Games and Economic Behavior 72 (2011) 100–122

Contents lists available at ScienceDirect

Games and Economic Behavior

www.elsevier.com/locate/geb

How to talk to multiple audiences ☆

Maria Goltsman, Gregory Pavlov *

Department of Economics, University of Western Ontario, Social Science Centre, London, Ontario N6A 5C2, Canada

ARTICLE INFO

Article history:
Received 11 June 2009
Available online 26 August 2010

JEL classification:
C72
C78
D74
D82

Keywords:
Communication
Information
Mechanism design
Cheap talk
Long cheap talk
Multiple audiences

ABSTRACT

We analyze the performance of various communication protocols in a generalization of the Crawford–Sobel (1982) model of cheap talk that allows for multiple receivers. We find that the sender prefers communicating by private messages if the receivers' average bias is high, and by public messages if the receivers' average bias is low and the receivers are sufficiently polarized. When both public and private messages are allowed, the sender can combine the commitment provided by public communication with the flexibility of private communication and transmit more information to the receivers than under either private or public communication scenarios. When the players can communicate through a mediator and the receivers are biased in the same direction, it is optimal for the sender to communicate with the receivers through independent private noisy communication channels.

1. Introduction

The problem of communicating effectively with several parties with diverse interests arises in many contexts. A firm's disclosure of information about the demand for its product may be simultaneously observed by the capital market, shareholders and competitors (Newman and Sansing, 1993; Gigler, 1994). A government bureaucrat may need to communicate with many policymakers with different policy preferences (Johns, 2007). During deliberations of a committee, each member discloses his private information to the other members in order to come to a joint decision (Austen-Smith, 1990; Li et al., 2001; Austen-Smith and Feddersen, 2006 and others), or a sponsor of a proposal wishes to convince the committee to accept it (Caillaud and Tirole, 2007). A lobbyist interacts with the members of the associated group and government officials (Ainsworth and Sened, 1993), or with two separate legislative bodies (Board and Dragu, 2008). A politician's statements are observed by the voters and by leaders of other countries (Levy and Razin, 2004; Kurizaki, 2007).

In all those settings, the informed party (the sender) faces the problem of selecting the most effective communication mode. An immediate question is whether the sender would prefer to make public announcements or to communicate

☆ We would like to thank the Editor and a referee for helpful comments and suggestions. We also thank Andreas Blume, Oliver Board, Dino Gerardi, Hsueh-Ling Huynh, Maxim Ivanov, Navin Kartik, Bart Lipman, Tymofiy Mylovanov, Hyun Song Shin, Al Slivinski and the seminar participants at Boston University, University of Toronto, CETC (Vancouver, 2008), World Congress of the Game Theory Society (Evanston, 2008) and Midwest Economic Theory Conference (Iowa City, 2009). We are grateful for financial support from The Social Sciences and Humanities Research Council of Canada. All remaining errors are ours.

* Corresponding author.
E-mail addresses: mgoltsma@uwo.ca (M. Goltsman), gpavlov@uwo.ca (G. Pavlov).

0899-8256/$ – see front matter
doi:10.1016/j.geb.2010.08.007

privately with each receiver. Also, it is possible that the sender could strictly improve upon both purely public and purely private communication by making some statements in public and some in private, and if so, one can ask which statements should be made in public and which in private. Another question is whether the sender would benefit from adopting a more complex communication protocol (e.g. multi-stage communication, or communication through a mediator), and what the optimal protocol would look like. The issue of comparison between public and private communication has been addressed by some of the papers listed above.[1] Farrell and Gibbons (1989) compare public and private communication in a more abstract model of cheap talk with multiple receivers, which we discuss later.[2,3]

In this paper we analyze the performance of various communication protocols (private and public communication, combined private and public communication, as well as mediated and multi-stage communication) in a model which is a natural extension of the classic cheap-talk model by Crawford and Sobel (1982) to the setting with multiple receivers. The sender privately observes a realization of the state of the world from a continuum of possible states, and communicates with two receivers, each of whom chooses an action from the real line. In order to focus on purely informational externalities between the receivers, we assume that each receiver's payoff depends only on his own action and the state, and that the sender's payoff depends on the actions of both receivers and the state and is separable in the two actions.

In Section 3 we compare public and private communication. In the private communication game, the sender can send individual private messages to the receivers; in the public communication game, the sender's messages are commonly observed by both receivers. For private communication, we show that the amount of information the sender transmits to a given receiver is the same as in the game between the sender and this receiver alone (Proposition 1). As in the model of Crawford and Sobel (1982), the amount of information revealed to a receiver depends on the extent to which the preferences of the receiver diverge from the preferences of the sender. In the public communication game, the sender's set of strategies is more restricted than in the private communication game, as it is no longer possible to reveal different information to different receivers. But restricting the set of strategies can sometimes be a good thing, because it allows the sender to commit not to tell each receiver a different lie. The fact that the messages are publicly observed by both receivers thus forces the sender to find a compromise between possibly conflicting incentives for misrepresentation of information to different receivers. We show that the amount of information the sender transmits to the receivers in the public communication game is the same as in the game between the sender and a single 'representative' receiver whose preferences are 'between' the preferences of the receivers (Proposition 2).

Whether the sender would like to communicate with the receivers privately or publicly depends on the extent to which the preferences of the receivers and the preferences of the representative receiver are different from the preferences of the sender. Simple examples are provided in Section 3.1, and Section 3.4 contains a detailed discussion of possible cases.

In Section 4 we study communication when the sender can send both private and public messages to the receivers (the 'combined' communication protocol). We identify and characterize two possible classes of equilibria, monotonic and nonmonotonic. In monotonic equilibria, the sender's public announcement partitions the state space into intervals, and the private announcements are used to provide further information to the receivers individually. In nonmonotonic equilibria, the sender's public announcement divides the state space into subsets which are not intervals. We show that both types of equilibria of the combined communication game often allow the sender to transmit more information to the receivers than under either the private or the public communication scenarios (Propositions 4 and 5).

One reason for the superiority of combined communication is that it has the advantages of both private and public communication. The sender can make use of the commitment the public announcements provide, and at the same time reveal different information to different receivers via private messages. Combining private and public communication can introduce subtle strategic effects: the sender may benefit from revealing less information than maximally possible at the private communication stage (Example 1); when the receivers are identical, the sender may benefit from providing the receivers with different information at the private communication stage (Example 2).

In Section 5 we study mediated communication and communication under multi-stage protocols. The starting point here is the revelation principle (Myerson, 1982), which states that the outcome of any communication protocol can be replicated by the procedure whereby the sender makes a secret report to a neutral trustworthy mediator, who then makes a private non-binding recommendation (possibly stochastic) to each receiver of what action to take. When the receivers are biased in the same direction, we show that it is optimal for the sender to communicate with the receivers through independent private noisy communication channels (Proposition 6). In this case we also show that there exists an unmediated protocol which implements the optimal mediation rule (Proposition 7). When the receivers are biased in the opposite direction, we show that it may be optimal to take advantage of pooling the sender's truthtelling constraints across the receivers (Example 3).

[1] See Newman and Sansing (1993), Gigler (1994), Kurizaki (2007). Galeotti et al. (2009) study public and private communication on a network; Hagenbach and Koessler (2010) study formation of a communication network where the agents can exchange private messages. A different strand of literature compares private and public contracts in a multilateral contracting environment (e.g. McAfee and Schwartz, 1994; Segal, 1999). See also Koessler and Martimort (2008) for a setting with non-transferable utility.

[2] Their analysis is extended for a setting with verifiable information by Koessler (2008) and Ozmen (2004).

[3] A related strand of literature studies signaling models with multiple audiences. For example, a firm's choice of financial structure is simultaneously observed by the capital market and competitors (Gertner et al., 1988), or by the capital market and a regulator (Spiegel and Spulber, 1997).

The paper most related to ours is Farrell and Gibbons (1989). They compare private and public communication in a cheap-talk model where there are two possible states of the world and each receiver has two possible actions. For this model, Farrell and Gibbons introduce the classification of the equilibria of private and public communication games that we use in Section 3.4. However, in our model, unlike theirs, the possible cases can be conveniently interpreted as depending on whether the sender's audience is polarized or homogeneous, extremist or moderate. Also the Farrell and Gibbons model is not rich enough to address some interesting questions. In particular, because their model has only two states, the sender has only two possibilities in a pure-strategy equilibrium: either to reveal the truth completely, or to reveal nothing at all. On the other hand, in our model it is possible to have a situation where the sender communicates some information under either communication protocol, but the informativeness of the statements differs across protocols. Finally, the Farrell and Gibbons model is not well-suited for studying combined private and public communication.

In coincident work, Koessler and Martimort (2008) provide a partial comparison of private and public communication. They do not allow for combining private and public communication, or analyze the optimal communication protocol. In another recent work, Golosov et al. (2008) study a one sender, one receiver cheap-talk model where the receiver has to take several actions over time. The two-period version of their model can be interpreted as a special case of our model, where the sender engages in combined communication with two identical receivers. They describe equilibria similar to our Examples 1 and 2.

2. Environment

There are three players, one sender and two receivers. The sender observes the state of the world $\theta \in \Theta = [0, 1]$, while the receivers do not observe θ. The common prior over the states of the world is a continuous distribution F on Θ. Each receiver i can choose an action $a_i \in \mathbb{R}$.

We assume that the utility function of the sender is $u(a_1, a_2, \theta) = -l_1(|a_1 - \theta|) - l_2(|a_2 - \theta|)$, where l_i is twice continuously differentiable with $l_i'(x) > 0$, $l_i''(x) > 0$, $\forall x > 0$, and the utility function of receiver i is $v_i(a_i, \theta) = -L(|a_i - \theta - b_i|)$, $L'(x) > 0$, $L''(x) > 0$, $\forall x > 0$, where $b_i \in \mathbb{R}$. Given these preferences, the sender's most preferred actions in state θ are $a_1 = a_2 = \theta$; receiver i's most preferred action is $a_i = \theta + b_i$. The utility of each party in state θ decreases in the distance from the preferred action(s) given θ to the action(s) that is(are) actually taken. A special case are the quadratic preferences ($l_i(x) = L(x) = x^2$, $i = 1, 2$), which are assumed in many applications.[4]

Before the receivers take their actions, the sender can send them payoff-irrelevant messages (cheap talk). We consider three ways to organize communication: public communication, private communication and combined communication. When communication is *public*, the sender is allowed to send only messages that are publicly observed by both receivers. When communication is *private*, the sender is allowed to send individual messages to each receiver. When communication is *combined*, the sender can send both public and private messages. In all these scenarios the receivers are not allowed to communicate either with each other or with the sender.[5] In Section 5 we discuss more complicated communication arrangements: communication through a mediator and communication where the receivers actively participate in the conversation (long cheap talk). The aim of the paper is to compare equilibria of various modes of communication and find the communication arrangement that maximizes the sender's ex ante utility.

For an equilibrium of a given game between the sender and the receivers, a function $a : \Theta \to \Delta(\mathbb{R}^2)$ will be called the *equilibrium outcome function* if the probability distribution over the actions of the receivers for the sender of type θ in this equilibrium is given by $a(\theta) \in \Delta(\mathbb{R}^2)$. When we say that $a(\theta) = a \in \mathbb{R}^2$, we mean that type θ gets the vector of actions a for sure.

3. Pure modes of communication

3.1. Examples

Suppose the utility function of the sender is $-(a_1 - \theta)^2 - (a_2 - \theta)^2$, the utility function of receiver i is $-(a_i - \theta - b_i)^2$, and the state θ is distributed uniformly on $[0, 1]$. We will compare the outcomes of two games, the private communication game and the public communication game.

Let $b_1 = 0$ and $b_2 = \frac{1}{2}$, and consider the private communication game. Since the sender's utility is separable in the actions of the two receivers, and neither of the receivers is affected by the other receiver's action, the sender in effect faces two 'separable' information transmission problems, one with each receiver. Therefore in equilibrium the sender will communicate with each receiver as she would if the other receiver was not present (Proposition 1). Since the interests of receiver 1 are perfectly aligned with the sender's, it is incentive compatible for the sender to tell receiver 1 exactly what the state is. However, the bias of receiver 2 is so large that it will be impossible for the sender to communicate any information to him in any equilibrium.

[4] See for example Gilligan and Krehbiel (1989), Grossman and Helpman (2001), Krishna and Morgan (2001), Stein (1989).

[5] Any equilibrium outcome in the above scenarios remains an equilibrium even when the receivers are allowed to communicate, because the receivers can always be prescribed to use uninformative (babbling) communication strategies.

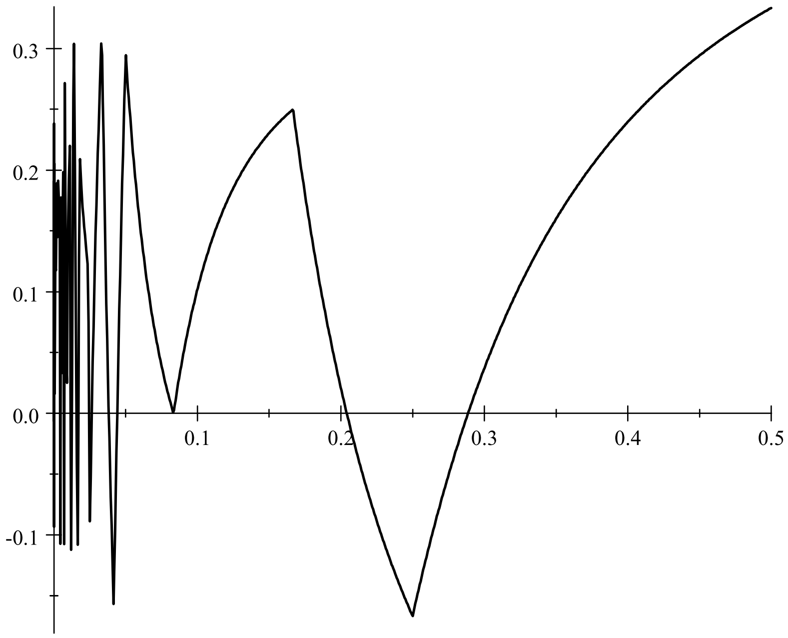

Fig. 1. Difference in the sender's ex ante payoff between the best private and the best public communication equilibria as a function of b_2 when $b_1 = 0$, normalized by $(b_2)^{-2}$.

