The public economics of forecasting and insurance*

James B. Davies and Al Slivinski
Department of Economics
University of Western Ontario

July 29, 2013

Abstract

Individuals deal with risk through insurance contracts as well as through self-protection. Recent interest in such behavior relates to meteorological events, for which private insurance is often incomplete, and governments provide additional ex-post compensation. Reliable warnings of such events can be important for the provision of self-protection. However, a warning system increases the return to engaging in (ex-post) self-protection from weather damage, while insurance reduces that return. We develop a model of insurance/warning system co-existence to analyze their contradictory incentives for self-protection, and find that insurance implies that a warning system must be more reliable and more costly if warnings are to induce ex-post self-protection. Further, heterogeneity in individual damages implies some individuals will rationally ignore warnings generated by an optimal forecast system.

Weather-risks vary by location, so we analyze the influence of insurance and warning systems on ex-ante self-protection via location choice and find that publicly funded but incomplete insurance causes individuals and capital to be over-allocated to riskier locations. Introducing a forecast system causes a further allocation of residents and productive factors into the risky region. This increases expected insurance payouts, but the forecast system may still increase societal expected wealth by increasing the social return to living and investing in riskier regions.

*We thank The Institute for Catastrophic Loss Reduction and the Social Science and Humanities Research Council of Canada for research support, Gordon McBean, Paul Kovacs and participants in a UWO Economic Policy Research Institute Seminar for helpful comments, and Ye Jia for able research assistance. All opinions expressed in the paper are the authors', and do not necessarily reflect those of the ICLR.

1. Introduction

The vast literature on insurance has long recognized that individuals have available a variety of ways to deal with risk. In particular, they can purchase insurance contracts and they can engage in activities that will reduce the losses from random events, and/or reduce the probability that these events occur. These latter activities are often referred to as self-insurance, or self-protection. (See Ehrlich and Becker, 1972, Lakdawalla and Zanjani, 2005, and Lohse, 2012) One type of risk which has recently been a focus of policy discussion arises from meteorological events such as hurricanes, tornadoes and flooding.[1] Individuals can engage in self-protection from such weather-related events, but the availability of private insurance against such damages is not universal. Most home insurance policies, for example, do not cover certain types of weather-related losses, or require customers to pay additional premiums before such losses are covered. However, hurricanes, tornadoes or floods often result in some government agency providing compensation to those who suffer damage. Such compensation may be forthcoming because of insurance contracts offered by government, or may simply be the result of an ex-post decision to provide compensation using public funds. (See e.g. Priest, 1996, 2003; Barnett, 1999; and Michel-Kerjan, 2010)

A key element in the ability of individuals to protect themselves from weather-related losses is the existence and quality of warning systems. Reliable and timely warnings about hurricanes, flooding and developing storms make it possible for individuals to mitigate the damage they will suffer when these events occur. Put differently, a warning system increases the return to individuals and organizations from engaging in self-protection from weather events, while the existence of insurance against such damage reduces that return. Because both warning systems and insurance are often the result of governmental decisions, it is important to develop our understanding of how these institutions jointly influence individual decision making.

The broad purpose of this paper is to develop a model in which insurance and a forecasting system co-exist, so as to understand how their apparently contradicting incentives interact to influence individual behavior.[2] We consider both private and publicly provided insurance systems, with the important difference that public systems are typically funded from general tax revenues, rather than fees paid by those

[1] See e.g. Kunreuther and Michel-Kerjan (2009) and Simmons and Sutter (2011). Extreme weather is just one form of catastrophic risk, of course. There has also been increased concern recently about earthquake risk (see e.g. Anbarci et al., 2005 and Petseti and Nektarios, 2012), and terrorism risks (see e.g. Lakdawalla and Zanjani 2005). Our formal analysis can be applied to the whole field of catastrophic risks. However, we focus on the role of forecasting, which is more important for weather than for other catastrophic risks.

[2] While there is an extensive literature on weather insurance and weather derivatives (see e.g. Kunreuther, 1996; Richards, Manfredo and Sanders, 2004; Kapphan, Calanca and Holzkaemper, 2012; Hardle and Cabrera, 2012), we are not aware of any formal modeling of the interaction between weather forecasting, insurance, and self-protection. Other work at the interface between economics and meteorology includes a large literature estimating the economic value of weather forecasts (see e.g. Katz and Murphy, 1997; Lazo, Morss and Demuth, 2009; and Rollins and Shaykewich, 2003.) Campbell and Diebold (2005) have "pushed the envelope" by exploring the use of time series econometrics to provide weather forecasts that would assist participants in the market for weather derivatives.

affected. We also analyze two kinds of self-protection activity. We first consider 'ex-post' self-protection, meaning actions that can be taken to mitigate damage when a weather event occurs. It is clear that a warning system increases the return to this type of self-protection. However, it is also true that the occurrence of weather-related damages is not uniformly distributed across locations; individuals can influence the ex-ante risk they face by choosing where to live, work and invest. The existence of a warning system that enhances the productivity of ex-post self-protection therefore influences these location decisions, as does the existence of insurance. Thus, individual and commercial location decisions can be seen as a form of 'ex-ante' self-protection, which we also analyze.

We start by analyzing the socially optimal (in the sense of aggregate wealth-maximizing) level of expenditure on a forecast system. We find that, when individuals have access only to ex-post self-protection, the existence of actuarially fair but incomplete insurance against weather damage has no effect on the optimal level of expenditure on a warning system, despite the apparent contradictory incentive effects mentioned above. Less surprisingly, we also find that a warning system must be more reliable (and hence more costly) in the presence of insurance than in its absence, if warnings are to induce individuals to engage in ex-post self-protection. As a corollary, we note that the greater is the heterogeneity in damage resulting from weather events, the more likely it is that some subset of the population will (rationally) ignore the warnings generated by an optimal forecast system.

We then look at the influence of insurance and warning systems on ex-ante self-protection via location decisions. Here, we focus on insurance systems that are publicly funded, since in our model actuarially fair private insurance has no influence on location decisions, even though its impact on ex-post self-protection is the same as detailed above. We find that a publicly funded insurance scheme that reimburses (incompletely) the losses of those who locate in a region subject to weather damage induces individuals and productive factors to be *over-allocated* to the risky region. That is, there are more residents living in, and productive factors employed in the risky region than would maximize societal wealth. Indeed, we show that such an insurance scheme results in a Pareto-inefficient allocation of residents and factors. We then show that introducing a forecast system into this environment causes a further allocation of residents and productive factors into the risky region. Although this has the obvious effect of increasing the expected payout of the public insurance system, it is nonetheless possible that the forecast system will increase societal wealth. This is true because it increases the social return to living and investing in the risky region.

2. Basics: Insurance and ex-post self protection

The basic set-up consists of a set of N individuals, who are assumed to care only about expected wealth; that is, they are assumed to be risk-neutral, so that their expected utility is identical to their expected wealth, and we will use the terms interchangeably. We assume risk-neutrality because the issues in which we are interested have to do with moral hazard; the effect of insurance on individual decisions to take risk. These behavioral effects occur even for risk-neutral individuals, hence we choose to avoid the complications that would be introduced by risk-aversion.[3]

The ex-ante wealth of each individual is denoted as W.

There are, for simplicity, two ex-post states of the world each period (where the 'period' in question is best thought of as a day). The unconditional ex-ante probability of state S_1 is p. S_1 is the state in which damage of known amount d - meaning a loss of wealth of that amount - occurs. The no-damage state is S_0.

Thus, in the absence of any protective activity or insurance, each individual's (daily) expected utility is:

$$EU = p(W - d) + (1 - p)W = W - pd$$

However, it is possible for the individual to spend an amount a on protective activity, which is assumed sufficient to eliminate any ex-post damage no matter what the true state turns out to be[4]. If the individual does make this expenditure, their expected wealth is;

$$EU_a = W - a$$

Clearly, risk-neutral individuals will make this expenditure if and only if $pd > a$, or alternatively, if $p > a/d$. The probability of the bad state must be sufficiently high, relative to the cost of protection as a fraction of the damage. In order for the possibility of inducing self-protection to be relevant, we will assume that in fact $pd < a$, so that individuals do not engage in self-protection if p is the only information they have.

