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Correlation, Consumption, Confusion, or Constraints: Why Do Poor Children Perform so Poorly?*

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Abstract

Early developing and persistent gaps in child achievement by family income combined with the importance of adolescent skill levels for schooling and lifetime earnings suggest that a key component of intergenerational mobility is determined before individuals enter school. After documenting important differences in early child investments by family income, we study four leading mechanisms thought to explain these gaps: intergenerational ability correlation, consumption value of investment, information frictions, and credit constraints. We evaluate whether these mechanisms are consistent with other stylized facts related to the marginal returns on investments and the effects of parental income on child investments and skills.

Keywords: Credit constraints; human capital; intergenerational mobility; uncertainty

JEL classification: *D*84; *D*91; *I*24; *I*26; *J*24

I. Introduction

Adolescent skill and achievement gaps by parental income can explain a substantial share of subsequent differences in educational attainment and lifetime earnings (Keane and Wolpin, 1997; Cameron and Heckman, 1998; Carneiro and Heckman, 2002), suggesting that an important component of intergenerational economic and social mobility is determined by the time

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children reach adolescence. Perhaps more troubling, sizeable differences in achievement by parental income are already evident by very young ages, persisting throughout childhood (Carneiro and Heckman, 2002; Cunha *et al.*, 2006; Cunha and Heckman, 2007; Cunha, 2013). This raises the possibility that a generation's fate might be sealed by the time it enters school.[1] Altogether, this evidence suggests that a complete understanding of intergenerational mobility and its implications for economic and social policy requires convincing answers to the vexing question: why do poor children perform so poorly?

Given the importance of family investments for early child development (Todd and Wolpin, 2007; Cunha and Heckman, 2008; Cunha *et al.*, 2010; Del Boca *et al.*, 2014; Pavan, 2014), we concentrate on understanding why low-income families invest so much less in their young children compared to higher-income families (Guryan *et al.*, 2008; Kaushal *et al.*, 2011). We consider four broad mechanisms often thought to explain early investment and achievement gaps by family income.

1. The natural ability of children and parents might be correlated (Becker and Tomes, 1979, 1986). If child achievement is an increasing function of own ability, then a positive intergenerational ability correlation can generate the income–achievement gradients documented in the literature.
2. Parents might enjoy making investments in their children. If investments provide a direct benefit to parents above and beyond the future labor market returns to children, then parents will choose to invest more as their income rises, in a similar way as they would purchase more of any other normal good (Lazear, 1977). It is also possible that low- and high-income families place different intrinsic value on investments or human capital more generally (Abbott *et al.*, 2013).
3. Low-income parents might be poorly informed about the productivity of, or returns to investments in, their children (Cunha *et al.*, 2013; Cunha, 2014; Dizon-Ross, 2015). For example, poor parents might incorrectly believe that investments in their young children are unproductive (or poorly rewarded in the labor market), or they might simply face greater uncertainty in the productivity of or returns to investments. Alternatively, poor parents might recognize the importance of investing in their children, but they might not know which types of investment activities/goods are most productive.
4. Poor families might be unable to invest efficiently in their children due to limits on their capacity to borrow against their own future income

[1] Recent studies show that these early achievement and educational attainment gaps have been growing in the United States for decades (Belley and Lochner, 2007; Reardon, 2011).

or against the potentially high returns on investments in their children (Becker and Tomes, 1979, 1986; Caucutt and Lochner, 2006, 2012; Cunha *et al.*, 2006; Cunha and Heckman, 2007; Cunha, 2013; Lee and Seshadri, 2014).

We use a simple framework of dynamic human capital investment to formally examine whether these mechanisms are also able to account for other important stylized facts in the literature on child development. In particular, we focus on four well-established findings related to the marginal returns to early investment and the role of family income: (i) the high marginal returns to early investments for economically disadvantaged children; (ii) lower returns on marginal investments for higher-income children; (iii) exogenous increases in family income leading to greater investments in children and improved childhood outcomes; (iv) the impacts of income on child investments, achievement, and educational attainment being greater if the income is earned (or received) when children are young.[2] While our analysis is not intended to determine which mechanism is most important for explaining income-based achievement gaps, it is useful for helping us to understand which mechanisms are needed to provide a more complete picture of the child development process and the role of family income.[3] This is important because the different mechanisms can have very different policy implications. For example, if investment and achievement gaps are driven only by intergenerational ability correlations or a "consumption" value of investment, then investments in children are likely to be economically efficient (in the absence of human capital externalities) and policies designed to improve equity will be inefficient.[4] By contrast, either information-based or credit market frictions can lead to inefficiently low investments in economically disadvantaged children. In this case, it might be possible to simultaneously improve both equity and efficiency through well-designed policies.

We organize this paper in the following way. In Section II, we briefly document differences in child achievement and investment levels by family

[2] We also briefly discuss other evidence related to specific mechanisms in Sections V–VIII where those mechanisms are considered in detail.

[3] See Cunha (2014) for a novel effort to empirically decompose the relative importance of a similar set of mechanisms using unique data on parental perceptions and stated choices about investments in children under different hypothetical budget sets. While we do not empirically evaluate the relative importance of different mechanisms, our theoretical analysis is based on a more general dynamic human capital investment model. We consider a wide range of information frictions and explicitly model intertemporal borrowing constraints.

[4] Of course, it might be socially desirable to encourage investment beyond the privately optimal amount due to human capital externalities in production (Moretti, 2004a, 2004b) or related to crime (Lochner and Moretti, 2004) or citizenship (Milligan *et al.*, 2004).

income, using data from the Children of the National Longitudinal Survey of Youth (CNLSY). Then, we summarize evidence on four additional stylized facts from the literature on child development in Section III. In Sections IV–VIII, we develop and analyze a unified framework of dynamic skill investment that incorporates all four potential mechanisms commonly thought to drive investment and achievement gaps by family income. We use this framework to formally examine whether the explanations are consistent with the stylized facts in Section III as well as other evidence in the child development literature. In Section IX, we conclude with a summary of our main results and their implications for future research.

II. Child Achievement and Investment Gaps by Family Income

In this section, we document differences in child achievement and investment behavior by family income using data from the CNLSY. A longitudinal survey that links mothers with their children, the CNLSY contains excellent measures of family background and income (starting in 1979), as well as biennial measures of child mathematics and reading achievement and family investments in children (beginning in 1986).[5]

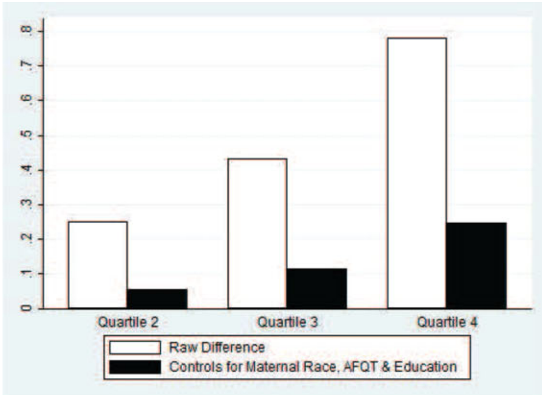
We use background measures of maternal education, race/ethnicity, and "ability" as measured by the Armed Forces Qualifying Test (AFQT).[6] As a medium-run measure of family income, we average all available reports of earnings by the mother and her spouse (if married) from the child's birth to ages 6 or 7 (depending on which of these ages achievement and investments in children were measured).[7] Child achievement is measured by the Peabody Individual Achievement Tests (PIAT) in mathematics, reading recognition, and reading comprehension; these measures are standardized to have a mean of zero and standard deviation of one at each age. A number of child investment activities/inputs are also reported in the CNLSY, as we discuss below.

Figure 1 documents sizeable differences in mathematics and reading achievement at ages 6–7 by family income quartile. The white bars represent raw differences in achievement between the reported parental income

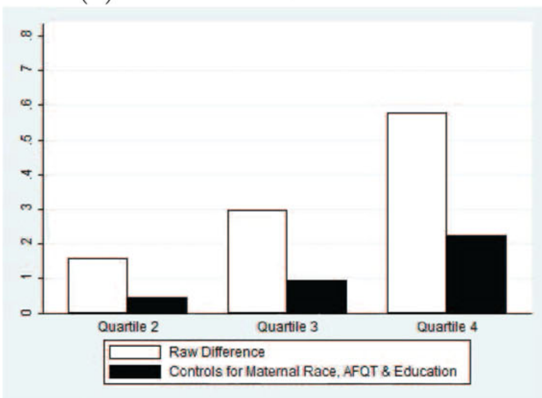
[5] As is standard in studies using the CNLSY, our (unweighted) analysis is based on children born to mothers from the random sample of the NLSY79. Thus, distributions are based on children born to a random sample of American women born between 1957 and 1964, which might differ from distributions for any specific cohort of children.

[6] (Nearly) all CNLSY mothers, born between 1957 and 1964, took the AFQT in 1980 as part of the survey. The AFQT tests basic mathematics and verbal/reading skills.

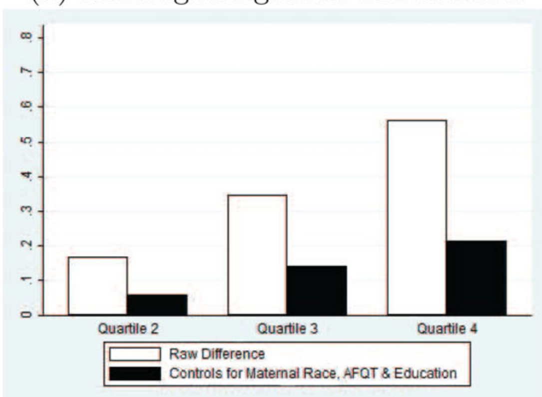
[7] Before averaging across time, we discount all income back to the child's birth year using a 5 percent discount rate, so our earnings measure reflects average discounted family earnings from the child's birth up to ages 6–7. Individuals are dropped if fewer than three income reports are available.



(a) Mathematics achievement



(b) Reading recognition achievement



(c) Reading comprehension achievement

Fig. 1. Achievement gaps at ages 6–7 by parental income quartile (relative to quartile 1)

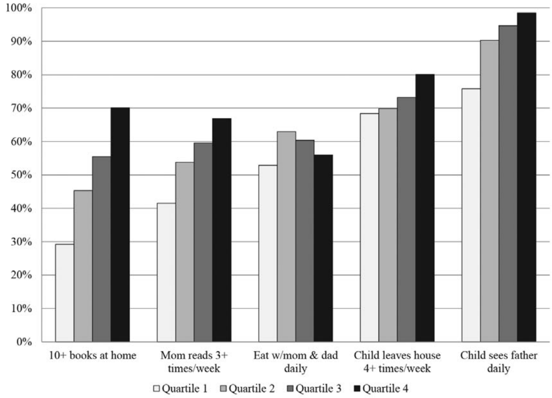
quartile and the bottom income quartile, while the black bars report differences after controlling flexibly for maternal race/ethnicity, AFQT, and educational attainment.[8] Raw gaps by income are sizeable: mathematics and reading scores of children with parents in the highest income quartile are all more than half of a standard deviation higher than those with parents in the lowest income quartile. Controlling for other important maternal characteristics substantially reduces these gaps (by as much as three-quarters), but does not eliminate them – parental income still has economically (and statistically) significant effects on child achievement.

Figures 2 and 3 document a number of early childhood family investment measures by parental income at different ages. For all measures except "eat with mom and dad daily" (ages 0–1, 2–3, and 4–5) and "family meets friends/relatives two or more times per month" (ages 6–7), investments are monotonically increasing with parental income. For a number of measures, the differences are substantial. For example, mothers of young children from the highest income quartile are over 50 percent more likely to read to their child three or more times per week compared to mothers from the lowest income quartile. High-income mothers with children aged 0–1 are more than twice as likely to have 10 or more books in the home. Among children aged 6–7, those from high-income families are more than twice as likely to be enrolled in special lessons or extracurricular activities.