Let us now consider the public communication game. Intuitively, since the sender is restricted to sending a public message which will be seen by two receivers with different interests, the sender should treat the two receivers as a single audience whose interests lie somewhere between the interests of the two receivers. Indeed, it is possible to prove (Proposition 2) that the sender should act as if she is facing a single receiver whose bias is the average of the two receivers' biases. In this case, the average is equal to $\frac{1}{4}$, which means that all equilibria with public communication will be uninformative.

To summarize, in case of private communication, the sender is able to transmit her information perfectly to receiver 1 but no information to receiver 2, while in case of public communication she is unable to transmit any information to either receiver. The high bias of receiver 2 *subverts* the possibility of informative public communication in this example. Therefore private communication is better than public for the sender: it provides the sender with the ability to tailor the message to the personality of the receiver, which may be valuable if the receivers have different preferences.

However, it is not always the case that this ability is beneficial for the sender. To illustrate, suppose $b_2 = \frac{1}{2}$ as above, but $b_1 = -\frac{1}{2}$. Now the sender will not be able to transmit any information to any of the receivers in the private communication game, because both biases are too high in absolute value. However, the average bias is equal to 0, so the public communication game has an equilibrium where the sender's message is perfectly informative of the state. This phenomenon is called *mutual discipline*: the presence of receiver 1 disciplines the communication with receiver 2 and vice versa, making the sender's public announcement of the state credible. In other words, the fact that the message is public can be viewed as a commitment device for the sender, allowing her to sustain perfectly informative public communication when no information transmission through private communication is possible.

The comparison of public and private communication for the sender becomes more complicated when both modes of communication admit informative equilibria. Suppose $b_1 = 0$ and $b_2 \in (\frac{1}{4}, \frac{1}{2})$. As in the first example, the best private communication equilibrium involves full revelation of information to receiver 1 and no information to receiver 2. But the average of receivers' biases is now low enough to allow for existence of a public communication equilibrium with two distinct informative messages. This is the case of *one-sided discipline*: some information can be credibly transmitted to a biased receiver 2 because of the presence of an unbiased receiver 1. The comparison between the two modes of communication for the sender depends on the informativeness of public communication: when b_2 is close to $\frac{1}{2}$, public communication is not very informative (i.e. the sender sends the same message in most states) and the sender prefers private communication; when b_2 is close to $\frac{1}{4}$, public communication is sufficiently informative to outweigh the benefits of private communication. This is illustrated in Fig. 1: the horizontal axis measures b_2; the vertical axis measures $\frac{1}{(b_2)^2}(U_{private}(b_2) - U_{public}(b_2))$, where $U_{private}(b_2)$ is the sender's ex ante payoff from the best private communication equilibrium and $U_{public}(b_2)$ is the sender's ex ante payoff from the best public communication equilibrium.

When $b_1 = 0$ and $b_2 \in (0, \frac{1}{4})$, it becomes possible to sustain private information transmission to receiver 2. Fig. 1 shows that private and public communication keep alternating as the best communication arrangement as b_2 decreases, with the local extrema corresponding to the values of b_2 at which either public or private communication equilibrium with a greater number of informative messages becomes feasible.

More generally, if the receivers are extremist on average ($|\frac{b_1+b_2}{2}|$ is high), then private communication is at least as good as public: if both receivers are biased in the same direction, then neither communication mode can sustain information transmission, while if only one of the receivers is an extremist, then the sender can at least communicate with the moderate receiver (as in the case $b_1 = 0$, $b_2 = \frac{1}{2}$). If the receivers are moderate on average ($|\frac{b_1+b_2}{2}|$ is low) and polarized ($|b_1 - b_2|$ is high), then public communication is preferred to private (as in the case $b_1 = -\frac{1}{2}$, $b_2 = \frac{1}{2}$). If the receivers are moderate on average and relatively homogeneous ($|b_1 - b_2|$ is low), then the comparison in general is ambiguous (as in the case $b_1 = 0$, $b_2 \in (0, \frac{1}{2})$), but if the receivers are perfectly homogeneous ($b_1 = b_2$), then private and public communication are equivalent.

3.2. Private communication

In this section we consider Bayesian-Nash equilibria of the following *private communication game*. At the first stage, after observing the state θ the sender sends two messages, m_1 and m_2, to the receivers. Receiver i is able to observe only the message m_i. At the second stage, receivers independently choose their actions a_1 and a_2.[6] A sender's strategy in this game maps the states into probability distributions over pairs of messages. Receiver i's strategy maps the messages into actions.[7]

As in the example in Section 3.1, we show that the sender will communicate with each receiver in private as she would in a model where only that receiver is present. Let us introduce the *Crawford–Sobel* (*CS*) *game* between one sender and one receiver. The sender privately observes the state $\theta \in [0, 1]$ distributed according to $F(\theta)$. She can send one payoff-irrelevant message to the receiver, who then takes an action $a \in \mathbb{R}$. The utility functions of both parties depend on the state and the receiver's action.

Proposition 1.

(i) *Suppose there exists an equilibrium of the private communication game with an outcome function $a(\theta)$. Then for $i = 1, 2$ there exists an equilibrium of the CS game between the sender with utility function $-l_i(|a_i - \theta|)$ and the receiver with utility function $-L(|a_i - \theta - b_i|)$ with the outcome function $a_i(\theta) := marg_{a_i} a(\theta)$.*

(ii) *Suppose for $i = 1, 2$ there exists equilibria of the two CS games with payoffs as in* (i) *with outcome functions $a_i(\theta)$. Then there exists an equilibrium of the private communication game with the outcome function $a(\theta) = (a_1(\theta), a_2(\theta))$.*

Crawford and Sobel (1982) characterize the equilibria of the CS game under more general assumptions on preferences. They prove that if $b_i = 0$, there exists an equilibrium where the state is completely revealed to the receiver. If $b_i \neq 0$, any equilibrium is characterized by a finite sequence of cutoff types $0 = \theta_0 < \theta_1 < \cdots < \theta_N = 1$ such that the equilibrium outcome function is constant on each interval (θ_{i-1}, θ_i). If there exists an equilibrium of size N, then there also exist equilibria of any size smaller than N. As a consequence, for any fixed value of b_i, there exists an equilibrium of the greatest size.

In our model, this translates into the fact that any equilibrium generates two interval partitions of $[0, 1]$, each partition corresponding to an equilibrium of the CS game with receiver i. For any values of b_1 and b_2, there exists an equilibrium where each receiver takes more distinct actions than in any other equilibrium.

3.3. Public communication

In this section we consider Bayesian-Nash equilibria of the following *public communication game*. At the first stage, after observing the state θ the sender sends a message m that is observed by both receivers. At the second stage, the receivers choose their actions a_1 and a_2. A sender's strategy in this game maps the states into probability distributions over messages. Receiver i's strategy maps the messages into actions a_i.

The following proposition establishes that the equilibria of the public communication game have an interval partitional form, like in the CS game. If in addition the sender's loss functions from the interaction with each receiver are identical, like in the example in Section 3.1, then the sender behaves as if she is facing a single 'representative' receiver with the bias equal to the average of the receivers' biases ($\bar{b} = \frac{b_1+b_2}{2}$).

[6] In cheap-talk games the set of equilibrium outcomes remains unchanged if one uses standard refinements of Bayesian-Nash equilibrium, like Perfect Bayesian equilibrium. See, for example, Section 3 in Farrell (1993). Though the sender can send a message to each receiver only once, it is straightforward to show that the set of equilibrium outcomes does not change if the sender is allowed to send several messages sequentially, as long as the receivers are not allowed to send messages. See, for example, discussion on p. 153 in Krishna and Morgan (2004). Similar argument applies to the public communication game considered in Section 3.3.

[7] Note that our assumptions guarantee that the equilibrium strategy of each receiver is pure. After any equilibrium message m_i, receiver i solves

$$\min_{a_i} \int_{\theta} L\big(|a_i - \theta - b_i|\big)\, dF_{m_i}(\theta)$$

where F_{m_i} is the posterior distribution of θ following message m_i. The solution is unique by strict convexity of L.

Proposition 2.

(i) *Suppose either $b_1 \neq b_2$ and $l_1'(|b_1|) \neq l_2'(|b_2|)$, or $b_1 = b_2 \neq 0$. Any equilibrium of the public communication game is characterized by a sequence of cutoff types $0 = \theta_0 < \theta_1 < \cdots < \theta_N = 1$ such that the equilibrium outcome $a(\theta)$ is a constant action pair on every interval (θ_k, θ_{k+1}) for $i = 1, 2$.*
(ii) *Suppose $l_i \equiv l$, $i = 1, 2$, and $b_1 \neq -b_2$. There is an equilibrium of the public communication game characterized by cutoff types $0 = \theta_0 < \theta_1 < \cdots < \theta_N = 1$ if and only the CS game between the sender with utility function $-l(|a - \theta|)$ and the receiver with utility function $-L(|a - \theta - \frac{b_1+b_2}{2}|)$ has an equilibrium with the same cutoff types.*

When $b_1 \neq b_2$ and $l_1'(|b_1|) = l_2'(|b_2|)$ (which is equivalent to $b_1 = -b_2$ if $l_i \equiv l$, $i = 1, 2$), there exists an equilibrium of the public communication game where every state is revealed truthfully by the sender, regardless of the absolute value of b_1 and b_2. To see this note that if the sender claims that the state is θ and the receivers expect her to be truthful, the optimal action of receiver 1 equals to $\theta + b_1$ and the optimal action of receiver 2 equals to $\theta + b_2$. Hence if the sender reports the state θ truthfully, her utility is $-l_1(|b_1|) - l_2(|b_2|)$; if she misreports the state to be $\theta + \Delta$, her utility is $-l_1(|\Delta + b_1|) - l_2(|\Delta + b_2|)$. This function is strictly concave, and since $l_1'(|b_1|) = l_2'(|b_2|)$ it is maximized at $\Delta = 0$. Thus the utility from telling the truth is higher than utility from any misreporting ($\Delta \neq 0$).

There seems to be no natural way to generalize part (ii) of this proposition when $l_1 \neq l_2$ except for one special class of environments described next. Suppose the preferences of the sender satisfy $l_1 = \lambda l$ and $l_2 = (1 - \lambda)l$, for some loss function l and $\lambda \in [0, 1]$. If l is quadratic, then it is easy to show that in the public communication game the sender behaves as if she is facing a single representative receiver with a bias $\bar{b} = \lambda b_1 + (1 - \lambda)b_2$. However if l is not quadratic then such a result does not hold, unless $\lambda = \frac{1}{2}$.

Note also that the public communication scenario, when $l_i = l$ for $i = 1, 2$, can be reinterpreteted as a model of communication with a single receiver whose bias is uncertain. Suppose that there is a sender with utility function $-l(|a - \theta|)$ and a single receiver with utility function $-L(|a - \theta - \hat{b}|)$, where $\hat{b}$ is either b_1 or b_2 with equal probabilities. The equilibrium characterization given in Proposition 2 is valid for this model.[8]

3.4. Comparison between private and public communication

For the remainder of the section we assume that $l_i = l$ for $i = 1, 2$ and F is uniform on $[0, 1]$, so that Assumption (M) of Crawford and Sobel is satisfied. Under this assumption, in the private communication game the maximal number of distinct actions that receiver i can take in equilibrium is a nonincreasing function of $|b_i|$.[9] In the public communication game the number of distinct actions taken in equilibrium is the same for both receivers and is a nonincreasing function of $|\bar{b}| = |\frac{b_1+b_2}{2}|$. Consequently, there exist a threshold $b^* \in \mathbb{R}_+$ such that there exists a private communication equilibrium where receiver i takes at least two different actions if and only if $|b_i| \leqslant b^*$, and there exists a public communication equilibrium where any receiver takes at least two different actions if and only if $|\bar{b}| \leqslant b^*$. We will say that public communication is better than private if the ex ante Pareto optimal equilibrium with public communication (i.e. the most informative one) gives higher utility to the sender than the ex ante Pareto optimal equilibrium with private communication (i.e. the one that is most informative with each of the receivers).

Depending on b_1 and b_2, there are the following five cases:

1. No communication ($|b_1|, |b_2|, |\bar{b}| \geqslant b^*$). Both the private communication and the public communication game have only uninformative (babbling) equilibria, and thus are welfare equivalent. This occurs when either both of the receivers are very biased in the same direction, or the receivers are very biased in the opposite directions and the magnitude of one of the biases is much larger than that of the other. See Fig. 2 for the range of parameters when this case, as well as the other cases, occurs.

2. Subversion ($|b_i| < b^*$; $|b_j|, |\bar{b}| \geqslant b^*$). There exist informative private communication equilibria with only one of the receivers, while all public communication equilibria are babbling, which implies that private communication is better than public. This case occurs when only one of the receivers has a bias which allows to sustain informative private communication. The other receiver is biased to the extent that the magnitude of the average bias is prohibitively large, and no informative public communication is possible.

3. Mutual discipline ($|b_1|, |b_2| \geqslant b^*$; $|\bar{b}| < b^*$). There are no informative private communication equilibria with either of the receivers, but there are informative public communication equilibria, which implies that public communication is better than private. This situation occurs when the receivers are biased in the opposite directions, but the magnitudes of their biases are of comparable sizes. As a result, in private communication, the sender wants to significantly overstate the true state to one receiver and understate it to the other one, which precludes informative communication, but in public communication these countervailing incentives result in an existence of an informative equilibrium. If $b_1 = -b_2$ then, as illustrated in the example in Section 3.1, a fully separating public communication equilibrium is feasible.

[8] Though there are a number of papers on cheap talk when the sender has uncertain bias (see for example Li and Madarász (2008) and references therein), we are not aware of papers where the receiver has uncertain bias.
[9] See Section 5 in Crawford and Sobel (1982).

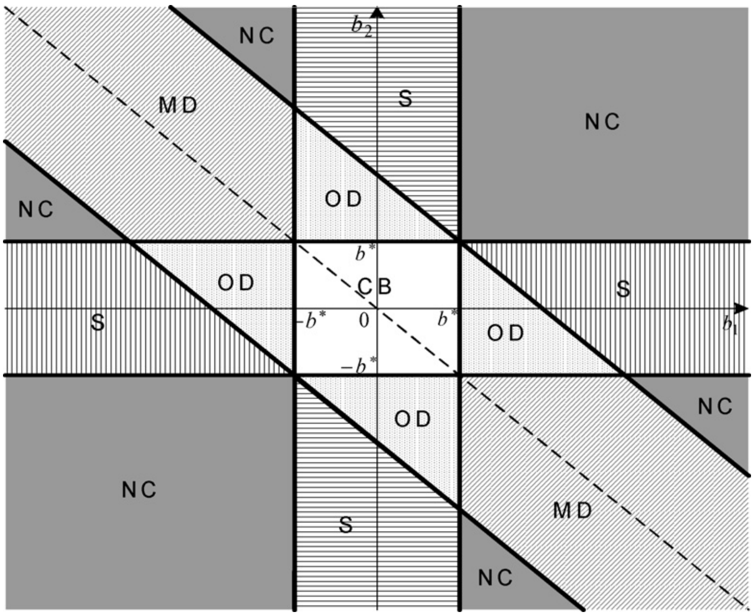

Fig. 2. Classification of private and public communication equilibria: NC = 'no communication', S = 'subversion', MD = 'mutual discipline', OD = 'one-sided discipline', CB = 'communication with both'.

