INTERNATIONAL ECONOMIC REVIEW
Vol. 43, No. 1, February 2002

EDUCATIONAL VOUCHERS WHEN THERE ARE PEER GROUP EFFECTS—SIZE MATTERS*

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In this article, I study the effects various educational voucher policies have on the sorting of children across schools and the per-student expenditure levels at these schools, when a child's peer group matters and students differ over income and ability. I find that, depending on the magnitude of the voucher, switching from a public system to a voucher system could entail either welfare gains or losses. All voucher policies under consideration lead to greater inequality than the public system; however, these increases are not monotone in the voucher size.

1. INTRODUCTION

Several states are considering enacting or have enacted educational voucher plans, which entail giving parents money to send their children to private schools. Both the cities of Milwaukee and Cleveland have started educational voucher programs. During the 1996–1997 school year, around 1650 students in Milwaukee received vouchers worth \$4400, at a total cost of over \$7 million. In the same year, Cleveland spent \$6.4 million, distributing an average voucher of around \$2000, to 1996 students. Other smaller scale voucher plans have sprouted up in school districts around the country. Proponents claim vouchers give poor parents educational choice for their children and give poor children a chance to succeed where the public schools fail them. Opponents argue that by funding vouchers, resources will be funneled out of the public schools to a small number of students and that the majority of students who are left behind in the public schools will suffer.

In order to evaluate these proposals, a framework is needed that incorporates the factors important to educational outcomes. Along with family background and educational expenditure, many empirical studies have shown that the peer group plays a significant role in educational achievement. Consider a change in educational policy, for example, adopting a voucher system. Such a switch will affect not only

* Manuscript submitted August 1999; revised February 2000.

[1] I greatly appreciate the comments of Raquel Fernandez, Rick Hanushek, Antonio Merlo, Edward Prescott, Richard Rogerson, Richard Romano, and an anonymous referee. Of course, any errors are mine. Please address correspondence to: Elizabeth M. Caucutt, Department of Economics, University of Rochester, 238 Harkness Hall, Rochester, NY 14627-0156. Phone: 585-275-0093. Fax: 585-256-2390. E-mail: *Caucutt@usc.edu*; *ecau@trai.cc.rochester.edu.*

educational expenditures but also the school a student chooses to attend, and hence a school's peer group. Therefore, it is essential to have a theory that endogenizes school formation, in order to predict the consequences of alternative educational proposals. In this article, I use the theory and the computational methods developed in Caucutt (2001) to analyze how equilibrium outcomes, in terms of the sorting of students across schools and the student expenditures at these schools, are affected by various voucher policies. I also consider the welfare and distributional ramifications of these changes, using a public system as a point of comparison.

In the framework that I work with, the number of types of private schools operating in equilibrium is not restricted, and private schools can charge different types of students different prices. The latter feature allows the peer group externality to be internalized. In equilibrium, good students pay less than bad students to attend the same school. The environment is static general equilibrium, with four types of parents who each have one child. There is heterogeneity in wage and ability across parents. Parents pass their ability on to their children. A parent receives utility from consumption and the human capital that her child acquires. A child obtains human capital by attending school, and how much he gets depends on his type, and the school's peer group and educational expenditures. There is a finite number of exogenously given school types. Instead of choosing a specific school type, a parent randomizes over these schools, choosing the probability that her child attends each of the school types. A parent cannot borrow to finance her child's education. The schools are profit maximizing, choosing how big to be, given their type and tuition levels.

There is one public school. This school is free and is financed by a given proportional income tax. I calibrate the model and predict how students will sort across schools and what levels of educational spending will arise at these schools. I then consider the effect on the equilibrium outcome of an alternate form of educational financing, specifically a voucher. Here a parent who sends her child to a private school receives a lump sum voucher to help defray tuition. This reduces the cost of leaving the public school system and encourages parents to send their children to private schools. I find that, depending upon the magnitude of the voucher, a switch from a public school system to a voucher system could have either welfare gains or losses. Low voucher levels are associated with welfare losses that decrease monotonically and become welfare gains as the voucher grows. Switching from the public school system to a voucher system leads to increases in income inequality; however, these increases are not monotone in the voucher size. These two facts imply that if a voucher policy is implemented, care needs to be taken when selecting the size of the voucher. I also look at the effects of switching to the completely private system of schooling that is initially discussed. Lastly, I consider a plan that targets the voucher to the poor. While this policy entails similar increases in inequality and slightly greater welfare costs or smaller welfare gains than the corresponding full voucher policy, the welfare losses are shifted from the poor to the rich. I do several sensitivity analyses.

There is a rich empirical literature that presents some strong evidence that the peer group is important to educational achievement. The principle finding of the 1966 Coleman Report, *Equality of Educational Opportunity,* is that a student's educational achievement is strongly and positively related to the educational background and aspirations of his or her classmates (Coleman et al., 1996). In that exhaustive study, in which almost 20,000 school teachers distributed surveys in their classrooms, this relationship is found to be much stronger for disadvantaged students. In other words, a disadvantaged student benefits more from an increase in the ability of his or her peer group than does an advantaged student. Summers and Wolfe (1977) use data from the classroom level. They, like Coleman, conclude that the peer group effect plays a significant role in educational outcomes. Using Canadian data, Henderson et al. (1978) also find strong evidence that the peer group is an important input in the educational process. But they conclude that the effect the peer group has on educational achievement is similar across students of differing ability. Strong students gain from an increase in the quality of the average student in the class, as do weak students. More recently, Black (1999) finds that parents are willing to pay a significant amount more for a house across the street from another if residing in that house implies attending a school with higher average test scores. Since these houses are in the same school district, educational expenditures are constant. Therefore, most of the effect she finds is attributable to unobservable parental characteristics and the peer group.

Not all studies find that peer effects are important. Evans et al. (1992) address the estimation of peer effects when measures of peer influence are potentially endogenous variables. When controlling for this endogeneity they find peer effects insignificant. They conclude that it is not that peer effects do not matter, but that the potential endogeneity problem needs to be taken into account. Hanushek (1986), in a large review of the education literature, claims that the findings on peer effects are ambiguous, due to the difficulty of separating peer effects from unobservable family background characteristics. Aaronson (1996) investigates the effect that neighborhoods have on high school graduation rates, using sibling data to correct for the potential endogeneity problem. The principle conclusion of the article is that, contrary to Evans et al. (1992), corrections for neighborhood selection biases do not necessarily eliminate the potential for significant community effects.

The theoretical literature on peer group effects is growing. Epple and Romano (1998a) consider a model with students differing continuously over ability and income. The education production function that they use depends on the average ability of the students in the school and the student's own ability, but not on the expenditures of the school. They find that under a voucher system the achievement gains of those high-ability students who switch from the public school to the private school, and hence a better peer group, are great, while the losses of those left behind are small. However, the number of students falling into the latter category is much greater. Their model also yields monotonically increasing welfare gains in the size of the voucher.[2] de Bartolome (1990) constructs an environment with two types of individuals, two types of communities, and schools where peer effects matter. Because his school system is public, the peer group effect is not priced and is therefore an externality. Benabou (1996) extends this model. He includes a capital market and

[2] This is true up until vouchers are very large and schools get inefficiently small.

derives several conditions under which the communities are stratified and when this is efficient. Depending upon the specification of the production function for human capital, he finds that a policy to equalize school budgets could either lead to integration or cause no change in mixing across the segregated communities. If communities remain segregated, the poor are better off due to the policy, but the rich are much worse off, implying a reduction in average achievement. Community integration could lead to an increase in average achievement. Nechyba (1996) considers a three-community model with public and private schools, migration, voting over expenditure level, and peer effects. The private sector is relatively passive with regard to peer effects because each private school can only charge one price, and therefore can only attract one type of student. He finds that as a result of a voucher policy, school-based stratification increases, but residential stratification decreases. This is because parents who send their children to private schools migrate to find communities with lower tax rates.

The article proceeds as follows: In Section 2, I lay out the the basic structure of the model and discuss the computational method used. In Section 3, I outline the kinds of schools, in terms of the peer group, that can arise in equilibrium. In Section 4, I add a public school to the private school framework, calibrate the model, and discuss the results of several policy changes. In Section 5, I do various sensitivity analyses, and I conclude in Section 6.

2. MODEL

In this section, I specify the model. The key contribution of this framework is that it incorporates peer group effects into the educational decision of the parent. A parent cares about the human capital that her child obtains. The human capital that a child acquires depends on three factors. First, it depends on his personal characteristics, including family background and innate ability to learn. These characteristics are referred to as his type. Second, his human capital depends on the per-student input of resources, or expenditures, at his school. Lastly, it depends on the relative numbers of the various types of students attending his school, or student body composition. This is the peer group.

