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Mediation, arbitration and negotiation

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Abstract

We compare three common dispute resolution processes – negotiation, mediation, and arbitration – in the framework of Crawford and Sobel [V. Crawford, J. Sobel, Strategic information transmission, Econometrica 50 (6) (1982) 1431–1451]. Under negotiation, the two parties engage in (possibly arbitrarily long) face-to-face cheap talk. Under mediation, the parties communicate with a neutral third party who makes a non-binding recommendation. Under arbitration, the two parties commit to conform to the third party recommendation. We characterize and compare the optimal mediation and arbitration procedures. Both mediators and arbitrators should optimally filter information, but mediators should also add noise to it. We find that unmediated negotiation performs as well as mediation if and only if the degree of conflict between the parties is low.

JEL classification: C72; C78; D74; D78; D82

Keywords: Communication; Information; Mechanism design; Cheap talk; Long cheap talk; Arbitration; Mediation; Negotiation

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1. Introduction

The purpose of this paper is to characterize the optimal structure of alternative dispute resolution procedures and compare their performance within the classic model introduced by Crawford and Sobel [7]. In this model, a player with private information about a state of the world, drawn uniformly from the unit interval, sends a message to an uninformed player who then takes an action in the real line. Players have quadratic preferences, with the difference in their bliss points parameterized by a bias parameter b. The Crawford and Sobel model highlights the difficulties of sustaining informative communication between asymmetrically informed parties. It has been a foundation for theoretical and applied work on communication in a variety of fields, including political economy (Grossman and Helpman [13]), finance (Morgan and Stocken [21]), organizational design (Dessein [9]).

Within this set-up, we consider the three most common means by which parties resolve disputes outside of courts.[1] Under *arbitration*, a neutral third-party renders a decision after hearing proofs and arguments from each party. While both the agreement to arbitrate and the presentation of these arguments is voluntary, the arbitrator's decision is binding, in the sense that courts will enforce it against a reluctant party. In contrast, under *mediation*, the neutral third-party has no authority to impose a settlement. Instead, he merely suggests an agreement that must be acceptable to the decision-maker. Finally, in unfacilitated *negotiation*, or cheap talk, the two parties directly and voluntarily exchange information back and forth, until the decision-maker makes his final decision. In Crawford and Sobel's original paper, communication is one-shot: the sender sends a message to the receiver once and for all. However, it is known that this assumption is not without loss of generality (see Krishna and Morgan [16]). Accordingly, when considering negotiation, we allow both parties to engage in an arbitrarily large but finite number of rounds of communication.

For sake of comparison, we assume that these procedures are designed to maximize the ex-ante welfare of the party without private information, the *decision-maker*. In the case of negotiation and mediation, it is known that this turns out to also maximize the ex-ante welfare of the other party, the *informed party.*[2] As should be clear, arbitration cannot do worse than mediation (because commitment can only help) which in turn weakly improves upon negotiation (as the revelation principle applies). Our purpose is therefore to understand when one procedure strictly outperforms another one, and characterize the optimal procedures.

We derive three main results:

- *Arbitration*: Among all possibly stochastic arbitration rules, the optimal one is deterministic. Therefore, it coincides with the arbitration rule identified in Melumad and Shibano [18].
- *Mediation*: We determine the welfare achieved by optimal mediation rules. This allows us to show that the mechanism introduced by Blume, Board and Kawamura [5] is optimal. Other optimal mediation rules exist. In all optimal mediation rules, the mediator introduces noise in the communication between the informed party and the decision-maker.

[1] Unlike Krishna and Morgan [17], we assume that the parties cannot sign contracts establishing transfers between each other.

[2] We are interested in designing mediation and arbitration rules that are optimal behind a veil of ignorance, i.e. before the informed party is assigned her type. Another possible optimality criterion is an interim optimality criterion, whereby the informed party chooses the optimal rule after she is informed of the state of the world. Evidently, this is a more involved problem, as the choice of the rule may signal information about the informed party type.

– *Negotiation*: Bounded negotiation rules do as well as mediation if and only if the intensity of conflict is sufficiently low or sufficiently high, i.e. if and only if $b \notin (1/8, 1/2)$. When the conflict is low, i.e. $b < 1/8$, the two-stage cheap talk equilibrium described by Krishna and Morgan [16] does as well as an optimal mediation rule. If the conflict is high, $b > 1/2$, mediation cannot improve upon the babbling equilibrium.

Despite the tractability of the model, open questions remain. First, we do not provide a description of the entire set of optimal mediation rules, but rather, a method to verify whether any given rule is optimal. Second, we are unable to characterize the performance of optimal negotiation mechanisms for biases in the interval $(1/8, 1/2)$.

The importance of providing an economic analysis of arbitration has long been stressed (see Crawford [8], for instance). Within the framework of Crawford and Sobel, it has been studied in the literature on delegation (Holmström [14]; Melumad and Shibano [18]; Dessein [9]; Alonso and Matouschek [1,2]). However, all these earlier contributions have restricted attention to deterministic mechanisms. We allow for stochastic arbitration mechanisms, and prove that the optimal protocol is deterministic. Therefore, it is the one identified in the earlier papers. This finding is not obvious, as noise potentially relaxes the incentive constraints faced by the sender. (Indeed, the optimal mediation mechanism is stochastic.) In coincident work, and following a different approach, Kováč and Mylovanov [15] generalize this last result to more general environments.

Mediation has also been studied before. Ganguly and Ray [11] provide a class of mediation rules that improve upon Crawford and Sobel's equilibrium, and a numerical tool to compute such rules in discrete environments has been developed by Myerson (crawfsob.xls, http://home.uchicago.edu/~rmyerson/research/index.html). Our paper is the first to solve for the optimum within the framework of Crawford and Sobel's model.

When the bias is not too large ($b < 1/2$) we prove that in any optimal mediation protocol, the mediator must choose his recommendation randomly for some reports. As mentioned, the mechanism of Blume, Board and Kawamura [5] is shown to be optimal. The informed party reports the state of the world truthfully to the mediator. The mediator coarsens this information, and adds noise. More precisely, he randomizes between two recommendations. The low recommendation is the same for all reports, while the high recommendation is constant over intervals of reports; it is equal to the low recommendation on the lowest interval, and it is increasing as we move to higher intervals.

The intuition for why a mediator must randomize over recommendations is simple. A mediator that would be a mere relay or censor of information would be of no value here, as such transmission or censoring of information could be directly performed by the informed party.[3] In particular, a mediator could not improve upon the (most informative) equilibrium outcome by Crawford and Sobel. In our environment, the mediator can only create value by controlling the flow of information between the parties. This role of mediation has already been pointed out in other contexts by Brown and Ayres [6], Ayres and Nalebuff [4] and Mitusch and Strausz [20].

Negotiation, finally, has been examined before by Krishna and Morgan [16], within the context of Crawford and Sobel's model. They provide two interesting classes of equilibria involving information revelation over two periods, but do not prove whether these equilibria are optimal.

[3] Censoring by the mediator may be valuable in situations in which both parties have private information, as the censoring may require knowledge of both reports, and could not be performed by either party on its own.

We characterize optimal negotiation for small biases ($b < 1/8$) by showing that one of these equilibria achieves the optimum from the mediation problem. We further show that finite communication cannot replicate mediation for larger biases (i.e. $b \in (1/8, 1/2)$). The proof of this result is more involved, and we defer discussion of it until Section 5.

There are several related papers that consider the problem of implementing mediated outcomes of finite Bayesian games as correlated equilibria of arbitrarily long negotiation protocols. An early reference is Forges [10]. See also Gerardi [12]. Vida [23,24] provides significant recent progress. These papers show how to dispense with mediation by using sunspots, but their results do not apply directly to our framework.

The paper is organized as follows. Section 2 introduces the set-up and formally defines optimal arbitration, mediation and negotiation. Section 3 studies arbitration. Section 4 analyzes mediation, and Section 5 examines negotiation. Concluding comments are in Section 6. All formal proofs are in Appendices A–C.

2. Model

There are two players, the informed party and the decision-maker. The payoffs of both players depend on the state of nature $\theta \in \Theta = [0, 1]$ and the action $y \in Y = \Re$. The informed party knows θ; the decision-maker does not know θ, and his prior is uniform on Θ. The decision-maker has the capacity to execute an action in Y.

We assume that the utility function of the decision-maker equals $v(y, \theta) = -(y - \theta)^2$, and that of the informed party equals $u(y, \theta) = -(y - (\theta + b))^2$ where $b > 0$. For any given θ, the informed party's preferred action is $y = \theta + b$, while the decision-maker's preferred action is $y = \theta$. The utility of each party in state θ decreases in the distance from the preferred action given θ to the action that is actually taken.

In this setting, we will study three different classes of communication procedures: arbitration, mediation and negotiation. Let us consider them in turn.