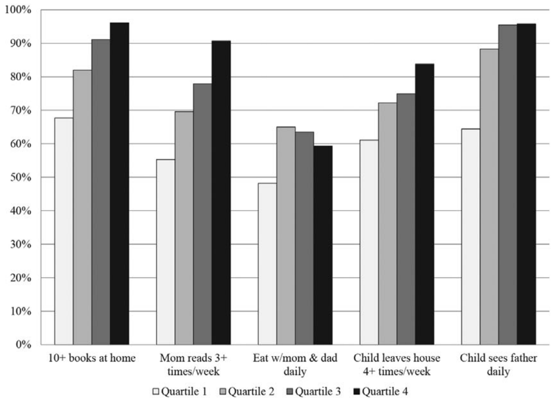
One interpretation of the investment measures reported in Figures 2 and 3 is that they represent different types of investment inputs that influence child development. An alternative interpretation is that they all represent noisy measures of a single underlying "investment". Under the latter interpretation, factor analysis can be used to uncover a more precise measure of the latent investment (Cunha and Heckman, 2008; Cunha *et al.*, 2010).[9] Based on this insight, we employ principal factor analysis using the measured inputs reported in Figures 2 and 3 to create age-specific predicted

[8] Standard multivariate regressions are used to control for maternal race/ethnicity (white, black, hispanic), AFQT quartiles, and educational attainment (high-school dropout, high-school graduate, some college, completed college) by including three-way interactions of all three sets of indicators along with indicator variables for parental income quartile. Sample sizes for raw differences by income are 3,449 for mathematics, 3,436 for reading recognition, and 3,267 for reading comprehension. Approximately 80 observations are dropped due to missing covariates when controlling for maternal characteristics.

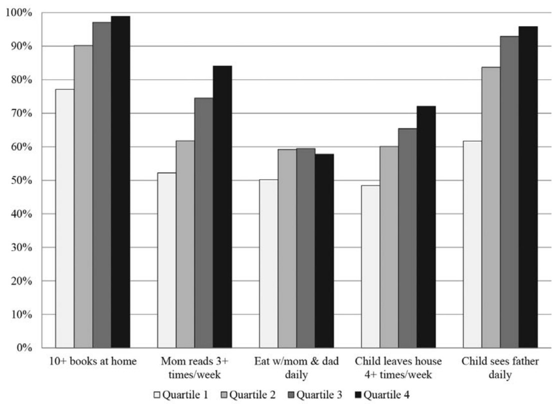
[9] These interpretations are not necessarily mutually exclusive if families face the same relative prices and productivity of inputs (assuming families maintain correct beliefs about the relative productivity of inputs). In this case, all inputs (in a given period) will be proportional to total child investment expenditure (that period), with all families choosing the same proportional mix. One can then think about the various reported measures as noisy measures of that specific input or of total investment expenditure (multiplied by the factor share for that input). We study the link between multiple early inputs and total early investment expenditures in Section VII along with the consequences of family mis-perceptions about relative and overall early input productivity levels.



(a) Children aged 0–1



(b) Children aged 2–3



(c) Children aged 4–5

Fig. 2. Family investments in children aged 0–1, 2–3, and 4–5 by parental income quartile

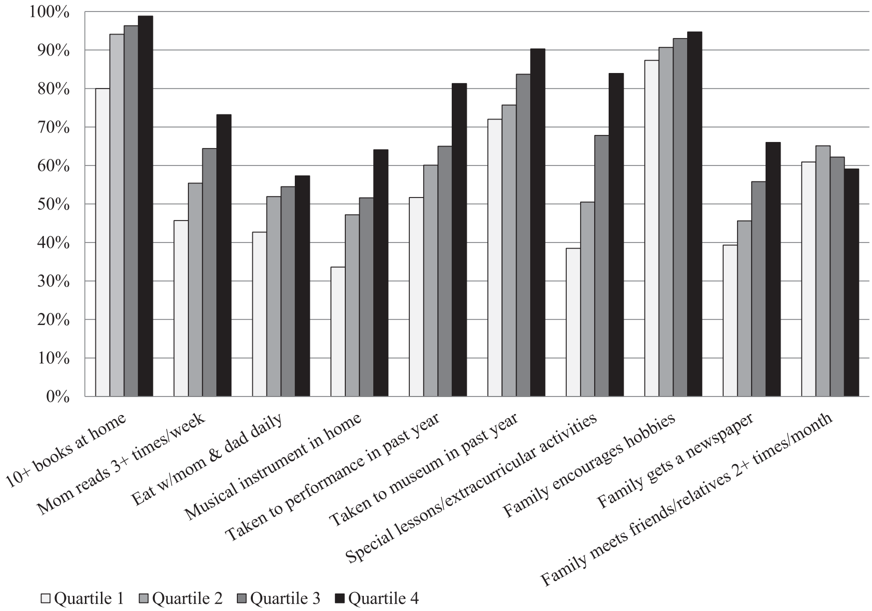


Fig. 3. Family investments in children aged 6–7 by parental income quartile

investment factor scores for each child (see Online Appendix A). For interpretation purposes, we normalize scores to have a mean of zero and standard deviation of one, plotting average scores by age and parental income quartile in Figure 4. The figure reveals sizeable differences in investment factor scores by parental income that are already evident at very young ages. Investments are roughly a full standard deviation higher among children from high-income families relative to low-income families. Figure 5 shows that these gaps shrink by as much as 50 percent but remain sizeable when controlling for maternal race, AFQT, and educational attainment.[10] Comparing Figure 1 with Figure 5 reveals that maternal characteristics explain a greater share of the income-based gaps in achievement than in investments.

III. Additional Stylized Facts on Child Development

In this section, we discuss four stylized facts on the marginal returns to investment in children and the role of family income in child development. Because of their general nature, these facts can be compared against

[10] Sample sizes for raw income differences are 2,324 (ages 0–1), 2,749 (ages 2–3), 2,813 (ages 4–5), and 3,493 (ages 6–7). When controlling for maternal characteristics, between 50 and 80 observations are dropped due to missing covariates.

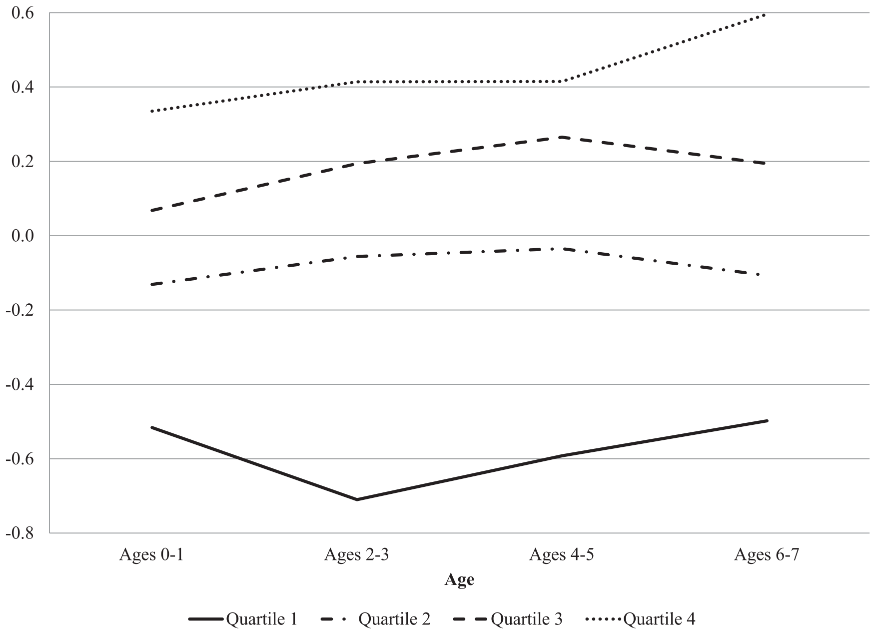


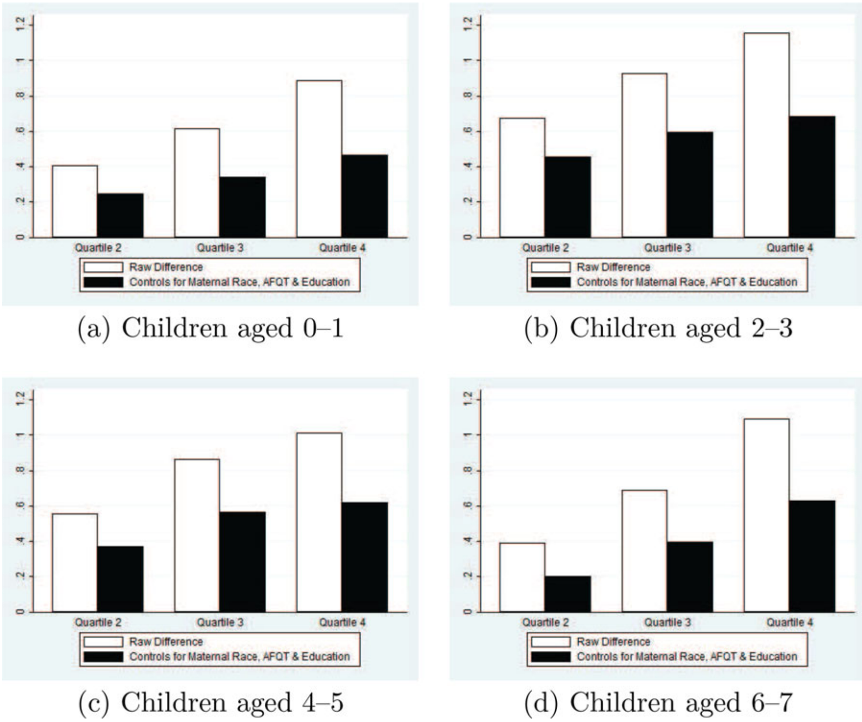
Fig. 4. Family investment factor scores by child age and parental income quartile

the predictions of any investment-based model of skill formation. Other, more mechanism-specific findings in the literature are briefly discussed in Sections V–VIII.

Fact 1

The returns to early marginal investments are higher than the return to savings for economically disadvantaged children. A number of comprehensive surveys (Karoly *et al.*, 1998; Blau and Currie, 2006; Cunha *et al.*, 2006; Almond and Currie, 2011; Heckman and Kautz, 2014; Kautz *et al.*, 2014) document both the short- and long-term impacts of numerous early childhood interventions in the US. Most notably, an experimental evaluation of the Perry Preschool program followed participants in the early 1960s up to age 40, measuring the program's impacts on a wide array of outcomes, including cognitive achievement, educational attainment, earnings, and crime.[11] Based on program costs and impacts measured from ages 15 to 40, Heckman *et al.* (2010) estimate a private internal rate of

[11] Perry Preschool provided daily high-quality preschool (2.5 hours per day) and weekly home visits for two years to children aged 3 and 4. The randomized control trial sample was drawn from low IQ children from families of low socioeconomic status.



(a) Children aged 0–1 (b) Children aged 2–3

(c) Children aged 4–5 (d) Children aged 6–7

Fig. 5. Family investment gaps by parental income quartile (relative to quartile 1)

return to Perry Preschool participants of around 8 percent.[12] The Abecedarian Project offered high-quality full-day preschool to a (randomly assigned) sample of mostly African American children born in the mid-1970s who were at risk of delayed intellectual and social development. Follow-up evaluations of Abecedarian to age 21 reveal significant long-term benefits that exceed the program's costs by a factor of roughly 2.5 (Barnett and Masse, 2007).[13] Researchers have also extensively analyzed the long-term impacts of Chicago's Child–Parent Center (CPC) preschool program, following a sample of low-income, mostly African American participants from the mid-1980s to the present. Rough calculations based on program impacts

[12] Social returns are even higher, largely due to benefits from crime reduction.