My work is most closely related to Epple and Romano (1998a). However, there are a few key differences. First, they have a continuum of different types of people, who differ over income and ability. I have four types of parents, with two different income levels and two different ability levels. While this sacrifices some realism, computational limitations restrict the number of types I can deal with. Having two different income levels and two different ability levels does allow me to predict how various policies will affect the four groups of interest. Second, because they have an integer number of schools, they encounter existence problems that often arise in club economies. As a result, they are constrained to considering cases where, although each school is maximizing profits locally, globally profits are not zero, but less than some epsilon. In my framework, because parents randomize over schools, the problem is convex. Existence is established in Caucutt (2001). Third, in their work, a student's achievement depends on his own ability and the ability of his peers. In my model, the human capital a student acquires from a school also depends on the educational expenditure level at that school. This can play an important role in the results. Including expenditure variation allows for another margin of tension across types. With only one expenditure level, able students and unable students of different income levels are more likely to mix. This is because they do not also have different preferences over educational expenditures. A principle result of this article is that small vouchers are often welfare decreasing. This is in contrast to Epple and Romano (1998a), who find welfare gains for all voucher sizes.

I begin by considering a completely private system of schools. Given this framework, it is relatively simple to introduce a public school. This is done in Section 4.

2.1. *Parents.* There are four types of parents, who differ over human capital endowments, h^p and h^r, with $h^p < h^r$, and learning ability endowments, a^u and a^a, with $a^u < a^a$. A direct relationship between human capital and wage is assumed. There is measure λ^i of each type of parent, $i = pu, pa, ru, ra$, with $\sum_i \lambda^i = 1$. These parental types will be referred to as poor, unable; poor, able; rich, unable; and rich, able. Each parent has one child who is endowed with the same learning ability as his parent. The learning ability is assumed to be perfectly observable. The human capital that a child accumulates depends on his learning ability and the school that he attends. An able student attending a given school accumulates more human capital than an unable student attending the same school. A parent receives utility from consumption and the human capital that her child accumulates. I assume that the parent considers the level of human capital her child obtains important and not the utility her child receives from that level.[3] A parent cannot borrow to finance her child's education.

There is an exogenously given, but very large, finite set of schools a parent can send her child to. A randomizing mechanism is introduced so that instead of choosing which school her child attends, the parent chooses the probability that her child attends each school in the set. All lottery equilibria are Arrow–Debreu equilibria, and by assuming a lottery technology, all gains from mutually beneficial gambles are exhausted (Rogerson, 1988). This assumption also simplifies the analysis by convexifying the parent's problem and allowing me to restrict attention to type identical allocations. In this environment, a commodity vector is made up of consumption and a set of four vectors; the ith vector's components correspond to the probability that a type i parent sends her child to each of the possible schools. Each parent has a set of four vectors in her commodity vector, even though three of the four will contain all zeros. This is because the probability that a type i parent sends her child to a school is a different commodity and will therefore be priced differently than the probability that a type j parent sends her child to the same school, $i \neq j$.[4]

[3] This is a common assumption in this literature; see Epple and Romano (1998a), Fernandez and Rogerson (1996, 1998), and Glomm and Ravikumar (1992).

[4] Recall that a student's type is public information.

The problem of a type i parent is given by

$$\max_{c,\,\pi} \log(c^i) + \xi \sum_s \pi_s^i \log(h_s'^i) \tag{1}$$

$$\text{s.t.} \quad c^i + \sum_s \pi_s^i p_s^i \leq h^i$$

$$\sum_s \pi_s^i = 1$$

$$\pi_s^i \geq 0, \qquad \forall s$$

Here c^i is the consumption of parent i, and π_s^i is the probability that parent i sends her child to school s. The human capital the child of a type i parent receives from attending school s is $h_s'^i$. This human capital will depend on the ability of the child, his school's peer group, and his school's expenditure level. Note that $h_s'^i$ is exogenous; the parent chooses the probability associated with each level of human capital, but not the actual determinants of the human capital. The total utility the parent receives from her child's human capital accumulation is then $\xi \sum_s \pi_s^i \log(h_s'^i)$. The price, or tuition, that a type i parent pays to school s is p_s^i. So the total expenditure of a type i parent on her child's education is given by $\sum_s \pi_s^i p_s^i$. Recall that the human capital of the parent is the income of the parent as well.

2.2. *Schools.* There is a finite set, S, of school types. Each parent must be able to afford to send her child to at least one school. In the computation, this set of schools will be chosen to be very large so that in practice each parent will have a wide choice. A school type, $s \in S$, is defined by the fraction of each student type attending, n_s^{pu}, n_s^{pa}, n_s^{ru}, n_s^{ra}, and its per-student expenditures, e_s. These inputs are combined to produce human capital in the following manner:

$$h'^i_s = B(a^i)^{\alpha} \left(\sum_i n_s^i a^i \right)^{\gamma} (e_s)^{\psi}, \qquad \forall i,\ s$$

A school, s, is normalized to a size of one student, and the number of those schools, z_s, is allowed to vary. So while the set of possible school types, S, is exogenous, the measure of each school in S, z_s $\forall s$, is endogenous. Tuition, p_s^i, depends on the type of the child.

In equilibrium, given prices p_s^i, per-student expenditures, e_s, and enrollments, n_s^i, all schools operating will have zero profits,

$$\sum_i p_s^i n_s^i - e_s = 0, \qquad \forall s$$

If the school were earning negative profits it would shut down, and if it were earning positive profits there would be an infinite number of such schools and therefore not an equilibrium.

2.3. *Resource Constraints.* The first resource constraint is the consumption resource constraint:

$$\sum_i \lambda^i c^i + \sum_s z_s e_s \leq \sum_i \lambda^i h^i$$

This ensures that total resources allocated to consumption plus total resources allocated to education are not more than the total endowment. The remaining resource constraints are the probability resource constraints:

$$\pi_s^i = \frac{z_s n_s^i}{\lambda^i}, \quad \forall i,\ s$$

These constraints guarantee that the probabilities the parents choose match the measures the schools choose.

2.4. *Equilibrium.* A competitive equilibrium is a set of allocations, c, π, z, and prices, p, such that the parents are solving their problems, the operating schools are earning zero profits, and the resource constraints hold.

2.5. *Computation of Equilibrium.* Because the commodity space is so large, I do not solve for the equilibrium directly, using the parents' problems and the schools' problems. I instead use the social planner's problem. I create a mapping from the set of weights from the social planner's problem to the set of transfers that support the corresponding Pareto allocations as competitive equilibria with transfers. This is referred to as a Negishi mapping. I then search for a set of weights whose transfers are zero. A Pareto allocation associated with these weights, along with the appropriately chosen prices, is a competitive equilibrium. As mentioned previously, an equilibrium exists; however, it is not guaranteed to be unique.[5]

The θ-weighted social planner's problem is given by

$$\begin{aligned} \max_{c,z} \sum_i \lambda^i \theta^i & \left[\log(c^i) + \xi \sum_s \tfrac{z_s n_s^i}{\lambda^i} \log(h_s^{\prime i})\right] \\ \text{s.t.} \sum_i \lambda^i c^i &+ \sum_s z_s e_s \le \sum_i \lambda^i h^i \\ \sum_s \tfrac{z_s n_s^i}{\lambda^i} &= 1, \quad \forall i \\ z_s &\ge 0, \quad \forall s \end{aligned} \tag{2}$$

This problem can be rewritten, moving the consumption resource constraint into the maximization and letting Γ be the Lagrange multiplier on the consumption resource constraint:

$$\begin{aligned} \max_{c,z} \sum_i \left[\lambda^i \theta^i \log(c^i) - \Gamma \lambda^i (c^i - h^i)\right] &+ \sum_s z_s \left[\xi \sum_i \theta^i n_s^i \log(h_s^{\prime i}) - \Gamma e_s\right] \\ \text{s.t.} \sum_s \frac{z_s n_s^i}{\lambda^i} &= 1, \quad \forall i \\ z_s &\ge 0, \quad \forall s \end{aligned} \tag{3}$$

[5] Using a variety of computational methods and initial values, etc., I have yet to find a case with more than one equilibrium.

Given Γ, the individual consumption levels follow from the first-order conditions with respect to consumption. The problem is then linear in z. I use standard linear programming techniques to solve (3), iterating over Γ until the consumption resource constraint holds. The solution to this problem is a θ-Pareto allocation. Prices that support this allocation as a competitive equilibrium are then calculated following Caucutt (2001). A set of transfers that support this Pareto allocation as a competitive equilibrium with transfers immediately follows, creating a Negishi mapping. This mapping is a correspondence, so a version of Scarf's algorithm is used to find a competitive equilibrium.