Arbitration requires that the players can find a neutral trustworthy third party (the arbitrator), to whom the players can send private or public messages. After having heard the messages, the arbitrator announces an action in Y. This announcement serves as a binding recommendation to the decision-maker: that is, the decision-maker cannot execute any action that is different from the recommended one.

To specify an arbitration rule as informally described above, one has to choose two message spaces, one for the informed party and one for the decision-maker. One also has to specify the protocol for the communication with the arbitrator (the sequence in which the parties can talk, and whether their messages are public or private) and a function (possibly a random one) that maps sequences of messages sent by the players to actions recommended by the arbitrator. From the above description, one can see that the set of arbitration rules is very large. Fortunately, the revelation principle (Myerson [22]) applies here. It says that without loss of generality, we can restrict attention to arbitration rules whereby only the informed party communicates with the arbitrator, sending a single message, which is a report on the state of nature. As a consequence, we can define an arbitration rule as a function that maps reported states of nature to lotteries on actions. The game proceeds as follows: first, the informed party privately reports a state of nature to the arbitrator; then the arbitrator selects which action to recommend according to the lottery that corresponds to the informed party's report and publicly announces the recommendation; finally, the decision-maker executes the action recommended by the arbitrator. Furthermore, the revelation principle implies that without loss of generality we can require that in equilibrium the informed party should find it optimal to report the true state. The arbitration rules that have a truthful equilibrium will be called incentive compatible.

With mediation, the parties communicate with a neutral trustworthy third party (the mediator) who then makes a recommendation of what action to take. The difference with arbitration is that the mediator's recommendation is not binding – that is, the decision-maker is free to choose an action that is different from the recommended one. The revelation principle applies to mediation as well, so without loss of generality we can restrict attention to mediation rules whereby the informed party sends a single private message to the mediator, which is a report on the state of nature. So, similarly to an arbitration rule, a mediation rule can be defined as a function that maps reported states of nature into lotteries on actions. Given a mediation rule, the game proceeds as follows: first, the informed party privately reports a state of nature to the mediator; then the mediator selects which action to recommend according to the lottery that corresponds to the informed party's report and publicly announces the recommendation; finally, the decision-maker chooses what action to execute. The revelation principle also implies that without loss of generality, reporting the true state should be optimal for the informed party, and obeying the mediator's recommendation should be optimal for the decision-maker (the last requirement is absent in the case of arbitration, because there the decision-maker cannot disobey the recommendation by definition). The mediation rules that have an equilibrium where the informed party always reports the truth and the decision-maker always obeys the recommendation will be called incentive compatible.

Finally, negotiation means that the informed party and the decision-maker engage in several rounds of unmediated communication, sending a message to the other party at each round. To describe a negotiation protocol formally, one needs to specify two message sets, one for each party, and a number T, which is the length of the protocol. Communication proceeds as follows: at stages $1, \dots, T$, the informed party and the decision-maker simultaneously choose a message, and their choices become commonly known at the end of the stage. At stage $T + 1$, the decision-maker selects an action.

From these descriptions, one can see that any outcome that can be achieved with mediation can be replicated with arbitration. Indeed, any mediation rule that induces truthful reporting from the informed party will also induce truthful reporting if applied to the arbitration game. Also, the revelation principle implies that any equilibrium of the game induced by any negotiation protocol is outcome equivalent to a truthful equilibrium of some incentive compatible mediation rule. So any outcome that can be achieved with negotiation can be replicated with mediation.

The aim of the paper is to find the communication procedure in each of the three classes that is optimal for the decision-maker. In this connection, the following fact is worth noting. Consider any equilibrium of any incentive compatible mediation rule, and let V be the ex-ante expected utility of the decision-maker, and $U(\theta)$ be the expected utility of the informed party in state θ in that equilibrium. Crawford and Sobel [7] prove that the incentive compatibility for the decision-maker implies

$$V = E_\theta U(\theta) + b^2. \tag{1}$$

The reason why this equality is true is that the decision-maker has a quadratic loss function, so the optimal action for him is equal to the expected value of the state conditional on his information (i.e. on the fact that this action is recommended to him by the mediator). Hence the equilibrium loss of the decision-maker is equal to the residual variance of the state after hearing the recommendation, and the equilibrium loss of the informed party is equal to the residual variance of the state plus the square of the bias.

As a consequence, an incentive compatible mediation rule p ex-ante Pareto dominates an incentive compatible rule q if and only if the decision-maker's ex-ante expected utility under p is higher than under q. This fact will allow us to maximize the expected utility of the decision-maker, with the understanding that the resulting mediation rule will be Pareto optimal. Since any equilibrium of any negotiation protocol is outcome equivalent to a truthful equilibrium of some incentive compatible mediation rule, the same statement holds also for the optimal negotiation protocol. One should note that this property does not hold for arbitration, because the recommended action need not be equal to the expected value of the state conditional on the fact that the action is recommended.

3. Arbitration

Let us formally introduce the optimization problem that is solved in case of arbitration. As mentioned in the previous section, the revelation principle allows us to restrict attention to arbitration rules whereby the informed party sends a single message to the arbitrator, which is a report on the state of nature. Further, in equilibrium the informed party should find it optimal to announce the true state. Hence we can define an arbitration rule to be a family $(p(\cdot|\theta))_{\theta\in\Theta}$, where for each $\theta \in \Theta$, $p(\cdot|\theta)$ is a probability distribution on Y. The interpretation is that upon hearing the report of θ from the informed party, the arbitrator selects his recommendation according to $p(\cdot|\theta)$.

Definition 1. An **optimal arbitration rule** $p = (p(\cdot|\theta))_{\theta\in\Theta}$ is family of probability distributions on Y that solves the following problem:

$$\max_{(p(\cdot|\theta))_{\theta\in\Theta}} V = -\int\limits_{Y\times\Theta} (y-\theta)^2\, dp(y|\theta)\, d\theta$$

subject to

$$\theta = \arg\max_{\hat{\theta}\in\Theta}\left[-\int\limits_{Y}\big(y-(\theta+b)\big)^2\, dp(y|\hat{\theta})\right], \quad \forall\theta\in\Theta. \tag{IC-IP}$$

The constraint (IC-IP) (IP stands for "informed party") reflects the fact that the informed party should find it optimal to tell the truth. An arbitration rule that satisfies (IC-IP) is called incentive compatible.

To solve for the optimal arbitration rule, we first develop a tractable way to deal with the incentive compatibility constraint for the informed party. Let $y(\hat{\theta}) = \int_Y y\, dp(y|\hat{\theta})$ and $\sigma^2(\hat{\theta}) = \int_Y (y - y(\hat{\theta}))^2\, dp(y|\hat{\theta})$ be the conditional expectation and the variance of y given a message $\hat{\theta}$. Then an expected payoff of the informed party of type θ who reported a message $\hat{\theta}$ in the mechanism p is

$$\int\limits_{Y} -\big(y-(\theta+b)\big)^2\, dp(y|\hat{\theta}) = -\sigma^2(\hat{\theta}) - \big(y(\hat{\theta}) - (\theta+b)\big)^2.$$

Namely, the fact that the informed party has a quadratic loss function implies that the informed party cares only about the expectation and the variance of the action.

One apparent benefit of this representation is that the constraint (IC-IP) can be stated in terms of $(y(\theta), \sigma^2(\theta))$ only. In addition, notice that the variance of y enters the utility function of the informed party in a quasi-linear way, and thus it does not interact with the type θ. Taking advantage of this fact, we can show (Lemma 1 below) that the incentive compatibility for the informed party is equivalent to two conditions: the expected action is non-decreasing in the state, and the informed party's equilibrium payoff in any state θ can be expressed as a function of his payoff in state 0 and of the expected action in the states below θ. This result is analogous to a well-known result in mechanism design for environments where the preferences are quasi-linear in money.

Lemma 1. $\{y(\theta), \sigma^2(\theta)\}_{\theta \in \Theta}$ *satisfy* (IC-IP) *if and only if*

(i) $y(\theta)$ *is non-decreasing*;
(ii) $-\sigma^2(\theta) = U(\theta) + (y(\theta) - (\theta + b))^2$, *and* $U(\theta) = U(0) + \int_0^\theta 2(y(\tilde{\theta}) - (\tilde{\theta} + b))\, d\tilde{\theta}$.

This representation allows us to prove the following theorem.

Theorem 1. *The optimal arbitration rule selects the preferred action of the informed party in the set* $[0, \max\{1 - b, \frac{1}{2}\}]$. *Formally, it satisfies*:

$$y(\theta) = \begin{cases} \theta + b, & \text{if } \theta \in [0, \max\{1 - 2b, 0\}), \\ \max\{1 - b, \frac{1}{2}\}, & \text{if } \theta \in [\max\{1 - 2b, 0\}, 1], \end{cases}$$

$$\sigma^2(\theta) = 0, \quad \forall \theta \in [0, 1],$$

$$U(0) = \begin{cases} 0, & \text{if } b \leqslant \frac{1}{2}, \\ -(\frac{1}{2} - b)^2, & \text{if } b > \frac{1}{2}. \end{cases}$$

Observe that when the preference divergence parameter b is above $\frac{1}{2}$, the optimal arbitration rule is a flat one (the same decision is enforced no matter what the informed party reports). For future reference notice that for these values of the parameter communication is useless in the arbitration model, and, consequently, it is useless in the mediation and negotiation models.