[13] As is common in this literature, these calculations assume an annual real discount rate of 3 percent. The benefit calculations project lifetime earnings impacts based on average earnings differences by educational attainment and the significant effects of Abecedarian on educational attainment. An age 30 follow-up study (Campbell *et al.*, 2012) estimates a sizeable but statistically insignificant increase in annual earnings for participants ($33,000 versus $21,000). Estimated effects on employment rates and use of public aid at age 30 are statistically significant.

measured to age 26 suggest an average (private) benefit/cost ratio of 3.6 (Reynolds *et al.*, 2011a).[14] Finally, a number of studies document significant long-term impacts of Head Start (Currie and Thomas, 1995, 1999; Garces *et al.*, 2002; Ludwig and Miller, 2007; Deming, 2009; Carneiro and Ginja, 2014); however, its full rate of return has not been systematically estimated.

Other family-based investments aimed at improving mother–child interactions and maternal parenting skills can also serve as productive early investments in the child development process, producing significant long-run benefits for children. For example, the Nurse–Family Partnership provided regular pre- and post-natal (up to age 2) home visits to low-income mothers with the goals of improving pregnancy outcomes and maternal health, improving the health and development of their children through proper care, and enhancing parental life-course development. Studies of the program in three US cities estimate long-term benefits from these investments on a number of child outcomes (Olds *et al.*, 2002; Eckenrode *et al.*, 2010; Kitzman *et al.*, 2010). Long-term benefits of similar family-based interventions have also been documented in Jamaica and Colombia, where home visitation programs provided one-hour weekly visits (for up to two years) aimed at improving mother–child interactions and developing child cognitive, language, and psychosocial skills (Gertler *et al.*, 2014; Attanasio *et al.*, 2015).

Fact 2

The returns to marginal investments are lower for more economically advantaged children. Because most experimental and government-subsidized early childhood programs serve low-income families, less is known about the lifetime returns to early investments in children from higher-income families. However, a number of studies estimate short- and medium-term impacts of early childhood interventions by family income or socioeconomic status (SES). These studies typically report greater benefits for more disadvantaged children. For example, Duncan and Sojourner (2013) estimate that the Infant Health Development Program (IHDP), which provided the Abecedarian preschool curriculum to an economically diverse sample of children aged 1–2 years who had low birth weight, yielded significantly greater improvements in IQ at age 5 for the subsample of children from

[14] As with the Abecedarian cost–benefit analysis of Campbell *et al.* (2012), Reynolds *et al.* (2011a) use a 3 percent discount rate and project lifetime earnings benefits from estimated impacts on educational attainment at age 26; they also incorporate benefits from reductions in childcare costs and child abuse/neglect. In a subsequent follow-up, Reynolds *et al.* (2011b) show that the program significantly increased earnings at age 28 by 7 percent.

low-income families relative to those from higher-income families.[15] A recent analysis of Head Start (Puma *et al.*, 2012) estimates significantly greater impacts on third grade cognitive and learning outcomes for children from "high risk" (i.e., low SES) households relative to lower-risk households. Estimated impacts of the Chicago CPC preschool program on educational attainment and earnings at age 28 are also higher for children from high-risk families (Reynolds *et al.*, 2011b). A few studies that estimate the impacts of introducing universal early childcare subsidies in Canada and Norway find negligible or even adverse impacts on children from middle- and high-income families, likely due to the substitution of lower-quality subsidized/free childcare in place of higher-quality unsubsidized family or informal care (Baker *et al.*, 2008; Havnes and Mogstad, 2014; Kottelenberg and Lehrer, 2014).[16]

Taking a very different approach, Cunha *et al.* (2010) apply dynamic factor models using multiple noisy measurements of child investments and skill levels to estimate the technology of human capital production from birth up to the end of school. Their estimated technology suggests that the most efficient allocation of educational investments would provide more to young disadvantaged children. The fact that actual investments are much lower for disadvantaged children (see Figures 2–5) coupled with diminishing marginal returns, suggests that returns on the margin are higher for the most disadvantaged.

Fact 3

Exogenous increases in parental income lead to greater investments in children and improvements in childhood outcomes. A number of recent studies have attempted to address concerns about endogeneity in estimating the effects of exogenous changes in family income on children. Dahl and Lochner (2012) exploit expansions of the Earned Income Tax Credit (primarily over the mid-1990s) to estimate the effects of additional family income on cognitive achievement. Their instrumental variable estimates suggest that an additional $1,000 in family income raises combined mathematics and reading scores by 6 percent of a standard deviation. Estimated effects also appear to be larger for children from more disadvantaged families. Milligan and Stabile (2011) estimate that expansions of child tax benefits in Canada led to similar improvements in child cognitive and educational outcomes as well as improvements in child and maternal health.

[15] Brooks-Gunn *et al.* (1992) estimate greater effects of IHDP on IQ at age 3 for families with lower maternal education.
[16] See Baker (2011) for a careful discussion of recent universal early childcare initiatives.

Combining data from ten welfare and anti-poverty experiments, Duncan *et al.* (2011) attempt to separately identify the effects of changes in family income from employment and other effects induced by different programs. Their analysis reaches similar conclusions, regarding the impacts of income on child achievement, as Dahl and Lochner (2012) and Milligan and Stabile (2011). Finally, Løken (2010) and Løken *et al.* (2012) estimate the impact of family income on Norwegian children using regional variation in the economic boom following the discovery of oil as an instrument for income. The latter study estimates that income has sizeable impacts on education and IQ for children from low-income families but much weaker effects for children from higher-income families.[17]

Changes in income can affect children in many ways. In this paper, we focus on investment-based theories, so it is important to know whether changes in family income cause families to make different investment choices. A few studies suggest that this is the case. Following the approach of Milligan and Stabile (2011), Jones *et al.* (2015) examine how Canadian parents altered their household expenditures in response to an expansion of child tax benefits. Their estimates suggest that low-income families, on average, spent 13 cents out of every additional dollar in benefits on education-related items (e.g., tuition, computers).[18] Carneiro and Ginja (2014) estimate models of income dynamics in the US, examining the extent to which family investments in children respond to permanent and transitory income shocks. Their results suggest modest positive responses to permanent shocks but negligible responses to transitory shocks. Effects appear to be largest for younger children and those with less-educated parents. Among children whose mothers had not attended college, a 10 percent increase in permanent income is estimated to increase measures of cognitive stimulation and time investments by about 0.02 standard deviations. Cunha *et al.* (2010) and Pavan (2014) estimate both the technology of skill formation for children (from birth to later school ages) and the extent to which family income as well as maternal and child skills affect investments in children. Their estimated investment functions suggest that increases in family income lead to significantly higher investments in children.

[17] Studies on the effects of parental job displacement on children also suggest that family income might have important effects on child schooling and labor market earnings (Oreopoulos *et al.*, 2008; Stevens and Schaller, 2011); however, parental job displacement might also affect child development through other channels (e.g., family dissolution).

[18] Interestingly, their results for all Canadian families suggest negligible effects on average education-related expenditures, so poor families appear to increase education-related spending much more than the typical family.

Fact 4

The timing of income matters for child development: increases in income at early ages (compared to later ages) lead to larger increases in investments and achievement/educational outcomes. The estimated child investment functions of Pavan (2014) imply significantly greater effects of family income on investments at very early ages relative to older ages. Other studies estimate the effects of family income received at different child ages on adolescent achievement or educational outcomes. For example, Duncan and Brooks-Gunn (1997), Duncan *et al.* (1998), and Levy and Duncan (1999) all estimate that income received at earlier ages has a greater impact on adolescent achievement than income received at later ages. However, Carneiro and Heckman (2002) correctly point out that (undiscounted) early income should have a larger effect than (undiscounted) later income due purely to discounting – something not taken into account in previous analyses.[19] More recent studies address this concern by discounting all income measures back to the year of birth (Carneiro and Heckman, 2002; Caucutt and Lochner, 2006, 2012).[20] Caucutt and Lochner (2006) report results consistent with the earlier literature, finding that income received at young ages has a greater effect than income received at older ages on subsequent child achievement. Caucutt and Lochner (2012) further show that family income earned when children are younger has a significantly greater effect on college attendance than does income earned at later ages; however, Carneiro and Heckman (2002) cannot reject that income has the same effects on college enrolment regardless of the age at which it was received. While both of these studies use data from the CNLSY, the former benefits from a sample size that is roughly twice as large, allowing for greater precision. Furthermore, because Carneiro and Heckman (2002) are more concerned with the importance of borrowing constraints at college-going ages, they control for age 12 mathematics achievement levels, which might absorb much of the effect of early income.

IV. Understanding Investment and Skill Gaps by Family Income

We now develop a general model of dynamic human capital investment in order to study four mechanisms thought to generate child investment and skill gaps by family income. Within this framework, we explore the extent to which these mechanisms are also capable of explaining the additional stylized facts just discussed. The problem is written as a life-cycle problem

[19] That is, with perfect credit markets, income received at age 0 should have an effect that is $(1+r)^a$ times larger than income received at age a, where r is the annual interest rate.

[20] These studies all use a 5 percent rate to discount income back to the year of a child's birth.

in which individuals invest in their human capital, while borrowing and saving (potentially subject to borrowing constraints) in an effort to finance investments and smooth consumption over time. However, the problem can also be interpreted as a "family" investment problem, where altruistic parents make investments in their children and family borrowing/saving decisions to smooth family consumption.[21]

We assume that people live through three stages in their lives. Human capital investment takes place in the first two stages (i.e., childhood), followed by the final stage, adulthood. Adulthood can last for many periods; however, its length is inconsequential for most of our analysis. We are largely agnostic about the form that investments might take, instead focusing primarily on total investment expenditures at different ages and the dynamic nature of skill production. Conceptually, investments can include various forms of goods inputs such as computers and books, parental time in child development activities, formal schooling, and other time inputs by older children.[22] When considering parental time as an investment, if the marginal product of parental time investment (in children) is proportional to the parent's labor market productivity (i.e., human capital), then investment is simply given by the parent's forgone earnings.[23]

Technology for Human Capital Production

We denote a child's ability to learn by $\theta > 0$. Investment expenditures in periods 1 and 2 are given by i_1 and i_2, respectively. Early investment produces an interim level of human capital,

$$h_2 = z i_1, \tag{1}$$

where $z > 0$ is the productivity of early investment. Together, late investment and this interim human capital produce stage 3 (adult) human capital:

$$h_3 = \theta f(h_2, i_2). \tag{2}$$

[21] See Caucutt and Lochner (2012) for a direct mapping between this life-cycle problem and a more explicit intergenerational problem.
[22] They might also include more general investments in families or mother–child interactions that are designed to facilitate child development (e.g., home visitation programs as discussed in Section III).
[23] To the extent that time investments are important, we implicitly assume that parents can flexibly adjust their labor supply at a fixed wage. In particular, they can adjust their labor supply downwards without incurring an hourly wage penalty. Otherwise, distortions similar to those observed for borrowing constraints (discussed below) might arise if parental time is a key (and non-substitutable) input for young children, and if parents cannot fully reduce their work hours to make desired time investments.

The human capital production function $f(\cdot)$ is strictly increasing and strictly concave in both of its arguments. To guarantee that appropriate second-order conditions hold in the decision problems described below, we assume the following throughout our analysis (without explicit reference).

Assumption 1.

$$f_{12}^2 < f_{11} f_{22} \quad \textit{and} \quad f_{12} > \max \left\{ f_{22} \left(\frac{f_1}{f_2} \right), f_{11} \left(\frac{f_2}{f_1} \right) \right\}.$$

The first condition limits the degree of dynamic complementarity in investments and ensures strict concavity of the production function. The second condition implies that the least costly way to produce additional human capital h_3 is to increase both early and late investments. Most plausible specifications for human capital production would entail dynamic complementarity (i.e., $f_{12} \geq 0$), satisfying this condition (Cunha *et al.*, 2010; Caucutt and Lochner, 2012); however, the condition holds much more generally.[24] We also assume standard Inada conditions to ensure interior solutions.[25]

At times, our analysis will employ a CES human capital production function of the form

$$f(h_2, i_2) = \left[a^{1-b} h_2^b + (1-a)^{1-b} i_2^b \right]^{d/b}, \tag{3}$$

where $a \in (0, 1)$, $b < 1$, and $d \in (0, 1)$; however, most of our analysis does not rely on any particular functional form. Assumption 1 holds for this production function.