3. PRIVATE SCHOOL SYSTEM

In this section, I consider the case of a completely private system of schools, described in Section 2. Here schools can price discriminate, charging students of different types different tuition. I begin by showing that some kinds of schools will never arise in equilibrium. I then prove that, without loss of generality, attention can be restricted to the set of schools with no more than two types of students, who differ in ability, attending. Lastly, I find an equilibrium when there are no government interventions.

It is important, for the computation, to understand what kinds of mixing might occur in equilibrium and what kinds of mixing would not occur in equilibrium. In Proposition 1, I show that two students of the same ability level, but with different income levels, will never mix in a school that contains only those two types of students. This is due to the fact that both types prefer a different expenditure level, and given that they have the same ability, there is no room for a subsidy. They are better off separated into two homogeneous schools. In Proposition 2, and Lemma 2, I show that any school with a mix of three or four types of students can be represented by a convex combination of schools with a mix of two types of students, with differing ability. These results are extremely useful computationally. They allow the set of possible schools to be restricted to only those schools that contain a mix of two types of students, with differing ability, and those schools that are completely homogeneous. That means the grid of possible school types is greatly reduced.

LEMMA 1. *If a type only attends one school and if that school is homogeneous, the preferred educational expenditure level is not a function of ability, but is a function of income.*

PROPOSITION 1. *No school composed of two types will ever contain a mix of two different types of the same ability level. In other words, the rich and poor, unable types will never form a school, and the rich and poor, able types will never form a school.*

LEMMA 2. *Every school with a mix of three or four types can be represented as a convex combination of four schools with a mix of two types (whose two types are of differing ability).*

PROPOSITION 2. *Without loss of generality, the only schools that need to be considered are those made up of combinations of two types, where those two types differ over ability.*

It is possible to describe all schools that have mixes of three or four types of students with schools that have mixes of two types of students, because all of these schools charge the same price and yield the same human capital outcome. A parent is indifferent between putting all of her probability on one school and mixing her probability over several of these schools.

4. A PUBLIC SCHOOL SYSTEM AND POLICY CHANGES

In the United States, a system of completely private schooling is far from the norm. Most state constitutions guarantee some form of equal opportunity in education. Because of credit constraints, redistribution for education is therefore necessary. One method of redistribution is to create a public school system. In this section, I add a public school option to the set of school types from which a parent can choose. I predict an equilibrium outcome and then impose a policy change. The policy changes that I study include implementing a voucher system and switching to a completely private system of education.

The equilibrium outcomes of these policy exercises depend crucially on the parameters of the human capital production function. There is considerable debate in this literature over the significance and magnitude of these parameters. One faction believes that there is little evidence that either peer effects or educational expenditures matter much to educational achievement and future earnings. However, most who believe that these inputs do not affect educational outcomes would nevertheless argue that parents believe that they do, and act accordingly when making educational decisions. Therefore, there should be little controversy in comparing the equilibrium school formation under various policies. On the other hand, when calculating welfare and distributional effects, a stand must be taken on how much these inputs truly matter to future earnings. While I am aware of the debate within this literature, I find it hard to believe that parents continually behave irrationally, investing in their children when it does not matter. I therefore report welfare and distributional effects of these policies at the end of the section. Results from a sensitivity analysis over the peer group parameter, γ, are reported in Section 5.

Measuring welfare costs in an environment with heterogeneity is not straightforward. I choose to measure the welfare cost of a policy, relative to the base case, by adding up the consumption each type requires, or is willing to give up after the policy change, in order to attain the utility level that she has under the base case. I then divide this sum by the total resources available in the economy to get the welfare cost as a fraction of total resources. This is referred to in the literature as the compensating variation and is the obvious generalization of Lucas (1987) to an economy with heterogeneity.

Note that compensating variation generally refers to the amount of extra income a person would need to make them just as well off. I calculated both the welfare cost in terms of consumption needed and income needed, and the results were almost identical. I use consumption because using income would entail each parent re-optimizing over consumption and schooling. I consider four measures pertaining to next period's distribution of human capital. The first is the average human capital acquired. The remaining three attempt to measure the inequality of the distribution. The coefficient of variation is the standard deviation of the distribution divided by the mean, and the range is merely the difference between the two extreme points of the distribution. The last measure is the standard Gini coefficient, measuring inequality on a scale of zero to one, with zero being complete equality and one being complete inequality.

4.1. *Public School System.* There is one public school. It is assumed that the public school and private schools have identical human capital production technologies. It is financed by a proportional tax on income. Any student can attend free of charge. If the student attends a private school, the parent pays tuition to that school on top of the taxes she is already paying to the public school system. While political economy issues are obviously relevant to school financing, I abstract from them here in order to concentrate on the principle interactions of the model. I assume a constant tax rate. Because total resources are fixed, this implies a fixed amount of tax revenues for the public school. These revenues are split evenly among those who attend the public school. The fewer the number of students in the public school, the higher the per-student expenditures. In the last section, I investigate the sensitivity of the results to this financing assumption. I contrast the results of this section with those that arise assuming a polar financing assumption. This alternative assumption is that per-student public educational expenditures are fixed and the tax rate adjusts to balance the budget.

In terms of the framework of Section 2, here a parent continues to choose the probability that her child attends each of the private schools, but now those probabilities are no longer required to sum to one. The residual probability is placed on the public school. The commodity space remains the same, since no new probabilities have been introduced. Equilibria here are constrained efficient. Total public school spending is held fixed, so given the financing scheme, the equilibria are Pareto optimal. The computational method discussed earlier can still be used since the Second Welfare Theorem holds given the financing assumption. This depends crucially on the assumption that there is just one public school. If another public school were added, parents would have to choose one of the public school probabilities directly. If that probability were not priced by ability, the peer group externality would not be internalized, the Second Welfare Theorem would no longer hold, and the equilibrium would have to be computed directly.[6]

The public school system described above is calibrated. There is a mapping from a student's type and the school's peer group and per-student expenditures to the human capital that the child receives from the school. The mapping is the same for all types in the sense that able and unable learners both benefit from an increase in

[6] A detailed Appendix is available from the author upon request.

the quality of their peer groups, as in Henderson et al. (1978). In all of the exercises here, the peer group is measured as the average learning ability in a school, $\sum_i n_s^i a^i$. Because parental income does not directly affect the human capital accumulated by a child, there are only two human capital outcomes associated with each school, one for the unable learners and one for the able learners:

$$h'^i_s = B(a^i)^\alpha \Big(\sum_i n_s^i a^i\Big)^\gamma (e_s)^\psi, \quad \forall i,\ s$$

My parameter choice is guided by the empirical literature on the determinants of educational outcomes. The parameter ψ, the elasticity of earnings with respect to educational expenditures, is chosen to be 0.1. This is based upon estimates of Card and Krueger (1992), Altonji and Dunn (1996), and Grogger (1996). The remaining two parameters of the human capital production function, α and γ, are harder to pin down. I choose γ, the parameter on the peer group, using an estimate from Black (1999). She finds that parents are willing to pay 2.1 percent more for a house associated with a school with 5 percent higher average test scores. People spend approximately 15 percent of consumption on housing services. I choose γ so that the difference in equilibrium price charged to a parent of a given type, across schools with identical expenditure levels but with a 5 percent difference in peer groups, is approximately 2.1 percent of 15 percent of equilibrium consumption. This yields a γ of 0.1. I choose the parameter on learning ability, α, following Henderson et al. (1978). In their study of peer effects, they find that last period's achievement (test score) has large effects on this period's achievement (test score). Their parameter estimates range from 0.5 to 0.7. Given this, I choose α to be 0.5. The parameter B is chosen so that next period's human capital is of the same magnitude as this period's human capital, $B = 11{,}500$. In the United States in 1992, 89 percent of eligible children attended public schools. The parameter on the parent's utility over her child's education, ξ, is chosen to yield 90 percent of the population attending public school, $\xi = 1$.

There are four types of parents, those endowed with low human capital and low learning ability, those endowed with low human capital and high learning ability, those endowed with high human capital and low learning ability, and those endowed with high human capital and high learning ability. Since there are only two income levels, the distribution of earnings is approximated by a two-point distribution. Suppose that 80 percent of the population is poor and 20 percent of the population is rich. Taking data from the 1992 census on total money earnings of full-time workers in the United States, the earnings of the poor are \$19,325, and the earnings of the rich are \$57,065. I assume that low learning ability corresponds to $a = 1$, and high learning ability corresponds to $a = 4$. Because the parameter on learning ability, α, is 0.5, this means that here the high-ability students get twice as much human capital from the same schooling. I assume that half of the poor and half of the rich are endowed with high learning ability. Therefore, $\lambda^{pu} = 0.4$, $\lambda^{pa} = 0.4$, $\lambda^{ru} = 0.1$, and $\lambda^{ra} = 0.1$. Human capital and learning ability endowments are then, $h^{pu} = 19{,}325$, $h^{pa} = 19{,}325$, $h^{ru} = 57{,}065$, $h^{ra} = 57{,}065$, and $a^{pu} = 1$, $a^{pa} = 4$, $a^{ru} = 1$, $a^{ra} = 4$. In the last section I discuss how sensitive the results are to the initial distribution, and the assumed learning ability differences.

