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Can private giving promote economic segregation?

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Abstract

This paper explores the theoretical relationship between tax relief for private giving and individual location choice. Tax relief for giving may receive political support at the local level because of its distributional effects; however, through its effects on public provision choices, such relief may affect individual location decisions and, in so doing, has an impact on the jurisdictional configurations that can arise in equilibrium. For some demographic parameters it will promote economic segregation rather than integration, while for others, the opposite is true. In the former scenario, a ban on local tax incentives for giving would be Pareto improving and would thus be sanctioned by a majority-supported federal tax constitution.

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1. Introduction

In policy and political circles there appears to be a belief that voluntarism is a valuable tool for community building and social capital enhancement. One need not look far to find statements to this effect made by politicians and other governmental authorities. For example, George Bush Sr., in accepting the nomination for President in 1988, made the phrase a "thousand points of light" synonymous with an America consisting of diverse individuals working together to keep the country strong. Other examples abound. In 1997, Sheila Copps, the Minister of Canadian Heritage developed an action plan called "Strengthening and Celebrating Canada for the New

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Millennium". A key component of both plans was to develop strategies to encourage community, civic and citizen participation in activities that are partial substitutes for governmental policies. A variety of arguments have been put forward as to why such private activity is beneficial, but the common thread in all of them is the idea that individual involvement in voluntary activities in the community generates positive group externalities. The promotion of policies intended to foster private voluntary behavior is thus based on the supposition not just that the involvement of private citizens in community-level activities yield benefits for those communities but also that policies designed to promote such involvement yield the desired outcomes. While the former supposition may or may not be valid, the latter may be questioned. Economists are all familiar with the phenomenon of 'unintended consequences of policy decisions'. What this paper investigates is the possibility that policies intended to promote voluntary action – whatever may be the community-level externalities such action generates – can actually produce social division as an unintended effect, which in turn leads to a reduction in voluntary activity. In fact we find this to be the case in suitably large societies, while also finding that such policies can enhance private activity in smaller populations.

Our analysis starts from the observation that the structure of communities is endogenous, and can itself be affected by public policy — a point that has been recognized since the work of Tiebout (1956). The basic premise of all Tiebout-type models of local public choice is that individuals 'vote with their feet' choosing the community in which they reside on the basis of the package of taxes and local goods each community offers. It follows that policies that alter these mixes will affect individual location decisions, and thereby influence the demographic structure of communities. The specific focus of the present paper is on the impact that subsidies for private provision of local public goods have on the structure of local jurisdictions; specifically, it investigates whether subsidies lead to a world in which communities are more diverse or less diverse? As far as we know this focus is novel. In order to examine the connections between private provision and community building, our model must allow for endogenous jurisdiction formation. Very little literature exists on the implications of private provision for segregation and jurisdiction formation. Horstmann and Scharf (2002) focus on the impact of income inequality on private provision and community segregation. The paper does not explore the reverse linkage — the impact of private provision on community building. Much of the rest of the literature on noncooperative giving behavior assumes an exogenous jurisdiction structure and focuses on various aspects of individual free riding.[1] There are other literatures concerned with group formation by mobile agents, including a large literature on clubs, and a more recent literature on the formation of nations.[2] However, none of this work analyzes the interaction between public and private activity.

Our analysis shows that subsidies for private provision shift the burden for public good provision off certain groups of individuals and on to other groups. This shift affects the incentives that individuals have to locate in particular communities and, by so doing, has an impact on the incentives for individuals to reside in jurisdictions with particular demographic structures. In some circumstances this will induce dissimilar groups of individuals to reside in separate communities, and the subsidies can then be seen as promoting economic segregation rather than integration. In other circumstances they induce heterogeneous individuals to reside in a single community, thereby promoting integration.

[1] See for example, Andreoni (1988, 1990), Andreoni and Bergstrom (1996), Bergstrom, Blume and Varian (1986), Bernheim (1986), Roberts (1987), and Warr (1982, 1983).

[2] See, for example, Alesina and Spolaore (2003) for a summary.

In order to illustrate our points, we describe a stylized political economy model of private and public provision of a single public good. The model has two types of individuals, heterogeneous only in their willingness to pay for the public good. The public good can be financed through taxation and/or private contributions. Although our analysis applies to all demographic configurations, we concentrate on the case in which individuals who value the public good less are in a majority in the population. This is because when the opposite is true, there is never any private giving in any equilibrium of our model. Individuals choose where to live – there is endogenous jurisdiction formation – then vote over tax/public good packages. Subsequently, each individual has the option to engage in noncooperative private provision of the public good.

We first focus on a scenario where no tax-financed subsidies for private contributions are permitted. One can think of this case as a situation in which there is a constitutional ban on such subsidies. We show that there are two kinds of potential jurisdictional equilibria in this case. One has all individuals living together in a single jurisdiction while the other has individuals segregated into two separate jurisdictions, with segregation based on willingness to pay. The single jurisdiction outcome is always an equilibrium; segregation occurs as an additional outcome when there are sufficiently many high willingness to pay individuals. In the single jurisdiction case, private provision occurs when the total population is small and tax provision occurs when the population is large. The segregated outcome always has tax provision in both jurisdictions.

We then examine a scenario where a majority can choose to adopt tax incentives for private giving as part of the tax/public good package. Our finding here is that there are population distributions for which segregation is an equilibrium when tax incentives are allowed but is not an equilibrium when tax incentives are banned. This occurs when the overall population is relatively large, so that we view it as the leading case. In essence, the availability of tax incentives for private giving can lead to segregation. The reason for this outcome is that subsidies for giving are a cheaper way for the low willingness to pay majority to provide the public good than are universal taxes. Subsidies induce the high willingness to pay individuals to engage in private provision and so load a disproportional share of the cost of the public good on these individuals. High willingness to pay individuals recognize that this tax shifting will occur if they live together with low willingness to pay individuals and so they choose to segregate under demographic conditions that would induce integration if subsidies were not available. It is also true that for relatively small populations, there are population distributions for which segregation is an equilibrium when tax incentives are not allowed while segregation cannot arise in equilibrium when they are. This happens when the low-preference majority would choose to rely entirely on private provision when there is a single heterogeneous jurisdiction; a phenomenon only likely to arise in a very small population. In this case, the availability of a subsidy will again be taken advantage of by the majority, but here it lifts some of the burden of provision from the high-preference minority, and so renders them less interested in segregating.

Finally, we look at the political incentives for individuals to adopt a ban on tax incentives for giving. We find that the ban will be approved by a majority of individuals (in fact will be Pareto improving) if and only if we are in the situation in which the ban would result in segregation no longer being an equilibrium. This finding implies that promoting private provision through the local tax system could actually produce a perverse effect on segregation and that, because of this possible effect, they would be banned by a majority-supported federal tax constitution. Tax incentives for giving offered at the level of central taxation, on the other hand, do not have any effect on segregation, and would therefore be used. This prediction is consistent with the features of real-world federal tax systems, where tax relief for private giving is generally offered at the central rather than at the local level despite the fact that a large proportion of private donations is towards public goods and services that are local in nature.[3]

2. The model

Consider an economy in which there are $N>2$ individuals, each of whom consumes a private consumption good and a single pure local public good. Each individual has an endowment of income, w, which is the same for all individuals. Individuals differ in their preferences over the private consumption and public goods. For simplicity, we assume that there are two types of individuals, $i=1$, 2, and that all individuals of a given type have identical preferences. The preferences for a type i individual are represented by the utility function

$$U_i(c_i,\ z) = c_i + m_i(z), \quad i = 1,\ 2, \tag{1}$$

where c_i is private consumption for an individual of type i, and z is the level of collective consumption. The functions $m_i\ (z)$ are increasing in z, strictly concave and such that $\lim_{z\to 0} m'(z)=\infty$. We assume that $m_2'(z)>m_1'(z)$ for all z: type 2 individuals value the public good comparatively more at the margin than type 1 individuals do. The population is comprised of N_i individuals of type i, $i=1$, 2, with $N_1+N_2=N$. We assume that $N_1>N_2+1$, i.e., the low-preference individuals are a non-trivial majority in the overall population. The reason for this assumption is that, when $N_2>N_1+1$, the equilibria of the game never involve private provision and so the issues in which we are interested do not arise. Thus, we consider only situations with the low-preference type as the overall majority.

Individuals can choose the jurisdiction in which they live, with jurisdiction membership determining individual access to the local public good. The number of individuals of each type in a jurisdiction is public information, but their identity is private information. As a result, taxes in each jurisdiction cannot be directly conditioned on an individual's type. In what follows we assume that the tax authorities are not able (or do not find it cost effective) to use some preference revelation mechanism to induce tax discrimination so that taxes within a jurisdiction are uniform. Each agent can be a member of at most one jurisdiction and obtains no utility from the public good provided in other jurisdictions.

