropout	13.2	9.6	4.1	4.2	1.0	23.0	16.2	4.7	9.1	1.7
HS Graduate	13.4	10.0	3.2	8.4	1.2	8.4	6.5	2.2	3.4	0.5
Some College	18.9	15.8	2.0	9.1	3.0	5.2	5.5	1.5	1.0	0.9
College Graduate	6.3	6.1	-0.3	0.1	1.9	0.0	0.7	-0.3	-1.0	0.1

Table 13: Effects of Fully Relaxing All Borrowing Limits

Parental Education	Short-Run Effects (% Change) Avg. i_1	Avg. i_2	HS+	Some Coll+	Avg. W_3	Long-Run Effects (% Change) Avg. i_1	Avg. i_2	HS+	Some Coll+	Avg. W_3
All Levels	72.5	63.2	12.5	31.0	11.7	111.1	89.6	14.4	48.8	17.7
HS Dropout	151.0	112.9	31.1	85.5	11.8	54.1	46.7	17.7	24.7	4.4
HS Graduate	161.4	116.4	14.4	74.8	14.0	183.5	126.3	13.8	71.7	13.4
Some College	102.9	90.9	7.3	24.0	16.3	40.9	44.1	4.6	0.8	7.5
College Graduate	16.8	13.0	4.7	-0.4	4.5	12.1	9.7	4.5	-1.5	3.3

Notes: These results report percentage changes relative to the baseline for the counterfactual case with $\gamma = 0.99$.

Table 14: Effects of Early and Late Investment Subsidies

Policy	Short-Run Effects (% Change) Avg. i_1	Avg. i_2	HS+	Some Coll+	Coll Grad	Avg. W_3	Long-Run Effects (% Change) Avg. i_1	Avg. i_2	HS+	Some Coll+	Coll Grad	Avg. W_3
Announced early:												
$s_1 = .10$	63.6	22.5	0.8	13.5	43.2	6.5	76.2	30.1	1.7	18.8	56.4	8.4
$s_2 = .026$	12.9	25.9	15.9	17.7	39.3	3.6	17.2	29.6	16.2	20.6	45.5	4.4
Announced late:												
$s_2 = .026$	0.0	15.4	15.9	15.4	15.1	1.6	17.2	29.6	16.2	20.6	45.5	4.4

Table 15: Short-Run Effects of Welfare-Maximizing Budget Neutral Changes in Early and Late Investment Subsidies

	Avg. i_1	Avg. i_2	HS+	Some Coll+	Coll Grad	Avg. W_3
% Change in Averages/Probability	330.2	-47.0	-15.2	-19.5	-90.5	11.1
% Change in Gaps by Parental Education	192.5	-51.6	20.8	-4.1	-81.5	16.9

Notes: Results reflect changes from baseline economy to an economy with a constant increase in s_1 to 0.43 and a constant reduction in $S_2(i_2)$ of $s_2 = -0.356$. This is the welfare-maximizing level of s_1 where s_2 is adjusted to keep the discounted present value of government expenditures constant. Gaps by parental education are differences between the children of college graduates and high school dropouts.

Online Appendix

O.1 Multiple Investment Inputs Each Period

In this Appendix, we show how our model can be generalized to include multiple investments each period, where i_j would then reflect total investment expenditures in period j given optimal choices about different inputs each period.

Suppose human capital production depends on two inputs each period: purchased goods g_j and parental time τ_j (scaled by effective parental human capital $h_j^p \equiv \Gamma_{j+2}h^p$). Define the child's human capital production function as:

$$h = \theta f(x_1, x_2) \tag{26}$$

where

$$x_j = \chi_j(g_j, \tau_j h_j^p), \quad j = 1, 2.$$

Notice that this technology assumes parental human capital increases the productivity of parental time inputs in the same way it increases productivity in the labor market. This is analogous to the neutrality assumption of Ben-Porath (1967) only with respect to investments in child human capital rather than own human capital.

Next, consider maximizing per period human capital inputs x_j subject to total expenditure i_j that period. We assume input prices (p for the price of goods and w for the price of human capital) are stable across periods and that individuals are at an interior point in their time budget (i.e. $\tau \in (\tau_{min}, \tau_{max})$). Define the following maximized period j input:

$$x_j^*(i_j; p, w) = \max_{g_j, \tau_j} \chi_j(g_j, \tau_j h_j^p) \qquad \text{subject to} \qquad pg_j + w\tau_j h_j^p = i_j \tag{27}$$

for total investment expenditures i_j in each period.