A school is defined by its per-pupil expenditures and the fraction of each type of student attending. In creating the set of possible school types, I allow the fraction of each type attending a school to vary by 0.1, between 0 and 1, and the expenditure level to vary by $100, between 0 and some nonbinding upper bound. The grid of possible school types generally contains over 10,000 schools.

The tax rate is chosen to be 0.088, which implies 10.5 percent of total resources are spent on education, with 1.7 percent spent on private education, and 8.8 percent spent on public education. In the United States in 1990, the breakdown was 0.08 percent on private education and 8.8 percent on public education. The model overshoots on the amount spent on private education. This discrepancy could be because of the fact that most private schools in the United States are affiliated with a church. It is likely that not all resources used at these schools are fully recorded as expenditures.[7]

The equilibrium is described in Table 1. There are two schools operating, the public school and a private school. Only the rich, able learners attend the private school, and per-student expenditures are $4700. Both of the poor types and the rich, unable type attend the public school. The per-student expenditures are $2628. In reality, the per-student spending in private schools is not generally twice that in public schools. As mentioned earlier, the spending in private schools in the model overshoots that in the data; see Footnote 7. Here 90 percent of the students are attending public schools, which is close to the 89 percent that attended public schools in the United States in 1992 and follows from the calibration of the parameter ξ.

4.2. *Vouchers.* Suppose the government wants to give poor students the same opportunity to attend private schools as the rich.[8] One way to do this is to provide an educational voucher that can help defray the costs of private school tuition. In this exercise, parents who send their children to a private school receive an exogenously chosen lump sum voucher of $1500, which comes out of the public school tax revenues.[9] The tax remains an 8.8 percent proportional income tax. There are two channels through which the voucher affects educational expenditures at the public school. First, implementing a voucher system can lead to a major reduction in resources allocated to the public school because vouchers, financed from the public school budget, are given to those students who were already in a private school. Under this voucher plan, that implies a transfer from the public school budget to the rich, able students, causing per-student public school expenditure to fall. Second, when students leave the public school system as a result of the voucher policy, they take a fraction of the resources that is smaller than that which would be allocated to them if they stayed. So when students who were in the public school leave, the

[7] For example, think of schools in church buildings, church staff teaching, and church members volunteering.

[8] The only role for vouchers here is to give transfers to those attending private schools; vouchers do not provide competition that may cause the public school system to become more efficient.

[9] For computational reasons, I do not constrain parents to spend the entire voucher on education, although in most of the computational results they choose to do so. The parent sends her child to the private schools with probability $\sum_s \pi_s^i$, so the voucher that she receives is $v \sum_s \pi_s^i$.

Table 1
SCHOOLS: PUBLIC SYSTEM

	Public School	Private School
Measure	0.9	0.1
Expenditures	2628	4700
Peer group	2.33	4.0
Fraction of poor unable	0.44	
Poor unable price		
Fraction of poor able	0.44	
Poor able price		
Fraction of rich unable	0.11	
Rich unable price		
Fraction of rich able		1
Rich able price		4700

per-student public school expenditure rises. Whether per-student public school expenditure rises or falls will depend on how many students are originally in a private school, how many students leave the public school system as a result of the policy, and the size of the voucher.

The equilibrium schooling structure is given in Table 2. The rich, able type continues to attend a homogeneous private school. The expenditure level is greater at this school than at the private school without vouchers. This reflects the transfer that the rich, able type receives. There is also a new private school. The rich, unable students and the poor, able students leave the public school and form another private school. The expenditure level at this school is lower than that at the original public school and lower than that at the public school post–policy change. Notice that these two types are now attending a school with lower expenditures but a higher quality peer group. The poor, unable students, on the other hand, are left in the public school with higher expenditures than they had pre–policy change but also with a lower quality peer group.

At lower voucher levels, only the rich, able type makes use of the voucher. This is not a very interesting case, in that there is no change in schooling except that the

Table 2
SCHOOLS: VOUCHER OF $1500

	Public School	School 1	School 2
Measure	0.4	0.5	0.1
Expenditures	3664	2400	4900
Peer group	1.00	3.40	4.00
Fraction of poor unable	1		
Poor unable price			
Fraction of poor able		0.8	
Poor able price		1936	
Fraction of rich unable		0.2	
Rich unable price		4255	
Fraction of rich able			1
Rich able price			4900

elite private school has a higher expenditure level, while the public school has a lower expenditure level. There is a transfer from those attending the public schools to those already attending private schools. At higher voucher levels, the poor, unable type begins to use the voucher. When the voucher is \$2000, 13 percent of the poor, unable students leave the public school and form a private school with some of the poor, able students. There continues to be an elite homogeneous private school and a private school with a mix of the rich, unable students and the poor, able students. When the voucher is \$2300, the percentage of the poor, unable students that leave the public school jumps to 74 percent.

4.3. *Completely Private System.* In this section I return to the original model of a completely private system of schools. Table 3 contains the private regime equilibrium schooling structure. Three kinds of schools are being operated. The first school is composed of a mix of the rich types. Expenditures at that school are \$5200. The unable learners are subsidizing the able learners, paying prices \$9167 and \$4759, respectively. The second school has a mix across income levels and ability levels. The rich, unable are subsidizing the poor, able. The expenditure level at this school is \$3200. The last school is made up of the poor types. The expenditure level is lower, \$1700. And, as in the other schools, the unable types are subsidizing the able types. The total expenditure on schooling (average) is \$2422, which corresponds to 9 percent of total resources being spent on education. The resulting private schools almost mirror the three private schools that arise when the voucher is \$2300. The expenditure levels are slightly different due to the redistributive nature of the voucher.

The mixing that we see in this example is intuitive. The poor, unable types do not mix with the rich, able types, because not only would they have to subsidize the rich, they would also have to attend a school with a much higher expenditure level than they would prefer. By higher expenditure level than they would prefer, I mean that all else constant, poor types prefer lower expenditure levels. The mixing within income levels occurs because both types have the same preferences over expenditure levels, and both types gain from the subsidy. The mixing between rich, unable learners and poor, able learners occurs because even though both types prefer a

TABLE 3
SCHOOLS: PRIVATE SYSTEM

	School 1	School 2	School 3
Measure	0.11	0.22	0.67
Expenditures	5200	3200	1700
Peer group	3.7	2.8	2.2
Fraction of poor unable			0.6
Poor unable price			2688
Fraction of poor able		0.6	0.4
Poor able price		1840	218
Fraction of rich unable	0.1	0.4	
Rich unable price	9167	5239	
Fraction of rich able	0.9		
Rich able price	4759		

different level of expenditures, the rich gain from a better peer group, and the poor gain from the subsidy.

4.4. *Welfare and Distributional Effects.* Table 4 contains the welfare effects of switching from a public school system.[10] Keep in mind that in this framework all equilibria are Pareto efficient given the financing scheme. Looking at the measure of total welfare in Table 4, there are both welfare gains and losses associated with a switch from the public school system to a voucher system. The size of the voucher matters. The welfare costs of a voucher system decrease as the size of the voucher increases, starting at a 1.2 percent loss and ending with a 0.6 percent gain. This is due to the fact that under the smaller voucher policies, the poor, able students leave the public school, reducing the quality of the public school, but the voucher is not big enough to facilitate certain types of student mixing, which larger vouchers and the private system do, that is, mixing between the two poor types or mixing between the two rich types. Some of these gains can be attributed to the redistributional aspect of the voucher as well. This is a principle result of the article: small vouchers are often welfare decreasing. This is in contrast to Epple and Romano (1998a), who find welfare gains for all voucher sizes.

The welfare measure that is used masks how each individual type fares under a policy change. The last four columns of Table 4 contain information on how much each type needs to be paid in order to compensate for the policy change. There are two effects that need to be considered. First, there is the redistribution effect and how it is altered under the various policies. Recall that the tax bill associated with all of the policy changes (aside from the private system) is the same as in the public system. However, how the tax revenues are redistributed changes. I will refer to the second effect as the peer group pricing effect. Under the public school system, the parents of the poor, able students and the parents of the poor, unable students pay the same price. The parents of the poor, able students are not compensated for the positive peer group externality that their children bring to the school, while the parents of the poor, unable students are getting this positive externality for free.

