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Communication in Cournot oligopoly ☆

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Abstract

This paper studies communication in a static Cournot duopoly model under the assumption that the firms have unverifiable private information about their costs. We investigate the conditions under which the firms cannot transmit any information through cheap talk, and show that when these conditions are violated, it may be possible to construct informative cheap-talk equilibria. If the firms can communicate through a third party, communication can be informative even when informative cheap talk is impossible. We exhibit a simple mediated mechanism that ensures informative communication and interim Pareto dominates the uninformative equilibrium for the firms.

JEL classification: C72; D21; D43; D82; D83

Keywords: Cournot oligopoly; Communication; Information; Cheap talk; Mediation

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1. Introduction

It is well recognized in both the theoretical literature and the antitrust law that information exchange between firms in an oligopolistic industry can have several effects (see, for example, [34] and [27]). On the one hand, more precise information about the market allows the firms to make more effective decisions. On the other hand, information exchange may facilitate collusion and increase barriers to entry, which reduce consumer surplus. Therefore, assessing the effects of communication on equilibrium prices and production is both interesting from the theoretical point of view and important for developing guidelines for competition policy. This paper contributes to the discussion by studying the possibility of informative communication in a Cournot oligopoly model where the firms have unverifiable private information about their costs.

There is a large literature on information exchange in oligopoly with private information about costs. In a typical scenario, the firms participate in information exchange before playing a one-shot Cournot game. Information is assumed to be verifiable, i.e. a firm can conceal its private information but cannot misrepresent it. Examples include Fried [15], Li [29], Gal-Or [18], Shapiro [40], Okuno-Fujiwara, Postlewaite and Suzumura [37], Raith [38] and Amir, Jin and Troege [1].[2] Most of these papers assume that each firm decides whether to share its information or not before it observes the cost realization. The conclusion from this literature is that in a Cournot oligopoly with linear demand, constant marginal cost and independently distributed cost shocks, each firm finds it profitable to commit to disclose its private information. The paper by Okuno-Fujiwara, Postlewaite and Suzumura [37] assumes that each firm decides whether to reveal its cost realization after observing it. In this case, it is shown that, due to an unravelling argument, under the standard conditions the firms will fully disclose their private information about costs in all sequential equilibria.[3]

However, the assumption that private information is costlessly verifiable may be restrictive. Ziv [46] notes that information about a firm's cost function "is part of an internal accounting system that is not subject to external audit and not disclosed in the firm's financial statements" (p. 456), which makes it potentially costly or impossible to verify, and that even if the verification took place, punishment for misrepresenting the information is unavailable in a one-shot game, because contracts that prescribe such punishment may violate antitrust law. In some cases, external verification of information is impossible in principle, as when the communication between firms takes the form of planned production preannouncements (an empirical investigation of information exchange via production preannouncements can be found in [11]). Therefore, one may wish to examine whether the conclusions of the literature on information sharing in oligopoly are robust to the assumption that information is verifiable.

Ziv [46] addresses this question in the framework of a Cournot duopoly with linear demand and constant marginal costs. He assumes that the marginal costs are private information, and each firm can send a cheap-talk message to its competitors before choosing its output. He shows that if the information is unverifiable, the conclusion that each firm will be willing to share the information no longer holds. To understand this result, suppose that there exists an equilibrium where each firm announces its cost realization truthfully, the competitors take each announce-

[2] A related strand of literature [35,44,17,25] studies information sharing between firms having private information about demand; Li [29], Raith [38] and Amir, Jin and Troege [1] cover both cost uncertainty and demand uncertainty.

[3] These results have been further generalized in [42].

ment at face value, and the output of each type of each firm is positive. Then, regardless of the true cost realization, each firm would like to deviate and announce the lowest possible cost in order to appear more aggressive and thus make the competitors reduce their output.

Various mechanisms to make unverifiable cost announcements credible have been considered in the literature. For instance, different announcements can be accompanied by appropriate levels of 'money burning' [46]. Alternatively, the announcements can determine the amount of side payments in a collusive contract [10] or the level of future 'market-share favors' from the competitors in repeated settings [9].

In this paper, we consider a Cournot duopoly model which generalizes the linear demand-constant marginal cost setting that is considered in almost all previous work. Each firm has unverifiable private information about the value of its marginal cost. We assume that the game is played only once, the firms cannot commit to information disclosure ex ante, and the communication between the firms cannot be substantiated by any costly actions.

First, we address the question of whether informative communication through cheap talk is possible in our model. While the intuition behind the impossibility of informative communication in [46] is compelling, the techniques of that paper are not applicable to a nonlinear setting. More importantly, there are results in the cheap-talk literature that show that informative communication is possible in some games where all the sender's types have the same preference ordering over the receiver's actions.[4] Nevertheless, in Theorem 1 we show that no information transmission is possible through one round of cheap talk in the environments where several assumptions are satisfied, including: (i) all cost types always find it optimal to produce; (ii) the firms' cost types are independently distributed; (iii) the inverse demand of a firm is additively separable in outputs of all firms and is linear in the opponents' output.

More generally, we prove that no cheap-talk game that lasts for a pre-determined finite number of rounds has an informative equilibrium (Theorem 2). We also show that assumptions (i)–(iii) are important for the impossibility of informative cheap-talk communication. If either assumption is not satisfied, then there may exist equilibria with informative cheap talk (Examples 2–4).

Next, we show that informative communication is possible even in the environments that satisfy assumptions (i)–(iii) if the firms are allowed to use more complex communication protocols than one-shot cheap talk. In particular, we consider the scenario where the firms can communicate through a neutral and trustworthy third party (a mediator). The mediator can both receive costless and unverifiable reports from the firms about their cost realizations and send messages back to the firms. In this setting, we show that for a range of parameters there exists a simple communication protocol that makes information transmission possible in equilibrium (Theorem 3) and leaves every type of every firm better off than in the Bayesian–Nash equilibrium without communication (Theorem 4).[5] The reason for this is that the mediator can play the role of an information filter between the firms: a firm does not get to see the competitor's cost report directly, and the amount of information that it gets about the competitor's cost depends on its own

[4] See [39,4,5].

[5] Liu [30] considers communication protocols that make use of a third party (correlated equilibria) in a Cournot oligopoly with complete information. He shows that the possibility of communication does not enlarge the set of possible outcomes: the only correlated equilibrium is the Nash equilibrium. We show that a similar result holds in our model too (Lemma 3). Therefore, for informative communication through a mediator to be possible, the mediator has to be able not only to send messages to the firms, but to receive cost reports from them as well.

report to the mediator.[6] Therefore, even though a higher cost report may lead to higher expected output by the competitor, it can cause the mediator to disclose more precise information about the competitor, which can make truthful reporting by the firms incentive compatible.[7] Finally, we generalize Theorem 3 to the case of more than two firms (Theorem 5), and show that when the number of firms is large enough, our communication protocol can be implemented without the help of the mediator (Theorem 6).

Our results have two implications for competition policy. First, they add a new aspect to the question of whether firms should be allowed to exchange disaggregated versus aggregate data. This issue is currently viewed mainly from the perspective of determining which of the regimes is more conducive to sustaining collusive equilibria when the firms interact repeatedly. From this point of view, the exchange of disaggregated data may be more harmful than the exchange of aggregate statistics, because, in case of a deviation from the collusive agreement, the former regime allows to establish the identity of the deviator [27]. For this reason, the competition policy views the exchange of aggregate statistics more favorably.[8] What we show is that information aggregation can have another effect: it can relax the incentive compatibility constraint of the participants of the data exchange and thus lead to more information revelation.[9]

Second, our results contradict the notion that efficiency-enhancing exchange of unverifiable information is infeasible, and therefore the only possible purpose for the exchange of such information is to sustain a collusive agreement.[10] We show that this is not necessarily true, and that exchange of unverifiable information can be efficiency-enhancing.

The rest of the paper is organized as follows. In Section 2 we describe an example that illustrates the ideas behind some of our results. Section 3 contains a description of the model. In Section 4 we analyze unmediated public communication (cheap talk) and show that under certain assumptions it cannot result in informative communication, while informative cheap-talk communication may be possible if these assumptions are not satisfied. In Section 5 we exhibit a simple mediated mechanism that ensures informative communication. Section 6 contains various extensions; in particular, we show that our mechanism can be implemented without a mediator when the number of firms is large. All proofs are relegated to Appendix A and the Online Appendix unless stated otherwise.

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6 The idea that introducing noise into communication in sender-receiver games can improve information transmission was introduced by Myerson [33] and analyzed in detail by Blume, Board and Kawamura [8].

7 The idea that an informed party may be induced to reveal information by making the amount of information it gets about its competitor contingent on its own message appears in Baliga and Sjöström [5], although the model and the results of that paper significantly differ from ours.

8 For example, Kühn and Vives [27] note that the European Commission "has no objection to the exchange of information on production or sales as long as the data does not go as far as to identify individual businesses".

9 In their narrative analysis of the Sugar Institute, a cartel of sugar refiners that operated in the US in 1928–1936, Genesove and Mullin [19] note that the confidentiality procedures adopted by the Institute in gathering and aggregating the data may have been adopted to ensure incentive compatibility for participating firms. To our knowledge, this insight has never before been formalized within a theoretical oligopoly model.

10 For example, the 2010 OECD report on "Information Exchanges between Competitors under Competition Law" [36] states:

> One important factor that the literature points out is that communications between firms may have little value in facilitating coordination unless the information is verifiable. Information which is not verifiable can be dismissed as "cheap talk" and therefore disregarded. However, some have suggested that "cheap talk" can assist in a meeting of minds and allow firms to reach an understanding on acceptable collusive strategies. (p. 34)

2. Example

Consider two symmetric firms producing a homogeneous good, the inverse demand for which is $P(Q) = 3 - Q$. Each firm has a linear cost function, the value of the marginal cost being its private information. Specifically, each firm can be either of type L, with the marginal cost of 0, or H, with the marginal cost of 2. The types are independently and identically distributed, and the probability of type L is $p \in (0, 1)$. Regardless of the type realization, each firm has a capacity constraint of x units, where $x \in (\frac{1}{3}, 1)$.