Taxes within jurisdictions are chosen by majority voting and it is assumed that the only tax instruments available to regional governments are lump-sum taxes. If the government in jurisdiction j sets a lump-sum tax, t_j, and there are N_j individuals in jurisdiction j, then tax revenues in j are $N^j t_j$. The provision of public goods can be funded by tax revenues and/or voluntary contributions. Voluntary contributions towards collective consumption made by a representative individual of type i in jurisdiction j are denoted as v_i^j, for $i=1$, 2. If N_i^j denotes the

[3] In the US, local income taxation is on the rise yet tax incentives for giving are only offered at the federal (or sometimes state) level in spite of some estimates of 97% of contributions (non-religious) aimed at the community in which the giver lives. The only states that do not use income taxes are Alaska, Florida, Nevada, New Hampshire and Tennessee (which has taxes only on dividends and interest income), Texas, Washington and Wyoming. Of the others, rates range from .75% (Ohio) to 6% (North Carolina). Rhode Island and Vermont charge 25% and 24% of the federal tax liability. With respect to local income taxes, as of 1997, 15 states are authorized to use them. Others are currently working on authorization (e.g. Virginia). Powers of cities to tax in these states vary: Colorado, Delaware, NY, Washington restrict taxation power to specific municipalities; Baltimore, NYC, St. Louis, Kansas City, Philadelphia, Columbus among others all use local income taxes, with rates typically being quite low (between 1% and 3%). Nevertheless, as far as we are aware, none offer relief for charitable contributions.

number of type i individuals in jurisdiction j, then total voluntary contributions by type i individuals in jurisdiction j are $N_i^j v_i^j$.

The focus of our analysis is the effect of government incentives for private provision on jurisdiction configurations. There are many ways in which government policy can be used to alter the incentives for private provision. In the relatively simple environment used here, the most natural instrument to consider is the granting of a subsidy to private giving, financed from general tax revenues. The ad valorem subsidy rate, s, can be thought of as the proportion of private contributions that citizens can deduct from their tax bills. This is in fact the actual tax treatment of private donations in many countries, including the US.[4] Given the jurisdiction j's government policy vector, $\theta_j=(t_j, s_j, g_j)$, private consumption of a type i citizen in jurisdiction j is $c_i^j=w-t_j-(1-s_j)v_i^j$, the level of z in jurisdiction j is $g_j+N_1^j v_1^j+N_2^j v_2^j$, and the government budget constraint is $g_j+s_j (N_1^j v_1^j+N_2^j v_2^j)=N^j t^j$.

We will compare two regimes: one in which subsidies to private provision are not allowed (i.e., we impose the constraint $s_j=0$ for all j) and one in which s_j is a policy choice within jurisdictions.

The sequencing of moves is as follows:

0. It is exogenously determined whether jurisdiction-level subsidies for private provision are allowed ($\sigma=s$), or not ($\sigma=0$).
1. Individual agents simultaneously choose a jurisdiction in which to live. There are N homogeneous possible locations for jurisdictions.
2. Given a population composition $n^j=(N_1^j, N_2^j)$ for each jurisdiction j, the tax rate t_j, level of public spending on z, g_j, and the subsidy rate (if $\sigma=s$) s_j, are set (and committed to) in each jurisdiction. These policy parameters, denoted as $\theta_j=(t_j, s_j, g_j)$, are chosen by a citizen of the majority type, and must be chosen from the set B_σ (N^j), where:

$$B_0(N^j) = \{t_j,\ s_j,\ g_j | s_j = 0, t_j \in [0,\ w], g_j = N^j t_j\}$$

and

$$B_s(N^j) = \{t_j,\ s_j,\ g_j | s_j \in [0, 1],\ t_j \in [0, w],\ \text{and}\ g_j = t_j N^j - s_j \Psi(s_j, g_j)\}$$

These two sets contain the policy choices that balance the government's budget along the equilibrium path under the no-subsidy and subsidy regimes respectively. The variable $\Psi(s_j, g_j)$ gives the equilibrium level of private provision from stage 3 of the game given the stage 2 choices of s_j, g_j and given that tax revenue must cover the subsidy to this private giving.[5]
3. Agents in each j observe θ_j and simultaneously choose non-negative levels of private provision for the public good. Total provision of the local public good in jurisdiction j is the sum of public and private funding in j, which we denote as g_j and V^j, respectively.

In each subsidy regime, we will determine the set of symmetric, sub-game perfect equilibria, and then compare the equilibrium sets for the two regimes. We impose symmetry only to determine the levels of individual private provision at stage 3, as there are an infinity of combinations of individual giving by agents of each type that can arise as equilibrium

[4] Actual tax systems, as well as any subsidies to private provision, are of course considerably more complicated than this. In particular, there is often an upper limit on the subsidy a taxpayer can earn through private donations. We will ignore such complications in our analysis.

[5] It will be shown below that in fact s_j, g_j are the only parameters that influence equilibrium contributions in Stage 3.

continuations. This multiplicity is the result of our assumption that individual preferences are quasi-linear in c. All combinations result in the same level of V^j.

3. Preliminary results

In this section, we develop a series of preliminary results that serve to limit the set of possible equilibrium continuation strategies for our game. These results reveal the basic logic that drives our main results in the following section and make those results simpler to derive. In what follows, when it is clear that we are analyzing behavior in a particular jurisdiction, we will drop the jurisdictional subscripts and superscripts from most variables. The term 'equilibrium' means 'subgame perfect equilibrium' and equilibrium values of choice variables will be denoted with a "*". We begin by analyzing behavior in the last stage of the game.

3.1. Private provision choices

Consider an arbitrary jurisdiction j, consisting of N_1^j individuals of type 1 and N_2^j individuals of type 2. Suppose that the public policy regime is given by $\theta=(t, s, g)$. Then, the private provision choice for an individual of type i, v_i, must be a solution to:

$$m_i'(g + V_{-i} + v_i) - (1-s) \leq 0, \;\; i = 1, 2; \tag{2}$$

$$[m_i'(g + V_{-i} + v_i) - (1-s)]\; v_i = 0, \quad i = 1, 2; \tag{3}$$

where V_{-i} is private giving by all other agents in the jurisdiction. We will assume in what follows that when (3) is satisfied for type 2 individuals, so that $v_2>0$, w is large enough that: $v_2\,(g, s, t) < (w-t)/(1-s)$ for all $(g, s, t) \in B_s(N^j)$. This simple observation leads to our first result.

Lemma 1. *In any jurisdiction j,we have that; (i) if $N_2^j>0$, then for any $(g, s, t) \in B_\sigma(N^j)$ the equilibrium continuation strategy of any type 1 residents of j must have $v_1(g, s, t)=0$, and (ii) if $N_2^j=0$ then there is no $(g, s, t) \in B_\sigma(N^j)$ which type 1 individuals can choose that yields a higher continuation payoff than $(g_1^t(N^j), 0, g_1^t(N^j)/N^j)$, where $g_1^t(N^j) = \text{argmax}_g \left\{ w - \dfrac{g}{N^j} + m_1(g) \right\}$.*

Lemma 1(i) means that we need only consider the high-valuation (type 2) individuals as potential private contributors to z in any heterogeneous jurisdiction. (ii) states the obvious fact that in a jurisdiction consisting of only type 1 individuals they can do no better than to set an optimal tax. They can in fact realize the same outcome with an appropriate subsidy, $s_t>0$, however we will assume they simply use pure public provision. Together these results mean that only type 2 individuals private giving behavior needs to be considered.

Further, we know that if type 2 individuals are engaged in private giving, it must be that total private contributions, V, satisfy

$$m_2'(g + V) = 1-s. \tag{4}$$

An implication of this condition is that, if there is private provision in equilibrium, the total amount contributed depends only on the values of g and s chosen in the jurisdiction. We let $\Psi_2(s, g)$ denote total giving by the type 2 individuals given policy choices (s, g). Since, from (4), $g+V_2=\rho_2(s)$, where $\rho_2(s)$ is the inverse of m_2', the value of $\Psi_2(s, g)$ is given as

$$\Psi_2(s, g) = \max\{0, \rho_2(s) - g\}. \tag{5}$$

The function Ψ_2 captures the equilibrium aggregate donation behavior of the type 2 individuals in any jurisdiction for any s, g chosen.

Note that the maximization of utility by type 2 individuals only determines the total level of private provision. It is here that we use the symmetry restriction to define private giving by each type 2 individual in jurisdiction j as $\Psi_2(s_j, g_j)/N_2^j$.

3.2. Policy choices

Stage 2 of the game is the political stage in which the policy vector θ is determined. In general the policy outcome in any jurisdiction will be a function of which regime is in force – subsidies for giving permitted or not – as well as of the demographic make-up of the jurisdiction — the number of type 1 and type 2 individuals. We show in what follows that there are three broad categories of policy regimes within a jurisdiction that can emerge in any equilibrium. We describe each below and then delineate the situations in which each might occur.

The simplest case is one in which government plays no role in the provision of z: $s=t=g=0$. This is the case of pure private provision in which the policy vector is given by $\theta=(0, 0, 0)$. In this case $z=\Psi_2(0, 0)$ and the jurisdiction gets only the level of public good that its type 2 citizens are willing to finance privately, with no tax subsidy. We will let $\Psi_2(0, 0)=z_2^v$ and note that $m_2'(z_2^v)=1$.

The second possibility is pure public provision. This is the case in which g is sufficiently high, given s, that $\Psi_2(s, g)=0$. In this case the value of g is given by total tax revenues so that $g=N^j t$. With pure public provision, and if type i individuals are in the majority, their optimal choice of g is

$$g_1^t\left(N^j\right) = \arg\max_g \left\{w - \frac{g}{N^j} + m_i(g)\right\}, \tag{6}$$

and the corresponding tax is $t_i\,(N^j)=g_i^t(N^j)/N^j$. The level of z is, as a consequence, determined by the condition:

$$m_i'\left(g_i^t(N^j)\right) = 1/N^j, \tag{7}$$

where i is the majority type. For future reference, we denote the policy vector for this case as $\theta=(t_i(N^j), 0, g_i^t(N^j))$.[6]

The third possible outcome is a subsidized private provision outcome and occurs only if subsidies to giving are permitted. For this case, the policy vector takes the form $\theta=(t, s, 0)$ and it induces an outcome of pure *subsidized* private provision. All taxes collected are used solely to finance the subsidy on private provision: $s\Psi_2(s, 0)=tN^j$. The level of z in a jurisdiction that adopts such a policy is equal to $\Psi_2(s, 0)$ and is therefore determined solely by the value of s. As a result, the general structure of such a policy choice is $\theta=(s\Psi_2(s, 0)/N^j, s, 0)$.