TABLE 4
WELFARE COMPARISONS TO PUBLIC SYSTEM

Case	Welfare Cost	Poor Unable	Poor Able	Rich Unable	Rich Able
Voucher = 1500	0.012	\$931	−\$60	\$1288	−\$1494
Voucher = 2000	0.006	\$1268	−\$564	\$771	−\$1998
Voucher = 2300	−0.004	\$1212	−\$1073	\$811	−\$2324
Private	−0.006	\$1881	−\$579	−\$1708	−\$5072

[10] I also compute the welfare consequences of switching to alternate public systems associated with different tax rates. None yields the welfare gains of switching to the private system, $\tau = 0$. However, there is a set of tax rates that give rise to a welfare gain relative to the benchmark.

Under the private system of education, because each type faces a different price, the peer group externality is priced.

The poor, unable parents are hurt by all of the policy changes under consideration. Each poor, unable parent needs between \$931 (when the voucher is \$1500) and \$1881 (when there is a private system) in consumption to be as well off as she is in the public system. This follows from considering both of the two effects discussed above. Under a voucher some of the tax revenue that formerly went to the public school is now being redistributed to the rich, able parents whose children attend the private school. This loss is weighed against the possible gain in expenditures resulting from a decrease in the number of students in the public school. Here the gain outweighs the loss, and the expenditure level at the public school rises. However, the greater the voucher, the larger the role the private schools play, and the more the peer group externality is priced. In fact, all of the able students leave the public school, and the unable students no longer receive a higher quality peer group for free. So even though the expenditure level rises at the public school, the decrease in the quality of the peer group is more costly.

The poor, able parent is happier under a voucher system, as long as the voucher is large enough for her to use. The higher the voucher, the greater the redistribution, and the closer the system comes to the private system, allowing the parents of the poor, able students to extract subsidies from the parents of the unable students. The poor, able parents can give up between \$60 (when the voucher is \$1500) and \$1073 (when the voucher is \$2300) each in consumption to stay as well off as they are under the public school system. Because there is no redistribution associated with the private system, they are only willing to give up \$579 in consumption in the private system to stay as well off as they are under the public school system. This gain is entirely due to the peer group pricing effect.

Interestingly, the rich, unable parents are worse off under a voucher plan even though they make use of the voucher. They need between \$771 (when the voucher is \$2000) and \$1288 (when the voucher is \$1500) each in consumption to stay as well off as they are under the public school system. They are worse off due to the fact that they now must subsidize the parents of poor, able students in order to get a better peer group. However, as the voucher increases, they are not paying more in taxes, but they receive more in the voucher. So the negative effect of the redistribution is mitigated. They are better off under a private system, because even though they have to subsidize the able learners, there is no redistribution. They are willing to give up \$1708 each in consumption in the private system and remain as well off as they are under the public school system.

The rich, able parents are better off under all of the policy changes. When a voucher is imposed, this is a transfer from the public school to the parents of the rich, able students, all of whom are in the private school. A switch to the private school system is extremely beneficial in that it ends redistribution and allows them to fully extract subsidies for their child's ability. Rich, able parents can give up \$5072 each in consumption in the private school system and remain as well off as they are under the public school system. Note that this is essentially their tax bill when there is redistribution.

The average human capital accumulated when there is a public school is 41,913. The range of the distribution is 34,031, the Gini coefficient is 0.1781, and the coefficient of variation is 0.3465. These measures of inequality, under several levels of vouchers and under a completely private system, are given in Table 5. The percentage change of each inequality measure in Table 5 is in parentheses. All policies lead to an increase in inequality. But surprisingly these changes are not monotone in the size of the voucher. The larger voucher policy and the private system generally imply smaller increases in inequality. There are strong similarities between the voucher of \$1500 and the voucher of \$2000, and between the voucher of \$2300 and the private system, because the students mixing in each case are almost identical. Under the larger voucher and the private system there is a private school with the two rich types mixing, and when the voucher is \$2300, very few of the poor, unable type continue to attend the public school.

4.5. *Targeted Voucher Policies.* Most of the voucher policies that have been proposed in the United States involve giving lump sum transfers only to the poor.[11] In the previous examples, the voucher policy was extreme in that all types were eligible for the transfer. The biggest winner in all cases is the rich, able type. This is generally not the segment of the population that educational policy makers are trying to target with vouchers. Therefore, in the next example I look at what happens if only the poor types are granted the voucher for private school attendance.[12] The voucher considered is \$2000. When the voucher is \$1500, no one uses it, and the outcome is the same as under the public system. Recall that with a full voucher system, a voucher of \$1500 is used by the poor, able students. Here they do not use it, because the rich, unable students (who do not get the voucher) are unwilling to leave the public school and form a private school where they subsidize the poor, able students. One of the reasons the rich, unable students are unwilling to leave the public school system, aside from the fact that they do not receive the voucher, is that now the expenditures at the public school are not driven down by transfers to the rich, able students in the private school.

The equilibrium schooling structure, for a targeted voucher of \$2000, is contained in Table 6. There are four schools, the public school and three private schools. The poor, able type takes advantage of the voucher and leaves the public school. In order to get a better peer group, 70 percent of the rich, unable students follow them. Unlike under the full voucher plan, the other 30 percent of the rich, unable students remain in the public school. The rich, able type stays in the homogeneous school and is totally unaffected by the voucher policy, relative to the public system. The poor, unable students remain in the public school. They have a worse peer group, but higher expenditures, than under the public school system. The expenditures are higher because even though the voucher system takes resources from the public

[11] Recent research on non-flat-rate voucher systems includes Nechyba (1999, 2000) and Epple and Romano (1998b).

[12] Here that means the bottom 80 percent, which is not much different from the full voucher plan. Section 5 contains targeted voucher results for a variety of initial distributions.

TABLE 5
COMPARISONS TO PUBLIC SYSTEM, PERCENTAGE CHANGES IN PARENTHESES

Case	Mean h'	Cv h'	Range h'	Gini h'	Exp
Public school	41,914	0.3465	34,030	0.1781	2835
Voucher = 1500	42,105	0.3711	35,668	0.1916	3156
	(0.45)	(6.63)	(4.59)	(7.05)	
Voucher = 2000	41,774	0.3720	36,147	0.1950	2845
	(−0.34)	(6.85)	(5.86)	(8.67)	
Voucher = 2300	41,302	0.3556	35,748	0.1889	2551
	(−1.48)	(2.56)	(4.81)	(5.72)	
Private	41,105	0.3564	35,500	0.1894	2422
	(−1.97)	(2.78)	(4.14)	(5.97)	

TABLE 6
SCHOOLS: TARGETED VOUCHER = 2000

	Public School	School 1	School 2	School 3
Measure	0.43	0.24	0.23	0.10
Expenditures	3638	2100	2400	4700
Peer group	1.00	3.70	3.40	4.00
Fraction of poor unable	0.93			
Poor unable price				
Fraction of poor able		0.9	0.8	
Poor able price		1918	2004	
Fraction of rich unable	0.07	0.1	0.2	
Rich unable price		3742	3984	
Fraction of rich able				1
Rich able price				4700

schools, it also takes students. And once again, resources are not being transferred to the rich, private school attendees.

When the targeted voucher is increased to \$2300, the elite homogeneous school disappears. The rich, able students now mix with the rich, unable students as in the completely private system. There continues to be mixing between the rich, unable students and the poor, able students, but now there is just one such school. A new school with both types of poor students arises. The public school expenditures fall to \$3096, even though all of the rich, unable students leave the public school. The fact that they leave has no effect on the peer group in the public school, and in isolation would cause the per-student expenditure level in the public school to rise. However, the voucher rises by \$300, and all the poor, able students who are already in the private school receive this increase. This causes the per-student expenditure level to fall in the public school.

Tables 7 and 8 contain welfare and inequality measures for the two targeted voucher plans discussed and their corresponding full voucher plans. The targeted plan does no better than the full voucher plan in terms of inequality. The welfare costs associated with a targeted plan are greater than the corresponding full voucher

TABLE 7

COMPARISONS ACROSS SYSTEMS, PERCENTAGE CHANGES IN PARENTHESES

Case	Mean h'	Cv h'	Range h'	Gini h'	Exp
Public school	41,914	0.3465	34,030	0.1781	2835
Targeted voucher = 2000	41,943	0.3722	35,429	0.1918	3090
	(0.07)	(6.90)	(3.95)	(7.14)	
Voucher = 2000	41,774	0.3720	36,147	0.1950	2845
	(−0.34)	(6.85)	(5.86)	(8.67)	
Targeted voucher = 2300	41,458	0.3636	35,492	0.1909	2762
	(0.35)	(6.60)	(4.00)	(6.95)	
Voucher = 2300	41,302	0.3556	35,748	0.1889	2551
	(−1.10)	(4.70)	(4.12)	(6.71)	

TABLE 8

WELFARE COMPARISONS TO PUBLIC SYSTEM

Case	Welfare Cost	Poor Unable	Poor Able	Rich Unable	Rich Able
Target = 2000	0.016	$944	−$495	$2565	$0
Voucher = 2000	0.006	$1268	−$564	$771	−$1998
Target = 2300	0.012	$974	−$875	$2787	$0
Voucher = 2300	−0.004	$1212	−$1073	$811	−$2324

plan. This is due to the fact that both of the rich types are worse off under the targeted plan as compared to the full plan; the poor, able type is a little better off under the full plan, which leaves the poor, unable type as the only type better off under the targeted plan than under the full voucher plan.