Suppose that firm i's expectation of the opponent's output is Q_{-i}. Then firm i's optimal output maximizes its profit function $\pi_i(q_i, Q_{-i}, c_i) = (3 - q_i - Q_{-i} - c_i)q_i$, where c_i is the marginal cost of firm i. It is easy to check that for a firm of type L, the capacity constraint binds whenever its expectation of the opponent's output does not exceed 1, and such a firm will find it optimal to produce x. On the other hand, the capacity constraint never binds for a firm of type H, and its optimal output is $q_i(Q_{-i}) = \frac{1-Q_{-i}}{2}$, which results in the profit of $(\frac{1-Q_{-i}}{2})^2$.

To start, consider the Bayesian–Nash equilibrium of the Cournot game where the firms simultaneously choose their outputs. In this equilibrium, a firm of type L chooses x and a firm of type H chooses q_H that satisfies the equation

$$q_H = \frac{1 - (px + (1 - p)q_H)}{2}$$

The solution to this equation is $q_H = \frac{1-px}{3-p}$.

Now suppose that the firms can commit to truthfully disclosing their cost realization to the competitor before making their production decisions. In this case, if the firms learn that both of them are of type H, both will produce $\frac{1}{3}$; if they learn that one of the firms is of type H and the other one of type L, the type-H firm will produce $\frac{1-x}{2}$. As before, a type-L firm will produce x regardless of what it knows about the opponent. It is straightforward to check that in this case, the ex ante expected profit of each firm is higher than in the case where the costs are private information.[11] Therefore, if the firms could participate in such an information-sharing agreement, they would have an incentive to do so.

Suppose, however, that such an information-sharing agreement is infeasible, and all a firm can do is make a public announcement about its marginal cost realization before choosing its output level. The announcements are made simultaneously, and are costless and unverifiable ("cheap talk"): a firm has no way to check whether its opponent has told the truth about its marginal cost. Let us show that in this case, the firms will not reveal their information truthfully in equilibrium.

Indeed, suppose a truthful equilibrium exists. In such an equilibrium, if a firm truthfully announces type H, it will find it optimal to produce $\frac{1}{3}$ if the opponent announces H as well, and $\frac{1-x}{2}$ if the opponent announces L. A firm of type L that truthfully discloses its type will find it optimal to produce x no matter what the opponent announces. Suppose that a type-H firm discloses its type truthfully. Then with probability p it will learn from its opponent's announcement that the opponent will produce x, and with the remaining probability it will learn that the opponent will produce $\frac{1}{3}$. But suppose that a type-H firm deviates and announces that its type is L;

[11] The difference in the ex ante expected profits between the complete information and the incomplete information case equals $\frac{p(1-p)^2(3x-1)(81x+5p-21-21px)}{36(3-p)^2}$, which is strictly positive for any $p \in (0, 1)$ and $x \in (\frac{1}{3}, 1)$.

then with probability p it will still learn that the opponent will produce x, but with the remaining probability it will learn that the opponent will produce $\frac{1-x}{2} < \frac{1}{3}$. Because the firm prefers the opponent to produce less, this deviation is profitable, and a truthful equilibrium does not exist. Therefore, even though the firms have an ex ante incentive to share their information, sharing it truthfully through cheap-talk messages is impossible: a high-cost firm will have an incentive to pretend that its cost is low in order to scare the opponent into producing less.[12,13]

To counteract this incentive, let us amend the information exchange scheme as follows. Suppose that, instead of announcing their types to each other, the firms report them privately to a neutral trustworthy third party (a mediator). We still assume that the reports are costless and unverifiable. If both firms have reported that they are of type H, the mediator makes a public announcement to that effect; otherwise the mediator remains silent. We will show that in equilibrium, both firms will have an incentive to report truthfully, and their ex ante welfare will be higher than without communication.

Indeed, if both firms have truthfully announced that they are of type H, then they learn that this is the case, and each of them chooses to produce $\frac{1}{3}$. If a firm of type H has truthfully reported its type, but the mediator remains silent, then the firm learns that the opponent is of type L, and thus best responds with $\frac{1-x}{2}$. A firm of type L always finds it optimal to produce x. Therefore, conditional on any type profile, the equilibrium outputs are the same as in the case when the firms commit to disclosing their types truthfully, and therefore the ex ante profit is also the same. Let us now check whether reporting truthfully is incentive compatible. Suppose a firm of type H reports truthfully. Then, as in the case of full revelation, with probability p it will learn that the opponent will produce x (and best respond with $\frac{1-x}{2}$), and with the remaining probability it will learn that the opponent will produce $\frac{1}{3}$ (and best respond with $\frac{1}{3}$). If a type-H firm deviates and reports L, its opponent's output will be equal to x with probability p and $\frac{1-x}{2}$ with probability $1-p$, just as in case of full revelation; but unlike that case, the firm will have to choose how much to produce without the benefit of knowing how much the opponent will produce. Its best response to the lottery over the opponent's output is to produce $\frac{1}{2}(1-(px+(1-p)\frac{1-x}{2}))$. The deviation is unprofitable if

$$p\left(\frac{1-x}{2}\right)^2+(1-p)\left(\frac{1}{3}\right)^2 \geq \left(\frac{1-(px+(1-p)\frac{1-x}{2})}{2}\right)^2$$

which is true if $p \geq \frac{3x+7}{9(3x-1)}$. It is also easy to check that a type-L firm will find it profitable to report truthfully for any values of $p \in (0,1)$ and $x \in (\frac{1}{3}, 1)$.

The intuition for why the mechanism above is incentive compatible is that, at the reporting stage, it makes the firms face a tradeoff between inducing the opponent to produce less in expectation (by sending message L) and learning exactly how much the opponent is going to produce (by sending message H). Different types of the firm resolve this tradeoff differently. A type-H firm values information about how much the opponent will produce; in contrast, a type-L firm always finds it optimal to choose the same output level and thus faces no need to coordinate

[12] If the private information about cost was verifiable, then type-H firms would not be able to mimic the announcement of type-L firms, and there would exist an equilibrium with full information revelation.

[13] In principle, the cheap-talk game could have a mixed-strategy equilibrium where the messages were partially informative about the types; however, in this example such equilibria do not exist.

with the opponent. This makes it possible for the firms to truthfully reveal their information and improve their expected profit relative to the no-communication case.[14]

3. The model

We consider a model of Cournot competition between two firms, A and B, with differentiated products. The inverse demand curve for firm i's product is given by $P(q_i, q_{-i}) = \max\{\rho(q_i) - \beta q_{-i}, 0\}$, where q_i is the output of firm i. We assume that $\rho(0) > 0$ and $-\rho'(q_i) \geq \beta > 0$ for every $q_i \geq 0$. The interpretation is that the products of the two firms are perfect or imperfect substitutes, and "own effect" on demand is greater than the "cross effect".[15] Firm i's cost function is $C(q_i, c_i)$ such that $C(0, c_i) = 0$, $\frac{\partial C(q_i,c_i)}{\partial q_i} \geq 0$ with strict inequality for $q_i > 0$, and $\frac{\partial^2 C(q_i,c_i)}{\partial q_i^2} \geq 0$. A higher value of the parameter c_i is associated with higher firm i's total cost and marginal cost: $\frac{\partial C(q_i,c_i)}{\partial c_i} \geq 0$ and $\frac{\partial^2 C(q_i,c_i)}{\partial c_i \partial q_i} \geq 0$. We assume that c_i is privately observed by firm i, and that c_A and c_B are independently distributed on $C = [0, \bar{c}]$ according to continuous distribution functions F_A with density $f_A > 0$ and F_B with density $f_B > 0$.

Lemma A.1 in the Online Appendix shows that rational behavior by the firms always results in strictly positive prices, and thus we can take $P(q_i, q_{-i}) = \rho(q_i) - \beta q_{-i}$ from now on. The profit of firm i of type c_i when it produces q_i and its competitor produces q_{-i} is

$$\pi_i(q_i, q_{-i}, c_i) = \big(\rho(q_i) - \beta q_{-i}\big)q_i - C(q_i, c_i) \tag{1}$$

Let $q(q_{-i}, c_i)$ be the set of best responses of firm i of type c_i to the opponent's output q_{-i}:

$$q(q_{-i}, c_i) = \underset{q_i \geq 0}{\arg\max}\, \pi_i(q_i, q_{-i}, c_i) \tag{2}$$

We will impose the following conditions on the best response correspondence q:

$$q(q_{-i}, c_i) \text{ is single-valued, continuous everywhere, } C^1 \text{ on } \big\{(q_{-i}, c_i):\ q(q_{-i}, c_i) > 0\big\} \tag{C1}$$

$$\text{If } q(q_{-i}, c_i) > 0, \text{ then } \frac{\partial q(q_{-i}, c_i)}{\partial c_i} \leq 0 \text{ and } \frac{\partial q(q_{-i}, c_i)}{\partial q_{-i}} \in (-1 + \delta, 0) \text{ for some } \delta > 0 \tag{C2}$$

$$q\big(q(0,0), 0\big) > 0 \tag{C3}$$

To guarantee (C1) and (C2), it is enough to assume that the components of the profit are twice continuously differentiable and that ρ is "not too convex" (see Lemma A.1 in the Online Appendix for the precise statement). In particular, the best response is nonincreasing in c_i and q_{-i} because of $\frac{\partial^2 C(q_i,c_i)}{\partial c_i \partial q_i} \geq 0$ and $\beta > 0$. Condition (C3) simply requires that the most efficient type never chooses to shut down, even if facing the most efficient opponent who chooses the monopoly output.

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[14] Furthermore, it can be shown that for a range of parameters in this example, this mechanism maximizes the ex ante joint profit of the firms in the class of all incentive compatible communication mechanisms. The proof is available upon request.

[15] This is a standard assumption: see for example, [18].

For some results in the next section, we will require that all types always choose strictly positive output:

$$q(q_{-i}, c_i) > 0 \text{ for every } q_{-i} \in [0, q(0,0)] \text{ and every } c_i \in C \tag{C4}$$

This can be guaranteed, for example, by assuming $\frac{\partial C(0,c_i)}{\partial q_i} = 0$ for every $c_i \in C$ (see Lemma A.1 in the Online Appendix).