There are of course many other feasible θ choices in any jurisdiction. However, only the above three types of policy vectors can arise as equilibrium outcomes. The next two lemmas demonstrate this result as well as the conditions under which each of the three policy vector types arise.

Lemma 2. *In either subsidy regime and in any jurisdiction j, in which type 2 individuals are the majority, the equilibrium policy choice $\theta=(t, s, g)$ will be such that $\Psi_2(s, g)=0$.*

[6] Since there is no private giving, the level of s, if $s>0$ is permitted, is indeterminate in this case: all s such that $\Psi_2(s, g_i^t(N^j))=0$ yield the same outcome. We assume, without loss of generality, that when pure public provision is the optimal policy choice, $s=0$ is chosen.

We already know from Lemma 1 that only type 2 individuals ever engage in private provision. This lemma indicates that a type 2 majority always finds it advantageous to rely on the tax system rather than private provision. The tax system forces type 1 individuals to contribute to z, something they will not do if there is private provision.

The equilibrium policy choices of a type 1 majority are more varied, depending on both the subsidy regime and the demographic make-up of a jurisdiction. The next result provides the details.

Lemma 3. *(a) If subsidies for private provision are not allowed, and type 1 is in the majority in a given jurisdiction, then there exists a $N^0 > 1$ such that*

$$\theta^* = \begin{cases} (t, g) = (0, 0) & \text{if} \quad N^j < N^0 \ \text{and} \ N_2^j > 0, \\ (t, g) = (t_1(N^j), g_1^t(N^j)) & \text{if} \quad N^j \geq N^0 \ \text{or} \ N_2^j = 0. \end{cases} \tag{8}$$

(b) If subsidies for private provision are permitted and type 1 is in the majority in a given jurisdiction in which $N^j > 0$, then there is a N^s such that

$$\theta^* = \begin{cases} (t, s, g) = (0, 0, 0) & \text{if} \quad N^j < N^s, \\ (t, s, g) = (s^*(N^j)\Psi_2(s^*(N^j), 0)/N^j, s^*(N^j), 0) & \text{if} \quad N^j \geq N^s, \end{cases} \tag{9}$$

where

$$s^*(N^j) = \arg\max_s \left\{ w - \frac{s\Psi_2(s, 0)}{N^j} + m_1(\Psi_2(s, 0)) \right\}; \tag{10}$$

if $N_2^j = 0$, then $\theta^* = (t, s, g) = (t_1(N^j), 0, g_1^t(N^j))$.

Part (a) of the lemma lays out the two policy choices a type 1 majority might adopt in equilibrium, if subsidies to giving are not permitted. When the jurisdiction is small ($N^j < N^0$), the tax price of the public good, $1/N^j$ is large. As a result, the majority relies entirely on the private giving of the type 2 minority. The majority's utility in this case is

$$\Upsilon_1^v \equiv w + m_1(z_2^v). \tag{11}$$

Once the jurisdiction is large enough, the tax-price of the public good is low enough that the majority is better off using tax financing. Doing so gives them a payoff of

$$\Upsilon_1^t(N^j) \equiv w - \frac{g^t(N^j)}{N^j} + m_1(g_t(N^j)). \tag{12}$$

This payoff is increasing in N^j while the payoff to the majority under a pure private provision policy, Υ_1^v, is independent of N^j. As a result, there is a critical value N^0 for the jurisdictional population at which the optimal policy choice changes for a type 1 majority.

Part (b) of the lemma demonstrates one of the key insights of our model. Once the option of setting subsidies for private provision is opened up, the low-preference majority *never* uses the tax system to finance z if the jurisdiction includes any high-preference individuals: g is always zero. All provision is done privately by the type two individuals and the only issue for the majority is the size of the subsidy to provide. The tax system is used only to raise the funds needed to pay the subsidy.

The reason that g is zero in this case is familiar from the literature on private provision. With $g>0$, and $\Psi_2>0$, the majority can always do better by lowering g, since the type 2 citizens will respond at Stage 3 by increasing their giving to completely offset the loss of g (this is clear from the expression (5)). Total taxes will drop by $(1-s)$ dollars for every dollar decrease in g, but z will not change. A tax-financed subsidy is a better instrument for the majority than direct tax financing of g, since taxes are levied uniformly, whereas a subsidy, although available to all, is 'targeted' at high-preference citizens via self-selection. By creating differential contributions to the public good, the subsidy shifts the cost for provision from the majority to the minority.

The utility of a (majority) type 1 individual from the optimal positive subsidy is

$$w-\frac{s^*(N^j)\Psi^2(s^*(N^j),0)}{N^j}+m_1\left(\Psi^2\left(s^*\left(N^j\right),0\right)\right)\equiv\Upsilon_1^s\left(N^j\right). \tag{13}$$

One can show that s^* increases with N^j and that Ψ_2 increases with s. The former follows from the fact that the individual tax burden of subsidization is less the larger is N^j; the latter is a result of the fact that $(1-s)$ is the subsidized price for z and z is normal. Since the payoff to a type 1 individual is increasing in N^j when s is fixed, it also must be when s is adjusted optimally: $\Upsilon_1^s(N^j)$ is increasing in N^j. Hence, there again exists a critical value, N^s, at which the policy choice of the type 1 majority changes to one of active subsidization.

3.3. Jurisdictional outcomes

The results above detail the possible equilibrium choices in the last two stages of the game. In principle, there are a large array of jurisdictional configurations that can arise at stage 1, since we do not limit the set of locations in any way. However, the results of Lemma 3 allow us to prove a final preliminary result which indicates that in fact there are only two possible configurations in equilibrium. The proof of this result, as well as some of those that follow, are made simpler if we make two additional assumptions regarding the elasticity of demand for collective consumption.

Consider a type i citizen elected as political decision maker in jurisdiction j choosing a utility-maximizing level of g, taking as given that $s=0$ and knowing that no private provision will result. The level of g in this situation is $g_i^t(N^j)$. It can be thought of as the level of spending on z desired by a type i citizen facing a *tax price* of $1/N$ for g. The 'tax-price elasticity' of this demand is

$$-N^j\frac{\partial g_i^t(N^j)/\partial N^j}{g_i^t(N^j)}\equiv\varepsilon_i^t\left(N^j\right). \tag{14}$$

Analogously, when a set of type 2 individuals is privately providing z, faced with a subsidy set at any $s\geq 0$ and a level of public provision of $g=0$, $\Psi_2(s, 0)$ is the level of z they will provide at the *subsidized price* $1-s$. In this case the 'subsidy price elasticity' of this demand is

$$-(1-s)\frac{\partial\Psi_2(s,0)/\partial s}{\Psi_2(s,0)}\equiv\varepsilon^s(s). \tag{15}$$

Our results so far imply that these elasticities are all negative. Throughout the rest of the analysis, we impose the restriction that they are less than one in absolute value. This assumption of price-inelasticity is sufficient for the results we derive but is by no means necessary. Under this restriction we can prove the following:

Lemma 4. *There are at most two equilibrium jurisdictional configurations: (i) a single jurisdiction containing all citizens (denoted J1); and (ii) two jurisdictions, each containing citizens of a single type (denoted J2).*

4. Equilibrium configurations and subsidies

In this section we analyze the jurisdictional equilibria, the situations in which they arise and how they are affected by whether or not the majority can employ subsidies for giving. In this way, we can examine the impact that such subsidies have on economic segregation. We begin with the case in which subsidies are constitutionally disallowed; we follow with the situation in which subsidies for giving are allowed and are chosen in the way described above. Finally, we ask whether or not individuals would want to impose a constitutional ban on subsidies for giving.

4.1. The no-subsidy case

From Lemma 4 we know that there are only ever two possible jurisdiction configurations: either all citizens reside in a single jurisdiction, which must therefore have type 1 as its majority, or there are two jurisdictions populated by the two different types. To determine whether either is an equilibrium configuration, we must determine whether any citizen would prefer to change their location, taking into account the equilibrium continuation that would follow from their doing so.

Consider the single jurisdiction outcome first. If subsidies for giving are not permitted, then Lemma 3(a) above applies. The policy outcome in equilibrium will be pure private provision if $N<N^0$, and $\theta=(t,\ g)=(t_1(N),\ g_1^t(N))$ if $N\geq N^0$. The only possible deviations from such an equilibrium are that an individual forms a separate jurisdiction. It is immediate that a type 1 individual never finds such a deviation profitable since the policies in the single jurisdiction are optimal for type 1 and there are more citizens to tax.

A type 2 individual in the single jurisdiction gets utility of

$$\Upsilon_2^v(N^2) = w - \frac{z_2^v}{N_2} + m_2(z_2^v), \tag{16}$$

when $N<N^0$; and

$$\Upsilon_2^t(N) = w - \frac{g_1^t(N)}{N} + m_2(g_1^t(N)), \tag{17}$$

when $N\geq N^0$. It is clear then that defecting from this single jurisdiction cannot be profitable for a type 2 individual when $N<N^0$, as the deviator would face private provision with no co-contributors.