4. One-sided discipline ($|b_i| \geqslant b^*$; $|b_j|, |\bar{b}| < b^*$). There exist informative private communication equilibria with only one of the receivers, as well as informative public communication equilibria. This occurs when one of the receivers has a low bias, and the other receiver has a bias that is high enough to preclude the possibility of informative private communication with him, but not high enough to prevent public communication. Proposition 3 below confirms that for quadratic payoffs the same qualitative pattern as in the example in Section 3.1 holds: private communication is better than public as long as receiver i is sufficiently biased.

5. Communication with both ($|b_1|, |b_2|, |\bar{b}| < b^*$). There exist both informative private communication equilibria with each of the receivers and informative public communication equilibria. The outcomes of the private and public communication equilibria are not equivalent in our model, unless the biases of the receivers exactly coincide. The welfare comparison between public and private communication in this case in general is ambiguous.

The following result provides a partial welfare comparison for cases 4 and 5 when the utility functions are quadratic.

Proposition 3. *Let* $l_i(x) = L(x) = x^2$, $i = 1, 2$.

(i) *Suppose* $|b_i| \geqslant \frac{1}{4}$; $|b_j|, |\bar{b}| < \frac{1}{4}$ *(the case of 'one-sided discipline'). There exist continuous functions* $\overline{B} : [-\frac{1}{4}, \frac{1}{4}] \to [\frac{1}{4}, \frac{1}{2}]$, $\underline{B} : [-\frac{1}{4}, \frac{1}{4}] \to [-\frac{1}{2}, -\frac{1}{4}]$ *such that public communication is better than private if and only if* $b_i \in [\underline{B}(b_j), -\frac{1}{4}] \cup [\frac{1}{4}, \overline{B}(b_j)]$.

(ii) *Suppose* $|b_1|, |b_2|, |\bar{b}| < \frac{1}{4}$ *(the case of 'communication with both'). Public communication is better than private if* $|b_1|, |b_2| \in (\frac{1}{2N(N+1)}, \frac{1}{2N(N-1)})$ *for some* $N = 2, \ldots$.

The question of whether public or private communication leads to better information transmission has been addressed before by Farrell and Gibbons (1989) in a setting where there are two states of the world, and each of the two receivers has a choice between two possible actions (hereafter the *FG model*). As in our model, the payoffs to each receiver are independent of the action of the other receiver. Focusing on pure strategy equilibria, Farrell and Gibbons provide conditions for existence of separating equilibria in the private communication game and in the public communication game. They arrive at a classification of cases that is the same as the one used above. In particular, neither in the FG model, nor in our model it is possible to have 'mutual subversion', i.e. a case where there exist informative private communication equilibria with each of the receivers but there are no informative public communication equilibria. This case becomes possible if one goes beyond the cheap-talk model and allows the sender to make certifiable statements (see Koessler, 2008; Ozmen, 2004).

Farrell and Gibbons study only pure strategy equilibria, but under some circumstances in their model there are interesting mixed strategy equilibria as well. For example, it can be shown that there are mixed strategy public communication equilibria which support some information transmission in cases when neither informative public communication, nor informative private communication with either receiver is possible (the 'no communication' case).[10] In contrast, in our model all private and public communication equilibria are essentially equivalent to partitional pure strategy equilibria. Also, in the informative equilibria of our model, the sender may reveal only some but not all the information, which cannot happen in pure strategy equilibria of the Farrell and Gibbons model. So, if both public and private informative equilibria exist in our model, they generally differ in their informativeness and resulting welfare.

4. Combined communication

4.1. Preliminaries

In this section we consider the game where the sender can send both public and private messages, and the receivers are not allowed to communicate either with each other or with the sender. Formally, we consider Bayesian-Nash equilibria of the following *combined communication game*. At the first stage, after observing the state θ the sender sends a message m that is observed by both receivers. The sender also sends two private messages, m_1 and m_2, to the receivers, such that receiver i is able to observe only the message m_i. At the second stage, receivers independently choose their actions a_1 and a_2. A sender's strategy in this game maps the states into probability distributions over messages. Receiver i's strategy maps messages into actions.[11] When discussing this game it is convenient to separate the communication stage into two: public communication stage and private communication stage.[12]

Because it is always possible to sustain uninformative communication at any stage of the combined communication game, any equilibrium outcome function of either the private or the public communication game can be achieved in an equilibrium of the combined communication game. Therefore, combined communication cannot be worse than private or public communication. We will be interested in whether combined communication can strictly improve on both.

Farrell and Gibbons (1989) do not study combined communication. Moreover, for every equilibrium of the combined communication game in their framework that we were able to find, there exists an equilibrium of either the private communication game or the public communication game that is equivalent or Pareto dominates it. Given the positive results for our model we present in this section, this suggests that the FG model is not well-suited for studying combined communication.

We begin characterizing the structure of the combined communication equilibria by noticing that, conditional on a public message, the private messages partition the state space into intervals.

Lemma 1. *Suppose m is a public message sent in a combined communication equilibrium, and suppose $\Theta(m)$ is the set of types that send m. Then, for $i = 1, 2$,*

$$\forall \theta, \theta' \in \Theta(m),\ \theta < \theta', \quad a_i(\theta) = a_i(\theta') = a_i \quad \Rightarrow \quad a_i(\theta'') = a_i \quad \forall \theta'' \in (\theta, \theta') \cap \Theta(m)$$

The lemma is proved by simply noting that, conditional on any public message, the argument for the CS model goes through, and the private messages partition the state space into intervals. The latter statement does not hold for public messages: in Example 2 public messages divide the state space into subsets that are not intervals.

Another property of any equilibrium of the combined communication game is given by the following lemma.

Lemma 2. *Let $a_1(\theta), a_2(\theta)$ be equilibrium outcome functions of some combined communication equilibrium. Then $\theta' > \theta$ implies that either $a_1(\theta') \geqslant a_1(\theta)$, or $a_2(\theta') \geqslant a_2(\theta)$, or both.*

Hence it is natural to distinguish the following two classes of combined communication equilibria.

Definition 1. A combined communication equilibrium is called *monotonic* if $a_1(\theta)$ and $a_2(\theta)$ are monotonically nondecreasing in θ; otherwise it is called *nonmonotonic*.

4.2. Monotonic equilibria

First we show that the equilibria from this class have the interval partition structure.

Lemma 3. *Any monotonic combined communication equilibrium is interval partitional.*

[10] One of the types of the sender mixes between a 'fully revealing' message and a 'pooling' message, while the second type always sends the 'pooling' message. The mixing probabilities are chosen so that the posterior after the 'pooling' message makes one of the receivers indifferent between his actions, so it is possible to choose a mixed strategy for this receiver to support such an equilibrium. The details are available upon request.

[11] In this section we focus on equilibria with deterministic outcome functions. Contrary to the public and private communication scenarios, in case of combined communication there may exist mixed strategy equilibria with a non-degenerate random outcome function.

[12] It is straightforward to show that the set of equilibrium outcomes does not change if we explicitly introduce the sequential timing for the messages. See footnote 6 for discussion of a related issue.

Hence in the monotonic equilibria the sender first makes public announcements which partition the state space into intervals, and then further refines the information of the receivers by privately communicating with each of them. Notice that all public communication equilibria and all private communication equilibria belong to this class. Despite the fact that the monotonic equilibria have an intuitive structure, a full characterization of all such equilibria is hard. Instead we settle on deriving a set of necessary conditions for the existence of monotonic equilibria with both informative communication at the public stage and informative communication at the private stage with at least one of the receivers.

Lemma 4.

(i) *Suppose F is uniform. If there exists a monotonic combined communication equilibrium with informative private communication with receiver i, then $|b_i| < \frac{1}{4}$.*
(ii) *Suppose F is uniform and $l_i(x) = x^2$, $i = 1, 2$. If there exists a monotonic combined communication equilibrium with informative public communication, then $|\bar{b}| < \frac{1}{4}$.*

Therefore, if a monotonic equilibrium where both the private and the public stages are informative exists, then both the conditions for the existence of an informative public communication equilibrium (as in Proposition 2) and the conditions for the existence of an informative private communication equilibrium with at least one of the receivers (as in Proposition 1) must be satisfied.

Next we consider an example of a monotonic equilibrium which performs strictly better than any public or private communication equilibrium.

Example 1. Suppose F is uniform, $l_i(x) = x^2$, $i = 1, 2$, and $(b_1, b_2) = (0, \frac{1}{4})$. The sender sends two public messages: '*Low*' if $\theta \in [0, x)$ and '*High*' if $\theta \in [x, 1]$, where $x = \sqrt{3} - 1 \approx 0.732$. After both public messages, the sender sends an uninformative message to receiver 2. The sender sends a fully revealing message to receiver 1 following the message '*High*' and an uninformative message following the message '*Low*'.

First we show that this communication arrangement constitutes an equilibrium. If the sender follows the strategy described above and the receivers play their best response, the outcome function is as follows:

$$\big(a_1(\theta), a_2(\theta)\big) = \begin{cases} \left(\frac{1}{2}x, \frac{1}{2}x + \frac{1}{4}\right) & \text{if } \theta \in [0, x) \\ \left(\theta, \left(\frac{1}{2}x + \frac{1}{2}\right) + \frac{1}{4}\right) & \text{if } \theta \in [x, 1] \end{cases}$$

Let us check incentive compatibility for the sender. Type x is indifferent between sending the public message '*Low*' and sending the public message '*High*' (and consequently communicating with receiver 1) if

$$-\left(\frac{1}{2}x - x\right)^2 - \left(\frac{1}{2}x + \frac{1}{4} - x\right)^2 = \max_{\hat{\theta} \in [x,1]} -(\hat{\theta} - x)^2 - \left(\frac{1}{2}x + \frac{3}{4} - x\right)^2 = -\left(\frac{1}{2}x + \frac{3}{4} - x\right)^2$$

Solving for x, we get $x = \sqrt{3} - 1$ as claimed above. The ex ante utility of the sender in this equilibrium is $\frac{1}{4}\sqrt{3} - \frac{9}{16} \approx -0.129$.

The best private communication equilibrium in this example involves full revelation of information to receiver 1, and no information revelation to receiver 2. The ex ante utility of the sender is $-\frac{7}{48} \approx -0.146$, which is smaller than the utility in the above combined communication equilibrium.

The best public communication equilibrium has a partition of two intervals, $[0, \frac{3}{4})$ and $[\frac{3}{4}, 1]$. The ex ante utility of the sender is $-\frac{13}{96} \approx -0.135$, which is smaller than the utility in the above combined communication equilibrium.[13]

Clearly the outcome of this combined communication equilibrium cannot be replicated by any public communication equilibrium, because receiver 1 must have more precise information than receiver 2. The reason why this outcome cannot be replicated by a private communication equilibrium is more subtle. Suppose there was a private communication equilibrium that resulted in the same outcome functions. Then type x would induce the actions $(a_1, a_2) = (x, \frac{1}{2}x + \frac{3}{4})$. But this type has a profitable deviation: it could achieve a higher utility by sending to receiver 1 a message which induces action x, and to receiver 2 a message which induces action $\frac{1}{2}x + \frac{1}{4}$. This deviation is unavailable to the sender in the combined communication game, because at the public stage it is made common knowledge whether the state is above or below x. This illustrates the role of having the public communication stage, which is to reduce the number of deviations available to the sender.

Another unusual feature of this combined communication equilibrium is that it prescribes uninformative communication with receiver 1 after the public message '*Low*' despite the fact that the preferences of the sender and of receiver 1 are

[13] There might be other combined communication equilibria that are better for the sender than the one described above. We do not solve for the optimal equilibrium here, but Proposition 6 describes the optimal mediation protocol for this example.

perfectly aligned.[14] Assume for a moment that the sender fully reveals the state of the world to receiver 1 after every public message. In this case, the public messages carry useful information only for receiver 2, and thus every combined communication equilibrium is equivalent to some private communication equilibrium. However, it is impossible to sustain any information revelation to receiver 2 because his bias is too high.[15]

Next we show that for a range of parameters it is possible to construct a monotonic combined communication equilibrium which outperforms all public communication equilibria from the ex ante perspective of the sender.

Proposition 4. *Suppose F is uniform and $l_i(x) = x^2$. For any $b_2 \in (-\frac{1}{2}, \frac{1}{2})\backslash\{0\}$, there exists $\varepsilon(b_2) > 0$ such that whenever $|b_1| \leqslant \varepsilon(b_2)$, there exists a monotonic combined communication equilibrium that gives the sender strictly higher ex ante utility than the best public equilibrium.*

The proof works as follows. We show that whenever there exists a public communication equilibrium of size N, it is possible to improve the sender's utility by constructing a combined communication equilibrium where the sender sends N public messages which partition the state space into intervals and then communicates informatively with one of the receivers.

Proposition 4 cannot be literally generalized to include private communication equilibria, because for any neighborhood of the zero vector in $\mathbb{R}^2$, there exists a countable subset of this neighborhood such that whenever (b_1, b_2) belong to that subset, the private communication equilibria result in the highest possible payoff among all possible communication mechanisms (including combined communication equilibria). This statement is made precise in Lemma 5 in Section 5.

4.3. Nonmonotonic equilibria

In the nonmonotonic combined communication equilibria the public messages divide the state space into subsets which are not intervals. Let us consider an example of a nonmonotonic equilibrium which performs strictly better than any public or private communication equilibrium or any monotonic equilibrium.

Example 2. Suppose F is uniform, $l_i(x) = x^2$, $i = 1, 2$, and $(b_1, b_2) = (\frac{1}{4}, \frac{1}{4})$. The sender sends two public messages: '*Outside*' if $\theta \in [0, x) \cup [z, 1]$ and '*Inside*' if $\theta \in [x, z)$, where $x \approx 0.021$ and $z \approx 0.932$. The sender sends an uninformative message to receiver 2 after both messages. Following the message '*Outside*' the sender reveals to receiver 1 whether $\theta \in [0, x)$ or $\theta \in [z, 1]$, and sends an uninformative message following the message '*Inside*'.