Now, suppose there is a Forecasting Service, (FS) which spends the amount e on obtaining and analyzing data and then producing and disseminating a forecast, or more appropriately for our purposes, a *warning*, which results in individuals updating the probability of the two states. The quality of this forecast is $Q(e)$. This function characterizes the quality of the forecast in the following sense. The FS's activity generates a *signal*, F, which can take on the value 0 or 1, and which has the property that:

$$\Pr\{F = 0|S = S_0\} = \Pr\{F = 1|S = S_1\} = Q(e) \in [0, 1]$$

We are assuming here that the probability the forecast correctly predicts the true state is the same for both states. This symmetry assumption is easily relaxed at the cost of some additional notation. We further assume that expenditures on forecasting affect both types of forecast reliability symmetrically, and this feature is also easily generalized.

Given this very simple forecasting technology, an individual who is told by FS that the signal $F = 1$ has been observed (this can be thought of as hearing a 'warning' issued by FS) now believes that

$$\Pr\{S = S_1|F = 1\} \equiv q_1(e, p) = \frac{Q(e)p}{Q(e)p + (1 - Q(e))(1 - p)}$$

[3] Risk neutrality has been assumed previously in modeling self-protection and its implications for societal decisions. See e.g. Anbarci et al. (2005).

[4] Nothing important changes if self protection eliminates only a portion of the damage. What is then important is how much damage is avoided in comparison with the amount of any insurance reimbursement.

using Bayes Rule. Further, we have that q_1 is strictly increasing in Q for $0 < p < 1$, and

$$q_1(e,p) > p \Leftrightarrow Q(e) > 1/2.$$

That is, the warning causes individuals to believe the probability of damage is higher than p (which was their prior belief) if and only if the signal is *informative* in the usual sense; namely, the signal is correct more than 1/2 the time. Since we are assuming that $p < a/d$, this means that a sufficiently informative forecast that $F = 1$ will induce individuals to engage in self-protection. This motivates the following assumptions:

A1: $Q(e) = 1/2$ for $e \leq e_m$ where $e_m > 0$ is some minimum level of expenditure needed to obtain a reliable forecast.

A2: For $e > e_m$, $Q'(e) > 0 > Q''(e)$, with $\lim_{e\to\infty} Q(e) = Q^0 < 1$.

Note that A2 implies $q_1(e,p)$ as well as $Q(e)$ are strictly increasing in e for $e > e_m$.

Of course a forecast can also be in error. Specifically,

$$\Pr\{S = S_0 | F = 1\} \equiv 1 - q_1(e,p) = \frac{(1-Q(e))(1-p)}{Q(e)p + (1-Q(e))(1-p)}$$

is the probability a warning is issued and no damaging event actually occurs. This possibility is relevant to individuals if the warning is sufficiently reliable that it causes them to alter their behavior in any way. In terms of our model, it is relevant precisely when the warning induces individuals to incur the protection cost, a, as $1 - q_1(e,p)$ is then the probability they incur that cost 'unnecessarily'.

We can define $\Pr\{S = S_1 | F = 0\} \equiv q_0(e,p)$, as the posterior probability an individual attaches to the occurrence of the bad state when they hear that the forecast indicates there will not be one (put differently, when no warning has been issued). For the sake of illustration we will identify the bad state as one in which a hailstorm occurs. The bad state will be referred to simply as "hail" and the good state as "no hail". We assume that a hailstorm will cause damage d to one's car unless it is parked under cover, at cost a.[5]

Now the posterior probability of no hail, when that is what the forecast indicates, is:

$$\begin{aligned} \Pr\{S = S_0 | F = 0\} &\equiv 1 - q_0(e,p) = \frac{Q(e)(1-p)}{Q(e)(1-p) + (1-Q(e))p} \\ &> 1 - p \Leftrightarrow Q(e) > 1/2, \text{ also.} \end{aligned}$$

This also implies that $q_0(e,p) < p \Leftrightarrow Q(e) > 1/2$, which is important for answering one further question. If the forecast is known to be informative, does it follow that hearing no warning implies individuals will not take care? Clearly they will not if $q_0(e,p) < a/d$, and since we have assumed that $p < a/d$ this will be the case. Note

[5] We use hailstorms as an example in large part because self-protection (by auto owners) takes a simple form in this case. Hailstorms are an important form of severe weather. Changnon, Changnon and Hilberg (2009) estimate annual hail damage in the U.S. to be $850 million.

however, that if we had started by assuming that $p > a/d$, so that individuals would take care in the absence of further information, there would still be a role for informative forecasts. This is because with probability $1 - p$ individuals are in this case engaging in costly self-protection when it turns out to be unnecessary ex-post. An informative forecast that raised $1 - q_0(e, p)$ sufficiently would induce individuals to avoid paying the cost a when they heard no warning, and would thereby be socially beneficial. A similar analysis to the one being done here can be pursued for that case, and would also be relevant in a world of heterogeneous agents, some of whom engage in self-protection and some of whom do not. For now we leave that issue aside and continue to assume that $p < a/d$.

Maintaining the assumption that $p < a/d$ implies that it is socially wasteful to spend any $e < e_m$. Further, it is wasteful to spend any $e < e_0$, where e_0 is such that:

$$q_1(e_0, p)d = a$$

since for $e < e_0$ we have $q_1(e, p)d < a$, and therefore no one alters their behavior upon hearing that $F = 1$. (This social waste claim of course assumes the forecast has no other value to anyone.)

We can now ask: What is the ex-ante expected wealth of an individual once an amount $e > e_0$ is spent, so that they in fact incur the protection cost whenever they are told $F = 1$ by the FS? We let $Q(e) = Q$ here, for ease of notation.

From the ex-ante perspective there is a probability p of hail, and in that state, the forecast will correctly predict that with probability Q, in which case the cost to the individual will be a, but it will be wrong with probability $1 - Q$, and the cost to the individual will be d. Symmetrically, there is a probability $1 - p$ that the true state will be "no hail", and the forecast will correctly predict that with probability Q, meaning no costs are incurred, while it will be wrong with probability $1 - Q$, in which case the cost of a will be incurred needlessly. The overall expected wealth is thus

$$\begin{aligned} EU(e) &= W - p\,[Qa + (1-Q)d] - (1-p)\,[Q \cdot 0 + (1-Q)a] \\ &= W - [p(1-Q)d + (pQ + (1-p)(1-Q)\,a] \end{aligned}$$

On the other hand, if $e = 0$ is chosen, then the individual will get no warning, will engage in no self-protection activity (by assumption) and so will have expected utility of

$$EU(0) = W - pd$$

The question then is: when is the individual better off because e is spent? A bit of algebra shows that:

$$EU(e) > EU(0) \Longleftrightarrow Q(e) > \frac{a(1-p)}{a(1-p) + p(d-a)}$$

That is, the expenditure of e must be enough to make the forecast sufficiently reliable.

A little more algebra, using the fact that

$$q_1(e,p) = \frac{Q(e)p}{Q(e)p + (1-Q(e))(1-p)},$$

shows that this is equivalent to the condition that $q_1(e,p)d > a$, or $e > e_0$,which is just the condition needed to get an individual to act upon the forecast of hail. Thus, we have the following:

Result 1: An individual is made ex-ante better off by an expenditure of e iff the expenditure produces a warning quality $Q(e)$ which is high enough to induce him to engage in protection.

Note, however, that Result 1 does not imply that an expenditure, e, sufficient to induce self-protection is socially beneficial, since it does not account for the cost of producing the forecast, e. Thus, the question of whether an individual is made ex-ante better off by the existence of forecasting turns on whether it is true that:

$$EU(e) - s_i e - c_i \geq EU(0)$$

where s_i is a particular individual's *share* of the total expenditure of e on the forecast, and c_i is any individual cost incurred to acquire access to the warnings issued by FS. Either or both of these costs may be zero for some individuals, depending on the method of providing and financing such forecasts. However, even if this inequality is reversed for some particular individual, meaning they would prefer that no forecasting be financed, it is still the case that if an amount $e > e_o$ is collected (perhaps via the tax system) and spent on forecasts, such an individual will heed any forecast warning, and will be better off for having done so. Ex-ante he may prefer that e not be spent, but if it is, ex-post he will take advantage of what it buys.

3. Insurance

3.1. Insurance with identical individuals

Now suppose that insurance against weather damage is available. An individual can pay a premium m, in return for a payment of θd in the event damage of d is incurred. Thus, $\theta \in [0,1]$ is the co-payment rate; the amount of the loss covered by the insurance policy[6].