Let us illustrate these conditions with an example.

Example 1. Let $\rho(q_i) = K - q_i$, $C(q_i, c_i) = \frac{c_i}{\gamma} q_i^\gamma$ such that $K > 0$, $\gamma \geq 1$, and $\beta \in (0, 1]$. If $\gamma > 1$, then $q(q_{-i}, c_i)$ equals 0 if $K - \beta q_{-i} \leq 0$, and solves the first-order condition

$$K - 2q - \beta q_{-i} - c_i q^{\gamma-1} = 0$$

otherwise. It is easy to check that (C1)–(C4) are satisfied. If $\gamma = 1$, then $q(q_{-i}, c_i) = \max\{0, \frac{1}{2}(K - \beta q_{-i} - c_i)\}$. It is easy to check that (C1)–(C3) are satisfied, while (C4) is satisfied if $\bar{c} < \frac{K}{2}$.

Substituting $q(q_{-i}, c_i)$ into the expression for the profit (1), we obtain the indirect profit function of firm i:

$$\Pi_i(q_{-i}, c_i) = \max_{q_i \geq 0} \pi_i(q_i, q_{-i}, c_i) = \pi_i\big(q(q_{-i}, c_i), q_{-i}, c_i\big) \tag{3}$$

4. Unmediated communication

4.1. Impossibility results

In this section, we allow the firms to communicate directly with each other using costless and unverifiable messages before choosing their output levels. First, to provide a benchmark, we describe what happens in the game with no communication. After that, we investigate the consequences of allowing one round of cheap talk communication. Finally, we look at games with any pre-determined finite number of rounds of cheap talk communication.

It is well-known that in the complete-information Cournot game with two firms, the unique intersection of the firms' best responses determines not only the unique Nash equilibrium strategy profile, but also the unique outcome of the iterated elimination of strictly dominated strategies.[16] In our setting, we have an analogous result for the game with no communication.

Lemma 1. *Suppose that conditions (C1)–(C3) hold. Then in the game with no communication the profile of strategies where each firm plays according to*

$$q_i^{NC}(c_i) = q\big(Q_{-i}^{NC}, c_i\big) \text{ for every } c_i,$$
$$\text{where } Q_i^{NC} = \int q_i^{NC}(c_i)\,dF_i(c_i),\ i \in \{A, B\} \tag{4}$$

is the unique Bayesian–Nash equilibrium and the unique profile of strategies that survives iterated elimination of interim strictly dominated strategies.

[16] See for example Chapter 2 in [16].

The proof of Lemma 1 can be found in the Online Appendix.

Note that in games with multiple equilibria, one possible role for preplay communication is to allow the players to coordinate among equilibria. Given Lemma 1, preplay communication in our setting cannot be used purely for coordination, but has to involve some information revelation.

We consider the following game where the firms can engage in cheap-talk communication before making their output choices. Let M_A and M_B be the sets of possible messages for firms A and B. Each firm i sends a costless message $m_i \in M_i$, and the messages are publicly observed. Firm i's pure strategy is thus a pair of functions $(m_i(c_i), q_i(m_i, m_{-i}, c_i))$, where $m_i : C \to M_i$ is a message strategy and $q_i : M_i \times M_{-i} \times C \to \mathbb{R}_+$ is the output strategy in the continuation game following a pair of messages (m_i, m_{-i}) being observed.

Let us first consider the continuation game after a pair of messages (m_i, m_{-i}) is observed. Let $F_i(\cdot|m_i)$ be the c.d.f. of firm $-i$'s equilibrium beliefs about c_i after it has observed firm i's message m_i.[17] Similarly to Lemma 1, we can characterize what happens in such a continuation game.[18]

Lemma 2. *Suppose that conditions (C1)–(C3) hold. Then, in the game with one round of cheap-talk communication after a pair of messages (m_i, m_{-i}) is observed, the profile of strategies given by*

$$q_i(m_i, m_{-i}, c_i) = q\big(Q_{-i}(m_i, m_{-i}), c_i\big) \text{ for every } c_i,$$

$$\text{where } Q_i(m_i, m_{-i}) = \int q\big(Q_{-i}(m_i, m_{-i}), c_i\big) dF_i(c_i|m_i), \ i \in \{A, B\}$$

is the unique Bayesian–Nash equilibrium and the unique profile of strategies that survives iterated elimination of interim strictly dominated strategies.

Next we consider how each firm chooses which message to send. Using Lemma 2 and the formula for the indirect profit function (3), we can compute the expected profit following any pair of messages (m_i, m_{-i}). Hence, the problem of firm i of type c_i is to choose m_i to maximize $E_{m_{-i}}[\Pi_i(Q_{-i}(m_i, m_{-i}), c_i)]$.

First, note that by the Envelope theorem $\frac{d}{dq_{-i}}\Pi_i(q_{-i}, c_i) = -\beta q(q_{-i}, c_i)$, and thus every cost type that chooses to produce is strictly better off if the opponent produces less. Next, suppose there exists a message (or a set of messages) for firm i that minimizes $Q_{-i}(m_i, m_{-i})$ simultaneously for every message m_{-i} that is sent in equilibrium with positive probability. Then all types of firm i only send such message(s), and thus no informative communication is possible. Such a situation occurs, for example, when firm $-i$ plays a "babbling" strategy: every type of firm $-i$ plays the same message strategy which leads to $Q_{-i}(m_i, m_{-i})$ being independent of m_{-i} for every m_i. Another case when it may be possible to minimize $Q_{-i}(m_i, m_{-i})$ for every m_{-i} is when Q_{-i} is additively separable in (m_i, m_{-i}).

The question whether informative cheap talk between oligopolists is possible has been considered by Ziv [46] in the context of a symmetric model with undifferentiated products, linear demand and constant marginal cost (which corresponds to Example 1 with $\beta = \gamma = 1$ and $F_A \equiv F_B$). Ziv's Proposition 3 shows that if the parameters are such that all cost types always find

[17] F_i does not depend on c_{-i}, because the types are independently distributed.

[18] The proof follows from Lemma 1.

it optimal to produce, no informative equilibrium exists.[19] Specifically, he shows that firm $-i$'s expected equilibrium output depends on its expectation of firm i's cost, $E[c_i \mid m_i]$, as follows

$$Q_{-i}(m_i, m_{-i}) = A + \alpha_i E[c_i \mid m_i] - \alpha_{-i} E[c_{-i} \mid m_{-i}]$$

where $A, \alpha_i, \alpha_{-i} > 0$. The higher the expectation of firm i's cost, the more firm $-i$ will choose to produce, regardless of its cost type. Thus all types of firm i only send messages that minimize $E[c_i \mid m_i]$, and as a result no informative communication is possible.

When the demand or the cost functions are nonlinear, then Q_{-i} is unlikely to be additively separable, and thus there may not exist a message m_i that minimizes $Q_{-i}(m_i, m_{-i})$ simultaneously for every m_{-i}. Despite this, we are able to show that if all cost types always find it optimal to produce (condition (C4)), informative equilibria do not exist in the game with one round of cheap talk.

Theorem 1. *Suppose that conditions (C1), (C2) and (C4) hold. Then the game with one round of cheap talk has no informative equilibrium. That is, following any equilibrium message profile* (m_i, m_{-i})*, the expected output of each firm i satisfies* $Q_i(m_i, m_{-i}) = Q_i^{NC}$*, and firm i plays the same strategy as in the game without communication:* $q(Q_{-i}(m_i, m_{-i}), c_i) = q_i^{NC}(c_i)$*, for every* c_i*,* $i = A, B$*.*[20]

To illustrate the proof, let us show why in a symmetric environment there cannot be an informative equilibrium which is symmetric and where each firm chooses between two messages. Let $F_i = F$ for $i = A, B$, and suppose there is a symmetric equilibrium where each firm sends two messages m and m'. Let

$$BR(q_{-i}|\widehat{m}) = \int q(q_{-i}, c_i)dF(c_i|\widehat{m})$$

be the "expected" best response of a firm which has sent $\widehat{m} \in \{m, m'\}$. Denote by $Q(\widehat{m}, \widetilde{m})$ the expected output of a firm that has sent message $\widehat{m}$ and received message $\widetilde{m}$. Then

$$\begin{aligned} &Q(m,m) = BR\big(Q(m,m)\big|m\big), \qquad Q\big(m,m'\big) = BR\big(Q\big(m',m\big)\big|m\big),\\ &Q\big(m',m'\big) = BR\big(Q\big(m',m'\big)\big|m'\big), \qquad Q\big(m',m\big) = BR\big(Q\big(m,m'\big)\big|m'\big). \end{aligned} \tag{5}$$

Condition (C2) implies that the slope of BR is negative. If $Q(m', m) = Q(m, m') = Q$, then (5) implies that $Q(m, m) = Q(m', m') = Q$ as well, and thus the equilibrium is uninformative. Let $Q(m', m) \neq Q(m, m')$, and without loss of generality suppose $Q(m', m) < Q(m, m')$. Then (5) implies that $Q(m, m), Q(m', m') \in (Q(m', m), Q(m, m'))$. Thus $Q(m, m) < Q(m, m')$ and

[19] Formally, Proposition 3 states that a fully revealing equilibrium does not exist; however, what is in fact proved is that no information transmission is possible through cheap talk.

[20] We cannot, however, claim that in equilibrium the messages sent by the firms are independent of their types. There may exist equilibria where different types use distinct message strategies, leading to the posterior probability distribution over types being dependent on the reported message, as long as the expected output conditional on every equilibrium message profile remains the same as in the equilibrium without communication. To illustrate, consider the setting of Example 1 with $\gamma = 1$, and suppose the parameter values are such that in equilibrium of the game without communication, all types produce. Then in any equilibrium of the game with cheap talk, the outputs after any pair of messages depend only on the expectation of the marginal costs conditional on the messages. Thus any message strategy that satisfies $E[c_i|m_i] = E[c_i]$ for every i and m_i can be part of an equilibrium (e.g. if $c_i \sim U[0, 1]$, one such message strategy is $m_i(c_i) = m$ if $\frac{1}{4} < c_i < \frac{3}{4}$, and $m_i(c_i) = m'$ otherwise).