When $N>N^0$, the type 1 majority prefers a policy of pure public provision to one of pure private provision. Since the type 1 individual pays taxes under a pure public provision policy, it must be that the pure public provision policy provides a higher level of the public good than does the pure private provision policy: $g_1^t(N)>z_2^v$. This fact in turn implies that a type 2 individual who stays in the single jurisdiction gets more g than if he defects and gets it at a lower price ($1/N<1$). This, coupled with the fact that $g_1^t(N)<g_2^t(N)$, imply that the type 2 individual is better off staying in the single jurisdiction. This analysis implies the following result:

Proposition 1. *A single heterogeneous jurisdiction is always an equilibrium outcome if subsidies to private provision are not allowed.*

As for the two-jurisdiction outcome, arguments similar to the above demonstrate that it is never in any individual's interest to live in autarky. It is also not in the interest of a type 1 individual to move to the type 2 jurisdiction. The reason is that the type 1 individual pays both a higher tax-price for z – by Lemma 2 the type 2 majority always chooses a tax regime and $1/N_1<1/(N_2+1)$ – and consumes more g than is individually optimal for him at this higher price since $g_1^t(N')<g_2^t(N')$ for any N'.

If $N_1+1<N^0$, then a type 2 individual who moves to the type 1 jurisdiction will be 'exploited' by the type 1 majority. In this case, as indicated by Lemma 3(a), the policy is pure private provision by the lone type 2 individual. Because the price of the public good is 1 rather than $1/N_2$ in this case, the deviating type 2 individual provides less of the public good than type 2 individuals provide in the type 2 jurisdiction — z_2^v rather than $g_2^t(N_2)$. The deviating individual also pays the entire cost. As a result, a type 2 individual has no incentive to move in this situation.

If $N_1+1>N^0$, then a pure public provision policy is adopted in the type 1 jurisdiction even if a type 2 individual moves there. In this case, even though the type 2 individual does not obtain the preferred amount of g in the type 1 jurisdiction, the tax price is lower than in the type 2 jurisdiction: $1/(N_1+1)<1/N_2$. As a result, a move to the type 1 jurisdiction may make a type 2 individual better off. This is more likely to be the case when N_2 is small relative to N_1 since, in this situation, a type 2 individual obtains a large reduction in the tax price by moving. More specifically, the move will pay for a type 2 individual if and only if

$$\Upsilon_2^t(N_1+1) = w - \frac{g_1^t(N_1+1)}{N_1+1} + m_2\left(g_1^t(N_1+1)\right) > w - \frac{g_2^t(N_2)}{N_2} + m_2\left(g_2^t(N_2)\right) \equiv \Gamma_2^t(N_2) \tag{18}$$

If we define the function $n_2(N_1)$ as $n_2(N_1)\equiv\min\{N_2|\Gamma_2^t(N_2)\geq\Upsilon^t(N_1+1)\}$, then the move pays if $N_2\leq n_2(N_1)$ and does not otherwise.[7]

[7] The function $n_2(N_1)$ gives the smallest value of N_2 such that it pays for a type 2 individual to stay in a jurisdiction with N_2 type 2 members rather than move to a type 1 jurisdiction with N_1 members. Note that $n_2(N_1)$ is increasing in N_1 and such that $N_1>n_2(N_1)$. The latter follows from the fact that the type 1 jurisdiction underprovides g from the type 2's perspective. To see the former, note that

$$\frac{\partial \Gamma_2^t(N_2)}{\partial N_2} = \frac{g_2^t(N_2)}{(N_2)^2} > 0,$$

using the fact that $m_2'(g_1^t(N_2))=1/N_2$. Also, we have that

$$\frac{\partial \Upsilon_2^t(N_1+1)}{\partial N_1} = g_1^{t\prime}(N_1+1)\left[m_2'(g_1^t(N_1+1)) - \frac{1}{(N_1+1)}\right] + \frac{g_1^t(N_1+1)}{(N_1+1)^2} > 0,$$

since $m_2'(g_1^t(N_1+1))>m_1'(g_1^t(N_1+1))=1/(N_1+1)$.

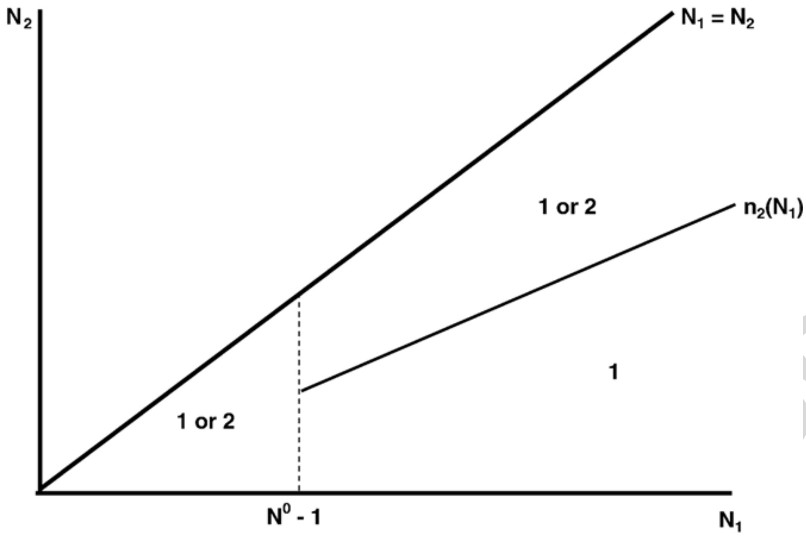

Fig. 1.

We have, then, the following result:

Proposition 2. *Two homogenous jurisdictions is an equilibrium outcome when subsidies to private provision are not allowed if and only if either:*

1. $N_1 < N^0 - 1$; *or*
2. $N_1 > N^0 - 1$ *and* $N_2 > n_2(N_1)$.

In essence, the two-jurisdiction configuration arises for two sorts of demographics, those in which the population has small numbers of low-valuation (and so also high-valuation) individuals and those in which the population is large and there are sufficiently similar numbers of low and high valuation individuals. Fig. 1 depicts the set of 'demographic parameter' configurations, (N_1, N_2), for which a configuration with two homogenous jurisdictions (i.e., $J2$) is an equilibrium when subsidies for private provision are not allowed. $J1$ is always an equilibrium configuration, as shown.

4.2. Subsidies for private provision

Suppose now that subsidies for private provision are constitutionally permissible (that is, $s > 0$ is permitted). What effect does the availability of this policy instrument have on jurisdiction configurations? For the case of the single mixed jurisdiction, it is immediate that a type 1 individual will never defect from such a jurisdiction, by the same reasoning used when subsidies were not available. Further, if $N < N^s$, a type 2 individual will not defect, again by the same reasoning as before, since there is no subsidy to private provision in the single jurisdiction in this situation (see Lemma 3(b)). If $N > N^s$, the utility of a type 2 individual in the mixed jurisdiction is

$$w - \left(\frac{s}{N} + \frac{1-s}{N_2}\right)\Psi_2(s) + m_2(\Psi_2(s)) \equiv \Omega_2(s, N, N_2), \tag{19}$$

with $s = s^*(N)$.[8] This same individual's utility in autarky is

$$\Omega_2(0,1,1) = w - \Psi_2(0) + m_2(\Psi_2(0)), \tag{20}$$

[8] Since we know that in equilibrium, $g = 0$ always, we will henceforth drop the argument g from the function $\Psi_2(s, g)$.

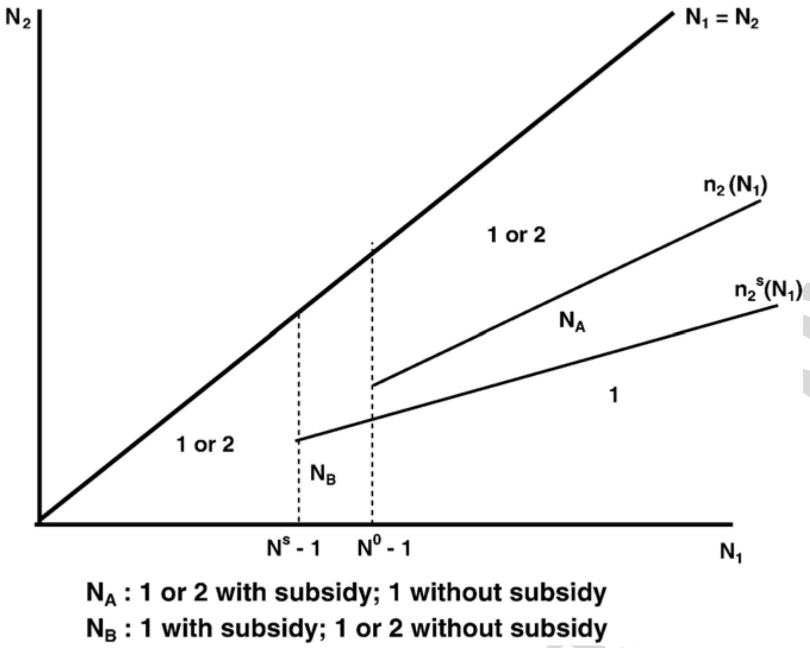

Fig. 2.