First, if we continue to assume that $p < a/d$, the individual who buys such a policy in the absence of any forecasting activity will not also take care, since it will only pay to do so if $p > a/(1-\theta)d > a/d$. More generally, insurance and self-protection will never both be engaged in by any individual if they are both available. An insurance policy with premium/co-payment combination (m, θ) will be chosen over complete self-protection with cost a if and only if: $a > m + p(1-\theta)d$. Thus, in particular, m must be strictly lower than a if the insurance policy does not fully cover losses (i.e., if $\theta < 1$).

[6] Given our assumption of a fixed loss, d, there is no distinction between a co-payment and a deductible.

Second, since we are assuming that doing nothing is preferable to self-protection when there is no FS, an individual would voluntarily buy the policy only if $W - m - p(1-\theta)d > W - pd$, or $p > m/\theta d$. This requires that $a/d > p > m/\theta d$, which in turn implies that $a > m/\theta$. Thus, the premium cannot be too high, nor the co-payment rate too low.

Risk-neutral individuals have no reason to purchase insurance unless it is beneficial in an actuarial sense. This means it must be that $p\theta d \geq m$; the expected payment from the policy must be at least as much as the premium. We consider here a system of mandatory publicly provided insurance, and assume that it is actuarially fair; that is, the premium payment is $m = p\theta d$, which means individuals are made no better or worse off by this insurance system. We continue to assume they prefer not to engage in self-protection.

The previous analysis of the effect of expenditures e on behavior is now modified as follows. Spending e on forecasting allows the FS to produce warnings of quality $Q(e)$, as determined by the current technology of forecasting. However, in order for this forecasting to have any effect on individual behavior, it must now be the case that an amount e has been spent such that:

$$
\begin{aligned}
a &< q_1(p,e)(1-\theta)d, \\
& \text{or} \\
q_1(p,e) &\geq q_1(p,e_\theta) \equiv \frac{a}{(1-\theta)d} > \frac{a}{d} \equiv q_1(p,e_0)
\end{aligned}
$$

which implies that $e_\theta > e_0$. The intuition is simple: once individuals have even partial insurance, warnings must be more reliable before they are heeded, because the damage one risks by not heeding them is reduced by the insurance payout. Therefore a useful forecast system must involve higher expenditure, and the greater is the coverage of damages, θ, the higher is e_θ. This gives:

Result 2: The existence of universal insurance coverage for weather related damage implies that a warning system must be more reliable, and hence more costly, than would be true without insurance before it can induce self-protection.

3.2. Heterogeneous individuals

We now consider some of the additional issues that arise when individuals are not homogeneous. There are many possible sources of heterogeneity, but our interest here is in heterogeneity that will induce individuals to respond differently to forecast warnings by the FS. We start by supposing that individuals vary in the level of damage they will sustain if state S_1 occurs. In particular, assume there are *high* types who sustain damage d_h in state S_1 and *low* types who sustain damage d_l, with $d_l < d_h$. We further assume that when there is no FS or insurance:

$$\frac{a}{d_h} < p < \frac{a}{d_l}$$

which implies that in the absence of warnings, the high types will incur the cost a while the low types will not. Further, there are two ways in which reliable (meaning

$Q > 1/2$) warnings that the state is likely to be S_1 can usefully alter behavior. Most obviously, such warnings may induce the low types to spend a to avoid damage d_l when S_1 is likely. Further, if warnings are known to be reliable, then the absence of a warning may induce the high types to save the cost a when in fact no warning is given. Since $q_1(p, e)$ is increasing in e, if the low types are to take care when a warning is issued, expenditure e on the FS must be such that

$$q_1(p, e) \geq \frac{a}{d_l}$$

while if the high types are to not take care when no warning is issued, it must be that

$$q_0(p, e) \leq \frac{a}{d_h}$$

Recall that q_0 is the posterior probability someone who hears no warning attaches to the state S_1 in which they are damaged, and it is easy to show that this is decreasing in e if $Q(e)$ is increasing in e. Thus, there is some sufficiently accurate warning system such that both of these conditions can be satisfied, and both types of individuals will therefore respond appropriately to warnings (and to the absence of warnings). As noted before, this does not mean that these individuals will have no regrets regarding their choices once the true state becomes apparent (that is, it either hails or it does not). By assumption it is not possible to spend enough to have $q_1(p, e) = 1$, and therefore $1 - q_1(p, e)$ will always be positive; this is the probability a warning is issued, all types incur the cost a, and no damage occurs . Similar remarks apply to the probability of hail occurring when no warning is issued.

If expenditure levels e' and e'' are such that $q_1(p, e') = \frac{a}{d_l}$ and $q_0(p, e'') = \frac{a}{d_h}$, respectively, then it must be that the maximum of e' and e'' is spent in order to have both types respond optimally to warnings. This makes it clear that in a society in which the damage from hail varies in the population between some $d_{\min}$ and $d_{\max}$, and the cost of protecting oneself against this damage also varies over some interval between $a_{\min}$ and $a_{\max}$, then getting all individuals to respond appropriately to warnings requires an e^+ such that

$$\begin{aligned} q_1(p, e^+) &\geq \max\{\frac{a}{d}\}, \text{ and} \\ q_0(p, e^+) &\leq \min\{\frac{a}{d}\} \end{aligned}$$

It is apparent that in an actual population of any size there will be wide variation in both these parameters, and thus the range of the parameter a/d is potentially very close to the entire interval $[0, 1]$.[7] It would then be the case that e^+ would have to be very high to satisfy both these conditions, if indeed it is even possible to spend enough on FS to do so. Such a level of expenditure may be neither politically feasible, nor even prudent if it is feasible. The actual level of e may be such that those with

[7]Note that the range cannot be greater than this so long as a is positive and less than d for all individuals; any individuals for whom $a > d$ will not take care under any circumstances, nor would society want them to.

an extremely low value of a/d take care all the time, irrespective of any issuance of warnings, and those with a very high a/d value ignore warnings and never incur their particular self-protection costs. Matters are rendered even more complex when it is recognized that in reality any FS will provide warnings regarding many different events, each of which affects many different individuals. Thus any FS, including one that is in some specified sense 'optimal', will necessarily provide warnings which some individuals will quite rationally ignore.

3.3. Optimal expenditures on warnings

We now analyze the determinants of the socially optimal level of expenditure, e, and how it is affected by the presence of universal insurance against weather-related losses. Since individuals are assumed to care only about their expected wealth, our optimality criterion is the sum of individuals' expected wealth minus the cost of the FS. We again consider a world in which all individuals are identical, as analyzed earlier.

In the absence of any insurance system, the socially optimal expenditure on e is that which maximizes

$$n\,[EU(e)] - e. \tag{3.1}$$

However, the value of $EU(e)$ depends on the responses of individuals to any warnings. If a value of $e < e_0$ is spent, then individuals' behavior (and therefore their ex-ante expected wealth) is not dependent on whether the FS issues a warning, even though it does cause them to revise the probability they attach to each state. If it is assumed that $p < a/d$, then any $e < e_0$ results in a $q_1(p, e) < p < a/d$, and from an ex-ante perspective (that is, before learning whether or not a warning has been issued) $EU(e) = W - pd = EU(0)$. It is then apparent that (3.1) has the value

$$n\,[W - pd] - e$$

which is decreasing in e. This implies that if $e < e_0$ is spent on FS, the only non-wasteful level of such expenditure is 0. On the other hand, if $e \geq e_0$ is spent, we know from above that:

$$\begin{aligned} EU(e) &= EU_a(e) = W - p\,[Q(e)a + (1 - Q(e))d] - (1 - p)\,[(1 - Q(e))a] \\ &= W - p\,(1 - Q(e))\,d - [pQ(e) + (1 - p)(1 - Q(e)]\,a, \end{aligned}$$

and therefore (3.1) takes the value:

$$n\,[W - p\,(1 - Q(e))\,d - [pQ(e) + (1 - p)(1 - Q(e)]\,a] - e \tag{3.2}$$

If $e^* > e_0$ is the level of expenditure on FS that maximizes this function, it must necessarily satisfy the condition:

$$Q'(e^*)\,(pd + (1 - 2p)a) - \frac{1}{n} = 0 \tag{3.3}$$

The second order condition for a maximum is that:

$$Q''(e^*)\,(pd + (1 - 2p)a) < 0$$

and since we are assuming that $Q'' < 0$, this requires the term in brackets to be positive. In fact it is possible that $(pd + (1-2p)a) < 0$, if $p > 1/2$. If this is the case, then the function (3.2) is decreasing in e for all $e > e_0$, and therefore the optimal level of e, conditional on $e \geq e_0$, is exactly $e^* = e_0$. Whether in this case e_0 is globally optimal depends on how the value of $nEU(0)$ compares to the value of $nEU(e_0) - e_0$. However, note that e_0 is precisely the value of e at which $Q(e_0)$ produces warnings accurate enough to leave individuals indifferent between spending a to avoid d and not doing so. This means $EU_a(e_0) = EU(0)$, and therefore

$$n\left[EU_a(e_0)\right] - e_0 = n\left[EU(0)\right] - e_0 < n\left[EU(0)\right]$$

Spending exactly e_0 on FS can never be optimal. Thus, there are only two possible socially optimal outcomes in this simple world: spend nothing, or spend an $e^* > e_0$ such that (3.3) is satisfied, and this latter can be the optimal solution only if $(pd + (1-2p)a) > 0$. This is all illustrated in Figure 1.