If we assume $\chi_j(\cdot, \cdot)$ are homogeneous of degree 1, then we can write

$$x_j^*(i_j; p, w) = \tilde{x}_j(p, w)i_j,$$

where $\tilde{x}(p, w)$ is the maximized output for a total expenditure of 1. Substituting this into equation (26) yields

$$h = \theta f(\tilde{x}_1(p, w)i_1, \tilde{x}_2(p, w)i_2).$$

Clearly, one can just re-write the production function in terms of total investment expenditures i_1 and i_2 as $\tilde{f}(i_1, i_2) = f(\tilde{x}_1(p, w)i_1, \tilde{x}_2(p, w)i_2)$ where the $\tilde{x}_j(p, w)$ are like technology parameters that depend on prices (p, w).

For the CES production function $f(x_1, x_2) = [ax_1^b + (1-a)x_2^b]^{d/b}$, we have the following:

$$\begin{aligned}
h &= \theta \left[a(\tilde{x}_1(p,w)i_1)^b + (1-a)(\tilde{x}_2(p,w)i_2)^b \right]^{d/b} \\
&= \theta \left\{ \left[\left(\frac{a\tilde{x}_1^b}{a\tilde{x}_1^b + (1-a)\tilde{x}_2^b} \right) i_1^b + \left(\frac{(1-a)\tilde{x}_2^b}{a\tilde{x}_1^b + (1-a)\tilde{x}_2^b} \right) i_2^b \right] \left(a\tilde{x}_1^b + (1-a)\tilde{x}_2^b \right) \right\}^{d/b} \\
&= \theta \left(a\tilde{x}_1^b + (1-a)\tilde{x}_2^b \right)^{d/b} \left[\left(\frac{a\tilde{x}_1^b}{a\tilde{x}_1^b + (1-a)\tilde{x}_2^b} \right) i_1^b + \left(1 - \frac{a\tilde{x}_1^b}{a\tilde{x}_1^b + (1-a)\tilde{x}_2^b} \right) i_2^b \right]^{d/b} \\
&= \tilde{\theta} \left[\tilde{a} i_1^b + (1-\tilde{a}) i_2^b \right]^{d/b}
\end{aligned}$$

where

$$\begin{aligned}
\tilde{\theta} &= \theta [a\tilde{x}_1^b(p,w) + (1-a)\tilde{x}_2^b(p,w)]^{d/b} \\
\tilde{a} &= \frac{a\tilde{x}_1^b(p,w)}{a\tilde{x}_1^b(p,w) + (1-a)\tilde{x}_2^b(p,w)}.
\end{aligned}$$

Thus, if (i) parental time investment is unconstrained (i.e. at an interior point), (ii) parental human capital is equally productive in child development and the labor market, and (iii) within period investment functions $\chi_j(\cdot,\cdot)$ are homogeneous of degree 1, then our CES human capital production function still represents the production process with i_j reflecting total investment expenditures in period j. The 'technology' parameters $\tilde{\theta}$ and $\tilde{a}$ now depend on input prices p and w in addition to true underlying technology parameters.

In general, variation in prices (w, p) can affect both total factor productivity $\tilde{\theta}$ and the relative productivity of early vs. late investments, $\tilde{a}$. Two interesting special cases yield variation in $\tilde{\theta}$ alone.

First, variation in price levels (but not relative prices) will only affect $\tilde{\theta}$. For example, consider two sets of prices (p, w) and (p', w') where $\frac{w'}{w} = \frac{p'}{p} = \delta$. In this case, it is easy to see that $\tilde{x}_j' = \tilde{x}_j/\delta$, so $\tilde{\theta}' = \tilde{\theta}\delta^{-d}$ and $\tilde{a}' = \tilde{a}$.

Second, if both within-period production functions are identical, so $\chi_j(\cdot,\cdot) = \chi(\cdot,\cdot)$ and $\tilde{x}_j(p,w) = \tilde{x}(p,w)$ are independent of period j, then differences in input prices (p, w) will generally lead to differences in $\tilde{\theta} = \tilde{x}^d\theta$ but not $\tilde{a}$, which equals a regardless of (w, p).

Special Case: CES $\chi_j(\cdot,\cdot)$

Suppose $\chi_j(g, \tau h^p) = [\psi_{jg} g^\phi + \psi_{j\tau}(\tau h^p)^\phi]^{1/\phi}$. In this case, it is straightforward to show that

$$\tilde{x}_j(p,w) = \frac{\left(\psi_{jg} \left[\left(\frac{\psi_{jg}}{\psi_{j\tau}} \right) \left(\frac{w}{p} \right) \right]^{\frac{\phi}{1-\phi}} + \psi_{j\tau} \right)^{1/\phi}}{p \left(\left[\left(\frac{\psi_{jg}}{\psi_{j\tau}} \right) \left(\frac{w}{p} \right) \right]^{\frac{1}{1-\phi}} + \frac{w}{p} \right)}.$$

O.2 Properties of the Value Function $V_3(\cdot,\cdot)$

In order to apply the proofs contained in CLP to our dynastic structure, we need to demonstrate that the properties of the lifecycle continuation utility are maintained with the dynastic value function. In particular, that $V_3(a_3, h)$ is strictly increasing and strictly concave in both assets and human capital.