General Decision Problem

We assume a period utility function over consumption, $u(c)$, that is strictly increasing, strictly concave, and satisfies standard Inada conditions. Tastes for early educational investment, i_1, are given by νi_1. The time discount rate is $\beta \in (0, 1)$, and the gross rate of return on borrowing and saving is $R > 0$. Assets saved in period j are given by a_{j+1}.

The individual/family receives exogenous income y_j during childhood periods $j = 1, 2$. We sometimes refer to these as (early and late) parental income; however, it can also include government transfers or earnings while older children are still enrolled in school (in period 2).[26]

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[24] For example, the condition holds for homothetic functions (e.g., CES) regardless of the degree of complementarity.

[25] That is, $\lim_{h_2 \to 0} f_1(h_2, i_2) = \infty$, $\forall i_2 \geq 0$, and $\lim_{i_2 \to 0} f_2(h_2, i_2) = \infty$, $\forall h_2 \geq 0$.

[26] Even if one considers the child to be the sole decision maker with y_1 and y_2 reflecting inter vivos transfers from parents, the interpretations in the text regarding parental income carry through as long as transfers are strictly increasing in parental income.

Children/families allocate their resources to consumption and skill investment, leaving some assets/debt for when the child grows up:

$$\max_{c_1,c_2,i_1,i_2,a_2,a_3} E[u(c_1) + \nu i_1 + \beta u(c_2) + \beta^2 V(a_3, h_3)], \tag{4}$$

subject to human capital production equations (1) and (2); budget constraints

$$a_{j+1} = Ra_j + y_j - i_j - c_j \quad \text{for } j = 1, 2; \tag{5}$$

initial assets a_1 given; and where $V(a_3, h_3)$ represents the child's utility in adulthood given a_3 and h_3.

In the next four sections, we consider the main mechanisms commonly thought to explain income-based gaps in early investment and skill levels. We analyze each mechanism separately, abstracting from the others, in order to highlight the key underlying forces of each mechanism and the extent to which it can explain other stylized facts discussed in Section III. In the next two sections, we abstract from uncertainty and other information problems, considering the consumption value of investment and the intergenerational correlation of ability. In Section VII, we study the implications of uncertainty and mis-information, at which point we describe information sets and variables over which expectations are taken in equation (4). Finally, in Section VIII, we introduce restrictions on borrowing of the form $a_{j+1} \geq -L_j$, where L_j is an upper limit on the amount that can be borrowed in period j. Until then, we assume that borrowing and saving are unrestricted.

V. Correlated Ability

We begin by studying the implications of a positive intergenerational correlation in ability, which is likely to generate a positive correlation between a child's ability and lifetime parental income (i.e., $\text{Cov}(\theta, Y) > 0$). Indeed, this is the starting point for many economic theories of intergenerational correlations in human capital and earnings (Becker and Tomes, 1979, 1986; Loury, 1981; Restuccia and Urrutia, 2004; Caucutt and Lochner, 2012; Cunha, 2013). To focus on this potential explanation for income-based gaps in investment and achievement, we study the effects of θ on investments, marginal returns to investment, and human capital, while abstracting from any consumption value of schooling, uncertainty, and credit constraints. Specifically, we assume $\nu = 0$ and that families have full and perfect information about the productivity of human capital investments. In the absence of borrowing constraints, the length of adulthood is irrelevant for our analysis, so we simply consider a three-period problem with $V(a_3, h_3) = u(Ra_3 + h_3)$ and a single lifetime budget constraint. For expositional purposes, we normalize $z = 1$.

With these assumptions, the problem can be written as

$$\max_{c_1,c_2,c_3,i_1,i_2} \{u(c_1) + \beta u(c_2) + \beta^2 u(c_3)\}$$

subject to the lifetime budget constraint

$$c_1 + R^{-1}c_2 + R^{-2}c_3 = Y - i_1 - R^{-1}i_2 + R^{-2}\theta f(i_1, i_2),$$

where $Y \equiv Ra_1 + y_1 + R^{-1}y_2$, which we often (loosely) refer to as lifetime parental income. Note that Y can also include initial family assets and government transfers; however, we can interpret the effects of changes in Y as changes in parental income holding these constant.

Optimal investments must satisfy the following first-order conditions:

$$\theta f_1(i_1, i_2) = R^2, \tag{6}$$

$$\theta f_2(i_1, i_2) = R. \tag{7}$$

When investments are made purely for investment purposes, they are chosen to equate the marginal labor market returns to investment, $\partial h_3/\partial i_j = \theta f_j(i_1, i_2)$ in both periods $j = 1, 2$, with the corresponding return to savings. This is the well-known result of Becker (1975): in the absence of borrowing constraints, uncertainty, and a direct utility value from investment, human capital investments simply maximize discounted lifetime earnings net of investment expenditures. Importantly, this relationship holds regardless of ability or family income. As such, investments are independent of family income, Y, given ability. The role of ability is summarized in the following proposition. (All proofs can be found in Online Appendix B.)

Proposition 1. *Optimal investments satisfy the following: (i) the marginal returns to investments are independent of ability; (ii) early and late investments are strictly increasing in ability; (iii) adult human capital h_3 is strictly increasing in ability.*

Not surprisingly, a positive correlation between parental income and child ability would produce a positive correlation between parental income and child investments and skills. Yet, the marginal return on investments should be unrelated to parental income, because investments in all children equate their marginal returns to the interest rate. This is inconsistent with both stylized Facts 1 and 2, which document returns to early investments for poor children that exceed standard interest rates as well as the returns for more economically advantaged children.

Additionally, the model implies no causal relationship between parental income and child investments/skills. Holding the child's ability constant, there should be no correlation between investments and parental income.[27] Furthermore, exogenous changes in parental income should have no effect on investments in children, contradicting stylized Facts 3 and 4.

VI. Consumption Value of Investment

Next, we explore the implications of a consumption/utility value associated with investment in children. Lazear (1977) provides an early analysis of education as a joint producer of human capital/earnings and utility. Keane and Wolpin (2001), Cunha *et al.* (2005), and Carneiro *et al.* (2011) emphasize and estimate the role of heterogeneity in the "consumption" value of schooling in explaining differences in schooling behavior, while Abbott *et al.* (2013) explicitly consider differences in tastes for schooling by parental wealth. To study the implications of this mechanism for investments in young children, we incorporate tastes for schooling $\nu \neq 0$ as in equation (4), while continuing to assume perfect information, no credit constraints with $V(a_3, h_3) = u(Ra_3 + h_3)$, and $z = 1$.

For simplicity, we assume that $\beta = R^{-1}$ so that optimal consumption profiles are flat: $c_t = c = B^{-1}[Y - i_1 - R^{-1}i_2 + R^{-2}\theta f(i_1, i_2)]$ for $t = 1, 2, 3$, where $B = 1 + R^{-1} + R^{-2}$. In the absence of any consumption value associated with late investment, i_2 is still determined from the first-order condition above (equation (7)); however, optimal early investment must now satisfy

$$\theta f_1(i_1, i_2) = \left[1 - \frac{\nu}{u'(c)}\right] R^2. \tag{8}$$

When investment provides a direct consumption or utility value to children or their families, this must be taken into account when making investment decisions, driving a wedge between the marginal labor market return to investment and the return to savings. For a positive consumption value ($\nu > 0$), early investment will have a low labor market return on the margin (i.e., $\theta f_1 < R^2$), because families will want to invest beyond the point where lifetime income is maximized.[28] The opposite is true if families/children dislike investment ($\nu < 0$).

When investment has a non-zero consumption value, the effects of parental income on investments and human capital, as well as the marginal

[27] Figures 1 and 5 suggest that there is likely a strong intergenerational ability correlation, as the relationship between family income and both achievement and investments is much weaker (although still non-trivial) once we control for maternal characteristics such as AFQT and educational attainment.

[28] This result also holds if parents value the child's final human capital level instead of early investment itself.

labor market return to early investment, are easily derived from the first-order conditions above and are summarized in the following proposition.

Proposition 2. *For $\nu > 0$ ($\nu < 0$), optimal investments satisfy the following: (i) the marginal return to early investment is strictly less (greater) than the return to savings and is strictly decreasing (increasing) in lifetime parental income Y; (ii) early investment is strictly increasing (decreasing) in lifetime parental income Y; (iii) later investment is increasing (decreasing) in lifetime parental income Y if and only if $f_{12} \geq 0$; and (iv) final human capital h_3 is strictly increasing (decreasing) in lifetime parental income Y.*

These results are intuitive. If families enjoy investing in their children ($\nu > 0$), they will invest beyond their income maximizing amounts and will invest more if their income rises. Thus, the positive relationship between family income and early childhood investment and skills requires a positive consumption value. Proposition 2 shows that if $\nu > 0$, the marginal labor market return to early investment should be low (inconsistent with Fact 1) and decreasing in lifetime parental income (consistent with Fact 2). A positive consumption value predicts that early investment should rise with exogenous increases in lifetime parental income (consistent with Fact 3); however, it predicts that the timing of that income is irrelevant (inconsistent with Fact 4).

One might reconcile the high estimated returns for early interventions targeted to economically disadvantaged children (Fact 1) by assuming that low-income parents find investment in their children costly (i.e., $\nu < 0$ for low-income families). However, this would then imply that investment in young children and their skill levels should decline when poor parents receive additional income, contradicting Fact 3.

VII. Confusion

Poor families might face greater uncertainty about the returns to investment, or they might simply maintain inaccurate beliefs about the productivity of various investments. We next examine how uncertainty and mis-information influence investment behavior, and we highlight the importance of accounting for the dynamic nature of skill production.

We begin this analysis by studying uncertainty about θ. In our general framework, this might reflect uncertainty about the child's ability to learn, parents' abilities to teach, or even the price of skill (including idiosyncratic variation, e.g., due to search frictions) in the labor market. We first consider the role of risk aversion, assuming that θ is revealed only after all investments have been made. We then consider uncertainty about θ, as well as the marginal productivity of early investments z, when that uncertainty is completely resolved after early investments but before late investments have been made. To focus on the nature of skill production and irreversibility of investments, these results abstract from risk aversion. Objective uncertainty (i.e., rational expectations) about θ (reflecting ability or skill prices) is most commonly assumed in the human capital literature; however, a growing number of studies highlight the role of subjective uncertainty, typically about the returns to education (Dominitz and Manski, 1996; Attanasio and Kaufmann, 2009; Stinebrickner and Stinebrickner, 2014; Wiswall and Zafar, 2015). We discuss both forms of uncertainty.

The literature on subjective beliefs about child development also explores biases in those beliefs. For example, Nguyen (2008) and Jensen (2010) document downward-biased beliefs (on average) about the returns to education in Madagascar and the Dominican Republic, respectively, further showing that these beliefs respond to newly provided information. Both studies show that actual schooling choices can be influenced by the provision of new information; however, Jensen (2010) estimates no behavioral responses among the most poor who might be financially constrained.[29] In this paper, we are more interested in the implications of biased beliefs about the productivity/value of earlier investments in children. Cunha *et al.* (2013) and Cunha (2014) show that a sample of mothers from Philadelphia underestimates, on average, the value of time investments for cognitive development in young children. Cunha (2014) further demonstrates that black mothers are more pessimistic about the productivity of these investments than white mothers, arguing that this difference might explain one-fourth of black–white early investment gaps. Dizon-Ross (2015) shows that parents in Malawi hold distorted beliefs about their children's school achievement levels with greater biases held by the least-educated parents. Furthermore, providing accurate information in a simple format for parents to understand leads to a reallocation in the types of investments they make in their children (e.g., purchasing remedial versus advanced workbooks) with greater responses observed among less-educated parents. Interestingly, the differential responses by parental education do not lead to corresponding differences in schooling outcomes (e.g., educational expenditures, school attendance).