$Q(m', m) < Q(m', m')$, i.e. message m' leads to a higher expected opponent's output than message m regardless of the opponent's message, which cannot happen in equilibrium.

The result of Theorem 1 extends to the setting where the firms can engage in finitely many rounds of cheap talk.[21] Specifically, suppose there are $T > 1$ possible communication stages, at each stage $t = 1, ..., T$ each firm simultaneously chooses a message, and their choices become commonly known at the end of the stage. After that, the firms choose outputs. We show that informative cheap talk is impossible in such a game with a pre-determined finite number of rounds.[22]

Theorem 2. *Suppose that conditions (C1), (C2) and (C4) hold. Then the game with finitely many rounds of cheap-talk communication has no informative equilibrium.*

The impossibility of informative cheap-talk communication in our model stands in contrast with a number of results on two-sided cheap talk with two-sided incomplete information. For example, informative cheap-talk equilibria have been shown to exist in the double auction game [12,31], in the arms-race game [5], and in the peace negotiations game [23]. However, in all these papers the underlying games have multiple equilibria, and the ability to have different continuation equilibria following different message profiles seems important for sustaining informative communication. In our setting, there is a unique continuation equilibrium for every posterior belief (Lemma 2), which makes it harder to sustain informative communication.

4.2. Examples with informative cheap talk

In this section we show that the cheap-talk game can have informative equilibria if we relax some of the assumptions of our basic model. First we consider the case when some of the firms' cost types are so unproductive that they prefer to shut down under all circumstances.

Example 2. Consider the setup of Example 1 when $\gamma = 1$, so that

$$q(q_{-i}, c_i) = \max\left\{0, \frac{1}{2}(K - \beta q_{-i} - c_i)\right\},$$

and let $\bar{c} > K$. Note that if $c_i \geq K$, then type c_i is so unproductive that it produces zero even if it is a monopolist: $q(q_{-i}, c_i) = 0$ for every $q_{-i} \geq 0$. There exists the following equilibrium with informative cheap talk: firm A sends message m when it is "productive" ($c_A < K$) and message m' otherwise; and firm B plays a babbling strategy. To see that this is an equilibrium, first note that the "unproductive" types of firm A are indifferent between sending either message because their profit is always zero. The productive types prefer to tell the truth, because firm B behaves as a monopolist if it believes that firm A is unproductive, and produces less if it believes that firm A is productive.[23]

[21] Games with multi-stage cheap talk have been studied both in the context of one-sided incomplete information [3,26], and two-sided incomplete information [2].

[22] It remains an interesting open question whether cheap talk can be informative when there is no pre-determined bound on communication length.

[23] Note that this equilibrium is not equivalent to the outcome under no communication. The productive types of firm A can credibly reveal their productivity, and thus enjoy lower expected output of firm B than in the case of no communication.

Next we show that informative cheap talk is possible if we perturb the original information structure. The firms observe auxiliary correlated signals, and in the constructed equilibrium the interpretation of the messages depends on the realizations of these signals. The idea of this example is similar to Example 2 in Forges [14] and Example 2 in Baliga and Morris [4], who study a model of preplay communication in a coordination game. It is also related to the model in Blume and Board [7], who study communication between players with differential privately known language competence.

Example 3. Consider the setup of Example 1 when $\gamma = 1$. Firm A has two equally likely cost types $\{c_L, c_H\}$, firm B is known to have cost c_L (such that $0 \le c_L < c_H < \frac{1}{2}K$). Suppose there is an auxiliary random variable x that is equally likely to be m or m', and x is independent of the cost type of firm A. Firm B observes the realization of x, while firm A observes x only if it has cost c_L.[24] There exists the following equilibrium with informative cheap talk: type c_L of firm A sends message equal to x, while type c_H evenly randomizes between m and m'; and firm B plays a babbling strategy. To see that this is an equilibrium, first note that if firm B receives a message that coincides with the realization of x, then its belief that firm A is of type c_L is revised to $\frac{2}{3}$; if the message does not coincide with x, then firm B learns that firm A is of type c_H. Type c_L of firm A has no incentive to deviate from the equilibrium because it wants firm B to believe that it has a lower expected cost. Type c_H of firm A would also like to send message equal to x, but, since it has an equal chance to guess x correctly with either message, it is willing to randomize.

Finally, we show that informative cheap talk is possible if we allow the inverse demand to be nonlinear in q_{-i} and nonseparable in (q_i, q_{-i}). As in the original setting each firm prefers the opponent to produce less. However, because of the nonlinearity of the inverse demand in q_{-i}, the firm now cares not just about the expected output of the opponent but also about other properties of the distribution of the opponent's output. Nonseparability of the inverse demand in (q_i, q_{-i}) leads different types of firm i to have different preferences over distributions over q_{-i}, which allows to sustain informative communication. The idea of this example is similar to Example 2 in Seidmann [39] in a sender-receiver setting, and Example 1 in Baliga and Morris [4]. Seidmann [39] conjectured that an example of this kind is possible in an oligopoly model with incomplete information.

Example 4. Let $P(q_i, q_{-i}) = 40 - q_i - \frac{1}{10}q_{-i} - \frac{1}{1000}q_{-i}q_i^2 - \frac{1}{1000}q_{-i}^2 q_i$ and $C(q_i, c_i) = c_i q_i$. Firm A has three equally likely types $\{c_L, c_M, c_H\}$, and firm B has two equally likely types $\{c_L, c_M\}$. For certain parameter values there exists the following equilibrium with informative cheap talk: type c_M of firm A sends message m, while types c_L and c_H send m'; and firm B plays a babbling strategy. We present here the main idea of the construction, and the details are in the Online Appendix.

Note that the expected inverse demand depends not only on the mean but also on the variance of the opponent's output. This is because $E[q_{-i}^2] = \mu_{-i}^2 + \sigma_{-i}^2$, where $\mu_{-i} = E[q_{-i}]$ and $\sigma_{-i}^2 = var(q_{-i})$. Firm i's profit decreases in both μ_{-i} and σ_{-i}^2, but different cost types of firm i may be willing to trade μ_{-i} and σ_{-i}^2 at different rates. Moreover, if we consider the maximized profit

[24] The Bayesian–Nash equilibrium of the game without communication remains unaffected by the presence of auxiliary random variable x. This is because x is payoff-irrelevant, and the game without communication is interim dominance solvable.

Π_i as a function of $(\mu_{-i}, \sigma^2_{-i}, c_i)$, the marginal rate of substitution of Π_i between μ_{-i} and σ^2_{-i} is nonmonotonic in c_i.

In the constructed equilibrium, types c_M of each firm value the reduction in the variance of the opponent's output relatively more than the other types. In the equilibrium, we have $E[q_B \mid m] > E[q_B \mid m']$ and $var(q_B \mid m) < var(q_B \mid m')$, which helps to induce type c_M of firm A to send message m, and types c_L and c_H to send m'. The mean and the variance of the output of firm B behave this way, because type c_L of firm B produces more after message m' than after message m, while type c_M produces more after message m than after message m'. The different types of firm B are induced to behave this way, because in the equilibrium $E[q_A \mid m] > E[q_A \mid m']$ and $var(q_A \mid m) = 0 < var(q_A \mid m')$.

5. Mediated communication

In this section, we assume that, before choosing how much to produce, the firms can communicate with a neutral and trustworthy third party (a mediator), which is initially ignorant of the firm's private information. Both firms, as well as the mediator, can send private or public messages according to a mediation rule, or mechanism, which specifies what messages the parties can send, in what sequence, and whether the messages are public or private. After the communication has ended, the firms simultaneously choose their outputs.

We assume that the mediator's role is limited to participating in communication between the firms and that it has no enforcement power over the firms' output choices. This distinguishes our setting from a standard mechanism design problem, where the mechanism designer can enforce the mechanism outcome, and makes it a mechanism design problem without enforcement. The literature on such problems, which dates back to Myerson [32], suggests that in certain settings, mediated communication allows the players to strictly improve upon cheap talk.[25]

First, we note that if the mediator is able only to send, but not to receive, messages from the firms, improving upon the uninformative Bayesian–Nash equilibrium outcome is impossible. More formally, suppose all the mediator can do is send the firms private messages m_A and m_B from some message sets M_A and M_B, generated according to a commonly known probability distribution $p \in \Delta(M_A \times M_B)$. (The Bayesian–Nash equilibria of communication games of this form are called the strategic form correlated equilibria of the game with no communication [14].) The following lemma is an immediate consequence of the fact, established in Lemma 1, that the game without communication is interim dominance solvable.

Lemma 3. *Under conditions (C1)–(C3), all strategic form correlated equilibria are outcome equivalent to the Bayesian–Nash equilibrium of the game without communication.*

If the mediator can also receive messages from the firms, this result is no longer valid, as the example in Section 2 suggests. What we will do next is generalize the mechanism described in the example, and provide sufficient conditions for it to result in informative communication in our model.

For the rest of the section, let us assume that the cost parameters are i.i.d. across the firms ($F_A \equiv F_B$). Let $c^* \in (0, \bar{c})$, and consider the mechanism which works as follows. Each firm i sends a private message $\hat{c}_i \in [0, \bar{c}]$, which is interpreted as the firm's report about its cost, to

[25] See, for example, [6,21,23].

the mediator. The mediator then publicly announces one message, m^0, if $\min\{\hat{c}_A, \hat{c}_B\} \le c^*$ and another message, m^1, otherwise. After that, the firms choose their outputs. Let us call such a mechanism the **"min" mechanism with threshold c***.[26]

This mechanism induces a game between the firms, where a pure strategy for firm $i \in \{A, B\}$ consists of a reporting strategy $\hat{c}_i(c_i)$ and an output strategy $q_i(c_i, \hat{c}_i, m)$, where $m \in \{m^0, m^1\}$. We will say that the mechanism is **incentive compatible** if it has an equilibrium where the firms report their types truthfully: $\hat{c}_i(c_i) = c_i$, $\forall c_i \in [0, \bar{c}]$, $i \in \{A, B\}$.