The argument in favor of the targeted voucher plan hinges on the individual welfare effects of the policy. Obviously, both rich types lose as compared to the full voucher plan. They are no longer receiving the voucher. The rich, able type is just as well off under the targeted plan as under no voucher plan (as long as their children do not attend the public school), but the rich, unable type is much worse off, as the voucher cuts into the public school resources. It may seem a bit odd that the poor, able students also prefer the full voucher plan to the targeted plan, given that they qualify for the voucher in both cases and fully make use of it in both cases. This is especially striking when the voucher is $2300. Because the public school expenditure level is not as adversely affected by the targeted voucher plan, over 60 percent of the poor, unable students remain in the public school. Whereas in the full voucher plan, (when the voucher is $2300) 79 percent of the poor, unable students leave the public school. When they leave the public school they join private schools and subsidize the poor, able students. Only the poor, unable types are better off under the targeted plan. If their children remain in the public school, the targeted plan siphons fewer resources from the public school into the voucher. If their children use the voucher, they benefit from a higher quality peer group. However, it is important to point out that the poor, unable types are always worse off as compared to the public system.

The big winner under a full voucher plan is the rich, able type. Most voucher policies are not aimed at this segment of the population. So the question then is, even though the overall welfare costs are slightly higher with targeting, is it better that they are now concentrated on the rich, unable type, as opposed to the poor, unable type?

5. SENSITIVITY

5.1. *Initial Distribution.* In all of the preceding analyses, there are two different income levels and two different ability levels. It is assumed that 80 percent of the population is poor and that 20 percent of the population is rich, and that able students receive two times more human capital, given the same educational expenditure level and peer group quality, than unable students. I perform identical policy experiments when 50 percent of the population is poor and 50 percent is rich, when 30 percent is poor and 70 percent is rich, and when the able students receive less than two times more human capital, fixing all other schooling inputs, than unable students.

Although some of the schooling structure outcomes change, for instance, when 50 percent of the population is poor and when 30 percent of the population is poor, all types attend the public school, some general patterns remain. First, vouchers in all cases can be associated with both welfare gains and losses, and the welfare loss decreases monotonically with the voucher size. The magnitudes range from around a 1 percent loss to around a 3 percent gain. Second, there are nonmonotonic increases in inequality resulting from vouchers. Depending upon the measure, these increases fall between a 0 percent increase and a 30 percent increase. Lastly, while targeted voucher plans have similar to slightly higher welfare costs, in all but one case, they also imply similar increases in inequality. When 30 percent of the population is poor, the targeted plan implies significantly lower increases in inequality. This leads me to believe that given a finer distribution of income, targeting vouchers to the poorest segment of the population could lead to smaller increases in inequality.

5.2. *The Peer Group Parameter.* There is uncertainty regarding the parameters in the human capital production function. In this section I look at the effect varying the peer group parameter, γ, has on the welfare and inequality implications of these voucher policies. The other parameters remain fixed. Table 9 contains the welfare costs of switching from a public system to several alternate systems when $\gamma = 0$, 0.05, 0.1, 0.15, and 0.2. The welfare cost is decreasing in the size of the voucher in all but one case. For values of $\gamma \geq 0.1$, initially there are welfare losses. Table 10 contains changes in one of the measures of inequality, the Gini coefficient, using the public system as a base. For values of $\gamma \geq 0.1$, there is less inequality

TABLE 9

SENSITIVITY TO PEER GROUP PARAMETER: WELFARE COSTS OF SWITCHING FROM A PUBLIC SYSTEM

Case	$\gamma = 0$	$\gamma = 0.05$	$\gamma = 0.1$	$\gamma = 0.15$	$\gamma = 0.2$
Voucher = 1500	−0.0046	−0.0046	0.0122	0.0158	0.0202
Voucher = 2000	−0.0076	−0.0004	0.0059	0.0062	0.0062
Voucher = 2300	−0.0085	−0.0074	−0.0036	−0.0038	−0.0033
Private	−0.0105	−0.0097	−0.0059	−0.0064	−0.0058

TABLE 10
SENSITIVITY TO PEER GROUP PARAMETER: PERCENTAGE CHANGES IN GINI COEFFICIENTS WITH PUBLIC SYSTEM AS A BASE

Case	$\gamma = 0$	$\gamma = 0.05$	$\gamma = 0.1$	$\gamma = 0.15$	$\gamma = 0.2$
Voucher = 1500	0.0375	0.0459	0.0705	0.1136	0.1510
Voucher = 2000	0.0325	0.0702	0.0867	0.1106	0.1288
Voucher = 2300	0.0392	0.0681	0.0572	0.0822	0.1193
Private	0.0496	0.0598	0.0597	0.0779	0.1193

associated with higher levels of voucher and the private system than with lower voucher levels. Note that all changes result in higher inequality than in the public system. For values of $\gamma \leq 1$, the change in the Gini coefficient is not monotone. The conclusion that a larger voucher dominates a smaller voucher on both welfare and inequality dimensions continues to hold.

5.3. *Financing Assumption.* In all of the policy exercises performed, it is assumed that the tax rate remains fixed and the expenditure level in the public school adjusts to balance the budget. This implies a fixed pool of resources devoted to the public school and possibly vouchers, which means that policy changes can affect the per-student expenditure level in the public school. An alternative approach is to fix the per-student expenditure level at the public school and adjust the tax rate to balance the budget. Instead of affecting the spending in the public school, policy changes will only affect consumption through the change in the tax rate.

I fix the expenditure level in the public school at \$2600 (it was \$2628 when the tax rate was fixed). The public system equilibrium is almost identical to the public system equilibrium previously, the only difference being the \$28 decrease in public school spending and its corresponding 0.1 percent tax decrease. Welfare measures associated with switching from the public system are given in Table 11. Compare these to those in Table 4. The welfare cost is lower in all cases when the expenditure level is fixed. Looking at the individual effects, the poor, unable students are worse off under the fixed expenditure assumption. Under the fixed tax rate assumption, when vouchers are implemented, it is possible that enough students leave the public school that the per-student expenditures actually rise. When the expenditure level is fixed, this cannot happen. And the poor, unable students remaining in the public school receive a lower expenditure level along with the reduction in their peer group. Both

TABLE 11
SENSITIVITY TO FINANCING ASSUMPTION: WELFARE COSTS OF SWITCHING FROM A PUBLIC SYSTEM

Case	Welfare Cost	Poor Unable	Poor Able	Rich Unable	Rich Able
Voucher = 1500	0.006	\$1273	−\$365	\$373	−\$2343
Voucher = 2000	−0.005	\$1257	−\$1030	\$363	−\$2724
Voucher = 2300	−0.006	\$1255	−\$1258	\$850	−\$2429
Private	−0.006	\$1879	−\$581	−\$1708	−\$5020

rich types appear to be better off under the fixed expenditure assumption. The last column of Table 12 contains the tax rates associated with each policy. The tax rate for all policy changes is lower than the tax rate in the public system. Obviously, the rich prefer less redistribution to more redistribution. The poor, able students prefer the fixed expenditure form of financing to the fixed tax rate form as well. When vouchers are implemented, the poor, able students are able to extract subsidies from the unable students. Under the fixed expenditure assumption relative to the fixed tax rate assumption, more of the unable students choose to leave the public schools. This follows from the fact that, in this case, per-student expenditures in the public schools cannot rise and compensate for the loss in the peer group. Therefore, the poor, able students can extract greater subsidies from the unable students. Despite these differences, the conclusion that smaller vouchers may imply welfare losses holds true under both financing assumptions.

Measures of inequality are given in Table 12. Compare these to those in Table 5. When the expenditure level is fixed, the inequality measures are monotone in the size of the voucher, but decreasing. When the voucher is \$2300 all inequality measures are lower than under the private system. Although the results are slightly different across financing assumptions, the same basic conclusion holds. The larger voucher policy and the private system generally imply smaller increases in inequality.

5.4. *Lottery Assumption.* The assumption that there is a mechanism through which parents can randomize over schooling choices allows all gains from trade to be exhausted. In addition, assuming a randomizing mechanism also convexifies preferences. This, along with several other assumptions (such as constant returns to scale technologies), ensures that an equilibrium exists. Cole and Prescott (1997) demonstrate the equivalence of a lottery equilibrium in this environment and a gambling equilibrium. Their gambling economy consists of two stages. In the first stage, a parent makes a fair gamble over wealth transfers, and in the second stage, conditional on her realized wealth level, the parent chooses her consumption and the single school her child will attend. The lottery assumption allows parents to engage in implicit wealth gambles. While the lottery assumption does guarantee an equilibrium exists, it does not guarantee uniqueness.