Motivated by this literature on subjective beliefs, we study early investments and their marginal returns when there are many types of early investment activities/inputs, and parents are mis-informed about the productivity of those activities/inputs. In particular, we consider the implications of both

[29] Consistent with the latter finding, Attanasio and Kaufmann (2009) estimate that subjective expectations about the returns to school are not significant predictors of college attendance for youth at the bottom of the income and wealth distribution in Mexico, while they are significant predictors among those that are not as poor. We discuss the role of borrowing constraints in Section VIII.

systematic and non-systematic bias. By systematic bias, we mean incorrect beliefs about the marginal productivity of all types of early investments, z, as emphasized by Cunha *et al.* (2013) and Cunha (2014). Non-systematic bias refers to incorrect beliefs about the relative productivity of different types of early investments, as discussed in Dizon-Ross (2015).

In the following analysis, we use the term beliefs to reflect a family's subjective probability distribution for some (productivity) parameter(s). For much of our analysis, it does not matter whether these beliefs reflect actual variation or simply subjective uncertainty. We use the term purely objective uncertainty to refer to the case where beliefs coincide with the actual probability distribution for the parameter(s) of interest. This is also commonly referred to as rational expectations. We use the term purely subjective uncertainty to refer to the case where beliefs are non-degenerate, even though the actual probability distribution is degenerate (i.e., if the true distribution for parameters were known, there would be no uncertainty). The distinction between purely objective or purely subjective uncertainty is mainly important for the realized marginal labor market returns to investments, because the former implies a distribution of ex post marginal returns for the same investments while the latter does not. We return to this point below.

Throughout this section, we continue to assume $\nu = 0$ and $V(a_3, h_3) = u(Ra_3 + h_3)$ in order to focus on the role of information frictions.

Risk Aversion and Uncertain Returns

We begin with a very natural form of purely objective uncertainty: both beliefs and the true distribution of θ are given by $\theta \sim \Phi(\theta)$, with $\bar{\theta} \equiv E(\theta)$. For this analysis, we assume that the true value of θ is not revealed until after all skill investments have been made. We continue to normalize $z = 1$.

If individuals are risk averse, then the expected return to risky investments should exceed the return on safe investments if individuals are to hold risky assets at all. With a concave human capital production technology, this means that skill investments will be lower under uncertainty (Levhari and Weiss, 1974).

The first-order conditions for investments satisfy

$$f_1(i_1, i_2) = Rf_2(i_1, i_2).$$

Given the separability between θ and $f(\cdot)$, there is no distortion between early and late investment, even if total investment spending is distorted. That is, for a given level of spending $i_1 + R^{-1}i_2$, early and late investments are chosen to maximize $f(i_1, i_2)$. Assumption 1 ensures that both i_1 and i_2 increase when total investment spending increases.

The level of total investment spending will equate the expected marginal benefit with the marginal cost of investment, so

$$\bar{\theta} f_1(i_1, i_2) + \frac{\mathrm{Cov}\{u'[c_3(\theta)], \theta\}}{E\{u'[c_3(\theta)]\}} f_1(i_1, i_2) = R^2,$$

where $c_3(\theta) = Ra_3 + \theta f(i_1, i_2)$ is optimal period 3 consumption in state θ. The expected marginal benefit of investment consists of a monetary return (first term) and a utility cost (second term). Because the marginal utility of consumption is low in states with a high return to investment, uncertain returns produce an additional utility cost of investment as reflected in the (negative) covariance term. As such, the expected marginal labor market returns to investment exceed the return to savings:

$$\bar{\theta} f_1(i_1, i_2) = R\bar{\theta} f_2(i_1, i_2) > R^2.$$

Risk-averse individuals facing uncertain returns invest less at all ages relative to those who know the return with certainty.

Moreover, if having greater resources makes people less risk averse, then investments are increasing and marginal labor market returns decreasing in lifetime parental income Y. Because no information about the value of θ is revealed until all investments have been made, choices depend only on the discounted present value of income over all investment years and not the timing of that income. The following proposition summarizes these results.[30]

Proposition 3. *When there is uncertainty in the final return to investment θ, optimal investments satisfy the following: (i) expected marginal returns to investment are strictly greater than the return to savings; (ii) expected marginal returns to investment are strictly decreasing in parental income Y if $u(\cdot)$ exhibits decreasing absolute risk aversion; and (iii) early and late investments are strictly increasing in parental income Y if $u(\cdot)$ exhibits decreasing absolute risk aversion.*

Uncertain Returns and the Irreversibility of Early Investment

We now consider the case in which families face uncertainty when making early investments in their children; however, that uncertainty is fully resolved before late investments are chosen. While early investments made under uncertainty are irreversible (i.e., families cannot go back in time to

[30] These results assume purely objective uncertainty; however, they also apply to the case of unbiased subjective uncertainty. The only difference in the latter case is that all individuals would experience the same marginal return to investments, given by the expected marginal returns in the case of purely objective uncertainty. Thus, the marginal returns to investment exceed the return on savings (and decline with income) regardless of the form of uncertainty.

modify ex post suboptimal early investment choices), families can base late investment decisions on the realizations of early investments and full knowledge of the human capital production process. Thus, families might be able to partially compensate for ex post suboptimal early investment through their choice of late investment. The extent to which this is effective depends crucially on the intertemporal complementarity/substitutability of investments.[31] It also depends on which features of technology are unknown. We consider uncertainty in the productivity of both investments, θ, as well as in the productivity of early investments alone, z, at the time early investments are made.

To focus on the implications of investment irreversibility and the dynamic nature of human capital productivity, we abstract from risk aversion. We continue to focus on purely objective uncertainty with the distribution of beliefs over (θ, z) reflecting the true variation in these productivity parameters; however, we comment briefly on the implications of purely subjective uncertainty at the end of our discussion.

It is useful to begin with the second period investment problem, which conditions on early investment and technology once θ and z are known. Let $\hat{i}_2(zi_1, \theta)$ denote the optimal second period investment conditional on i_1 and technology state (θ, z):

$$\hat{i}_2(zi_1, \theta) \equiv \underset{i_2}{\operatorname{argmax}}\{-i_2 + R^{-1}\theta f(zi_1, i_2)\}. \tag{9}$$

Optimal late investment equates the marginal labor market return with the return to savings (i.e., $\theta f_2(zi_1, i_2) = R$). From this, it is easy to see that second period investment is increasing in θ. However, late investment is increasing in z if and only if early and late investments are gross complements (i.e., $f_{12} \geq 0$), because z only affects the marginal return to late investment indirectly through $h_2 = zi_1$. Here, we begin to see the distinction between neutral and early-specific productivity as well as the importance of intertemporal complementarity/substitutability of investments. These factors are also important in determining the response of early investment to uncertainty about investment productivity.

Taking the late investment policy $\hat{i}_2(zi_1, \theta)$ as given, the net realized (or ex post) return to early investment for actual productivity parameters (θ, z) is given by

$$\Pi(i_1, \theta, z) \equiv -i_1 - R^{-1}\hat{i}_2(zi_1, \theta) + R^{-2}\theta f[zi_1, \hat{i}_2(zi_1, \theta)]. \tag{10}$$

[31] See Cunha and Heckman (2007) for a discussion of dynamic complementarity and the difficulty of compensating for low early investments by increasing late investments. Cunha *et al.* (2010) estimate and Caucutt and Lochner (2012) calibrate a strong degree of intertemporal complementarity for investments in children.

The following lemma establishes concavity of net realized returns in early investment and is useful for a number of results.

Lemma 1. *The net return to early investment* $\Pi(i_1, \theta, z)$ *is strictly concave in* i_1.

Because i_1 must be determined before (θ, z) is realized, optimal early investment maximizes the expected net return:

$$\tilde{i}_1 \equiv \underset{i_1}{\operatorname{argmax}}\, E[\Pi(i_1, \theta, z)],$$

where the expectation is taken over the distribution of (θ, z). The first-order condition equates the expected marginal labor market return to early investment with the return to savings:

$$E\{z\theta f_1[z\tilde{i}_1, \hat{i}_2(z\tilde{i}_1, \theta)]\} = R^2. \tag{11}$$

With purely objective uncertainty and risk neutrality, the expected marginal labor market return to early investment always equals the return to savings regardless of the type (i.e., θ or z) or extent of uncertainty – the expected marginal return is independent of the (θ, z) distribution. In contrast with Facts 1 and 2 of Section III, the average marginal labor market return should equal the interest rate for children from all backgrounds.

We are also interested in understanding how changes in the distribution of (θ, z) affect early investment amounts. We consider two notions of a change in the distribution that are widely used in economics: first-order stochastic dominance and mean-preserving spread (Rothschild and Stiglitz, 1970, 1971). In this framework, how $\tilde{i}_1$ changes with the distribution of productivity parameters depends on how $\partial\Pi(i_1, \theta, z)/\partial i_1$ varies with (θ, z). If $\partial\Pi(i_1, \theta, z)/\partial i_1$ is increasing (decreasing) in θ, a first-order stochastic dominance shift in θ will increase (decrease) $\tilde{i}_1$. If $\partial\Pi(i_1, \theta, z)/\partial i_1$ is concave (convex) in θ, then a mean-preserving spread in θ decreases (increases) $\tilde{i}_1$. The same is true for changes in the distribution of z. We first consider investment when θ is unknown, then turn attention to the case with z unknown. Some of our results assume a CES production function for human capital, as defined in equation (3).

Neutral Productivity Shock. We now consider uncertainty in the overall ability of a child (θ) that is fully resolved after early investments have been made but before late investments are chosen. We assume z is known.[32]

Proposition 4. *(i) A first-order stochastic dominance shift in* θ *increases early investment. (ii) For the CES production function* (3)*, a mean-preserving spread in* θ *reduces early investment if and only if* $b > d$.

[32] This case was originally considered by Hartman (1976) in the analysis of firm investment and labor demand under uncertain output prices.

It is not surprising that a first-order stochastic shift in θ unambiguously increases early investment, because θ directly raises the marginal return to investment for any given level of early and late investment. The effect of a mean-preserving spread in the distribution of θ is more complicated and depends on the degree of complementarity between investments. For a CES human capital production function, an increase in uncertainty about θ reduces early investment if and only if early and late investments are gross substitutes (i.e., $b > d \Leftrightarrow f_{12} < 0$). With strong intertemporal substitutability, families facing uncertainty about θ will choose to invest little in the first period and wait to learn the productivity of investment. If investment is highly productive, the family can easily compensate for inadequate early investment by investing more in the second period. In the more empirically relevant case where investments are gross complements ($f_{12} > 0$), it is too costly to make up for a lack of early investment by increasing late investment. As a result, early investment increases with the degree of uncertainty.

Early Investment-Specific Productivity Shock. Families might be more uncertain about the productivity of early investments in their children than they are about later investments, such as college attendance. To explore this possibility, we now assume θ is known from birth, and we consider the case where z is initially unknown but is revealed after early investments have been made.

A change in z has two opposing effects on the marginal return to early investment. An increase in z directly increases the productivity of i_1, but it also reduces the marginal return because $h_2 = zi_1$ is subject to diminishing returns in the production of adult human capital h_3. The latter effect is attenuated by adjustments in late investments when $f_{12} \neq 0$. The overall effect of z depends on the following condition.

Condition 1.

$$f_1 > \left(\frac{f_{11} f_{22} - f_{12}^2}{-f_{22}} \right) zi_1.$$

If the direct productivity effect (left-hand side) is greater than the diminishing return effect (right-hand side), then the marginal return to i_1 is greater for larger z. For the CES production function given in equation (3), this condition holds if $b \geq 0$.

Condition 1 is appealing, because it is equivalent to requiring that an increase in z raises the net marginal return to early investment.

Lemma 2. *The marginal net return to early investment $\partial \Pi(i_1, \theta, z)/\partial i_1$ is strictly increasing in z if and only if Condition 1 holds.*

When Condition 1 is satisfied, a better distribution of z produces a higher marginal return to i_1, on average, which makes it profitable to increase early investment. The following proposition formalizes this result and characterizes the effects of a mean-preserving spread in z.

Proposition 5. *For the CES production function* (3)*, (i) a first-order stochastic dominance shift in z increases early investment if* $b \geq 0$*, and (ii) a mean preserving spread in z reduces early investment if* $b \geq 0$.