When $b \leqslant \frac{1}{2}$, the optimal arbitration rule is deterministic. It implements the most preferred action of the informed party for low states of the world, and is constant at $1 - b$ for high states of the world. See Fig. 1 for an illustration.

Melumad and Shibano [18] already established the optimality of such a rule among deterministic mechanisms. The optimal mechanism can be viewed as a delegation of the decision to the informed party with a limited form of discretion: the informed party can enforce any decision he likes, as long as it does not exceed $1 - b$. Since the informed party's most preferred action in any state of the world is higher than that of the decision maker, it pays to impose an upper bound on the allowable actions. On the other hand, it turns out that the best way to make use of the informed party's information in case of the low states is to grant a complete freedom of choice of the action to the informed party.[4]

Our result demonstrates that this delegation rule remains optimal even if we allow for stochastic mechanisms. The tradeoff here is between an implementation of expected action functions which are more desirable for the decision maker and incentive costs due to an increased variance of the mechanism. It turns out that this tradeoff is always resolved in favor of using mechanisms with the smallest possible variance, i.e. deterministic mechanisms.

[4] For additional intuition and results on optimal delegation see also Holmström [14], Alonso and Matouschek [1,2].

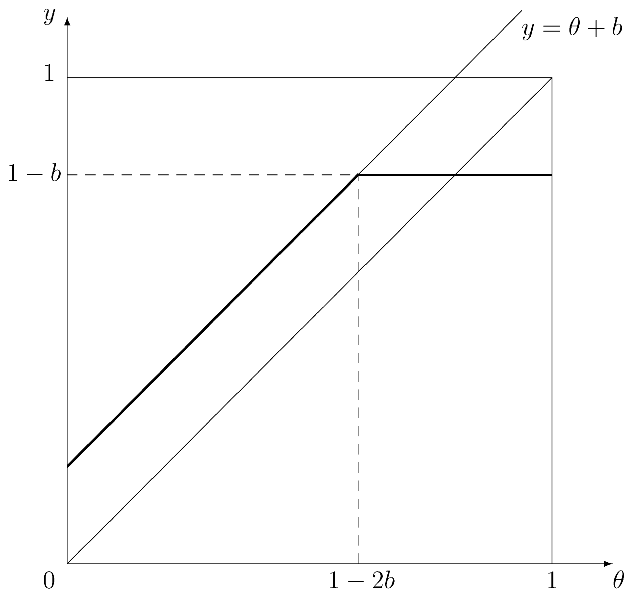

Fig. 1. Optimal arbitration.

To gain the intuition for why the optimal arbitration rule is deterministic, consider, for example, reducing $y(\theta)$ by a small $\delta > 0$ on an interval $[0, \bar{\theta}] \subseteq [0, 1-2b]$. The distance between the expected action and the decision-maker's most preferred action is thus reduced from b to $b-\delta$ on this interval. The expected gain for the decision-maker is

$$\int_0^{\bar{\theta}} \left(-(b-\delta)^2\right) d\theta - \int_0^{\bar{\theta}} \left(-b^2\right) d\theta = (2b-\delta)\delta\bar{\theta}.$$

In order to implement this new expected action function in an incentive compatible way we need to adjust a conditional variance function. First, we need to adjust $\sigma^2(\cdot)$ on the interval $[0, \bar{\theta}]$. Part (ii) of Lemma 1 implies

$$\frac{d}{d\theta}\sigma^2(\theta) = -2\big(y(\theta) - (\theta + b)\big)\frac{d}{d\theta}y(\theta)\bigg|_{y(\theta)=\theta+b-\delta} = 2\delta.$$

Intuitively, since we are implementing expected actions which are below the most preferred actions of the informed party, we need to discourage him from choosing higher actions by increasing their variance. The variance for the types on the interval $[0, \bar{\theta}]$ is thus $\sigma^2(\theta) = 2\delta\theta$.

Second, the variance for the types immediately above $\bar{\theta}$ cannot be lower than the variance $\sigma^2(\bar{\theta}) = 2\delta\bar{\theta}$, since otherwise the type $\bar{\theta}$ could slightly overstate his type and receive his most preferred expected action, $\bar{\theta} + b$, at a smaller variance. The incentive compatibility of the original mechanism implies that the variance for all the types above $\bar{\theta}$ is at least as large as the variance of the type $\bar{\theta}$.

The expected loss for the decision-maker from the variance is thus at least

$$\int_0^{\bar{\theta}} 2\delta\theta \, d\theta + \int_{\bar{\theta}}^1 2\delta\bar{\theta} \, d\theta = (2-\bar{\theta})\delta\bar{\theta}.$$

Hence the net benefit for the decision-maker is at most

$$\big((2b-\delta)-(2-\bar{\theta})\big)\delta\bar{\theta}<0,$$

where the inequality follows from $\delta>0$, $\bar{\theta}<1$ and $b\leqslant\frac{1}{2}$.

For completeness let us also comment on a symmetric arbitration problem which maximizes the ex-ante payoff of the informed party assuming that the informed party can commit to any announcement strategy as a function of the state while the decision-maker cannot commit. Eq. (1) implies that the upper bound on the ex-ante payoff of the informed part is $-b^2$. This upper bound can be achieved by truthful announcement of the state by the informed party to which the decision-maker best-responds with the action equal to the state.

4. Mediation

In this section, we look for the optimal mediation rule. We first note that the optimal arbitration rule (always recommending the action $y=\frac{1}{2}$) is feasible when $b>\frac{1}{2}$. Since the mediation problem is more constrained than the arbitration problem, this rule also has to be the optimal mediation rule. So we focus on finding a solution for $b\in(0,\frac{1}{2}]$.

By the revelation principle, one can restrict attention to mediation protocols whereby the informed party reports the state of the world to the mediator, and the mediator makes a recommendation to the decision-maker. Further, we can assume without loss of generality that the report is truthful, and the recommended action is incentive compatible (see Myerson [22]). Formally, a mediation rule is a family of probability distributions on Y, $(p(\cdot|\theta)_{\theta\in\Theta})$, with the interpretation that the mediator selects his recommendation according to $p(y|\theta)$ after hearing the report θ from the informed party.

Definition 2. An **optimal mediation rule** $p=(p(\cdot|\theta)_{\theta\in\Theta})$ is a family of probability distributions on Y that solves the following problem:

$$\max_{(p(\cdot|\theta)_{\theta\in\Theta})} V=-\int\limits_{Y\times\Theta}(y-\theta)^2\,dp(y|\theta)\,d\theta$$

subject to

$$\theta=\arg\max_{\hat{\theta}\in\Theta}\left[-\int\limits_{Y}\big(y-(\theta+b)\big)^2\,dp(y|\hat{\theta})\right],\quad\forall\theta\in\Theta;\qquad\text{(IC-IP)}$$

$$y=E_\theta[\theta|y],\quad\forall y\in Y.\qquad\text{(IC-DM)}$$

A mediation rule that satisfies (IC-IP) and (IC-DM) is called incentive compatible.

The constraint (IC-DM) states that the decision-maker never has an incentive to deviate from an action that is prescribed to him by the mediator (the right-hand side of the equality is the expectation of θ given recommendation y, which is the action that maximizes the decision-maker's payoff when the mediator recommends y). Given $p(y|\theta)$ and the unconditional distribution of θ, $E_\theta[\theta|y]$ is determined uniquely up to a zero-measure subset of Y.

We will proceed as follows. First, we will derive an upper bound on the objective function. Next, we show that some of the mechanisms already proposed in the literature achieve this upper bound for certain values of b.

Lemma 2.

(a) *If a mediation rule p is incentive compatible, then* $V \leqslant -\frac{1}{3}b(1-b)$;
(b) *An incentive compatible mediation rule is optimal if and only if the lowest sender's type gets its preferred decision, i.e.* $U(0) = 0$.

Lemma 2 can be compared to the revenue equivalence theorem in standard mechanism design. However, while in the revenue equivalence theorem, the revenue is pinned down by the utility of the lowest type and the allocation, here the welfare from an incentive compatible mechanism is determined only by the utility of the lowest type. The reason for this difference is that in our problem, the mechanism designer is facing two sets of incentive compatibility constraints, one for the informed party and one for the decision-maker. The incentive compatibility constraints for the decision-maker allow us to express $y(\theta)$, which plays the role of an "allocation", as a function of $U(0)$, the lowest type's utility.