TABLE 12

SENSITIVITY TO FINANCING ASSUMPTION: COMPARISONS TO PUBLIC SYSTEM, PERCENTAGE CHANGES IN PARENTHESES

Case	Mean h'	Cv h'	Range h'	Gini h'	Tax
Public school	41,876	0.3467	34,060	0.1782	0.087
Voucher = 1500	41,758	0.3830	36,674	0.1983	0.072
	(−0.28)	(9.48)	(7.13)	(10.14)	
Voucher = 2000	41,222	0.3560	36,193	0.1888	0.075
	(−1.59)	(2.61)	(5.89)	(5.61)	
Voucher = 2300	41,187	0.3545	35,108	0.1878	0.086
	(−1.67)	(2.20)	(2.99)	(5.11)	
Private	41,105	0.3564	35,500	0.1894	0
	(−1.88)	(2.72)	(4.06)	(5.91)	

TABLE 13
SCHOOLS: BENCHMARK WITH NO LOTTERY

	School 1	School 2
Measure	0.2	0.8
Expenditures	5200	1800
Peer group	2.50	2.50
Fraction of poor unable		0.5
Poor unable price		3034
Fraction of poor able		0.5
Poor able price		566
Fraction of rich unable	0.5	
Rich unable price	7696	
Fraction of rich able	0.5	
Rich able price	2704	

One obvious question to ask is, how much do the results depend on the lottery assumption? In the benchmark case, the completely private system, both the poor, able parents and the rich, unable parents are randomizing across schools in equilibrium. However, in the public school system and when the voucher is \$1500, no parent randomizes over schools. Each child attends one school with probability one. In these cases, although it is available, the lottery is not used. In order to quantify the importance of this assumption on my results, I only need to compute an equilibrium for the benchmark case without lotteries; see Table 13. An equilibrium is not guaranteed to exist without lotteries, but I have found one here.

The school with the poor, able students and the rich, unable students that operates when there is a lottery disappears when there is no longer access to a lottery. The two other schools, with mixes within income classes, remain. The expenditure levels are almost identical to those in the lottery case, while the peer groups are different (3.7 and 2.2 with the lottery and 2.5 and 2.5 without the lottery). The welfare loss associated with a switch from an environment with a randomizing device to an environment without one is 0.29 percent of consumption. Everyone is better off having access to a lottery.

The results shown in Table 14 can be compared to those in Table 5. The major difference is in the changes in the range and the Gini coefficient. With no lottery, a

TABLE 14
COMPARISONS TO PUBLIC SYSTEM WHEN THERE IS NO LOTTERY, PERCENTAGE CHANGES IN PARENTHESES

Case	Mean h'	Cv h'	Range h'	Gini h'	Exp
Public school	41,914	0.3465	34,030	0.1781	2835
Voucher = 1500	42,105	0.3711	35,668	0.1916	3156
	(0.45)	(6.63)	(4.59)	(7.05)	
Private	40,900	0.3547	32,639	0.1754	2480
	(−2.48)	(2.31)	(−4.26)	(−1.54)	

TABLE 15
WELFARE COMPARISONS TO PUBLIC SYSTEM WHEN THERE IS NO LOTTERY

Case	Welfare Cost	Poor Unable	Poor Able	Rich Unable	Rich Able
Voucher = 1500	0.012	$931	−$60	$1288	−$1494
Private	−0.004	$1887	−$582	−$1098	−$5237

switch to a private system from the public system actually reduces inequality as measured by the range and the Gini coefficient.

Table 15 contains the welfare effects of switching from the public system to a voucher or private system when there is no access to a lottery. Compare this table to Table 4 to see the consequences of the lottery assumption. Obviously, a switch from the public system to a voucher of $1500 is identical, because the lottery is not used in either case, even when it is available. Switching to a private system without a lottery increases welfare by 0.4 percent, as opposed to the 0.6 percent increase when there is a lottery.

6. CONCLUSION

In this article, I use a general equilibrium framework to predict the effects of implementing several voucher policies on how students sort themselves across schools and what expenditure levels these schools have. Given these changes at the school level, I look at the corresponding welfare and distributional effects of these policies. I begin with a perfectly private system of schooling. In equilibrium there are several mixed schools, in which the unable learners subsidize the able learners. I then incorporate a public school into the system of private schools. I find an equilibrium and then implement a voucher financing system. I compare the voucher equilibrium with the public school equilibrium.

I find that if the voucher is $1500, there is a welfare loss of 0.9 percent, and if the voucher is $2300, there is a welfare gain of 0.5 percent. Therefore, implementing a voucher can lead to either welfare gains or losses. The welfare loss decreases monotonically with the voucher size. All of the voucher policies imply an increase in inequality regardless of the measure of inequality used, the coefficient of variation, the Gini coefficient, or the range. The increases range from 2.5 to 8.7 percent. Interestingly, these increases are not monotone in the voucher size. When the voucher is $1500, inequality increases substantially, but it increases by less as the voucher gets larger. This is due to the fact that at first the voucher is large enough to motivate the poor, able students to leave the public school, but not large enough to imply all of the kinds of mixing that arise in the private schooling case. As the voucher gets larger, these mixes are exploited. Lastly, I consider a targeted voucher policy, where only the poor are eligible for a voucher. While this policy entails similar increases in inequality and slightly greater welfare costs or smaller welfare gains than the full voucher plan, the welfare losses are shifted from the poor, unable students to the rich, unable students. These findings highlight that the choice of voucher size can be important in terms of both welfare and inequality.

It is important to have a framework that includes peer effects when studying the consequences of various educational policies. As has been shown here, a voucher system can lead to a change in the quality of the peer group and expenditure level at the public school. These changes can imply either welfare gains or losses, although all are associated with increases in inequality. In order to concentrate on the effects of the peer group, I abstract from several other important issues. First, I assume that the private schools and the public school have identical technologies for producing human capital. It is often claimed that public schools are at a disadvantage because they have higher administrative costs and higher costs due to teachers' unions. It would be straightforward to incorporate that into this framework, by changing the technology of the public school. A related issue is competition between public and private schools. Perhaps if private schools are more efficient than the public schools, encouraging competition between the two would raise outcome levels at both types of schools. I leave this issue to future work. Second, I assume a relatively passive public sector, in the sense that it contains only one school. It would be interesting to explore the effects of these kinds of policies when there are several public schools, that is, suburban and inner city schools. In this case the equilibrium is not Pareto optimal, because the peer group effect is not priced, so my method of relying on the social planner's problem to find the equilibrium can no longer be used. A new computational method needs to be developed.

APPENDIX

PROOF OF LEMMA 1. If the parent is allowed to directly choose the expenditure level at the homogeneous school that her child attends with probability one, her problem is given by

$$\begin{aligned} \max_{c,e}\ & \log(c^i) + \xi \log(Ba^{i\alpha}\bar{a}_s^{\gamma}e_s^{\psi}) \\ \text{s.t.}\quad & c^i + e_s = h^i \end{aligned} \tag{A.1}$$

The first-order condition yields $e_s = h^i\xi\psi/(1+\xi\psi)$, which is independent of ability, but is dependent on income. ■

PROOF OF PROPOSITION 1.[13] Suppose we have rich and poor students of the same ability.

When attending a homogeneous school with probability one, the rich prefer an expenditure level e^r and pay a price $p = e^r$. When attending a homogeneous school with probability one, the poor prefer an expenditure level e^p and pay a price $p = e^p$. These are outcomes that each type can attain regardless of what others do; they can

[13] I assume that the set of possible school types is large enough to contain all the possible alternative schools considered in these proofs.

always separate out and only attend a homogeneous school. Note that if $h^r > h^p$, then $e^r > e^p$.

I will show that there is no expenditure level at which they will find it preferable to mix. This is because they both have the same ability, so there is no role for a subsidy. They get the same peer group if they mix as they do if they stay separate.

Pick an expenditure level e. If $e \neq e^p$, the price the poor students are willing to pay to attend this school must be less than e, otherwise they would have chosen e as their preferred expenditure level. If $e = e^p$, then the price the poor students are willing to pay is e.

The argument follows the same lines for the rich students. If $e \neq e^r$, the price the rich students are willing to pay to attend this school must be less than e, otherwise they would have chosen e as their preferred expenditure level. If $e = e^r$, then the price the rich students are willing to pay is e.