Let us show that this communication arrangement constitutes an equilibrium. If the sender follows the strategy described above and the receivers play their best response, the outcome function is as follows:

$$\big(a_1(\theta), a_2(\theta)\big) = \begin{cases} \left(\frac{1}{2}x + \frac{1}{4}, \left(\frac{x}{x+1-z}\left(\frac{1}{2}x\right) + \frac{1-z}{x+1-z}\left(\frac{1}{2}z + \frac{1}{2}\right)\right) + \frac{1}{4}\right) & \text{if } \theta \in [0, x) \\ \left(\left(\frac{1}{2}x + \frac{1}{2}z\right) + \frac{1}{4}, \left(\frac{1}{2}x + \frac{1}{2}z\right) + \frac{1}{4}\right) & \text{if } \theta \in [x, z) \\ \left(\left(\frac{1}{2}z + \frac{1}{2}\right) + \frac{1}{4}, \left(\frac{x}{x+1-z}\left(\frac{1}{2}x\right) + \frac{1-z}{x+1-z}\left(\frac{1}{2}z + \frac{1}{2}\right)\right) + \frac{1}{4}\right) & \text{if } \theta \in [z, 1] \end{cases}$$

Let us check incentive compatibility for the sender. Type x is indifferent between the 'low' strategy of sending the public message '*Outside*', with the consequent revelation to receiver 1 that her type is in $[0, x)$, and the 'intermediate' strategy of sending the public message '*Inside*' if

$$-\left(\frac{1}{2}x + \frac{1}{4} - x\right)^2 - \left(\frac{x}{x+1-z}\left(\frac{1}{2}x\right) + \frac{1-z}{x+1-z}\left(\frac{1}{2}z + \frac{1}{2}\right) + \frac{1}{4} - x\right)^2$$
$$= -\left(\frac{1}{2}x + \frac{1}{2}z + \frac{1}{4} - x\right)^2 - \left(\frac{1}{2}x + \frac{1}{2}z + \frac{1}{4} - x\right)^2$$

Type z is indifferent between the 'high' strategy of sending the public message '*Outside*', with the consequent revelation to receiver 1 that her type is in $[z, 1]$, and the 'intermediate' strategy of sending the public message '*Inside*' if

[14] One may argue that such an equilibrium is unnatural, because the sender has an incentive to communicate further with receiver 1 after the public message '*Low*' (Blume and Sobel, 1995, introduce the notion of 'communication-proof equilibria', which formalizes this idea). Since our goal is to study how various means of communication expand the set of equilibrium outcomes in cheap-talk games, we prefer not to address the issue of equilibrium selection here. See also the discussion following Example 2 below.

[15] Note that it is impossible to transmit any information to receiver 2 if the sender sends a fully revealing message to receiver 1 following the message '*Low*' and an uninformative message following the message '*High*'. The outcome in such an equilibrium would be as follows:

$$\big(a_1(\theta), a_2(\theta)\big) = \begin{cases} \left(\theta, \frac{1}{2}x + \frac{1}{4}\right) & \text{if } \theta \in [0, x) \\ \left(\frac{1}{2}x + \frac{1}{2}, \left(\frac{1}{2}x + \frac{1}{2}\right) + \frac{1}{4}\right) & \text{if } \theta \in [x, 1] \end{cases}$$

Solving for x, we get $x = 1$. Hence the resulting equilibrium is equivalent to the best private communication equilibrium, which involves full information revelation to receiver 1 and uninformative communication with receiver 2.

$$-\left(\frac{1}{2}z+\frac{3}{4}-z\right)^2-\left(\frac{x}{x+1-z}\left(\frac{1}{2}x\right)+\frac{1-z}{x+1-z}\left(\frac{1}{2}z+\frac{1}{2}\right)+\frac{1}{4}-z\right)^2$$
$$=-\left(\frac{1}{2}x+\frac{1}{2}z+\frac{1}{4}-z\right)^2-\left(\frac{1}{2}x+\frac{1}{2}z+\frac{1}{4}-z\right)^2$$

Let $d := z - x$. Straightforward but tedious calculations yield $x = \frac{1}{2}(1-d)(1+\frac{(1+d)}{(1-d)^2-4d})$, where d is the root of the following polynomial: $d^4 - \frac{37}{3}d^3 + 41d^2 - \frac{55}{3}d - \frac{26}{3}$. There exists a root $d \approx 0.910$, which yields $x \approx 0.021$ and $z \approx 0.932$. The ex ante utility of the sender in this equilibrium is approximately -0.266.

The best private communication equilibrium, the best public communication equilibrium, and the best monotonic combined communication equilibrium (by Lemma 4) are all babbling equilibria. The ex ante utility of the sender is $-\frac{7}{24} \approx -0.292$, which is smaller than the utility in the above nonmonotonic combined communication equilibrium.

Let us outline the logic behind the constructed equilibrium. Since both receivers have high positive biases, the sender is tempted to pretend to be a low type. To support informative communication, we need to reduce the sender's desire to do so. In the constructed equilibrium, the 'low' strategy of sending the public message '*Outside*' with the consequent revelation to receiver 1 that her type is in $[0, x)$ results in the action pair $(a_1, a_2) \approx (0.261, 0.991)$; the 'intermediate' strategy of sending the public message '*Inside*' results in the action pair $(a_1, a_2) \approx (0.727, 0.727)$. The intermediate sender types do not deviate to the 'low' strategy, because the resulting low action of receiver 1 is counterbalanced by the relatively high action of receiver 2. Similarly, the 'high' strategy of sending the public message '*Outside*' with the consequent revelation to receiver 1 that her type is in $[z, 1]$ results in the action pair $(a_1, a_2) \approx (1.216, 0.991)$; the unattractive high action of receiver 1 is counterbalanced by the action of receiver 2, so high types of the sender do not deviate to the 'intermediate' strategy.

As in Example 1, the key ingredient which allows to sustain informative communication in this equilibrium is that we handicap the sender in her ability to communicate with one of the receivers. If the sender is forced to reveal to receiver 2 whether the state is in $[0, x)$ or $(z, 1]$, then the construction breaks down and the resulting equilibrium is equivalent to an uninformative equilibrium.[16] To some extent, this feature of the equilibrium is a familiar one: in many dynamic settings the parties want to commit to ex post inefficient outcomes for some states of the world in order to support outcome functions which are Pareto superior in the ex ante sense. In our situation, commitment in the literal sense is not required, because the uninformative outcome at the private stage of communication is self-enforcing. There is a large literature that aims at constructing an equilibrium refinement that picks the most informative equilibrium in cheap-talk games.[17] In our environment, refining away less informative equilibria of some subgames may result in an ex ante Pareto inferior outcome.[18]

Equilibria of this sort can also be naturally sustained in the environments where the sender is unable to communicate privately with some of the receivers. For example, a firm may have an ability to hold a private meeting with a lender (or with a union), but be unable to communicate privately with numerous equity holders. Suppose that the firm is able to schedule a private meeting with the lender, and the equity holders observe whether the meeting is scheduled but do know what is discussed at the meeting. The firm schedules a meeting only in the extreme situations, i.e., when the business conditions are either very good or very bad, and at the meeting the firm reveals to the lender which one is the case. Thus the scheduling of the meeting plays the role of the public message '*Outside*', and the absence of the meeting plays the role of the public message '*Inside*'. The assumption that no private meeting between the firm and the equity holders takes place is non-controversial because it is plausible to assume that the lender could always send a spy there.[19]

Another key feature of our equilibrium is that the sender's message strategy with receiver 1 differs from the message strategy with receiver 2, and thus the receivers are induced to take different actions even though their preferences are identical. Such an effect can be replicated in a model with a single receiver if two-stage communication is possible, as shown by Krishna and Morgan (2004). Indeed, assume that the sender and a single receiver with the bias of $\frac{1}{4}$ have a fair coin. The sender sends two messages: '*Outside*' if $\theta \in [0, x) \cup [z, 1]$ and '*Inside*' if $\theta \in [x, z)$. Following the message '*Outside*' a coin is flipped. In case of 'heads' the sender reveals to the receiver whether $\theta \in [0, x)$ or $\theta \in [z, 1]$, in case of 'tails' no further information is revealed. This constitutes an equilibrium with the same values of x and z as above. Following 'heads', the receiver behaves as receiver 1 from our example, and following 'tails' he assumes the identity of receiver 2. Instead of an access to a coin, we could assume that the receiver is allowed to participate in the conversation with the sender, and thus they can perform a jointly controlled lottery which replicates the coin.[20] We continue the discussion of the benefits of such conversations in Section 5.

[16] The resulting equilibrium is equivalent to a public communication equilibrium of size $N = 3$, but we know that the public communication game has only uninformative equilibria.

[17] See for example Chen et al. (2008) and the references therein.

[18] This is also a feature of environments with multiple communication stages; see, for example, Aumann and Hart (2003) and Krishna and Morgan (2004).

[19] Another setting where such equilibria are natural is when there is a single receiver who has to take several actions over time (Golosov et al., 2008). While the information revealed by the sender in first period is 'publicly observed' by both first- and second-period receiver, the information revealed by the sender in the second period is 'privately observed' by the second-period receiver only. Nonmonotonic equilibria can also arise when the receiver observes a noisy signal of the state (Chen, 2009). In this case, the 'private' message corresponds to the value of the signal and is outside of the sender's control.

[20] See Aumann and Hart (2003) for a discussion of jointly controlled lotteries.

Next we generalize the message of Example 2. We show that for a range of parameters it is possible to construct a nonmonotonic combined communication equilibrium.

Proposition 5. *Suppose F is uniform, $l_i(x) = L(x) = x^2$, $i = 1, 2$. For any $b_2 \in \mathbb{R}$, there exists $\varepsilon(b_2) > 0$ such that whenever $|b_1| \leqslant \varepsilon(b_2)$ there exists a non-trivial nonmonotonic combined communication equilibrium.*

In the proof of this result we show that there always exists a nonmonotonic combined communication equilibrium of the same form as in Example 2 as long as the preferences of one of the receivers are closely aligned with the preferences of the sender. This is a surprising finding, because in both public equilibria and monotonic combined communication equilibria it is possible to communicate some information to an extremely biased receiver only in the situations of 'mutual discipline', i.e. when the average bias is small enough (Proposition 2 and Lemma 4). In the constructed nonmonotonic equilibria the public messages are informative for any value of the average bias.

We do not claim that the constructed nonmonotonic equilibria are generally better for the sender than other communication arrangements. However, Example 2 shows that there are situations when this is the case.

5. Mediated communication and long cheap talk

5.1. Mediated communication

In this section we introduce the possibility of mediated communication, whereby the players communicate with a neutral trustworthy party (the mediator) who then sends back private messages to the players. The mediator does not know the state of the world and does not have the power to impose what actions the players are to take.

The value of studying mediated communication is twofold. First, it is interesting to find out when it is beneficial to invite an outside mediator to facilitate communication between the players. Second, according to the revelation principle (Myerson, 1982), any equilibrium outcome of any communication protocol (mediated or unmediated) can be replicated by the procedure whereby the sender secretly reports the state of world to a neutral trustworthy mediator, who then makes non-binding private recommendations (possibly stochastic) to each receiver of what action to take. Hence, when looking for the optimal (according to some criterion) communication protocol, it is enough to optimize within this class. After that one can check whether the outcome can be replicated by some unmediated communication protocol.

Formally, a mediation rule is a family $(p(\cdot|\theta))_{\theta\in\Theta}$, where for each $\theta \in \Theta$, $p(\cdot|\theta)$ is a probability distribution on the space of action pairs $\mathbb{R}^2$. Given a mediation rule, the game proceeds as follows. At the first stage, after observing the state θ, the sender privately reports a state $\hat{\theta}$ to the mediator. Upon hearing the report from the sender, the mediator selects the individual recommended actions a_1 and a_2 according to $p(\cdot|\hat{\theta})$ and privately announces them to each receiver. The revelation principle implies that without loss of generality reporting the true state should be optimal for the sender, and obeying the mediator's recommendation should be optimal for each receiver. The mediation rules that have an equilibrium where the sender always reports the truth and each receiver always obeys the recommendation will be called *incentive compatible*.[21]

We are looking for incentive compatible mediation rules that maximize the ex ante utility of the sender, and focus on the case when F is uniform and the payoffs are quadratic.

Definition 2. An *optimal mediation rule* $p = (p(\cdot|\theta))_{\theta\in\Theta}$ is a family of probability distributions on $\mathbb{R}^2$ that solves the following problem:

$$\max_{p(\cdot|\theta)_{\theta\in\Theta}} \int\limits_{\mathbb{R}^2\times\Theta} \left(-(a_1-\theta)^2-(a_2-\theta)^2\right) dp\,(a_1,a_2|\theta)\,d\theta$$

subject to

$$\theta = \arg\max_{\hat{\theta}\in\Theta}\left[\int\limits_{\mathbb{R}^2\times\Theta} \left(-(a_1-\theta)^2-(a_2-\theta)^2\right) dp\,(a_1,a_2|\hat{\theta})\right], \quad \forall\theta\in\Theta \tag{IC-S}$$

$$a_i = E_\theta[\theta|a_i] + b_i, \quad \forall a_i \in \mathbb{R},\ i = 1, 2 \tag{IC-R}$$

The constraints (IC-S) say that the sender should find it optimal to tell the truth. The constraints (IC-R) state that each receiver has no incentive to deviate from the action that is recommended to him by the mediator. The right-hand side of the equality is the expectation of θ given the recommendation a_i corrected by the bias of receiver i, which is the action that maximizes the payoff of receiver i when the mediator recommends a_i. Given $p(a_1, a_2|\theta)$ and the unconditional distribution of θ, $E_\theta[\theta|a_i]$ is determined uniquely up to a zero-measure subset of $\mathbb{R}$.

[21] The incentive compatible mediation rules are sometimes called *communication equilibria* (Forges, 1990; Myerson, 1991).

Proposition 6.