It is straightforward to apply the results in Stokey, Lucas, and Prescott (1989) (SLP) to show that the dynastic value function is unique, strictly increasing and strictly concave. We can rewrite the dynastic problem to be consistent with SLP:

$$V(a_3,h) = \max_{c_3,c_4,c_6,a_4,a_5,i_1',a_3',h'} \hat{U}(a_3,h,c_3,c_4,c_6,a_4,a_5,i_1',a_3',h') + \rho\beta^2 V(a_3',h') \tag{28}$$

subject to:

$$Ra_3 + W_3(h) + y_3 - a_4 - i_1' - c_3 - c_1' > 0,$$

$$Ra_4 + W_4(h) + y_4 + W_2 - a_5 - a_3' - i_2'(i_1',h') - c_4 - c_2' > 0,$$

$$Ra_5 + W_5(h) - R^{-1}c_6 > 0,$$

$$a_4 \geq -L_3,$$

$$a_5 \geq -L_4,$$

$$a_3' \geq -L_2,$$

$$h' = \theta' f(i_1', i_2'),$$

where

$$\hat{U}(a_3,h,c_3,c_4,c_6,a_4,a_5,i_1',a_3',h') = u(c_3) + \beta u(c_4) + \beta^2 u(Ra_5 + W_5(h) - R^{-1}c_6) + \beta^3 u(c_6) +$$

$$\rho[u(Ra_3 + W_3(h) + y_3 - a_4 - i_1' - c_3) + \beta u(Ra_4 + W_4(h) + y_4 + W_2 - a_5 - a_3' - i_2'(i_1',h') - c_4)].$$

Note that $i_2(i_1', h')$ is the i_2' that satisfies, $h' = \theta' f(i_1', i_2')$.

The following assumptions (see SLP, chapter 4) hold for this problem:

- A4.3 The state space (a_3, h) is a convex subset of R^2 and the constraint set is non-empty, compact-valued and continuous.
- A4.4 The function $\hat{U}$ is bounded and continuous and $\rho\beta^2 < 1$. Because $\hat{U}$ is derived from u it is bounded and continuous. The latter condition holds when $\beta < 1$ and $\rho < 1$.
- A4.5 The function $\hat{U}$ is strictly increasing in a_3 and h. It is clear that $\hat{U}$ is strictly increasing in a_3 and h because $u'(\cdot) > 0$, and arguments of u are increasing in a_3 and h.

A4.6 The constraint set is monotone: As either state variable a_3 or h increases, the set of possible choice variables contains the original set.

A4.7 The function $\hat{U}$ is concave. Because u and f are concave, $\hat{U}$ is concave.

A4.8 The constraint set is convex. Convexity of the constraint set follows because f is concave.

A4.9 The function $\hat{U}$ is continuously differentiable with respect to a_3 and h.

Given these assumptions, we have (see SLP, chapter 4):

Theorem 4.6 If A4.3 and A4.4 hold there exists a unique V.

Theorem 4.7 If A4.3-A4.6 hold, V is strictly increasing.

Theorem 4.8 If A4.3-A4.4 and A4.7-A4.8 hold, V is strictly concave.

Theorem 4.11 If A4.3-A4.4 and A4.7-A4.9 hold, V is continuously differentiable.

Therefore, there exists a unique V, that is strictly increasing, strictly concave, and continuously differentiable. We do not know if V is twice, continuously differentiable. What we need is that V is twice differentiable at an optimum (at least one-sided). If this is the case, then $V_{22} < 0$, due to the concavity of V.

O.3 Calibration Sensitivity Analysis

We conduct a comprehensive sensitivity analysis for our calibration, counterfactual, and policy simulations. In particular, we re-calibrate our model imposing different assumptions about (i) the extent of dynamic complementarity (i.e. different values for b), (ii) greater borrowing opportunities (i.e. $\gamma = 0.5$), (iii) no effect of parental human capital on the child's ability (i.e. $\pi_2 = 0$), and (iv) no unmeasured costs of high school (i.e. $\zeta_1 = 0$). We also re-calibrate our model using a 'full' family income measure that adjusts for the possibility that mothers may work part-time in order to spend time investing in their children. In all cases, we repeat our main counterfactual and policy simulations with the restricted/new parameter sets obtained through the same simulated method of moments procedure.