Since $e^r > e^p$, at most one student type is willing to pay e, and at least one student type is only willing to attend for a price below e. Therefore, the school is unable to make positive profits at an expenditure level of e and will not operate. Hence no school composed of two types will contain poor and rich students of the same ability level. ■

Proof of Lemma 2. Let $S = (n_1,\ n_2,\ n_3,\ 1 - n_1 - n_2 - n_3,\ e)$ be a school with a mix of three or four types, so at most one of the following is true: $n_1 = 0,\ \ n_2 = 0,\ \ n_3 = 0,\ \ 1 - n_1 - n_2 - n_3 = 0$.

Consider the following four schools with a mix of two types:

$$S_1 = (n_1 + n_3,\ 1 - n_1 - n_3,\ 0,\ 0,\ e)$$
$$S_2 = (0,\ 1 - n_1 - n_3,\ n_1 + n_3,\ 0,\ e)$$
$$S_3 = (0,\ 0,\ n_1 + n_3,\ 1 - n_1 - n_3,\ e)$$
$$S_4 = (n_1 + n_3,\ 0,\ 0,\ 1 - n_1 - n_3,\ e)$$

Note that all five schools, $S,\ S_1,\ S_2,\ S_3$, and S_4, have the same expenditure level, e, and average ability level, $(n_1 + n_3)a^u + (1 - n_1 - n_3)a^a$. They, therefore, imply the same human capital outcomes.

Take the convex combination:

$$x \cdot S_1 + y \cdot S_2 + z \cdot S_3 + (1 - x - y - z) \cdot S_4 = S$$

This yields the following two equations:

$$y = \frac{n_3}{n_1 + n_3} - z \tag{A.2}$$

$$x = \frac{n_2}{1 - n_1 - n_3} - \frac{n_3}{n_1 + n_3} + z \tag{A.3}$$

It will next be shown that $\exists z \geq 0$ s.t. Equations (A.2) and (A.3) hold, $x + y + z \leq 1,\ \ x \geq 0$, and $y \geq 0$.

$$x + y + z \leq 1, \quad \text{Equations (A.2) and (A.3)} \ \Rightarrow\ z \leq \frac{1 - n_1 - n_2 - n_3}{1 - n_1 - n_3}$$

$$y \geq 0, \quad \text{and Equation (A.2)} \;\Rightarrow\; z \leq \frac{n_3}{n_1 + n_3}$$

$$x \geq 0, \quad \text{and Equation (A.3)} \;\Rightarrow\; z \geq \frac{n_3}{n_1 + n_3} - \frac{n_2}{1 - n_1 - n_3}$$

Therefore,

$$\frac{n_3}{n_1 + n_3} - \frac{n_2}{1 - n_1 - n_3} \leq z \leq \min\left(\frac{1 - n_1 - n_2 - n_3}{1 - n_1 - n_3}, \; \frac{n_3}{n_1 + n_3}\right)$$

Case 1. If

$$\min\left(\frac{1 - n_1 - n_2 - n_3}{1 - n_1 - n_3}, \; \frac{n_3}{n_1 + n_3}\right) = \frac{n_3}{n_1 + n_3}$$

then obviously there is a $z \geq 0$ such that

$$\frac{n_3}{n_1 + n_3} - \frac{n_2}{1 - n_1 - n_3} \leq z \leq \frac{n_3}{n_1 + n_3}$$

Case 2. If

$$\min\left(\frac{1 - n_1 - n_2 - n_3}{1 - n_1 - n_3}, \; \frac{n_3}{n_1 + n_3}\right) = \frac{1 - n_1 - n_2 - n_3}{1 - n_1 - n_3}$$

is there a $z \geq 0$ such that

$$\frac{n_3}{n_1 + n_3} - \frac{n_2}{1 - n_1 - n_3} \leq z \leq \frac{1 - n_1 - n_2 - n_3}{1 - n_1 - n_3}$$

Take

$$\frac{n_3}{n_1 + n_3} - \frac{n_2}{1 - n_1 - n_3} \leq \frac{1 - n_1 - n_2 - n_3}{1 - n_1 - n_3}$$

Add $n_2/(1 - n_1 - n_3)$ to both sides to get

$$\frac{n_3}{n_1 + n_3} \leq 1$$

Therefore, there is a $z \geq 0$ such that

$$\frac{n_3}{n_1 + n_3} - \frac{n_2}{1 - n_1 - n_3} \leq z \leq \frac{1 - n_1 - n_2 - n_3}{1 - n_1 - n_3}$$

Given this $z \geq 0$, y and x can be constructed from Equations (A.2) and (A.3), respectively. Therefore, S can be represented as a convex combination of schools S_1, S_2, S_3, and S_4. ■

PROOF OF PROPOSITION 2. All of the schools that comprise the convex combination yield the same level of human capital, and charge the same price, as the original school. Therefore, given the linearity of utility over schooling, Proposition 2 follows from Lemma 2. ■

REFERENCES

ALTONJI, J. G., AND T. A. DUNN, "Using Siblings to Estimate the Effect of School Quality on Wages," *Review of Economics and Statistics* 78 (1996), 665–71.

AARONSON, D., "Using Sibling Data to Estimate the Impact of Neighborhoods on Children's Educational Outcomes," Working Paper Series, Federal Reserve Bank of Chicago, WP-96-19, 1996.

BENABOU, R., "Equity and Efficiency in Human Capital Investment: The Local Connection," *Review of Economic Studies* 63 (1996), 237–64.

BLACK, S. E., "Do Better Schools Matter? Parental Valuation of Elementary Education," *Quarterly Journal of Economics* 114 (1999), 577–99.

CARD, D., AND A. KRUEGER, "Does School Quality Matter? Returns to Education and the Characteristics of Public Schools in the United States," *Journal of Political Economy* 100 (1992), 1–40.

CAUCUTT, E. M., "Peer Group Effects in Applied General Equilibrium," *Economic Theory* 17 (2001), 25–51.

COLE, H. L., AND E. C. PRESCOTT, "Valuation Equilibria with Clubs," *Journal of Economic Theory* 74 (1997), 19–39.

COLEMAN, J. S., E. Q. CAMPBELL, C. J. HOBSON, J. MCPARTLAND, A. M. MOOD, F. D. WEINFELD, AND R. L. YORK, *Equality of Educational Opportunity* (Washington: U.S. Department of Health, Education, and Welfare, National Center Educational Statistics, 1966).

DE BARTOLOME, C. A. M., "Equilibrium and Inefficiency in a Community Model with Peer Group Effects," *Journal of Political Economy* 98 (1990), 110–33.

EPPLE, D., AND R. E. ROMANO, "Competition between Private and Public Schools, Vouchers and Peer Group Effects," *American Economic Review* 88 (1998a), 33–62.

———, AND ———, "Educational Vouchers and Cream Skimming," mimeo, University of Florida, 1998b.

EVANS, W. N., W. E. OATES, AND R. M. SCHWAB, "Measuring Peer Group Effects: A Study of Teenage Behavior," *Journal of Political Economy* 100 (1992), 966–91.

FERNANDEZ, R. AND R. ROGERSON, "Income Distribution, Communities, and the Quality of Public Education," *Quarterly Journal of Economics* 61 (1996), 135–64.

———, AND ———, "Public Education and Income Distribution: A Dynamic Quantitative Evaluation of Education-Finance Reform," *American Economic Review* 88 (1998), 813–33.

GLOMM, G., AND B. RAVIKUMAR, "Public versus Private Investment in Human Capital: Endogenous Growth and Income Inequality," *Journal of Political Economy* 100 (1992), 818–34

GROGGER, J., "School Expenditures and Post-Schooling Earnings: Evidence from High School and Beyond," *Review of Economics and Statistics* 78 (1996), 628–37.

HANUSHEK, E. A., "The Economics of Schooling," *Journal of Economic Literature* 24 (1986), 1141–77.

HENDERSON, J. V., P. MIESZKOWSKI, AND Y. SAUVAGEAU, "Peer Group Effects and Educational Production Functions," *Journal of Public Economics* 10 (1978), 97–106.

LUCAS, R. E., JR., *Models of Business Cycles* (Oxford: Basil Blackwell, 1987).

NECHYBA, T. J., "Public School Finance in a General Equilibrium Tiebout World: Equalization Programs, Peer Effects and Private School Vouchers," Working Paper 5642, National Bureau of Economic Research, 1996.

———, "School Finance Induced Migration Patterns: The Impact of Private School Vouchers," *Journal of Public Economic Theory* 1 (1999), 5–50.

———, "Mobility, Targeting and Private School Vouchers," *American Economic Review* 90 (2000), 130–46.

ROGERSON, R., "Indivisible Labor, Lotteries and Equilibrium," *Journal of Monetary Economics* 21 (1988), 3–16.

SUMMERS, A., AND B. WOLFE, "Do Schools Make a Difference?," *American Economic Review* 67 (1977), 639–52.