The function $\Omega_2(s, N, N_2)$ is increasing in both N and N_2 and increases when s^* is adjusted optimally as N increases (see the Proof of Lemma 4). Therefore, it must be that $\Omega_2(s^*(N), N, N_2) > \Omega_2(0, 1, 1)$, and so it can never pay for a type 2 individual to move from the single jurisdiction when $N > N^s$.

We have then the following proposition:

Proposition 3. *A single jurisdiction is an equilibrium outcome for all values of* (N_1, N_2), *when subsidies for giving are permitted*.

The implication of Proposition 3 is that any impact that subsidies for giving have must be on the tendency for individuals to segregate into separate jurisdictions. The issue is whether subsidies expand the set of circumstances in which segregation occurs or reduces this set. To answer this question we need to identify the set of demographics for which the two jurisdiction configuration is an equilibrium.

So consider a two jurisdiction outcome. As before, a type 1 individual has no incentive to move to a type two jurisdiction. The reason is as before; namely that, by Lemma 2, the type 2 jurisdiction only ever uses pure public provision and a so type 1 individual pays a higher tax price and obtains a less preferred level of the public good by moving to a type 2 jurisdiction.

As for a move by a type 2 individual to the type 1 jurisdiction, if $N_1 + 1 < N^s$, then Lemma 3(b) implies that such a move produces a pure private provision policy in which the lone type 2 individual is the sole provider of the public good. As before, this outcome makes the type 2 individual worse off. If $N_1 + 1 > N^s$, then the type 2 individual faces a subsidised private provision policy when moving to the type 1 jurisdiction. In this case, the type 2 individual obtains utility of payoff

$$\Omega_2(s^*(N_1 + 1), N_1 + 1, 1) \tag{21}$$

when moving to the type 1 jurisdiction. The move pays if and only if this utility is greater than $\Gamma_2^t(N_2)$, the utility from staying in the segregated type 2 jurisdiction. Analogously to before, we can define the function $n_2^s(N_1)$ giving the smallest value of N_2 such that a type 2 individual prefers staying in the type 2 jurisdiction to moving to a type 1 jurisdiction with N_1 individuals. This function is increasing in N_1 and is defined by

$$n_2^s(N_1) \equiv \min\{N_2 | \Gamma_2^t(N_2) \geq \Omega_2(s^*(N_1+1), N_1+1, 1)\}. \tag{22}$$

As before, the type 2 individual moves if $N_2 \leq n_2^s(N_1)$. We have, therefore, the following proposition:

Proposition 4. *When subsidies to private giving are allowed, there is an increasing function $n_2^s(N_1)$ such that a two-jurisdiction configuration is an equilibrium outcome if and only if either*

1. *$N_1 < N^s - 1$; or*
2. *$N_1 \geq N^s - 1$ and $N_2 \geq N_2^s(N_1)$.*

We are now in a position to analyze how the equilibrium set changes when subsidies for giving are allowed. As noted above, our answer turns on a comparison of Propositions 2 and 4. This comparison in turn depends on the magnitude of N^s relative to N^0 and on the value of $n_2(N_1)$ relative to n_2^s (N_1) for any N_1.

Proposition 5. *$N^0 > N^s$, and $n_2(N_1) > n_2^s(N_1)$ for any N_1.*

The result implies that when N_1 is large ($N_1 > N^0$), subsidies for private provision enlarge the set of demographics for which the two jurisdiction configuration is an equilibrium. In Fig. 2, the set of (N_1, N_2) for which subsidies allow for the existence of the separate jurisdictional equilibrium when it would not exist without subsidies, is marked as N_A. The existence of this region gives the sense in which subsidies for private provision increase economic segregation. The reason is essentially the following. Tax-financed subsidies for private provision of public goods are a way of inducing high-demanders to self-select into providing a larger share of the financing of public goods. If a low-demand majority is given the right to implement such subsidies, then the availability of this policy tool increases the incentive for high-demanders to segregate themselves into communities consisting of other high-demanders. Doing so lets them avoid the differential burden that would result if they lived in a mixed community. Thus, for large N, the subsidies can lead to communities that are more homogeneous and in this sense, decrease the amount of social cohesion.

When N_1 is small ($N_1 < N^0$) but not 'too small' ($N_1 > N^s$) and high-valuation individuals are a sufficiently small minority ($N_2 < n_2^s(N_1)$) then the opposite is true; the existence of subsidies eliminates the possibility of a two-jurisdiction equilibrium when it would exist in their absence. The set of (N_1, N_2) for which subsidies have this effect is depicted as N_B in Fig. 2. The intuition for this phenomenon is that with a small polity all living in one jurisdiction, tax-financed provision of g is too costly for the low-valuation majority. Thus, the equilibrium outcome in the single jurisdiction is pure private provision. If subsidies are allowed, they are used, and this reduces the burden on the high-valuation minority relative to what they would bear without subsidies, and this prevents them from segregating. In this 'small polity' situation, then, subsidies do promote integration.

It is highly intuitive that in a polity with a relatively low population, the burden of public provision will be high; the self-selection that subsidies allow for increases the efficiency of public provision, making it the preferred political choice when it would otherwise not be. This allows public provision to dominate pure private provision which in turn allows the advantage of large numbers to overcome the disadvantages (to high-valuation citizens) of a heterogeneous community.

4.3. Constitutional choices

Up to now we have treated the existence of tax-financed subsidies to private provision as exogenous. It is reasonable to ask what the model can say about the outcome of an ex-ante political decision to allow such subsidies. That is, what would happen if, at stage zero of our game, individuals could vote on a constitutional ban against the adoption of subsidies by local jurisdictions? We pursue this question here under the supposition that individuals know their type when they vote and that the outcome of this single vote will determine whether or not subsidies are banned in all jurisdictions that might form at later stages.

The answer to this question depends on the demographic makeup of the overall polity. More specifically, we can conclude from the preceding analysis that there are three distinct sets of demographics that must be considered. If we let $\mathcal{N} \equiv \{(N_1, N_2) | N_1 \geq N_2 + 1\}$, these three sets are defined as:

$$\mathcal{N}_{\mathrm{A}} \equiv \{(N_1, N_2) \in \mathcal{N} | N_1 > N^0 - 1 \text{ and } n_2(N_1) > N_2 > n_2^s(N_1)\}; \tag{23}$$

$$\mathcal{N}_{\mathrm{B}} \equiv \{(N_1, N_2) \in \mathcal{N} | N^s - 1 < N_1 < N^0 - 1 \text{ and } N_2 < n_2^s(N_1)\}; \tag{24}$$

$$\mathcal{N}_0 \equiv \mathcal{N} \setminus (\mathcal{N}_{\mathrm{A}} \cup \mathcal{N}_{\mathrm{B}}); \tag{25}$$

The set $\mathcal{N}_{\mathrm{A}}$ is the set of demographic parameters for which the introduction of subsidies to private provision makes the $J2$ configuration possible in equilibrium when it would otherwise not be. Alternatively, it is the set of n for which banning subsidies eliminates the possibility of a $J2$ equilibrium. N_{B} is the set of parameters for which banning subsidies makes the $J2$ configuration a possible equilibrium outcome when it would otherwise not be. N_0 is the set for which the presence of subsidies has no effect on the set of possible equilibrium configurations.

In $\mathcal{N}_0$ and under the assumption that individuals do not directly coordinate their location choices on the constitutional structure (i.e., the equilibrium outcome itself is not affected by a constitutional ban), Type 1 individuals weakly prefer no ban on subsidies while type 2 individuals weakly prefer a ban. To see this, note that both types are indifferent to a ban if the equilibrium configuration is $J2$ since subsidies are not used. Both types are also indifferent if the equilibrium configuration is $J1$ but subsidies are not adopted by the type 1 majority, as occurs when $N < N^s$. When the equilibrium configuration is $J1$ and subsidies are employed, if permitted, their use makes type 1 individuals strictly better off and type 2 individuals strictly worse off. Therefore, if a majority vote is required to ban subsidies for giving, they will not be banned for $n \in \mathcal{N}_0$.

In $\mathcal{N}_{\mathrm{B}}$, banning subsidies makes $J2$ an equilibrium configuration when it would not be in the presence of subsidies. Thus, the equilibrium configuration will necessarily be $J1$ if subsidies are permitted. If the ban on subsidies leaves the equilibrium outcome $J1$, then the type 1 individuals are worse off: they are no longer able to use a subsidy which they find optimal without the ban. Type 2 individuals are also worse off if $N < N^0$ since, in this case, the policy under the ban is pure private provision so that they obtain less of the public good and pay a higher price for it. If, on the other hand, $N \geq N^0$ then the ban makes type 2 individuals better off since the policy regime is pure tax provision under a ban but subsidised private provision without it. If the ban results in a change to a $J2$ equilibrium configuration, both types of individuals are worse off. The reason is obvious for the type 1 individuals. As for type 2 individuals, the fact that $J2$ is not an equilibrium configuration with subsidies means that a type 2 individual prefers to be in the type 1 jurisdiction even if this individual is the lone but subsidized contributor to the public good. As a consequence, type 2 individuals must certainly prefer to be together in a single jurisdiction with subsidies. We have then that, for $\mathcal{N}_{\mathrm{B}}$, a constitutional ban on subsidies is majority opposed and is unanimously opposed if $N < N^0$.