(i) *Let* $(b_1, b_2) \in (-\frac{1}{2}, 0)^2 \cup (0, \frac{1}{2})^2$. *The optimal mediation rule is characterized by two sequences of cutoff types* $0 = \theta_{1,0} < \theta_{1,1} < \cdots < \theta_{1,N_1} = 1$ *and* $0 = \theta_{2,0} < \theta_{2,1} < \cdots < \theta_{2,N_2} = 1$ *where* N_i *is such that* $|b_i| \in [\frac{1}{2}(N_i)^{-2}, \frac{1}{2}(N_i - 1)^{-2})$, *and by two numbers* $\mu_1, \mu_2 \in [0, 1]$. *If* $\theta \in [\theta_{i,k}, \theta_{i,k+1})$, $k = 1, \ldots, N_i - 2$, *receiver i is recommended action* 0 (*if* $(b_1, b_2) < 0$) *or* 1 (*if* $(b_1, b_2) > 0$) *with probability* μ_i, *and action* $a_{i,k} = \frac{1}{2}(\theta_{i,k} + \theta_{i,k+1}) + b_i$ *with probability* $1 - \mu_i$. *If* $\theta \in [0, \theta_{i,1})$, *receiver i is recommended action* 0 *with probability* 1 *if* $(b_1, b_2) < 0$ *or action* 1 *with probability* μ_i *and action* $\frac{1}{2}\theta_{i,1} + b_i$ *with probability* $1 - \mu_i$ *if* $(b_1, b_2) > 0$. *If* $\theta \in [\theta_{i,N_i-1}, 1]$, *receiver i is recommended action* 1 *with probability* 1 *if* $(b_1, b_2) > 0$ *or action* 0 *with probability* μ_i *and action* $\frac{1}{2}(\theta_{i,N_i-1} + 1) + b_i$ *with probability* $1 - \mu_i$ *if* $(b_1, b_2) < 0$.

(ii) *Let* $b_i \in (-\frac{1}{2}, 0) \cup (0, \frac{1}{2})$ *and* $b_j = 0$. *The optimal mediation rule makes recommendations to receiver i as in the rule described in* (i) *and recommends action* θ *to receiver j for every* $\theta \in \Theta$.

(iii) *Let* $b_1 = b_2 = 0$. *The optimal mediation rule recommends to receiver i action* θ *for every* $\theta \in \Theta$, $i = 1, 2$.

(iv) *Let* $b_1 = b_2 \in \mathbb{R} \backslash (-\frac{1}{2}, \frac{1}{2})$. *The optimal mediation rule recommends to receiver i a constant action* $\frac{1}{2} + b_i$ *for every* $\theta \in \Theta$.

Mediation rules similar to the one in Proposition 6 appeared in the literature on cheap-talk games before. For the CS model with a single receiver, Blume et al. (2007) introduced the mediation rule which is otherwise identical to ours, and Goltsman et al. (2009) proved its optimality. Thus in the cases covered by Proposition 6 the optimal mediation rule with two receivers is equivalent to the twice-replicated optimal mediation rule with a single receiver. It can thus be implemented with the help of two mediators, such that mediator i is allowed to communicate only with the sender and receiver i (in particular, the recommendation of mediator i has to be independent of that of mediator $j \neq i$ and of the sender's report to mediator $j \neq i$). The mediation rules that can be implemented in this fashion will be called *private mediation rules*. In particular, note that all equilibrium outcomes of the private communication game can be replicated with private mediation rules.

Besides the (IC-S) and (IC-R) constraints, the incentive compatible private mediation rules satisfy the condition that the sender has to report the state of the world truthfully to each of the two mediators. If the optimal mediation rule is private, this means that there are no benefits from pooling together the sender's incentive constraints across receivers. Proposition 6 shows that this is the case when the receivers' biases are of the same sign and of moderate magnitude.[22]

For the cases when the receivers' biases are of the same sign that are not covered by Proposition 6, the optimal mediation rule is unknown. We conjecture that the optimal mediation rule also belongs to the class of private rules and is equivalent to the twice-replicated optimal mediation rule for the model with a single receiver.

When the receivers' biases are of the opposite sign, the optimal mediation rule is unlikely to be private, because, similarly to the 'mutual discipline' case in private communication, it may now be valuable to pool the sender's truthtelling constraints across receivers. One special class of mediation rules that takes advantage of pooling the sender's truthtelling constraints across receivers is when the mediator recommends actions to each of the receivers publicly rather than in a private manner. Such mediation rules will be called *public mediation rules*.[23] Note that all equilibrium outcomes of the public communication games can be achieved with public mediation rules. Also, similarly to the case of equilibria of the public communication game, the public mediation rules can be shown to be equivalent (from the point of view of the sender) to a mediation rule between the sender and a single receiver with a bias equal to the average of the two biases, i.e. $\bar{b} = \frac{b_1+b_2}{2}$.[24]

It is easy to show that if $b_1 + b_2 = 0$, the optimal mediation rule is public and recommends to receiver i action θ for every $\theta \in \Theta$. We do not know whether the optimal mediation rules belong to the class of public rules for other values of the receivers' biases. However it is possible to show that the ex ante payoff of the sender from the optimal public mediation rule is higher than from the optimal private mediation rule when the receivers' biases are of the opposite sign and are close in absolute values.[25]

Next we show that in some cases neither private nor public mediation rules are optimal. We present an example of a monotonic equilibrium of the combined communication game which performs better than any private or public mediation rule.

Example 3. Let $(b_1, b_2) = (\frac{1}{40}, -\frac{11}{40})$. The sender sends two public messages: '*Low*' if $\theta \in [0, x)$ and '*High*' if $\theta \in [x, 1]$, where $x \approx 0.261$. The sender sends an uninformative message to receiver 2 after both public messages. Following the message '*Low*' the sender to receiver 1 whether $\theta \in [0, t)$ or $\theta \in [t, x)$, where $t \approx 0.180$, and sends an uninformative message following the message '*High*'.

---

[22] More specifically, first we use the (IC-S) and (IC-R) constraints to derive an upper bound on the sender's ex ante utility (see Lemmas 11 and 12 in Appendix A), and then we show that mediation rule given in Proposition 6 achieves this upper bound.

[23] Our definition of public mediation rules differs from the one in Lehrer and Sorin (1997). We assume that the sender submits to the mediator a report about the state of the world, while Lehrer and Sorin (1997) allow for more general reports.

[24] See Proposition 2.

[25] The proof is available upon request.

This communication arrangement constitutes an equilibrium, and the ex ante utility of the sender is approximately -0.146. The ex ante payoffs of the sender from the best private and the best public mediation rule are approximately -0.151 and -0.149, respectively.[26]

The optimal mechanism for the case described in Example 3 is not known. However the fact that the given monotonic equilibrium performs better than any private or public mediation rule suggests that the optimal mechanism must both take advantage of pooling the sender's truthtelling constraints across receivers, as well as transmit some of the information to the receivers in a private manner.

5.2. Unmediated communication protocols

In this section we discuss whether it is possible to implement optimal mediation rules by some communication schemes between the players without the use of mediator. We begin by noting that under some values of the biases, the optimal mediation rules can be implemented as equilibria of the private communication game described in Section 3.

Lemma 5. *Let b_1 and b_2 be of the same sign and either $|b_i| = \frac{1}{2}(N_i)^{-2}$ for some $N_i \in \mathbb{N}^*$ or $b_i = 0$ for $i = 1, 2$. The optimal mediation rule is outcome-equivalent to the most informative equilibrium of the private communication game.*

The result follows from the fact that the optimal mediation rule in Proposition 6 becomes deterministic for such values of the biases. Note, however, that when the optimal mediation rule in Proposition 6 is stochastic, then it is not possible to implement it as an equilibrium of any communication protocol from Sections 3 and 4. Since each receiver's best response is always a singleton, the randomization must be performed by the sender, but there can be at most a single type of the sender that is indifferent between any two given actions of receiver i.[27]

Let us now turn to more complicated protocols with active participation of the receivers. A general model of such protocols, or *long cheap talk*, was introduced by Aumann and Hart (2003).

Proposition 7. *Let $(b_1, b_2) \in (-\frac{1}{2}, 0]^2 \cup [0, \frac{1}{2})^2$. The optimal mediation rule from Proposition* 6 *can be achieved with long cheap talk.*

In contrast, in the game with one receiver, there exists an optimal mediation rule that is implementable with long cheap talk only if the absolute value of the bias is less than $\frac{1}{8}$, not $\frac{1}{2}$.[28] Therefore, the presence of the second receiver makes it possible to extend the range of biases for which there exists an optimal unmediated communication protocol. The reason for this is that it is possible to use each receiver to play the role of a correlation device (as in Forges, 1988) in the communication between the sender and the other receiver. Moreover, this can be done in such a way that a receiver does not learn anything about the state of the world while facilitating communication between the other players.

The construction discussed above does not work when the optimal mediation rule does not belong to the class of private rules. Though there exists a literature which studies the problem of implementing mediated outcomes of Bayesian games as correlated equilibria of long cheap talk protocols (see, for example, Forges, 1990), to the best of our knowledge it deals with the games with finite action and type spaces. We think that the results from this literature will carry through in our model for mediation rules such that, ex ante, positive probability is placed only on a finite number of lotteries, each with a finite support (such as the optimal mediation rules described in part (i) of Proposition 6).

6. Conclusion

We have analyzed communication via various protocols between the sender and two receivers in a natural extension of the framework of Crawford and Sobel (1982). Throughout the paper we have assumed that the payoffs of each receiver are independent of the action of the other receiver, and that the sender's payoff is separable in the actions of the two receivers. Hence the only thing that links two otherwise 'separable' problems of information transmission (one between the sender and receiver 1, and the other between the sender and receiver 2) is the state of the world which is privately known by the sender.

We have identified several means by which the incentives for information transmission can be affected by simultaneous communication with both receivers in this environment. In Section 3 we have shown that using public announcements has a commitment value, because it reduces the number of deviations available to the sender. In Section 4 we have shown that under the combined communication scenario it may be beneficial to reveal less information at the private communication stage in order to improve incentives for information revelation at the public communication stage. In Section 5 we have

[26] See Appendix A for calculations.

[27] Mitusch and Strausz (2005) emphasize that the advantage of using mediator comes from the possibility to implement stochastic outcomes without imposing constraints that the sender must be indifferent between the receivers' actions.

[28] See Theorem 1 in Krishna and Morgan (2004) and Theorem 3 in Goltsman et al. (2009) for details.

shown that it may be beneficial to use noisy communication channels, which can be replicated using multi-stage plain conversation protocols between the players.

In the environments where our 'separability' assumptions do not hold, one can expect the following additional effects to come into play. Relaxing payoff independence between the receivers will bring in an element of strategic interaction at the action choice stage. The sender will have to take into account that her message announcements induce a particular information structure into the game to be played between the receivers (which might have multiple equilibria). Relaxing the separability of the sender's payoff in the actions of the receivers will complicate the receivers' inference problem when the messages are private.

One interesting topic for future research is to extend the model to more than two audiences. While we expect that the comparison between the outcomes of games with public communication with all audiences and the games with private communication with each audience to be similar the case of two audiences, there are also many intermediate communication arrangements like organizing the audiences into different groups, so that the messages of the sender are commonly observed by the members of the same group. Optimal design of such groups depending on the preferences of the receivers is an interesting question, and the results of the current paper can serve as a building block in providing the optimal way of organizing communication within any two-member group.

There are other topics for future research that we find interesting. First, one can analyze communication with multiple receivers in an environment where messages are costly. To the best of our knowledge, the existing models of signalling with multiple audiences do not allow for the possibility of private or combined communication. Another avenue for future research is studying communication through other realistic communication channels (for example, using private messages which become publicly known with some probability, or using the 'blind carbon copy' option for private communication). One can also extend our model to allow for communication between the receivers, or for an endogenous choice between communication modes by the sender.

Appendix A

A.1. Proofs of Section 3

Proof of Proposition 2. (i) After any equilibrium message m, receiver i solves

$$\min_{a_i} \int_{\theta} L\big(|a_i - \theta - b_i|\big)\,dF_m(\theta)$$

where F_m is the posterior distribution of θ following message m. The solution a_i clearly belongs to $[b_i, 1+b_i]$ and is unique by strict convexity of L. The first-order conditions are

$$\int_0^{a_i-b_i} L'(a_i-\theta-b_i)\,dF_m(\theta) = \int_{a_i-b_i}^{1} L'(\theta+b_i-a_i)\,dF_m(\theta)$$

Therefore the actions of the receivers are related as follows: $a_2 - b_2 = a_1 - b_1$. Thus we can rewrite the utility of the sender as depending on a_1 only:

$$-\Lambda(a_1-\theta) = -l_1\big(|a_1-\theta|\big) - l_2\big(|a_1+b_2-b_1-\theta|\big) \tag{1}$$

Consider a CS game between a sender with utility function $-\Lambda$ and a receiver with utility function $-L$. Since Λ is convex, in every equilibrium of the public communication game, if a_1 is induced with positive probability in equilibrium by types θ and θ'', then a_1 must be an equilibrium action for every $\theta' \in (\theta, \theta'')$. Furthermore, by Lemma 1 in Crawford and Sobel (1982) the set of equilibrium actions is finite if the most preferred actions of the sender and the receiver remain distinct, i.e. $|a_1^{\Lambda}(\theta) - (\theta + b_1)| > \varepsilon$ for some $\varepsilon > 0$ for every θ, where $a_1^{\Lambda}(\theta)$ is the minimizer of Λ. If $b_1 \neq b_2$ then $a_1^{\Lambda}(\theta)$ satisfies $l_1'(|a_1^{\Lambda}(\theta) - \theta|) = l_2'(|a_1^{\Lambda}(\theta) + b_2 - b_1 - \theta|)$, and the above condition can be rewritten as $l_1'(|b_1|) \neq l_2'(|b_2|)$. If $b_1 = b_2 = b$ then $a_1^{\Lambda}(\theta) = \theta$, and thus the set of equilibrium actions is finite if $b \neq 0$.

(ii) Note that the condition $l_1'(|b_1|) \neq l_2'(|b_2|)$ is equivalent to $b_1 \neq -b_2$ if $l_i \equiv l$, $i = 1, 2$. Thus every equilibrium of the public communication game is of interval partitional form.