Lemma 2 immediately implies that some of the procedures that have been proposed in the literature as improvements upon one-shot negotiation are, in fact, optimal. One of them is described below.

Theorem 2. *For every* $b < \frac{1}{2}$, *an optimal mediation rule is such that the mediator randomizes between two actions in each state. With some probability* μ, *he recommends action* b, *and with probability* $1-\mu$ *he recommends action* a_i *when* $\theta \in [\theta_i, \theta_{i+1})$, $i = 0, \ldots, N-1$, *where*

$$\theta_0 = 0;$$
$$\theta_i = 2bi^2 - \left(2bN^2 - 1\right)\frac{2i-1}{2N-1}, \quad i = 1, \ldots, N;$$
$$a_i = b(i+1) - 2bi(N-i) + \frac{(2-b)i}{2N-1}, \quad i = 0, \ldots, N-1;$$
$$\mu = 1 - \frac{1-2b}{4(1-b)}\left(\frac{1}{N-1} - \frac{1}{N} - \frac{2-b}{bN-1} + \frac{2-b}{bN-b+1}\right)$$

and N *is such that*

$$\frac{1}{2N^2} \leqslant b < \frac{1}{2(N-1)^2}.$$

It is straightforward to verify that this mediation rule is incentive compatible. The fact that it is optimal follows from the fact that it results in $U(0) = 0$ (since type 0 gets action b with probability one) and Lemma 2. See Fig. 2 for an illustration.

It is immediate to verify that, as the bias tends to zero, so does the probability μ. However, μ is not monotonic in the bias. Rather, for each value of N, it is concave and equal to zero for the two extreme values of bias that are consistent with N. For these extreme (and non-generic) values, the mediation rule replicates the most informative equilibrium of Crawford and Sobel.

The above rule appears in Blume, Board and Kawamura [5], who propose it as an improvement upon the most informative Crawford and Sobel equilibrium, but do not prove that it is optimal, or interpret it as a mediation procedure. They propose the following simple interpretation. Imagine that the informed party sends one message from the interval $[0, 1]$ to the decision-maker, but the decision-maker gets his message only with probability $1-\mu$: with probability μ, the message that the decision-maker gets is a random draw from the uniform distribution

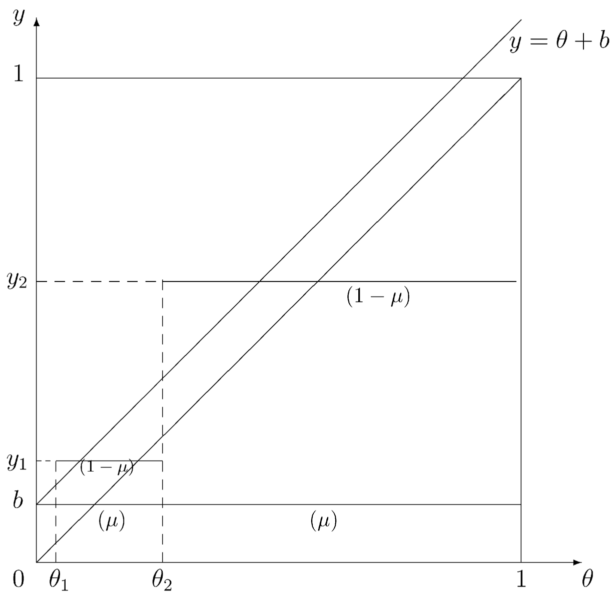

Fig. 2. Optimal mediation.

on [0, 1]. It is straightforward to show that this game has an equilibrium that is outcome equivalent to the truthful equilibrium of the mediation rule described above.

Theorem 2 highlights the fact that the primary role of the mediator is to filter the information provided by the informed party. In fact, the optimal mediator's only function is to introduce noise into communication. As Blume, Board and Kawamura [5] note, introducing noise can have opposing effects on the amount of information transmitted. On the one hand, the direct effect of noise is to make the message received by the decision-maker less informative. On the other hand, the presence of noise relaxes the incentive compatibility for the informed party by weakening the link between his message and the decision-maker's reaction, which makes it easier to motivate the informed party to transmit more information. Theorem 2 shows that the second effect dominates the first one; moreover, simply introducing an optimal amount of noise into communication is optimal in the class of all mediation rules.

The optimal mediation rule described above is not unique. In particular, Lemma 2 implies that another mediation rule that has been proposed in the literature is also optimal when $b \leqslant \frac{1}{8}$. This is the mediation rule of Krishna and Morgan [16], which can be implemented with two rounds of cheap talk and is discussed in more detail in the next section.

Observe also that in the optimal mediation problem the constraints are convex in p, and the objective function is linear. Therefore, the set of optimal mediation rules is convex, so that there is in general a continuum of optimal mediation rules.

5. Negotiation

In our setting, negotiation means that the informed party and the decision-maker engage in several rounds of unmediated communication, sending a message to the other party at each round. Similarly to Aumann and Hart [3], a negotiation protocol will include two sets, I and D, and $T \in N \cup \{\infty\}$, where I and D are the sets of admissible messages of the informed party and the decision-maker, respectively, and T is the length of the protocol. The protocol will define a game with incomplete information with $T+2$ stages that proceeds as follows. At stage 0, nature selects the state θ and informs the informed party. At each of the stages $1, \dots, T$, the informed party and the decision-maker simultaneously choose a message, and their choices become commonly known at the end of the stage. At stage $T+1$, the decision-maker selects an action. The payoffs for the decision-maker and the informed party are $v(y, \theta)$ and $u(y, \theta)$ respectively, where y is the action, and θ is the true state of nature. A negotiation protocol will be called finite if $T < \infty$.

Definition 3. An **optimal negotiation protocol** (I, D, T) is a solution to the following problem:

$$\max_{p(\cdot), I, D, T} V = -\int\limits_{Y \times \Theta} (y-\theta)^2 p\,(dy, d\theta)$$

subject to

p is the outcome distribution of a Bayesian–Nash equilibrium of the game induced by the protocol (I, D, T).

The central result of this section is the following one.

Theorem 3. *Finite negotiation achieves the optimal mediated outcome if and only if $b \leqslant 1/8$.*

The "if" part of the theorem is easy to show on the basis of our Lemma 2. The optimal 'monotonic' equilibrium of Krishna and Morgan [16] in a two-period negotiation protocol exists if and only if $b \leqslant 1/8$, and achieves value $U(0) = 0$: the type-0 informed party achieves the optimal utility. In light of Lemma 2, for an incentive compatible mediation scheme to be optimal, it is necessary and sufficient that $U(0) = 0$. Hence the optimal monotonic two-period negotiation equilibrium by Krishna and Morgan performs as well as the optimal mediation scheme.

For purposes of illustration, we sketch the construction of the optimal 'monotonic' equilibrium developed by Krishna and Morgan [16]. The reader is referred to that paper for the details in the construction. In the first period of the negotiation protocol, the informed party signals whether the state is above or below some threshold θ^*. Simultaneously, the informed party and the decision-maker exchange messages in a meeting, so as to emulate a public randomization device with probabilities p and $1-p$.[5] In the second round of communication, if the informed party's message indicates that the state is below θ^*, a partitional equilibrium is played, as in Crawford and Sobel [7]. Given the number of elements in the partition N such that $1/[2(N+1)^2] \leqslant b < 1/[2(N)^2]$, and the set of thresholds $\{\theta_i\colon i = 0, \dots, N-1\}$, with $\theta_i = 2bi^2$, $\theta_{N-1} = \theta^*$, the informed party reports in which interval $[\theta_i, \theta_{i+1}]$ the state lies, and the decision-maker takes the corresponding action $a_i = [\theta_i + \theta_{i+1}]/2$. These thresholds assure that $\theta_0 = 0$, $\theta_1 = 2b$ and hence $a_0 = b$ so that, optimally, $U(0) = 0$.

If the informed party's message reported that the state is above $\theta^* = \theta_{N-1}$ in the first round of communication, the continuation play depends on the outcome of the simultaneous exchange of messages. With some probability p, no further communication occurs and the decision-maker takes his action accordingly: $a^*_{N-1} = [1 + \theta_{N-1}]/2$. With probability $1-p$, the informed party

[5] Such meetings in which parties simultaneously exchange messages is called a jointly controlled lottery. The reader is referred to Aumann and Hart [3] and Krishna and Morgan [16] for a formal definition.

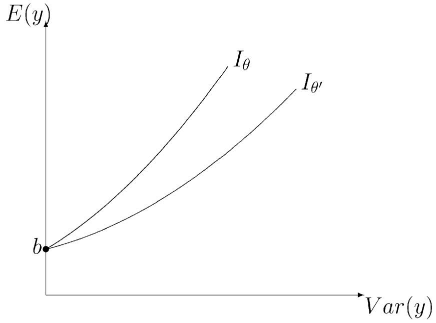

Fig. 3. Indifference curves for θ, θ' ($\theta' > \theta > 0$).

further signals whether the state is in some lower interval $[\theta_{N-1}, \theta_N]$, or upper interval $[\theta_N, 1]$, upon which the decision-maker takes his action: $a_{N-1} = [\theta_{N-1} + \theta_N]/2$ and $a_N = [1 + \theta_N]/2$ respectively.