In $\mathcal{N}_{\mathrm{A}}$, $J1$ is the only equilibrium configuration if subsidies are banned. If, when subsidies are permitted, $J1$ is also the equilibrium outcome, then the ban makes type 1 individuals worse off and type 2 individuals better off. The reason is as above. If, however, the equilibrium is $J2$ when subsidies are permitted, then a ban necessarily changes the outcome to $J1$. Both types of individuals are made better off in this case and so a constitutional ban on subsidies will receive universal support.

5. Conclusion

Our analysis shows that the availability of subsidies for private giving can lead individuals to segregate into separate jurisdictions. Whether or not such segregation occurs depends on the size of the polity. When the polity is relatively large, the public good will be provided solely through taxes if subsidies for giving are not available and both high and low willingness-to-pay individuals reside in the same jurisdiction. In this case, high willingness-to-pay individuals find it inexpensive to live in a mixed jurisdiction relative to segregating into a separate jurisdiction. When subsidies are available, they will be used by a low willingness-to-pay majority in order to shift the tax burden off themselves and on to the high willingness-to-pay minority should both types of individuals reside in the same jurisdiction. When the polity is relatively large, the high willingness-to-pay individuals find this tax shifting expensive relative to segregating in a separate jurisdiction. The result is that subsidies for giving can result in segregation between groups.

When the polity is relatively small, the availability of subsidies for giving can be a force for integration instead. This is the case whenever the polity is so small that, even when both types reside in a single jurisdiction, all provision of the public good is through private provision by high willingness-to-pay individuals. In this case, the availability of a subsidy for private giving shifts some of the burden for provision off the high willingness-to-pay individuals and on to the low willingness-to-pay ones. In so doing the subsidy acts as a force for integration, making a mixed jurisdiction more attractive to high willingess-to-pay types. While these results were derived under the assumption of quasi-linear preferences so that mixed public and private provision never occurred, the basic intuition underlying the results applies for more general preferences. Even under mixed provision, a subsidy shifts the cost of provision off one group and onto another; this cost shifting feature of the subsidy drives the location decisions. So for instance, even under mixed provision and with a large polity, a subsidy shifts the cost of provision from the low willingness-to-pay individuals and on to the high willingness-to-pay ones. This potential cost shifting is what deters a high willingness-to-pay individual from locating in a jurisdiction with low willingness-to-pay types and so supports the segregation outcome.

Whether or not individuals wish to institute a constitutional ban on subsidies for giving depends on which sort of polity they expect to prevail. As a general principle, individuals prefer a constitutional regime that promotes mixed jurisdictions and discourages segregation. If the polity is a relatively small one, so that subsidies for giving discourage segregation, then individuals unanimously prefer subsidies. If, on the other hand, the polity is a large one and individuals anticipate that segregation will occur if subsidies for giving are permitted, then they unanimously prefer a ban on subsidies for giving. These results suggest a potential explanation for the observation that tax deductions for charitable giving are uncommon at the local level and are more common at higher jurisdictional levels.

Finally, the reader may wonder why we restrict attention to the case of $N_1 > N_2 + 1$. Analysis of the case $N_2 > N_1 + 1$ is certainly possible. Note, however, that the results for stages 2 and 3 of the game are unchanged as they do not depend on the demographic assumption. At Stage 1, the set of possible equilibrium jurisdictional configurations is also the same. The equilibrium outcomes are different in two ways. First, in no equilibrium is there any private provision or nontrivial use of subsidies? Second, although private provision and subsidies (when allowed) may be used off the equilibrium path for some (N_1, N_2) pairs, the presence or absence of subsidies has no impact on the equilibrium correspondence. Given the irrelevance of the availability of subsidies for the structure of equilibrium with this set of demographic parameters, the analysis was not included in the paper.

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Appendix A

Proof of Lemma 1. Note first that in any jurisdiction, given any (g, s, t) chosen for that jurisdiction, the equilibrium continuation strategies of a type i citizen must satisfy:

$$v_i(g,s,t) = \arg\max_v \{w - t - (1-s)v + m_i(V_{-i}(g,s,t) + v + g)\}$$

where $V_{-i}(g, s, t) = \Sigma_{l \neq i} v_l(g, s, t)$. Therefore, if $v_i(g, s, t) > 0$, it must be that $m_i'(g + V(g, s, t)) \geq 1 - s$, where $V(g, s, t) = \Sigma_l v_l(g, s, t)$.

(i) Suppose then that $N_2^j > 0$ in some jurisdiction j, and $v_1(g, s, t) > 0$. Then it must be that $m_2'(g + V(g, s, t)) > m_1'(g + V(g, s, t)) \geq 1 - s$, which is inconsistent with the type 2 individuals choosing equilibrium levels of donations, unless $(1-s)v_2(g, s, t) = w - t$, and we have assumed w is large enough that this cannot occur.

(ii) Suppose that $N_2^j = 0$ in some jurisdiction. Then it is immediate from the definition of $g_1^t()$ that no θ with $s = 0$ can be better for type 1 individuals than $(g, s, t) = (g_1^t(N^j), 0, g_1 t(N^j)/N^j)$.When $\sigma = s$, note that $g_1^t\,(N^j)$ satisfies $m_1'(g_1^t(N^j)) = 1/N^j$, which implies that the resulting allocation is Pareto efficient. If, however, an $s > 0$ is set that does induce private giving, the resulting level of z must satisfy $m_1'(z) = 1 - s$, which cannot be Pareto efficient unless $1 - s = 1/N^j$. Since in any of these equilibrium continuations, all type 1 individuals get the same payoff, it must be that for any s, the outcome is weakly Pareto dominated by the payoff attained with optimal taxes, $w - g_1^t(N^j)/N^j + m_1(g_1^t(N^j))$,and so this payoff is at least as great as any with private giving. □

Proof of Lemma 2. Suppose that $\sigma = 0$. Then it is immediate from the definition of $g_2^t(N^j)$ that a type 2 citizen can do no better than choosing $(g_2^t(N^j), 0, g_2^t(N^j)/N^j)$.

Suppose then that $\sigma=s$ and $N_1^j=0$. Then, by the same argument as in the proof of Lemma 1, a type 2 citizen cannot get a higher payoff than that which results from the same policy choice. We assume that is what is chosen.

Suppose instead that $\sigma=s$ but that $N_1^j>0$. Suppose also that the equilibrium choice $(g, s, t)\in B_s(N^j)$ is such that $\Psi_2(s, g)>0$. Then by definition $tN^j=g+s\Psi_2(s, g)$. Now, define an alternative policy choice, $s'=0$, $t'=t+(1-s)\Psi_2(s, g)/N^j$ and $g'=N^jt'$. Then if $\Psi_2(0, g')=0$, we have that:

$$\begin{aligned} z' = g' = N^j t' &= N^j t + \frac{N^j}{N_2^j}(1-s)\Psi_2(s,g) > N^j t + (1-s)\Psi_2(s,g) \\ &= N^j t - s\Psi_2(s,g) + \Psi_2(s,g) = g + \Psi_2(s,g) = z \end{aligned}$$

Since $m_2'(g+\Psi_2(s, g))=1-s$, it follows that $m_2'(g')<1-s<1$, so that $\Psi_2(0, g')=0$, in fact. Thus, by construction we have that $N^jt'=g'+s'\Psi_2(s', g')$ and so $(g', s', t')\in B(N^j)$ and we have also that:

$$x_2' = w - t' = w - t - \left(\frac{1-s}{N_2^j}\right)\Psi_2(s,g) > w - t - \frac{\Psi_2(s,g)}{N_2^j} = x_2$$

So $u_2(x, z)<u_2(x', z')$, and the original (g, s, t) could not therefore be an equilibrium continuation strategy choice for a type 2 majority in any j in which $N_1^j>0$. □

Proof of Lemma 3. The following results are for any given jurisdiction.

(a) Suppose $s=0$ is imposed. The payoff to a type 1 citizen if there is pure private provision, $\theta=(0, 0)$, is $\Upsilon_1^v=w+m_1(z_2^v)$. If there is pure public provision the maximal payoff to a type 1 citizen is $\Upsilon_1^t(N^j)=w-g_1^t(N^j)/N^j+m_1(g_1^t(N^j))$. From the definition of $g_1^t(N^j)$ we have that $\partial g_1^t(N^j)/\partial N^j=-1/m_1''(g)(N^j)^2>0$, so that $\partial\Upsilon_1^t(N^j)/\partial N^j=g/(N^j)^2>0$. Further, $m_1'(g)<m_2'(g)$ for all g implies that $\Upsilon_1^t(1)<\Upsilon_1^v$. As long as there is a value of N^j sufficiently large such that $g_1^t(N^j)>z_2^v$, then $\Upsilon_1^t(N^j)>\Upsilon_1^v$ for some N^j. Define N^0 as the value of N^j at which the inequality switches. The result then follows as long as there are no mixed provision outcomes. Since under mixed provision, $m_2'(g+\Psi_2(0, g))=1$ for all g, the type 1 citizen can reduce g with no impact on z. As a reduction in g lowers the type 1 citizen's taxes, type 1 can increase utility by setting $g=0$, implying that mixed provision does not occur.

(b) Suppose $s>0$ is allowed. We first show that it is always preferable for the type 1 majority to choose s and g such that $\Psi_2(s, g)>0$. Suppose by way of contradiction, that the s and g chosen are such that $\Psi_2(s, g)=0$. Then it must be that $g>\rho_2(s)$, from the definition of $\Psi_2(s, g)$, and $t=g/N^j$. However, letting $s'=1-m_2'(g)$, it follows that if $\theta'=(s'g/N^j, s', 0)$, then $z=g=\Psi_2(s', 0)$ and $t'<t$. Thus, θ' results in greater consumption for the type 1 citizen and the same z as in θ, so this is superior.