Let (a_1, a_2) be a pair of actions chosen by type θ in a given equilibrium of the public communication game, and (a_1', a_2') be a pair of actions that is chosen by some other type. Then, using (1), it must be the case that $\Lambda(a_1 - \theta) \leqslant \Lambda(a_1' - \theta)$. Since Λ is convex and symmetric around its minimum, $-(\frac{b_2-b_1}{2})$, this condition can be written as

$$\left|a_1 + \frac{b_2-b_1}{2} - \theta\right| \leqslant \left|a_1' + \frac{b_2-b_1}{2} - \theta\right| \tag{2}$$

Now consider the CS model with a receiver with the loss function $L(|a_i - \theta - \frac{b_1+b_2}{2}|)$. Let us check that this model has an equilibrium characterized by the same cutoffs. Indeed, if the sender follows the same strategy as in the original equilibrium, the receiver after given message will take the action equal to $(a_1 - b_1) + \frac{b_1+b_2}{2} = a_1 + \frac{b_2-b_1}{2}$, where a_1 is the action of receiver 1 in the original equilibrium. By (2), the sender's original strategy is the best response to these actions of the receivers. □

Lemma 6. *Let F be uniform and $l_i(x) = L(x) = x^2$, $i = 1, 2$. The most informative public communication equilibrium is of size $N(\bar{b}) = \lceil -\frac{1}{2} + \frac{1}{2}\sqrt{1 + \frac{2}{|\bar{b}|}}\rceil$, where $\bar{b} = \frac{b_1+b_2}{2}$; the ex ante utilities of each receiver and the sender in this equilibrium are*

$$-\frac{1}{12}\big(N(\bar{b})\big)^{-2} - \frac{1}{3}(\bar{b})^2\big(\big(N(\bar{b})\big)^2 - 1\big)$$

and

$$-\frac{1}{6}\big(N(\bar{b})\big)^{-2} - \frac{2}{3}(\bar{b})^2\big(\big(N(\bar{b})\big)^2 - 1\big) - (b_1)^2 - (b_2)^2$$

respectively.

Proof. By Proposition 2, the equilibrium cutoff types are the same as in the CS game with a receiver with bias $\bar{b}$. Using the results of Crawford and Sobel (1982), the most informative equilibrium is of size $N(\bar{b}) = \lceil -\frac{1}{2} + \frac{1}{2}\sqrt{1 + \frac{2}{|\bar{b}|}}\rceil$; the ex ante utility of receiver i in an equilibrium of size N:

$$\begin{aligned}-E\big((a_i - \theta - b_i)^2\big) &= -\sum_{k=1}^{N}\int_{\theta_{k-1}}^{\theta_k}\left(\frac{1}{2}(\theta_{k-1} + \theta_k) - \theta\right)^2 d\theta = -\frac{1}{12}\sum_{k=1}^{N}(\theta_k - \theta_{k-1})^3 \\ &= -\frac{1}{12}\sum_{k=1}^{N}\left(\frac{1}{N} + 2\bar{b}(1 + N - 2k)\right)^3 = -\frac{1}{12}N^{-2} - \frac{1}{3}(\bar{b})^2\big(N^2 - 1\big)\end{aligned}$$

The ex ante utility of the sender is

$$\begin{aligned}&-E\big((a_1 - \theta)^2\big) - E\big((a_2 - \theta)^2\big) \\ &\quad = -E\big((a_1 - \theta - b_1)^2\big) - (b_1)^2 - E\big((a_2 - \theta - b_2)^2\big) - (b_2)^2 \\ &\quad = -\frac{1}{6}N^{-2} - \frac{2}{3}(\bar{b})^2\big(N^2 - 1\big) - (b_1)^2 - (b_2)^2 \qquad \square\end{aligned}$$

Proof of Proposition 3. Define $f(b) = -\frac{1}{12}(N(b))^{-2} - \frac{1}{3}b^2(N(b)^2 - 1)$, where $N(b)$ is the number of distinct actions in the most informative equilibrium of the CS game where the receiver's bias is b. It is straightforward to show that f is a continuous function, and that it is decreasing in the absolute value of b. The ex ante utility of the sender in the most informative equilibrium of a CS game where the receiver's bias is b equals to $f(b) - b^2$ (see Section 4 of Crawford and Sobel, 1982). By Proposition 1 and Lemma 6, private communication is better than public if and only if

$$\Delta(b_1, b_2) := f(b_1) + f(b_2) - 2f\left(\frac{b_1 + b_2}{2}\right) \geqslant 0$$

(i) Consider the region where $b_1 \in (-\frac{1}{4}, \frac{1}{4})$ and $b_2 \geqslant \frac{1}{4}$ (the other cases are symmetric). This implies $\frac{b_1+b_2}{2} \in [0, \frac{1}{4})$ and $b_2 \in [\frac{1}{4}, \frac{1}{2} - b_1)$.

First note that $N(b_2) = 1$, and thus $f(b_2) = -\frac{1}{12}$. Since f is decreasing on nonnegative domain, we have that $\Delta(b_1, b_2)$ is strictly increasing in b_2. Note that $\Delta(b_1, b_2)$ is a continuous function, since f is continuous. Thus to prove the result it is enough to show that $\Delta(b_1, \frac{1}{4}) \leqslant 0 \leqslant \Delta(b_1, \frac{1}{2} - b_1)$ for every $b_1 \in (-\frac{1}{4}, \frac{1}{4})$.

Let $b_2 = \frac{1}{2} - b_1$. Then $\frac{b_1+b_2}{2} = \frac{1}{4}$, and thus $f(\frac{b_1+b_2}{2}) = -\frac{1}{12}$. Then for every $b_1 \in (-\frac{1}{4}, \frac{1}{4})$ we have

$$\Delta\left(b_1, \frac{1}{2} - b_1\right) = f(b_1) - \frac{1}{12} + \frac{1}{6} = f(b_1) + \frac{1}{12} \geqslant 0$$

where the inequality is due to the fact that the range for f is $[-\frac{1}{12}, 0]$.

Let $b_2 = \frac{1}{4}$. Then $\frac{b_1+b_2}{2} \in (0, \frac{1}{4})$, which implies that $N(\frac{b_1+b_2}{2}) \geqslant 2$. This implies that $f(\frac{b_1+b_2}{2}) \geqslant -\frac{1}{48} - (\frac{b_1+b_2}{2})^2$, and thus

$$\begin{aligned}\Delta\left(b_1, \frac{1}{4}\right) &= f(b_1) - \frac{1}{12} - 2f\left(\frac{b_1}{2} + \frac{1}{8}\right) \leqslant f(b_1) - \frac{1}{12} - 2\left(-\frac{1}{48} - \left(\frac{b_1}{2} + \frac{1}{8}\right)^2\right) \\ &= -\frac{1}{3}\left(N(b_1)^2 - \frac{5}{2}\right)(b_1)^2 + \frac{1}{4}b_1 - \frac{1}{12}\big(N(b_1)\big)^{-2} - \frac{1}{96}\end{aligned}$$

If $b_1 \in (-\frac{1}{4}, 0]$ then $N(b_1) \geqslant 2$, and thus the above expression is negative. If $b_1 \in (0, \frac{1}{12}]$ then $N(b_1) \geqslant 3$, and we have

$$\Delta\left(b_1, \frac{1}{4}\right) \leqslant -\frac{13}{6}(b_1)^2 + \frac{1}{4}b_1 - \frac{1}{96} < 0$$

where the last inequality is true because such polynomial has no real roots. Finally, if $b_1 \in (\frac{1}{12}, \frac{1}{4})$ then $N(b_1) = 2$, which implies

$$\Delta\left(b_1, \frac{1}{4}\right) \leqslant -\frac{1}{2}\left(b_1 - \frac{1}{4}\right)^2 \leqslant 0$$

(ii) Notice that we have $N(b_1) = N(b_2) = N$, and

$$|\bar{b}| \leqslant \frac{1}{2}|b_1| + \frac{1}{2}|b_2| \leqslant \frac{1}{2N(N-1)}$$

where the first inequality is by the triangle inequality. Thus there exists a public equilibrium of size N. Hence

$$\Delta(b_1, b_2) = -\frac{1}{6}\left(N^2 - 1\right)(b_1 - b_2)^2 < 0$$

unless $b_1 = b_2$. $\square$

A.2. Proofs of Section 4

Proof of Lemma 2. Suppose $\theta' > \theta$. From incentive compatibility for the sender:

$$\begin{aligned}
&-l_1\left(\left|a_1(\theta) - \theta\right|\right) - l_2\left(\left|a_2(\theta) - \theta\right|\right) \geqslant -l_1\left(\left|a_1(\theta') - \theta\right|\right) - l_2\left(\left|a_2(\theta') - \theta\right|\right) \\
&-l_1\left(\left|a_1(\theta') - \theta'\right|\right) - l_2\left(\left|a_2(\theta') - \theta'\right|\right) \geqslant -l_1\left(\left|a_1(\theta) - \theta'\right|\right) - l_2\left(\left|a_2(\theta) - \theta'\right|\right)
\end{aligned}$$

Add up and rearrange to get

$$\begin{aligned}
0 &\leqslant \left(l_1\left(\left|a_1(\theta') - \theta\right|\right) - l_1\left(\left|a_1(\theta) - \theta\right|\right)\right) - \left(l_1\left(\left|a_1(\theta') - \theta'\right|\right) - l_1\left(\left|a_1(\theta) - \theta'\right|\right)\right) \\
&\quad + \left(l_2\left(\left|a_2(\theta') - \theta\right|\right) - l_2\left(\left|a_2(\theta) - \theta\right|\right)\right) - \left(l_2\left(\left|a_2(\theta') - \theta'\right|\right) - l_2\left(\left|a_2(\theta) - \theta'\right|\right)\right) \\
&= \int_{\theta}^{\theta'} \left(\int_{a_1(\theta)}^{a_1(\theta')} l_1''\left(|\tilde{a} - \tilde{\theta}|\right) d\tilde{a} + \int_{a_2(\theta)}^{a_2(\theta')} l_2''\left(|\tilde{a} - \tilde{\theta}|\right) d\tilde{a} \right) d\tilde{\theta}
\end{aligned}$$

Note that $l_i''(|x|) > 0$ for every x and $i = 1, 2$. Hence, we cannot have both $a_1(\theta') < a_1(\theta)$ and $a_2(\theta') < a_2(\theta)$. $\square$

Proof of Lemma 3. Suppose the equilibrium is not partitional, i.e. $\exists i \in \{1, 2\}$, $\theta, \theta' \in [0, 1]$, $\theta'' \in (\theta, \theta')$ such that $a_i(\theta) = a_i(\theta') = a_i$, $a_i(\theta'') \neq a_i$. Suppose $a_i(\theta'') < a_i$ (the opposite case is treated similarly). Then $a_i(\theta'') < a_i(\theta) = a_i(\theta')$, which is a contradiction to the equilibrium being monotonic. $\square$

Proof of Lemma 4. (i) There must exist a public message such that there is further communication with receiver i. If following this public message receiver i takes an infinite number of different actions in equilibrium, then $b_i = 0 \in [-\frac{1}{4}, \frac{1}{4}]$. Suppose receiver i takes a finite number of actions in equilibrium, say, action $\underline{a}_i$ is taken when $\theta \in (x, y)$ and $\bar{a}_i$ is taken when $\theta \in (y, z)$. Because F is uniform, $\underline{a}_i = \frac{1}{2}x + \frac{1}{2}y + b_i$, $\bar{a}_i = \frac{1}{2}y + \frac{1}{2}z + b_i$.

Since the utility of the sender is separable in the actions of the two receivers, she can optimize over private messages to be sent to receiver i independently of which messages she plans to send to receiver j. Thus type y is indifferent between $\underline{a}_i$ and $\bar{a}_i$ if $l_i(|\underline{a}_i - y|) = l_i(|\bar{a}_i - y|)$, which implies that

$$y = \frac{1}{2}x + \frac{1}{2}z + 2b_i$$

Note that we need $z - y \geqslant 0$, which implies $\frac{1}{4}(z - x) \geqslant b_i$. Hence, $\frac{1}{4} \geqslant b_i$. Also note that we need $y - x \geqslant 0$, which implies $b_i \geqslant -\frac{1}{4}(z - x)$. Hence, $b_i \geqslant -\frac{1}{4}$.

(ii) If communication with both receivers is informative, then

$$\left|\frac{1}{2}b_1 + \frac{1}{2}b_2\right| \leqslant \frac{1}{2}|b_1| + \frac{1}{2}|b_2| \leqslant \frac{1}{4}$$

where the second inequality follows from the proof of (i).

Assume there is informative communication at the public stage, as well as informative private communication only with receiver 1 (the argument for receiver 2 is similar). Then there exist a, x, y, z, c such that $0 \leqslant a \leqslant x \leqslant y \leqslant z \leqslant c \leqslant 1$ and receiver 1 gets informed whether $\theta \in (x, y)$ or $\theta \in (y, z)$, whether receiver 2 gets informed whether $\theta \in (a, y)$ or $\theta \in (y, c)$. This implies

$$\big(a_1(\theta), a_2(\theta)\big) = \begin{cases} \left(\frac{1}{2}x + \frac{1}{2}y + b_1, \frac{1}{2}a + \frac{1}{2}y + b_2\right) & \text{if } \theta \in (x, y) \\ \left(\frac{1}{2}y + \frac{1}{2}z + b_1, \frac{1}{2}y + \frac{1}{2}c + b_2\right) & \text{if } \theta \in (y, z) \end{cases}$$

The indifference condition for type y implies

$$y = \frac{(z-x)}{(z-x)+(c-a)}\left(\frac{1}{4}x + \frac{1}{2}y + \frac{1}{4}z + b_1\right) + \frac{(c-a)}{(z-x)+(c-a)}\left(\frac{1}{4}a + \frac{1}{2}y + \frac{1}{4}c + b_2\right)$$

Denote $\lambda := \frac{(z-x)}{(z-x)+(c-a)}$ and $B := \lambda b_1 + (1-\lambda)b_2$. Then

$$y = \lambda\left(\frac{1}{4}x + \frac{1}{2}y + \frac{1}{4}z\right) + (1-\lambda)\left(\frac{1}{4}a + \frac{1}{2}y + \frac{1}{4}c\right) + B \in \left[\frac{3}{4}y + B, \frac{3}{4}y + \frac{1}{4} + B\right]$$

which implies $B \in [\frac{1}{4}y - \frac{1}{4}, \frac{1}{4}y] \subseteq [-\frac{1}{4}, \frac{1}{4}]$. Thus

$$\left|\frac{1}{2}b_1 + \frac{1}{2}b_2\right| = \frac{1}{2}\left|\frac{1-2\lambda}{1-\lambda}b_1 + \frac{1}{1-\lambda}B\right| \leqslant \frac{1}{2}\left(\frac{1-2\lambda}{1-\lambda}|b_1| + \frac{1}{1-\lambda}|B|\right) \leqslant \frac{1}{2}\left(\frac{1-2\lambda}{1-\lambda}\frac{1}{4} + \frac{1}{1-\lambda}\frac{1}{4}\right) = \frac{1}{4}$$

where the equality follows from the definition of B, the first inequality follows from the triangle inequality and the fact that $\lambda \leqslant \frac{1}{2}$, the second inequality uses the facts that $|b_1| \leqslant \frac{1}{4}$ (follows from part (i)) and $|B| \leqslant \frac{1}{4}$ (derived above). □

To prove Proposition 4 we construct the following monotonic equilibrium of the combined communication game. Consider a sequence $0 = \theta_0 < t < \theta_1 < \cdots < \theta_N = 1$. We say that $(\theta_0, t, \theta_1, \ldots, \theta_N)$ constitute a *type-I equilibrium* of size N if there exists an equilibrium of the following form:

1) At the public stage, the sender announces an element of a partition $[\theta_k, \theta_{k+1}], k = 0, \ldots, N-1$.
2) At the private stage, if $\theta \in [0, \theta_1]$, the sender announces to receiver 1 whether $\theta \in [0, t]$ or $\theta \in [t, \theta_1]$.