We can now ask: how is the socially optimal value of e altered by the presence of universal, actuarially fair insurance? We saw above that such insurance does not make individuals any better off under risk-neutrality. However, we also saw that its presence increases the level of expenditure needed to induce individuals to act on any warnings issued by the FS. Thus, by analogy with the analysis above, any level of $e \leq e_\theta$ results in a value for (3.1) of:

$$n\left[EU(0)\right] - e$$

Suppose an amount $e \geq e_\theta$ is spent, however. First, note that independently of the insurance premium an individual pays, this will induce them to spend a to avoid the uninsured loss $(1-\theta)d$. However, once this happens, then the insurance company will in fact only face an expected loss from a policy sold to these individuals of $p(1-Q(e))\theta d$. This is because the insured will incur a loss only when the forecast system is wrong, in that it does not predict the bad state correctly. We assume here that the policy does not pay for the cost of avoiding damage. This means that the insurance premium will now depend on the reliability of the warning system, and so we write it as[8]:

$$m(e)\begin{cases} = p(1-Q(e))\theta d, \text{ if } e \geq e_\theta \\ = p\theta d, \text{ if } e < e_\theta \end{cases}.$$

This is obviously decreasing in e when $e \geq e_\theta$, providing a second benefit to individuals from any improvement in the reliability of the warning system. For values of $e \geq e_\theta$, the value of social wealth can be written as:

$$\Phi_\theta(e) = nEU_\theta(e) - e = n\left[W - p\left(1-Q(e)\right)(1-\theta)d - \left[pQ(e) + (1-p)(1-Q(e)\right]a - m(e)\right] - e$$

and using the definition of $m(e)$, this is in fact equal to:

$$n\left[W - p\left(1-Q(e)\right)d - \left[pQ(e) + (1-p)(1-Q(e)\right]a\right] - e = nEU_a(e) - e = \Phi_a(e)$$

[8]This specific fact is the result of our assuming that spending a eliminates *all* damage. If we had assumed that it only reduces the damage done, the insurance company would also have to make a payment when there was a warning in the bad state, but an actuarially fair premium would still depend on e.

This has the following implications. First, social welfare has the same value, with and without insurance, for any $e < e_0$ and any $e > e_\theta$. However, for $e \in [e_0, e_\theta]$, the value of social welfare under insurance remains $n(W - pd) - e$, because e is not high enough to induce self-protection, while without insurance it is, and social welfare is thus $\Phi_a(e) > n(W - pd) - e$. This in turn means that the value of social welfare in the presence of insurance has a discontinuity at e_θ, increasing discretely because the change in individual behavior causes a discontinuous decrease in the premium for actuarially fair insurance[9]. This then implies that the two social welfare functions must, when $(pd + (1 - 2p)a) > 0$, look as in Figure 2. Thus, the optimal level of expenditure on such a warning system is unaffected by the introduction of actuarially fair insurance.

This gives us our major finding with regard to warning systems.

Result 3: With identical individuals the existence of universal actuarially fair insurance against weather damage implies that:

i) the minimum expenditure required to develop a warning system that affects individual behavior is higher than it would be without the insurance system, however

ii) the availability of actuarially fair insurance has no effect on the optimal level of expenditure on a warning system.

4. Ex-ante self protection

Up to now we have focused on an environment in which the only way individuals can respond to weather hazards is to buy insurance beforehand, or to react to warnings when they are issued. However, it is clear that weather hazards are often not uniformly distributed across the regions of a political jurisdiction. For the US, as an example, hurricanes are not an issue for those who live in Minnesota but are for those in Florida or Louisiana, whereas flooding rivers are not a concern for (most) individuals who live outside a flood plain. That suggests that the existence and nature of both insurance schemes and weather warning systems will have an impact on long-term decisions by individuals regarding where they will live, work and invest.[10] These decisions therefore include an element of ex-ante protection, and that is the issue we take up in this section.

4.1. A model of regional weather risk

Consider then a country divided into two regions. We let the regions be denoted as A and B, and be indexed by $j = a, b$. Production can take place in both regions, and individuals can live in either region. The technological possibilities in each region are given by a production function $F_j(N_j, K_j)$ where N_j is the number of workers in region j, while K_j are the units of capital devoted to production in region j. N and K differ in that it is assumed that workers must live in the region in which they

[9] Alternatively, if the premiums in a government-run insurance program were not reduced, it would run a surplus once $e > e_\theta$, and this could be used to provide something which individuals valued.

[10] See Barnett (1999), Kunreuther and Michel-Kerjan (2009) and Michel-Kerjan (2010) for discussion of how some of these effects have worked out in practice in the U.S.

supply their labor services, but this is not the case for capital owners. Both workers and capital units are assumed to be infinitely lived. All workers are assumed equally skilled, for simplicity, and capital is also homogeneous. The single good produced in each region is assumed identical and freely transportable between regions, so it has a common price that we take to be 1 without loss of generality.

Workers in region j are paid a wage $w_j = F'_{jN}(N_j, K_j)$ and so the value of their stock of human capital if they locate in region j is Rw_j where $R = (1+r)/r$, with r being the interest rate, which we assume fixed and exogenous. Capital is paid its marginal product, $\rho_j = F'_{jK}(N_j, K_j)$, also. Assuming zero depreciation, it has a value of $R\rho_j$.

All individuals get a return of v_j from living in region j, in addition to the returns to the factors they provide in production. More will be said below about what comprises v_j for each j, but we note here that v_j can be interpreted as the wealth equivalent of the utility one gets from living in region j. Thus, $v_a - v_b$ is the wealth an individual would give up to live in region A rather than B.

Then the lifetime wealth of an individual who lives and works in region j is

$$v_j + RF'_{jN}(N_j, K_j)$$

and the lifetime wealth of a capitalist who lives in region i and invests his capital in region j is

$$v_i + RF'_{jK}(N_j, K_j)$$

where to keep the notation simple, we have assumed that each capitalist owns one unit of capital.

We assume that the two regions differ in two ways. First, as indicated by the notation above, the production technology may differ between regions. Second, region B can be in either of two states, as previously, whereas in region A there is no uncertainty. In state 0, workers in region B get a lifetime payoff of

$$v_b + RF'_{bN}(N_b, K_b)$$

However, in state 1, we assume a damaging weather event, which we will refer to as a hurricane, occurs, and region B workers get a lifetime payoff of:

$$v_b - d + R_d F'_{bN}(N_b, K_b)$$

where $R_d < R$. Note that, for simplicity, we assume that any severe weather is a "one off" event.

The idea is that the hurricane does damage to the productive facilities in region B which interrupts the flow of wages a worker in region B gets, and this is why the present value of lifetime earnings is $R_d w_j$.[11] This formulation allows us to forego modeling the process of destruction and re-building directly. In addition, the hurricane causes a reduction in the direct utility from living in region B. This reduction can

[11] The fact that $R_d < R$ is not due to any change in the discount rate, r, which we have assumed is fixed and exogenous - - as for a small country operating in a world capital market in long-run equilibrium. The fall from R to R_d reflects a loss of real earnings for some period of time.

in principle have as many dimensions as does v_j itself. In particular, v_j will typically include the utility flow from private durable goods that are damaged or destroyed by the hurricane. We denote the utility flow as W_j, and the reduction in this flow is denoted as d_D. v_j may also include the utility flow from various public attributes of region j, whether these are natural, or man-made (in which case they are typically referred to as 'infrastructure'). This can be denoted as g_j and the reduction in it denoted as d_g. Finally, there may be a direct utility loss due to injury or death of oneself or one's loved ones. This damage can be denoted as d_u. Thus, we have that $v_j = W_j + g_j + u$, and $d = d_D + d_g + d_u$.