Krishna and Morgan prove that such equilibria exist for $b \leqslant 1/8$ with the values θ_N and p that satisfy the following two indifference conditions. The type-θ_N sender is indifferent between the outcome a_{N-1} and a_N, and the type-θ_{N-1} sender is indifferent between the outcome a_{N-2} and the lottery determining the outcome a_{N-1} with probability $(1 - p)$ and the outcome a^*_{N-1} with probability p.

The "only if" part is considerably more involved, and its proof is relegated to Appendix C. To gain some intuition for this part, recall from Lemma 2 that for an incentive compatible mediation scheme to be optimal, it is necessary and sufficient that the lowest informed party's type be mapped into the action b (with probability 1). Since preferences are quadratic, any lottery y over actions can be summarized by its first two moments. We may thus represent the preferences of the different informed party's types by their indifference curves in the plane defined by the lotteries' expectation and variance. Fig. 3 shows the indifference curves going through the lottery that is degenerate on the action b for two different types $\theta' > \theta > 0$. The indifference curve for θ' is lower than for θ because $v_{12} > 0$. Observe that if the informed party's type θ is indifferent between the (degenerate) lottery b and some non-degenerate lottery, then type θ' strictly prefers this non-degenerate lottery to b. To put it differently, if in some equilibrium the informed party's type θ' is mapped into the action b, then so must be all lower types. Furthermore, in equilibrium, there can be at most one type indifferent between the action b and some non-degenerate lottery.

Suppose that, in an equilibrium of some negotiation protocol, action b is chosen when the state is 0, so that the optimal mediated outcome is achieved. Then we prove that it must be the case that, for some θ^*, the action b is finally chosen for almost all states in $[0, \theta^*]$. Furthermore, we prove that action b cannot be finally adopted with positive probability when the state is larger than θ^*. For this to conform with the decision-maker's equilibrium beliefs, it must be that $\theta^* = 2b$.

Consider now the choice of the informed party when the state is $\theta^* - \varepsilon$ for $\varepsilon > 0$ small enough. By following the equilibrium strategy, the informed party gets action b with probability one. But this type of informed party may be better off by upsetting the equilibrium, deviating and mimicking a type higher than $2b$. If the sender follows such a strategy, then by the final stage of communication the receiver will believe with probability one that the state is higher than $2b$ (because all types in the interval $[0, 2b]$, and only them, get the same action b). So such a strategy by the sender will ensure that the action is at least $2b$. Also, the sender can choose a strategy that will lead to actions no larger than $(\theta^* + 1)/2$ – the expected value over types in $(\theta^*, 1)$. Therefore, in equilibrium type θ^* must prefer action b for sure to some lottery with the support contained in $[2b, (\theta^* + 1)/2]$. Because his preferences are single-peaked at $2b$, this means that he must prefer action b for sure to action $(\theta^* + 1)/2$ for sure:

$$\big(b - (\theta^* - \varepsilon) - b\big)^2 \leqslant \left(\frac{\theta^* + 1}{2} - (\theta^* - \varepsilon) - b\right)^2, \quad \forall \varepsilon > 0 \text{ or } b \leqslant 1/8,$$

because $\theta^* = 2b$.

More succinctly, the 'only if' part of the above theorem follows because quadratic preferences imply that such an equilibrium be monotonic, in the sense that the set of states for which the action b should finally be chosen constitute an initial interval (i.e., an interval containing 0). If the bias is large, this imposes a significant cost on the informed party when the state is close enough to the upper end of this interval, the informed party may be better off pretending that the state of the world is larger. This intuition suggests that the result should extend to the case of communication of unbounded length. But we have no proof for this, and cannot rule out that some almost surely finite negotiation protocol achieves the mediation outcome.

6. Discussion and conclusion

We have compared the performance of three common dispute resolution processes – arbitration, mediation and negotiation – in the framework of Crawford and Sobel [7]. Under arbitration, the two parties commit to conform to the decision of a neutral third party. Under mediation instead, compliance with the third party's suggested settlement is voluntary. Finally, under unfacilitated negotiation, the two parties engage in (possibly arbitrarily long) face-to-face cheap talk. We have characterized and compared the optimal arbitration and mediation schemes, and identified necessary and sufficient conditions for negotiation to perform as well as mediation. The optimal mediation scheme corresponds to the communication protocol developed by Blume, Board and Kawamura [5]. Thus, we find that mediators may act optimally by filtering the unmediated communication and introducing noise to it. We have found that mediation performs better than negotiation when the conflict of interest is intermediate, whereas a mediator is unnecessary and two rounds of communication suffice when the conflict of interest is low.

In terms of welfare, our findings can be summarized by Fig. 4, which represents the cost (i.e., the opposite of the sender's ex-ante payoff) of arbitration (dashed line), optimal mediation (solid line), and the upper bound on the cost of cheap talk for the range of biases $[0, 1/\sqrt{8}]$. (Up to $b = 1/8$, this bound is given by Krishna and Morgan's two-period equilibrium described earlier, which was shown to be optimal in this range; in the range $[1/8, 1/\sqrt{8}]$, this bound is given by the other 'non-monotonic' equilibrium that can be found in Krishna and Morgan, and has not been discussed so far. This equilibrium might or might not be optimal. Finally, no non-babbling cheap talk equilibrium is known for biases above $1/\sqrt{8}$.)

Most of our results have been derived within the standard uniform quadratic framework. As mentioned, in concomitant work, Kováč and Mylovanov [15] have shown that the results for arbitration remain valid more generally. We are able to extend some of our results on mediation somewhat, to the class of distributions with linear hazard rate, and have proved an analog of Lemma 2, as well as shown that an optimal mediation mechanism of the form presented in Theorem 2 exists (details available upon request). While we have not shown that the results in the case of negotiation extend to those distributions, this does not appear implausible. Indeed, for small biases, the characterization of the two-stage equilibrium of Krishna and Morgan [16]

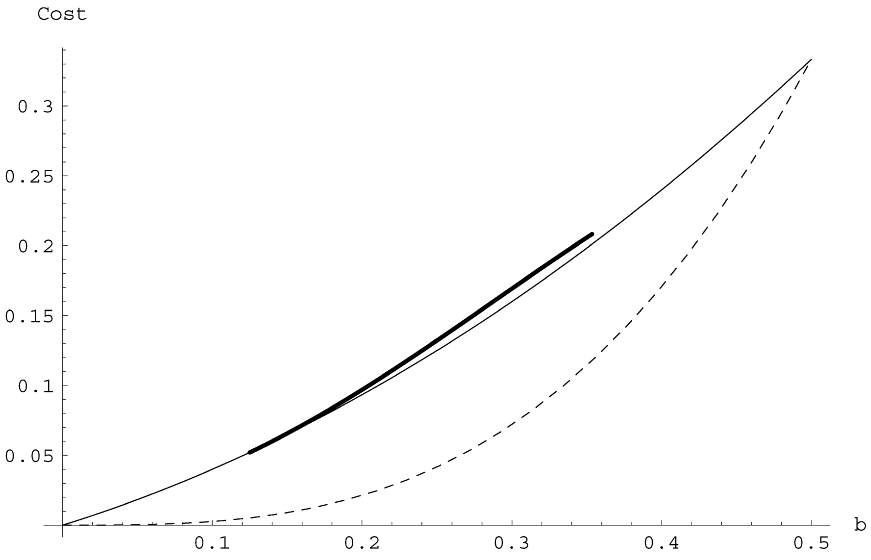

Fig. 4. Cost (the negative of the sender's payoff) achieved by optimal arbitration (dashed line), optimal mediation (solid line), and upper bound on the cost of cheap talk (bold solid line) for $b \in [1/8, 1/\sqrt{8}]$.

reduces to the solution of a system of non-linear equations with one more unknown than equations. Therefore, we expect that, within some range of parameters, this unknown can be chosen to set the lowest type's utility to zero.