The effect of a mean-preserving spread in z is similar to its counterpart for θ (Proposition 4), except that uncertainty in z discourages early investment more than does uncertainty in θ. In contrast with an increase in uncertainty about θ, an increase in uncertainty about z can reduce early investment even when early and late investment are gross complements (e.g., $0 \leq b \leq d$). Intuitively, diminishing marginal returns to $h_2 = zi_1$ in the production of adult human capital lessens the benefits of high z realizations for the marginal return to early investment. This force moderates the costs of under-investment and discourages early investment when z is uncertain, but it is absent with uncertainty in θ.

Purely Subjective Uncertainty, Investments, and Marginal Returns. The previous analysis assumes individuals have purely objective uncertainty about heterogeneous productivity levels (θ, z). Even if all children have the same productivity levels, they might have different subjective beliefs about the true productivity of investments. For example, the poor might have downward-biased beliefs or they might have unbiased beliefs with greater subjective uncertainty. Regardless, Propositions 4 and 5 characterize the effects of changes in beliefs on early investment choices.

More interestingly, purely subjective uncertainty has different implications from purely objective uncertainty for observed marginal returns in the labor market. In the latter case, families facing the same distribution of productivity levels make the same early investment choices, but they experience different labor market outcomes due to heterogeneous productivity levels. As discussed earlier (see the discussion surrounding equation (11)), the average realized marginal labor market return under purely objective uncertainty always equals the return to savings. The case of purely subjective uncertainty is quite different. All families with the same beliefs and ability/productivity will make the same early investment choices and will, therefore, experience the same labor market returns. Strict concavity of $\Pi(i_1, \theta, z)$ in i_1 (Lemma 1) directly implies that under purely subjective uncertainty, the observed marginal labor market return to early investment is strictly decreasing in the level of early investment. Together with Propositions 4 and 5, this directly implies the following corollary.

Corollary 1. *Assume the CES production function given in equation* (3). *Under purely subjective uncertainty, a mean-preserving spread in the distribution of beliefs about θ (z) increases the marginal labor market return to early investment if and only if $b > d$ (if $b \geq 0$).*

Families with greater subjective uncertainty about θ will have a higher marginal labor market return to early investment if and only if early and late investments are gross substitutes. Marginal labor market returns will be increasing in the amount of subjective uncertainty about z under a modest amount of dynamic complementarity in investments.

Biased Beliefs about Human Capital Production

Thus far, we have focused on the extent of uncertainty about the productivity of investments. It is also possible that parents might have little subjective uncertainty about the productivity of investments, but their beliefs may be biased. This possibility seems particularly likely to arise when there are many different potential inputs/activities families can engage in to raise the human capital of their children (e.g., reading to children, taking them to museums, teaching them to play musical instruments, playing with them). Even if parents are correct in gauging the average productivity across different inputs, they might easily misjudge their relative productivity. To explore this issue, we now assume homogeneity in the productivity of different early inputs across families; however, we allow for the possibility that families might hold biased beliefs about the productivity of any or all early child inputs. To simplify the analysis, we abstract away from any form of uncertainty – families are certain but might be wrong. We further assume that families learn the true outcomes of their early investments, h_2, before they need to make later investment decisions.

Assume early investment consists of n different activities $\boldsymbol{x} = (x_1, \ldots, x_n)$ that produce h_2 according to the following CES production function:

$$h_2 = z \left(\sum_{j=1}^{n} w_j^{1-\phi} x_j^{\phi} \right)^{1/\phi}, \tag{12}$$

where $\phi \in (0, 1)$ and $\boldsymbol{w} = (w_1, \ldots, w_n) \geq 0$ satisfies $\sum_{j=1}^{n} w_j = 1$.[33] Here, z reflects the total factor productivity of early investments. Changes in z have no effect on the relative productivity or optimal composition of different inputs. Productivity weights w_j determine the relative importance of

[33] Vector equality and inequality are defined as follows: (i) $\tilde{\boldsymbol{x}} = \boldsymbol{x}$ if $\tilde{x}_j = x_j$ for all $j = 1, \ldots, n$; (ii) $\tilde{\boldsymbol{x}} \neq \boldsymbol{x}$ if $\tilde{x}_j \neq x_j$ for some $j = 1, \ldots, n$; (iii) $\tilde{\boldsymbol{x}} \leq \boldsymbol{x}$ if $\tilde{x}_j \leq x_j$ for all $j = 1, \ldots, n$.

each input as well as their optimal expenditure shares. It is straightforward to show that demand for input x_j conditional on total early investment spending $i_1 = \sum_{j=1}^{n} x_j$ is given by

$$x_j = w_j i_1.$$

By substituting these conditional demands into the production function, we obtain the indirect production function (as a function of total early expenditure i_1) equivalent to that assumed earlier:

$$h_2 = z \left[\sum_{j=1}^{n} w_j^{1-\phi} (w_j i_1)^{\phi} \right]^{1/\phi} = z i_1.$$

To investigate the implications of incorrect beliefs about early investment productivity, it is useful to distinguish beliefs from actual productivity parameters. Let $\tilde{z}$ and $\tilde{\boldsymbol{w}}$ denote a family's beliefs about z and $\boldsymbol{w}$, respectively. Without loss of generality, we assume $\sum_{j=1}^{n} \tilde{w}_j = 1$. We say that a belief is biased if $\tilde{z} \neq z$ or $\tilde{\boldsymbol{w}} \neq \boldsymbol{w}$. When $\tilde{z} \neq z$, the bias is systematic in the sense that families are, on average, biased about the productivity of early investments. When $\tilde{z} = z$ but $\tilde{\boldsymbol{w}} \neq \boldsymbol{w}$, the bias is non-systematic, because beliefs are, on average, correct even though they are wrong about the relative productivity of different early inputs.

Let $(\tilde{\boldsymbol{x}}, \tilde{i}_1, \tilde{h}_2, \tilde{h}_3)$ be the optimally chosen investments and realized human capital of children with family beliefs $(\tilde{z}, \tilde{\boldsymbol{w}})$. Let $(\boldsymbol{x}^*, i_1^*, h_2^*, h_3^*)$ reflect these same variables when beliefs are unbiased. Families first choose investments $\boldsymbol{x}$ and i_1 based on their beliefs $(\tilde{z}, \tilde{\boldsymbol{w}})$:

$$\tilde{i}_1 = \underset{i_1}{\operatorname{argmax}} \ \Pi(i_1, \theta, \tilde{z})$$

$$\tilde{x}_j = \tilde{w}_j \tilde{i}_1, \quad \forall j = 1, \ldots, n,$$

where the child's lifetime income net of investments, $\Pi(\cdot, \cdot, \cdot)$, is defined by equation (10). Note that total early investment spending $\tilde{i}_1$ is only affected by $\tilde{z}$ and not by $\tilde{\boldsymbol{w}}$, because $\boldsymbol{w}$ does not affect total factor productivity.

Next, interim human capital $\tilde{h}_2$ is realized based on investment choices $(\tilde{\boldsymbol{x}}, \tilde{i}_1)$ and the true technology $(z, \boldsymbol{w})$:

$$\tilde{h}_2 = z \left(\sum_{j=1}^{n} w_j^{1-\phi} \tilde{x}_j^{\phi} \right)^{1/\phi} = z \tau(\tilde{\boldsymbol{w}}) \tilde{i}_1,$$

where

$$\tau(\tilde{\boldsymbol{w}}) \equiv \left(\sum_{j=1}^{n} w_j^{1-\phi} \tilde{w}_j^{\phi} \right)^{1/\phi} \leq 1$$

reflects the distortion due to a suboptimal allocation of expenditures across inputs. When $\tilde{\boldsymbol{w}} \neq \boldsymbol{w}$, early investment spending is less productive than it should be ($\tau(\tilde{\boldsymbol{w}}) < 1$), so interim human capital is low ($\tilde{h}_2 < z\tilde{i}_1$).

We assume that i_2 is chosen knowing the actual realization for $\tilde{h}_2$. That is, families are able to evaluate their child's skill/achievement, effectively learning that their beliefs were mistaken.[34] Given the resulting interim human capital, late investments are determined as in the previous subsection (see equation (9)). Finally, adult human capital $\tilde{h}_3$ is produced based on actual $\tilde{h}_2$ and late investment:

$$\tilde{h}_3 = \theta f[\tilde{h}_2, \hat{i}_2(\tilde{h}_2, \theta)].$$

where $\hat{i}_2(\cdot, \cdot)$ is defined in equation (9).

We first study how systematic bias affects early investment and human capital accumulation.

Proposition 6. *(i) $\tilde{i}_1$ is strictly increasing in $\tilde{z}$ if and only if Condition 1 holds for $(h_2, i_2) = [\tilde{z}\tilde{i}_1, \hat{i}_2(\tilde{z}\tilde{i}_1, \theta)]$. (ii) Suppose that $\tilde{\boldsymbol{w}} = \boldsymbol{w}$. If and only if $\tilde{i}_1 \leq i_1^*$, then $\tilde{\boldsymbol{x}} \leq \boldsymbol{x}^*$, $\tilde{h}_2 \leq h_2^*$, $\tilde{h}_3 \leq h_3^*$, and $z\theta f_1[\tilde{h}_2, \hat{i}_2(\tilde{h}_2, \theta)] \geq R^2$.*

Families with a biased belief about z behave as if the true productivity of early investment is $\tilde{z}$ rather than z. As shown in Lemma 1, higher productivity in early investment does not necessarily lead to more early investment due to the diminishing return effect. However, when this effect is weak so that Condition 1 is satisfied (e.g. modest dynamic complementarity or substitutability), individuals with downward-biased beliefs under-invest in all early inputs/activities, resulting in low levels of human capital. Moreover, under-investment implies a high observed marginal labor market return to early investment. By contrast, when Condition 1 does not hold, families with downward-biased beliefs may over-invest (in all inputs) and obtain high levels of human capital. In this case, providing information that shifts beliefs upwards towards the truth would actually reduce early investment and human capital. Figure 6 demonstrates this possibility with CES production function (3) and $b < 0$. In this example, Condition 1 does not hold for high values of z, so moving downward-biased beliefs $\tilde{z}$ from the middle of the graph towards the true (higher) value of z would result in lower (but more efficient) levels of early investment.

Next, consider the effects of non-systematic bias.

Proposition 7. *Suppose that $\tilde{z} = z$ and $\tilde{\boldsymbol{w}} \neq \boldsymbol{w}$. Then (i) $\tilde{x}_j \leq x_j^*$ if and only if $\tilde{w}_j \leq w_j$; (ii) $\tilde{i}_1 = i_1^*$, $\tilde{h}_2 < h_2^*$, and $\tilde{h}_3 < h_3^*$; (iii)*

[34] Note that this does not necessarily require that parents learn the true productivity values (z, w); although, it is too late to matter.

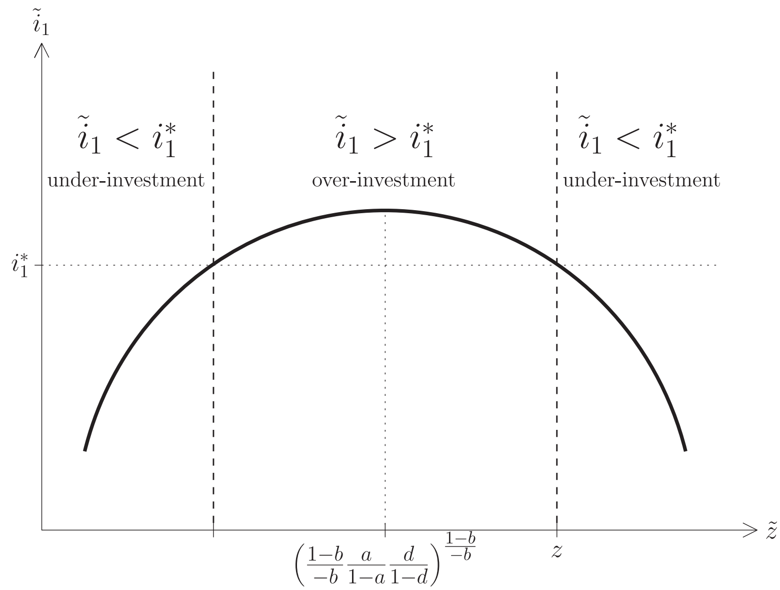

Fig. 6. Systematic bias and early investment (CES with $b < 0$)

$z\tau(\tilde{\boldsymbol{w}})\theta f_1[\tilde{h}_2, \hat{i}_2(\tilde{h}_2, \theta)] < R^2$ *if Condition 1 holds for all* $(h_2, i_2) = [z' i_1^*, \hat{i}_2(z' i_1^*, \theta)]$ *where* $z' \in [z\tau(\tilde{\boldsymbol{w}}), z]$.