If the probability of a hurricane's occurrence is p, then expected lifetime wealth of a worker in region B is

$$v_b - pd + \{pR_d + (1-p)R\}F'_{bN}(N_b, K_b)$$

A capitalist who invests in region B gets an expected return of:

$$\{pR_d + (1-p)R\}F'_{bK}(N_b, K_b)$$

It is not important for our results that the value of R_d for workers and capitalists be the same, but in the absence of any reason to make them different, we will assume that they are. It is also clear that if all capitalists have identical returns from living in the two regions, then all will choose to live in the same region, with the choice depending on the relative magnitude of v_a and $v_b - pd$. So as to avoid having to deal with possible changes in the residence choice of *every* capital owner, we will assume that in fact capital owners' v_j are distributed over a set $V^k = \{(v_a^k, v_b^k) | v_j^k \in [0, \overline{v}]$, for $j = a, b\}$ and let $l_a(pd) = \{k | v_a^k > v_b^k - pd\}$, and $l_b(pd) = \left\{k | v_a^k \leq v_b^k - pd\right\}$, with $k_a(pd) = \# l_a(pd)$.[12] Then our assumption that each capitalist owns one unit of capital, means that $k_b(pd) = K - k_a(pd)$, where K is the total capital stock available. Note also that the fact that capital owners need not live where their capital is invested means that their residential decisions are determined entirely separately from the other decisions in the model.

The total expected payments to factors of production employed in region b will be:

$$N_b\{pR_d + (1-p)R\}F'_{bK}(N_b, K_b) + K_b\{pR_d + (1-p)R\}F'_{bN}(N_b, K_b)$$

We assume the regional production functions F_j exhibit constant returns to scale (CRS), meaning there is free entry into production in each region. This implies that these expected region B payments are equal to:

$$\{(1-p)R + pR_d\}F_b(N_b, K_b)$$

which is the expected value of total production in the region. Similarly, $RF_a(N_a, K_a)$ is the present value of production in region A and is equal to payments to the factors

[12] We could also introduce heterogeneity in (v_a, v_b) across workers, but this would add notational complexity without adding any new insights.

employed there. Thus, there are no other payments to account for in either region, and the expected value of societal wealth can be written:

$$\sum_{k \in l_a(pd)} v_a^k + \sum_{k \in l_b(pd)} (v_b^k - pd) + N_a v_a + N_b(v_b - pd) + \qquad (4.1)$$
$$\{(1-p)R + pR_d\} F_b(N_b, K_b) + RF_a(N_a, K_a)$$

as a function of the allocation of factors and residents across the two regions. As noted above, the residence decisions of capitalists will be based on a binary comparison that is independent of the decisions of workers and independent of their own investment decisions, so we have left this part of capital owners' payoffs out of this expression for now.

With a given supply of workers, N, and of capital, K, in equilibrium workers will choose to locate in the region that maximizes their lifetime wealth, and capital owners will supply their capital to whichever region maximizes its contribution to their lifetime wealth. This means that the equilibrium allocation of productive factors, if both factors are employed in both regions, will have to satisfy:

$$v_a + RF'_{aN}(N_a, K_a) = v_b - pd + \{pR_d + (1-p)R\}\, F'_{bN}(N_b, K_b) \qquad (4.2)$$

and

$$RF'_{aK}(N_a, K_a) = \{pR_d + (1-p)R\}\, F'_{bK}(N_b, K_b) \qquad (4.3)$$

so long as all the second derivatives, F''_{jNN} and F''_{jKK} are negative, so that any reallocation of either factor causes a reduction in the return to the reallocated factor's owner. In addition of course it will have to be that:

$$\begin{aligned} N_a + N_b &= N \\ K_a + K_b &= K \end{aligned}$$

Note that if we let $pR_d + (1-p)R \equiv R(p)$, then the fact that $R(p) < R$ means that the return to capital will have to be higher in region B in order to compensate those who invest in the riskier region. Similarly, either v_b must be higher than v_a for workers, or wages must be higher in region B, or both, in order to compensate workers for living and working in that region.

If an allocation is to maximize the total of societal wealth across the two regions, however, it will have to maximize the expression (4.1) above subject to these last two resource constraints. The necessary first-order conditions for this constrained maximization problem are:

$$\begin{aligned} v_a + RF'_{aN}(N_a, K_a) &= \lambda_N \\ v_b - pd + R(p)F'_{bN}(N_b, K_b) &= \lambda_N \\ RF'_{aK}(N_a, K_a) &= \lambda_K \\ R(p)F'_{bK}(N_b, K_b) &= \lambda_K \end{aligned}$$

Which imply exactly the conditions (4.2) and (4.3). We can sum this up as follows:

Result 4: In the absence of any insurance scheme or weather warning system, individuals will make location decisions that maximize societal wealth.

4.2. Insurance

If private insurance contracts were available to residents of region B, they would not typically provide full coverage of the entire loss $d = d_D + d_g + d_u$. We can imagine then a residential policy that pays any purchaser some amount θd, where $\theta \in (0, 1)$. An actuarially fair premium for such a 'residential' policy would be $m = p\theta d$, and it is easy to show again that such a policy will have no impact on the utility of anyone locating in region B who buys it. Thus, individuals who locate in region B will be indifferent about purchasing such a policy, and its availability will have no effect on individual location decisions.

Similarly, privately offered 'business insurance' policies, paying those who lose wage or capital income in state 1 in region B some amount c_s, where for workers, $0 \leq c_w < (R - R_d)F'_{bN}$ and for capital owners, $0 \leq c_\rho < (R - R_d)F'_{bK}$, will have no effect.

If actuarially fair public insurance were provided it also would have no effect on location decisions. In the U.S. the National Flood Insurance Program (NFIP) provides mandatory insurance for any residence covered by a federally insured mortgage. This coverage is, however, explicitly and implicitly subsidized. (See Barnett, 1999, and http://www.fema.gov/nfip/qanda.shtm.) NFIP policies may be thought of as bundling two policies: one that is actuarially fair and additional coverage paid for by taxpayers. The latter coverage may affect locational decisions. It is an example of a type of public "insurance" that is widely seen, for example in disaster relief programs.

Suppose that (still incomplete) residents' coverage is made available by the highest level of government to all who choose to locate in region B, and this scheme is financed by a general tax on *all* individuals.[13] We denote the expected per-person tax payments required to finance such a plan by t, and assume the policy pays for the proportion $\theta \in (0, 1)$ of losses. There are at least two reasons to set $\theta < 1$. First, it may be that a publicly funded insurance system does not provide full coverage, just as with private insurance. In particular, public plans typically cover at most some part of the loss d_D as defined above. Second, it may be that the public plan is not actually an insurance program that guarantees ex-ante to provide any reimbursement for losses. Rather, it may simply be that individuals understand that there is a positive probability the government will step in to provide some level of disaster relief *if* a hurricane or flood causes damages. Thus, we can write $\theta = \eta\alpha$ to indicate that the government will reimburse the proportion $\eta < 1$ of losses with probability $\alpha < 1$. However θ is interpreted, the government budget constraint implies that t must be set so that :

$$\{(K - k_a^k) + (N - N_a)\}p\theta d_D = (N + K)t$$

This means that the 'expected tax payment per capita' required to finance such a

[13] As explained above, part of NFIP coverage in the U.S. is effectively of this type. Further coverage is provided in the form of disaster relief by the Small Business Administration (SBA) and the Federal Emergency Management Agency (FEMA). Disaster relief is contingent on an official disaster declaration, and may take the form of low-interest loans or housing grants. See Kunreuther and Michel-Kerjan (2009) and Michel-Kerjan (2010).

scheme will depend on the ultimate equilibrium location decisions, as:

$$t(N_a, k_a^k, N, K, p\theta d) = \frac{\{(K - k_a^k) + (N - N_a)\}p\theta d_D}{N + K}$$

which is less than $p\theta d_D$, unless $N_a = k_a^k = 0$. (From here on we let $d_{-D} = d_u + d_g$.)