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Appendix A. Arbitration

A.1. Proof of Lemma 1

Proof. (Only If)

(i) From incentive compatibility for every $\theta, \theta' \in \Theta$ we have

$$-\sigma^2(\theta) - \big(y(\theta) - (\theta + b)\big)^2 \geqslant -\sigma^2(\theta') - \big(y(\theta') - (\theta + b)\big)^2;$$
$$-\sigma^2(\theta') - \big(y(\theta') - (\theta' + b)\big)^2 \geqslant -\sigma^2(\theta) - \big(y(\theta) - (\theta' + b)\big)^2.$$

Adding up and rearranging we get

$$(\theta - \theta')\big(y(\theta) - y(\theta')\big) \geqslant 0.$$

(ii) Note that we can express

$$-\sigma^2(\theta) = U(\theta) + \big(y(\theta) - (\theta + b)\big)^2.$$

By the generalized Envelope Theorem (Corollary 1 in Milgrom and Segal [19]) we have

$$U(\theta) = U(0) + \int_0^{\theta} 2\big(y(\tilde\theta) - (\tilde\theta + b)\big)\, d\tilde\theta.$$

(If)
We need to show that for every $\theta, \theta' \in \Theta$,

$$\big(-\sigma^2(\theta) - \big(y(\theta) - (\theta + b)\big)^2\big) - \big(-\sigma^2(\theta') - \big(y(\theta') - (\theta + b)\big)^2\big) \geqslant 0.$$

Notice that

$$\begin{aligned}
-\sigma^2(\theta') - \big(y(\theta') - (\theta + b)\big)^2 &= -\sigma^2(\theta') - \big(y(\theta') - (\theta' + b)\big)^2 - 2y(\theta')(\theta' + b) \\
&\quad + (\theta' + b)^2 + 2y(\theta')(\theta + b) - (\theta + b)^2 \\
&= U(\theta') - \int_{\theta}^{\theta'} 2\big(y(\theta') - (\tilde\theta + b)\big)\, d\tilde\theta.
\end{aligned}$$

So

$$U(\theta) - U(\theta') + \int_{\theta}^{\theta'} 2\big(y(\theta') - (\tilde\theta + b)\big)\, d\tilde\theta = \int_{\theta}^{\theta'} 2\big(y(\theta') - y(\tilde\theta)\big)\, d\tilde\theta \geqslant 0. \qquad \square$$

A.2. *Proof of Theorem 1*

By Lemma 1 the optimal arbitration rule has to solve the following simplified problem:

$$\max_{y(\cdot), \sigma^2(\cdot), U(0)} V = \int_0^1 \big(-\sigma^2(\theta) - \big(y(\theta) - \theta\big)^2\big)\, d\theta$$

subject to

$$y(\theta) \text{ is non-decreasing;} \tag{MON}$$

$$\sigma^2(\theta) = -U(0) - \int_0^{\theta} 2\big(y(\tilde\theta) - (\tilde\theta + b)\big)\, d\tilde\theta - \big(y(\theta) - (\theta + b)\big)^2; \tag{ENV}$$

$$\sigma^2(\theta) \geqslant 0, \qquad U(0) \leqslant 0. \tag{NONNEG}$$

The proof of Theorem 1 proceeds through a series of lemmas.

Lemma 3. *If* $(y(\theta), \sigma^2(\theta), U(0))$ *are feasible, then*

$$V\big(y(\theta), \sigma^2(\theta), U(0)\big) = U(0) + 2\int_0^1 y(\theta)(1 - \theta - b)\, d\theta + b^2 - \frac{1}{3}.$$

Proof. Substitute constraint (ENV) into the objective function and change the order of integration in the double integral. □

Lemma 4. *Let* $b \in [0, \frac{1}{2}]$. *Mechanism* $(y(\theta), \sigma^2(\theta), U(0))$ *is optimal.*

Proof. Assume there exists a mechanism $(\hat{y}(\theta), \hat{\sigma}^2(\theta), \widehat{U}(0))$ which achieves a strictly higher welfare than the mechanism $(y(\theta), \sigma^2(\theta), U(0))$.

By Lemma 3 we have

$$0 < V\big(\hat{y}(\theta), \hat{\sigma}^2(\theta), \widehat{U}(0)\big) - V\big(y(\theta), \sigma^2(\theta), U(0)\big)$$
$$= \widehat{U}(0) - U(0) + 2\int_0^1 \big(\hat{y}(\theta) - y(\theta)\big)(1 - \theta - b)\,d\theta.$$

Also

$$0 < V\big(\hat{y}(\theta), \hat{\sigma}^2(\theta), \widehat{U}(0)\big) - V\big(y(\theta), \sigma^2(\theta), U(0)\big)$$
$$= -\int_0^1 \big(\big(\hat{y}(\theta) - \theta\big)^2 + \hat{\sigma}^2(\theta)\big)\,d\theta + \int_0^1 \big(\big(y(\theta) - \theta\big)^2 + \sigma^2(\theta)\big)\,d\theta$$
$$\leqslant -\int_0^1 \big(\hat{y}(\theta) - \theta\big)^2\,d\theta + \int_0^1 \big(y(\theta) - \theta\big)^2\,d\theta$$
$$= 2\int_0^1 \big(\hat{y}(\theta) - y(\theta)\big)\big(\theta - y(\theta)\big)\,d\theta - \int_0^1 \big(\hat{y}(\theta) - y(\theta)\big)^2\,d\theta$$
$$< 2\int_0^1 \big(\hat{y}(\theta) - y(\theta)\big)\big(\theta - y(\theta)\big)\,d\theta.$$

Adding up two inequalities,

$$0 < \widehat{U}(0) - U(0) + 2\int_0^1 \big(\hat{y}(\theta) - y(\theta)\big)\big(1 - b - y(\theta)\big)\,d\theta.$$

Substituting $y(\theta)$ and using (ENV) we get

$$0 < \widehat{U}(0) - U(0) + 2\int_0^{1-2b} \big(\hat{y}(\theta) - y(\theta)\big)\big(1 - 2b - \theta\big)\,d\theta$$
$$= \widehat{U}(0) + \int_0^{1-2b} \frac{d\widehat{U}(\theta)}{d\theta}(1 - 2b - \theta)\,d\theta - U(0) - \int_0^{1-2b} \frac{dU(\theta)}{d\theta}(1 - 2b - \theta)\,d\theta$$

$$= \widehat{U}(0) - \widehat{U}(0)(1-2b) + \int_0^{1-2b} \widehat{U}(\theta)\,d\theta - U(0) + U(0)(1-2b) - \int_0^{1-2b} U(\theta)\,d\theta$$

$$= 2b\big(\widehat{U}(0) - U(0)\big) + 2\left(\int_0^{1-2b} \big(\widehat{U}(\theta) - U(\theta)\big)\,d\theta\right).$$

However, this is not possible since $\widehat{U}(\theta) \leqslant U(\theta) = 0$ for every $\theta \in [0, 1-2b]$. □

Lemma 5. *Let $b > \frac{1}{2}$. If $(\hat{y}(\theta), \hat{\sigma}^2(\theta), \widehat{U}(0))$ are optimal, then $\hat{y}(\theta)$ is constant on $(0,1)$.*

Proof. Suppose that $\hat{y}(\theta)$ is not constant on $(0,1)$, i.e. $\exists\theta, \theta' \in (0,1)$ such that $\theta' > \theta$, $\hat{y}(\theta') > \hat{y}(\theta)$.

Consider the following policy:

$$y_1(\theta) = \hat{y}(0) \quad \text{for every } \theta,$$
$$\sigma_1^2(\theta) = \hat{\sigma}^2(0),$$
$$U_1(0) = \widehat{U}(0).$$

Obviously $(y_1(\theta), \sigma_1^2(\theta), U_1(0))$ satisfy constraint (MON) and (NONNEG). Moreover, this policy achieves a strictly higher value of the objective function than the original policy, since, by Lemma 3,

$$V\big(\hat{y}, \hat{\sigma}^2\big) = \widehat{U}(0) + 2\int_0^1 \hat{y}(\theta)(1-\theta-b)\,d\theta + b^2 - \frac{1}{3}$$
$$< \widehat{U}(0) + 2\int_0^1 \hat{y}(\theta)\,d\theta \int_0^1 (1-\theta-b)\,d\theta + b^2 - \frac{1}{3}$$
$$< \widehat{U}(0) + 2\hat{y}(0)\int_0^1 (1-\theta-b)\,d\theta + b^2 - \frac{1}{3}$$
$$= V\big(y_1, \sigma_1^2\big).$$

The first inequality is due to (MON) and the fact that $\hat{y}(\theta)$ is not constant on $(0,1)$; the last inequality is due to (MON) and the fact that $\int_0^1 (1-\theta-b)\,d\theta = \frac{1}{2} - b < 0$. So the original policy is suboptimal. □

Lemma 6. *Let $b > \frac{1}{2}$. Mechanism $(y(\theta), \sigma^2(\theta), U(0))$ is optimal.*

Proof. By Lemma 5, if $\hat{y}(\theta)$ is a part of an optimal policy, then it is constant on $(0,1)$. Without loss of generality, we can restrict attention to policies such that $\hat{y}(\theta)$ is constant on $[0,1]$. Take any such policy $(\hat{y}(\theta), \hat{\sigma}^2(\theta), \widehat{U}(0))$. Then

$$V\big(y(\theta), \sigma^2(\theta), U(0)\big) - V\big(\hat{y}(\theta), \hat{\sigma}^2(\theta), \widehat{U}(0)\big)$$
$$= \int_0^1 \big(-\sigma^2(\theta) - \big(y(\theta) - \theta\big)^2\big)\, d\theta - \int_0^1 \big(-\hat{\sigma}^2(\theta) - \big(\hat{y}(\theta) - \theta\big)^2\big)\, d\theta$$
$$\geqslant -\int_0^1 \big(y(\theta) - \theta\big)^2\, d\theta + \int_0^1 \big(\hat{y}(\theta) - \theta\big)^2\, d\theta = \hat{y}^2(0) - \hat{y}(0) + \frac{1}{4} \geqslant 0,$$

where the first inequality follows from the fact that $\hat{\sigma}^2(\theta) \geqslant \sigma^2(\theta) = 0$. □

The proof of Theorem 1 follows from Lemmas 4 and 6.