Table O-1 reports the calibrated parameter values for all cases, while Table O-2 reports the mean weighted squared error (MWSE) for the different subsets of moments. Tables O-3 to O-6 report measures of investment and the fraction of families borrowing up to their limits in the re-calibrated economies. Tables O-7 to O-12 reproduce the main results from our counterfactual and policy simulations for each calibration.

Section 5 of the paper discusses the analysis and findings for our results imposing different values for b (-0.5, 0, 0.5) and using the 'full income' measure in creating our sets of moments (iii)-(v) for investment and period 3 wage outcomes conditional on family income and maternal education. Here, we provide a brief discussion of results for the other three cases.

When re-calibrating the model fixing any single parameter (i.e. $\gamma = 0.5$ or $\pi_2 = 0$ or $\zeta_1 = 0$), most other parameter estimates are quite similar to those of our baseline calibration (Table O-1). One exception is the smaller value for γ when imposing $\zeta_1 = 0$, implying fewer borrowing opportunities than in the baseline case. Given the importance of dynamic complementarity for many of our results, it is also worth noting the somewhat higher values (compared to the baseline calibration) for b when we impose $\pi_2 = 0$ or $\zeta_1 = 0$ and lower value when we set $\gamma = 0.5$. In terms of fit (Table O-2), imposing $\gamma = 0.5$ produces a poor fit for late investments and wage distributions, while imposing $\pi_2 = 0$ leads to a poor fit for early and late investments conditional on parental education and family income. Imposing $\zeta_1 = 0$ fits slightly worse than the baseline for all moments, but is not particularly bad for any subset. In all cases, the investment ratios for children of college graduates vs. high school dropouts are comparable to the baseline calibration (Table O-5). The proportions of families up against their borrowing or transfer constraints (Table O-6) are also quite similar to those reported for the baseline calibration with one exception: far fewer old parents are borrowing constrained when imposing $\gamma = 0.5$. The fraction of young parents up against their borrowing limit is quite similar to the baseline case even with the much higher γ. Other parameters adjust to fit the data in a way that still yields a non-trivial fraction of borrowing constrained young parents.

Table O-7 reports the anticipated and unanticipated short-run effects of a \$10,000/year income transfer to old parents. In all cases, the effects of an anticipated transfer are much greater than an unanticipated transfer; however, the the differences are more modest when $\pi_2 = 0$ or $\zeta_2 = 0$ are imposed. These more muted differences are consistent with the greater substitutability implied by the higher estimated values for b in these cases, much as we see for the case imposing $b = 0.5$.

Tables O-8 and O-9 reproduce the counterfactual analyses aimed at understanding the importance of ability transmission and market frictions for intergenerational mobility. In all cases, child ability accounts for a comparable share of the investment gaps by parental income, while eliminating lifecycle borrowing constraints would have similar or stronger effects (compared to the baseline calibration). There is a greater discrepancy between calibration cases in the implied role of ability vs. market frictions when we simulate the economy with zero intergenerational ability correlation (Table O-9). Assuming greater opportunities for borrowing than estimated by our baseline calibration (imposing $\gamma = 0.5$) produces a much greater role for ability transmission relative to market frictions.

As shown in Tables O-10 and O-11, we obtain very similar effects of relaxing borrowing constraints (one-by-one or completely eliminating all constraints) for all of our restricted calibration sets, even when $\gamma = 0.5$ is assumed. In all cases, completely eliminating all lifecycle borrowing constraints has substantial effects on investments and post-school earnings – much greater than the effects of relaxing any single borrowing limit by itself.[80]

Finally, Table O-12 reports the short-run effects of fiscally equivalent early and late investment subsidies. In all calibration cases, we consider the impacts of increasing s_1 to 0.1, as well as increasing s_2 by an amount that produces the same total expenditure on all investment subsidies. Our main conclusions hold for all parameterizations: (i) early investment subsidies have greater effects than late subsidies, and (ii) the effects of late subsidies are much greater when the subsidies are announced early so early investment can respond. Perhaps surprisingly, the effects of subsidies are greatest when $\gamma = 0.5$. They are smallest when $\pi_2 = 0$.

[80]Note that Table O-10 studies the short-run effects of increasing borrowing limits by $1,500 rather than $2,500 as in the paper, because increasing borrowing limits (at one stage) by the latter amount (in two calibration cases) would extend them beyond the natural borrowing limits for some families due to subsequent constraints.