The imposition of such a scheme implies that the utility portion of the payoff to someone choosing to locate in region B is now:

$$v_b - \left\{\frac{(K - k_a^k) + (N - N_a)}{N + K}\right\} p\theta d_D - p\left[(1-\theta)d_D + d_{-D}\right] > v_b - pd$$

Writing the tax payment more simply as $t(N_a, k_a^k)$, the equilibrium allocation of resources in the presence of such mandatory, publicly funded insurance will be determined by the conditions:

$$v_a + RF'_{aN}(N_a, K_a) - t(N_a, k_a^k) = v_b - p\left[(1-\theta)d_D + d_{-D}\right] + R(p)F'_{bN}(N_b, K_b) - t(N_a, k_a^k)$$

and

$$RF'_{aK}(N_a, K_a) = R(p)F'_{bK}(N_b, K_b)$$

which can be re-written as:

$$v_a - v_b + p\left[(1-\theta)d_D + d_{-D}\right] = R(p)F'_{bN}(N_b, K_b) - RF'_{aN}(N_a, K_a) \tag{4.4}$$

and

$$0 = RF'_{aK}(N_a, K_a) - R(p)F'_{bK}(N_b, K_b) \tag{4.5}$$

The second of these is not different from the condition that determined equilibrium without insurance, but the first condition in the absence of insurance, was:

$$v_a - v_b + pd = R(p)F'_{bN}(N_b, K_b) - RF'_{aN}(N_a, K_a)$$

Suppose for the moment that in fact capital is not allocated differently with and without insurance. Then, since $pd > p[(1-\theta)d_D + d_{-D}]$, it must be that introducing the public insurance plan causes labor to be allocated so as to decrease the value of $R(p)F'_{bN} - RF'_{aN}$ relative to the value without insurance, and our assumption that $F''_{jNN} < 0$ then requires that workers be reallocated from region A to B. If we assume that inputs are complementary in each F_j[14], then this in turn means that (4.5) cannot be satisfied unless capital is reallocated from region A to B, also. We can therefore conclude that such an insurance scheme has the effect of reallocating some of both factors of production from region A to B.

In addition, the original set of capital owners who chose to reside in region A rather than B would be those for whom the condition

$$v_a^k - v_b^k \geq pd$$

[14] Specifically, we assume that $\partial^2 F_j / \partial N \partial K > 0$ for both j, meaning an increase in either input increases the marginal product of the other factor.

holds, whereas now the region A resident capital owners would be those for whom the condition

$$v_a^k - v_b^k \geq p[(1-\theta)d_D + d_{-D}]$$

holds. Again, this implies a possible shift in the residence choices of capitalists from A to B, because $\theta > 0$.

However, it is easy to show that the presence of such a publicly funded insurance program does not change the value of societal wealth for *any* given allocation of factors and residents across regions, as it is a purely redistributive program. This implies that the allocation induced by a publicly funded insurance system cannot be wealth-maximizing, since it entails a reallocation of factors and residents from region A into region B, compared to the wealth-maximizing allocation. More simply, this new equilibrium allocation cannot satisfy the conditions necessary for wealth-maximization.

In principle, it could still be true that the public insurance induced allocation is Pareto efficient, even though it does not maximize societal wealth. However, this is not the case. Consider first a capitalist who is induced to move his place of residence from region A to region B by the institution of the publicly funded insurance scheme. For such an individual it must be the case that

$$v_b^k - p[(1-\theta)d_D + d_{-D}] > v_a^k > v_b^k - pd$$

The first inequality implies he prefers region B with the insurance in place, the second implies that he prefers region A when it is absent. Now imagine offering this capitalist the following deal: the government insurance system will continue to pay him a grant of θd_D if state 1 occurs in region B, even if he moves to region A. This makes his expected payoff if he moves to region A:

$$v_a^k + p\theta d_D$$

which is greater than his payoff above from staying in region B, since it is $v_b^k - pd + p\theta d_D < v_a^k + p\theta d_D$, by the second inequality above. This arrangement thus makes this individual better off if he stays in region A, while keeping the insurance system completely funded, as neither tax payments nor the payout in either state have changed. This is therefore a Pareto improvement over the insurance-induced allocation of individuals across regions, *if* the insurance system actually induced some capitalists to move from region A to B. However, it is possible in our model that V and p, θ, d are such that all capitalists locate in the same region both with and without insurance. Further, this argument does not establish that factors are being used inefficiently in production.

However, a similar argument establishes that labor, because it is 'attached' to workers who must decide where to live, is being allocated inefficiently. Consider a worker who lives and works in region B in the equilibrium induced by the insurance scheme. Her expected payoff there is

$$v_b - p[(1-\theta)d_D + d_{-D}] - t + R(p)F'_{bN} = v_a - t + RF'_{aN}.$$

Now imagine offering her the same deal as above: an agreement to pay her θd_D in state 1 if she moves to region A and goes to work there. It is immediate that this increases her expected payoff in region A to be more than it is currently in B, since they were equal in equilibrium, and so she will accept the deal and live and work in region A. The insurance program again remains balanced, and the net change in the lifetime value of production due to her move is: $RF'_{aN} - R(p)F'_{bN}$, which is exactly what is required to accommodate the change in wages she receives due to her change in region of employment. Thus, this worker is better off, and no one else is affected, meaning again that the insurance-induced allocation cannot be Pareto efficient.

This establishes a further result:

Result 5: A publicly funded insurance scheme induces a re-allocation of factors of production and residents into regions that are subject to greater weather risk. This reallocation reduces aggregate expected wealth, and results in an allocation of residents and factors that is Pareto inefficient.

4.3. Warnings and insurance

Now we consider the effect of a warning system of quality $Q(e)$, just as above. Assume that the residents of region B can completely avoid the utility loss d_u in state 1, at a cost a, where $d_u > a$. They may do so, for example, by fleeing when they receive a hurricane warning.[15] We will assume that the warning does not enable individuals to avoid any of the other losses incurred in state 1, for simplicity. Then just as before, there will be a level of expenditure, e_u, which satisfies:

$$q_1(e_u, p)d_u = a,$$

which has the property that individuals who hear a warning of quality $Q(e)$ will incur cost a to avoid the loss d_u if and only if $e > e_u$. If the warning is of this quality or higher, then the payoff (gross of any costs related to the warning system) to an individual who lives in region B becomes:

$$\begin{aligned} & v_b - p\left[Qa + (1-Q)d_u + d_{-u}\right] - (1-p)(1-Q)a \\ = \quad & v_b - pd + pQ(d_u - a) - (1-p)(1-Q)a^{16} \end{aligned}$$

plus the returns to their factor supply. This is clearly increasing in Q and therefore in e, when $e > e_u$.

The analysis of previous sections can be applied to show once again that if there is an actuarially fair private insurance system available to residents of region B, they

[15] In practice, it sometimes becomes difficult to flee a region where a hurricane is predicted to strike due to congestion on the roads. This is an example of the kind of negative externalities in self-protection discussed by Ehrlich and Becker (1972) and Lakdawalla and Zanjani (2005). We do not introduce such externalities here explicitly. They may be thought of as increasing the self-protection cost, a, which would lead to an equilibrium with fewer people locating in region B. This is a case where a negative externality in ex post self-protection could help to reduce a distortion caused by insurance.

will be indifferent about purchasing it. Furthermore, it will have no impact on their decision regarding where to locate. For those who locate in region B and do purchase this private insurance (and therefore for all residents, if the scheme is compulsory) a level of e greater than the value e_u defined above will have to be spent before they heed any warnings and incur the cost a.

We will ignore in what follows any private insurance, and consider the joint impact of a warning system and a public insurance scheme, since the latter is such a common feature of government policy.

Suppose then that a public insurance scheme is in place as above, with θ again being the proportion of the loss d_D that the government will (or might) cover in the event that state 1 occurs. As shown above, this will have the effect of increasing the number of individuals who locate in region B, as well as shifting inputs and production into the region, relative to the levels that would occur without this scheme.

With no warning system, the equilibrium location and factor employment decisions will be determined by the following conditions:

$$v_a + RF'_{aN}(N_a^0, K_a^0) = v_b - p\zeta_0 + R(p)F'_{bN}(N_b^0, K_b^0) \tag{4.6}$$

$$RF'_{aK}(N_a^0, K_a^0) = R(p)F'_{bK}(N_b^0, K_b^0) \tag{4.7}$$

$$l_a(\zeta_0) = \{k | v_a^k > v_b^k - p\zeta_0\} \tag{4.8}$$

where $\zeta_0 = (1-\theta)d_D + d_{-D}$.