Non-systematic bias does not affect total early investment spending $\tilde{i}_1$, but it reduces the actual return to early investment due to the misallocation of resources to the wrong inputs. As such, it leads to low levels of human capital. In this case, providing more precise information will not affect total early investment expenditures, but it will lead to more efficient human capital production and, consequently, greater human capital. Because non-systematic bias reduces the productivity of early investment while leaving total investment expenditure unaffected, its effect on the marginal return to early investment depends on Condition 1. When Condition 1 holds, families with non-systematic bias have lower marginal labor market returns to early investment due to misallocation.

Information Problems and the Stylized Facts

The nature of an information problem is important for understanding its effects on human capital investment behavior. While none of the information problems we study are able to explain why the timing of income is important for human capital investment (Fact 4), some are more consistent with the other stylized facts in Section III.

Uncertainty about child ability θ (or labor market returns to human capital) coupled with risk aversion causes families to under-invest in their children. With decreasing absolute risk aversion, under-investment is worse among the poor, and an increase in lifetime parental income would be met with an increase in child investments (Fact 3). Expected marginal returns exceed the return to savings (Fact 1) and are especially high for children from low-income families (Fact 2). These results apply whether uncertainty is objective or subjective.

We also explore the implications of uncertainty resolved after early investments have been made but in time for late investment choices to respond. Here, we abstract from risk aversion in order to emphasize the role of early investment irreversibility and the technology of skill formation. If early and late investments are mildly complementary, then subjective uncertainty about the productivity of early investments, z, can lead to under-investment in young children and high marginal returns to early investment (relative to interest rates). If poor families face greater subjective uncertainty about z, then they will invest less in their young children than higher-income families, stopping investment when marginal returns are relatively high.[35] Thus, with modest dynamic complementarity, differences in subjective uncertainty by parental income can help to explain Facts 1 and 2. While this form of uncertainty can explain the positive correlation between parental income and child investments, it does not help us understand why changes in parental income lead to contemporaneous changes in child investments and achievement (Facts 3 and 4) unless income brings new information with it.[36] Uncertainty about θ (with risk neutrality) resolved after early childhood is inconsistent with a positive parental income–child investment relationship and other stylized facts, unless early and late investments are substitutes.[37]

Finally, we consider the possibility that families are simply mistaken about the productivity of early investment activities as documented in Cunha (2014) and Dizon-Ross (2015). With modest dynamic complementarity, we show that poor families that systematically underestimate the productivity of early investments will under-invest in their young children and have a high marginal return to early investment relative to the return to savings and the marginal return for high-income families with accurate beliefs (Facts 1 and 2). By contrast, non-systematic bias

[35] With purely objective uncertainty and risk neutrality, expected marginal returns to investment always equal the return on savings.

[36] Changes in income might lead to changes in information and, therefore, investment behavior, if information about the productivity value of investment can be purchased by families. We do not explicitly model this possibility.

[37] Available evidence suggests dynamic complementarity for investments (Cunha *et al.*, 2010; Caucutt and Lochner, 2012; Attanasio *et al.*, 2015).

(i.e., overestimation of the productivity of some inputs offset by underestimation of the productivity of others) has no effect on total early investment expenditures. Instead, it results in a mis-allocation across early inputs, which tends to reduce the marginal return to early expenditures.[38] Thus, non-systematic bias among poor families cannot explain the basic correlation between family income and child investment/achievement, nor can it explain the high marginal return to investment among the poor (Facts 1 and 2). Neither form of bias helps to explain the responsiveness of early investment and achievement to changes in income (Facts 3 and 4).[39]

VIII. Borrowing Constraints

Lastly, we consider the possibility that families might be unable to borrow against future earnings to efficiently finance investments in their children. Studies of intertemporal consumption behavior frequently document patterns consistent with borrowing constraints, with many finding stronger evidence of binding constraints among younger households (e.g., Meghir and Weber, 1996; Alessie *et al*, 1997; Stephens 2008).[40] A number of studies also estimate the empirical impacts of borrowing constraints on college-going behavior; however, there is less evidence on the extent to which borrowing constraints distort early investments in children. See Lochner and Monge-Naranjo (2012) for a recent survey of this literature.

In this section, we use our framework to analyze the implications of borrowing constraints at different stages of child development for human capital investment. Consistent with Cunha and Heckman (2007), our analysis demonstrates the importance of dynamic complementarity of investments for the impacts of both borrowing constraints and family resources.

We incorporate borrowing constraints by imposing upper limits on the total debts families can accumulate in any period. Specifically, we restrict assets carried into any period $j+1$ to satisfy the constraint $a_{j+1} \geq -L_j$. To focus on the role of borrowing constraints, we consider the problem of Section IV (now with borrowing constraints) assuming $z = 1$, $\nu = 0$, and perfect information.

Because we are not only concerned with borrowing constraints during the investment period, but also later in life, we interpret the continuation

[38] The prediction of overinvestment in some inputs and underinvestment in others conflicts with Figures 2 and 3, which shows that the poor generally invest less than the rich in nearly all inputs.

[39] Systematic bias in z might be consistent with Fact 3 if $b > 0$ and beliefs about z respond to changes in family income. See footnote 36.

[40] In related work, Attanasio *et al.* (2008) conclude that many younger and middle-age American households are likely to be borrowing constrained based on differential car loan demand elasticities with respect to interest rates and loan maturity.

utility $V[a_3, \theta f(h_2, i_2)]$ as the solution to the asset allocation problem for individuals entering adulthood, allowing for the possibility of binding future constraints. We assume that individuals live to age T and that adult earnings depend on human capital acquired through childhood investments h_3, growing exogenously thereafter with

$$h_j = \Gamma_j h_3, \qquad j \in \{4, \dots, T\}. \tag{13}$$

Individuals entering adulthood with human capital h_3 and assets a_3 allocate consumption across their remaining life in the following way:

$$V(a_3, h_3) = \max_{c_3,\dots,c_T} \sum_{j=3}^{T} \beta^{j-3} u(c_j), \tag{14}$$

subject to budget constraints $a_{j+1} = Ra_j + h_j - c_j$ for $j \in \{3, \dots, T\}$, borrowing constraints $a_{j+1} \geq -L_j$ for $j \in \{3, \dots, T-1\}$, and $a_{T+1} = 0$.[41]

Given the value function defined in equation (14), families solve the maximization problem (4) subject to budget constraints (5), initial assets a_1, and borrowing constraints $a_2 \geq -L_1$ and $a_3 \geq -L_2$.

Investment Behavior

Consumption allocations satisfy $u'(c_j) \geq \beta R u'(c_{j+1})$, $\forall j = 1, \dots, T-1$, where the inequality is strict if and only if the borrowing constraint for period j $(a_{j+1} \geq -L_j,)$ binds. First-order conditions for investment are given by

$$u'(c_1) = \beta^2 \theta \frac{\partial V(a_3, h_3)}{\partial h_3} f_1(i_1, i_2), \tag{15}$$

$$u'(c_2) = \beta \theta \frac{\partial V(a_3, h_3)}{\partial h_3} f_2(i_1, i_2), \tag{16}$$

where $\partial V(a_3, h_3)/\partial h_3 = \sum_{j=3}^{T} \beta^{j-3} \Gamma_j u'(c_j) > 0$ with $\Gamma_3 = 1$. Taking the ratio of these equations reveals that optimal investment equates the technical rate of substitution in the production of human capital with the marginal rate of substitution for consumption:

$$\frac{f_1(i_1, i_2)}{f_2(i_1, i_2)} = \frac{u'(c_1)}{\beta u'(c_2)} \geq R.$$

Unconstrained optimal investments, i_1^* and i_2^*, satisfy $\chi \theta f_1(i_1^*, i_2^*) = R^2$ and $\chi \theta f_2(i_1^*, i_2^*) = R$, where $\chi = \sum_{j=3}^{T} R^{3-j} \Gamma_j$ reflects the discounted

[41] Online Appendix B shows that $V_2 > 0$, $V_{22} < 0$, and $V_{21} < 0$. These properties are used repeatedly in proving the results below.

present value of an additional unit of human capital. As in Section V, unconstrained investments maximize the discounted present value of lifetime earnings net of investment costs. They are independent of the marginal utility of consumption and income/transfers, because individuals can optimally smooth consumption across periods. This is not true when borrowing constraints bind, as shown in the next proposition.

Proposition 8. *(i) If and only if any borrowing constraint binds, then optimal early investment is strictly less than the unconstrained amount, the marginal return to early investment is strictly greater than the return to savings, and adult human capital is strictly less than the unconstrained level. (ii) If any borrowing constraint binds and either (a) the period one constraint does not bind or (b)* $f_{12} > 0$*, then optimal late investment is strictly less than the unconstrained amount. (iii) If and only if any borrowing constraint in period two or later binds, then the marginal return to late investment is strictly greater than the return to savings.*

Early investment is always low and its marginal labor market return high (relative to the unconstrained case) when any borrowing constraints bind. Late investment is also low and its marginal return high if constraints at that age or later are binding and either the early constraint does not bind or early and late investments are complementary.

The complementarity of investments across periods plays a central role in determining individual responses to borrowing constraints and changes in parental income. If investments are very substitutable, individuals can shift investment from constrained periods to unconstrained periods with little loss to total acquired human capital. Their ability to do this is diminished as investments become more complementary. In particular, the following dynamic complementarity condition is important for a number of results.

Condition 2.

$$\frac{f_{12} f}{f_1 f_2} > \frac{-V_{22}(-RL_2, h_3) h_3}{V_2(-RL_2, h_3)}.$$

If preferences are given by the constant intertemporal elasticity of substitution (IES) form $u(c) = c^{1-\sigma}/(1-\sigma)$ (where $1/\sigma$ is the IES) and credit constraints are non-binding throughout adulthood, then this condition simplifies to something very intuitive:

$$\underbrace{\frac{f_1 f_2}{f_{12} f}}_{\text{Hicksian elasticity of substitution}} < \underbrace{\frac{1}{\sigma}}_{\text{IES}} \underbrace{\left(1 - \frac{RL_2}{\chi h_3}\right)}_{1 - \frac{\text{maximum debt}}{\text{lifetime income}}}.$$

See Online Appendix B for details.[42] As the Hicksian elasticity of substitution between early and late investments declines (i.e., investments become more complementary) or the consumption intertemporal elasticity of substitution increases (i.e., individuals become less concerned about maintaining smooth consumption profiles), this inequality is more likely to hold. More generally, when individual preferences for smooth consumption are strong, Condition 2 requires strong complementarity between early and late investments.