Appendix B. Mediation

B.1. Proof of Lemma 2

Proof. By (IC-DM) and the fact that θ is uniform on [0, 1],

$$\int_{\Theta} y(\theta)\, d\theta = \int_{Y \times \Theta} y\, dp(y|\theta)\, d\theta = \int_{Y \times \Theta} \theta\, dp(y|\theta)\, d\theta = \frac{1}{2}. \tag{1}$$

By (IC-DM),

$$\operatorname{cov}\big(\theta, y(\theta)\big) = \operatorname{cov}(\theta, y) = \operatorname{cov}\big(E_\theta[\theta|y], y\big) = \operatorname{cov}(y, y) = \operatorname{var}(y). \tag{2}$$

By Lemma 3 (see Appendix A.2) and Eqs. (1) and (2),

$$V = U(0) + 2\int_0^1 y(\theta)(1 - \theta - b)\, d\theta + b^2 - \frac{1}{3}$$
$$= U(0) - 2\int_0^1 y(\theta)\theta\, d\theta + 1 - b + b^2 - \frac{1}{3}$$
$$= U(0) - 2\operatorname{var}(y) + \frac{1}{6} - b + b^2. \tag{3}$$

On the other hand,

$$V = -E(y - \theta)^2$$
$$= -E\left[\left(\left(\theta - \frac{1}{2}\right) - \left(y - \frac{1}{2}\right)\right)^2\right]$$
$$= -\operatorname{var}(\theta) + 2\operatorname{cov}(y, \theta) - \operatorname{var}(y)$$
$$= \operatorname{var}(y) - \operatorname{var}(\theta) = \operatorname{var}(y) - \frac{1}{12}, \tag{4}$$

where the second equality follows from (1), the third equality follows from (2), and the last equality holds because θ is uniformly distributed.

Combining (3) and (4), we get

$$U(0) = 3\,\mathrm{var}(y) - \frac{1}{4} + b - b^2.$$

Since $U(0) \leqslant 0$, we have

$$\mathrm{var}(y) \leqslant \frac{1}{12} - \frac{1}{3}b + \frac{1}{3}b^2. \tag{5}$$

Substituting (5) into (4), we get

$$V \leqslant -\frac{1}{3}\left(b - b^2\right).$$

This holds with equality if and only if $U(0) = 0$. □

Appendix C. Negotiation

C.1. Proof of Theorem 3

First, let us prove an auxiliary result about a helpful property of quadratic preferences.

Lemma 7. *Let $\theta_1, \theta_2 \in [0, 1]$, $\theta_1 < \theta_2$. Let l be a lottery on Y such that l does not put probability one on action b, and θ_1 weakly prefers l to action b. Then θ_2 strictly prefers l to action b.*

Proof. Recall that the utility of a lottery l with mean y and variance σ^2 for the informed party of type θ equals $U(\theta) = -\sigma^2 - (y - (\theta + b))^2$. Consequently, type θ weakly prefers l to action b if and only if

$$\sigma^2 + (y - b)^2 \leqslant 2\theta(y - b),$$

which implies that $y \geqslant b$, no matter what θ is. So if $\theta_2 > \theta_1$ and the inequality above holds weakly for θ_1, then it has to hold strictly for θ_2. □

We restrict attention to canonical equilibria in the sense of Aumann and Hart [3]: that is, equilibria in which revelations by the informed party alternate with jointly controlled lotteries. For expositional simplicity, let us suppose that the players, instead of conducting jointly controlled lotteries, have access to a randomization device that sends messages at the jointly controlled lottery stages, so that at each stage, either the informed party or the device sends one public message.

First, let us introduce some notation. Let p be an optimal mediation rule, and suppose that p is implementable with finite cheap talk. Let $\Theta_1 := \{\theta \in \Theta\colon\ p(b|\theta) = 1\}$. We know that $\Theta_1 \neq \emptyset$, because $0 \in \Theta_1$. Let N be the set of all possible sequences of messages that can be observed in the equilibrium that implements p, and let $\mu(\cdot|\theta)$ be the probability distribution over N conditional on the state being θ. Let $P(\cdot|n)$ be the decision-maker's posterior upon observing $n \in N$, that is, for $\Theta \subseteq [0, 1]$, $n \in N$,

$$P(\Theta|n) = \frac{\int_{\Theta} \mu(n|\theta)\,dF(\theta)}{\int_{[0,1]} \mu(n|\theta)\,dF(\theta)}, \quad \text{if} \int_{[0,1]} \mu(n|\theta)\,dF(\theta) > 0.$$

Let us also assume that

$$P(\Theta|n) = 1 \quad \text{if} \int\limits_{[0,1]} \mu(n|\theta)\, dF(\theta) = 0 \text{ and } \big[\mu(n|\theta) > 0 \Rightarrow \theta \in \Theta\big].$$

The last assumption implies that if a particular path of play can only appear in one state θ^*, then upon observing this path of play, the DM concludes that the state is θ^* with probability one (this restriction on conditional probabilities seems somewhat arbitrary, but it is commonly made in signaling models with a continuum of types when talking about separating equilibria).

Finally, let $n(t)$ be the restriction of sequence $n \in N$ to the first t stages (including stage t), and let n_t be the message sent at stage t according to sequence n. We can also define $\mu(n(t)|\theta) := \int_{n' \in N:\, n'(t)=n(t)} d\mu\,(n|\theta)$, the probability that $n(t)$ realizes in equilibrium given θ.

Lemma 8. $\Theta_1 = [0, 2b]$.

Proof. For any period $t = 0, \dots, T$ and partial history $n(t)$, let

$$A\big(n(t)\big) = \big\{\theta \in [0,1]\colon \exists n' \in \text{support } \mu(\cdot|\theta),\ n'(t) = n(t)\big\}$$

be the set of types whose equilibrium behavior is consistent with partial history $n(t)$. Let $\Theta_1(n(t)) = \{\theta \in A(n(t))$: for a.e. (with respect to $\mu(\cdot|\theta)$) $n' \in N$ s.t. $n'(t) = n(t), E(\theta|n') = b\}$ be the set of types that, following the history $n(t)$, get action b with probability one. Let us prove that for every $t = 0, \dots, T$ and $n(t)$ such that $\Theta_1(n(t)) \neq \emptyset$,

(a) $\Theta_1(n(t)) = [0, \theta(n(t))] \cap A(n(t))$, for some $\theta(n(t)) \geqslant b$;

(b) $E(\theta|n(t), \Theta_1(n(t))) = b$.

The proof will be by induction, starting from $t = T$. Take any partial history $n(T-1)$ such that $\Theta_1(n(T-1)) \neq \emptyset$. Suppose, without loss of generality, that T is a revelation stage. By Lemma 1 of Crawford and Sobel [7], the equilibrium of the subgame following the history $n(T-1)$ is partitional. In particular, since $\Theta_1(n(T-1)) \neq \emptyset$, there exists an interval $[a(n), \theta(n)\rangle$ (closed or open on the right) such that, after the history $n(T-1)$, all types in this interval, and only them, choose messages that lead to action b; that is, $\Theta_1(n) = [a(n), \theta(n)\rangle \cap A(n(t))$. Moreover, $a(n)$ can be taken to be 0. Suppose not, that is, $a(n) > \theta$, for some $\theta \in A(n(t))$. Then in the partitional equilibrium of the subgame that we consider, type θ achieves an action lower than b, which is strictly worse for it than action b. But it could have achieved action b if it played like type $a(n)$ – a contradiction. It also has to be the case that $E(\theta|n, \Theta_1(n)) = E(\theta|n) = b$, and, consequently, that $\theta(n) \geqslant b$.