As in Section 2, the idea behind this mechanism is to give each firm a choice between having the competitor produce less in expectation and getting more information about how much the competitor will produce. Specifically, suppose that firm i reports $\hat{c}_i \le c^*$. Then, if firm j has reported $\hat{c}_j > c^*$, the mediator will announce message m^0, and firm j will learn that firm i has reported its cost to be low. This will make firm j produce less in expectation, which is favorable to firm i. However, firm i reporting $\hat{c}_i \le c^*$ also deprives it of an opportunity to learn anything about firm j's report, because the mediator will announce m^0 regardless of firm j's report. Conversely, reporting $\hat{c}_i > c^*$ will result in firm j producing more in expectation, but will enable firm i to learn whether $\hat{c}_j$ is above or below c^*. The mechanism will be incentive compatible if different types of the firm resolve this tradeoff differently: types above c^* value additional information about the opponent more than the reduction in the opponent's expected output, while types below c^* exhibit the reverse preference.[27]

To guarantee the incentive compatibility of our mechanism, we will impose the following additional condition on the best response functions:

$$q(q_{-i}, c_i) \text{ is } C^2, \text{ and } \frac{\partial^2 \ln(q(q_{-i}, c_i))}{\partial c_i \partial q_{-i}} < 0 \text{ on } \{(q_{-i}, c_i) : q(q_{-i}, c_i) > 0\} \tag{C5}$$

The second part of condition (C5) is a joint requirement on the demand and the cost that ensures that the optimal output of the firm with a higher cost type is relatively more responsive to the changes in the expected output of the opponent than the optimal output of the firm with a lower cost type. This condition is more likely to be satisfied the "more concave" is the marginal revenue $\rho'(q_i)q_i + \rho(q_i) - \beta q_{-i}$, the "more convex" is the marginal cost C_q, and the "less convex" is the cost disadvantage from having a higher cost type C_c (see Lemma A.2 in the Online Appendix for the precise statement).

In addition, we will impose a condition that guarantees that each firm's output sufficiently varies with respect to its type.

$$\lim_{c_i \to \infty} q(q_{-i}, c_i) = 0 \quad \text{for every } q_{-i} \ge 0 \tag{C6}$$

We illustrate these conditions with examples.

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[26] This mechanism is similar to the AND mechanism analyzed by Lehrer [28], Gossner and Vieille [22] and Vida and Āzacis [43]. Hugh-Jones and Reinstein [24] suggest that a similar mechanism may improve welfare in a matching problem where a player suffers disutility in the event a prospective partner knows of his interest and rejects him.

[27] Similar logic lies behind the results of Seidmann [39] and Watson [45], who show that in a sender-receiver game with two-sided private information, an informative equilibrium can exist even if all the sender's types have the same preference ordering over the receiver's actions. This is because different types of the receiver respond differently to the sender's messages, and thus, from the sender's viewpoint, each message corresponds to a lottery over the receiver's actions. Informative communication is possible if different sender types have a different preference ranking over these lotteries. This effect has also been emphasized by Baliga and Sjöström [5] in the context of an arms-race game. Unlike our model, however, these settings admit informative cheap talk.

Example 1 *(continued).* In this example, $\frac{\partial^2 \ln q(q_{-i},c_i)}{\partial c_i \partial q_{-i}} = \frac{2\beta(\gamma-2)q_i^{\gamma-1}}{(2q_i+c_i(\gamma-1)q_i^{\gamma-1})^3}$. Therefore, (C5) holds if $\gamma < 2$, and (C6) is always satisfied.

Example 5. Let $C(q_i,c_i) = c_i q_i$. Then $\frac{\partial^2 \ln q(q_{-i},c_i)}{\partial c_i \partial q_{-i}} = -\frac{\beta}{q_i^2}\frac{2\rho'(q_i)+4\rho''(q_i)q_i+\rho'''(q_i)q_i^2}{(2\rho'(q_i)+\rho''(q_i)q_i)^3}$. Therefore, a sufficient condition for (C5) to hold is $\rho'(q_i), \rho''(q_i), \rho'''(q_i) < 0$ for every $q_i \geq 0$, and (C6) is always satisfied.

To interpret condition (C5), note that

$$\frac{\partial^2 \ln q(q_{-i},c_i)}{\partial c_i \partial q_{-i}} = \frac{\partial}{\partial c_i}\left(\frac{\frac{\partial q(q_{-i},c_i)}{\partial q_{-i}}}{q_i(q_{-i},c_i)}\right) = -\frac{\partial}{\partial c_i}\left(\left|\frac{\frac{\partial^2 \Pi_i}{\partial q_{-i}^2}}{\frac{\partial \Pi_i}{\partial q_{-i}}}\right|\right)$$

The denominator of the latter expression measures how much the indirect profit of firm i changes with the expected output of the opponent, so it shows how much firm i values a reduction in the opponent's output. The numerator measures how convex the indirect profit function is, and thus how much the firm values information about the opponent's output. Condition (C5) is a "single-crossing condition" on the firm's preferences: it says that the higher the firm's cost, the more it values information about the opponent relative to reduction in opponent's expected output.

Condition (C5) implies that to ensure that the "min" mechanism is incentive compatible, it is enough to choose threshold c^* to be the type that is indifferent between reporting $\hat{c} \leq c^*$ and $\hat{c} > c^*$: if type c^* is indifferent, then any type above c^* will strictly prefer reporting $\hat{c} > c^*$, and any type below c^* will strictly prefer reporting $\hat{c} \leq c^*$. The following theorem shows that when the support of the cost distribution is large enough, such c^* can be found.

Theorem 3. *Suppose that $F_A \equiv F_B \equiv F$, conditions (C1)–(C3), (C5) and (C6) hold, and $\bar{c}$ is large enough. Then there exists $c^* \in (0, \bar{c})$ such that the "min" mechanism with threshold c^* is incentive compatible.*

It remains an open question whether it is possible to construct an informative mechanism when conditions (C5) or (C6) do not hold. Suppose, for example, that (C5) holds with the reverse inequality for every (q_{-i}, c_i). A natural guess is that one could construct an informative "max" mechanism, whereby the mediator announces whether $\max\{\hat{c}_A, \hat{c}_B\} \leq c^*$. However, this guess is incorrect: if such a mechanism was in place, a low cost report would both lower the opponent's output and result in more information about the opponent, and therefore every cost type would have an incentive to send a low report. We conjecture that in that case, informative communication is impossible. We also conjecture that (C6) could be somewhat relaxed; however, sufficient heterogeneity in the behavior of different cost types seems essential for sustaining informative communication.

The next theorem shows that whenever a "min" mechanism is incentive compatible, it interim Pareto dominates the Bayesian–Nash equilibrium without communication for the firms.

Theorem 4. *Suppose that $F_A \equiv F_B \equiv F$. If an incentive compatible "min" mechanism exists, then every type of every firm is better off under this mechanism than in the Bayesian–Nash equilibrium without communication. If, in addition, condition (C4) holds, then every type of every firm is strictly better off.*

The intuition behind this theorem is that, when a "min" mechanism is in place, reporting $\hat{c} \le c^*$ results in higher expected profit for every type than the Bayesian–Nash equilibrium without communication. This is because in both cases, the firm gets no information, but reporting $\hat{c} \le c^*$ results in lower expected output by the opponent than the uninformative equilibrium. Since reporting $\hat{c} \le c^*$ is possible for every type and the mechanism is incentive compatible, in equilibrium every type's expected profit must be at least as high as the one guaranteed by this action.

While we are unable to provide a general result on how the total surplus and the consumer surplus under the "min" mechanism compare to those in the no-communication equilibrium, the following example shows that in some cases, the "min" mechanism results in a higher total surplus (although a lower consumer surplus).

Example 1 *(continued).* Suppose that $\beta = \gamma = 1$ and $c_i \sim U[0, \bar{c}]$. Then an incentive compatible "min" mechanism exists if and only if $\bar{c} > \frac{2}{3}K$. If $K \in (\frac{3}{2}\bar{c} - \varepsilon, \frac{3}{2}\bar{c})$, then every type's output is strictly positive both under the incentive compatible "min" mechanism and in the no-communication Bayesian–Nash equilibrium (the proof is in the Online Appendix). Under this condition, the ex ante expected total surplus in the no-communication equilibrium equals

$$TS^{NC} = \frac{4}{9}\left(K - \frac{\bar{c}}{2}\right)^2 + \frac{\bar{c}^2}{16}$$

and the total surplus under the incentive-compatible "min" mechanism equals

$$TS^{min} = \frac{4}{9}\left(K - \frac{\bar{c}}{2}\right)^2 + \frac{\bar{c}^2}{16} + \frac{c^*(\bar{c} - c^*)^2(17\bar{c} + 11c^*)}{144(\bar{c} + c^*)^2}$$

where c^* is the threshold of the incentive compatible "min" mechanism (which depends on K and $\bar{c}$). The ex ante expected consumer surplus in the no-communication equilibrium equals

$$CS^{NC} = \frac{2}{9}\left(K - \frac{\bar{c}}{2}\right)^2 + \frac{\bar{c}^2}{48}$$

and the consumer surplus under the incentive-compatible "min" mechanism equals

$$CS^{min} = \frac{2}{9}\left(K - \frac{\bar{c}}{2}\right)^2 + \frac{\bar{c}^2}{48} - \frac{c^*(\bar{c} - c^*)^2(5\bar{c} - c^*)}{144(\bar{c} + c^*)^2}$$

It is obvious that $TS^{NC} < TS^{min}$ and $CS^{NC} > CS^{min}$. Intuitively, information sharing makes oligopolists coordinate their outputs, which reduces the variability of aggregate output. This decreases consumer surplus, because it is a convex function of output.[28]

Other incentive compatible mechanisms exist in our model as well. For example, one can show that in the case of homogeneous good, linear demand and constant marginal cost (Example 1 with $\beta = \gamma = 1$), under certain conditions the following "N-step min mechanism" is incentive compatible and superior to the "min" mechanism in terms of ex ante profit: the mediator announces a public message m^k ($k = 0, 1, \ldots, N$) if $\min\{\hat{c}_A, \hat{c}_B\}$ is between c^k and c^{k+1}, where $0 = c^0 < c^1 < \ldots < c^N < c^{N+1} = 1$. It is also plausible that in some cases, mechanisms

[28] Note that if the firms could commit to revealing their information truthfully, the ex ante expected total surplus would also be higher and the consumer surplus lower than in the no-communication equilibrium: see e.g. [1].

where the mediator sends private messages may improve upon public mechanisms. For example, suppose that only firm A has private information about costs, and firm B's cost is commonly known. In this case, public or deterministic mechanisms cannot support informative communication: firm A can precisely anticipate firm B's output choice, and thus there is no residual uncertainty about firm B's output, which is essential for sustaining information revelation by firm A. Nonetheless, one can construct an informative mechanism of the following form. After receiving the cost report from firm A, the mediator sends a noisy (but informative) private signal to firm B, and, in addition, a blind carbon copy of this signal is sent to firm A if and only if its reported costs are high. As a result, the types of firm A that report high costs expect on average a higher output by firm B, but are compensated by information useful for predicting firm B's output.