Lemma 7. *Type-I equilibrium of size N takes the following form*:

$$\big(a_1(\theta), a_2(\theta)\big) = \begin{cases} \left(\frac{1}{2}t + b_1, \frac{1}{2}\theta_1 + b_2\right) & \textit{if } \theta \in (0, t) \\ \left(\frac{1}{2}(t+\theta_1) + b_1, \frac{1}{2}\theta_1 + b_2\right) & \textit{if } \theta \in (t, \theta_1) \\ \left(\frac{1}{2}(\theta_{k-1}+\theta_k) + b_1, \frac{1}{2}(\theta_{k-1}+\theta_k) + b_2\right) & \textit{if } \theta \in (\theta_k, \theta_{k+1}) \end{cases}$$

for $k = 2, \ldots, N$, *where* $\theta_k = \frac{k-1}{N-1} + (N-k)(k-1)(b_1+b_2) + \frac{N-k}{N-1}\theta_1$ *for* $k = 2, \ldots, N$, $t = \frac{1}{2}\theta_1 + 2b_1$, *and* θ_1 *solves*

$$(\theta_1)^2 + \left(\frac{2}{\frac{5}{8}(N-1)^2 - 1}\right)\theta_1 - \left(\frac{(1+N(N-1)(b_1+b_2))(1+(N-1)(N-2)(b_1+b_2)) - 6(N-1)^2(b_1)^2}{\frac{5}{8}(N-1)^2 - 1}\right) = 0$$

The proof is by straightforward calculation.

Lemma 8. *Let* $b_1 = 0$ *and* $b_2 \in (-\frac{1}{2}, 0)$. *If there exists a public communication equilibrium of size N then there exists a type-I equilibrium of size N.*

Proof. Note that to show that type-I equilibrium of size N exists it is enough to demonstrate that $\theta_{k+1} - \theta_k \geqslant 0$ for $k = 1, \ldots, N-1$ and $\theta_1 \geqslant 0$.

Let $N = 2$. By Lemma 7: $\theta_1 = \frac{8}{3}(1 - \sqrt{1 - \frac{3}{8}(1+2b_2)}) \in (0, 1)$ if $b_2 \in (-\frac{1}{2}, 0)$.

Let $N > 2$. Note that by Lemma 7:

$$\theta_{k+1} - \theta_k = \frac{1}{N-1}(1-\theta_1) + (N-2k)b_2 \quad \text{for } k = 1, \ldots, N-1$$

Since $b_2 < 0$ it is enough to show that $\theta_2 - \theta_1 \geqslant 0$, or $1 + (N-1)(N-2)b_2 \geqslant \theta_1$, and also $\theta_1 > 0$.

Note that by Lemma 6 a necessary condition for a public communication equilibrium of size N to exist is $1 + N(N-1)b_2 > 0$, which implies $1 + (N-1)(N-2)b_2 > 0$. Evaluating the quadratic polynomial given in Lemma 7 at $1 + (N-1)(N-2)b_2$ gives

$$\big(1+(N-1)(N-2)b_2\big)^2+\left(\frac{1-N(N-1)b_2}{\frac{5}{8}(N-1)^2-1}\right)\big(1+(N-1)(N-2)b_2\big)>0$$

Evaluating the quadratic polynomial given in Lemma 7 at 0 gives

$$-\frac{(1+N(N-1)b_2)(1+(N-1)(N-2)b_2)}{\frac{5}{8}(N-1)^2-1}<0$$

Hence, $1+(N-1)(N-2)b_2>\theta_1>0$. □

Lemma 9. *Let $b_1=0$ and $b_2\in(-\frac{1}{2},0)$. Assume there exist a public communication equilibrium of size N and a type-I equilibrium of size N. Then the latter yields a higher payoff to the sender than the former.*

Proof. By Lemma 12 it is enough to show that the equilibrium payoff of type $\theta=1$ in the type-I equilibrium of size N is higher than in the public communication equilibrium of size N. By Proposition 2 the payoff of type $\theta=1$ in the public communication equilibrium of size N is

$$-\left(\left(\frac{1}{2}\theta^*_{N-1}+\frac{1}{2}\right)-1\right)^2-\left(\left(\frac{1}{2}\theta^*_{N-1}+\frac{1}{2}+b_2\right)-1\right)^2$$

By Lemma 7 the payoff of type $\theta=1$ in the type-I equilibrium of size N is

$$-\left(\left(\frac{1}{2}\theta_{N-1}+\frac{1}{2}\right)-1\right)^2-\left(\left(\frac{1}{2}\theta_{N-1}+\frac{1}{2}+b_2\right)-1\right)^2$$

Since $\theta^*_{N-1}<1$, $\theta_{N-1}<1$ and $b_2<0$ it is enough to show that $\theta^*_{N-1}<\theta_{N-1}$. Using the formulas for θ^*_{N-1} and θ_{N-1} we get

$$\begin{aligned}\theta^*_{N-1}-\theta_{N-1}&=\left(\frac{N-1}{N}+(N-1)b_2\right)-\left(\frac{N-2}{N-1}+(N-2)b_2+\frac{1}{N-1}\theta_1\right)\\&=\frac{1}{N-1}\left(\frac{1}{N}+(N-1)b_2-\theta_1\right)\end{aligned}$$

Hence it is enough to show that $\frac{1}{N}+(N-1)b_2<\theta_1$.

Let $N=2$. Here we need to show that $\frac{1}{2}+b_2<\frac{8}{3}(1-\sqrt{1-\frac{3}{8}(1+2b_2)})$, or $0<(\frac{3}{8})^2(\frac{1}{2}+b_2)^2$, which is true since $b_2\in(-\frac{1}{2},0)$.

Let $N>2$. Evaluating the quadratic polynomial given in Lemma 7 at $\frac{1}{N}+(N-1)b_2$ gives

$$\begin{aligned}&\left(\frac{1}{N}+(N-1)b_2\right)^2-\frac{N(N-2)}{\frac{5}{8}(N-1)^2-1}\left(\frac{1}{N}+(N-1)b_2\right)^2\\&\quad=-\frac{3}{8}\left(\frac{(N-1)^2}{\frac{5}{8}(N-1)^2-1}\right)\left(\frac{1}{N}+(N-1)b_2\right)^2<0\end{aligned}$$

Hence, $\frac{1}{N}+(N-1)b_2<\theta_1$. □

Proof of Proposition 4. Let $b_2\in(-\frac{1}{2},0)$. By Lemmas 8 and 9 when $b_1=0$ there exists a type-I equilibrium which yields a strictly higher payoff than the best public communication equilibrium.

By Lemma 7 the cutoff values defining the type-I equilibrium are continuous functions of b_1. Hence for every $b_2\in(-\frac{1}{2},0)$ a type-I equilibrium exists and yields a strictly higher payoff than the best public communication equilibrium whenever b_1 is close enough to 0.

It is straightforward to prove an analogous statement for $b_2\in(0,\frac{1}{2})$ using a *type-Ib equilibrium* of the following kind. Consider a sequence $0=\theta_0<\theta_1<\cdots<\theta_{N-1}<t<\theta_N=1$.

1) At the public stage, the sender announces an element of a partition $[\theta_k,\theta_{k+1}]$, $k=0,\dots,N-1$.

2) At the private stage, if $\theta\in[\theta_{N-1},1]$, the sender announces to receiver 1 whether $\theta\in[\theta_{N-1},t]$ or $\theta\in[t,1]$.[29] □

To prove Proposition 5 we construct a nonmonotonic equilibrium of the combined communication game similar to the one in Example 2. Consider a pair (x,z) such that $0<x<z<1$. We say that (x,z) constitute a *type-II equilibrium* if there exists an equilibrium of the following form:

[29] The details are available upon request.

1) At the public stage, the sender announces whether $\theta \in [0,x) \cup [z,1]$ or $\theta \in [x,z)$.
2) At the private stage, if $\theta \in [0,x) \cup [z,1]$, the sender announces to receiver 1 whether $\theta \in [0,x)$ or $\theta \in [z,1]$.

Lemma 10. *Type-II equilibrium takes the following form*:

$$\big(a_1(\theta), a_2(\theta)\big) = \begin{cases} \big(\frac{1}{2}x + b_1, \big(\frac{x}{x+1-z}\big(\frac{1}{2}x\big) + \frac{1-z}{x+1-z}\big(\frac{1}{2}z+\frac{1}{2}\big)\big) + b_2\big) & \text{if } \theta \in [0,x) \\ \big(\big(\frac{1}{2}x+\frac{1}{2}z\big) + b_1, \big(\frac{1}{2}x+\frac{1}{2}z\big) + b_2\big) & \text{if } \theta \in [x,z) \\ \big(\big(\frac{1}{2}z+\frac{1}{2}\big) + b_1, \big(\frac{x}{x+1-z}\big(\frac{1}{2}x\big) + \frac{1-z}{x+1-z}\big(\frac{1}{2}z+\frac{1}{2}\big)\big) + b_2\big) & \text{if } \theta \in [z,1] \end{cases}$$

where $x = (1-d)(\frac{1}{2} + \frac{2(1+d)b_1}{(d^2-6d+1)})$, $z = x + d$, *and* d *solves*:

$$(1+d)\left((1-3d) - 16b_1^2\frac{(3-d)(3d^2-6d-1)}{(d^2-6d+1)^2} - 64b_1b_2\frac{1}{(d^2-6d+1)}\right) = 0$$

The proof is by straightforward calculation.

Proof of Proposition 5. First we show that there exists a type-II equilibrium when $b_1 = 0$ and $b_2 \in \mathbb{R}$.

When $b_1 = 0$ the equilibrium conditions (given in Lemma 10) simplify to $x = \frac{1}{2} - \frac{1}{2}d$, $z = \frac{1}{2} + \frac{1}{2}d$, where d solves $(1+d)(1-3d) = 0$. The only feasible solution is $d = \frac{1}{3}$, which gives $(x,z) = (\frac{1}{3}, \frac{2}{3})$. Hence,

$$\big(a_1(\theta), a_2(\theta)\big) = \begin{cases} \big(\frac{1}{6}, \frac{1}{2} + b_2\big) & \text{if } \theta \in \big[0, \frac{1}{3}\big) \\ \big(\frac{1}{2}, \frac{1}{2} + b_2\big) & \text{if } \theta \in \big[\frac{1}{3}, \frac{2}{3}\big) \\ \big(\frac{5}{6}, \frac{1}{2} + b_2\big) & \text{if } \theta \in \big[\frac{2}{3}, 1\big] \end{cases}$$

This equilibrium is outcome-equivalent to the private communication equilibrium where the first receiver receives 3 informative messages, and the second receiver receives no informative messages.

The left-hand side of the equation which determines d (given in Lemma 10) is continuously differentiable in (d, b_1) in an open neighborhood of $(\frac{1}{3}, 0)$ and has a nonzero partial derivative in d at $(\frac{1}{3}, 0)$. Hence, for b_1 close to zero there exists a feasible type-II equilibrium. Moreover, whenever $b_1 \neq 0$, this equilibrium involves informative communication with the second receiver:

$$\left(\frac{x}{x+1-z}\left(\frac{1}{2}x\right) + \frac{1-z}{x+1-z}\left(\frac{1}{2}z+\frac{1}{2}\right)\right) - \left(\frac{1}{2}x + \frac{1}{2}z\right) = \frac{1}{2} - \frac{1}{1-d}x = -\frac{2(1+d)b_1}{d^2-6d+1} \neq 0 \qquad \square$$

A.3. Proofs of Section 5

Let $\alpha_i(\hat{\theta}) = \int_{\mathbb{R}} a_i\, dp\,(a_1, a_2|\hat{\theta})$ and $\sigma_i^2(\hat{\theta}) = \int_{\mathbb{R}} (a_i - a_i(\hat{\theta}))^2\, dp\,(a_1, a_2|\hat{\theta})$ be the conditional expectation and the variance of a_i given a message $\hat{\theta}$. Then an expected payoff of the sender of type θ who reported a message $\hat{\theta}$ in the mediation rule p is

$$\begin{aligned} U(\theta, \hat{\theta}) &:= \int_{\mathbb{R}^2} \big(-(a_1-\theta)^2 - (a_2-\theta)^2\big)\, dp\,(a_1, a_2|\hat{\theta}) \\ &= -\big(\alpha_1(\hat{\theta}) - \theta\big)^2 - \big(\alpha_2(\hat{\theta}) - \theta\big)^2 - \sigma_1^2(\hat{\theta}) - \sigma_2^2(\hat{\theta}) \end{aligned}$$

The truthtelling conditions for the sender (IC-S) can be written as follows:

$$U(\theta) := U(\theta, \theta) \geqslant U(\theta, \hat{\theta}), \quad \forall \theta, \hat{\theta} \in \Theta$$

Before proving Proposition 6, we can prove the following two lemmas using the techniques in Goltsman et al. (2009).

Lemma 11. *A mediation rule* $\{\alpha_1(\theta), \alpha_2(\theta), \sigma_1^2(\theta), \sigma_2^2(\theta)\}_{\theta \in \Theta}$ *is incentive compatible for the sender if and only if*

(i) $\alpha_1(\theta) + \alpha_2(\theta)$ *is non-decreasing*;
(ii) $-\sigma_1^2(\theta) - \sigma_2^2(\theta) = U(\theta) + (\alpha_1(\theta) - \theta)^2 + (\alpha_2(\theta) - \theta)^2$, *and* $U(\theta) = U(0) + \int_0^\theta 2(\alpha_1(\tilde{\theta}) + \alpha_2(\tilde{\theta}) - 2\tilde{\theta})\, d\tilde{\theta}$.