We are now ready to study how family income during early and late childhood affect investment behavior. As noted above, changes in family income have no effect on investments for unconstrained individuals. The following proposition shows how constraints at different stages of child development determine the responsiveness of investment to changes in income at early and late ages. These results highlight how the timing of income/transfers can impact human capital investments and accumulation when individuals are constrained.[43]

Proposition 9. *(i) If borrowing constraints bind in late childhood, but not early childhood, then:*

$$\text{(a)}\frac{\partial i_1}{\partial y_1} = R\frac{\partial i_1}{\partial y_2} = \frac{\partial i_1}{\partial (R^{-1}y_2)} > 0;$$

$$\text{(b)}\frac{\partial i_2}{\partial y_1} = R\frac{\partial i_2}{\partial y_2} = \frac{\partial i_2}{\partial (R^{-1}y_2)} > 0;$$

$$\text{(c)}\frac{\partial h_3}{\partial y_1} = R\frac{\partial h_3}{\partial y_2} = \frac{\partial h_3}{\partial (R^{-1}y_2)} > 0.$$

(ii) If borrowing constraints only bind in early childhood, then:

$$\text{(a)}\frac{\partial i_1}{\partial y_1} > 0; \quad \text{and} \quad \frac{\partial i_1}{\partial y_2} < 0;$$

$$\text{(b)}\frac{\partial i_2}{\partial y_1} > 0 \iff f_{12} > 0; \quad \text{and} \quad \frac{\partial i_2}{\partial y_2} < 0 \iff f_{12} > 0;$$

$$\text{(c)}\frac{\partial h_3}{\partial y_1} > 0; \quad \text{and} \quad \frac{\partial h_3}{\partial y_2} < 0.$$

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[42] For the CES production function given in equation (3), the Hicksian elasticity of substitution between early and late investments (the left-hand side) is simply $d/(d-b)$. The condition cannot hold for $d \leq b$, but this only rules out very strong substitution between early and late investments such that $f_{12} \leq 0$.

[43] See Cunha and Heckman (2007) for a related analysis of the impacts of early versus late income on the early-to-late investment ratio i_1/i_2 when the early borrowing constraint binds.

(iii) If borrowing constraints bind during both early and late childhood, then:

$$\text{(a)}\frac{\partial i_1}{\partial y_1} > 0; \quad \text{and} \quad \frac{\partial i_1}{\partial y_2} > 0 \iff \text{Condition 2 holds;}$$

$$\text{(b)}\frac{\partial i_2}{\partial y_1} > 0 \iff \text{Condition 2 holds;} \quad \text{and} \quad \frac{\partial i_2}{\partial y_2} > 0;$$

$$\text{(c)}\frac{\partial h_3}{\partial y_1} > 0; \quad \text{and} \quad \frac{\partial h_3}{\partial y_2} > 0.$$

There are two key implications of this proposition. First, if the late constraint binds but the early constraint does not, then investments depend only on the discounted present value of family income $y_1 + R^{-1}y_2$, not the timing of income (conditional on discounting y_2). Second, when the early constraint binds, the response of investments to changes in income depends on the timing of income, the extent of dynamic complementarity, and whether late constraints are also binding. While constrained early investment is always increasing in y_1, this is not necessarily the case for changes in y_2. Because an increase in late income exacerbates the early borrowing constraint, early investment is unambiguously decreasing in y_2 when the late constraint does not also bind. Intuitively, families would like to consume some of the increased late income in the earlier period as well; however, if they are borrowing constrained, they can only do this by reducing early investments. When only the early constraint binds, the impacts of income on late investment depend entirely on its effect on early investment and whether early investment raises ($f_{12} > 0$) or lowers ($f_{12} < 0$) the marginal return to late investment. Perhaps surprisingly, when $f_{12} > 0$ and only the early constraint binds, then an increase in family income during late childhood reduces skill investments in both periods. When constraints are binding throughout (early and late) childhood, increases in income in any period increase investment in both periods if and only if there is sufficient dynamic complementarity.

To better understand the implications of policies aimed at expanding credit for educational investments, we consider the impacts of raising borrowing limits for families at different stages of child development, beginning with limits faced by families with older children (e.g., expanding student loan programs for higher education).

Proposition 10. *Assume that the borrowing constraint binds during late childhood (i.e., $a_3 = -L_2$). (i) If the early borrowing constraint does not bind (i.e., $a_2 > -L_1$), then $\partial i_1/\partial L_2 > 0$, $\partial i_2/\partial L_2 > 0$, and $\partial h_3/\partial L_2 > 0$. (ii) If the early borrowing constraint also binds (i.e., $a_2 = -L_1$), then $\partial i_1/\partial L_2 > 0$ if Condition 2 holds, $\partial i_2/\partial L_2 > 0$, and $\partial h_3/\partial L_2 > 0$.*

Relaxing the borrowing constraint during late childhood unambiguously increases late investment. If the early constraint is non-binding or if early and late investments are sufficiently complementary, then any increase in late investment also encourages additional early investment. Even in the case of strong intertemporal substitutability when early investment might decline, individuals acquire more adult human capital when the late constraint is relaxed. Altogether, these results are fairly intuitive.

Next, we show that relaxing borrowing constraints on families during early childhood (e.g., loans for preschool) can lead to more surprising effects on investment behavior depending on the extent of dynamic complementarity.

Proposition 11. *Assume that the borrowing constraint binds during early childhood (i.e., $a_2 = -L_1$). (i) If no other borrowing constraint binds, then $\partial i_1/\partial L_1 \in (0, 1)$, $\partial i_2/\partial L_1 > 0 \iff f_{12} > 0$, and $\partial h_3/\partial L_1 > 0$. (ii) If the late borrowing constraint also binds (i.e., $a_3 = -L_2$) and Condition 2 does not hold, then $\partial i_1/\partial L_1 > 0$ and $\partial i_2/\partial L_1 < 0$.*

When individuals are only constrained during early childhood, relaxing that constraint leads to an increase in early investment, which encourages late investment as long as the marginal productivity of i_2 is increasing in i_1.

When children are constrained in both periods, relaxing the early constraint effectively shifts resources from late to early childhood. If early and late investments are very complementary, they will both tend to move in the same direction. In most cases, investments will increase; however, it is possible that investments could actually decrease in both periods. Intuitively, if late investment is very productive, then relaxing the early borrowing constraint can starve that investment. By contrast, if investments are sufficiently substitutable over time, shifting resources from late to early childhood by relaxing the early constraint causes investment to shift from the late to the early period as well.

The stylized facts of Section III and other evidence in Caucutt and Lochner (2012) are most consistent with binding early and late constraints and sufficient dynamic complementarity (Proposition 9(iii)). In this case, investments increase with additional family income (Fact 3) and the timing of income matters (Fact 4). Because poor children are more likely to be borrowing constrained, Proposition 8 implies that they are likely to have marginal labor market returns that exceed the return to savings as well as the marginal returns for unconstrained children from higher income families (Facts 1 and 2). Finally, Propositions 10 and 11 suggest that policies designed to expand borrowing opportunities (at either stage of child development) can raise the investment and skill levels of children from constrained (i.e., low-income) families, improving both efficiency and equity.

IX. Summary and Conclusions

It is well known that poor children perform much worse academically and in achievement tests than their more economically advantaged counterparts. The most immediate explanation for these differences is that poor parents invest less in their young children. As we document, poor parents have fewer books in the home, read less to their young children, engage in fewer lessons and extracurricular activities, etc. Important differences in investment activities and achievement relating to family income remain even after controlling for maternal characteristics such as education, achievement, and race. In this paper, we ask the next logical question: why do poor parents invest so much less in their children?

While there are many competing theories for these investment and skill gaps, few studies attempt to sort amongst them.[44] We systematically study four leading investment-based theories/mechanisms thought to drive income-based skill gaps: an intergenerational correlation in ability, a consumption value of investment, information frictions, and credit constraints. In order to help understand which mechanisms drive family investments in children, we consider the extent to which they also explain other important stylized facts related to the marginal returns to investment and the effects of parental income on child investment and skills.

The main lessons from our theoretical analysis are summarized in Table 1, which shows considerable differences in the extent to which each mechanism explains important stylized facts about child development. While a positive intergenerational correlation in ability might be partially responsible for the relationship between family income and child investment and achievement, it is not helpful for understanding any of the other important stylized facts. A theory based only on a positive consumption value of investment can explain the positive causal effects of income on investment as well as decreasing marginal labor market returns in family income; however, it predicts overinvestment in skills such that the labor market returns to investment should be less than the return to savings. This mechanism offers no explanation for the importance of early income relative to late income.

Uncertainty coupled with risk aversion leads to underinvestment in human capital and high marginal returns to additional investment. With decreasing absolute risk aversion, investment disincentives are greater for the poor. Thus, this mechanism can explain the qualitative patterns for marginal

[44] Cunha (2014) is an important recent exception.

Table 1. *Summary of results*

	High MR to i_1 for poor	Lower MR to i_1 for rich	Increase in income causes an increase in i_1	Timing of income
Intergenerational ability correlation	No	No	No	No
Consumption value ($\nu > 0$)	No	Yes	Yes	No
Uncertainty with risk aversion	Yes	Yes	Yes	No
Poor more subjective uncertainty, risk neutrality, i_1 irreversible:				
Uncertainty in θ	No[a]	No[a]	No	No
Uncertainty in z	Yes[a,b]	Yes[a,b]	Only if info. changes with income[b]	No
Poor have biased beliefs:				
Systematic downward bias	Yes[b]	Yes[b]	Only if info. changes with income[b]	No
Non-systematic bias	No[b]	No[b]	No	No
Credit constraints	Yes	Yes	Yes	Yes

Notes: Where relevant, the results in the table assume gross complementarity (i.e., $f_{12} \geq 0$). [a]Under purely objective uncertainty, expected marginal returns equal return to savings for everyone. [b]Assumes that Condition 1 holds or that $b \geq 0$ in the case of CES production function.

returns documented in the literature as well as the evidence on causal effects of income on early child investments and achievement. Neither this mechanism nor any other information-based explanation we explore can explain why the timing of income is important. Even in the absence of risk aversion, subjective uncertainty in the productivity of early investments can lead to underinvestment and high marginal returns due to the irreversibility of investments. If poor families face greater subjective uncertainty than rich families, then predicted patterns for marginal returns are consistent with empirical evidence. This is also true if poor families simply underestimate the productivity of early investments compared to higher-income families. Unless changes in income directly improve the information of poor families, these mis-information problems only generate a correlation between family income and investment; they cannot explain why changes in income produce changes in investment or achievement.

The inability of poor families to borrow against future income can lead to underinvestment in their children, which can further explain high marginal returns to investment among the poor. For children in constrained families, improvements in income lead to increases in investment and higher skill levels. If constraints are binding for families with young children, the timing of income will be important. Thus, binding credit constraints are consistent with the four main stylized facts we consider.[45]

We caution that our comparison of model predictions with the evidence should not be taken as a score sheet, evaluating the importance of each mechanism by the number of facts it explains. A positive intergenerational correlation in ability is almost certainly important given the extent to which maternal characteristics help to explain income-based differences in investment and achievement (see Figures 1 and 5); yet, it offers no explanation for any of the other stylized facts. A number of recent studies also document important biases in beliefs about the productivity of early investments or labor market returns to education; however, the extent to which these biases explain differences in investment and achievement by parental income remains to be seen. Our results suggest that these biases are unlikely to explain why child achievement improves when family income rises or why the timing of income is important.

Our primary contribution is to help clarify key empirical predictions of different mechanisms that can be useful in thinking about policy options or in future empirical research aimed at quantifying the importance of those mechanisms. For example, our analysis suggests that information or

[45] Borrowing constraints cannot easily explain why many poor mothers hold biased beliefs about the productivity of investments in children (Cunha, 2013; Dizon-Ross, 2015); however, they can explain why schooling choices among the most poor are relatively unresponsive to differences in those beliefs or to new information (Attanasio and Kaufmann, 2009; Jensen, 2010).

credit market frictions are needed to explain the high marginal returns to early investment among the poor. This is important, because it means that appropriately designed policies might be able to reduce inequality while improving economic efficiency. We also show that evidence on the relative importance of early versus late family income for child investment and achievement is particularly useful for identifying the presence of credit market frictions, as none of the other mechanisms predict that the timing of income matters. More generally, a better understanding of the implications and limits of different mechanisms should be helpful in refining current theories and empirically sorting out their quantitative importance. In particular, future empirical research should attempt to exploit data/evidence on the types of relationships we highlight to aid in the identification of general models that incorporate multiple mechanisms.

Supporting Information

The following supporting information can be found in the online version of this article at the publisher's web site.

Online Appendix

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