6. Extensions and discussion

6.1. More than two firms

Our model can be extended to accommodate the case of $n > 2$ firms. Specifically, suppose that the inverse demand for firm i's product is $\max\{\rho(q_i) - \beta q_{-i}, 0\}$, where $q_{-i} = \sum_{j \neq i} q_j$ is the aggregate output of all firms other than i, and, as before, let $q(q_{-i}, c_i)$ be the best response function of each firm. Let the "min" mechanism with threshold $c^* \in (0, \bar{c})$ be the mechanism whereby each firm i sends a private message $\hat{c}_i \in [0, \bar{c}]$ to the mediator, who then publicly announces one message, m^0, if $\min\{\hat{c}_1, \dots, \hat{c}_n\} \le c^*$ and another message, m^1, otherwise. The following result generalizes Theorem 3 to the case of more than two firms.[29]

Theorem 5. *In the model with $n \ge 2$ firms, suppose that $F_i \equiv F$, $i = 1, \dots, n$, conditions (C1)–(C3), (C5) and (C6) hold, and that $\bar{c}$ is large enough. Then there exists $c^* \in (0, \bar{c})$ such that the "min" mechanism with threshold c^* is incentive compatible.*

Given the results in Section 4.1, plain cheap talk cannot sustain informative communication when $n = 2$. When $n > 2$, however, this is not necessarily the case: the literature on "universal mechanisms" [13,20] suggests that if n is large enough, any incentive compatible communication mechanism can be implemented without a mediator. The results in this literature are not directly applicable to our case, as they assume a finite number of possible types and actions for each player; nevertheless, the next theorem shows that they can be generalized to cover the "min" mechanism in our environment.

Theorem 6. *Suppose that $n \ge 5$ and a "min" mechanism is incentive compatible. Then there exists a game with finitely many rounds of cheap-talk communication, such that the firms are able to send private messages to a subset of other firms, that has a weak perfect Bayesian equilibrium that is outcome equivalent to the truthful equilibrium of the "min" mechanism.*

The proof of Theorem 6 consists of several steps. First, a "min" mechanism is constructed for a particular auxiliary game with finitely many cost types and possible outputs. By Theorem 2 of

[29] The proof is a straightforward generalization of the proof of Theorem 3 and can be found on the authors' webpages, as well as the proof of Theorem 6.

Gerardi [20], this auxiliary game can be augmented with a particular pre-play communication protocol so that it admits an equilibrium that is outcome equivalent to the "min" mechanism. Finally, it is shown that if the original game is augmented with the same communication protocol, then there exists an equilibrium that replicates the "min" mechanism.

The ability of each firm to send private messages to a subset of other firms is the key feature of the proof of Theorem 6. If the firms can only send public messages, then informative cheap talk may be impossible. Indeed, if the firms can only send public messages, then it is straightforward to show that the proofs of Theorems 1 and 2 go through if the second part of condition (C2) is replaced by a stronger assumption $\frac{\partial q(q_{-i},c_i)}{\partial q_{-i}} \in (-\frac{1-\delta}{n-1}, 0)$.[30]

6.2. Optimal mechanisms

One might ask which mechanism maximizes some particular objective, like the ex ante joint profit or the ex ante total surplus, in this environment. To address this question, we can use the revelation principle [32] and restrict attention to direct revelation mechanisms, whereby the firms privately report their costs to the mediator, who then makes private output recommendations. Furthermore, the revelation principle states that without loss of generality, one may consider only incentive compatible direct revelation mechanisms, where in equilibrium the firms report truthfully and follow the mediator's recommendations.[31]

Each of these conditions defines a continuum of constraints, and solving for the optimal mechanism is a difficult problem. However, if we consider cost distributions that are concentrated on a finite number of points, and restrict outputs to be chosen from a finite grid, it is possible to solve for the optimal mechanism numerically. We have performed calculations for the linear case (Example 1 with $\beta = \gamma = K = 1$), when c_i takes two values, 0 and $c \in \{0, \frac{1}{10}, ..., 1\}$, $\Pr\{c_i = 0\} \in \{0, \frac{1}{10}, ..., 1\}$, and outputs are allowed to be chosen from $\{0, \frac{1}{40}, ..., \frac{1}{2}\}$. The results indicate that the profit-maximizing mechanism is generally more complex than the "min" mechanism: whenever an informative mechanism is optimal, it is stochastic, features private messages,

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30 To see how the proof of Theorem 1 should be modified, fix any firm i, and let (m_i, m_{-i}) be a message profile. Let $BR_{-i}(q_i|m_{-i}) = \sum_{j\neq i} q_j$, where $(q_j)_{j\neq i}$ are a solution to the system of equations $q_j = BR_j(q_{-j}|m_j)$, $j \in \{1,\dots,n\} \setminus \{i\}$ (this solution, and therefore the function BR_{-i}, depends on m_i and q_i). Then define $(\underline{q}_i, \overline{q}_i, \underline{q}_{-i}, \overline{q}_{-i})$ analogously to $(\underline{q}_A, \overline{q}_A, \underline{q}_B, \overline{q}_B)$. As in Theorem 1, we get $\frac{1-\delta}{n-1}(\overline{q}_{-i} - \underline{q}_{-i}) \geq \overline{q}_i - \underline{q}_i$. On the other hand, it is easy to see that $\sum_{j\neq i}(\overline{q}_j - \underline{q}_j) \geq \overline{q}_{-i} - \underline{q}_{-i}$. Combining these inequalities and summing up over i results in $(1-\delta)(\sum_{i=1}^n(\overline{q}_i - \underline{q}_i)) \geq \sum_{i=1}^n(\overline{q}_i - \underline{q}_i)$, which is impossible unless $\overline{q}_i = \underline{q}_i$ for every i.

31 Formally, a direct revelation mechanism is a family of probability measures $g(\cdot|\hat{c}_A, \hat{c}_B)$ over the set of pairs of output recommendations ($\mathbb{R}^2_+$), indexed by the pair of cost reports submitted to the mediator ($(\hat{c}_A, \hat{c}_B) \in C^2$). A direct revelation mechanism $\{g(\cdot|\hat{c}_A, \hat{c}_B)\}_{\hat{c}_A,\hat{c}_B}$ is incentive compatible if every firm finds it optimal to report its true cost, conditional on the opponent reporting its true cost and following the mediator's recommendation:

$$E_{\hat{q}_i}\left[\Pi_i\left(E_{q_{-i}}[q_{-i} \mid c_i, \hat{q}_i], c_i\right) \mid c_i\right] \geq E_{\hat{q}_i}\left[\Pi_i\left(E_{q_{-i}}[q_{-i} \mid \hat{c}_i, \hat{q}_i], c_i\right) \mid \hat{c}_i\right] \quad \text{for every } c_i, \hat{c}_i \in C$$

and every firm that has reported its cost truthfully is willing to be obedient upon receiving the output recommendation, conditional on the opponent being truthful and obedient:

$$q_i = q\left(E_{q_{-i}}[q_{-i}|c_i, q_i], c_i\right) \quad \text{for every } c_i \in C, q_i \in \mathbb{R}_+$$

where $E_{q_{-i}}[q_{-i} \mid \hat{c}_i, \hat{q}_i]$ is the expected output of firm $-i$ conditional on firm i reporting $\hat{c}_i$ and receiving recommendation $\hat{q}_i$. (Distribution of $\hat{q}_i$ conditional on $\hat{c}_i$ and distribution of q_{-i} conditional on $(\hat{c}_i, \hat{q}_i)$ are derived from $\{g(\cdot|\hat{c}_A, \hat{c}_B)\}_{\hat{c}_A,\hat{c}_B}$ and the prior.)

and involves some information revelation to both types (although it is not clear whether the optimal mechanism that we find is the unique one). If the objective is to maximize the total surplus, the optimal mechanism often resembles the "min" mechanism in that it involves a public message informing the firms that both of them have high cost; however, the mechanism is typically stochastic and reveals some information to the low-cost type. We leave further investigation of optimal mechanisms for future research.

6.3. Other settings

Suppose that, instead of cost shocks, the firms face private demand shocks. In particular, suppose θ_i is a private (i.i.d.) demand shock that affects firm i as follows: $P(q_i, q_{-i}, \theta_i) = \max\{\rho(q_i, \theta_i) - \beta q_{-i}, 0\}$ with $\rho_\theta < 0$. Then we can define the best response function $q(q_{-i}, \theta_i)$, make the same assumptions (C1)–(C6) with θ_i in place of c_i, and replicate all the analysis.

The question of whether any of the results would extend to the case where cost or demand shocks are correlated is more difficult. To see why, suppose that each firm receives a signal about a common cost parameter. Now each firm might prefer to be perceived as having a high cost signal rather than a low cost signal, because if the opponent believes the report about the high cost signal, then it may decide to produce less. We leave this question for future research.