We now show that choosing $g=0$ is always optimal. If θ with $g>0$ and $\Psi_2(s, g)>0$ were chosen, then the level of g can be reduced to 0, with the result that $\Psi_2(s, g)$ increases by the amount g, (by the definition of $\Psi_2(s, g)$ in the main text). Then in turn total taxes can be reduced by the amount $(1-s)g$, which maintains a balanced government budget, and thus a type 1 individual gets greater c_1 and the same z. It follows from this that $t=s\Psi_2(s, 0)/N^j$ always, and the only remaining issue is the subsidy. $s^*(N^j)$ is the utility-maximizing subsidy of type 1, given that $g=0$. The payoff to a type 1 individual from a subsidy under these conditions is

$$\Omega_1(s, N^j) = w - \frac{s\Psi_2(s)}{N^j} + m_1(\Psi_2(s)),$$

where we have dropped the g from the argument of Ψ_2, since it will always be 0. If an $s>0$ is chosen, then it will satisfy the first-order condition

$$\Psi_2'(s)\left[m_1'(\Psi_2(s))-\frac{s}{N^j}\right]-\frac{\Psi_2(s)}{N^j}=0.$$

Note that the only parameter that the maximal s will depend on is N^j, and that it must also satisfy the second-order condition

$$\Psi_2''(s)\left[m_1'(\Psi_2(s))-\frac{1+s}{N^j}\right]+m_1''(\Psi_2(s))[\Psi_2'(s)]^2-\frac{\Psi_2'(s)}{N^j}<0.$$

If $s^*(N^j)>0$, then it can be shown that $\partial s^*/\partial N^j=A/B$ where $A=-\ [s\Psi_2'(s)+\Psi_2(s)]/(N^j)^2<0$, and the second-order condition above implies that $B<0$. Thus, s^* is increasing in N^j.

Note further that when $s=0$, the payoff is simply $\Omega_1(0, N^j)=\Upsilon_1^v$, which is in fact independent of N^j. If an $s>0$ is chosen for any N^j, then the payoff to a type 1 citizen is $w-s^*(N^j)\Psi_2(s^*(N^j))/N^j+m_1(\Psi_2(s^*(N^j)))$. Since $\Omega_1(s, N^j)$ is increasing in N^j for any given s, it follows that this last expression is increasing in N^j, since s is chosen optimally for each value of N^j. Thus, we let N^s be the value of N^j at which this payoff exceeds Υ_1^v. There must be such a value, since if s is fixed at any value $s'\in(0, 1)$, then $\Psi_2(s)>z_2^v$, and so for some sufficiently large N', $\Omega_1(s', N')>\Upsilon_1^v$. □

The following result will be useful in proving Lemma 4.

Lemma A. *If $N'>N''$ and $g_1^t(N')>z_2^v$, then $w-z_2^v/N''+m_2(z_2^v)<w-g_1^t(N')/N'+m_2(g_1^t(N'))$*

Proof. Since $m_2()$ is concave, we have that:

$$\begin{aligned}m_2(g_1^t(N'))-m_2(z_2^v)&>m_2'(g_1^t(N'))(g_1^t(N')-z_2^v)\\&>\frac{1}{N'}(g_1^t(N')-z_2^v)\ \text{(since } m_2'>m_1'\text{)}\\&>\frac{g_1^t(N')}{N'}-\frac{z_2^v}{N''}\ \text{(since } N'>N''\text{)}\end{aligned}$$

so $m_2\left(g_1^t(N')\right)-\dfrac{g_1^t(N')}{N'}>m_2\left(z_2^v\right)-\dfrac{z_2^v}{N''}$ which proves Lemma A. □

Proof of Lemma 4. This will be proved by proving a series of results.

Lemma 4A. *There can be at most two jurisdictions in any equilibrium.*

The proof consists of two parts. First, we show that any increase in the number of agents in a jurisdiction that leaves the majority type unchanged does not decrease the utility of that type. Next we show that this result implies that each jurisdiction must have a different type of citizen in the majority, which means there can be at most two jurisdictions in equilibrium. (i) *Majority type's utility non-decreasing in number of citizens in jurisdiction:* Consider a given jurisdiction with N^j agents and with type i agents being the majority. Let the continuation equilibrium tax/public goods package in the jurisdiction be $\theta^*=(t^*, s^*, g^*)$. Now consider the addition of another k agents to the jurisdiction, so that $N'^j=N^j+k$, $k\geq1$, such that type i continues to be the majority.

We consider two cases:

Case 1. $t^*>0$ A feasible tax/public goods package for this larger jurisdiction is $\theta'=(t', s', g')$ where $s'=s^*$, $g'=g^*$ and $t'=t^*N^j/(N^j+k)$. This package results in the same equilibrium levels of private provision at the last stage of the game, since $\Psi_2(s, g)$ depends only on s and g, and thus the level of z is the same, and the government budget balances. However, consumption is higher if $t^*>0$, since t' is lower. As a result, all the original citizens' utilities are strictly higher under θ', and so the majority type's utility must be higher with the addition of the k new citizens.

Case 2. $t^*=0$ Necessarily then $s^*=g^*=0$ also, and it must therefore be that either type 1 is the majority type, or $N^j=1$. If type 2 individuals were in the majority, then Lemma 2 implies that $v_2^*=v_1^*=0$, and $\theta=(g_2^t(N^j)/N^j,\ 0,\ g_2^t(N^j))$ produces a higher payoff for a type 2 citizen, when $N^j>1$. If $N^j=1$, it is immediate that the single type i's utility strictly increases if more citizens of any type are added and i remains the majority, since in the enlarged jurisdiction a $t>0$ can be imposed, with $s=0$, that makes the original solitary citizen strictly better off. If $t^*=0$ and $N^j>1$, it must be that type 1 is the majority type, and further, it must be the case that $N_2^j>0$, since if $N_2^j=0$ a tax $t_1(N^j)=g_1^t(N^j)/N^j>0$, would produce a higher payoff for type 1 citizens than does θ^*. Thus, if $t^*=0$ in any jurisdiction with a type 1 majority, it must be that the payoff to the majority is $w+m_1(z_2^v)$, and that $N_2^j>0$. Since the payoff to a type 1 majority is otherwise increasing in the size of the jurisdiction, it follows that once the payoff is greater than this, it is strictly increasing in N^j from then on.

(ii) *Must be a different majority type in each jurisdiction*: Suppose, by way of contradiction, that there are two jurisdictions, j and l, with type i the majority type in each. Assume, without loss of generality, that $N^j \geq N^l \geq 1$.

If $i=2$, then part (i) implies that type 2 citizens' utility is increasing in each jurisdiction if more citizens are added. Hence, a type 2 citizen who moves from l to j will strictly increase their utility, and so two such jurisdictions cannot be part of an equilibrium configuration.

If $i=1$, on the other hand, it follows from (i) that a type 1 citizen's utility cannot decrease as more citizens are added to a jurisdiction, so a type 1 citizen who moves from l to j cannot be worse off. If in fact the move does not increase his utility, then it must be that N_2^l, $N_2^j>0$ and that $t=0$ in l, and in j after he moves there. If that is the case, then all provision in both jurisdictions is done privately by type 2 individuals, with $s=0$. Assuming without loss of generality that $N_2^j \geq N_2^l$, a type 2 citizen who moves from l to j will get the same level of public good, z_2^v, but will have greater consumption, and so be strictly better off. Hence, two such jurisdictions still cannot be part of an equilibrium configuration, which proves part (ii).

This proves Lemma 4A. □

Lemma 4B. *If $s=0$ is required, and there are two jurisdictions, then each consists of all citizens of one type.*

Proof. We already know that each jurisdiction must have a different type in the majority, by Lemma 4A, so suppose that, without loss of generality, $j=1$ has type 1 in the majority, and $j=2$ has type 2. Now, suppose, by way of contradiction, that $j=1$ includes type 2 citizens. We know from Lemma 2 that the payoff to type 2 citizens in $j=2$ is increasing in N^2, so it must be that $U_2^1>U_2^2$, otherwise it would pay for a type 2 citizen to deviate from $j=1$ to $j=2$. However, the payoff to a type 2 citizen in $j=1$ is

$$U_2^1(N^1,N_2^1) = \begin{cases} w - z_2^v/N_2^1 + m_2(z_2^v) & \text{if} \quad N^1 < N^0, \\ w - g_1^t(N^1)/N^1 + m_2(g_1^t(N^1)), & \text{if} \quad N^1 \geq N^0. \end{cases}$$

Thus, U_2^1 is increasing in N^1 if $N^1 \geq N^0$, meaning a type 2 individual from $j=2$ would find it profitable to deviate to $j=1$. Thus it must be that $N^1 < N^0$. If $N^1+1 < N^0$, then again a type 2 citizen from $j=2$ would find it profitable to move to $j=1$. So, it must be that $N^1 < N^0 \leq N^1+1$.