Lemma 12. *Let* $U = E_\theta[U(\theta)]$ *be the ex ante payoff of the sender. In every incentive compatible mediation rule,*

$$U = \frac{1}{3}U(0) + \frac{1}{3}(b_1 + b_2) - \frac{2}{3}\big((b_1)^2 + (b_2)^2\big) = \frac{1}{3}U(1) - \frac{1}{3}(b_1 + b_2) - \frac{2}{3}\big((b_1)^2 + (b_2)^2\big)$$

Proof of Proposition 6. (i) Let $(b_1, b_2) \in (-\frac{1}{2}, 0)^2$. Constraints (IC-R) imply

$$a_{i,0} = \frac{\theta_{i,1}(\frac{1}{2}\theta_{i,1}) + \mu_i(1-\theta_{i,1})(\frac{1}{2}\theta_{i,1} + \frac{1}{2})}{\theta_{i,1} + \mu_i(1-\theta_{i,1})} + b_i$$

$$a_{i,k} = \frac{1}{2}(\theta_{i,k} + \theta_{i,k+1}) + b_i, \quad k = 1, \dots, N_i - 1$$

Constraints $(IC-S)$ can be shown to imply

$$\theta_{i,k} = \frac{1}{2}(a_{i,k-1} + a_{i,k}), \quad k = 1, \dots, N_i - 1$$

It is straightforward but tedious to show that the unique solution satisfying $a_{i,0} = 0$ is as follows:

$$\theta_{i,k} = 2|b_i|k^2 - \big(2|b_i|N_i^2 - 1\big)\frac{2k-1}{2N_i - 1}, \quad k = 1, \dots, N_i$$

$$a_{i,k} = |b_i|k - 2|b_i|k(N_i - k) + \frac{(2-|b_i|)k}{2N_i - 1}, \quad k = 0, \dots, N_i - 1$$

$$\mu_i = 1 - \frac{1-2|b_i|}{4(1-|b_i|)}\left(\frac{1}{N_i - 1} - \frac{1}{N_i} - \frac{2-|b_i|}{|b_i|N_i - 1} + \frac{2-|b_i|}{|b_i|N_i - |b_i| + 1}\right)$$

This mediation rule has a property $U(0) = 0$, and thus, by Lemma 12, it yields the highest possible ex ante utility of the sender.

The argument for the cases $(b_1, b_2) \in (0, \frac{1}{2})^2$ and (ii)–(iii) is similar.[30]

(iv) Let $b_1 = b_2 = b \in [\frac{1}{2}, +\infty)$ (the argument for the other case is symmetric). Suppose there exists a mediation rule $\{\alpha_1(\theta), \alpha_2(\theta), \sigma_1^2(\theta), \sigma_2^2(\theta)\}_{\theta\in\Theta}$ which gives the sender a strictly higher ex ante utility than the constant rule. By Lemma 12 the payoff of the highest type of the sender from this mediation rule must be higher than from the constant rule, i.e.,

$$-\big(\alpha_1(1) - 1\big)^2 - \big(\alpha_2(1) - 1\big)^2 - \sigma_1^2(1) - \sigma_2^2(1) > -\left(\frac{1}{2} + b - 1\right)^2 - \left(\frac{1}{2} + b - 1\right)^2 \tag{3}$$

By part (i) of Lemma 11 and (IC-R) we must have

$$\alpha_1(1) + \alpha_2(1) \geqslant E\big(\alpha_1(\theta) + \alpha_2(\theta)\big) = E(a_1 + a_2) = 1 + 2b$$

This together with the Jensen inequality gives

$$-\big(\alpha_1(1) - 1\big)^2 - \big(\alpha_2(1) - 1\big)^2 \leqslant -2\left(\frac{\alpha_1(1) + \alpha_2(1)}{2} - 1\right)^2 \leqslant -2\left(\frac{1}{2} + b - 1\right)^2$$

Since $\sigma_1^2(1), \sigma_2^2(1) \geqslant 0$ we get a contradiction with Eq. (3). □

Calculations for Example 3. First we show that this communication arrangement constitutes an equilibrium. Given the sender's strategy, the outcome function is as follows:

$$\big(a_1(\theta), a_2(\theta)\big) = \begin{cases} \big(\frac{1}{2}t + \frac{1}{40}, \frac{1}{2}x - \frac{11}{40}\big) & \text{if } \theta \in [0, t) \\ \big(\frac{1}{2}(t+x) + \frac{1}{40}, \frac{1}{2}x - \frac{11}{40}\big) & \text{if } \theta \in [t, x) \\ \big(\frac{1}{2}(x+1) + \frac{1}{40}, \frac{1}{2}(x+1) - \frac{11}{40}\big) & \text{if } \theta \in [x, 1] \end{cases}$$

Let us check incentive compatibility for the sender. Type t is indifferent between a strategy of sending the public message '*Low*', with a consequent revelation to receiver 1 that her type is in $[0, t)$, and a strategy of sending the public message '*Low*', with a consequent revelation to receiver 1 that her type is in $[t, x)$ if

$$-\left(\frac{1}{2}t + \frac{1}{40} - t\right)^2 - \left(\frac{1}{2}x - \frac{11}{40} - t\right)^2 = -\left(\frac{1}{2}(t+x) + \frac{1}{40} - t\right)^2 - \left(\frac{1}{2}x - \frac{11}{40} - t\right)^2$$

Type x is indifferent between a strategy of sending the public message '*Low*', with a consequent revelation to receiver 1 that her type is in $[t, x)$, and a strategy of sending the public message '*High*' if

[30] A mechanism of the same form appears in Proposition 9 in Blume et al. (2007) and in Theorem 2 in Goltsman et al. (2009).

$$-\left(\frac{1}{2}(t+x)+\frac{1}{40}-x\right)^2-\left(\frac{1}{2}x-\frac{11}{40}-x\right)^2=-\left(\frac{1}{2}(x+1)+\frac{1}{40}-x\right)^2-\left(\frac{1}{2}(x+1)-\frac{11}{40}-x\right)^2$$

The solution is $x=\frac{8}{3}-\frac{1}{30}\sqrt{5209}\approx 0.261$ and $t=\frac{83}{60}-\frac{1}{60}\sqrt{5209}\approx 0.180$. This results in $U(1)\approx -0.534$. Using Lemma 12, the ex ante payoff of the sender is approximately -0.146.

By Proposition 6 in the best private mediator the sender of type $\theta=1$ gets action 1 from receiver 1, and a lottery between actions 0 and $\frac{3}{10}$ with probabilities $\frac{2}{17}$ and $\frac{15}{17}$ from receiver 2. This results in $U(1)=-0.55$. Using Lemma 12, the ex ante payoff of the sender from the best private mediator is approximately -0.151.

By Lemma 5 and Proposition 6, in the best public mediator the sender of type $\theta=1$ gets action $\frac{13}{20}$ from receiver 1, and action $\frac{7}{20}$ from receiver 2. This results in $U(1)=-0.545$. Using Lemma 12 the ex ante payoff of the sender from the best public mediator is approximately -0.149.

Proof of Lemma 5. Note that μ_i in the proof of Proposition 6 is equal to 0 when $|b_i|=\frac{1}{2}(N_i)^{-2}$ for some $N_i=1,2,\ldots.$ Thus the mediation rule in Proposition 6 is equivalent to the most informative equilibrium of the CS game between the sender and receiver i. $\square$

Proof of Proposition 7. We construct a communication protocol that implements the optimal mediation rule when $(b_1,b_2)\in(-\frac{1}{2},0)^2$ (the other cases are similar). Take N_j, μ_i, and $(\theta_{j,1},\ldots,\theta_{j,N_j-1})$ to be the same as in Proposition 6 for $j=1,2$.

The protocol has three stages:

1. For $i=1,2$, receiver i produces $2N_j-2$ independent draws from the uniform distribution on $[0,1]$ (call them $x_1,\ldots,x_{N_j-1},y_1,\ldots,y_{N_j-1}$) and a draw from the Bernoulli distribution over outcomes α,β with probabilities $\mu_i, 1-\mu_i$.

2. If outcome α has realized, receiver i informs the sender privately of $x=(x_1,\ldots,x_{N_j-1})$, otherwise he informs the sender of $y=(y_1,\ldots,y_{N_j-1})$, but in either case he does not tell the sender whether the reported vector is x or y. Receiver i also informs receiver j of x privately.

3. Sender sends private messages to both receivers.

It is straightforward to verify that this protocol has the following equilibrium. Receiver i randomizes according to the description above. Sender's message to receiver j is a uniform draw from $[0,1]$ if $\theta\in[0,\theta_{j,1})$ and the kth element of the vector reported to her by receiver i if $\theta\in[\theta_{j,k},\theta_{j,k+1})$, $k=1,\ldots,N_j-1$. After receiving a message from the sender, receiver j takes action $a_j=0$ if the number he gets from the sender does not coincide with any of the N_j-1 numbers reported to him by receiver i at stage 2; receiver j takes action $a_{j,k}=\frac{1}{2}(\theta_{j,k}+\theta_{j,k+1})+b_j$ if the number he gets from the sender coincides with the kth element of the vector reported to him by receiver i at stage 2. $\square$

References

Ainsworth, S., Sened, I., 1993. The role of lobbyists: entrepreneurs with two audiences. Amer. J. Polit. Sci. 37 (3), 834–866.
Aumann, R., Hart, S., 2003. Long cheap talk. Econometrica 71 (6), 1619–1660.
Austen-Smith, D., 1990. Information transmission in a debate. Amer. J. Polit. Sci. 34 (1), 124–152.
Austen-Smith, D., Feddersen, T., 2006. Deliberation, preference uncertainty and voting rules. Amer. Polit. Sci. Rev. 100 (2), 209–217.
Blume, A., Sobel, J., 1995. Communication-proof equilibria in cheap talk games. J. Econ. Theory 65, 359–382.
Blume, A., Board, O., Kawamura, K., 2007. Noisy talk. Theoretical Econ. 2 (4), 395–440.
Board, O., Dragu, T., 2008. Expert advice with multiple decision makers. Mimeo, University of Pittsburgh.
Caillaud, B., Tirole, J., 2007. Building consensus: how to persuade a group. Amer. Econ. Rev. 97 (5), 1877–1900.
Chen, Y., 2009. Communication with two-sided asymmetric information. Mimeo, Arizona State University.
Chen, Y., Kartik, N., Sobel, J., 2008. Selecting cheap-talk equilibria. Econometrica 76 (1), 117–136.
Crawford, V., Sobel, J., 1982. Strategic information transmission. Econometrica 50 (6), 1431–1451.
Farrell, J., 1993. Meaning and credibility in cheap talk games. Games Econ. Behav. 5, 514–531.
Farrell, J., Gibbons, R., 1989. Cheap talk with two audiences. Amer. Econ. Rev. 79 (5), 1214–1223.
Forges, F., 1988. Can sunspots replace a mediator? J. Math. Econ. 17 (4), 347–368.
Forges, F., 1990. Universal mechanisms. Econometrica 58 (6), 1341–1364.
Galeotti, A., Ghiglino, C., Squintani, F., 2009. Strategic information transmission in networks. Mimeo, University of Essex.
Gertner, R., Gibbons, R., Scharfstein, D., 1988. Simultaneous signalling to the capital and product markets. RAND J. Econ. 19 (2), 173–190.
Gigler, F., 1994. Self-enforcing voluntary disclosures. J. Acc. Res. 32 (2), 224–240.
Gilligan, T., Krehbiel, K., 1989. Asymmetric information and legislative rules with a heterogeneous committee. Amer. J. Polit. Sci. 33, 459–490.
Golosov, M., Skreta, V., Tsyvinski, A., Wilson, A., 2008. Dynamic strategic information transmission. Mimeo, NYU Stern School of Business.
Goltsman, M., Hörner, J., Pavlov, G., Squintani, F., 2009. Mediation, arbitration and negotiation. J. Econ. Theory 144, 1397–1420.
Grossman, G., Helpman, E., 2001. Special Interest Politics. MIT Press, Cambridge.
Hagenbach, J., Koessler, F., 2010. Strategic Communication Networks. Rev. Econ. Stud. 77, 1072–1099.
Johns, L., 2007. A servant of two masters: communication and the selection of international bureaucrats. Int. Organ. 61, 245–275.
Koessler, F., 2008. Lobbying with two audiences: public vs. private certification. Math. Soc. Sci. 55, 305–314.
Koessler, F., Martimort, D., 2008. Multidimensional communication mechanisms: cooperative and conflicting designs. Mimeo, Toulouse School of Economics.
Krishna, V., Morgan, J., 2001. A model of expertise. Quart. J. Econ. 116, 747–775.
Krishna, V., Morgan, J., 2004. The art of conversation: eliciting information from informed parties through multi-stage communication. J. Econ. Theory 117, 147–179.
Kurizaki, S., 2007. Efficient secrecy: public versus private threats in crisis diplomacy. Amer. Polit. Sci. Rev. 101 (3), 543–558.
Lehrer, E., Sorin, S., 1997. One-shot public mediated talk. Games Econ. Behav. 20, 131–148.
Levy, G., Razin, R., 2004. It takes two: an explanation of the democratic peace. J. Europ. Econ. Assoc. 2 (1), 1–29.

Li, H., Rosen, S., Suen, W., 2001. Conflicts and common interests in committees. Amer. Econ. Rev. 91 (5), 1478–1497.
Li, M., Madarász, K., 2008. When mandatory disclosure hurts: expert advice and conflicting interests. J. Econ. Theory 139, 47–74.
McAfee, R.P., Schwartz, M., 1994. Opportunism in multilateral vertical contracting: nondiscrimination, exclusivity, and uniformity. Amer. Econ. Rev. 84 (1), 210–230.
Mitusch, K., Strausz, R., 2005. Mediation in situations of conflict and limited commitment. J. Law, Econ., Organ. 21 (2), 467–500.
Myerson, R., 1982. Optimal coordination mechanisms in generalized principal-agent problems. J. Math. Econ. 10 (1), 67–81.
Myerson, R., 1991. Game Theory: Analysis of Conflict. Harvard University Press, Cambridge.
Newman, P., Sansing, R., 1993. Disclosure policies with multiple users. J. Acc. Res. 31 (1), 92–112.
Ozmen, D., 2004. Information transmission to multiple audiences. Mimeo, Yale University.
Segal, I., 1999. Contracting with externalities. Quart. J. Econ. 114 (2), 337–388.
Spiegel, Y., Spulber, D., 1997. Capital structure with countervailing incentives. RAND J. Econ. 28 (1), 1–24.
Stein, J., 1989. Cheap talk and the Fed: a theory of imprecise policy announcements. Amer. Econ. Rev. 79, 32–42.