Finally, one may also ask whether the results of the paper apply to a Bertrand model with differentiated products. Because prices are strategic complements, each firm will have an incentive to overstate its type, which is the opposite of what happens in the Cournot model. Nevertheless, we believe that, when the assumptions are adjusted to reflect this change, the results of the paper will go through with the "max" mechanism (the mediator announcing whether the maximum of the cost reports exceeds a certain threshold) replacing the "min" mechanism in Theorem 3.

Appendix A

A.1. Proofs of Section 4

Before proving Theorems 1 and 2, we need some preliminary results.

Define

$$BR_i(q_{-i} \mid m_i) = \int q(q_{-i}, c_i) dF_i(c_i \mid m_i) \quad \text{for } i \in \{A, B\} \tag{6}$$

Suppose there exists an informative cheap talk equilibrium. Let $M_A = M$ and $M_B = N$ be the sets of equilibrium messages for firms A and B, respectively, and let (m, n) be a representative element of $M \times N$. The fact that the equilibrium is informative implies that $\max\{|M|, |N|\} \geq 2$. We will assume, without loss of generality, that every message induces a different distribution over the opponent's output. To state this assumption formally, let $\sigma_i(\cdot|c_i)$ be a probability distribution over M_i defining the message strategy of firm i, and let $G_{-i}(x|m_i) = \Pr(Q_{-i}(m_i, m_{-i}) \leq x|m_i) = \iint 1_{\{Q_{-i}(m_i, m_{-i}) \leq x\}} d\sigma_{-i}(m_{-i}|c_{-i}) dF_{-i}(c_{-i})$ be the distribution function of firm $-i$'s expected output conditional on firm i sending message m_i. Then we will assume that $G_{-i}(x|m_i) \neq G_{-i}(x|m_i')$, $\forall m_i, m_i' \in M_i$, $i \in \{A, B\}$.

Lemma 4. *Suppose (C1)–(C4) hold. For every $m, m' \in M$ such that $m \neq m'$, there exist $n, n' \in N$ such that $Q_B(m, n) > Q_B(m', n)$ and $Q_B(m, n') < Q_B(m', n')$. Symmetrically, for every $n, n' \in N$ such that $n \neq n'$, there exist $m, m' \in M$ such that $Q_A(m, n) > Q_A(m, n')$ and $Q_A(m', n) < Q_A(m', n')$.*

Proof. Suppose the conclusion of the lemma does not hold for $m, m' \in M$; e.g. $\forall n \in N$, $Q_B(m,n) \geq Q_B(m',n)$. This implies that $\forall x \geq 0$, $G_B(x|m) \leq G_B(x|m')$. Then the difference in expected profit of type c_A from sending message m as opposed to m' is

$$\int \Pi_A(q_B, c_A) dG_B(q_B \mid m) - \int \Pi_A(q_B, c_A) dG_B(q_B \mid m')$$
$$= \int \frac{d\Pi_A(q_B, c_A)}{dq_B}\big(1 - G_B(q_B \mid m)\big) dq_B - \int \frac{d\Pi_A(q_B, c_A)}{dq_B}\big(1 - G_B(q_B \mid m')\big) dq_B$$
$$= -\beta \int q(q_B, c_A)\big(G_B(q_B \mid m') - G_B(q_B \mid m)\big) dq_B \leq 0$$

where the first equality is obtained through integration by parts (the validity of integration by parts is guaranteed by Theorem II.6.11 of Shiryaev [41], which applies because the support of q_B is bounded and Π_A is decreasing in q_B), and the second equality is by the Envelope Theorem. Moreover, (C4) implies that $q(q_B, c_A) > 0$ for every (q_B, c_A), so, because $G_B(x|m) \neq G_B(x|m')$, the inequality is strict. Hence every type c_A strictly prefers sending message m' to message m, which is a contradiction. □

Lemma 5. *Suppose (C1)–(C4) hold. For every $n, n' \in N$ such that $n \neq n'$, $\exists q^*(n,n') = (q_A^*(n,n'), q_B^*(n,n'))$ such that $q_B^*(n,n') = BR_B(q_A^*(n,n')|n) = BR_B(q_A^*(n,n')|n')$. Moreover, $\exists m, m' \in M$ s.t. $q_A^*(n,n')$ is strictly between $Q_A(m,n)$ and $Q_A(m',n)$. A symmetric statement holds for any $m, m' \in M$ such that $m \neq m'$.*

Proof. By Lemma 4, there must exist $m, m' \in M$ such that $Q_A(m,n) > Q_A(m,n')$ and $Q_A(m',n) < Q_A(m',n')$.

Let

$$\psi(q_A) := BR_B(q_A \mid n') - BR_B(q_A \mid n)$$

and

$$\phi(q_A; \tilde{m}, \tilde{n}) := BR_B(q_A \mid \tilde{n}) - BR_A^{-1}(q_A \mid \tilde{m})$$

Function ϕ is increasing in q_A, since BR_A^{-1} is steeper than BR_B. In equilibrium, $Q_A(\tilde{m},\tilde{n}) = BR_A(Q_B(\tilde{m},\tilde{n})|\tilde{m})$ and $Q_B(\tilde{m},\tilde{n}) = BR_B(Q_A(\tilde{m},\tilde{n})|\tilde{n})$. Thus, $\phi(Q_A(\tilde{m},\tilde{n}); \tilde{m},\tilde{n}) = 0$ for every $(\tilde{m},\tilde{n})$.

Note that

$$\psi\big(Q_A(m,n)\big) = \phi\big(Q_A(m,n); m, n'\big) > \phi\big(Q_A(m,n'); m, n'\big) = 0 \tag{7}$$

where the equalities use (6); the inequality holds because $Q_A(m,n) > Q_A(m,n')$ and because ϕ is increasing. Similarly,

$$\psi\big(Q_A(m',n)\big) = \phi\big(Q_A(m',n); m', n'\big) < \phi\big(Q_A(m',n'); m', n'\big) = 0 \tag{8}$$

Since the best responses, and thus ψ, are continuous, from (7) and (8) it follows that there exists $q^*(n,n')$ at which $BR_B(\cdot \mid n)$ and $BR_B(\cdot \mid n')$ intersect, and $q_A^*(n,n')$ is strictly between $Q_A(m,n)$ and $Q_A(m',n)$ by construction. □

For $i \in \{A, B\}$, let $\underline{q}_i = \inf_{(m,n)\in M\times N} Q_i(m,n)$; that is, $\forall (m,n) \in M \times N$, $Q_i(m,n) \geq \underline{q}_i$, and $\forall \varepsilon > 0$, $\exists (m,n) \in M \times N$: $Q_i(m,n) \leq \underline{q}_i + \varepsilon$. Similarly, let $\overline{q}_i = \sup_{(m,n)\in M\times N} Q_i(m,n)$.

Note that $\overline{q}_i$ is finite, because $Q_i(m,n) \leq q_i(0,0) < \infty$. By definition, $\underline{q}_i \leq \overline{q}_i$; the fact that the equilibrium is informative implies that $\underline{q}_i < \overline{q}_i$ (indeed, if $\underline{q}_i = \overline{q}_i = q_i$, then $Q_i(m,n) = q_i$, $\forall (m,n) \in M \times N$; therefore, $Q_j(m,n)$ is also constant with respect to (m,n), and the equilibrium is uninformative).

Proof of Theorem 1. Suppose an informative equilibrium exists. Let us first prove that

$$(1-\delta)(\overline{q}_A - \underline{q}_A) \geq \overline{q}_B - \underline{q}_B \tag{9}$$

For this, it is sufficient to prove that for any $\varepsilon > 0$, however small,

$$(1-\delta)(\overline{q}_A - \underline{q}_A) > \overline{q}_B - \underline{q}_B - 2\varepsilon \tag{10}$$

Fix any $\varepsilon > 0$. By definition of $\overline{q}_B$, there exists $(m,n) \in M \times N$ such that $Q_B(m,n) \in (\overline{q}_B - \varepsilon, \overline{q}_B]$. Similarly, there exists $(m',n') \in M \times N$ such that $Q_B(m',n') \in [\underline{q}_B, \underline{q}_B + \varepsilon)$. Since $\underline{q}_B < \overline{q}_B$, $Q_B(m,n) > Q_B(m',n')$ if ε is small enough.

If $n = n'$, both $Q(m,n) = (Q_A(m,n), Q_B(m,n))$ and $Q(m',n') = (Q_A(m',n'), Q_B(m',n'))$ satisfy the equation $q_B = BR_B(q_A|n)$. Then by (C2), and since $Q_A(m,n) < Q_A(m',n')$, we have

$$(1-\delta)\big(Q_A(m',n') - Q_A(m,n)\big) > Q_B(m,n) - Q_B(m',n') \tag{11}$$

Since $Q_A(m',n') \leq \overline{q}_A$ and $Q_A(m,n) \geq \underline{q}_A$, we have $\overline{q}_A - \underline{q}_A \geq Q_A(m',n') - Q_A(m,n)$. By the choice of (m,n) and (m',n'), we also have $Q_B(m,n) - Q_B(m',n') > \overline{q}_B - \underline{q}_B - 2\varepsilon$. Combining this with (11), we get (10).

If $n \neq n'$, by Lemma 5 there exists $q^*(n,n') = (q_A^*(n,n'), q_B^*(n,n'))$ such that $q_B^*(n,n') = BR_B(q_A^*(n,n')|n) = BR_B(q_A^*(n,n')|n')$, and $q_A^*(n,n') \in (Q_A(\hat{m},n), Q_A(\tilde{m},n))$ for some $\hat{m}, \tilde{m} \in M$. There are three cases to consider.

Case 1: $Q_A(m,n) < q_A^*(n,n') < Q_A(m',n')$.

The first inequality, together with the fact that both $Q(m,n)$ and $q^*(n,n')$ satisfy the equation $q_B = BR_B(q_A|n)$, implies

$$(1-\delta)\big(q_A^*(n,n') - Q_A(m,n)\big) > Q_B(m,n) - q_B^*(n,n') \tag{12}$$

Similarly, the second inequality implies

$$(1-\delta)\big(Q_A(m',n') - q_A^*(n,n')\big) > q_B^*(n,n') - Q_B(m',n') \tag{13}$$

Summing up (12) and (13) gives (11), which, as when $n = n'$, implies (10).