These inequalities and the fact that $U_2^1 > U_2^2$, imply that in order for it to not be profitable for a type 2 in $j=1$ to deviate to $j=2$, it must be that

$$w - \frac{z_2^v}{N_2^1} m_2\left(z_2^v\right) > w - \frac{g_2^t(N^2)}{N^2} + m_2\left(g_2^t\left(N^2\right)\right) \geq w - \frac{g_1^t(N^1+1)}{N^1+1} + m_2\left(g_1^t\left(N^1+1\right)\right).$$

We also know that at N^0, $m_1(z_2^v) = m_1(g_1^t(N^0)) - g_1^t(N^0)/N^0$, and therefore $g_1^t(N^0) > z_2^v$, meaning $g_1^t(N^1+1) > z_2^v$, since g_1^t is increasing in N^1. This in turn means that in order for the inequality above to hold, it must be that $m_2(g_1^t(N^1+1)) - g_1^t(N^1+1)/(N^1+1) < m_2(z_2^v) - z_2^v/N^1$. However, Lemma A above implies that this cannot hold. Thus, it must be that $N^1=0$.

Suppose now that $j=2$ includes type 1 citizens. Part (i) of the proof of Lemma 4A (contained in this Appendix) implies that U_1^2 is increasing in N^2 so it must be that $U_1^2 < U_1^1$. Since the argument above implies that $N_2^1=0$, it follows that $U_1^1 = \Upsilon_1^t(N^1)$, which is strictly increasing in N^1, meaning a type 1 citizen in $j=2$ will be better off if he deviates to $j=1$.

This proves Lemma 4B. □

Lemma 4C. *If there are two jurisdictions and $s>0$ is allowed, each one contains citizens of only one type.*

Proof. Suppose again that there are two jurisdictions in equilibrium and that jurisdiction j has type j in the majority. From Lemmas 2 and 3 we know that the payoffs in each jurisdiction are as follows. In jurisdiction 1:

$$U_1^1\left(N^1\right) = \arg\max_s \left\{ w - \frac{s\Psi_2(s)}{N^1} + m_1(\Psi_2(s)) \right\};$$

$$U_2^1\left(s,N^1,N_2^1\right) = w - \frac{s\Psi_2(s)}{N^1} - \frac{(1-s)\Psi_2(s)}{N_2^1} + m_2(\Psi_2(s));$$

where $s=s^*(N)$, so that $s=0$ if $N^1 < N^s$.

In jurisdiction 2 we have

$$U_1^2\left(N^2\right) = w - \frac{g_2^t(N^2)}{N^2} + m_1\left(g_2^t\left(N^2\right)\right);$$

$$U_2^2\left(N^2\right) = w - \frac{g_2^t(N^2)}{N^2} + m_2\left(g_2^t\left(N^2\right)\right).$$

It is immediate that $U_1^1(N^1)$ is increasing in N^1 and that $U_2^2(N^2)$ is increasing in N^2. Further, we have that

$$\frac{\partial U_1^2(N^2)}{\partial (N^2)} = \frac{g_2^t(N^2)}{(N^2)^2}\left[1 - \frac{N^2 g_2^{t'}(N^2)}{g_2^t(N^2)}\right] + g_2^{t'}(N^2)m_1'(g_2^t(N^2)).$$

We know that the second term on the right hand side of the above is positive, and our elasticity assumption implies the same for the first term, hence $U_1^2(N^2)$ is increasing in N^2, also, and this immediately implies that there cannot be any type 1 citizens in $j=2$.

Now, note that U_2^1 is increasing in N_2^1 when $N<N^s$, since then

$$U_2^1(N_2^1) = w - \frac{\Psi_2(0)}{N_2^1} + m_2(\Psi_2(0)).$$

On the other hand, when $N \geq N^s$ so that $s^*(N)>0$, it must be that $s=s^*(N)$ satisfies the first-order condition

$$m_1'(\Psi_2(s))\Psi_2'(s) - \frac{1}{N^1}(s\Psi_2'(s) + \Psi_2(s)) = 0.$$

Further, we have that

$$\frac{\partial U_2^1(s,N,N_2^1)}{\partial s}\Big|_{s=s^*} = [m_2'(\Psi_2(s)) - m_1'(\Psi_2(s))]\Psi_2'(s) + \frac{\Psi_2(s)}{N_2^1}\left[1 - \frac{(1-s)\Psi_2'(s)}{\Psi_2(s)}\right].$$

And this is positive, because of our subsidy-price elasticity assumption and the fact that $m_2'(\Psi_2(s)) - m_1'(\Psi_2(s))>0$ for any s.

Then, noting that

$$\frac{\mathrm{d}U_2^1(s,N^2,N_2^1)}{\mathrm{d}N_2^1} = \left[\frac{\partial U_2^1(s,N^1,N_2^1)}{\partial s}\Big|_{s=s^*}\right]\frac{\partial s^*(N^1)}{\partial N^1}\frac{\partial N^1}{\partial N_2^1} + \frac{\partial U_2^1(s,N^1,N_2^1)}{\partial N^1}\frac{\partial N^1}{\partial N_2^1} + \frac{\partial U_2^1(s,N^2,N_2^1)}{\partial N_2^1},$$

it is obvious that $\partial N^1/\partial N_2^1 = 1$, and that both of the last two terms are positive. Also, we have that

$$\frac{\partial s^*}{\partial N^1} = -\frac{1}{A}\left[\frac{s\Psi_2'(s) + \Psi_2(s)}{(N^1)^2}\right] > 0$$

because the second-order condition associated with the maximizing choice of s^* by type 1 citizens implies that

$$A = \Psi_2'(s)\left[m_1'(\Psi_2(s)) - \frac{s}{N^1}\right] + \Psi_2''(s)\left[m_1''(\Psi_2(s)) - \frac{2}{N^1}\right] < 0.$$

Finally, note that the payoff to a type 2 citizen in $j=1$ changes continuously at $N=N^s$, as s simply increases (continuously, as the outcome of a maximization problem) from zero. Thus, $U_2^1(s, N^2, N_2^1)$ is increasing in N_2^1, and so there cannot be any type 2 citizens in $j=1$.

This proves Lemma 4C. □

Proof of Proposition 5. First, we will show that $N^0 > N^s$. Recall that, by the definition of N^s and N^0, $\Omega_1(s^*(N^s), N^s) = w + m_1(z_2^v) = \Upsilon_1^t(N^0)$. Further, a subsidy can always do better for the type 1 majority than any tax, t. If the tax raises g' in revenue, then $m_1'(g') = 1/N < m_2'(g')$. So, it is always possible to set s so that $\Psi_2(s') = g'$, and set taxes at $s'g'/N$, so the same level of z is provided, but consumption for a type 1 citizen is $w - s's'/N > w - s'/N$. Thus, $\Upsilon_1^t(N) < \Omega_1(s, N)$ for any N. Since we also know that both $\Omega_1(s, N)$ and $\Upsilon_1^t(\cdot)$ are increasing functions, it follows that $N^0 > N^s$.

We now prove that for any N_1 we have $n_2(N_1) > n_2^s(N_1)$. Fix N_1 and recall that if $n_2(N_1) = n'$ and $n_2^s(N_1) = n''$, then we have that $\Upsilon_2^t(N_1+1) = \Gamma_2^t(n')$, and $\Gamma_2^t(n'') = \Omega_2(s^*(N_1+1), N_1+1, 1)$. So we will show that in fact that $n' > n''$. This follows from the three Claims proved below.

Claim 1. *$\Psi_2(s^*(N)) < g_1^t(N)$ for any N.*

Proof. If $s^*(N) > 0$, then it must be that $g^s = \Psi_2(s^*(N))$ satisfies:

$$\left[m_1'(g^s) - \frac{s^*(N)}{N}\right]\Psi_2'(s^*(N)) = \frac{\Psi_2(s^*(N))}{N},$$

which implies that

$$m_1'(g^s) = \frac{1}{N}\left[s^*(N) + \frac{\Psi_2(s^*(N))}{\Psi_2'(s^*(N))}\right].$$

Our elasticity assumption is that

$$-(1-s)\frac{\Psi_2'(s)}{\Psi_2(s)} > -1,$$

which implies that

$$\frac{\Psi_2(s)}{\Psi_2'(s)} > 1 - s;$$

so

$$m_1'(g^s) = \frac{1}{N}\left[s^*(N) + \frac{\Psi_2(s^*(N))}{\Psi_2{}'(s^*(N))}\right] > \frac{1}{N}(s^*(N) + 1 - s^*(N)) = m_1'\big(g_1^t(N)\big);$$

and so $m_1'' < 0$ implies $g^s < g_1^t(N)$. This proves Claim 1. □

Claim 2. *$\Upsilon_2^t(N) > \Omega_2^s(s', N, N_2)$, if $\Psi_2(s') = g_1^t(N)$.*

Proof.

$$\Upsilon_2^t(N) - \Omega_2(s', N, N_2) = g_1^t(N)\left[\frac{s'-1}{N} - \frac{1-s'}{N^2}\right] > 0,$$

since $N_2 < N$ and $s' < 1$. This proves Claim 2. □

Claim 3. *For any N_1, $\Upsilon_2^t(N_1+1)>\Omega_2(s^*(N_1+1), N_1+1, 1)$.*

Proof. Let s' be such that $\Psi_2(s')=g_1^t(N_1+1)>\Psi_2(s^*(N_1+1))$, with the last inequality following from Claim 1. Then $\Psi_2'>0$ implies $s'>s^*(N_1+1)$. Then we have that $\Upsilon_2^t(N_1+1)>\Omega_2(s', N_1+1, 1)$ by Claim 2. And $\Omega_2(s', N_1+1, 1)>\Omega_2(s^*(N_1+1), N_1+1, 1)$ because $\partial\Omega_2/\partial s>0$. This proves Claim 3, and Proposition 5 then follows. □

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