Now suppose that the statement is true for all partial histories of length $t+1, \dots, T$, and let us prove it for partial histories of length t. Consider any $n(t)$ such that $\Theta_1(n(t)) \neq \emptyset$. By definition, $\Theta_1(n(t)) \subseteq A(n(t))$. There are two cases to consider:

(a) t is a revelation stage. We have to prove that

$$\theta', \theta'' \in \Theta_1\big(n(t)\big), \quad \theta \in (\theta', \theta'') \cap A\big(n(t)\big) \quad \Rightarrow \quad \theta \in \Theta_1\big(n(t)\big),$$

and that

$$\theta' \in \Theta_1\big(n(t)\big), \quad \theta \in \bigcap A\big(n(t)\big), \quad \theta < \theta' \quad \Rightarrow \quad \theta \in \Theta_1\big(n(t)\big).$$

Suppose $\theta', \theta'' \in \Theta_1(n(t))$ and $\theta \in A(n(t))$. Then both θ' and θ'' choose continuation strategies at stage t that guarantee action b with probability one. Incentive compatibility implies that θ also has to choose a continuation strategy that guarantees b with probability one – otherwise either θ' or θ'' has an incentive to imitate θ. This means that $\theta \in \Theta_1(n(t))$.

Now, suppose that $\theta' \in \Theta_1(n(t))$, $\theta \in \bigcap A(n(t))$ and $\theta < \theta'$. This means that θ' chooses a continuation strategy at stage t that guarantees action b with probability one. If θ chooses a strategy that results in a different lottery over actions, then, by Lemma 7, θ' should strictly prefer to imitate θ – a contradiction. This means that θ also chooses a continuation strategy at stage t that guarantees action b with probability one, so $\theta \in \Theta_1(n(t))$.

This proves that $\Theta_1(n(t)) = [0, \theta(n(t))] \cap A(n(t))$.

Since t is a revelation stage,

$$\Theta_1\bigl(n(t)\bigr) = \Biggl(\bigcup_{n' \in N:\ n'(t)=n(t)} \Theta_1\bigl(n'(t+1)\bigr)\Biggr) \Bigg\backslash B,$$

where $B \subseteq [0,1]$ includes at most one type. To see this, note that it follows from the definition that $\Theta_1(n(t)) \subseteq \bigcup_{n' \in N:\ n'(t)=n(t)} \Theta_1(n'(t+1))$. Now suppose that $\Theta_1(n(t)) \subset \bigcup_{n' \in N:\ n'(t)=n(t)} \Theta_1(n'(t+1))$, and take any $\theta \in \bigcup_{n' \in N:\ n'(t)=n(t)} \Theta_1(n'(t+1)) \setminus \Theta_1(n(t))$. By the definition of $\Theta(n(t))$, it must be the case that type θ is randomizing at stage t between messages that will result in action b with probability one, and messages that results in some other lottery. But with quadratic preferences, there can be at most one such type. To see this, suppose, by way of contradiction, that there are two types, θ_1 and θ_2, both in $\bigcup_{n' \in N:\ n'(t)=n(t)} \Theta_1(n'(t+1)) \setminus \Theta_1(n(t))$, such that after history $n(t)$, type θ_i is randomizing between messages that will result in action b with probability one, and messages that results in some other lottery (call it l_i). Without loss of generality, suppose that $\theta_1 < \theta_2$. Then it must be the case that θ_1 is indifferent between action b and lottery l_1, so, by Lemma 7, θ_2 strictly prefers l_1 to b and, consequently, to l_2. This means that imitating θ_1 is a profitable deviation for θ_2 – a contradiction. This proves that B contains at most one type.

So

$$\begin{aligned}
&E\bigl(\theta|n(t), \Theta_1\bigl(n(t)\bigr)\bigr)\\
&\quad= \int\limits_{n' \in N:\ n'(t)=n(t)} E\bigl[\theta|n'(t+1), \Theta_1\bigl(n(t)\bigr)\bigr]\, d\mu\bigl(n'(t+1)|n(t), \Theta_1\bigl(n(t)\bigr)\bigr)\\
&\quad= \int\limits_{n' \in N:\ n'(t)=n(t)} \bigl\{E\bigl[\theta|n'(t+1), \Theta_1\bigl(n'(t+1)\bigr)\bigr] P\bigl[\Theta_1\bigl(n'(t+1)\bigr)|n'(t+1), \Theta_1\bigl(n(t)\bigr)\bigr]\\
&\qquad + E\bigl[\theta|n'(t+1), \Theta_1\bigl(n(t)\bigr) \setminus \Theta_1\bigl(n'(t+1)\bigr)\bigr]\\
&\qquad \times P\bigl[\Theta_1\bigl(n(t)\bigr) \setminus \Theta_1\bigl(n'(t+1)\bigr)|n'(t+1), \Theta_1\bigl(n(t)\bigr)\bigr]\bigr\}\, d\mu\bigl(n'(t+1)|n(t), \Theta_1\bigl(n(t)\bigr)\bigr)\\
&\quad= b,
\end{aligned}$$

where the last equality follows from the fact that

$$P\bigl(\Theta_1\bigl(n'(t+1)\bigr)|n'(t+1), \Theta_1\bigl(n(t)\bigr)\bigr) = \begin{cases} 1, & \text{if } n'(t+1) \in \text{support } \mu(\cdot|\Theta_1(n(t)));\\ 0, & \text{otherwise} \end{cases}$$

and from the induction hypothesis. It follows immediately that $\theta(n(t)) \geqslant b$.

(b) t is a jointly controlled lottery stage. Then it follows from the definition of $\Theta_1(n(t))$ that

$$\Theta_1\bigl(n(t)\bigr) = \bigcap_{n' \in N:\ n'(t)=n(t)} \Theta_1\bigl(n'(t+1)\bigr) = \bigcap_{n' \in N:\ n'(t)=n(t)} \bigl[0, \theta\bigl(n'(t+1)\bigr)\bigr] \cap A\bigl(n'(t+1)\bigr).$$

If t is a jointly controlled lottery stage, then for any $n', n'' \in N$ such that $n'(t) = n''(t) = n(t)$, $A(n'(t+1)) = A(n''(t+1)) = A(n(t))$. So

$$\Theta_1\big(n(t)\big) = A\big(n(t)\big) \cap \bigcap_{n' \in N:\ n'(t)=n(t)} \big[0, \theta\big(n'(t+1)\big)\big] = A\big(n(t)\big) \cap \big[0, \theta\big(n(t)\big)\big]$$

where $\theta(n(t)) = \inf_{n' \in N:\ n'(t)=n(t)} \theta(n'(t+1))$. Furthermore, the first equality above, together with the fact that $\forall n' \in N$: $n'(t) = n(t)$, $E(\theta | A(n(t)) \cap [0, \theta(n'(t+1))]) = b$ implies that $E(\theta | A(n(t)) \cap [0, \theta(n(t))]) = b$.

So we have proved that for every $t = 0, \ldots, T$ and $n(t)$ such that $\Theta_1(n(t)) \neq \emptyset$,

(a) $\Theta_1(n(t)) = [0, \theta(n(t))] \cap A(n(t))$, for some $\theta(n(t)) \geqslant b$;

(b) $E(\theta | n(t), \Theta_1(n(t))) = b$.

In particular, if $t = 0$, then $n(t)$ is an empty history, $\Theta_1(t) = \Theta_1$ by definition, and $A(n(t)) = [0, 1]$. Consequently, $\Theta_1 = [0, \theta_0]$ for some $\theta_0 \geqslant b$, and $E(\theta | \Theta_1) = b$. It follows immediately that $\theta_0 = 2b$. □

Proof of Theorem 3 (*Only if*). Because any type θ smaller than and sufficiently close to $2b$ strictly prefers $(2b+1)/2$ over b, it follows that any such type θ also strictly prefers to the outcome b any non-degenerate distribution q over actions with support contained in $[b, (2b+1)/2]$.

Consider any strategy μ such that, after any history $n(t-1)$ such that t is a revelation stage and support $P(\theta | n(t-1)) \cap [2b, 1] \neq \emptyset$, the sender chooses a message m that minimizes $E[\theta | n(t-1), m']$ among the messages m' such that support $P(\theta | n(t-1), m') \cap [2b, 1] \neq \emptyset$. Then at the first stage (without loss of generality, suppose that this is a revelation stage), this strategy calls for sending a message m such that $E[\theta | m] \leqslant [2b+1]/2 = E(\theta | [2b, 1])$; and by the law of iterated expectations, for every stage t, $E[\theta | n(t-1), m] \leqslant E[\theta | n(t-1)]$. It is clear that since $\Theta_1 = [0, 2b]$, this strategy cannot lead to any action that is lower than $2b$. On the other hand, at the terminal stage T, for any history n that can realize if strategy μ is followed, $E(\theta | n) \leqslant E(\theta | n(t-1)) \leqslant \cdots \leqslant E(\theta | n(1)) \leqslant (2b+1)/2$, so the action that will be executed cannot exceed $(2b+1)/2$. It follows that the strategy μ induces a lottery over actions whose support is contained in $[2b, (2b+1)/2]$, and a type $\theta = 2b - \varepsilon$ for $\varepsilon > 0$ small enough will prefer following this strategy to the strategy that induces action b with certainty.

It follows that p is not incentive compatible. □

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