Case 2: $q_A^*(n,n') \leq Q_A(m,n) < Q_A(m',n')$.

Like in Case 1, $q_A^*(n,n') < Q_A(m',n')$ implies (13). Since $\underline{q}_A \leq Q_A(\hat{m},n) < q_A^*(n,n')$, we have $\overline{q}_A - \underline{q}_A \geq Q_A(m',n') - q_A^*(n,n')$. Since $q^*(n,n')$ and $Q(m,n)$ lie on the curve $q_B = BR_B(q_A|n)$, which is downward sloping, $q_B^*(n,n') \geq Q_B(m,n) > \overline{q}_B - \varepsilon$. Hence, $q_B^*(n,n') - Q_B(m',n') > \overline{q}_B - \underline{q}_B - 2\varepsilon$. Combining this with (13), we get (10).

Case 3: $Q_A(m,n) < Q_A(m',n') \leq q_A^*(n,n')$.

Like in Case 1, $Q_A(m,n) < q_A^*(n,n')$ implies (12). Since $q_A^*(n,n') < Q_A(\tilde{m},n) \leq \overline{q}_A$, we have $\overline{q}_A - \underline{q}_A \geq q_A^*(n,n') - Q_A(m,n)$. Since $q^*(n,n')$ and $Q(m',n')$ lie on the curve $q_B = BR_B(q_A|n')$, which is downward sloping, $q_B^*(n,n') \leq Q_B(m',n') < \underline{q}_B + \varepsilon$. Hence, $Q_B(m,n) - q_B^*(n,n') > \overline{q}_B - \underline{q}_B - 2\varepsilon$. Combining this with (12), we get (10).

Symmetrically, we can show

$$(1-\delta)(\overline{q}_B - \underline{q}_B) \geq \overline{q}_A - \underline{q}_A$$

which is in contradiction with (9) and the fact that $\delta \in (0, 1)$. □

Proof of Theorem 2. Suppose there exists no informative t-round cheap talk equilibrium. We will show that then every $t+1$-round cheap talk equilibrium is uninformative as well. Suppose the message profile in the first round is (m_A, m_B), and the posterior beliefs are $(F_A(\cdot \mid m_A), F_B(\cdot \mid m_B))$. The continuation game starting from period 2 has no informative cheap talk equilibrium. That is, the expected quantities are always the same as in the game without communication, (Q_A^{NC}, Q_B^{NC}) calculated for beliefs $(F_A(\cdot \mid m_A), F_B(\cdot \mid m_B))$:

$$Q_A^{NC} = BR_A\big(Q_B^{NC} \bigm| m_A\big), \qquad Q_B^{NC} = BR_B\big(Q_A^{NC} \bigm| m_B\big)$$

Thus if in $t+1$-round cheap talk game there exists an informative equilibrium, then there exists an outcome equivalent informative equilibrium where the firms use the same first-period communication strategies, and use babbling strategies in the remaining periods. However this implies that in one-round cheap talk game there exists an outcome equivalent informative equilibrium where the firms use the same first-period communication strategies as above, which is a contradiction with Theorem 1. □

A.2. Proofs of Section 5

Proof of Theorem 3. The proof will proceed by a series of lemmas, the proofs of which can be found in the Online Appendix.

Consider a "min" mechanism with threshold $c^* \in (0, \bar{c})$. After m^1 is announced, the expected output of firm $-i$ is $Q^{H2}(c^*)$ that solves

$$Q_{-i}^{H2} = \frac{1}{1-F(c^*)} \int_{c^*}^{\infty} q\big(Q_{-i}^{H2}, c_i\big)\, dF(c_i) \tag{14}$$

Lemma 6. *Suppose that conditions (C1)–(C3) and (C6) hold. For every c^*, there exists a unique $Q^{H2}(c^*)$ that solves (14), and thus there exists a unique continuation equilibrium following message m^1, which is symmetric. The function $Q^{H2}(c^*)$ is continuous and decreasing in c^*, $Q^{H2}(0) = Q^{NC}$, $\lim_{c^* \to \infty} Q^{H2}(c^*) = 0$.*

Let $Q^L(c^*)$ be the expected output of firm $-i$ if m^0 was announced and firm i reported $\hat{c}_i < c^*$, and let $Q^{H1}(c^*)$ be the expected output of firm $-i$ if m^0 was announced and firm i reported $\hat{c}_i > c^*$. Then $Q^L(c^*)$ and $Q^{H1}(c^*)$ solve

$$\begin{cases} Q_{-i}^{L} = \int_0^{c^*} q(Q_{-i}^{L}, c_i)\, dF(c_i) + \int_{c^*}^{\infty} q(Q_{-i}^{H1}, c_i)\, dF(c_i) \\ Q_{-i}^{H1} = \frac{1}{F(c^*)} \int_0^{c^*} q(Q_{-i}^{L}, c_i)\, dF(c_i) \end{cases} \tag{15}$$

Lemma 7. *Suppose that conditions (C1)–(C3) hold. For every c^* there exist unique $Q^L(c^*)$ and $Q^{H1}(c^*)$ that solve equations (15), and thus there exists a unique continuation equilibrium after public message m^0, which is symmetric. Both $Q^L(c^*)$ and $Q^{H1}(c^*)$ are continuous; $Q^L(c^*)$ is increasing and $Q^{H1}(c^*)$ is decreasing in c^*; $Q^L(c^*) \leq Q^{H1}(c^*)$; $Q^L(0) > 0$; $\lim_{c^* \to \infty} Q^L(c^*) = \lim_{c^* \to \infty} Q^{H1}(c^*) = Q^{NC}$.*

For firm i of type c_i, let $\Delta\Pi(c_i;c^*)$ be the gain from reporting $\hat{c}_i < c^*$ relative to reporting $\hat{c}_i > c^*$ when the "min" mechanism with threshold c^* is in place:

$$\begin{aligned}\Delta\Pi\big(c_i;c^*\big) &= \Pi_i\big(Q^L\big(c^*\big),c_i\big)\\ &\quad - F\big(c^*\big)\Pi_i\big(Q^{H1}\big(c^*\big),c_i\big) - \big(1-F\big(c^*\big)\big)\Pi_i\big(Q^{H2}\big(c^*\big),c_i\big)\\ &= \Pi_i\big(Q^L\big(c^*\big),c_i\big) - \Pi_i\big(Q^{H1}\big(c^*\big),c_i\big)\\ &\quad - \big(1-F\big(c^*\big)\big)\big(\Pi_i\big(Q^{H2}\big(c^*\big),c_i\big) - \Pi_i\big(Q^{H1}\big(c^*\big),c_i\big)\big)\end{aligned}$$

A"min" mechanism with threshold c^* is incentive compatible if $\Delta\Pi(c;c^*) \geq 0$ for $c \leq c^*$, and $\Delta\Pi(c;c^*) \leq 0$ for $c \geq c^*$. The next lemma ensures that if $\Delta\Pi(c^*;c^*) = 0$, then the "min" mechanism with threshold c^* is incentive compatible.

Lemma 8. *Suppose that conditions (C1)–(C3) and (C5) hold. If $\Delta\Pi(c;c^*) = 0$, then either $\Delta\Pi(c';c^*) = 0$, $\forall c' \geq c$; or $\frac{\partial\Delta\Pi(c;c^*)}{\partial c} < 0$.*

The next lemma establishes that under the conditions of Theorem 3, there exists a value of $c^* \in (0,\bar{c})$ such that $\Delta\Pi(c^*;c^*) = 0$.

Lemma 9. *Suppose that conditions (C1)–(C3), (C5) and (C6) hold. If $\bar{c}$ is large enough, then there exists $c^* \in (0,\bar{c})$ such that $\Delta\Pi(c^*;c^*) = 0$.*

The conclusion of the theorem follows from Lemma 9. □

Proof of Theorem 4. By Lemma 7, $Q^L(c^*) \leq Q^{NC}$; therefore $\pi_i(q_i, Q^L(c^*), c_i) \geq \pi_i(q_i, Q^{NC}, c_i)$, for every $q_i \geq 0$ and $c_i \in [0,\bar{c}]$, and $\pi_i(q_i, Q^L(c^*), c_i) > \pi_i(q_i, Q^{NC}, c_i)$ if $q_i > 0$. This implies that $\Pi_i(Q^L(c^*), c_i) \geq \Pi_i(Q^{NC}, c_i)$.

Consider firm i of type c_i. If $c_i < c^*$ and it reports its type truthfully, its interim expected profit equals $\Pi_i(Q^L(c^*), c_i) \geq \Pi_i(Q^{NC}, c_i)$. If $c_i \geq c^*$ and it reports its type truthfully, its interim expected profit equals $F(c^*)\Pi_i(Q^{H1}(c^*), c_i) + (1-F(c^*))\Pi_i(Q^{H2}(c^*), c_i) \geq \Pi_i(Q^L(c^*), c_i) \geq \Pi_i(Q^{NC}, c_i)$, where the first inequality follows from the incentive compatibility of the "min" mechanism.

By condition (C4), $q(q_{-i}, c_i) > 0$, for every $q_{-i} \in [0, q_i(0,0)]$, $c_i \in [0,\bar{c}]$. Therefore $q(Q^{NC}, c_i) > 0$, so $\Pi_i(Q^{NC}, c_i) < \pi_i(q_i(Q^{NC}, c_i), Q^L(c^*), c_i) \leq \Pi_i(Q^L(c^*), c_i)$. Thus $\max\{\Pi_i(Q^L(c^*), c_i), F(c^*)\Pi_i(Q^{H1}(c^*), c_i) + (1-F(c^*))\Pi_i(Q^{H2}(c^*), c_i)\} > \Pi_i(Q^{NC}, c_i)$, and every type is strictly better off under the "min" mechanism than in the Bayesian–Nash equilibrium without communication. □

Appendix B. Supplementary material

Supplementary material related to this article can be found online at http://dx.doi.org/10.1016/j.jet.2014.06.008.

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