Introducing the warning system and allowing individuals and factors to relocate if they wish results in a new equilibrium that is characterized by three conditions that are identical to those above, except that ζ_0 is replaced by

$$\zeta_Q = (1-\theta)d_D + Qa + (1-Q)d_u + (\frac{1-p}{p})(1-Q)a.$$

The same analysis as above shows that a warning system of quality Q will make those located in region B better off if and only if it induces them to incur the cost a whenever a warning is issued. If an amount e is spent such that $Q(e)$ is high enough for that to be the case, this implies that $\zeta_0 > \zeta_Q$, and therefore such a warning system will induce more individuals to locate in region B than is the case with just the public insurance scheme. Because N_b will rise, this will also induce an increase in K_b. It might then seem that introducing such a warning system must decrease social wealth; it is costly, as some sufficiently high e must be spent to have $Q(e)$ high enough, and the warning system then exacerbates what is already an over-allocation of individuals and factors into the risky region. In fact this is not the case.

To see this most easily, consider first an ideal warning system. That is, assume that it operates at no cost, so $e = 0$, and is completely accurate, so $Q = 1$. The introduction of such a warning system will induce an allocation of resources determined by the three conditions above, except that ζ_Q has the value: $\zeta_1 = (1-\theta)d_D + a$.

Since $a < d_u < d_{-D}$, it follows that $\zeta_1 < \zeta_0$, and this system will again induce a further re-allocation of factors and residents into region B even in the presence of the public insurance scheme. However, it will also unambiguously increase societal wealth. To see this, note that social wealth can be written as:

$$\begin{aligned}\Gamma(\zeta) &= \sum_{k\in l_a(\zeta)} v_a^k + \sum_{k\in K\setminus l_a(\zeta)} [v_b^k - p\zeta] + \\ &\quad N_a v_a + (N-N_a)(v_b - p\zeta) + RF_a(N_a, K_a) + R(p)F_b(N-N_a, K-K_a)\end{aligned}$$

The equilibrium value of this expression with insurance but no warning system will be $N_a^0, K_a^0, l_a(\zeta_0)$, as determined by the conditions given above. Introducing the 'ideal' warning system results in values determined by those same conditions, except that ζ_0 is replaced by ζ_1. Now, imagine that in fact the warning system is introduced in the presence of an existing public insurance scheme, but individuals and factors are prevented from relocating. It is immediate that the value of the first two sums of $\Gamma(\zeta_0)$ can be increased (or at least cannot decrease) if capital owners are allowed to change their region of residence if they wish. The payoff to living in region B is increased by this warning system, and so any relocating capital owner is better off, with no one else being affected by their move.

Suppose now that labor is allowed to move to region B if it wishes. The derivative of $\Gamma(\zeta_0)$ with respect to N_a is:

$$\frac{\partial\Gamma}{\partial N_a} = v_a - v_b + p\zeta_1 + RF'_{aN}(N_a^0, K_a^0) - R(p)F'_{bN}(N - N_a^0, K - K_a^0)$$

and a comparison with (4.6), and the fact that $\zeta_1 < \zeta_0$, means that this expression is negative. Social wealth can be increased by a decrease in N_a, starting from N_a^0. This in turn means that it can be further increased by a reduction in K_a from K_a^0, because of complementarity. It is clear that social wealth is increased by these relocations up to the point at which the conditions (4.6) to (4.8) - with ζ_0 replaced by ζ_1 - are satisfied.

Therefore, such an ideal warning system does increase social wealth, and it follows immediately that a warning system that can attain a sufficiently high value of Q, at a low enough cost, can also increase social wealth. It will of course be true that the relocation into region B induced by the warning system will cause the public insurance scheme's expected payouts to rise. That fact does not alter the findings above.

Result 6: An effective warning system will cause factors and residents to relocate into regions characterized by weather risk, even in the presence of a publicly funded insurance system. This relocation will increase the expected payouts of the public insurance scheme. However, such a warning system can still result in an increase in aggregate social wealth.

5. Conclusion

In this paper we have investigated the interaction between insurance and a warning system in a world in which individuals and enterprises face the risk of damages related to weather events. The analysis produced a number of findings which are important for decisions regarding public investment in meteorological forecast and warning systems. We showed that the existence of fair insurance against weather damages has no impact on the level of expenditure on warning systems that is optimal, despite the influence of insurance on individuals' incentive to respond to warnings. However, it was also shown that the fact that some individuals choose not to respond to warnings is not evidence that the warning system is inefficient or insufficiently informative. We also found that publicly funded insurance against weather damage induces an over-allocation of residents and enterprises to regions subject to weather risks like hurricanes and flooding. Not surprisingly, an effective warning system also can induce an increase in the number of residents and enterprises who locate in such regions, thereby increasing the expected payouts of the public insurance scheme. Despite this, we showed that investment in an effective weather warning system can still increase aggregate social wealth under these circumstances.

Much further work on these issues could be done. One interesting aspect is suggested by the analysis in the final section above, in which one of the determinants of individual location decisions is the level of public infrastructure in a region. Since such infrastructure is inevitably the result of public investment decisions, an analysis of the interaction between this type of investment and investments in meteorological warning systems would clearly be worthwhile.

6. References

Anbarci, Nejat, Monica Escaleras, and Charles A. Register, 2005, "Earthquake fatalities: the interaction of nature and political economy", *Journal of Public Economics* 89: 1907-1933.

Barnett, Barry J., 1999, "US Government Natural Disaster Assistance: Historical Analysis and a Proposal for the Future", *Disasters* 23(2): 139-155.

Campbell, Sean D. and Francis X. Diebold, 2005, "Weather Forecasting for Weather Derivatives", *Journal of the American Statistical Association* 100 (469): 6-16.

Changnon, Stanley A., David Changnon and Steven D. Hilberg, 2009, *Hailstorms Across the Nation, An Atlas About Hail and its Damages*, Contract Report 2009-12, Champaign, Illinois: Illinois State Water Survey.

Ehrlich, Isaac, and Gary S. Becker, 1972, "Market insurance, self-insurance, and self-protection", *Journal of Political Economy* 80 (4): 623-648.

Hardle, Wolfgang Karl and Brenda López Cabrera, 2012, "The Implied Market Price of Weather Risk", *Applied Mathematical Finance* 19 (1): 59-95.

Kapphan, Ines, Pierluigi Calanca, and Annelie Holzkaemper, 2012, "Climate Change, Weather Insurance Design and Hedging Effectiveness", *Geneva Papers on Risk and Insurance: Issues and Practice*, 37 (2): 286-317.

Katz, Richard W. and Allan H. Murphy (eds.), 1997, *Economic Value of Weather and Climate Forecasts*, Cambridge: Cambridge University Press.

Kunreuther, Howard, 1996, "Mitigating Disaster Losses Through Insurance", *Journal of Risk and Uncertainty* 12: 171-187.

Kunreuther, Howard C. and Erwann O. Michel-Kerjan, 2009, *At War with the Weather: Managing Large-Scale Risks in a New Era of Catastrophes*, Cambridge and London: MIT Press.

Lakdawalla, Darius, and George Zanjani, 2005, "Insurance, self-protection, and the economics of terrorism", *Journal of Public Economics* 89: 1891-1905.

Lazo, Jeffrey K., Rebecca E. Morss, and Julie L. Demuth, 2009, "300 Billion Served: Sources, Perceptions, Uses, and Values of Weather Forecasts", *Bulletin of the American Meteorological Society*, June: 784-798.

Lohse, Tim, 2012, "Self-Insurance and Self-Protection as Public Goods", *Journal of Risk and Insurance* 79 (1): 57-76.

Michel-Kerjan, Erwann O., 2010, "Catastrophe Economics: The National Flood Insurance Program", *Journal of Economic Perspectives* 24 (4): 165-186.

Petseti, Aglaia, and Milton Nektarios, 2012, "Proposal for a National Earthquake Insurance Programme for Greece", *Geneva Papers on Risk and Insurance: Issues and Practice* 37 (2) : 377-400.

Priest, George L., 1996, "The Government, the Market, and the Problem of Catastrophic Loss", *Journal of Risk and Uncertainty* 12: 219-237.

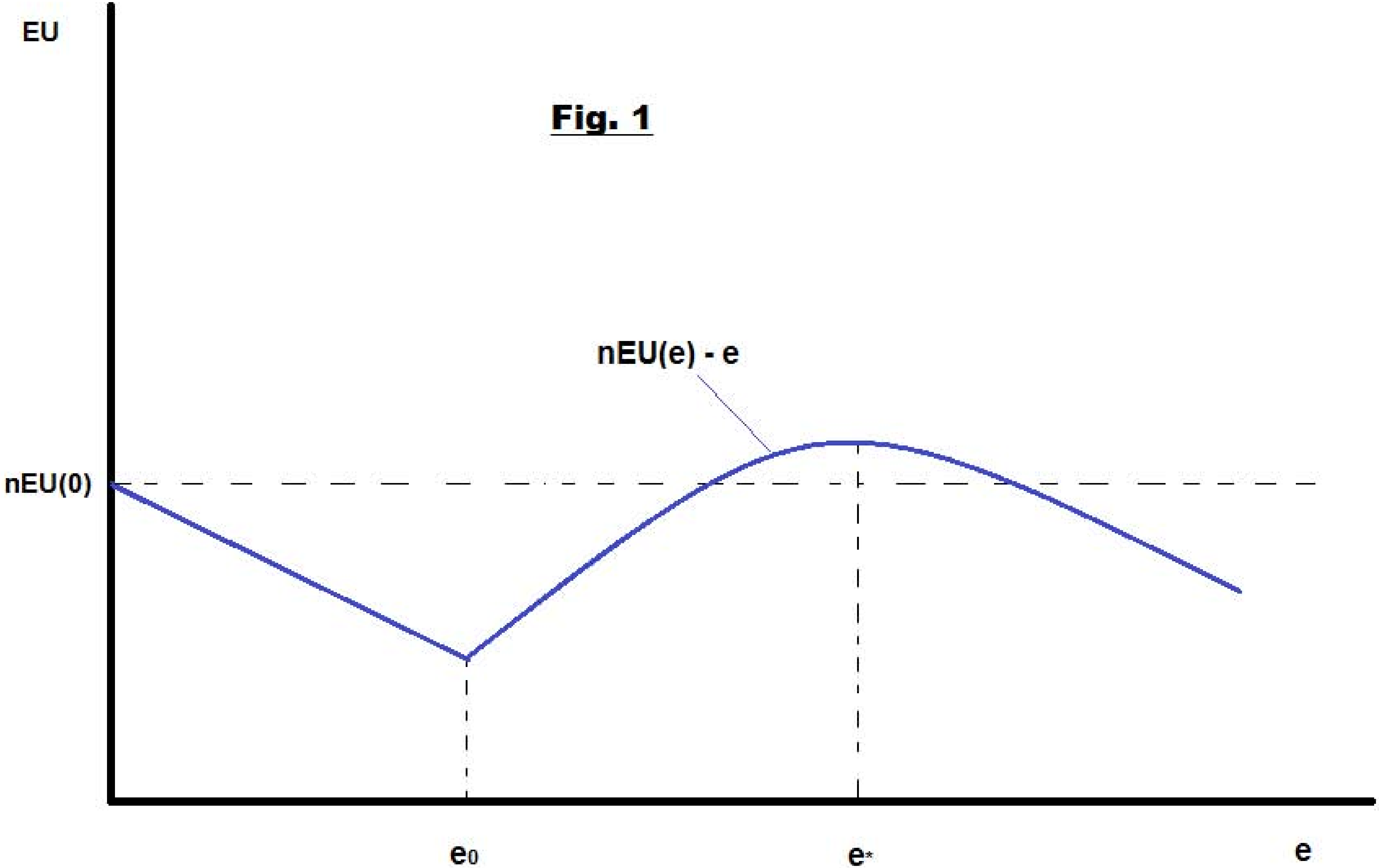
Priest, George L., 2003, "Government Insurance versus Market Insurance", *Geneva Papers on Risk and Insurance: Issues and Practice* 28 (1): 71-80.

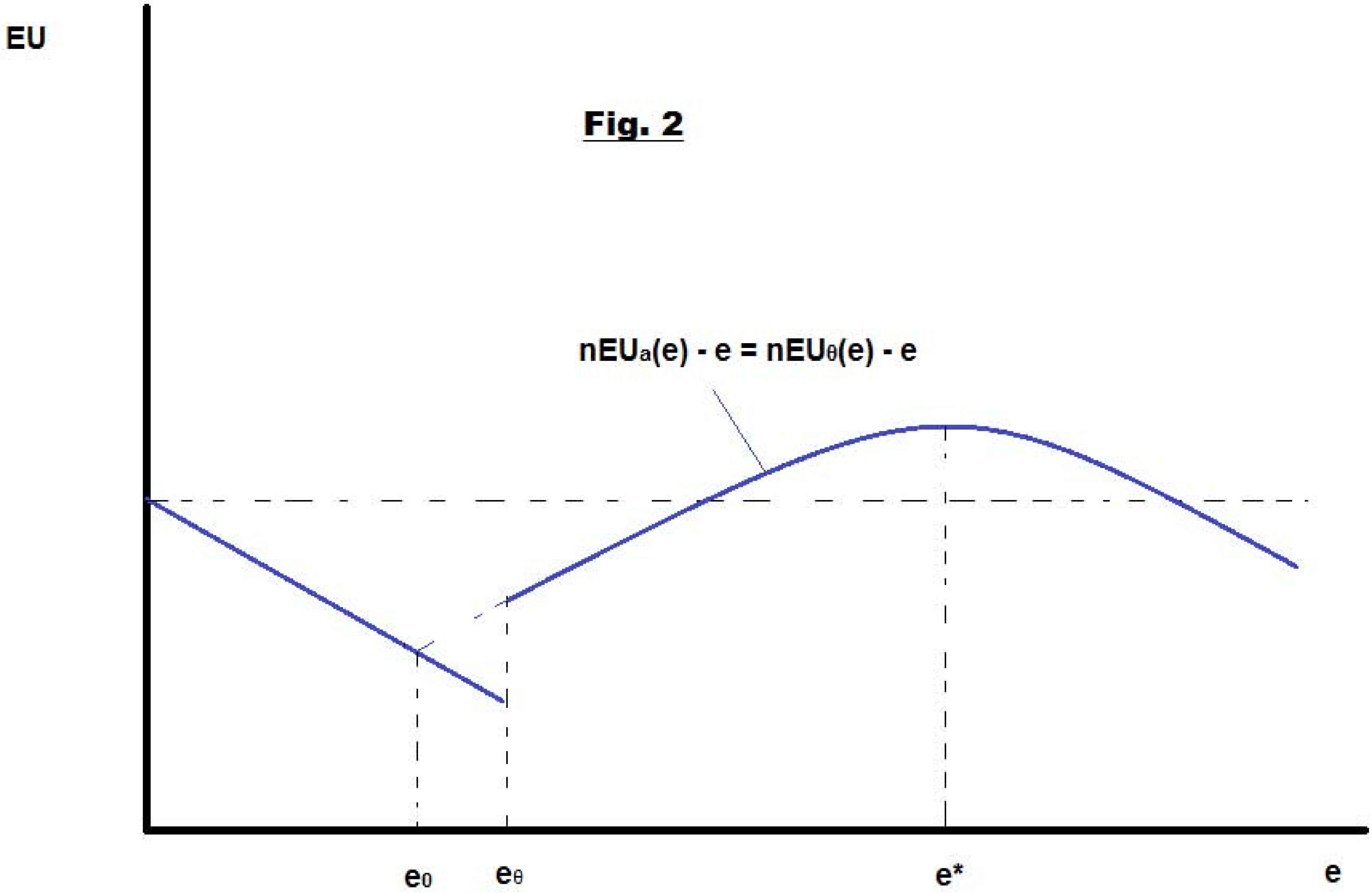
Richards, T., M. Manfredo, and D. Sanders, 2004, "Pricing weather derivatives", *American Journal of Agricultural Economics*, 86(4), pp. 1005-1017.

Rollins, K., and J. Shaykewich, 2003, "Using Willingness-to-pay to Assess the Economic Value of Weather Forecasts for Multiple Commercial Sectors, *Meteorological Applications* 10: 31-38.

Simmons, Kevin M. and Daniel Sutter, 2011, *Economic and Societal Impacts of Tornadoes*, Boston: American Meteorological Society.

Fig. 1




EU
Fig. 2
nEUa(e) - e = nEUθ(e) - e
e0
eθ
e*
e